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PERSPECTIVE

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Quantum Anomalies in Condensed Matter

Michael T. Pettes, Shi-Zeng Lin, Elizabeth A. Peterson, Jian-Xin Zhu, Laurel E. Winter, Johanna C. Palmstrom, Jinkyoung Yoo, Nicholas S. Sirica, Prashant Padmanabhan, Priscila F. S. Rosa, Sean M. Thomas, and Avadh Saxena**

Quantum materials provide a fertile ground in which to test and realize unusual phenomena such as quantum anomalies predicted by quantum field theory. There are three important symmetries that are broken when classical field theory is moved into the quantum regime, the scale anomaly, the axial (chiral) anomaly, and the parity anomaly. Several potential device applications may be realized by the discovery of quantum anomalies in condensed matter, enabled by the new physics they embody, including ultra-sensitive dark matter detectors, far infrared optical modulators, micro-bolometric detectors, low-dissipation ballistic transporters, terahertz-based qubits, terahertz polarization state controls, passive magnetic field sensors, stable topological superconductors that host Majorana fermions, and qubits topologically protected against decoherence. In this perspective article, the definition of these quantum anomalies is laid out, how little is known in the context of condensed matter, and how quantum anomalies are predicted to manifest as anomalous electronic, thermal, and magnetic behavior in experiments on topological quantum materials, including Weyl and Dirac semimetals. Furthermore, the importance that mechanical strain and defects will play in modifying signatures of quantum anomalies is discussed.

1. Introduction

When a classical field theory is quantized, some symmetries that are intact in the classical realm are broken in quantum field theory. This specific breaking of symmetry due to quantum fluctuations is referred to as a quantum anomaly. The term anomaly as related to particle or high energy physics, was first discovered in the triangle diagram of Adler-Bell-Jackiw[1,2] (**Figure 1**). All symmetries in nature can have an anomaly in principle as long as certain constraints (such as Wess-Zumino consistency conditions) are met. A good number of anomalies are known in quantum field theory. Some prominent examples are the chiral anomaly [U(1) charge conservation], the parity anomaly (parity symmetry), and the scale/trace/conformal/Weyl anomaly (scale invariance).

In this perspective, we focus on three important symmetries that exist in classical field theory but are broken when translated to the quantum regime in the context of topological materials: axial (or "chiral"),[3–7] parity,[8–11] and scale (or "conformal")[12,13] (summarized in **Table 1**). One important implication of Emmy Noether's theorem is that any form of broken symmetry will lead to a non-conserved current, J, or "signature."[14] Thus, the symmetry-breaking pro-

M. T. Pettes, S.-Z. Lin, J.-X. Zhu, J. Yoo, P. Padmanabhan, P. F. S. Rosa
Center for Integrated Nanotechnologies
Materials Physics and Applications Division
Los Alamos National Laboratory
Los Alamos, New Mexico 87545, U.S.A
E-mail: pettesmt@lanl.gov

S.-Z. Lin, E. A. Peterson, J.-X. Zhu, A. Saxena
Quantum and Condensed Matter Physics Group
Theoretical Division
Los Alamos National Laboratory
Los Alamos, New Mexico 87545, U.S.A
E-mail: avadh@lanl.gov

L. E. Winter, J. C. Palmstrom
National High Magnetic Field Laboratory – Pulsed Field Facility,
Materials Physics and Applications Division
Los Alamos National Laboratory
Los Alamos, New Mexico 87545, U.S.A

N. S. Sirica
Material Science and Technology Division
U. S. Naval Research Laboratory
Washington, DC 20375, U.S.A

P. F. S. Rosa, S. M. Thomas
Quantum Group
Materials Physics and Applications Division
Los Alamos National Laboratory
Los Alamos, New Mexico 87545, U.S.A

The ORCID identification number(s) for the author(s) of this article can be found under https://doi.org/10.1002/apxr.202400189

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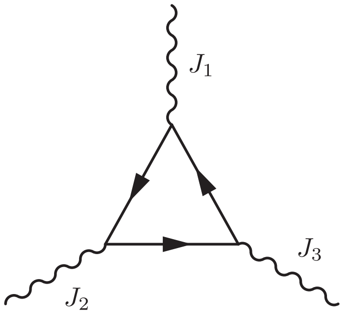

