SAND2024-09245C

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Modeling Shock in Solids Using a 10-Node Composite Tetrahedral Element

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July 25, 2023

USNCCM17

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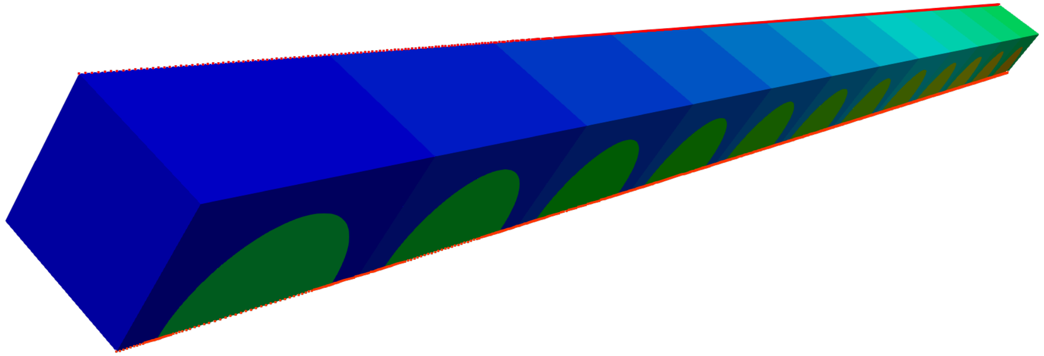
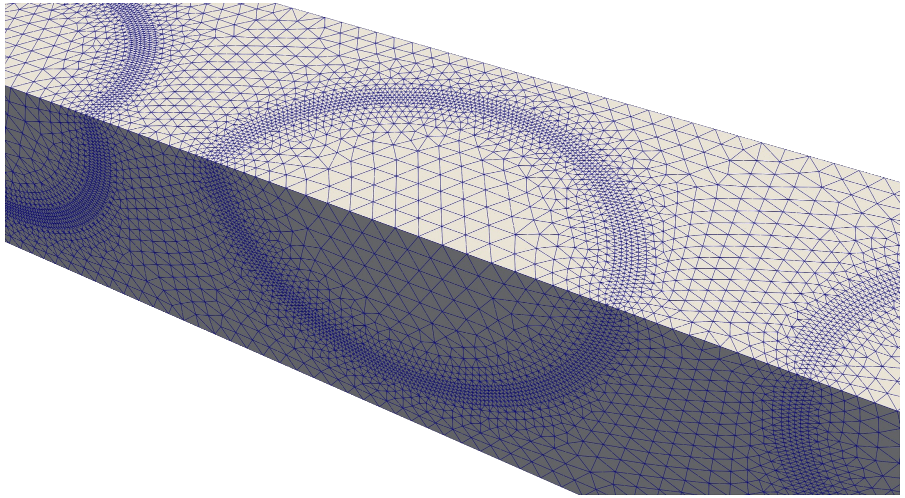
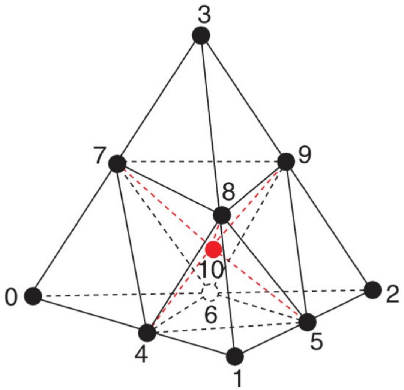
Overview

Goal is to provide computational capabilities necessary for modeling and simulation of shock in solids



- Complex material behavior – equation of state + strength
- Mesh generation/modification – Lagrangian tetrahedral elements



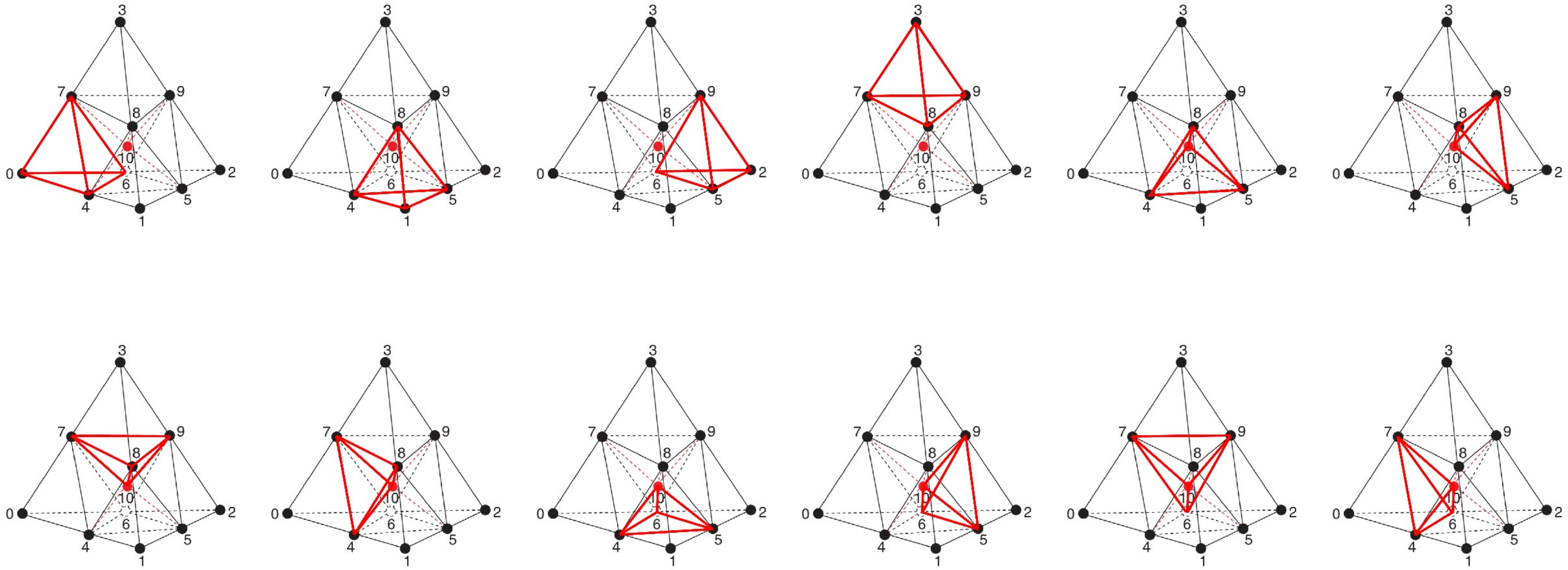


10-node composite tetrahedral element

Element Overview

Composite tetrahedron has 12 linear sub-tetrahedral elements

$$\int_{V_0} f \, dV_0 = \sum_{a=0}^{11} \int_{V_0^a} f \, dV_0^a$$



Element Formulation

Map gradient operator onto a linear space:

$$\bar{\mathcal{B}}_{iI} = \lambda_\alpha \left[\int_{V_0} \lambda_\alpha \lambda_\beta \, dV_0 \right]^{-1} \int_{V_0} \lambda_\beta \mathcal{B}_{iI} dV_0 = \lambda^\beta \int_{V_0} \lambda_\beta \mathcal{B}_{iI} dV_0$$

The functions λ_α span the space of linear functions on the element for our Composite Tet10 (but can be chosen otherwise).

