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Manipulation of Geographic Information in Global Seismology

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ABSTRACT

Geographic data, such as seismic event locations, station locations, etc., are generally given in geographic latitude ϕ', longitude θ, and depth below sea level, z, using the WGS84 ellipsoid as a reference. In software systems that use this type of geographic data, it is necessary to manipulate the data mathematically in order to perform such tasks as finding the angular distance or azimuth from one point to another, to find an array of points along a great circle, to rotate a point about a pole of rotation, to move a point some angular distance in a specified direction, to find the intersections of two great circles or to find the intersections of a great circle and a small circle. In this paper, equations are presented that convert geographic locations first to geocentric coordinates and then to Earth-centered Cartesian coordinates where many mathematical manipulations can be performed conveniently and efficiently.

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ACRONYMS AND TERMS

Acronym/Term	Definition
WGS84	World Geodetic System 1984

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1. INTRODUCTION

Much of the geographic information used by seismologists is presented in the form of geographic latitude, longitude and depth or elevation relative to sea level. This positional information is almost always described relative to the 1984 World Geodetic System ellipsoid. From this information seismologists need to calculate the angular distance and azimuth from one point to another and other geometrical relationships. There are several ways to accomplish these calculations. Vincenty (1975) described a method for computing the distance in km between two points on the surface of an ellipsoid by evaluating elliptical integrals. Another approach is to perform spherical trigonometry with spherical triangles connecting points on the surface of the ellipsoid. In this paper we take the approach of converting geographical locations to an Earth-centered coordinate system and performing the calculations using vector analysis.

2. WGS84 ELLIPSOID

The shape of Earth is reasonably well approximated by an oblate spheroid which is an ellipsoid generated by aligning the minor axis of an ellipse with the rotational axis of the Earth and rotating the ellipse about its minor axis. In general, ellipsoids require one to specify the lengths of its three principal axes, however, oblate spheroids only require specification of the polar and equatorial axes because the equator is assumed to be circular.

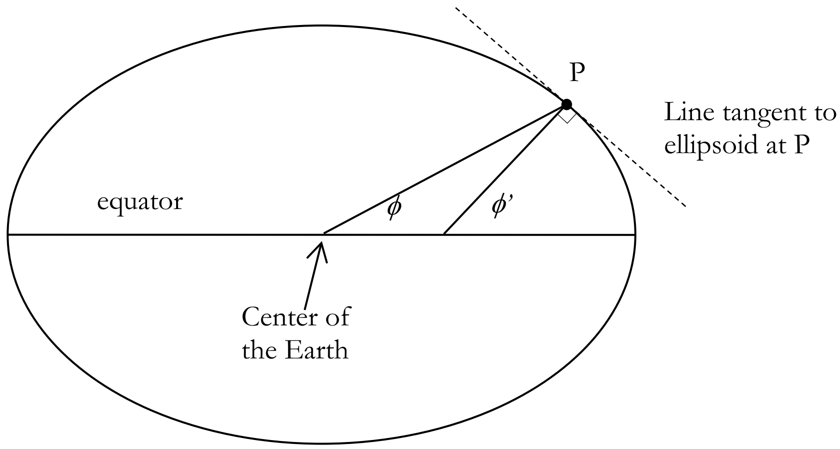
Over several centuries, geodesists have defined many ellipsoids for the Earth (Snyder, 1987) but the WGS84 ellipsoid is arguably the one most frequently encountered in seismology. The parameters that define the WGS84 ellipsoid are

$$
\begin{aligned}
a &= 6378.137 \text{ km} \\
b &= 6356.75231424518 \text{ km} \\
f &= \frac{a-b}{a} = 1/298.257223563 \\
e^2 &= \frac{a^2-b^2}{a^2} = 2f - f^2 = 0.0066943799901413165
\end{aligned}
\qquad (1)
$$

where a and b are the equatorial and polar radii of the Earth, respectively, f is the flattening parameter, and e is the eccentricity. Any two of these parameters are sufficient to uniquely define the ellipsoid. In the equations presented in this paper a and e^2 are used.

