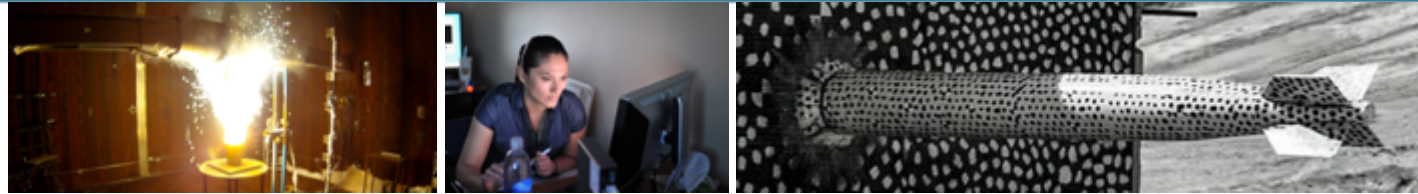




# Meshfree Modeling of Thermo-Mechanical Behavior and Damage Evolution in Binderized Energetic Materials



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GRC Energetic Materials

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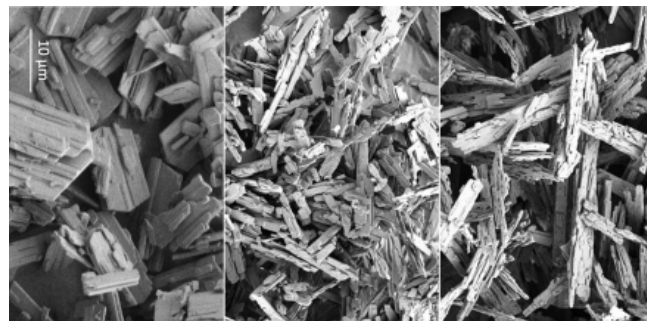
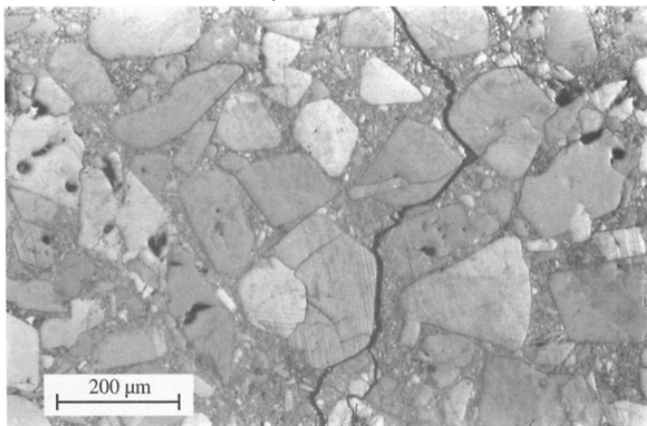
- Prof. Marcial Gonzalez
- Caleb Overstreet
- Tim Vander Woude
- Chelsea O'Donnell
- Sophia Knapp
- Tyler Falls
- Abhay Kumar

# Energetic Materials: A Modeling Challenge



- ❖ Complex material structure
- ❖ Chemically reactive (fast, exothermic)
- ❖ Inherently multi-physics

Plastic Bonded Explosive [Rae, 2002]



Energetic Crystals [Yarrington, 2018]

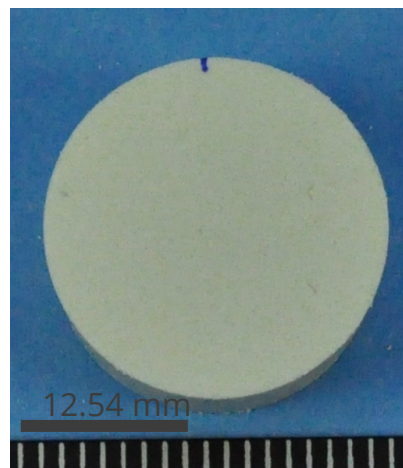
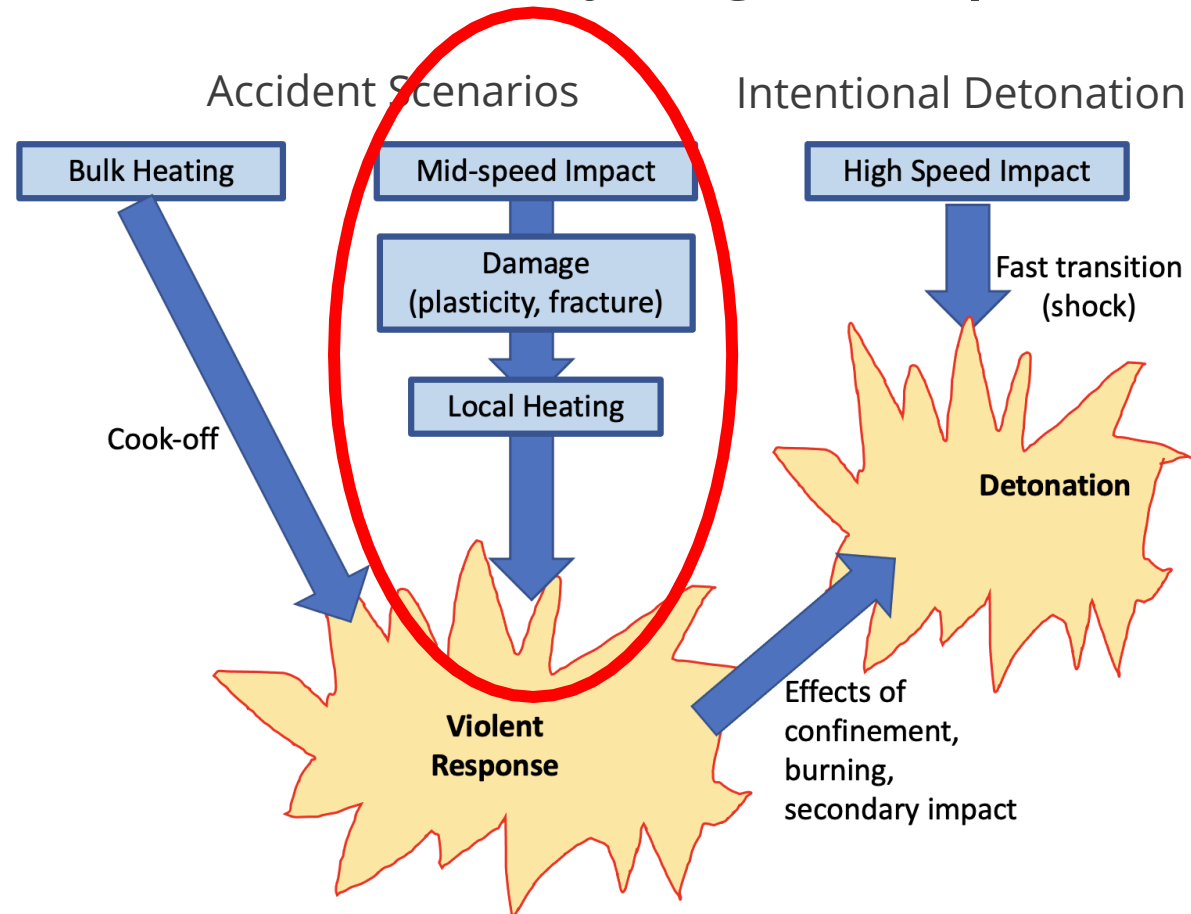


Image: courtesy Marcia Cooper

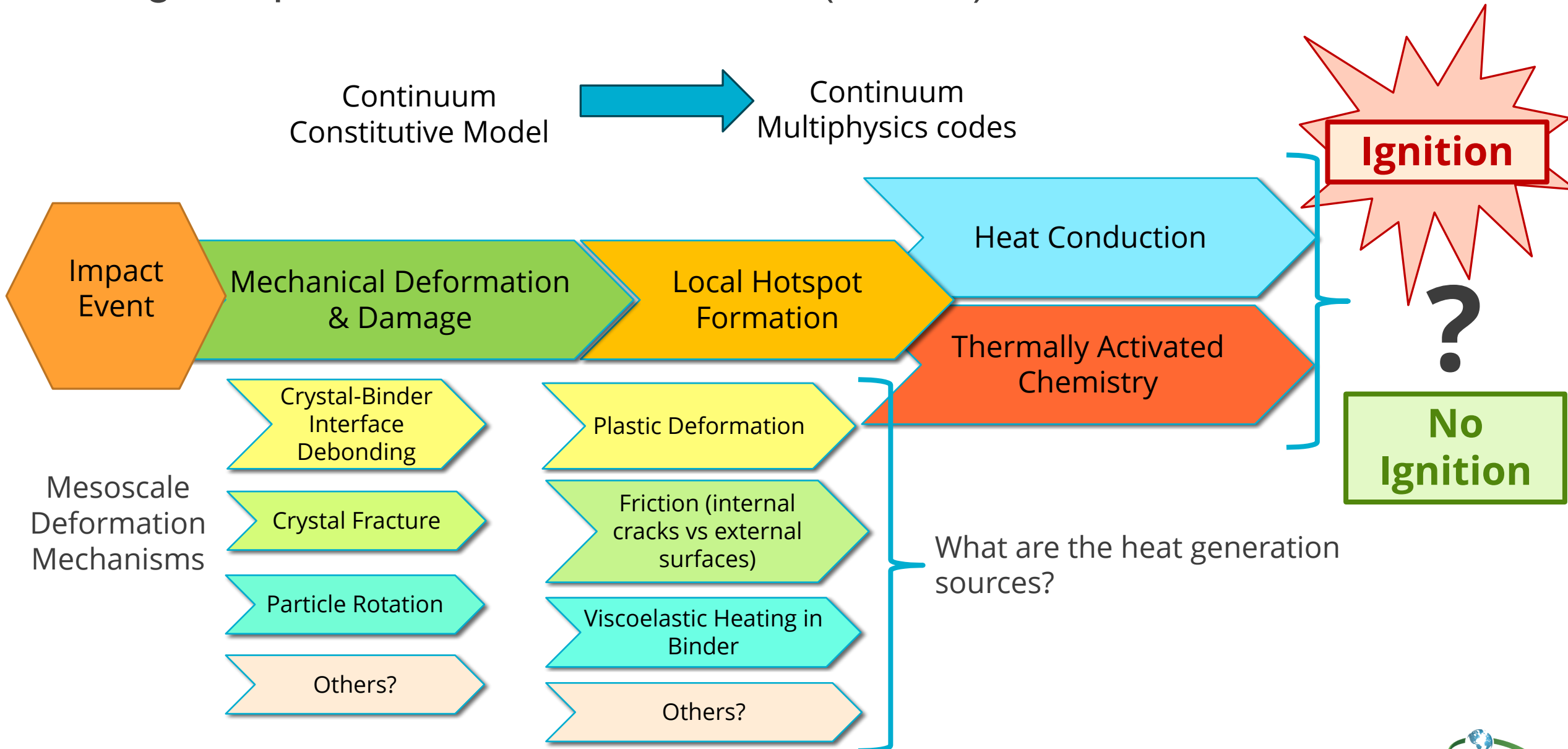
## A few different ways to get an explosion...