$\mathcal{B}_{iI}$ are the piecewise constant gradient operators: $\mathcal{B}_{iI} = \frac{\partial N_i}{\partial X_I}$

λ^α are dual functions to λ_α: $\int_{V_0} \lambda^\alpha \lambda_\beta dV_0 = \delta^\alpha_\beta$

Energy Integration

- Energy is coupled to stress (pressure) in equation of state:

$$\dot{E} = p\frac{\dot{\rho}}{\rho^2} + \dot{E}_s \qquad\qquad p = p(\rho, E)$$

- For a model that is linearly dependent on the energy, a central difference scheme can be used:

$$p = f_1(\rho) + f_2(\rho)E \;\Rightarrow E_{n+1} = \left[1 - f_{2,n+1}\left[\frac{d\rho}{2\rho^2}\right]_{n+1/2}\right]^{-1}\left[E_n + \left(f_{1,n+1} + p_n\right)\left[\frac{d\rho}{2\rho^2}\right]_{n+1/2} + dE_{s,n+1/2}\right]$$

- For an arbitrary dependence, a fixed-point iteration algorithm is used:

$$p_{n+1}^{(k+1)} = p\left(\rho_{n+1}, E_{n+1}^{(k)}\right) \qquad\qquad E_{n+1}^{(k+1)} = E\left(\rho_{n+1}, p_{n+1}^{(k+1)}\right)$$

Tensor Based Artificial Viscosity

$$\sigma_{\text{stab}} = \sigma + \sigma_a$$

$$\mathbf{D} = \text{Sym}(\nabla\mathbf{v}) = \frac{1}{2}\left(\nabla\mathbf{v} + (\nabla\mathbf{v})^{\text{T}}\right)$$

$$\text{tr}(\mathbf{D}) = \nabla \cdot \mathbf{v}$$

$$\varepsilon = b_1 - b_2^2 \Delta t \ \min(0, \text{tr}(\mathbf{D}))$$

- Scalar viscosity: artificial pressure
- Ref: Laursen, Attaway, Zadoks. SAND98-1760/3.

$$\sigma_a = (\varepsilon(\lambda + 2\mu)\Delta t \,\text{tr}(\mathbf{D}))\ \mathbf{I}$$

$$p_a = \varepsilon(\lambda + 2\mu)\Delta t \,\text{tr}(\mathbf{D})$$

$$\sigma_a = p_a\mathbf{I}$$

- Tensorial viscosity
- Common in literature, e.g. "Dobrev, Kolev, Rieben. SIAM J. Sci. Comput. Vol 34, No. 5, B606-B641"

$$\sigma_a = \varepsilon(\lambda + 2\mu)\Delta t \,\mathbf{D}$$

Stabilization

Penalize soft mode

$$\Phi^{\dagger}[\boldsymbol{\varphi}, \bar{\boldsymbol{F}}, \bar{\boldsymbol{P}}, \bar{J}^{\star}, \bar{p}^{\star}] = \Phi^{\star}[\boldsymbol{\varphi}, \bar{\boldsymbol{F}}, \bar{\boldsymbol{P}}, \bar{J}^{\star}, \bar{p}^{\star}] + \int_B K\left(\frac{\bar{J}^{\star}}{\bar{J}}\right) \mathrm{d}V$$

$$K_{log}(x) = \frac{1}{2} k \log^2\left(\frac{\bar{J}^{\star}}{\bar{J}}\right)$$

$$K_m(x) = \frac{1}{2} k (\theta_m)^2$$

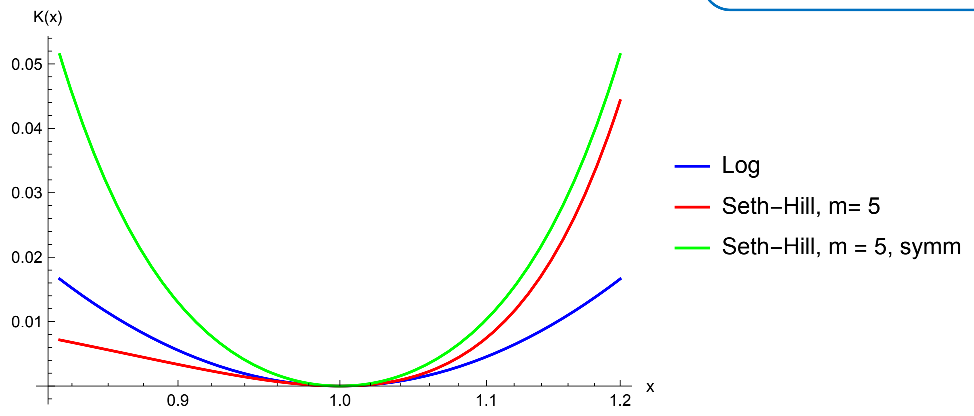
$$\theta_m = \frac{1}{m}\left[\left(\frac{\bar{J}^{\star}}{\bar{J}}\right)^m - 1\right]$$

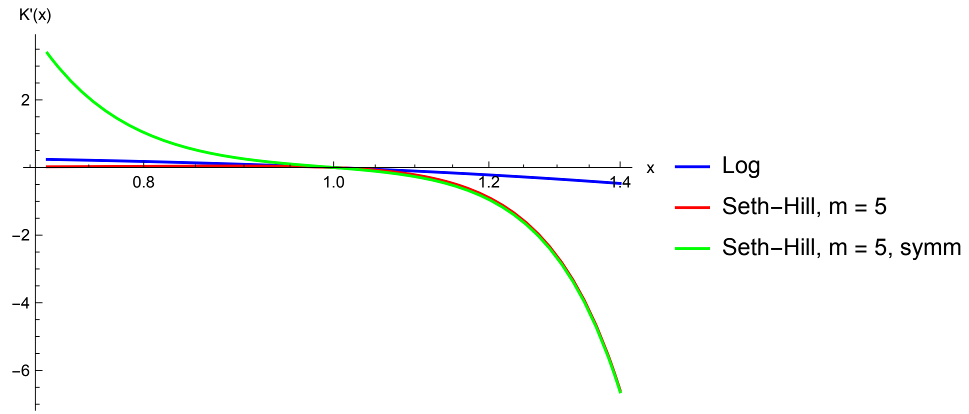
$$\bar{\theta}_m = \frac{1}{m}\left[\left(\frac{\bar{J}^{\star}}{\bar{J}}\right)^{-m} - 1\right]$$

