Investigating the Impact of Temporal and Directional Traffic Distribution on Crash Frequencies*

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ABSTRACT

Safety Performance Functions (SPFs) are mathematical models that establish relationships between the frequency of various crash types and site-specific characteristics, serving as essential tools for traffic safety analysis and roadway design. Traditional SPFs, however, often overlook the temporal fluctuations in traffic flow (such as peak-hour surges) and directional imbalances between opposing traffic streams. These traffic patterns can exacerbate congestion, disrupt driver behavior, and create unexpected conflict points, potentially leading to increased crash frequencies and more severe accidents. In light of this gap, this study aims to explore the potential of incorporating K-factors (representing peak-hour traffic proportions) and D-factors (reflecting the imbalance of directional traffic) into the development of SPFs to assess whether these factors can effectively represent the impact of temporal and spatial traffic distribution on roadway safety. Using crash data from Pennsylvania urban-suburban collector roadways, it is found that the D-factor plays a significant role in predicting the frequency of total crashes, fatal + injury crashes, and angle crashes, with positive coefficient signs indicating that higher directional imbalances correspond to increased crash risks. Similarly, the K-factor emerges as a critical predictor for fatal + injury crashes and rear-end crashes, with negative coefficients suggesting that a more pronounced traffic peak is associated with a reduction in expected crash frequencies. These results highlight the importance of accounting for uneven traffic distribution in both time and direction when developing SPFs, offering deeper insights into crash patterns and supporting more effective safety interventions and roadway designs.

INTRODUCTION

Traffic safety is a critical concern in transportation and is typically evaluated through crash frequency and severity. Crash frequency refers to the number of crashes occurring within a specific time period or location, while crash severity categorizes the crashes based on the level of damage or injury sustained, such as property damage only, minor injuries, major injuries, or fatalities (C. Wang et al., 2018).

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Safety Performance Functions (SPFs) are mathematical models that establish relationships between the frequency of various crash types and site-specific characteristics, serving as essential tools for traffic safety analysis and roadway design. Common types of SPFs include Poisson, Negative Binomial, and Zero-Inflated models, which account for the discrete nature of crash data and over-dispersion issues (Abdel-Aty & Radwan, 2000; Dong et al., 2014; Lord & Mannering, 2010; Mannering & Bhat, 2014; Poch & Mannering, 1996). Traditional SPFs, however, often overlook the temporal fluctuations in traffic flow (such as peak-hour surges) and directional imbalances between opposing traffic streams. These traffic patterns can exacerbate congestion, disrupt driver behavior, and create unexpected conflict points, potentially leading to increased crash frequencies and more severe accidents.

In light of this gap, this paper aims to explore the potential of incorporating K-factors (representing peak-hour traffic proportions) and D-factors (reflecting the imbalance of directional traffic) into the development of SPFs to assess whether these factors can effectively represent the impact of temporal and spatial traffic distribution on roadway safety. Using crash data from Pennsylvania urban-suburban collector roadways, it is found that the D-factor plays a significant role in predicting the frequency of total crashes, fatal + injury crashes, and angle crashes, with positive coefficient signs indicating that higher directional imbalances correspond to increased crash risks. Similarly, the K-factor emerges as a critical predictor for fatal + injury crashes and rear-end crashes, with negative coefficients suggesting that a more pronounced traffic peak is associated with a reduction in expected crash frequencies. These results highlight the importance of accounting for uneven traffic distribution in both time and direction when developing SPFs, offering deeper insights into crash patterns and supporting more effective safety interventions and roadway designs.

The remainder of this paper is organized as follows. First, we gave a brief literature on Safety Performance Functions (SPFs). Then, the methodology in this paper is discussed in detail. Next, we describe the data used for modeling the crash frequencies in the paper. This is followed by the results of SPFs. Finally, some discussion and concluding remarks are provided.

LITERATURE REVIEW

Safety Performance Functions (SPFs)

Safety Performance Functions (SPFs) are statistical models widely used in traffic safety analysis to predict the expected number of crashes at specific locations under given traffic and roadway conditions (Srinivasan et al., 2013). They serve as essential tools in identifying high-risk sites, evaluating countermeasures, and informing transportation planning.