When we use an ellipsoid as our figure of the Earth, we must distinguish between geographic latitude and geocentric latitude (Figure 1). Geographic latitude ϕ' is the acute angle between the equatorial plane and a line perpendicular to the plane that is tangent to the reference ellipsoid at the point of interest. Geodesic latitude is a synonym for geographic latitude. Geocentric latitude ϕ is the acute angle between the equatorial plane and a line from the center of the Earth to the point of interest. Geographic, geodesic and geocentric longitudes are all equivalent.

Figure 1. An exaggerated ellipsoid illustrating the difference between geographic latitude ϕ' and geocentric latitude ϕ.

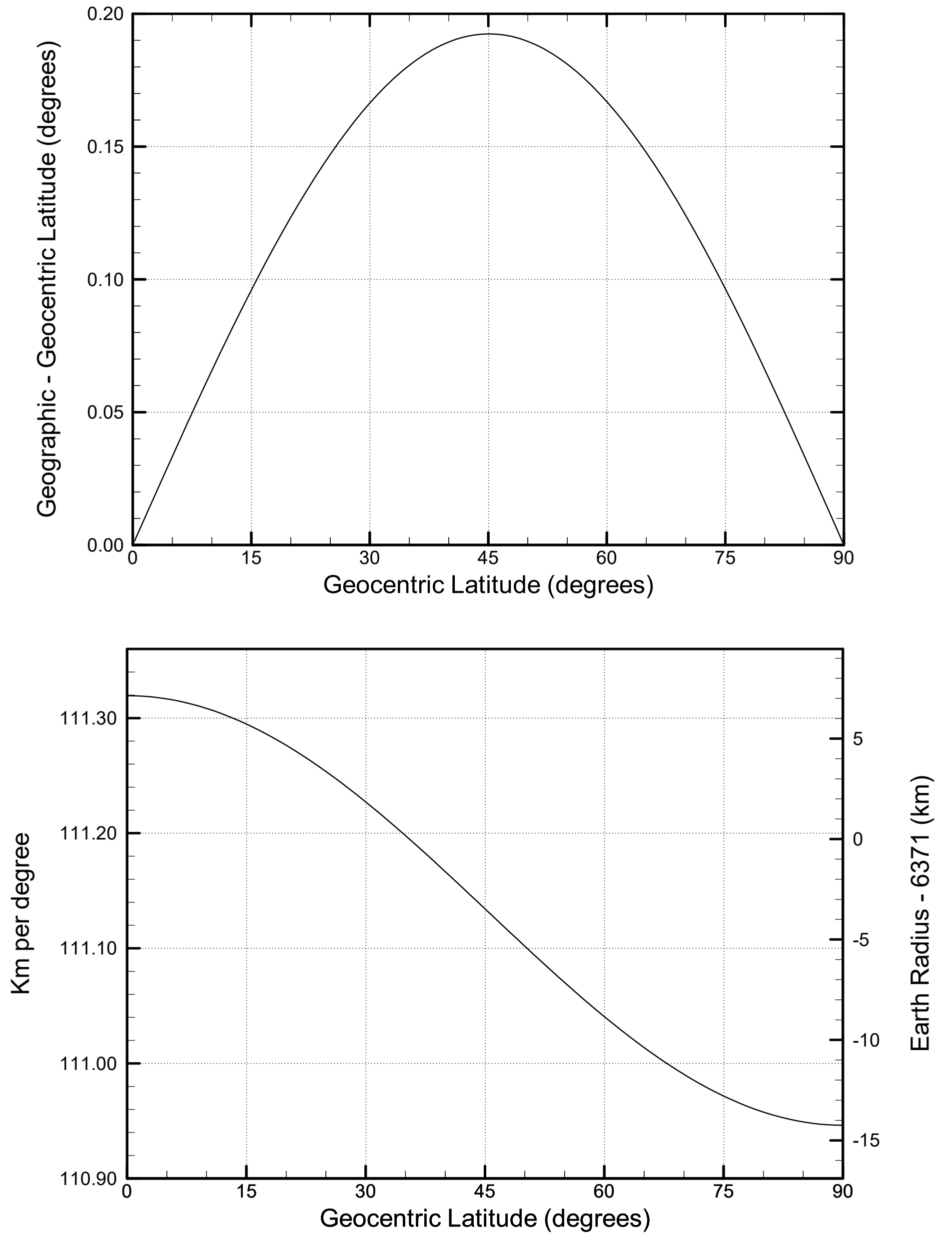


For points on the surface of the ellipsoid, we convert back and forth between geographic latitude and geocentric latitude using

$$\phi = \arctan\left((1 - e^2)\tan\phi'\right) \qquad (2)$$
$$\phi' = \arctan\left(\tan\phi/(1 - e^2)\right)$$

(Snyder, 1987). Geographic and geocentric latitudes at the surface of the ellipsoid are compared in Figure 2.

Figure 2. a) Comparison of geographic and geocentric latitudes for the WGS84 ellipsoid. b) Km per degree and Earth radius as function of geocentric latitude.



We also need to convert between depth, *z*, and radius, *r*, as a function of geocentric latitude.

$$r = R(\phi) - z \tag{3}$$

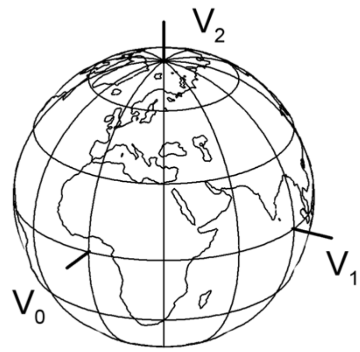
where *R(ϕ)*, the radius of the Earth at geocentric latitude ϕ is given by

$$R(\phi) = a\left(1 + \frac{e^2}{1 - e^2} sin^2\phi\right)^{-1/2} \tag{4}$$

3. EARTH-CENTERED CARTESIAN COORDINATES