"strain-like" penalty terms

$$K^{\star}(x) = \frac{1}{2} k [(\theta_m)^2 + (\bar{\theta}_m)^2]$$

symmetric Seth-Hill





See Jay Foulk's presentation Wednesday, 2:20 – 2:40 pm.
"Recent Advances in a 10-Node Composite Tetrahedral Framework for Shock and Mesh Adaptivity"

Isentropic Compression

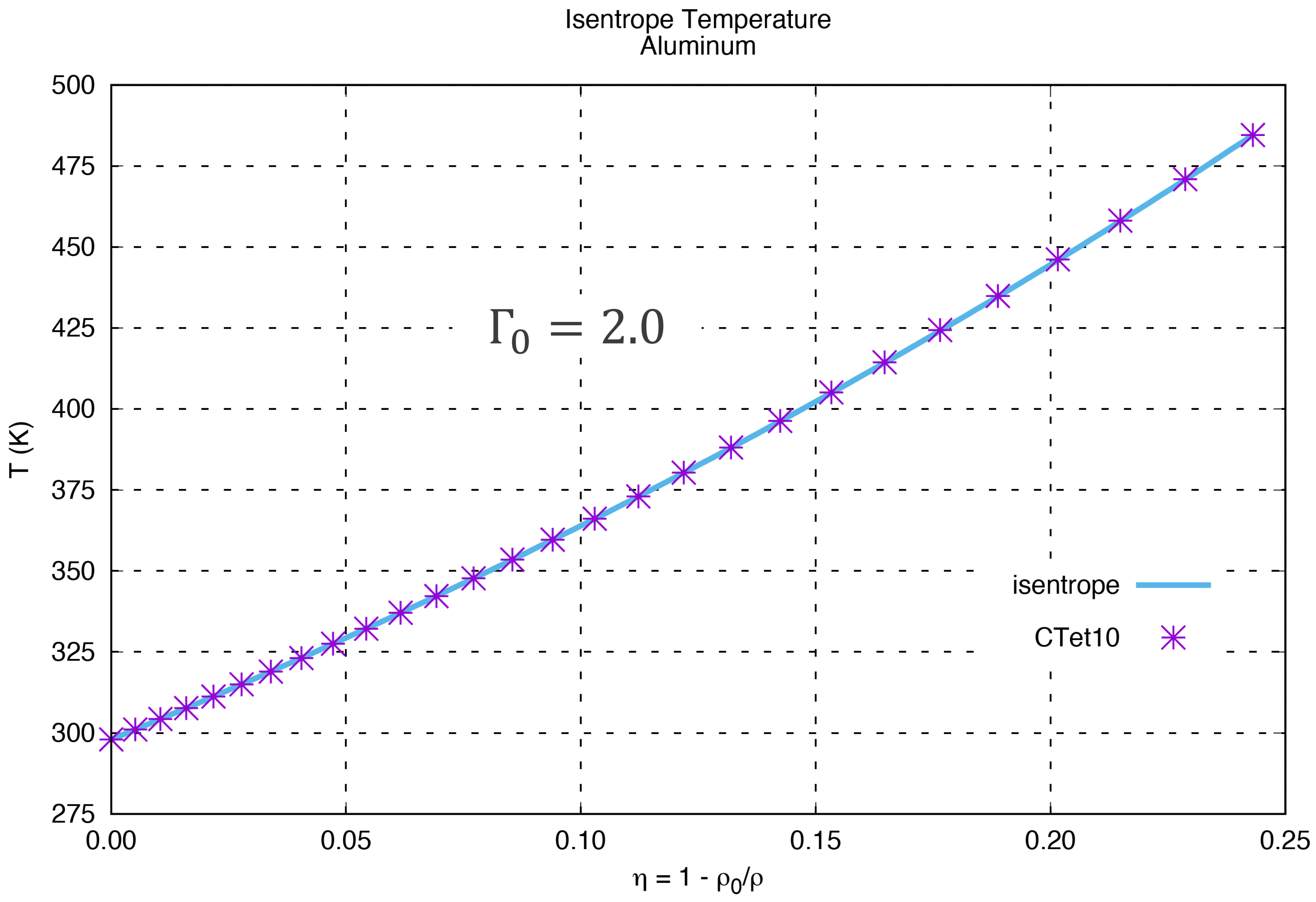
- Proper energy integration can be tested in isentropic compression

- We know the temperature on the isentrope for the following form of a linear Mie-Grüneisen model

$$P(\rho, E) = P_H(\rho) + \frac{\Gamma_0}{V_0}\left(E - E_H(\rho)\right)$$

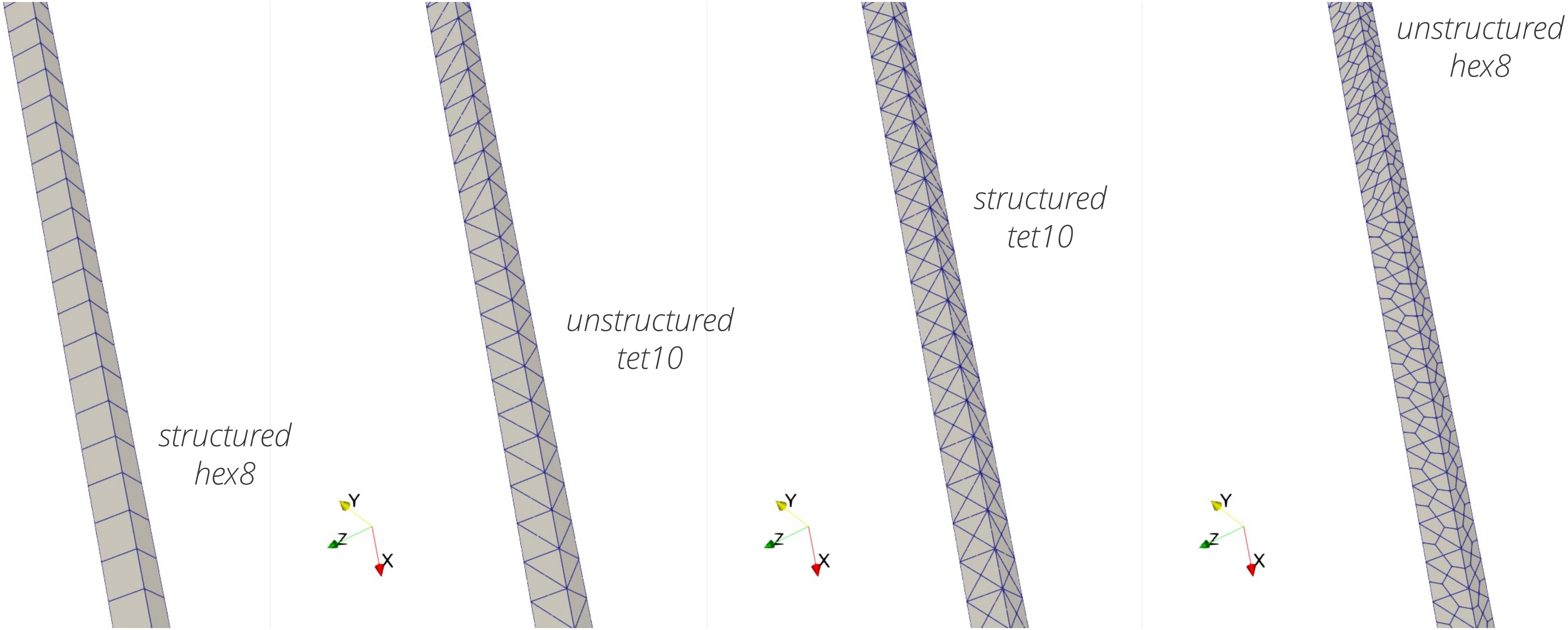
$$\frac{dT}{T} = -\frac{\Gamma_0}{V_0} dV \qquad \boxed{T = T_0 \exp(\Gamma_0 \eta)} \; ; \quad \eta = 1 - \frac{V}{V_0}$$