Figure 1. Triangle diagram of Adler-Bell-Jackiw denoting three interacting bosons (wavy lines) showing Green's functions (arrows). Each vertex represents the interaction of a gauge boson with a fermion line, thus forming a closed loop. The calculation of this Feynman diagram, if non-zero, leads to an anomaly. If the chiral symmetry is broken, i.e., left-handed and right-handed fermions are treated differently, a chiral (or axial) anomaly results.

cess from classical field theory to quantum field theory results in a "quantum anomaly," and each known quantum anomaly leads to a different non-conserved current (Table 1), which must be measurable. This paradigm[15] is gaining ground in both theory and experiments on topological quantum materials such as Dirac semimetals (DSMs) and Weyl semimetals (WSMs) as well as two-dimensional (2D) materials with promise for electronic devices, although it remains at the forefront of science with only a few high-impact publications to date.

The study of quantum anomalies in condensed matter carries an inherent and broader impact; few experimental tests exist whereby scientists can probe quantum theory in General Relativity. In this sense, condensed matter offers another experimental cradle to explore the fundamental physics of the universe at the lab scale. This potential arises because symmetry is an organizing principle of our world. Two aspects of symmetry that are important in this perspective are conservation (of a physical property) and breaking of symmetry in the classical and quantum worlds. The Noether theory[14] states that invariance under a certain operation (for example continuous translation) leads to a conservation law (for example momentum conservation at low energies), although we note that crystalline solids that break translational symmetry do retain discrete subgroups and that these discrete symmetries do not lead to Noether symmetries. The classical effect of the spontaneous breaking of symmetry is a phase transition due to thermal fluctuations, for example, from the crystalline ice of snowflakes to liquid water. In the quantum world, we have quantum fluctuations, and some states respect symmetry while others may break it. The Noether theory of conservation of classical currents ($\partial\mu J^{\mu} = 0$) is replaced by Ward identities[16] of quantum averages ($\langle \partial\mu J^{\mu} \rangle = 0$). Thus, symmetries are conserved in classical systems but can be broken in quantum systems, and the anomaly emerges as a new principle to organize and understand different quantum phases. The observable quantum effect is a symmetry violation due to the quantum anomaly. This is so important as a building principle of our world at a fundamental level that the charge states of quarks (+2/3, -1/3), which compose protons and neutrons, are defined by the charge needed to cancel quantum anomalies globally for the theory to be sensible.

Because the universe is quantum, a better understanding of these anomalies (and their signatures in different systems)[17–20] is pivotal to improving the overall quantum field theoretical framework. However, these anomalies are not readily testable in field theory in most cases, and their nature means that, in a quantum system, the coupling constants are dependent on measurement. For example, it is known that the scale anomaly depends on the energy at which it is measured, but the measurement is so technically challenging that the scale anomaly has never been definitively measured in any solid material. Likewise, the parity anomaly has only recently been observed in a single semimagnetic topological insulator (TI) heterostructure.[21] Thus, the challenge to the scientific community is two-fold: both new experimental methods are needed to enable the measurement of anomalous currents and new theoretical approaches are needed that will enable their interpretation in condensed matter.

One immediate consequence of the quantum anomalies is to dictate whether a quantum state can exist. The quantum state cannot exist if the quantum anomalies cannot be cancelled globally. As a simple example, the chiral anomaly forbids the existence of truly one-dimensional (1D) wires with only left or right-moving electrons. This arises from the fact that the number of left and right-moving modes must be the same because the electron en-

Table 1. Overview of the current state of research regarding the three main anomalies that arise when classical field theory symmetries are broken in the quantum regime.

Quantum Anomaly	Measured? / in condensed matter?	Expected Signature	Dependency: from predicted quantum anomaly to a measurement	Devices enabled by quantum anomalies
Axial Anomaly	yes / yes	negative longitudinal magneto resistance	$\boldsymbol{J} \propto (\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{B}$ Fukushima *et al.*, *Phys. Rev. D* **2008**, *78*, 074033; Nielsen & Ninomiya, *Phys. Lett. B* **1983**, *130*, 389–396.	• far infrared optical modulators • low-dissipation ballistic transporters • terahertz-based qubits • terahertz polarization state controls
Parity Anomaly	yes / yes	half-integer quantum anomalous Hall effect	$\boldsymbol{J} = (½\, e^2/h)\boldsymbol{E}_{Hall}$ Dudal *et al.*, *Sci. Rep.* **2022**, *12*, 5439.	• passive magnetic field sensors • stable topological superconductors that host Majorana fermions; noise-impervious topological quantum computers • qubits protected against decoherence • quantum resistance standard
Scale Anomaly	yes / no	anomalous magneto-thermoelectric current	$\boldsymbol{J} \propto \boldsymbol{B}\times(\nabla T/T)$ Chernodub *et al.*, *Phys. Rev. Lett.* **2018**, *120*, 206601.	• ultra-sensitive micro-bolometric detectors • dark matter detectors