The Highway Safety Manual (Highway Safety Manual 2010) outlines three different ways in which SPFs can be used by jurisdictions to make better safety decisions, including network screening, before-and-after studies, and economic evaluation of safety interventions. In network screening, SPFs identify sites with potential for safety improvement by comparing observed crash frequencies with expected values (Tegge et al., 2010). In before-and-after studies, SPFs facilitate the evaluation of safety treatments by estimating the counterfactual crash counts had no intervention occurred (Lyon et al., 2005). Finally, economic evaluations often integrate SPFs with cost-benefit analyses to prioritize investments in safety measures (Zarei et al., 2022).

Statistical Modeling of SPFs

SPFs are based on statistical models of relating crash frequencies to roadway and driver characteristics. Currently, the two most widely used statistical models are lognormal regression and loglinear regression models.

Lognormal regression models are used when the distribution of data is skewed (Gustavsson, 2015). This model proves to be especially effective when the data is inherently non-negative and the mean is relatively large. This type of distribution is common with intersections that have high volumes, such as at a signalized control (Bauer et al., 2000).

A log-linear model is a mathematical function where the logarithm of the outcome is expressed as a linear combination of the model's parameters. The two main types of loglinear models are the Poisson model and negative binomial model. The Poisson distribution predicts the probability of a specific number of events occurring over a given time, assuming events are independent and occur with a known probability (Katti & Rao, 1968). The negative binomial distribution extends this approach, addressing over-dispersion by allowing the variance to exceed the mean (Ross & Preece, 1985). Past studies have found that crashes are better modeled using the negative binomial distribution due to unaccounted variability in crash data (Park et al., 2010).

Variables in SPFs

Variables depicting roadway characteristics, driver behavior, climatic conditions, and traffic data are often incorporated into SPF development. For modeling purposes, these variables are categorized into two types: quantitative and categorical. Quantitative variables consist of discrete or continuous values that describe specific conditions, characteristics, or parameters. In previous studies, common quantitative variables used to develop SPFs for roadway segments include (E. Donnell et al., 2014, 2016; Gooch et al., 2018; Montella & Imbriani, 2015; Srinivasan et al., 2011, 2013; Tegge et al., 2010):

- Average Annual Daily Traffic (AADT)
- Segment length
- Grade
- Lane width
- Median width
- Number of lanes
- Percent heavy vehicles
- Radius of curvature
- Number of curves
- Number of intersections
- Access point density
- Segment length
- Shoulder width
- Speed (85th percentile or posted speed)
- …

The second type of variable used in developing SPFs is categorical variables, which are non-numerical and provide a descriptive representation of specific conditions or scenarios. In modeling, these variables are typically encoded as binary dummy variables to represent different possible cases or states. Common categorical variables incorporated into SPFs include (E. Donnell et al., 2014, 2016; Gooch et al., 2018; Montella & Imbriani, 2015; Srinivasan et al., 2011, 2013; Tegge et al., 2010):

- Roadside hazard rating
- Functionality classification of the roadway
- Auxiliary lanes (passing lane/climbing lane/other auxiliary lane)
- Terrain
- Median type
- Area type (rural/urban)
- Shoulder type
- …

However, no variable in traditional SPFs explicitly accounts for temporal fluctuations in traffic flow, such as peak-hour surges, or directional imbalances between opposing traffic streams. These dynamic traffic patterns can intensify congestion, influence driver behavior, and create unexpected conflict points, increasing the likelihood of crashes and the severity of accidents.

METHODOLOGY

Model Specification

Variable for temporal traffic unevenness

To evaluate the impact of temporal traffic volume unevenness on crash frequencies, K-factor is employed as a measure of the variability in traffic distribution over time. Federal Highway Administration defines K-factor as the proportion of Annual Average Daily Traffic (AADT) occurring during the Design Hour, i.e. the hour in which the 30th highest hourly traffic flow of the year takes place (Traffic Monitoring Guide 2013). It is used in design engineering for determining the peak loading on a roadway design that might have similar traffic volumes. The calculation for the K factor is given by the formula:

$$K - factor = \frac{DHV}{AADT} \tag{1}$$

where DHV is the design hourly volume (veh/hr) and $AADT$ is the annual average daily traffic (veh/day).

Variable for directional traffic imbalance

To analyze the impact of directional traffic volume imbalances on crash frequencies, the D-factor is utilized as a key metric in this paper. D-factor is also referred to as the directional distribution factor. It is the proportion of traffic traveling in the peak direction during a selected hour, usually expressed as a percentage (Traffic Monitoring Guide 2013). It is calculated by dividing the higher directional volume occurring in the 30th highest hour by the total roadway volume for that hour. The calculation for the D factor is given by the formula:

$$D - factor = \frac{DDHV}{DHV} \tag{2}$$

where $DDHV$ is higher directional design-hour volume (veh/hr).