Isentropic Compression

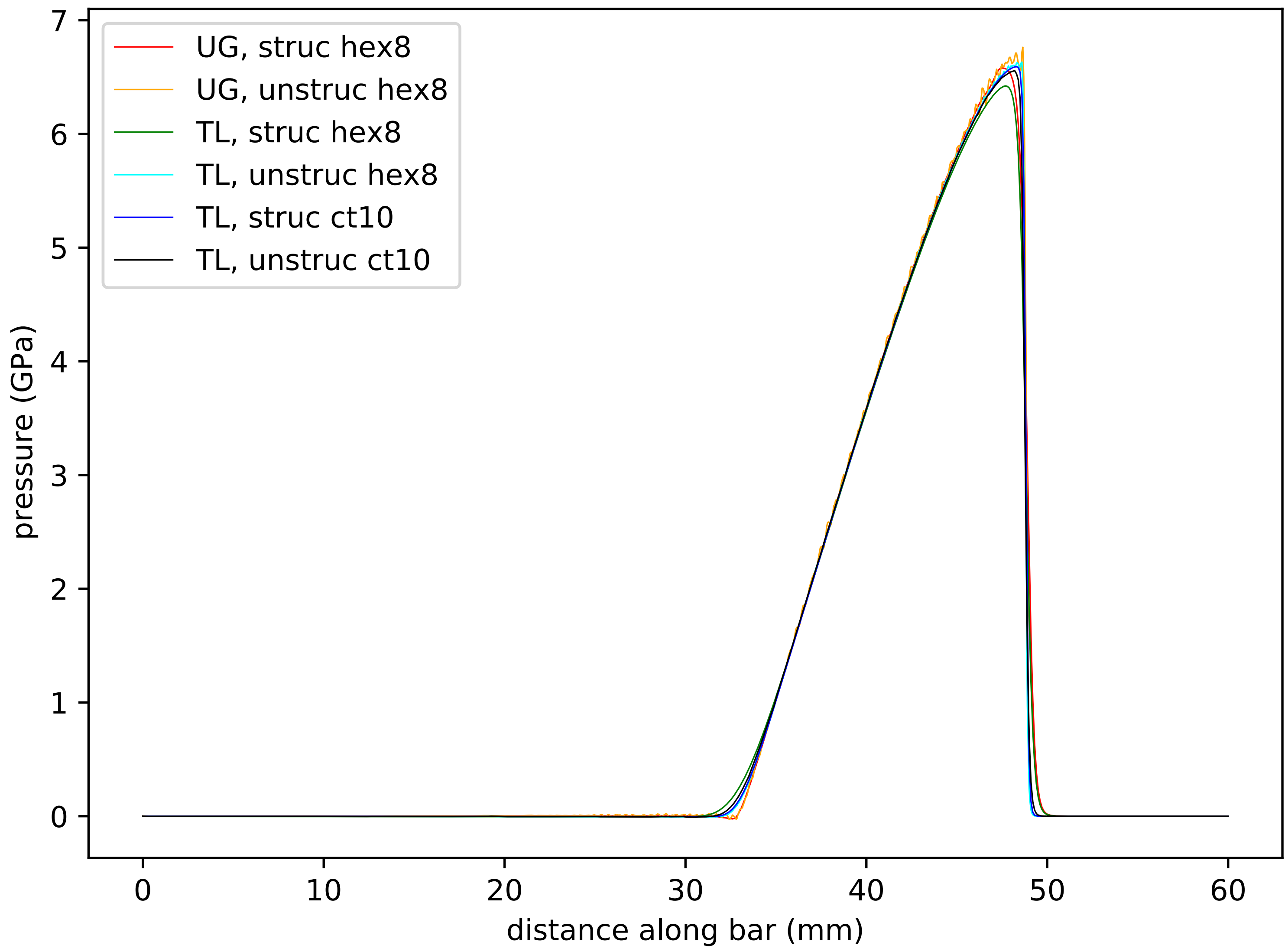


Examine structured & unstructured uniaxial bars for shock

Material: *aluminum* Model: *cth_ep* EOS: *cth_mgr* BC: *Apply 500 m/s sinusoidal loading to end*