Mathematical manipulation of geographic information is most easily accomplished if we first transform the information into an Earth-centered Cartesian coordinate system where points are defined by a unit vector, **v**, with its origin at the center of the Earth, and a radial distance from the center of the Earth, *r*, measured in km (Figure 3).

Figure 3. Earth centered coordinate system. The vector v_0 points from the center of the Earth towards the point on the surface with latitude and longitude 0°, 0°; v_1 points toward latitude, longitude 0°, 90° and v_2 points toward the north pole.



Conversion back and forth between geocentric and Cartesian coordinates is accomplished using

$$v_0 = \cos\phi\cos\theta \qquad (5)$$
$$v_1 = \cos\phi\sin\theta$$
$$v_2 = \sin\phi$$
$$\phi = \arcsin v_2$$
$$\theta = \arctan\left({}^{v_1}/_{v_0}\right)$$

(Zwillinger, 2003). These equations are valid for all points in space, not just those on the surface of the ellipsoid.

Once geographic information has been converted to unit vectors, a variety of useful calculations can be performed.

3.1. Epicentral distance between two points

Given two points defined by unit vectors, **u** and **v**, the angular separation of the two points is

$$\Delta = \arccos(\mathbf{u}\cdot\mathbf{v}) \qquad (6)$$

3.2. Azimuth from one point to another

The azimuth α from **u** to **v**, measured clockwise from north, is

$$\alpha = \arccos(|\mathbf{u}\times\mathbf{v}|\cdot|\mathbf{u}\times\mathbf{n}|) \qquad (7)$$

$$if\ (|\mathbf{u} \times \mathbf{v}| \cdot \mathbf{n} < \mathbf{0}) \qquad \alpha = 2\pi - \alpha$$

where **n** is the vector pointing to the north pole, **n** = [0, 0, 1]. Note that azimuth is undefined if **u** and **v** are parallel, anti-parallel, or if **u** corresponds to the north or south pole.