Statistical model

Negative binomial regression is used to develop the roadway segment SPFs in this paper to be consistent with the models developed in HSM (Highway Safety Manual 2010) . If K-factor and D-factor are found to have significant effects on crash frequencies, the final SPF will be in the form of:

$$\lambda_i = e^{\beta_0} \times L^{\beta_1} \times AADT^{\beta_2} \times e^{\beta_3 K + \beta_4 D + \beta_5 X_5 + \beta_6 X_6 + \ldots + \beta_n X_n} \tag{3}$$

Where:
λ_i - expected number of crashes on roadway segment i;
β_0 - regression coefficient for constant;
L - roadway segment length (miles);
$AADT$ - average annual daily traffic (veh/day);
K - K-factor;
D - D-factor;
β_1 - regression coefficient for segment length;
β_2 - regression coefficient for AADT;
β_3 - regression coefficient for K-factor;
β_4 - regression coefficient for D-factor;
β_5,...β_n - regression coefficients for explanatory variables and
X_5,...X_n - other explanatory variables.

Total crash frequencies, the sum of fatal and injury crash frequencies, rear-end crash frequencies, and angle crash frequencies are considered as the dependent variable (λ_i) respectively in the model above. These categories allow for a comprehensive assessment of how temporal traffic unevenness and directional traffic imbalance influence different levels of crash severity and crash types.

Model Assessment

Statistical Significance of Variables

To determine whether the K-factor and D-factor are statistically significant, a z-test is performed, which evaluates the difference between an observed value and the hypothesized value in units of standard error. This involves calculating the p-value, which represents the probability of observing an effect of the same magnitude or more extreme under the assumption that the null hypothesis is true. The null hypothesis is rejected if the p-value is less than the predetermined significance level, α. An α level of 0.05 is used in this paper.

Model Comparison

In this paper, the Akaike information criterion (AIC) is used as the criterion for model selection among the models; the model with the lowest AIC is preferred. AIC is an estimator of the relative quality of statistical models for a given set of data (Akaike, 2011). Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models. Thus, AIC provides a means for model selection. In estimating the amount of information lost by a model, AIC deals with the trade-off between the goodness of fitness of the model and the simplicity of the model.

DATA

Data Description

This paper leverages two key data sources to estimate Safety Performance Functions (SPFs) for roadway segments:

- *Pennsylvania Roadway Inventory and Crash Database:* This dataset, utilized in (E. T. Donnell et al., 2019), is derived from PennDOT's Roadway Management System and supplemented by the PennDOT Online VideoLog System. It also integrates Google Earth crash data files, providing comprehensive details about crash events, drivers, and vehicle occupants for all reported incidents on Pennsylvania's two-lane undivided urban-suburban collectors.
- *Pennsylvania Traffic Counts:* These datasets, provided by PennDOT, include key traffic metrics such as Average Annual Daily Traffic (AADT), K-factor, and D-factor. Data from the years 2016, 2017, and 2018 are utilized in this study.

The two datasets are merged using shared identifiers—state route ID and segment ID—ensuring alignment between roadway inventory, crash data, and traffic count metrics. This integration facilitates a comprehensive analysis for SPF estimation.

Statistical Summary

A total of 23,415 unique roadway segments were available in the two-lane undivided urban - suburban collector segment analysis file. Roadway elements from the data sources for the roadway segment analysis include:

- Linear reference information (county, route, and segment)
- Segment length
- Average annual daily traffic (vehicles/day)
- K-factor
- D-factor
- Paved roadway width (including all travel lanes)
- Number of travel lanes in both directions
- Posted speed limit
- Divisor type
- Left- and right-shoulder type
- Left- and right-shoulder paved width (feet)
- Left- and right-shoulder total width
- Presence of bicycle lanes
- Presence of on-street parking
- Presence of curb/sidewalk combinations
- Driveway density
- Presence of auxiliary lanes (e.g., turn lanes, bus lanes, etc.)