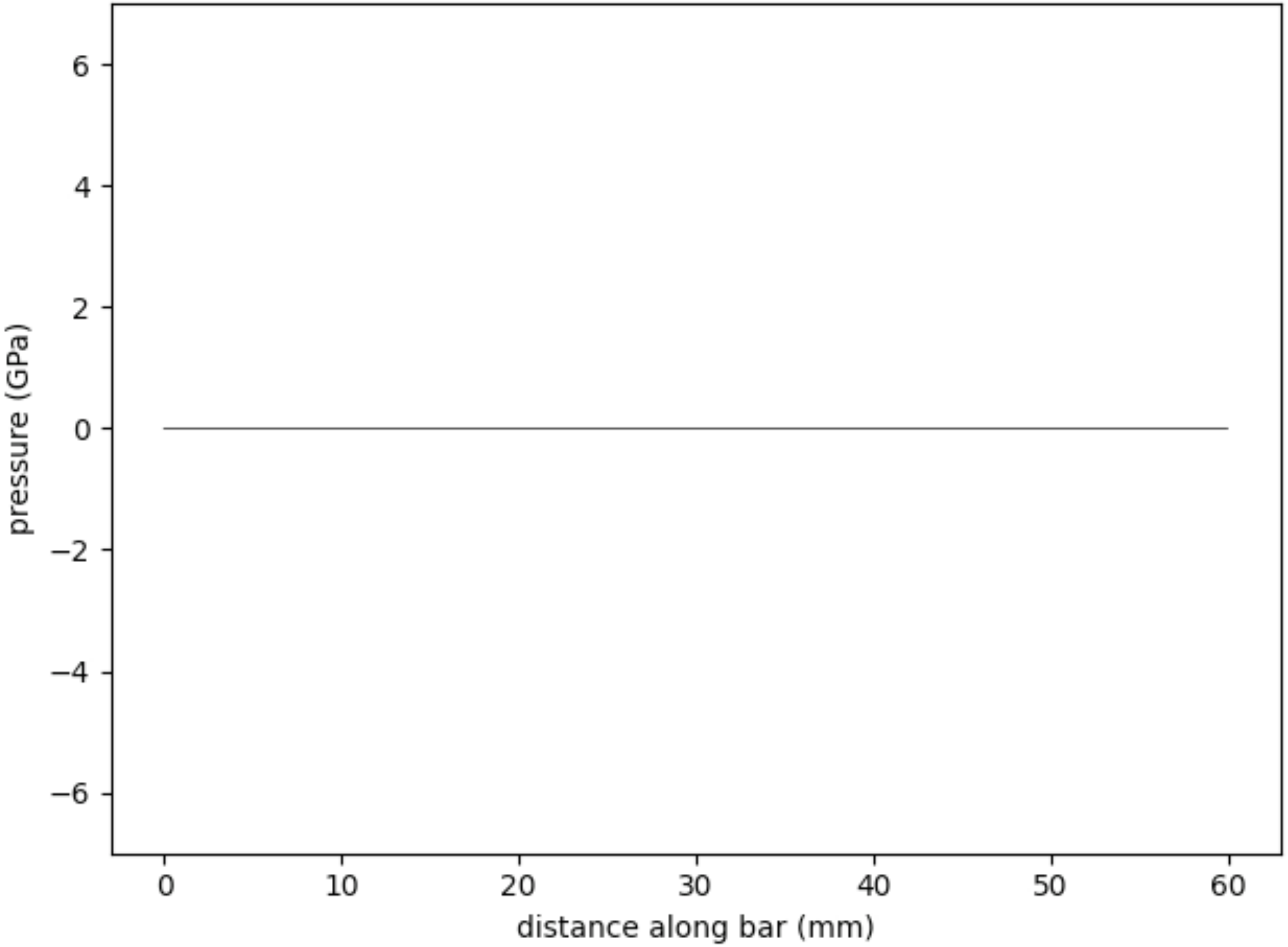


Examining the pressure for multiple element technologies



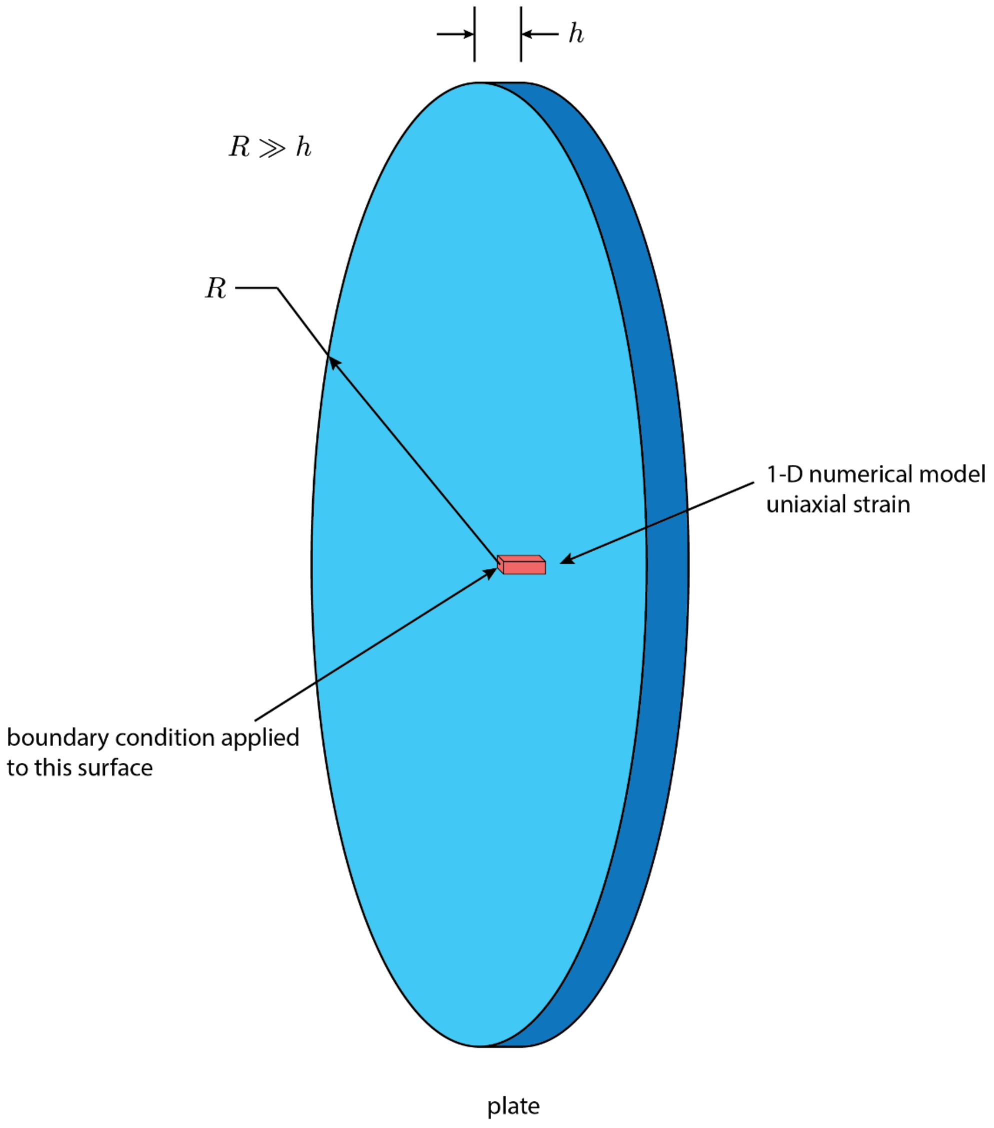
NOTE: Structured UG & CT solutions yield similar, smooth pressures. Unstructured CT solution smoother than unstructured UG/TL solutions.

