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An Investigation of Bose-Einstein Correlations in Muon-Nucleon Interactions at 490 GeV

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Abstract

An investigation of Bose-Einstein correlations amongst like-charged pions produced in muon-nucleon interactions at 490 GeV is presented. On top of a broader enhancement, a steep increase in the correlations at small four-momentum differences between the two pions is observed which may be explained by the contribution from decays of resonances (ρ -mesons). A two-dimensional analysis discriminates between two different parametrizations of the Bose-Einstein effect, strongly favoring the Lorentz-invariant parametrization over a parametrization based on a Gaussian source distribution in space and time.

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Bose-Einstein (BE) correlations were observed in particle physics for the first time by Goldhaber et al. [1]; they measured an enhanced probability for the production of pions of the same charge and of similar momenta in proton-antiproton annihilation. The effect arises from the symmetrization of the two-particle wave function, required by Bose-Einstein quantum statistics for identical bosons. It was pointed out by Kopylov, Podgoretskii and Cocconi [2] that this phenomenon can be used as a tool to measure the space-time structure of particle production in collisions of elementary particles. Since the first observation, BE correlations have been extensively studied in various types of reactions (hadron-hadron, e^+e^- , lepton-nucleon, hadron-nucleus, and nucleus-nucleus collisions). Reviews of more recent theoretical and experimental work can be found in [3, 4, 5]. In this paper new results on BE correlations in muon-nucleon interactions at 490 GeV from Fermilab experiment E665 are presented.

Experimentally the two-particle BE correlation is defined in terms of a ratio R which is related to the theoretical model describing the space-time structure of particle production by the following equation:

$$R(p_1, p_2) = \frac{\sigma(p_1, p_2)}{\sigma_0(p_1, p_2)} = 1 + \lambda |\tilde{\rho}(\mathbf{q}, q_0)|^2 . \quad (1)$$

Here $\sigma(p_1, p_2)$ is the measured two-particle density of the pairs of like-charged pions and $\sigma_0(p_1, p_2)$ denotes a “reference density” corresponding to $\sigma(p_1, p_2)$ in the absence of BE correlations. The following notation for the kinematical variables is used: $p = (\mathbf{p}, E)$ is the four-momentum of a particle, with \mathbf{p} and E being its three-momentum and energy; $q = (\mathbf{q}, q_0)$ with $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$, $q_0 = E_1 - E_2$ is the four-momentum difference of the two particles. Another variable which will be used in this analysis is the Lorentz-invariant quantity Q [1]

$$Q = \sqrt{-(p_1 - p_2)^2} = \sqrt{\mathbf{q}^2 - q_0^2} = \sqrt{M_{\pi\pi}^2 - 4m_\pi^2} , \quad (2)$$

where $M_{\pi\pi}$ and m_π are the effective $\pi\pi$ mass and pion mass respectively. The right hand side of eq. (1) describes the theoretical expectation, where $\tilde{\rho}(\mathbf{q}, q_0)$ is the Fourier transform of the space-time distribution $\rho(\mathbf{r}, t)$ of the particle emission source and λ ($0 \leq \lambda \leq 1$) an empirical parameter, describing the strength of the effect.

Experiment E665 [6] was performed with positive muons of energy 490 GeV at the Fermilab Tevatron. The muons impinged on a one meter long liquid hydrogen or deuterium target. The present analysis includes only fast charged particles which pass through the forward spectrometer magnet and which are associated with the primary vertex. All particle tracks are required to satisfy the basic selection criteria (e.g. a minimum χ^2 -probability of vertex and track fits; a maximum distance to the muon interaction vertex). The average fractional momentum resolution is $\sim 10^{-4}p/\text{GeV}$; tracks with a fractional error $> 2\%$ are excluded from the analysis. The reconstruction program can falsely construct two tracks from one, using nearby extra hits. Such double-counting can artificially enhance the BE effect at small momentum difference. Therefore a special cut, based on the Monte Carlo simulation of the experiment (see below), is applied on the number of shared hits to eliminate such pairs.