# High Explosive Violent Reaction (HEVR)



Continuum Constitutive Model  $\rightarrow$  Continuum Multiphysics codes



# Many Outstanding Modeling Challenges



1. Large Deformation (Global and/or localized) and Pervasive Damage/Fracture
  - Numerical method challenges
2. Complex mechanical behavior, requires unique constitutive models
  - Mesoscale mechanisms vs. macroscale effects
  - What are the heat generation mechanisms? What drives hot spots & ignition?
3. Multi-physics nature of HEVR event
  - Governing physics change throughout the event
  - Initially mechanics dominated, localized heat generation and transfer, transition to material flow and reactive burn

*We are exploring meshfree numerical methods as a computational tool to study mechanically induced HEVR*

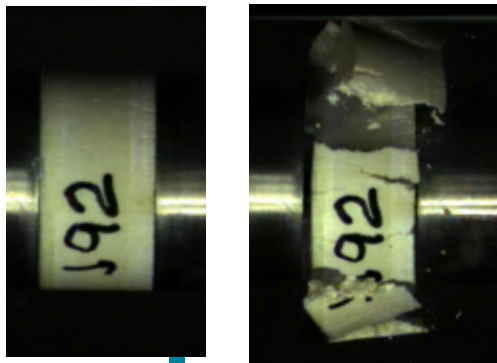
# 1. Large Deformation and Pervasive Fracture: The Case for Meshfree Numerical Methods

# The Case for Meshfree Methods: Macroscale



Numerical Method Should Accurately Predict:

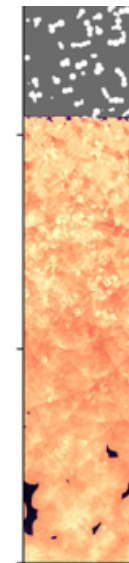
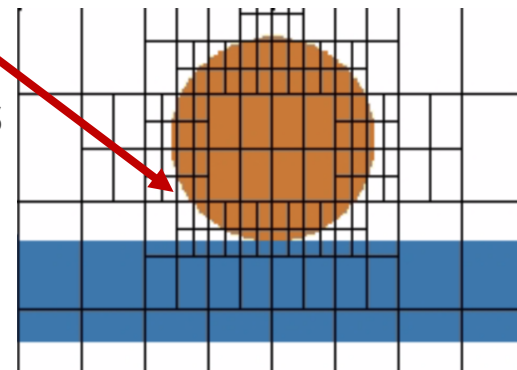
- Capture transition from solid to rubble
- Deformation-induced heating, chemistry



Example: Impact Test,  
Marcia Cooper

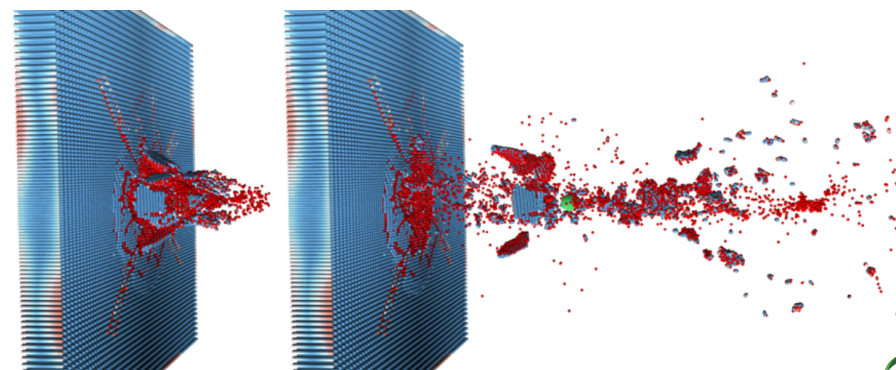
*Problem: poorly resolved strain fields and interface physics, averaging in mixed material cells*

**Hydrocode Methods**



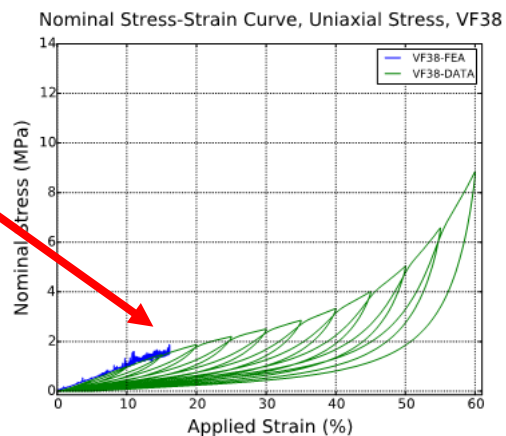
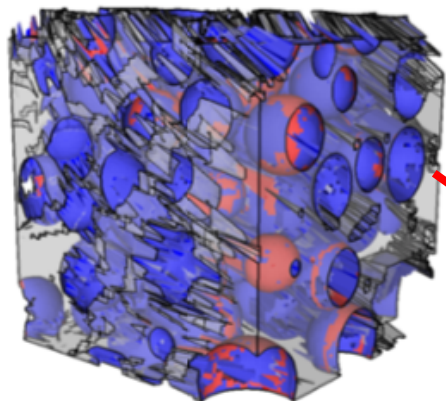
**Meshfree Methods**

Show promise in overcoming these problems at both meso and macro scales



**Mesh-based Methods (FEA)**

*Problem: Mesh entanglement at large deformations*



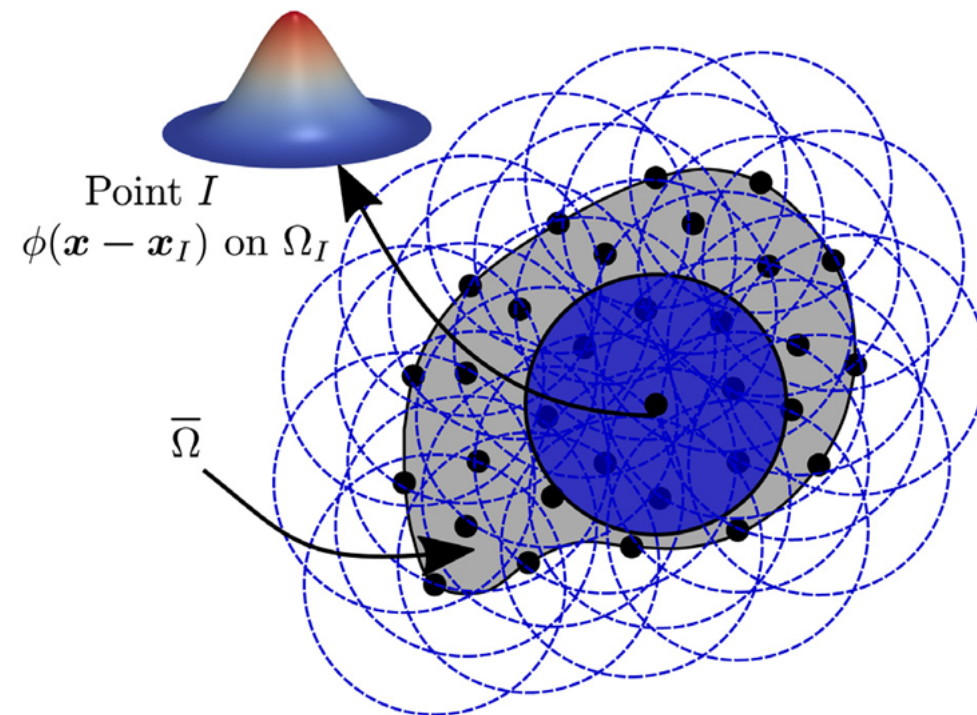
## Reproducing Kernel Particle Method

- Galerkin-based variational method using the reproducing kernel discretization
- Shape functions are the product of a window/kernel function and correction function

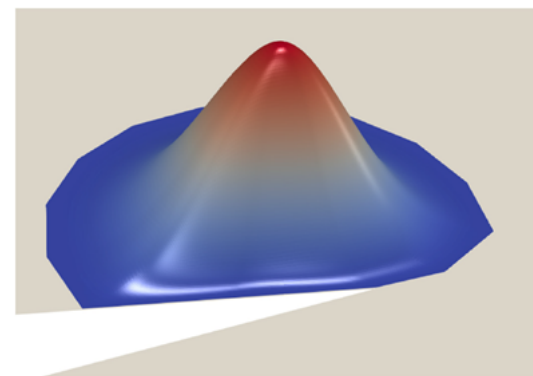
$$u^h(x) = \sum_{I=1}^{NP} \Psi_I d_I; \quad \Psi_I = C(x; x - x_I) \phi_a(x - x_I)$$

## Conforming Reproducing Kernel

- Graph distance informed window/kernels replace traditional Euclidian kernels to provide improved accuracy and robustness for nonconvex geometries and essential boundary conditions



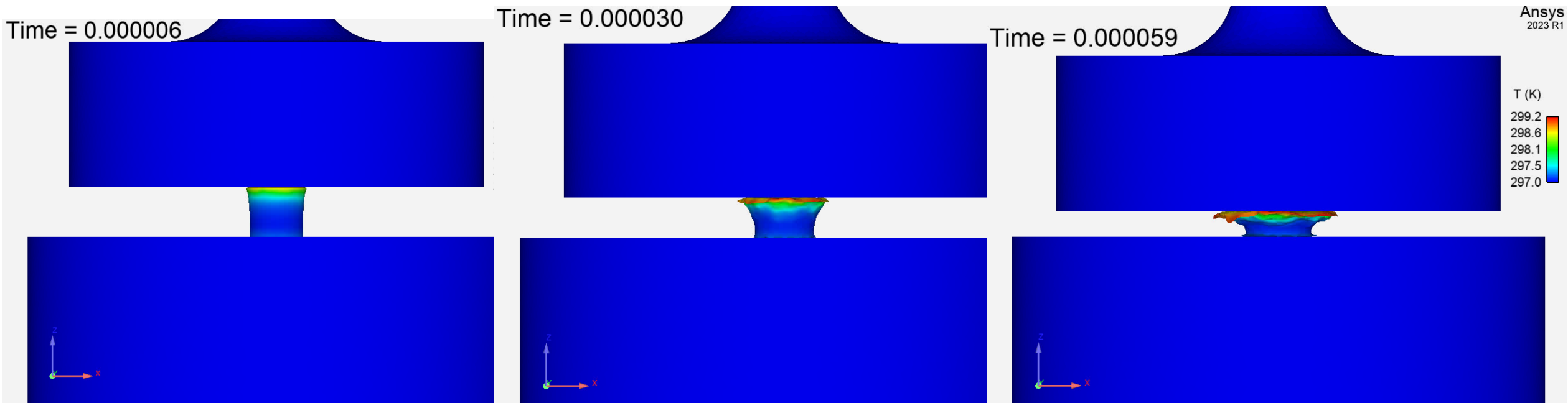
J. Koester, J.S. Chen, *Comput. Methods Appl. Mech. Engrg.* 347 (2019) 588–621