Smoothness in the structured 10-node tetrahedral solution



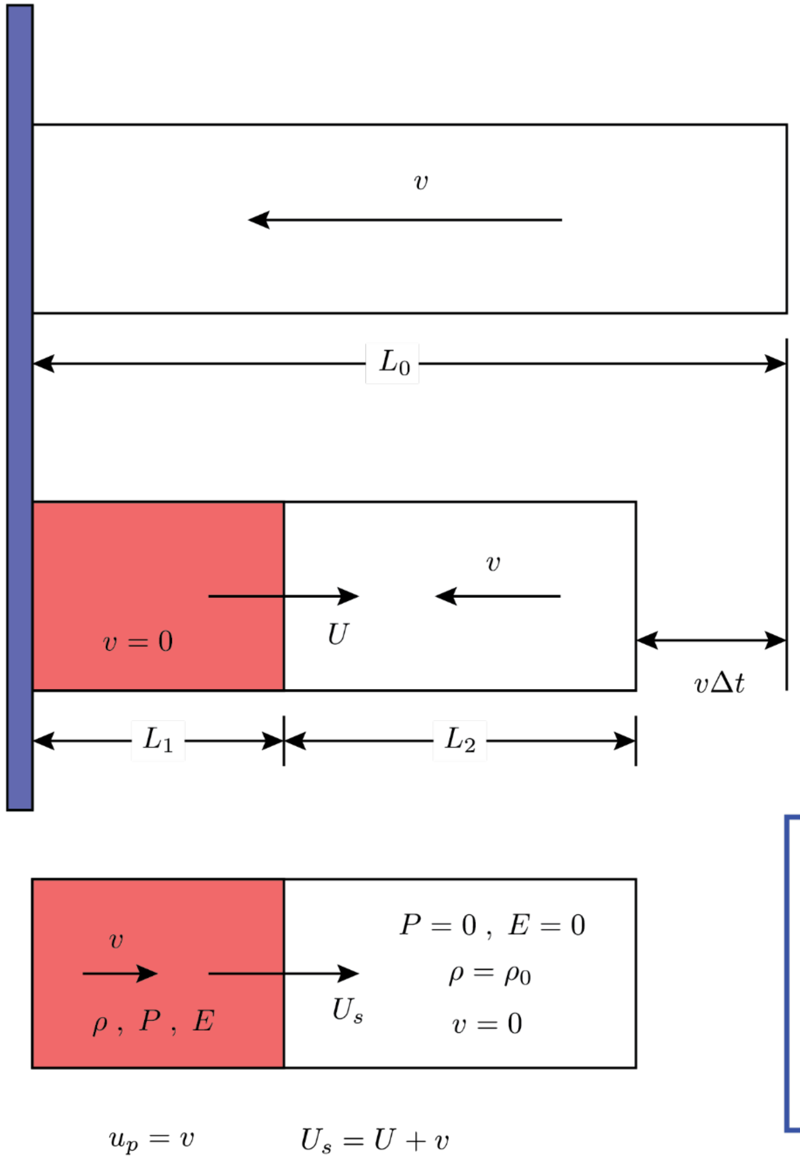
1D Wave Propagation Problems

- For 1D wave propagation in a thin plate, one can assume uniaxial strain
- To model uniaxial strain, the model is “cored” out of the center of the plate
- Wave propagation is modeled through the thickness of the plate



Reverse Ballistic – Rigid Wall

- Plate hits a rigid wall with some initial velocity
- Rigid wall provides infinite impedance
- Excellent problem for exploring the Hugoniot

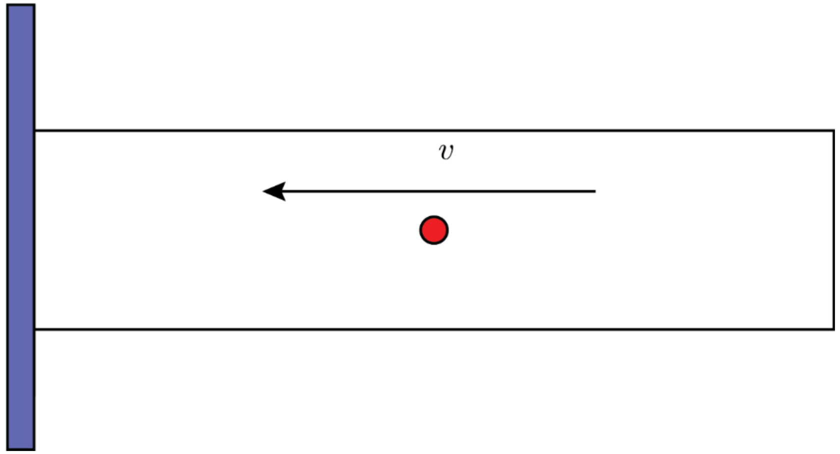


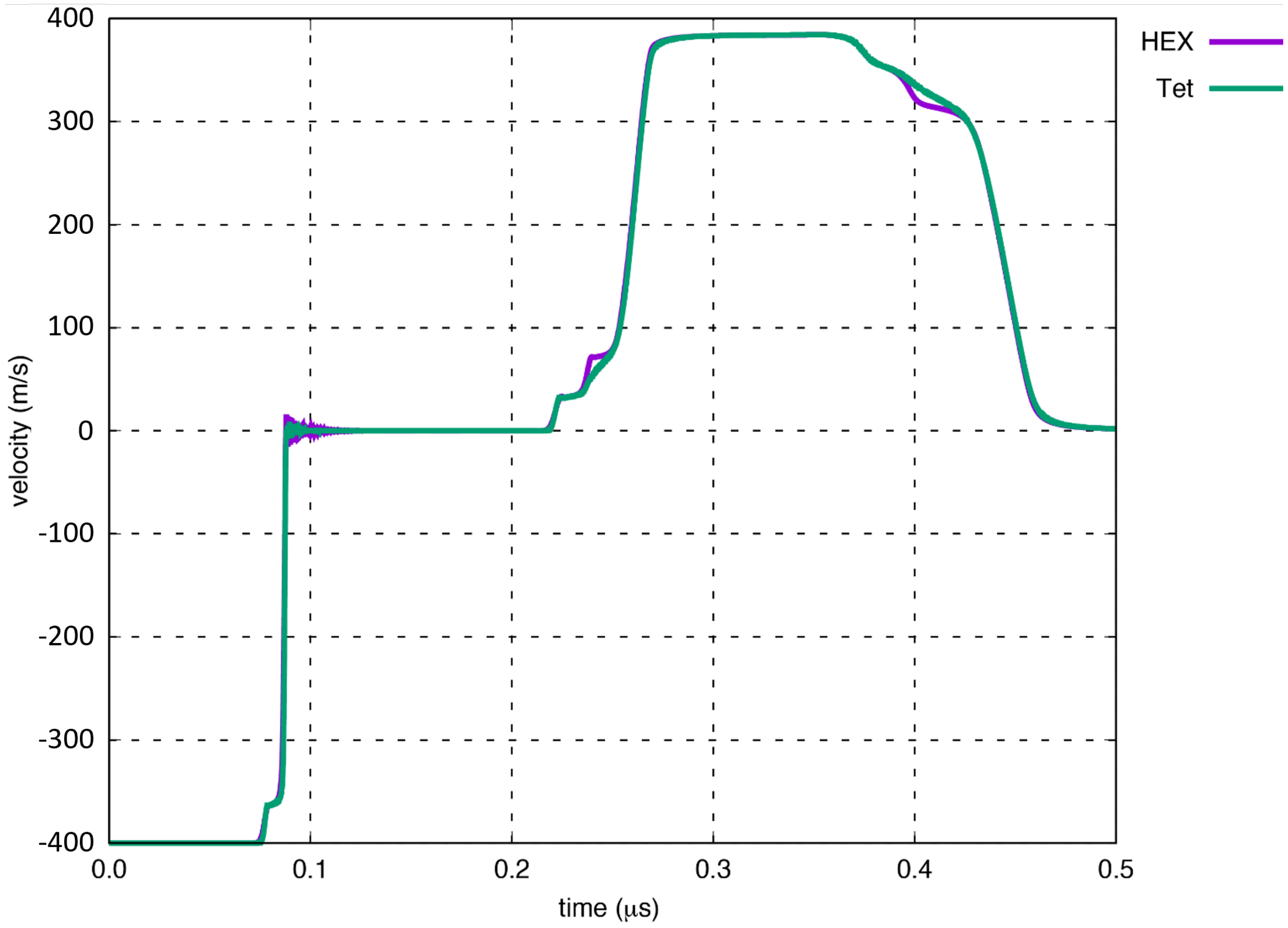
$$\rho_0 U_s = \rho \left(U_s - u_p\right)$$