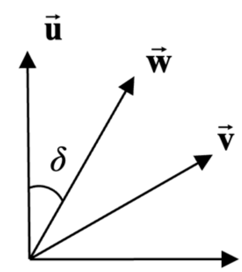
3.3. Points on a great circle

Given two points defined by unit vectors **u** and **v**, to find another point, **w,** that lies on the great circle defined by **u** and **v,** at some angular distance δ, measured from **u** in the direction of **v** (Figure 4)

$$\begin{aligned} \mathbf{t} &= |\mathbf{u} \times \mathbf{v} \times \mathbf{u}| \\ \mathbf{w} &= \mathbf{u}\cos\delta + \mathbf{t}\sin\delta \end{aligned} \tag{8}$$

Note that if many points are to be found along the same great circle defined by **u** and **v**, the normalized vector triple product, **t**, only needs to be computed once.

Figure 4. Calculation of w given u and v. All vectors are unit length and reside entirely in the plane of the figure.



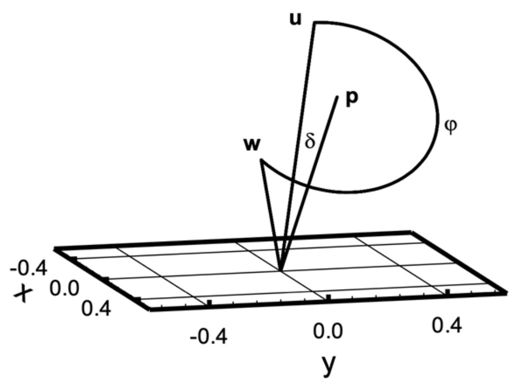
3.4. Rotate one vector around another

Vector **u** can be rotated angle φ about another vector **p** as follows (see Figure 5).

$$\mathbf{w} = \mathbf{u}\cos\varphi + \mathbf{p}(\mathbf{p} \cdot \mathbf{u})(1 - \cos\varphi) + (\mathbf{p} \times \mathbf{u})\sin\varphi \tag{9}$$

Note that φ is positive clockwise when viewed in the direction of vector **p** (right hand rule).

Figure 5. Start at point p, located at latitude, longitude 45°, 0°. Find a new point u δ = 20° north of p. Then rotate u φ = -235° around p to position w. Note that ∠pu = ∠pw = δ = 20°.



3.5. Finding a new point some distance and azimuth from another point

To find a point, **w**, that is some specified distance δ from **p** in direction φ, (Figure 5) we first find an intermediate point **u**, distance δ north of **p** by applying Equation 8 with **v** = **n** = [0, 0, 1]. Then **w** is found by rotating **u** around **p** by angle -φ.

3.6. Points of intersection of two great circles

Great circles g_1 and g_2 have, in general, two points of intersection located at $|\mathbf{n_1} \times \mathbf{n_2}|$ and $-|\mathbf{n_1} \times \mathbf{n_2}|$ where $\mathbf{n}_1$ and $\mathbf{n}_2$ are the unit normals to the two great circles.

3.7. Points of intersection of a great and a small circle

Given a great circle g with normal **n** and a small circle with center at **c** with radius r, we wish to find any points of intersection which might exist (Figure 6). We begin by finding vector **b**, which is normal to the great circle containing vectors **n** and c