# CRK Example: Dynamic Crushing of a PBX Material



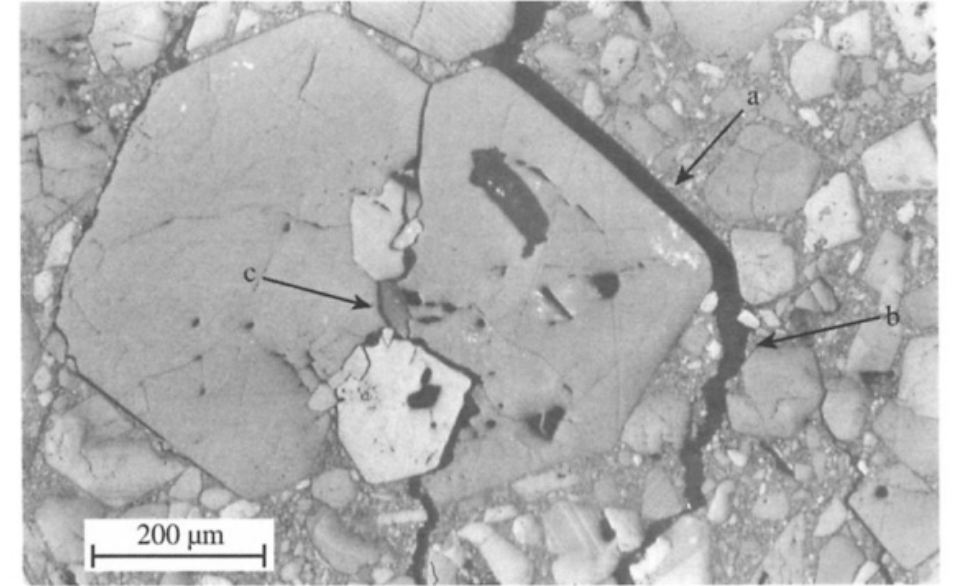
- Impact of top anvil at 60 m/s
- Method is numerically robust at very large deformations



# The case for meshfree methods: Mesoscale



- Many physical mechanisms:
  - Plasticity/viscoelasticity in binder (or grains)
  - Rotation/delamination/friction in grains
  - Crack nucleation, growth, coalescence
- Variety of length scales
  - Failure often dominated by binder phase:  
Can be much smaller/thinner than grains
  - Macro response depends on 100s-1000s of grains
  - Need to simulate an representative sample
- Mechanics of micron-sized grains may be dominated by defects, not well described by bulk continuum model
- Microcracks create many discontinuities



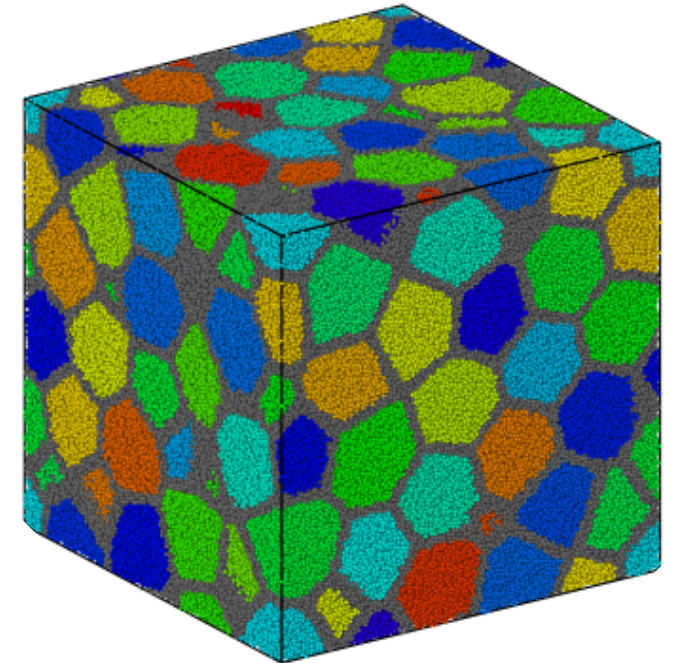
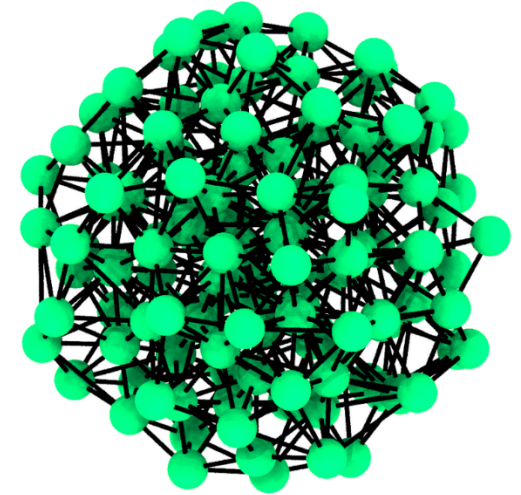
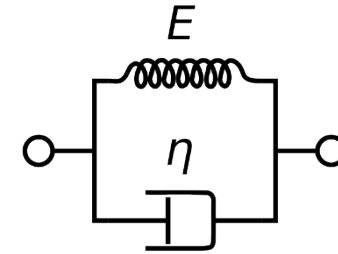
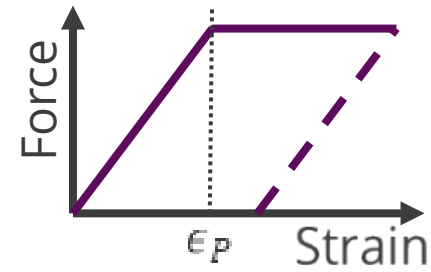
PJ Rae Proc. R. Soc. Lond. A. 2002

Need a flexible and cheap modeling approach!

# Bonded Particle Model (BPM)



- Lagrangian material points connected by a breakable network of bonds, AKA a spring network
- Solve Newtonian equations while obeying symmetries  
⇒ **Physics-based approach, focus on emerging behavior**
- Ideal for studying trends in abstract material classes, vary details of bond forces to capture different physics

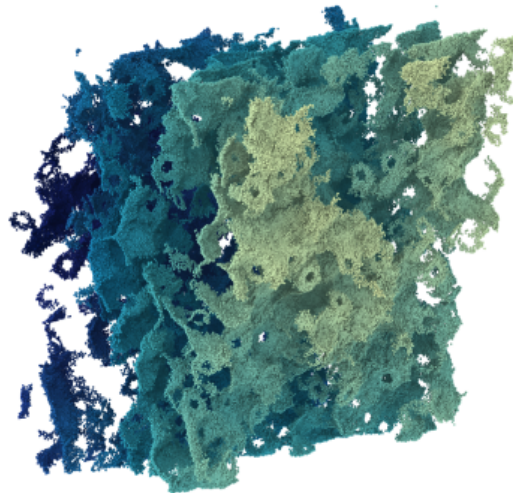
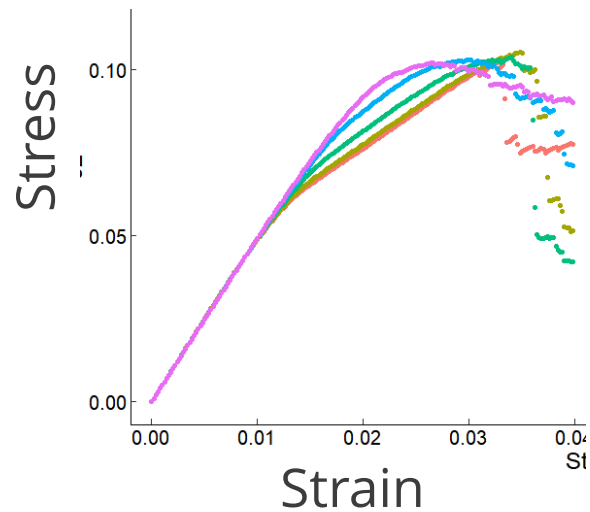


- **Simple and computationally cheap** – can simulate large domains at high resolution
- **Stably handles large deformation**, complex cracking, etc.

# BPM example: Compression of Crystals in Binder

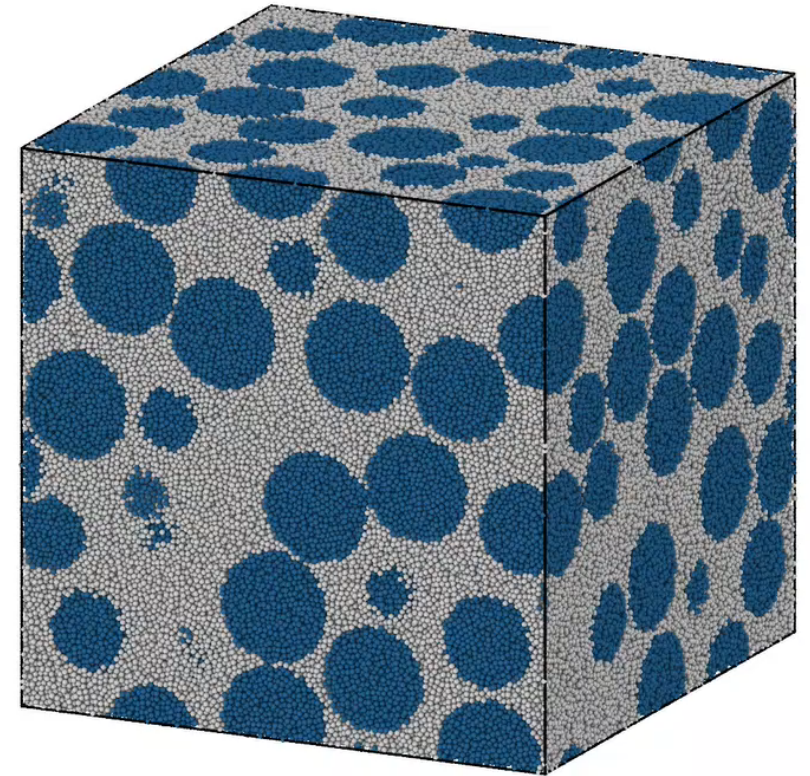


- Spherical, rigid crystals
- Elastic Binder,  $E_{\text{binder}} \ll E_{\text{crystals}}$
- Volume Fractions: Crystals ~64%, Binder ~36%
- Simulation Outputs:
  - Global homogenized stress/strain
  - Time-evolution of damage metrics: crack size, connected crack networks, etc.



Final percolating crack networks in binder

Uniaxial Compression

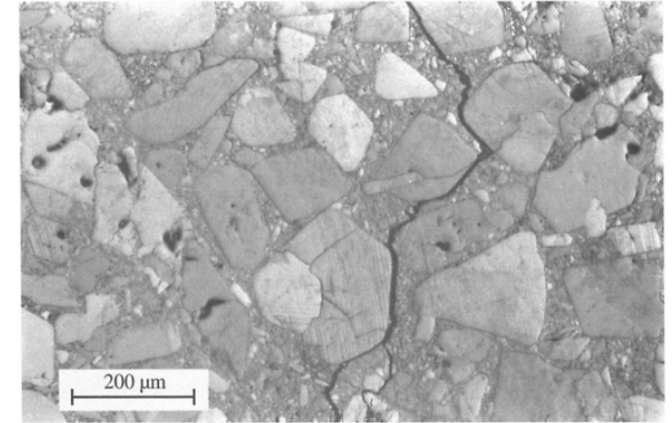


## 2. Complex mechanical behavior requires unique constitutive models

# Mechanical Response Characteristics



- Mechanical behavior of Binderized Energetics is complex:
  - Strain rate dependence
  - Temperature dependence
  - Pressure dependence
  - Tension-Compression Asymmetry
  - Loss of Stiffness (Damage)
  - Permanent Strains (Plasticity)
  - Shear-induced dilation
- Many inelastic deformation mechanisms driving these macroscopic behaviors:
  - Viscoelasticity (binders)
  - Cracking (intra- and inter-granular)
  - Porosity opening
  - Dislocation slip within crystals
  - Twinning (some energetic crystals)



Plastic Bonded Explosive [Rae, 2002]

*Macroscopic behavior  
(e.g. permanent strain) is  
produced by a  
combination of  
mesoscale mechanisms*