$$P = \rho_0 U_s u_p$$

$$E = \frac{P}{2}\left(\frac{1}{\rho_0} - \frac{1}{\rho}\right)$$

Reverse Ballistic – Rigid Wall

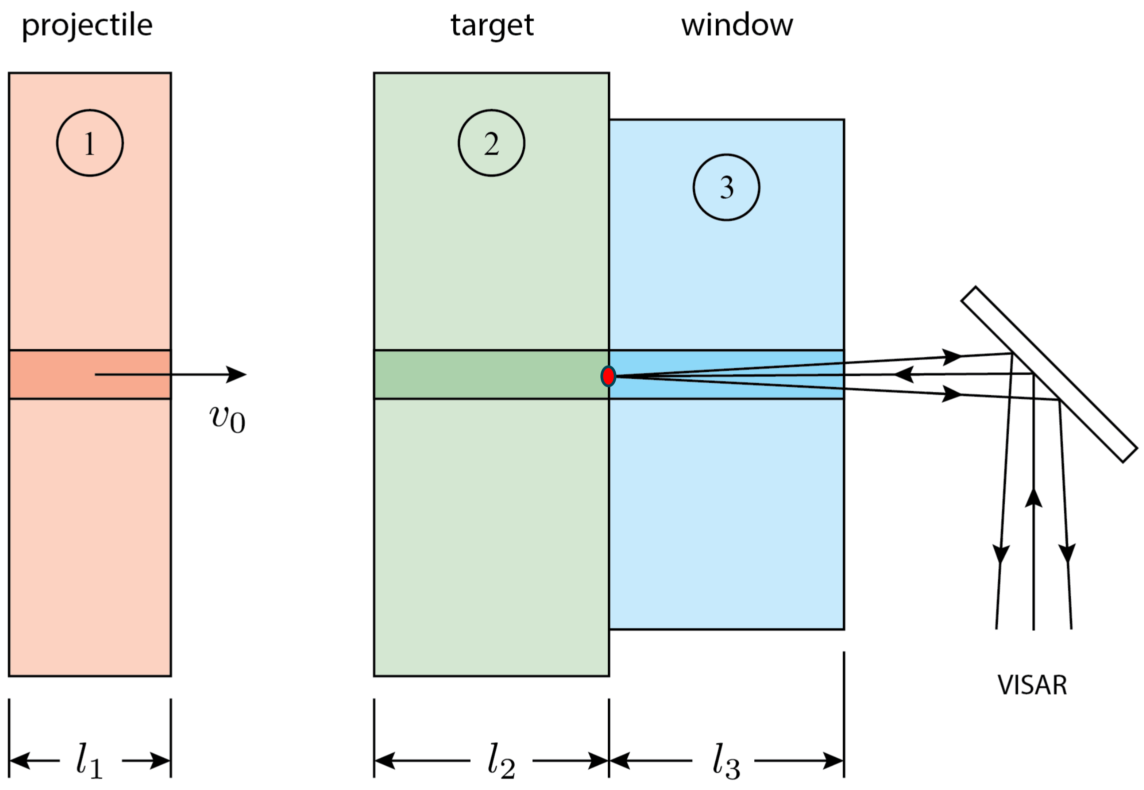




- Material is aluminum
- Velocity is computed at midpoint for both hexahedral and tetrahedral meshes