$$\mathbf{b} = |\mathbf{n} \times \mathbf{c}| \tag{10}$$

Then we find vector **a** which is the point on g which is closest to **c**

$$\mathbf{a} = |\mathbf{b} \times \mathbf{n}| \tag{11}$$

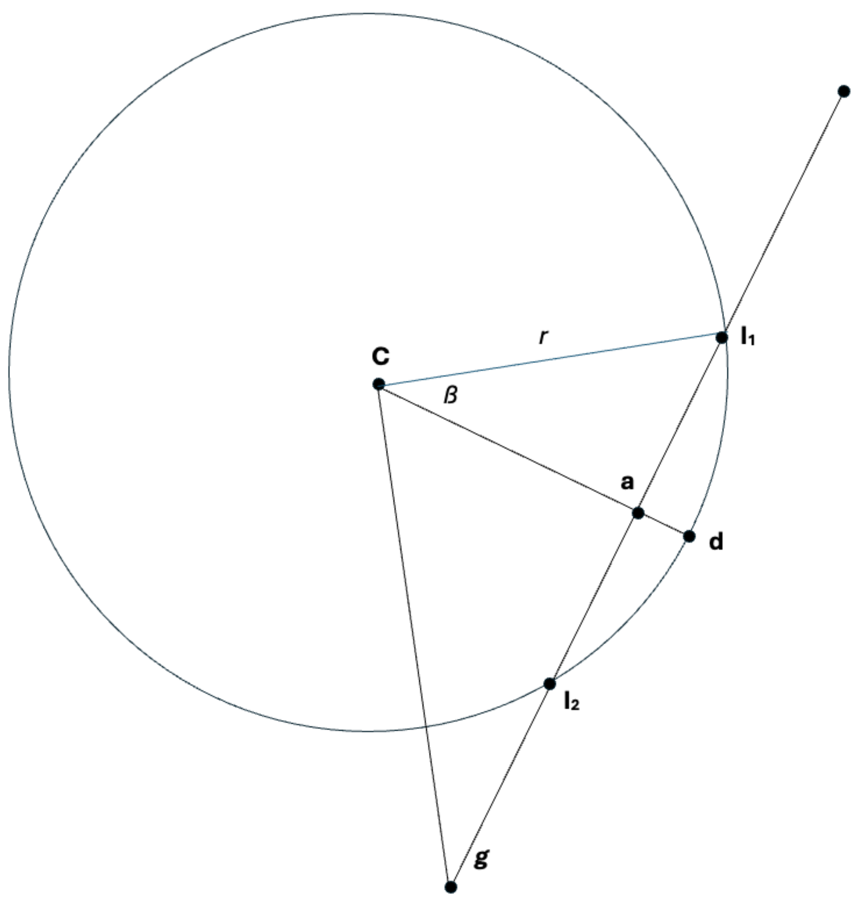
Angle α, given by $cos^{-1}(\mathbf{c} \cdot \mathbf{a})$, is the angle between vectors **c** and **a**. If α is greater than the radius of the small circle r, then no points of intersection exist. If $\alpha <= r$ then find vector **d** by rotating vector **c** around **b** by angle r as described in equation 9. **d** is the point of intersection of the small circle with a great circle through **c** and **a**.

Next, use spherical trigonometry to find angle β

$$\beta = cos^{-1}\left[\frac{\tan(\alpha)}{\tan(r)}\right] \tag{12}$$

which is the angle between the circular arc from **c** to **a** and the circular arc from **c** to one of our desired intersection points, $\mathbf{i}_1$. The two intersection points can then be found by rotating vector **d** around **c** by angles $\pm\beta$ using equation 9.

Figure 6. A small circle with radius *r* on a sphere. The great circle *g* intersects the small circle at two points, i_1 and i_2. a is the point on *g* that is closest to the center of the small circle c. d is located at the intersection of the small circle and the great circle that includes points c and a. *ß* is the angle between the circular arc from c to a and a circular arc from c to i_1. Other features discussed in the text cannot be displayed.



4. SUMMARY

We have described a mathematical framework for computing geometrical relationships between points in or near the Earth. Geographical information such as the positions of seismic stations, seismic events, etc., are generally reported in geographical coordinates relative to the WGS84 ellipsoid. The approach we use involves first converting the information to geocentric coordinates and then to Earth-centered Cartesian coordinates where geographical information can be efficiently manipulated using vector analysis.

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