All charged particles are regarded as pions. A small contamination of protons, kaons and electrons or wrongly associated tracks from secondary interactions reduces the strength of the effect (λ), but does not affect significantly the line-shape of the BE correlation [7]. No attempt is made to account for this effect on the measured value of λ . Practically all (98 %) of the reconstructed particles are going forward in the virtual-photon nucleon center-of-momentum system (hadronic cms). Only events with at least two charged hadrons are used in the analysis. The hydrogen and deuterium data samples were combined since the analyses described below, when performed with the hydrogen and deuterium samples separately, gave consistent results. The final sample consists of ~ 67000 events, with a mean multiplicity of charged hadrons of 2.9, an average effective mass W of the hadronic system of ~ 19 GeV and an average four-momentum transfer squared of ~ 5 GeV². The BE correlations are studied in the combined sample of $(++)$ and $(--)$ pairs.

The calculation of R , eq. (1), relies on the construction of a properly defined reference sample (σ_0). As the effect is very sensitive to this choice two different types of reference samples were considered: a) pairs of particles with unlike charge from the same event; b) an artificial sample of “mixed” pairs, combining like-charged particles from different events. Tracks are combined only from similar events, requiring the maximum difference of multiplicity and W to be 1 and 1 GeV, respectively. In order to reduce the statistical fluctuations a sample of mixed pairs ten times larger than the data sample of like-charged pairs was generated.

Both reference samples have some shortcomings, e.g. the effect of resonances in the first case, and the violation of the kinematical constraints in the second case. The bias due to these effects can be reduced by applying a correction based on the Monte-Carlo (MC) simulation of the experiment, assuming that the BE effect is independent of other types of correlations. The MC events were generated according to the Lund model [8] without BE correlations, were then modified according to a detailed simulation of the apparatus, and were passed through the same analysis chain as the data. The corrected correlation function is defined by dividing the ratio obtained from the data by the equivalent ratio from the MC simulation (as a function of the considered variable): $R(\text{corr}) = R(\text{data})/R(\text{MC})$. This double-ratio implicitly includes a correction for the imperfection of the pattern recognition procedure, e.g. for the reduced reconstruction efficiency for pairs of close-by like-charged tracks. In the case of the unlike-charge reference sample the double-ratio mitigates the effect of resonances. The effect of final-state Coulomb interactions [9] is negligible in the range covered by the data, except for the first data point ($Q \leq 40$ MeV). Because of several other experimental uncertainties, this point is not used in the analysis; therefore no Coulomb correction is applied. Unless stated otherwise, the sample of “mixed” pairs will be used as reference.

We start with a “one-dimensional” analysis, studying the BE correlation as a function of the Lorentz-invariant variable Q . The MC corrected correlation function $R(Q)$ is shown in Fig. 1a, which clearly exhibits the BE enhancement ($R > 1$) at small Q ($Q < 1$ GeV). First the data were fitted by the commonly used Gaussian expression [1]

$$R(Q) = 1 + \lambda \exp(-r^2 Q^2), \quad (3)$$

including in the fit (as in all other fits) a proper normalization constant as an additional free parameter. The parametrization (3) is merely empirical and describes the extension of the source(s) in space and time by a single “source-size” parameter r . It is worthwhile stressing that the region r has to be regarded as the region from which pions of similar momenta (as seen in their common rest frame) are emitted, which does not necessarily correspond to the whole production region [10]. The result of the fit of eq. (3) is shown in Fig. 1a (lower curve) and given in Table I (a). Despite a reasonable overall χ^2 ($\chi^2/NDF = 41/36$) for the fit, the data points in the region $Q < 0.2$ GeV are systematically above the fitted curve (see also last column in Table I which contains the χ^2 per fitted data point for $Q < 0.2$ GeV). A similar observation is reported in [11]. Therefore, in order to account for the steep rise observed in the low- Q region, fits were performed with a double-Gaussian and an exponential function, each of which is more peaked at low Q than the single Gaussian:

$$R(Q) = 1 + \lambda_1 \exp(-r_1^2 Q^2) + \lambda_2 \exp(-r_2^2 Q^2) \quad (r_1 < r_2) \text{ and} \quad (4)$$

$$R(Q) = 1 + \lambda \exp(-rQ). \quad (5)$$

The numerical results of the two fits are given in Table I (b,c). The fitted double-Gaussian function, eq. (4), is shown in Fig. 1a (upper curve). The exponential fit (not shown) has a shape which is practically identical to that of the the double-Gaussian. Both expressions (4) and (5) fit the data well as can be seen from the χ^2/NDF (NDF = number of degrees of freedom) values in Table I (b,c). It should be noted that recent theoretical models (e.g. [10]) predict a Q -dependence of the BE correlations at low Q , which can be well approximated by an exponential in Q .