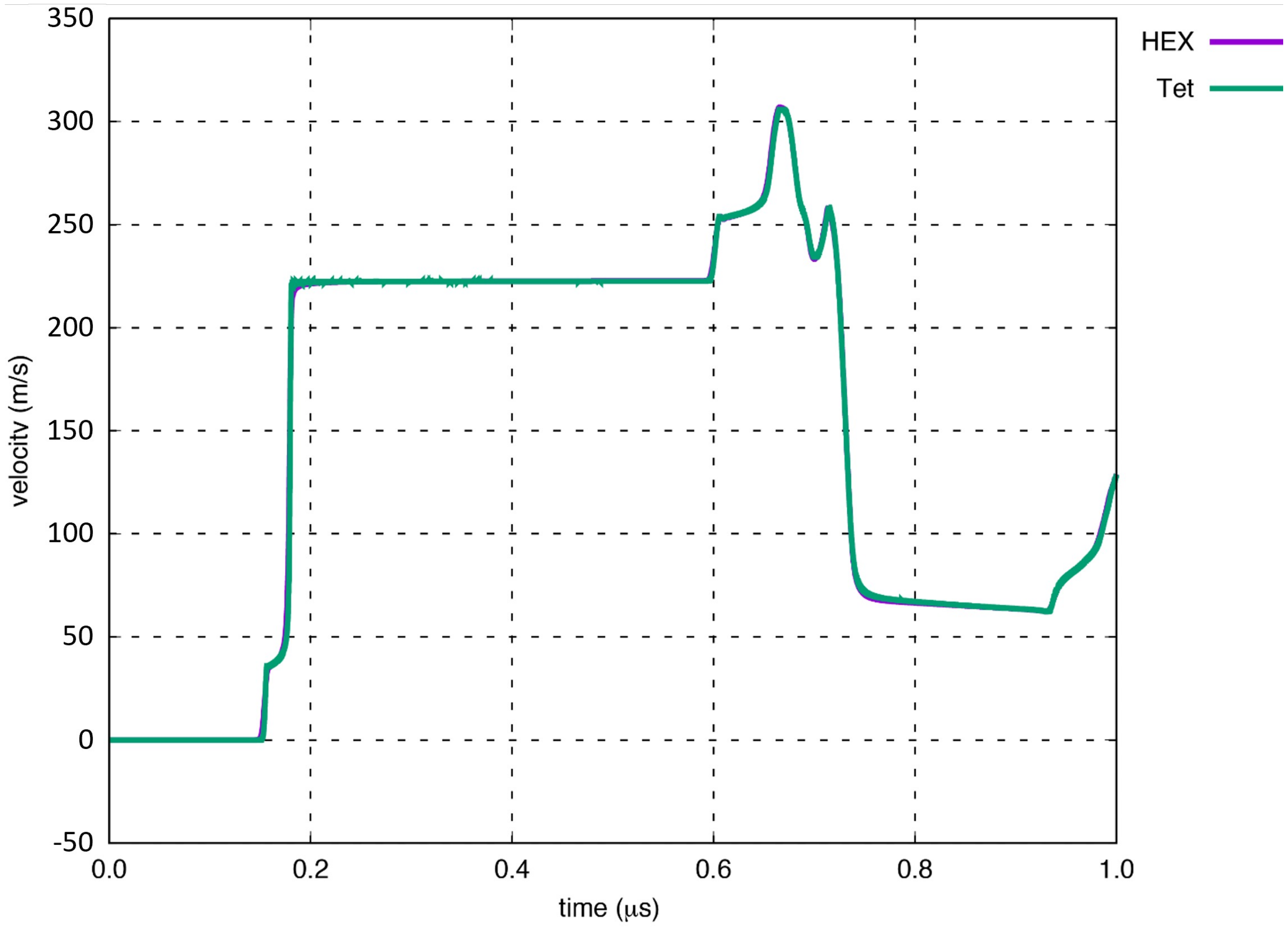
Forward Ballistic

- The forward ballistic model has a projectile and a target
- Can be modeled with or without a window
  - Without is usually a spall test
- Same problem set up can also be used for a reverse ballistic Hugoniot test
  - Projectile is material of interest
  - Target is a window
  - Measure interface velocity

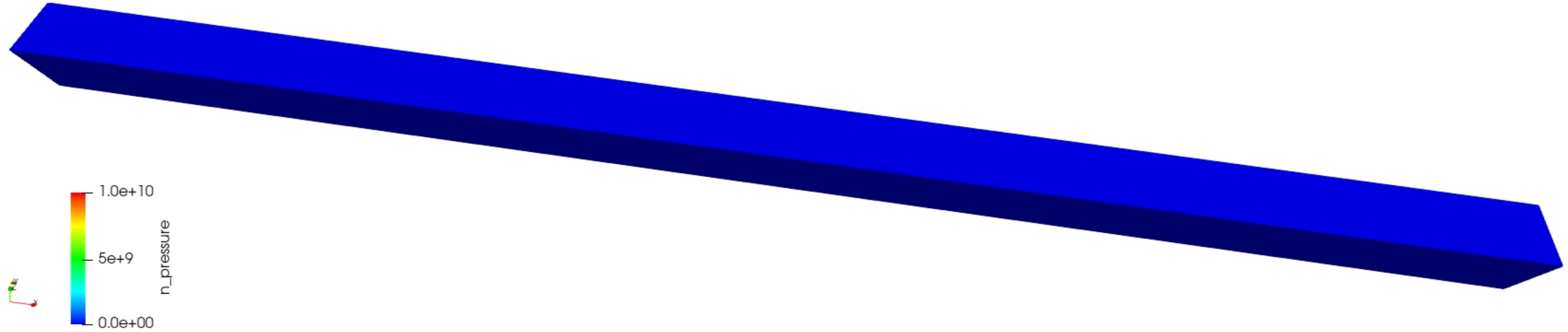


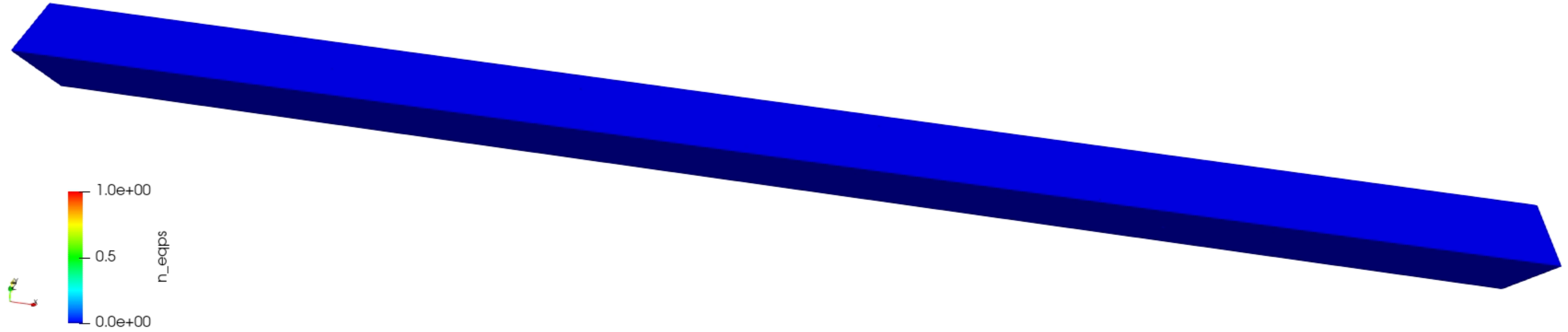
Forward Ballistic – Transmitted Wave

- Projectile and target materials are aluminum
- Window is LiF
- Simulated VISAR trace shows excellent agreement between simulations using hexahedral and tetrahedral elements



Mesoscale





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Conclusions and Future Work

- A robust capability to model shock in solids has been developed for a composite tetrahedral element.

- More work needs to be done verifying the capabilities.

- Need careful consideration of constant fields, specifically the energy.

- Materials with very low or no shear response manifest an additional low-energy mode that is not yet stabilized.

Acknowledgements

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Backup Slides

Material Properties

		aluminum	copper	LiF
density	ρ_0 (kg/m^3)	2703	8930	2703
sound speed	c_0 (m/s)	5220	3940	5220
slope of U_S-u_p curve	S_1 (−)	1.37	1.489	1.37
Grüneisen parameter	Γ_0 (−)	1.97	1.99	1.97
specific heat	C_V (J/kg · K)	922	393	922
Poisson's ratio	ν (−)	0.34	0.31	0.22
Johnson-Cook parameters	A (MPa)	265	90	
	B (MPa)	426	292	
	n (−)	0.34	0.31	
	C (−)	0.015	0.025	
	T_m (K)	800	1381	
	m (−)	1.0	1.09	