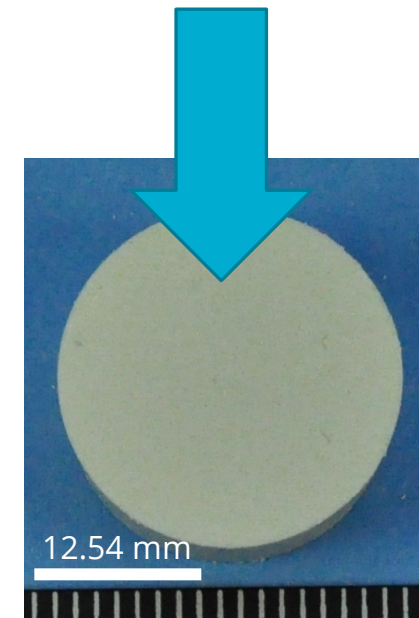
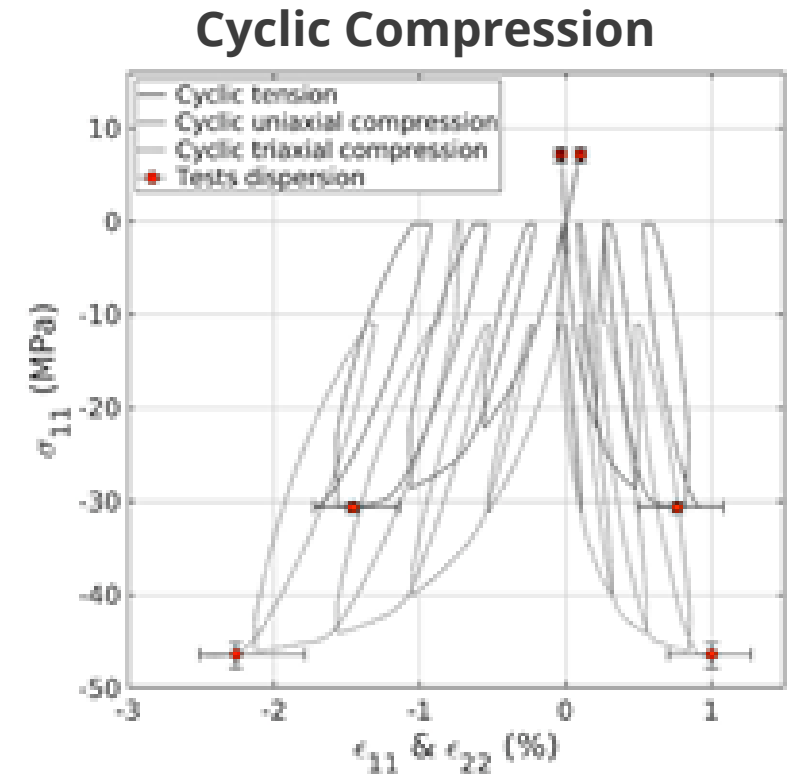
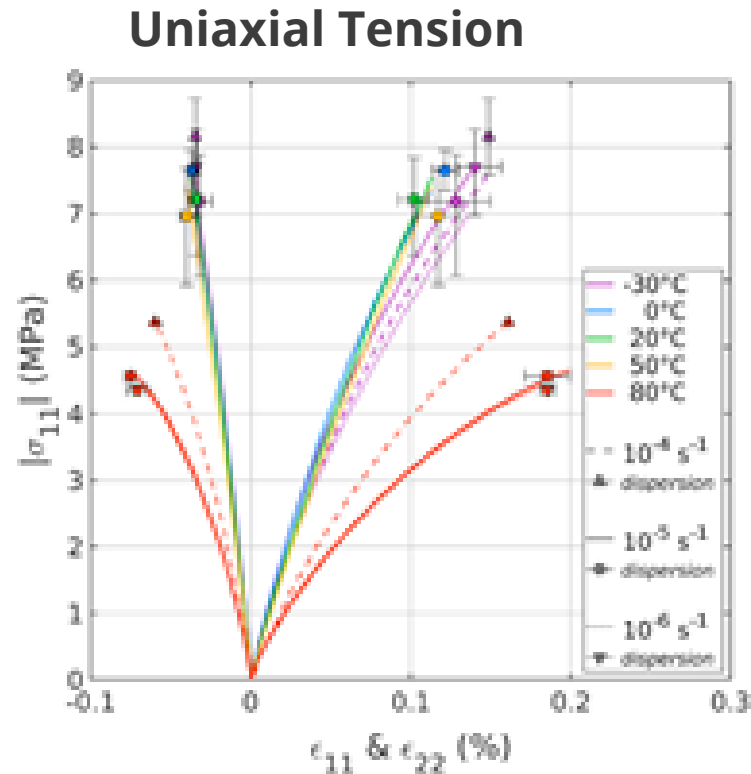
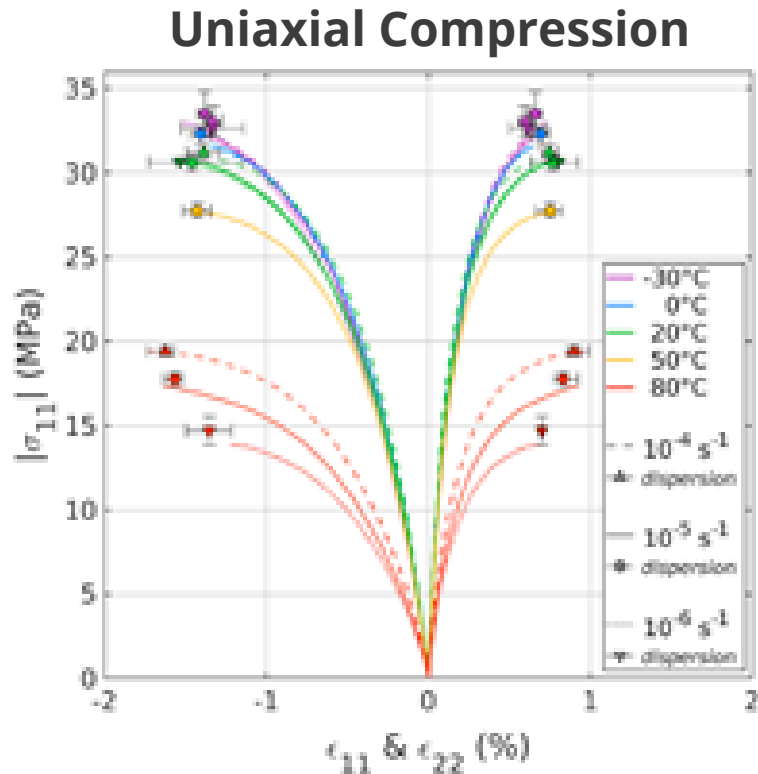


Image: courtesy Marcia Cooper

# Example Data from the Literature



- Excellent review paper by *Plassart et al., Mech. Mat., 150 (2020) 103561.*



*Monotonic uniaxial stress tests induce response governed by many deformation mechanisms—thus these are not enough to fully characterize behavior in an arbitrary stress state*

# Mapping observed macroscale behaviors to constitutive model features



## Macroscale Behavior:

- Strain rate dependence
- Temperature dependence
- Pressure dependence
- Tension-Compression Asymmetry
- Loss of Stiffness
- Permanent Strains (Deviatoric & Volumetric)
- Shear Induced Dilation

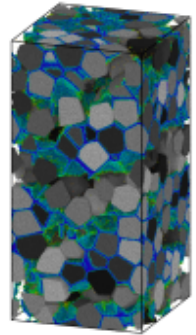
## Constitutive Model Feature:

- Volumetric Response
  - Compaction Curve
  - Equation of State
- Viscoelasticity
  - Time-temperature superposition
- Damage
  - Various formulations available
- ViscoPlasticity
  - Yield Surface Shape
  - Hardening Law
- Failure Criterion

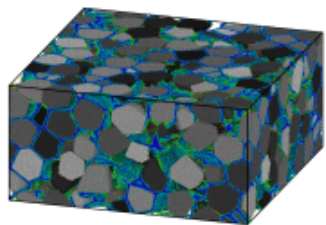
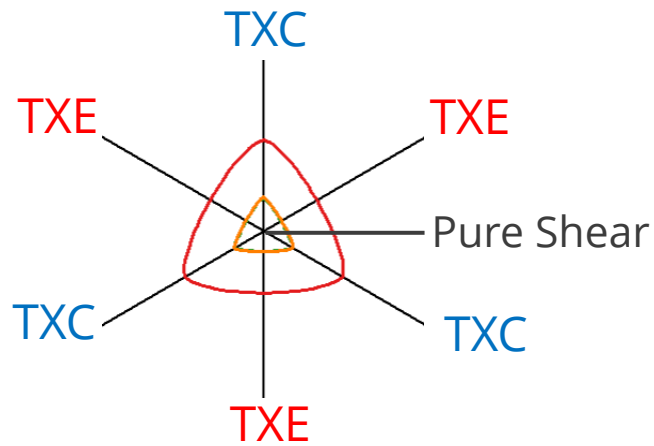
# Mesoscale Studies of Failure Surfaces in Multiaxial Stress Space



- Study Parameters: Binder deformation (elastic vs. elastic-plastic), Crystal-binder bond strength

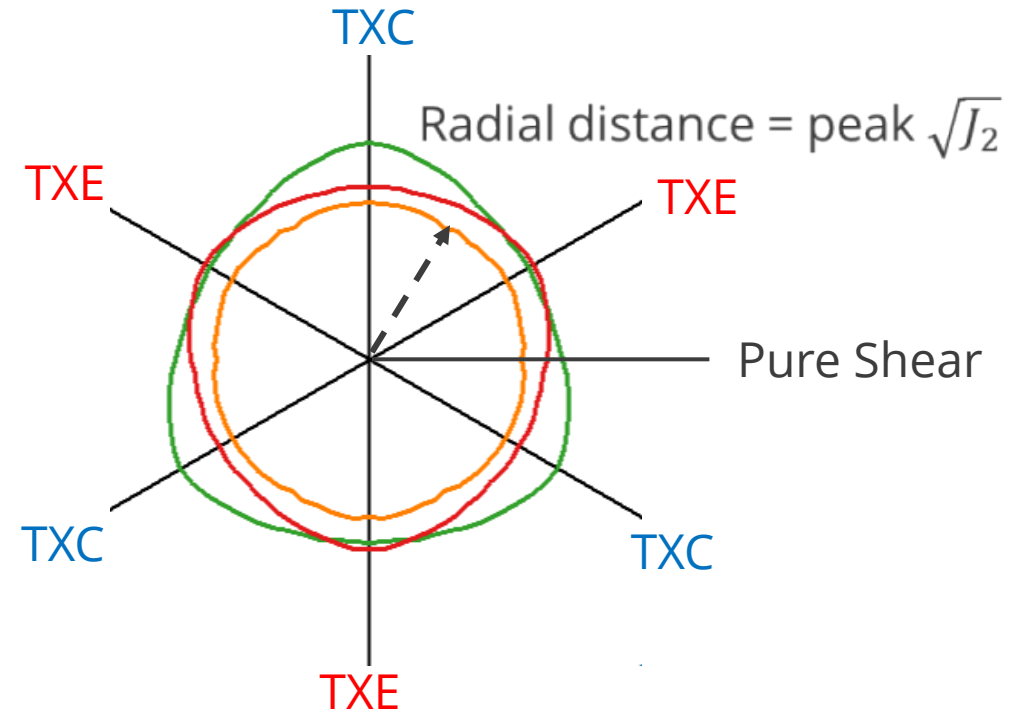


**Unconfined, mean P = 0**



- All cases are angular, Mohr-Coulomb-like, with strong tensile-compressive asymmetry