Further fits were performed to check the influence of the correction method and of the reference sample on the fit parameters. The results of the double-Gaussian fit to the uncorrected data with the “mixed” reference sample and to the MC corrected ratio with the “unlike” reference sample are also presented in Table I (d,e). In case of the “unlike” reference sample, eq. (4) was multiplied by an empirical factor $(1 + \delta \cdot Q)$ to account for an observed slow rise of R at higher values of Q . (Without this factor the χ^2/NDF deteriorates from 1.4 to 2.0). From the fit a value of $\delta = (0.19 \pm 0.06)$ GeV⁻¹ was obtained. The spread of fitted parameters in Table I is larger than one would expect from the statistical errors only. As a best estimate of the source parameters one may take an appropriate average of the fits from the “mixed” and “unlike” reference samples, Table I (b,e), which, however, are not statistically independent. Therefore, with some arbitrariness, the simple arithmetic average was calculated and the statistical error was taken as the larger of the two individual errors. The systematic error of the combined result is calculated as half the difference between the two individual values. As a result a statistically well established first source size of $r_1 = 0.44 \pm 0.06(stat) \pm 0.08(syst)$ fm (“wide enhancement”) and a somewhat less well defined second source size of $r_2 = 1.66 \pm 0.52(stat) \pm 0.10(syst)$ fm (“narrow enhancement”) is obtained. Other leptonproduction experiments (μp [7] and νD [12] scattering,) report similar values (~ 0.5 fm), while e^+e^- annihilation and hadron-hadron scattering data [3] show a source size in the range of 0.5 – 1 fm.

A possible interpretation [13] of the double-Gaussian shape, suggested by the data, has been investigated. It is well known that a large fraction of the final-state pions are not produced directly, but are rather decay products of resonances. An especially strong contribution comes from ρ -mesons, which have a mean path length of ~ 3.4 fm (as seen in the rest frame of one of the ρ -decay pions), but the K^* and ω resonances also contribute. When a direct pion from the primary source interferes with a pion of the same charge from a resonance decay, the typical resonance decay distance may show up in the BE correlation. In order to check this hypothesis, pions in the data sample were flagged according to the following criterion: When a pion, combined with any other pion of opposite charge in the same event, yields an effective mass in the region 0.5 to 1.0 GeV it is called an R pion (possible decay product of a resonance), otherwise it is called a D pion (direct). From the above interpretation one expects a strong suppression of the narrow enhancement in the case of like-charged DD pairs and a more complex structure in the case of like-charged DR and RR pairs. This is indeed observed in Fig. 1b (DR and RR pairs) where the data points are fitted by a single- and a double-Gaussian (lower and upper curve respectively): The data are seen to demand the second Gaussian (see also last column in Table I (f,g)). In Fig. 1c (DD pairs), on the other hand, the data points are sufficiently well described by a single wide Gaussian with a small r value. The fitted parameters for the two cases are given in Table I (f,g and h).

A two-dimensional analysis in terms of the two variables q^2 and q_0^2 has also been performed. From eqs. (2) and (3) one expects a non-vanishing correlation even if the two variables are large, under the condition that their difference is small. The experimental data are shown in Fig. 2a. For a better graphical presentation only the enhancement term $R(q^2, q_0^2) - 1$ is plotted; q and q_0 are measured in the hadronic cms. The data show a significant enhancement along the diagonal $q^2 \simeq q_0^2$ ($Q^2 \simeq 0$) up to high values of q^2, q_0^2 . In the orthogonal direction the enhancement decreases rapidly as described by eq. (3). The statistical significance of the enhancement at the high end of the diagonal is shown in Fig. 2b, where $R(Q)$ is plotted for $q^2, q_0^2 > 0.8$ GeV². This distribution is completely in accordance with that displayed in Fig. 1a and a fit of eq. (3) gives a similar source parameter ($r = 0.36 \pm 0.05$ fm). The two expressions