A statistical summary of the combined dataset is shown in **Table 1**:

Table 1. Summary of Variables

Variable	Number of Observations = 23415			
	Mean	Std. Dev.	Min	Max
Total Crashes	1.021	1.602	0	22
Fatal Plus Injury Crashes	0.460	0.945	0	20
Rear-end Crashes	0.159	0.499	0	12
Angle Crashes	0.342	0.890	0	14
AADT (veh/day)	3319.245	2814.396	0	20072
Segment Length (mile)	0.437	0.168	0.001	0.820
K-factor (%)	9.358	3.445	0	21
D-factor (%)	51.761	19.418	0	100
Degree of Curvature Per Mile (°/mile)	20.865	54.407	0	1379.865
Access Points and Intersections Per Mile	33	22	0	377
		Percent of Sites with Characteristic		Percent of Sites without Characteristic
Presence of on-street parking		9.660%		90.340%
Presence of curb		20.730%		79.270%
Posted speed limit 45 mph or above		22.960%		77.040%
Segment is less than 0.1 miles long		4.100%		95.900%

RESULTS

Total Crash SPF

First, **Table 2** presents the estimation results for the Total Crash Safety Performance Function (SPF). The model includes various explanatory variables, each accompanied by its coefficient, standard error, and statistical significance level , represented by the p-value (P > z). Among these, the D-factor, representing the directional distribution of traffic, demonstrates a statistically significant positive association with total crashes at the α = 0.05 significance level. This finding indicates that greater directional traffic imbalance will lead to an increase in total crash frequencies, suggesting that implementation of measures to balance traffic flows between directions may improve roadway safety. Potential countermeasures include dynamic lane allocation (Baskar et al., 2008; Hausknecht et al., 2011), where lanes are reassigned based on real-time traffic conditions, or the use of variable message signs (VMS) or variable speed limits (VSL) to manage traffic flow and redirect vehicles during peak periods (Almallah et al., 2021; Chatterjee & Mcdonald, 2004; Xu et al., 2024).

In contrast, the K-factor, which reflects temporal traffic unevenness, is not statistically significant. This result implies that variations in hourly traffic volume distribution do not exhibit a strong relationship with total crash frequencies in this model. The contrasting roles of the D-factor and K-factor reflects the importance of directional traffic patterns over temporal variations in influencing total crash frequency.

Table 2. Total Crash SPF

Variable Name	Coef.	Std. Err.	P>z
Natural logarithm of AADT	0.7226	0.0134	<0.001
Natural logarithm of segment length (mile)	0.6416	0.0253	<0.001
Degree of curvature per mile	-0.0009	0.0002	<0.001
D-factor	**0.0048**	**0.0013**	**<0.001**
Presence of on-street parking (1 if present; 0 otherwise)	0.2821	0.0343	<0.001
Presence of curb (1 if present; 0 otherwise)	0.2091	0.0266	<0.001
Posted speed limit 45 mph or above (1 if present; 0 otherwise)	-0.1109	0.0227	<0.001
Segment is less than 0.1 miles long (1 if true; 0 otherwise)	0.3266	0.0908	<0.001
Constant	-5.6548	0.1358	<0.001
Pseudo R^2 = 0.0772 Log-likelihood at convergence = -26935.876 AIC = 53892			

Sum of Fatal and Injury Crash Frequency SPF

Second, **Table 3** presents the fitted SPF for estimating the sum of fatal and injury crash frequency. Consistent with the findings in **Table 2**, D-factor demonstrates a statistically significant positive association with fatal and injury crashes at the α = 0.05 significance level. This result indicates that greater directional traffic imbalance contributes to an increase in the sum of fatal and injury crash frequencies, reinforcing the importance of addressing heavily directional traffic patterns to enhance safety. By comparing **Table 2** and **Table 3**, we can see an increase in the coefficient for D-factor in the SPF for fatal and injury crashes. This implies that greater directional traffic imbalance may have a stronger impact on severe crashes.

Conversely, K-factor exhibits a statistically significant negative association with fatal and injury crashes. This suggests that higher temporal traffic imbalances may reduce the likelihood of fatal and injury crashes. A possible reason is that temporal peaks in traffic may encourage lower speeds during high-volume periods, reducing crash severity.

Table 3. Sum of Fatal and Injury Crash SPF

Variable Name	Coef.	Std. Err.	P>z
Natural logarithm of AADT	0.7630	0.0186	<0.001
Natural logarithm of segment length (mile)	0.6787	0.0347	<0.001

Degree of curvature per mile	-0.0013	0.0003	<0.001
K-factor	**-0.0310**	**0.0118**	**0.008**
D-factor	**0.0076**	**0.0017**	**<0.001**
Presence of on-street parking (1 if present; 0 otherwise)	0.4102	0.0434	<0.001
Presence of curb (1 if present; 0 otherwise)	0.2777	0.0344	<0.001
Segment is less than 0.1 miles long (1 if true; 0 otherwise)	0.4102	0.1235	0.001
Constant	-6.6614	0.2280	<0.001
Pseudo R^2 = 0.0802 Log-likelihood at convergence = -17409.8330 AIC = 34840			

Rear-end Crash Frequency SPF

Next, **Table 4** presents the fitted SPF for estimating rear-end crash frequencies. In this model, the K-factor is statistically significant, while the D-factor is not. This result is intuitive, as a larger K-factor indicates more congestion during peak hours, which increases the risk of rear-end crashes. In contrast, uneven directional traffic distribution, captured by the D-factor, does not strongly influence rear-end crash occurrences, as it is less relevant to this crash type.