**Confined with mean P = 0.3**



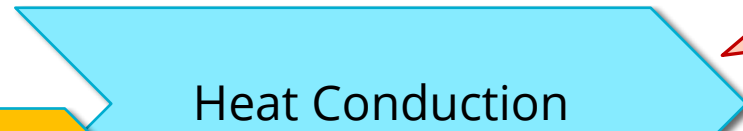
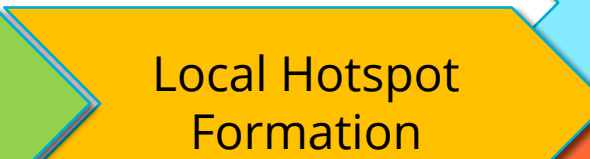
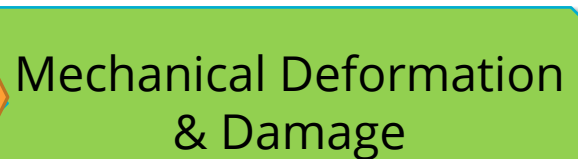
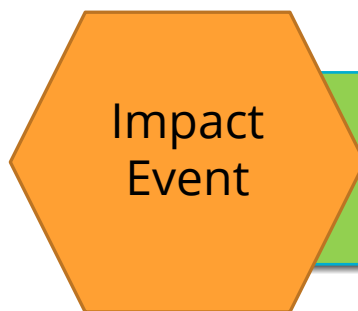
- All cases show hardening (larger yield surface) with confining pressure

### 3. High Explosive Violent Reaction is a Multi-Physics, Multi-Scale Event

# Revisiting the Full HEVR Event



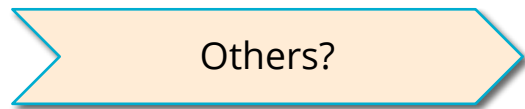
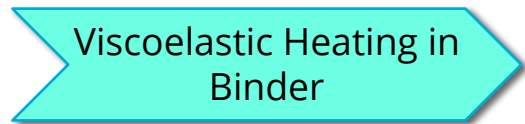
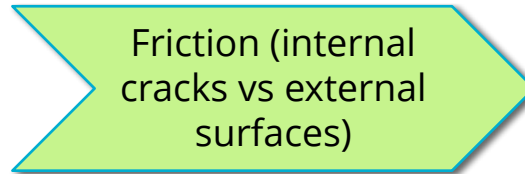
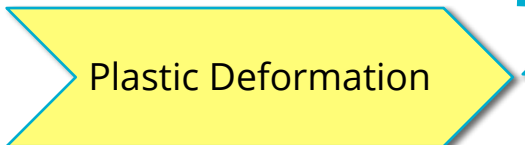
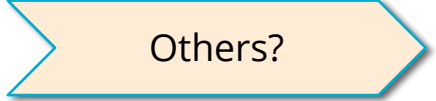
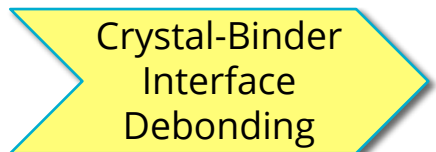
Continuum Constitutive Model → Continuum Multiphysics codes



?



Mesoscale Deformation Mechanisms



What are the heat generation sources?



## Meshfree Conforming Reproducing Kernel Continuum Code

Handshake solve for conservation of **momentum**:

$$\int_{\Omega} \mathbf{w} \cdot \rho \ddot{\mathbf{u}} d\Omega + \int_{\Omega} \mathbf{F}(\nabla \mathbf{w}) : \mathbf{P}(\nabla \mathbf{u}) d\Omega = \mathbf{f}^{\text{ext}}(\mathbf{u}),$$

and **energy**:

$$\int_{\Omega} w \rho C_P \dot{T} d\Omega + \int_{\Omega} \underbrace{\nabla w \cdot K \nabla T}_{\text{Thermal conduction}} d\Omega = \int_{\Omega} w \left( \underbrace{\dot{q}^{\epsilon^P}}_{\text{Adiabatic heating from plastic deformation}} + \underbrace{\dot{q}^{\text{species}}}_{\text{Chemical heating}} \right) d\Omega + \int_{\partial\Omega} w h d\Gamma.$$

*Thermal conduction*

*Adiabatic heating from plastic deformation*

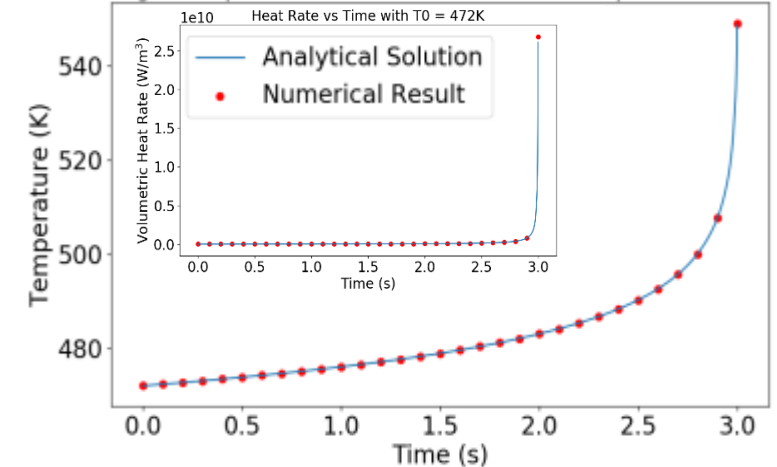
*Chemical heating*

- State variable from mechanical constitutive model

- Chemical heating from exothermic decomposition
  - Currently restricted to Arrhenius rate
  - More sophisticated models in progress

$$Q = \rho \Delta H Z e^{-E_a/RT}$$

Average Temperature vs Time with initial Temperature = 472K



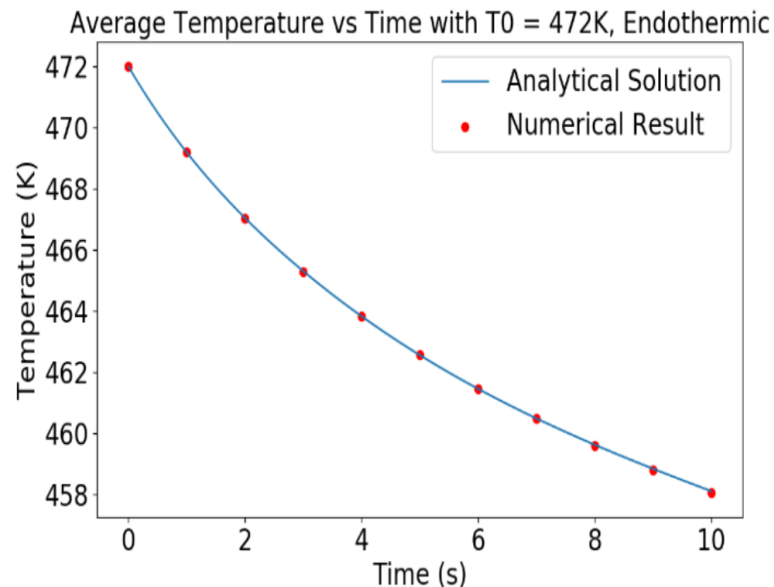
# Thermo-chemical coupling verification



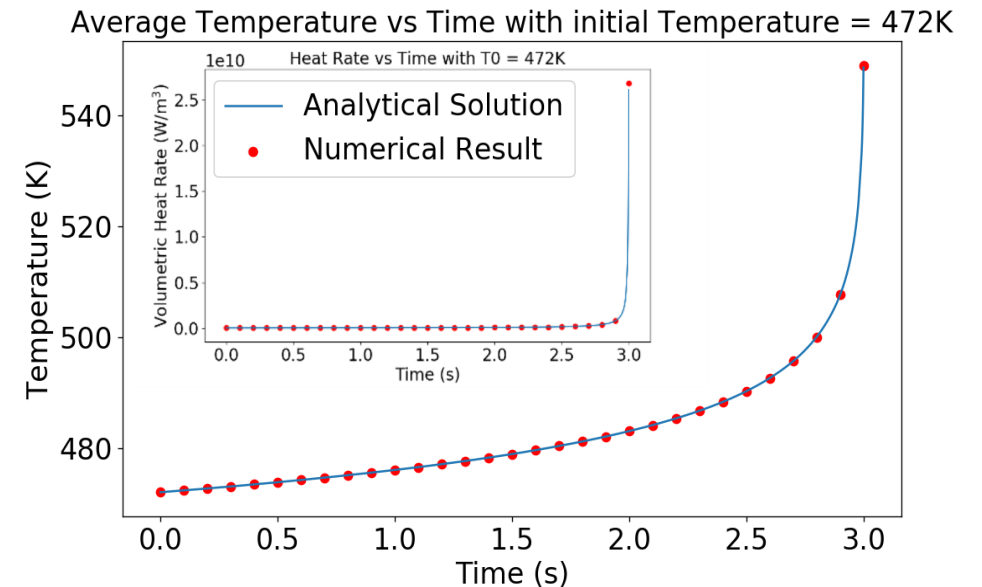
- **Temporal verification** based on Frank-Kamenetskii equation[1]:

$$-\lambda \nabla^2 T + \rho C \frac{dT}{dt} = \rho \Delta H Z e^{-E_a/RT} \xrightarrow{\text{Uniform temperature change (temporal variation only)}} \rho C \frac{dT}{dt} = \rho \Delta H Z e^{-E_a/RT}$$

## Endothermic Process



## Exothermic Process



<sup>1</sup>Cooper, Paul W.. Explosives Engineering, John Wiley & Sons, Incorporated, 1996.

# Thermo-chemical coupling verification

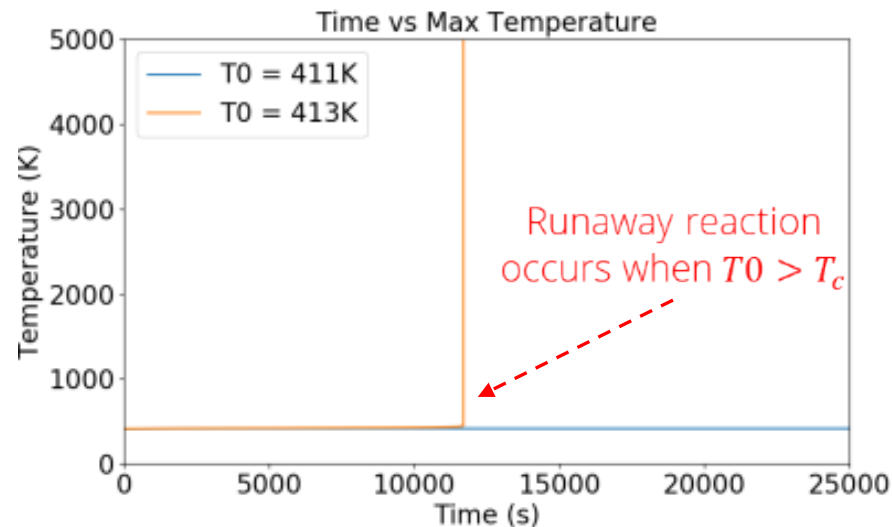
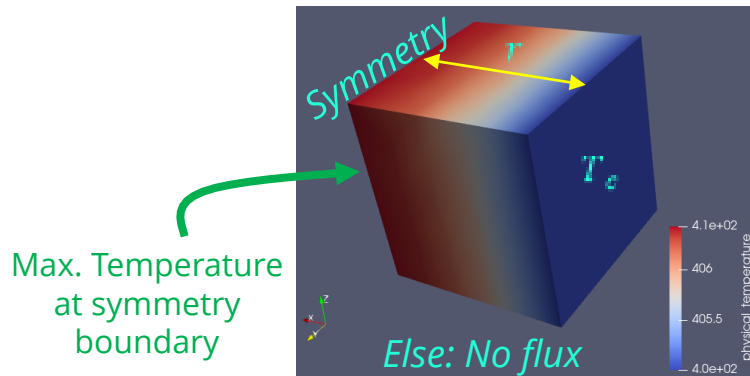