$$R(\mathbf{q}, q_0) = 1 + \lambda \exp(-r^2 \mathbf{q}^2 + r^2 q_0^2) \text{ and} \quad (6)$$

$$R(\mathbf{q}, q_0) = 1 + \lambda \exp(-r^2 \mathbf{q}^2 + r_0^2 q_0^2) \quad (7)$$

were fitted to the data in the two-dimensional plot. Eq. (6) is identical to eq. (3) (see eq. (2)), while eq. (7) is somewhat more general, admitting different correlation lengths in q^2 and q_0^2 . The results of the fits are given in Table I (i,j) and the fitted function (6) is plotted in Fig. 2c. Both equations give a reasonably good description of the data. It is worth noting that in case of eq. (7) the fit yields $r \simeq r_0$, which supports the Lorentz-invariant form (6).

It is interesting to confront the data also with the prediction of a model discussed extensively in the literature [4]: For a Gaussian source distribution in space and time and assuming the times of emission from the different sources to be indepen-

dent ($\rho(\mathbf{r}, t) \propto \exp(-\mathbf{r}^2/2r^2 - t^2/2r_0^2)$), one obtains according to eq. (1):

$$R(\mathbf{q}, q_0) = 1 + \lambda \exp(-r^2 \mathbf{q}^2 - r_0^2 q_0^2). \quad (8)$$

The striking difference between eqs. (6) and (8) is the opposite sign in front of the q_0^2 term. The results of the fit of eq. (8) and the fitted function are shown in Table I (k) and in Fig. 2d, respectively. The data clearly rule out this form for $R(\mathbf{q}, q_0)$ and the best fit gives $r_0 = 0$ for the time parameter. Similar conclusions were reported in [14].

In summary, Bose-Einstein correlations amongst like-charged pions produced in muon-nucleon interactions at 490 GeV have been investigated. The steep Q -dependence of the correlations at low Q may be explained by the contribution of pions from the decay of resonances (ρ -mesons). Furthermore, a two-dimensional analysis discriminates between two different parametrizations: The Lorentz-invariant parametrization (6) is in agreement with the data, whereas the parametrization (8), based on a Gaussian source distribution in space and time, is ruled out.

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Table I:

Results of various fits. All fits were performed in the ranges of the variables as shown in Figs. 1 and 2, respectively. In the one-dimensional fits the first data point at $Q = 20$ MeV has not been used.

Type of fit/ display in Figure	Refer. sample	MC corr.	λ, λ_1	r, r_1 (fm)	λ_2	r_2, r_0 (fm)	χ^2/NDF	χ^2 per fitted data point for Q < 0.2 GeV
1-dim., variable: Q								
(a) eq. (3)/Fig. 1a	mixed	yes	0.35 ± 0.02	0.39 ± 0.02	41/36	1.70
(b) eq. (4)/Fig. 1a	mixed	yes	0.32 ± 0.02	0.36 ± 0.03	0.27 ± 0.13	1.76 ± 0.52	32/34	0.18
(c) eq. (5)	mixed	yes	0.58 ± 0.03	0.50 ± 0.06	36/36	0.22
(d) eq. (4)	mixed	no	0.32 ± 0.03	0.43 ± 0.03	0.22 ± 0.08	1.52 ± 0.35	45/34	0.05
(e) eq. (4)	unlike	yes	0.44 ± 0.07	0.52 ± 0.06	0.47 ± 0.16	1.56 ± 0.31	46/33	0.53
(f) eq. (3)/Fig. 1b	mixed	yes	0.35 ± 0.02	0.39 ± 0.03	44/36	2.21
(g) eq. (4)/Fig. 1b	mixed	yes	0.31 ± 0.03	0.34 ± 0.03	0.39 ± 0.15	1.68 ± 0.38	32/34	0.08
(h) eq. (3)/Fig. 1c	mixed	yes	0.39 ± 0.04	0.41 ± 0.04	33/36	0.35
2-dim., variables: q^2, q_0^2								
(i) eq. (6)/Fig. 2c	mixed	yes	0.35 ± 0.04	0.31 ± 0.04	141/117	
(j) eq. (7)	mixed	yes	0.38 ± 0.05	0.30 ± 0.04	...	0.28 ± 0.04	137/116	
(k) eq. (8)/Fig. 2d	mixed	yes	0.33 ± 0.05	0.25 ± 0.04	...	0.00 ± 0.04	216/116	