Table 4. Rear-end Crash SPF

Variable Name	Coef.	Std. Err.	P>z
Natural logarithm of AADT	1.4234	0.0341	<0.001
Natural logarithm of segment length (mile)	0.6008	0.0462	<0.001
Degree of curvature per mile	-0.0022	0.0006	<0.001
K-factor	**-0.0435**	**0.0190**	**0.022**
Access_density	0.0028	0.0010	0.004
Presence of on-street parking (1 if present; 0 otherwise)	0.1677	0.0665	0.012
Presence of curb (1 if present; 0 otherwise)	0.2922	0.0509	<0.001
Constant	-12.8936	0.3722	<0.001
Pseudo R2 = 0.1398 Log-likelihood at convergence = -8182.6285 AIC = 16383			

Angle Crash Frequency SPF

Finally, the fitted SPF for estimating angle crash frequency is given in **Table 5**. In this model, the D-factor is statistically significant, while the K-factor is not included. This result also aligns intuitively with the nature of angle crashes, as a larger D-factor, reflecting uneven directional traffic distribution, is more likely to contribute to angle crashes, which often occur at intersections or locations with conflicting traffic flows. In contrast, temporal traffic unevenness, captured by the K-factor, is less relevant to the occurrence of angle crashes.

Table 5. Angle Crash SPF

Variable Name	Coef.	Std. Err.	P>z
Natural logarithm of AADT	0.8871	0.0250	<0.001
Natural logarithm of segment length (mile)	0.5103	0.0346	<0.001
Degree of curvature per mile	-0.0027	0.0005	<0.001
D-factor	**0.0122**	**0.0022**	**<0.001**
Access_density	0.0053	0.0008	<0.001
Presence of on-street parking (1 if present; 0 otherwise)	0.4087	0.0570	<0.001
Presence of curb (1 if present; 0 otherwise)	0.4452	0.0453	<0.001
Posted speed limit 45 mph or above (1 if present; 0 otherwise)	-0.0988	0.0415	0.017
Constant	-8.9336	0.2532	<0.001
Pseudo R2 = 0.0781 Log-likelihood at convergence = -14003.331 AIC = 28027			

CONCLUSIONS

This study highlights the significance of considering temporal fluctuations in traffic flow (such as peak-hour surges) and directional imbalances between opposing traffic streams in the development of Safety Performance Functions (SPFs). Using crash data from Pennsylvania urban-suburban collector roadways, we demonstrate that the D-factor, representing directional traffic imbalance, is a key predictor for predicting total crashes, fatal and injury crashes, and angle crashes. Its positive association with crash frequency indicates the importance of addressing directional traffic imbalances to mitigate safety risks. Similarly, the K-factor, reflecting temporal traffic fluctuations, is identified as a critical factor for predicting fatal and injury crashes and rear-end crashes. Interestingly, its negative association with crash frequency suggests that higher peak-hour traffic proportions may reduce crashes, potentially due to speed reduction during congestion. By integrating K-factor and D-factor, SPFs can more effectively capture the dynamic interactions between traffic patterns and traffic safety, offering insights that traditional models might overlook. This enhanced understanding supports the development of targeted safety interventions, such as dynamic traffic management strategies and roadway designs tailored to address temporal and directional imbalances.

Due to data availability, this study focuses exclusively on developing SPFs for urban-suburban collector roadways. Future research could expand on this work by exploring the role of these factors across diverse roadway types and regions (Xu & Gayah, 2023). In addition, future work should also consider other variables that capture the uneven distribution of traffic in time and space. Future studies could also investigate the impact of temporal and directional traffic distribution at intersections, incorporating factors such as turning ratios (Z. Wang et al., 2023; Xu et al., 2020) and signal timing (Yu et al., 2022) into SPFs. These would further refine the application of SPFs in traffic safety analysis and contribute to safer, more efficient roadway systems.

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