- Verification test based on Frank-Kamenetskii critical temperature[1]:

$$-\lambda \nabla^2 T + \rho C \frac{dT}{dt} = \rho \Delta H Z e^{-E_a/RT} \quad \xrightarrow{\text{Steady state solution (spatial variation only)}} \quad -\lambda \nabla^2 T = \rho \Delta H Z e^{-E_a/RT}$$

$$\frac{E_a}{T_c} = R \ln \left( \frac{r^2 \rho \Delta H Z E_a}{T_c^2 \lambda \delta R} \right)$$

Cubic domain with no flux boundary except one surface with constant  $T_c$  (infinite slab)



Solve for critical ambient temperature ( $T_c$ ) before runaway reaction<sup>1</sup>

Analytical  
Critical  
Temperature:  
~412K

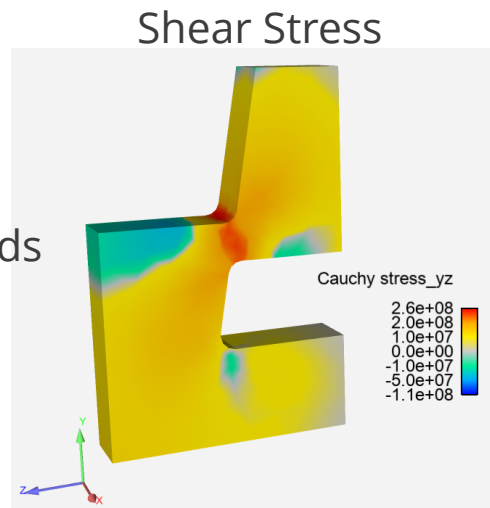
<sup>1</sup>Cooper, Paul W.. Explosives Engineering, John Wiley & Sons, Incorporated, 1996.

# Inert Material Study: Shear-Induced Heating in Steel

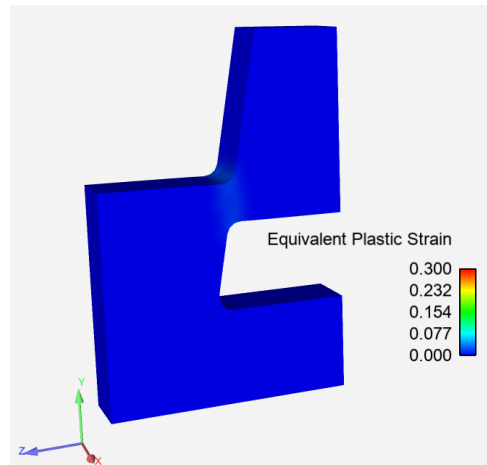


- Top-hat geometry designed to induce high localized shear
- Material properties: steel

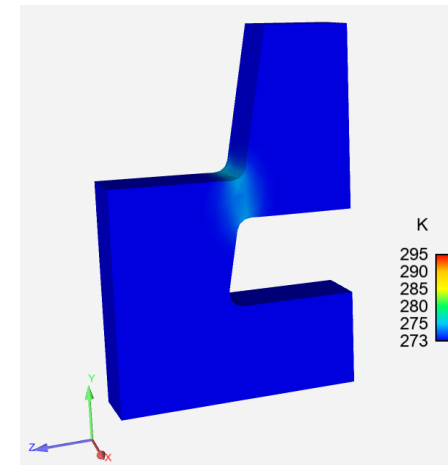
Time = 65  
microseconds



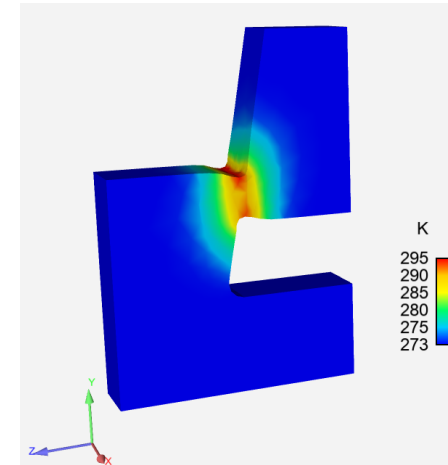
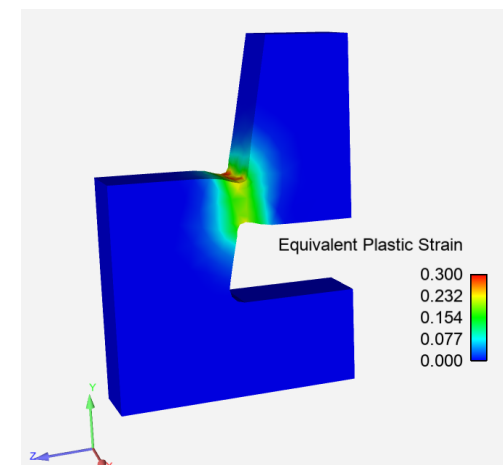
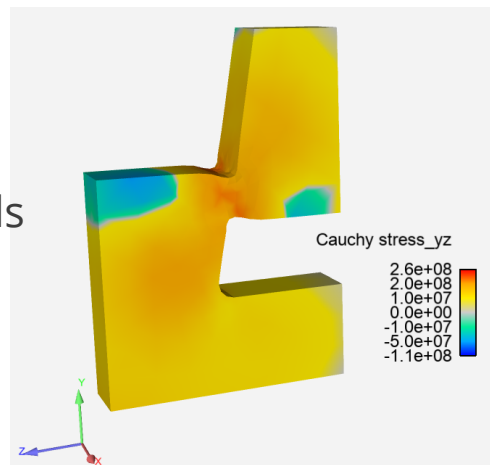
Equivalent Plastic Strain



Temperature



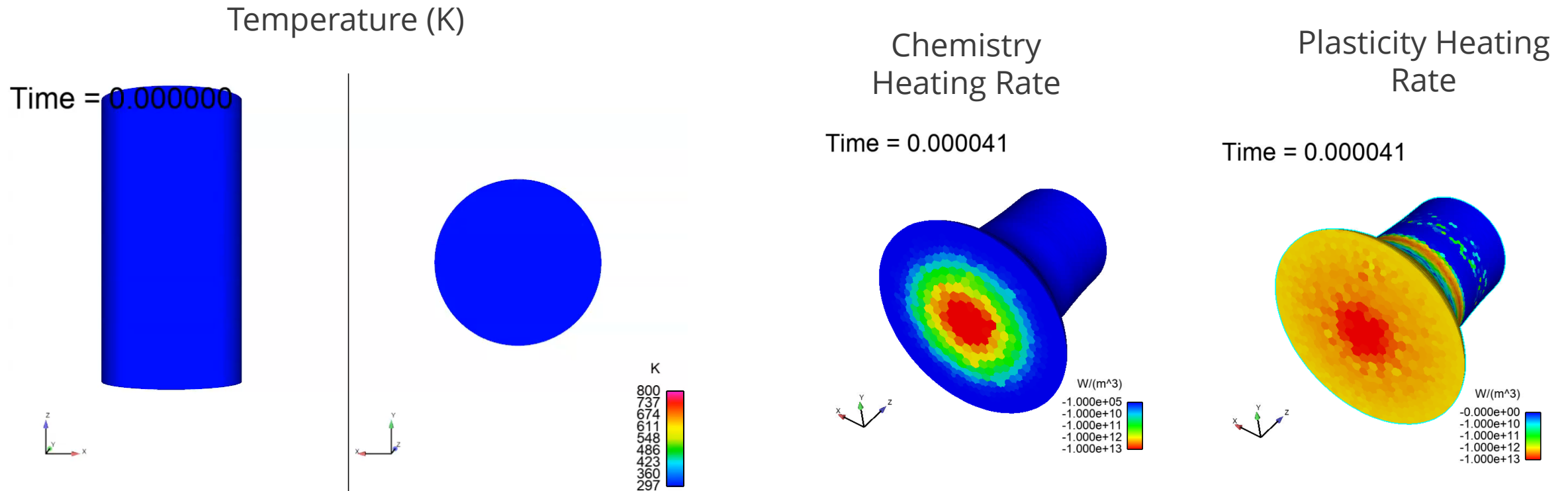
Time = 250  
microseconds



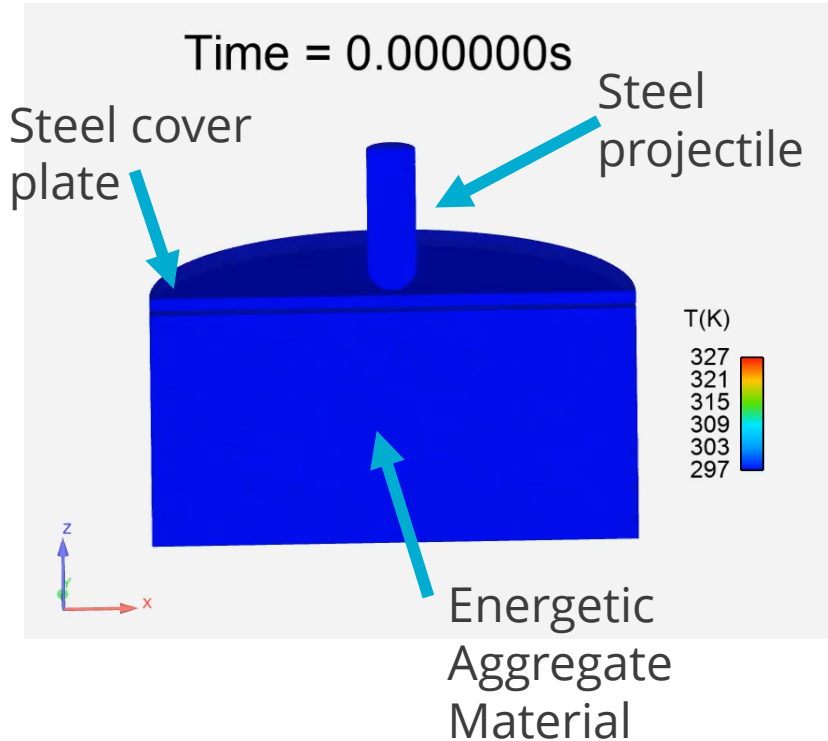
# Mechanically Induced Thermal Runaway



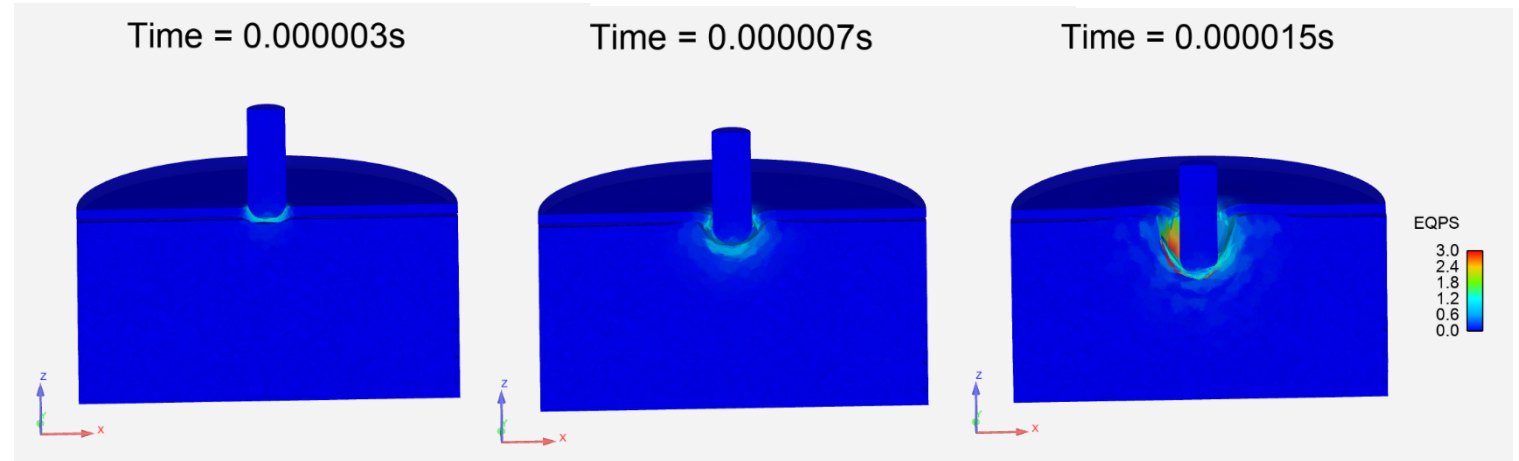
- Taylor Bar Impact: 450 m/s
- Material properties: pure energetic crystals
- Energy dissipated due to plastic deformation raises temperature enough to start runaway chemistry