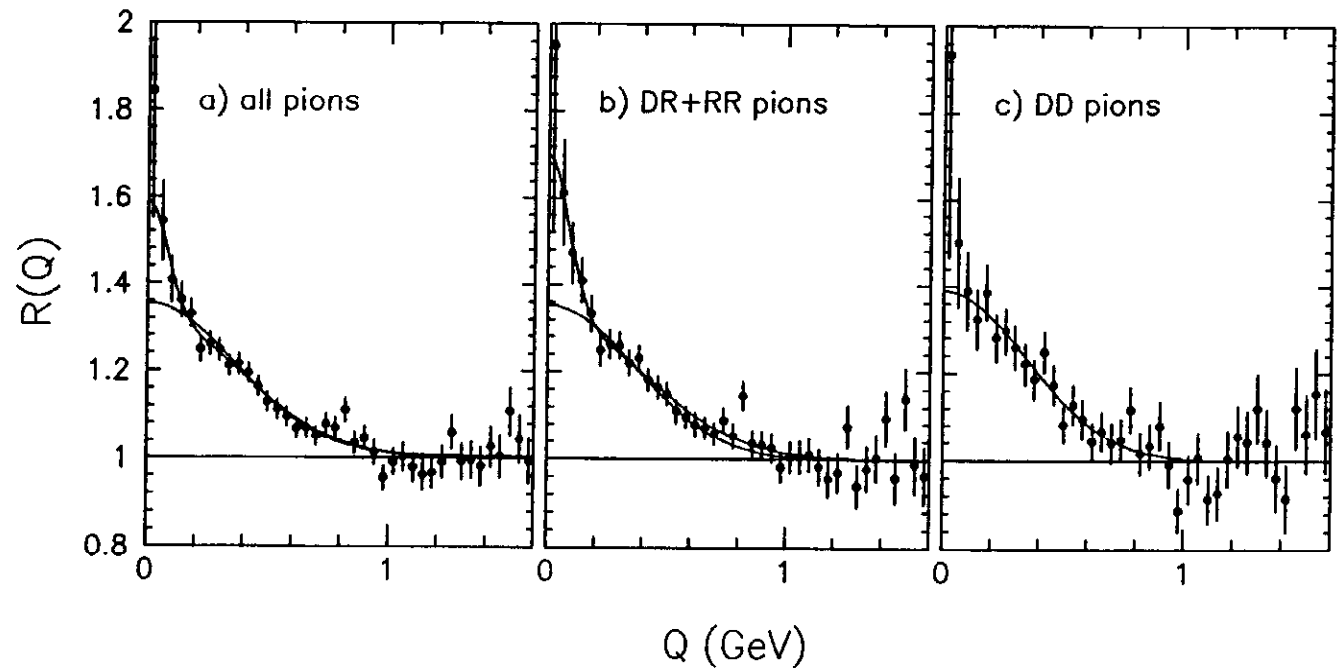


Fig. 1: Ratio R of like-charged pairs to mixed pairs versus Q (MC corrected and properly normalized). a) all pairs; b) pairs for which at least one pion is possibly a decay product of a resonance; c) pairs of “direct” pions (see text). In a,b the lower (upper) curves are the results of fits of eq. (3) (eq. (4)), the curve in c shows the fit of eq. (3). (The first points in the figures have not been included in the fits.)