ergy dispersion is periodic in momentum space. However, topology comes to our rescue and makes the seemingly impossible task possible. The chiral anomaly is cancelled by bulk topology, which results in the quantum Hall effect in 2D systems. Consequently, the edges of the quantum Hall system represent a new quantum state that does not exist in a truly 1D lattice, leading to dissipationless conducting channels at the edges. This phenomenon can also be realized in Chern insulators without applying a magnetic field and is known as the quantum anomalous Hall effect (QAHE).[22,23]

One issue that arises in the field of quantum anomalies in topological materials is that research approaches are based on methods that rely significantly on chance encounters of the coupling constants to acquire data of interest under lab-scale perturbations. As an example, some established magneto-thermal measurements in low-direct current (DC) magnetic fields rely on thermal gradients (e.g., the Nernst or Seebeck effect),[24–26] but none have been performed in ultra-high pulsed magnetic fields where exotic physics governs transport.[27] Although challenges exist in adapting magneto-thermal measurements techniques in low fields to these ultra-high magnetic fields, many can be mitigated through the use of pulse magnets that produce a flat-top field on the order of 1–2 s.[28,29] Using such longer pulse magnet systems while also controlling the electronic band structure through strain in strain-tunable systems may allow measurements of quantum anomalous currents at ultra-high magnetic fields.

2. Discussion

2.1. Chiral Anomaly

The anomalous non-conservation of a chiral current in a physical system is referred to as the chiral anomaly. In the case of particle physics, the "charge-parity (CP) violation" – or CP non-conservation – provides direct evidence for the existence of the chiral anomaly. Evidently, the very fact that the observable universe contains vastly more matter than antimatter is likely a consequence of the chiral anomaly. The chiral anomaly was first explained by the calculations of Adler-Bell-Jackiw[1,2] in the context of quantum electrodynamics. The chiral or axial symmetry is broken by quantum corrections calculated via the triangle diagram (e.g., Figure 1).

To illustrate the basic notion of the chiral anomaly, let us consider a 1D metal. Its Brillouin zone is also 1D, and periodic in momentum, k. Its energy dispersion $E(k)$ is a periodic function in k, which means that the number of times that $E(k)$ crosses the Fermi energy must be an even number. In condensed matter systems, we are usually only interested in low-energy states around the Fermi energy. Suppose we linearize the dispersion around the Fermi energy, we obtain an equal number of left and right-moving chiral modes, as shown in **Figure 2**a. Imagine we do not have the knowledge about the full spectrum of the system, and we only know these low-energy chiral modes near the Fermi energy. It may appear that the number of fermions for each chiral mode is separately conserved. Indeed, in the low-energy Hamiltonian, we can define a symmetry operator such that it commutes with the Hamiltonian. However, in the presence of an electric field, **E**, it is obvious from the full spectrum

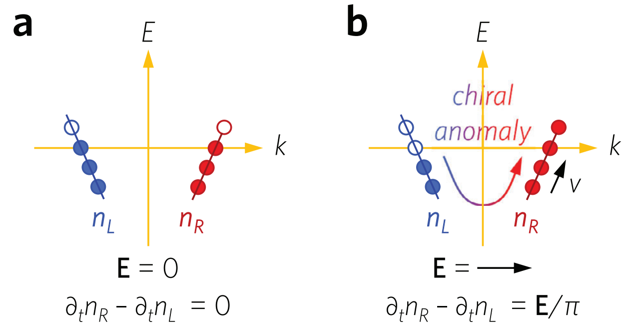

Figure 2. Chiral anomaly in one space dimension. a) Left- (n_L, blue) and right- (n_R, red) moving fermions are conserved without an electric field for charged fermions such as electrons. b) In the presence of an external electric field, **E**, 1D chiral fermions are pumped in the direction of the field at a rate of $\mathbf{E}/\pi$, which results in a non-conservation of left and right-moving charged fermions separately.

of $E(k)$ that electrons are pumped from one chiral branch to the opposite chiral branch. However, if we only know the spectrum around the Fermi energy, then it becomes mysterious that fermions at one branch disappear and reappear at the other branch, which seems to be anomalous, hence the name chiral anomaly. The rate of transferring electrons from one branch to the other branch is given by[17] $\partial_t n_R - \partial_t n_L = \mathbf{E}/\pi$, where n_L and n_R are the number of left and right-moving electrons, respectively. In terms of the full $E(k)$ spectrum, there is nothing anomalous, which means that the anomaly must cancel globally.