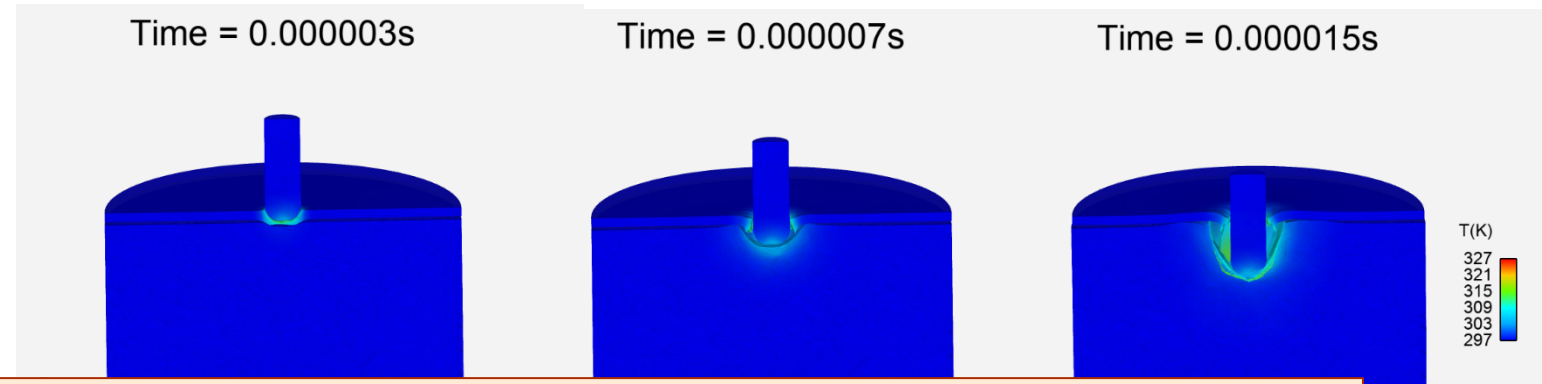
# Multi-material impact example



## Equivalent Plastic Strain



## Temperature



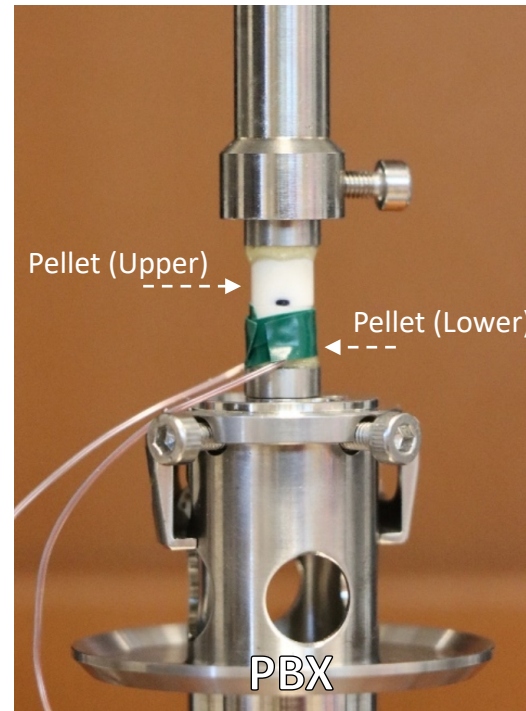
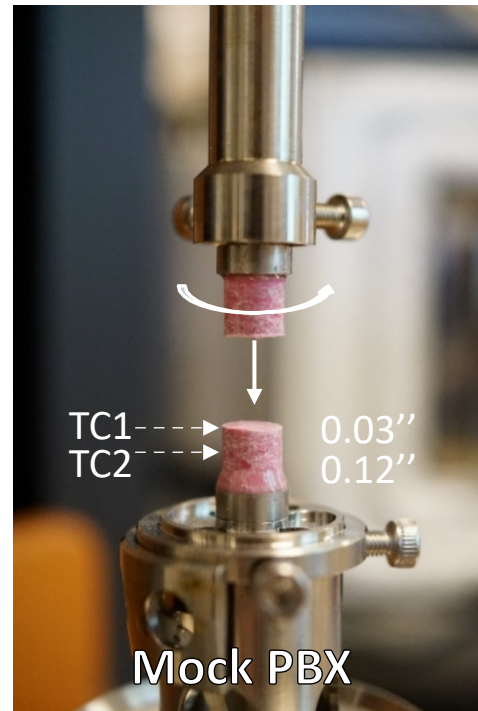
Extreme numerical robustness coupled with multi-physics enables new simulation capability

# Mechanism Discovery: Frictional Heating Experiments



## Objective: Isolate and measure heating due to friction

The *TA Discovery Hybrid Rheometer* was used to frictionally shear two explosive pellets at a controlled angular sliding speed and applied normal force.



## Procedure

- Fix pellets to Rheometer fixtures
  - ½" dia. x ½" height pellets
- Insert into rheometer grips
- Place TC in contact with sample surface at known locations and secure with tape
- Wait for thermal equilibrium in environmental chamber
- Set axial load and run (3 rad/s, 180s)
  - 2.5N, 5N, 10N (triplicate)

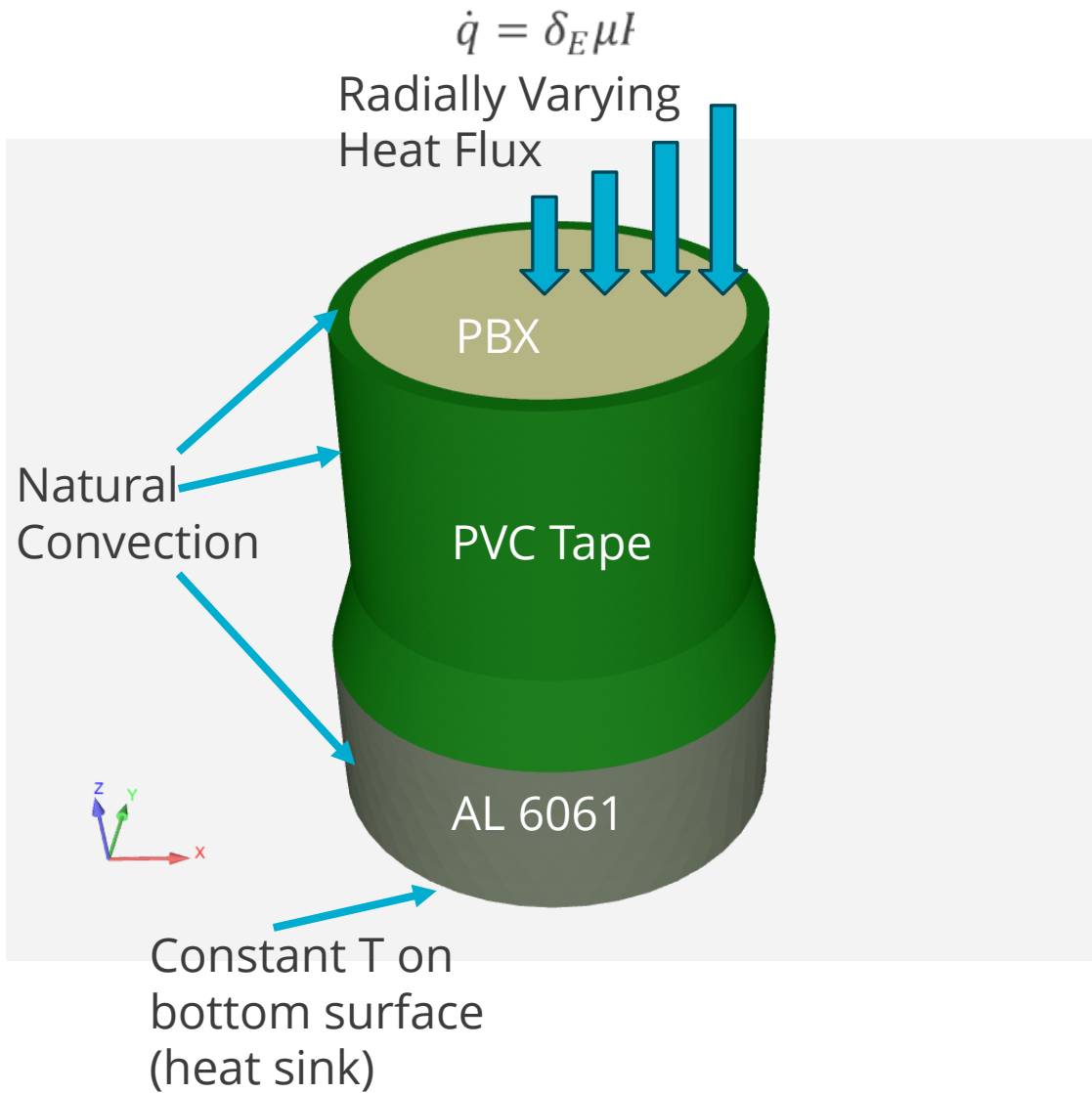
## Measurements

- Normal force, Torque, Chamber temperature
- Sample temperature on outer surface using T-type thermocouples (X2)