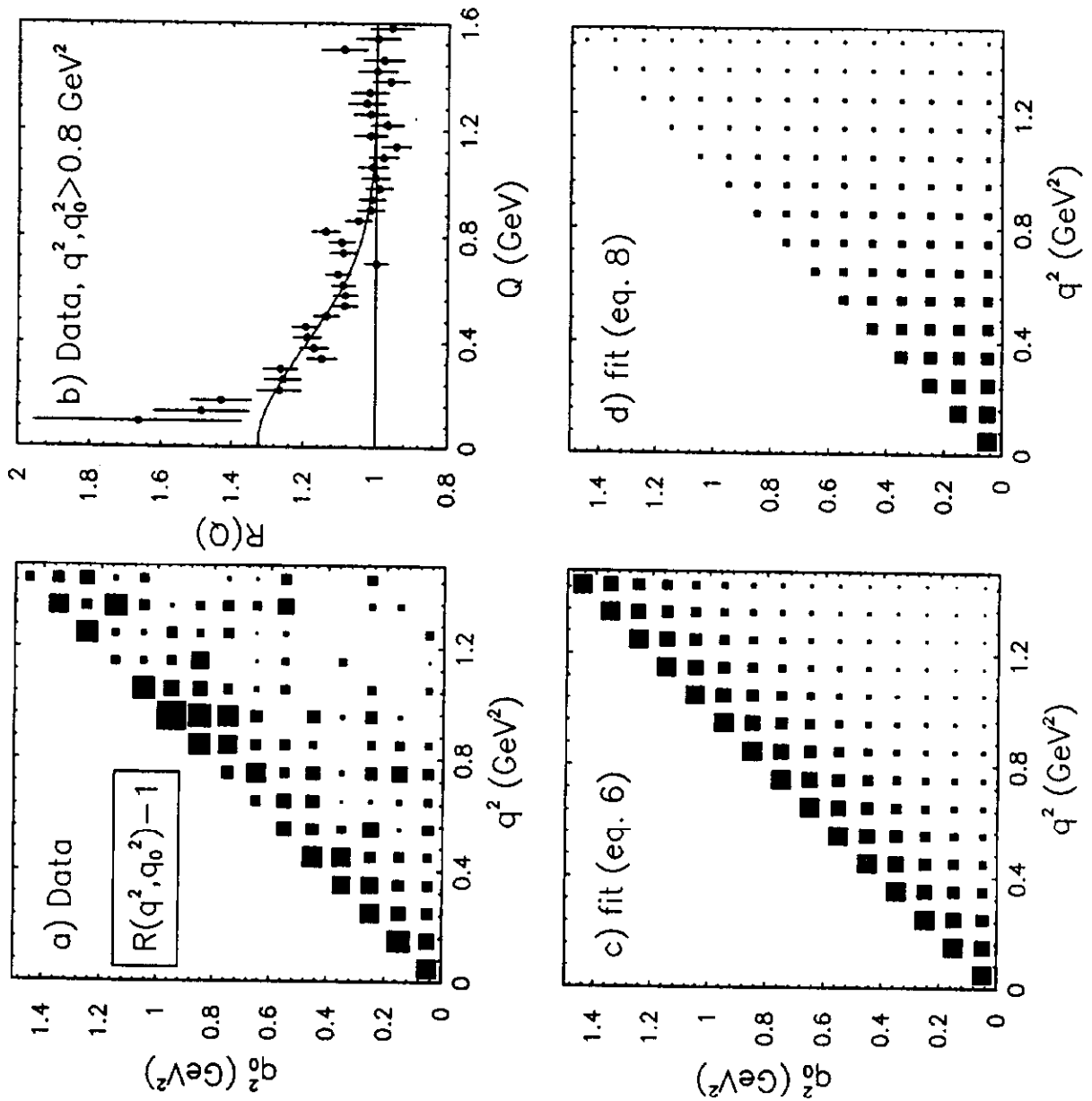


Fig. 2: a Display of the BE enhancement $R - 1$ in the $q^2 - q_0^2$ plane, where R is the MC corrected and properly normalized ratio of like-charged pairs to mixed pairs. (The side lengths of the boxes are proportional to $R - 1$). b $R(Q)$ for $q^2, q_0^2 > 0.8$ GeV². c and d Display of eqs. (6) and (8), respectively, fitted to the data in a.