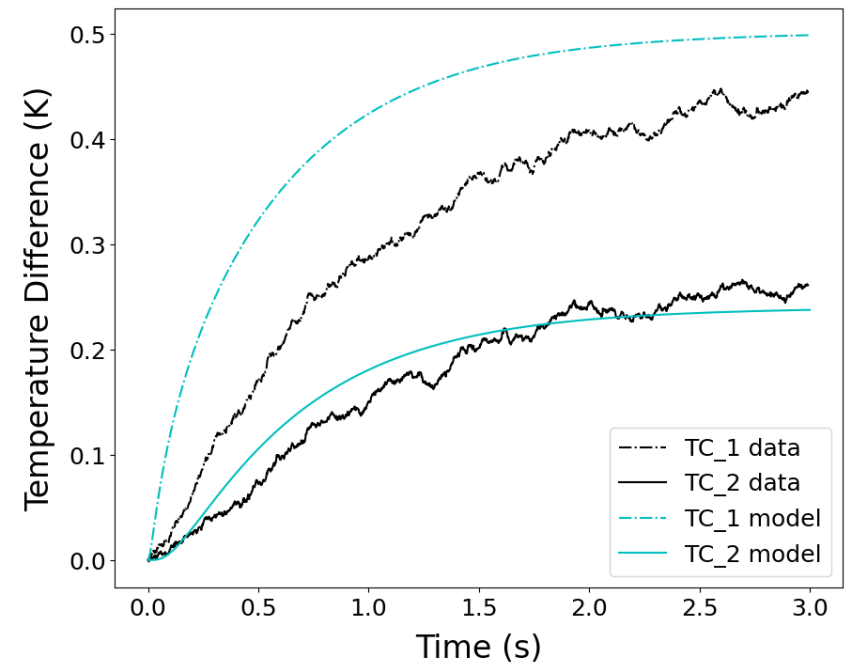
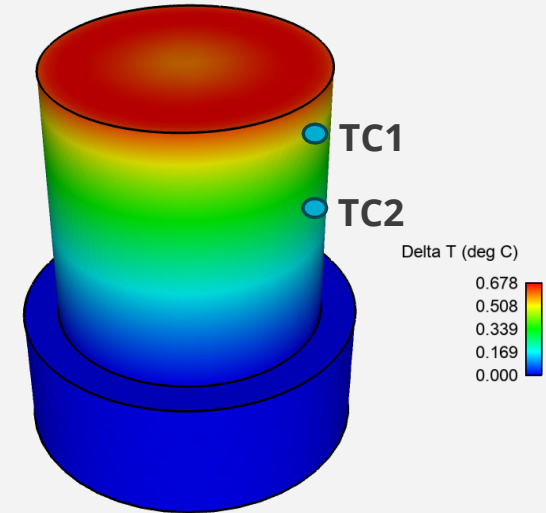
## Outcome

- Temperature rise due to heat generated by friction
- Calculated coefficient of friction

# Frictional Heating Model (thermal only)



Time = 180.0

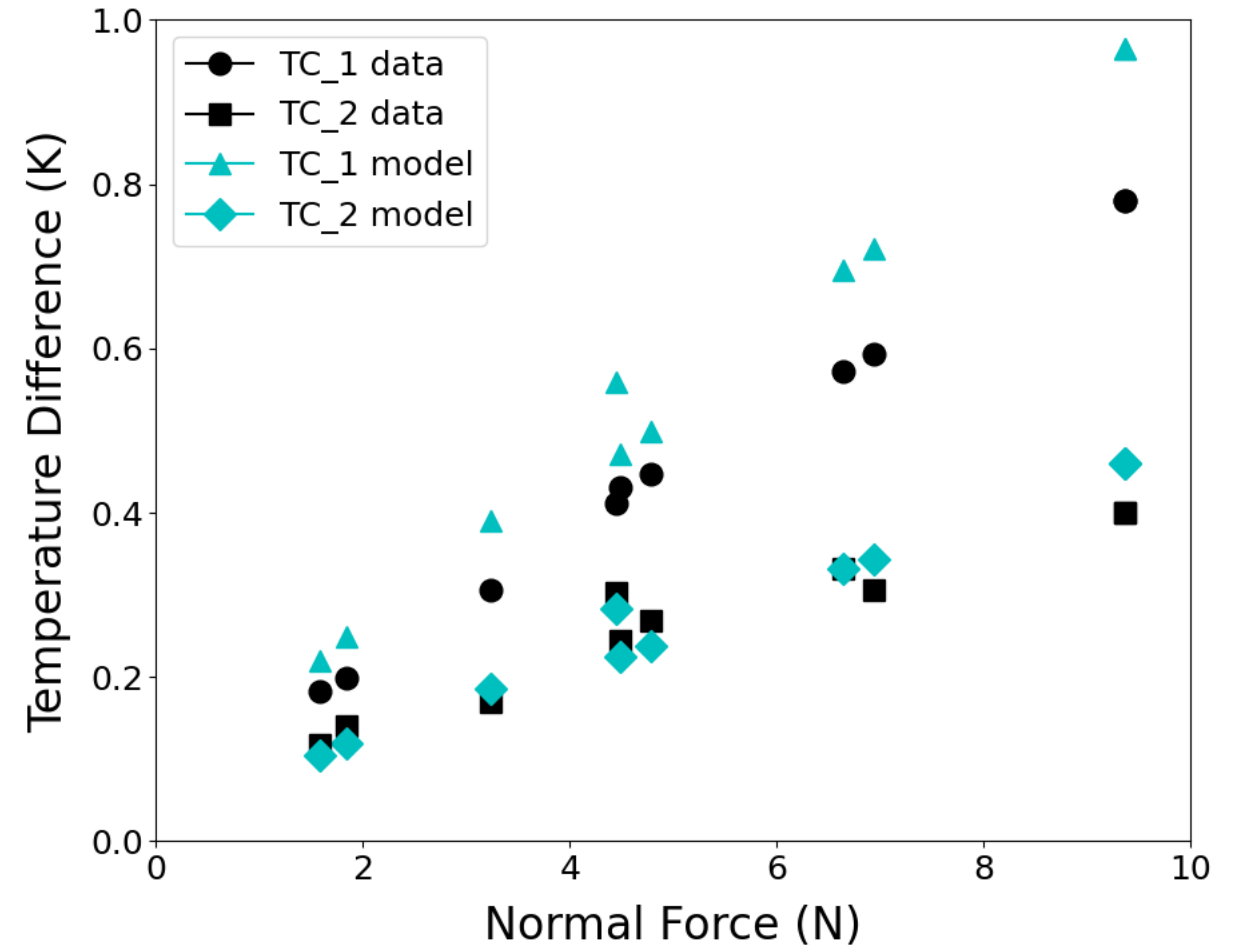
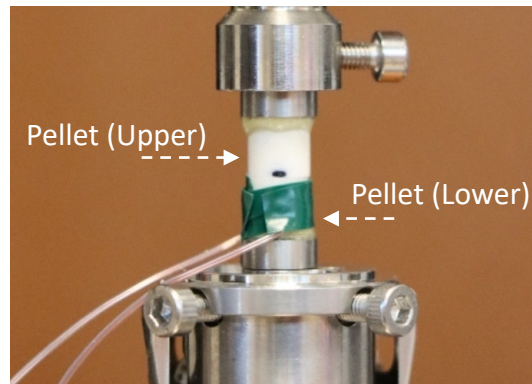
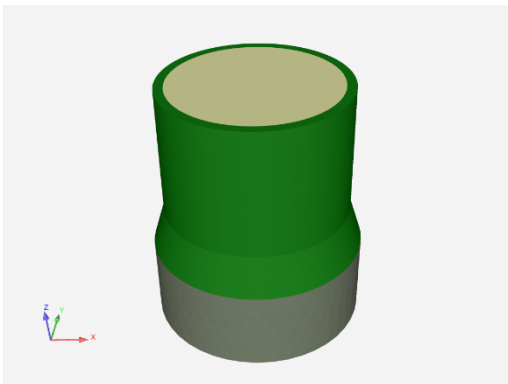


# Frictional Heating Model (thermal only)



## Takeaways:

- Linear global temperature rise with axial loading predicted by model
- Good match with experimental data considering the modeling uncertainties
  - E.g. properties of PVC tape, convection coefficient, small temperature rises
  - Tape geometry differences
  - Lumping of glue in with the tape



# Wrap Up & Discussion Points



## 1. Large Deformation (Global and/or localized) and Pervasive Damage/Fracture

- Meshfree numerical methods (continuum and mesoscale) show much promise here
  - Bonded Particle Methods enable computationally cheap, large mesoscale studies
  - Conforming Reproducing Kernel (CRK) Method as a powerful tool for *Lagrangian* continuum mechanics at massive strains and coupled physics

## 2. Complex mechanical behavior, requires unique constitutive models

- Need for characterization beyond the uniaxial stress-strain curve
- Need for flexible, efficient mesoscale models to study linkages between microscopic inelastic mechanisms and macroscale effective behaviors
  - Inform macroscale constitutive model features and balance between them (yield & failure surfaces, damage formulation, etc.)

## 3. Multi-physics nature of HEVR event

- Meshfree CRK method with coupled physics shows promise for pre-ignition phase where thermo-mechanics dominates
- Discovery experiments characterizing heat generation mechanisms still needed!



**Thank You!**

# Some Relevant Literature



- **CRK Numerical Method:** *J. Koester, J.S. Chen, Comput. Methods Appl. Mech. Engrg. 347 (2019) 588–621*
- **Review of Mechanical Behavior of HMX and TATB-based PBX Materials:** *Plassart et al., Mech. Mat., 150 (2020) 103561.*
- **BPM Study of Pressure Dependence in Plastic Bonded Energetic Materials:** *J.T. Clemmer, K.N. Long, J.A. Brown, Mech. Mater. 184 (2023) 104693.*

➤ Kinematics:

$$\boldsymbol{\epsilon} = \boldsymbol{e} + \frac{1}{3} \epsilon_{\text{vol}} \boldsymbol{I} \quad \sigma_m = K \epsilon_{\text{vol}}$$

$$\boldsymbol{e} = (\boldsymbol{e}^{ve} + \boldsymbol{e}^D) + \boldsymbol{e}^P$$

➤ Viscoelasticity

$$\dot{\boldsymbol{s}} = 2G^\infty \dot{\boldsymbol{e}}^{ve} + \sum_{\kappa=1}^N \left( 2G^{(\kappa)} \dot{\boldsymbol{e}}^{ve} - \frac{\boldsymbol{s}^{(\kappa)}}{\tau^{(\kappa)}} \right)$$

Prony series of shear moduli and relaxation times

$$\dot{\boldsymbol{s}}^{(\kappa)} = 2G^{(\kappa)} (\dot{\boldsymbol{e}} - \dot{\boldsymbol{e}}^P) - \frac{\boldsymbol{s}^{(\kappa)}}{\tau^{(\kappa)}} - \frac{G^{(\kappa)}}{G_0} \left[ \frac{3}{a} \left( \frac{c}{a} \right)^2 \dot{c} s + \left( \frac{c}{a} \right)^3 \dot{s} \right]$$

➤ SCRAM Damage

$$\boldsymbol{e}^D = \frac{1}{2G_0} \left( \frac{c}{a} \right)^3 \boldsymbol{s}$$

$$\dot{c} = \begin{cases} v_{res} \left( \frac{K_I}{K_1} \right)^m & \text{for } K_I < K' \\ v_{res} \left[ 1 - \left( \frac{K_{0\mu}}{K_I} \right)^2 \right] & \text{otherwise} \end{cases}$$

$$D = \frac{\left( \frac{c}{a} \right)^3}{1 + \left( \frac{c}{a} \right)^3}$$

➤ Drucker-Prager Plasticity

$$f(\sigma_{ij}) = \sigma_e + A \cdot \sigma_m - \sigma_y$$

$$g(\sigma_{ij}) = \sigma_e + B \cdot \sigma_m - \sigma_y$$

$$\dot{\boldsymbol{e}}^P = \lambda \frac{\partial g}{\partial \boldsymbol{\sigma}}$$

$$\lambda = \frac{1}{\tilde{\tau}} \left\langle \frac{f(\sigma_{ij})}{\sigma_0} \right\rangle_{\tilde{m}} \quad \sigma_e = \sqrt{\frac{3}{2} s_{ij} s_{ij}}$$