A single chiral mode cannot appear in a 1D system due to the chiral anomaly. However, it can appear as a boundary of a 2D system as manifested in quantum Hall systems, such as depicted in **Figure 3**. For an electron gas under a strong perpendicular magnetic field, the electrons' motion forms closed Landau orbitals, and the spectrum of electrons becomes Landau levels. Each Landau level has a band topology characterized by a topological invariant called a Chern number. When electrons fully occupy a certain number of Landau levels, the system develops precisely quantum Hall conductance in units of the quantum conductance, e^2/h, where e is the elementary charge and h is the Planck constant. It is the chiral edge mode at the boundary of the Hall system that is responsible for the Hall quantization, and the number of chiral edge modes is determined by the bulk topology. These chiral edge modes are truly 1D in nature and exhibit the chiral anomaly at the edge. In contrast to the modes in a 1D system, where an arbitrarily weak disorder causes the modes to be localized, these topologically protected chiral edge modes cannot be localized by disorder. These chiral edge modes are allowed because of the bulk topology, which allows for the cancellation of the chiral anomaly at the edge and is known as anomaly inflow. Specifically, in the presence of an electric field in the x direction, the left-moving electrons are pumped into the right-moving electrons, similar to the process shown in Figure 2b. In quantum Hall systems, this pumping is through the bulk and is nothing but the transverse Hall current. The consequence is that the quantum Hall system as a whole does not have an anomaly. Our discussion here only focuses on non-interacting physics, and we will not discuss the possibility of the fractional quantum Hall effect

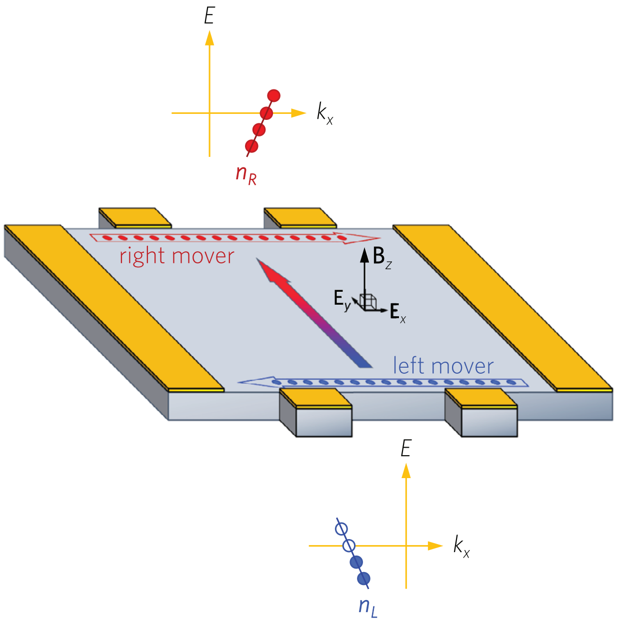

Figure 3. Chiral anomaly in a quantum Hall insulator. The boundary of the material is anomalous, hosting 1D chiral edge modes, which separates left and right movers along opposite edges. The chiral anomaly at the boundary is canceled by bulk topology through anomaly inflow, i.e., bringing about charge pumping from left movers (blue) to right movers (red) through the Hall current due to bulk topology. This schematic reveals the deep and close connection between anomaly and topology.

due to electronic correlations. The discussion here is also applicable to Chern insulators.

The chiral anomaly also exists in three-dimensional (3D) systems. One important example is that of WSMs. In these materials, we have an even number of topologically protected band crossings, known as Weyl nodes. These Weyl nodes serve as sinks or sources for Berry curvature in momentum space. In the presence of a magnetic field, the motion of the Weyl fermions in the plane transverse to the applied magnetic field is confined, and the system forms Landau levels while the motion along the field direction is unrestricted. This results in dispersive Landau levels as shown in **Figure 4**. What makes Weyl fermions special is that the zero-th Landau level is chiral, similar to the chiral modes in 1D wires (lattices) and 2D quantum Hall systems. In other words, the application of a magnetic field effectively reduces the dimension of the system from 3D to 1D due to the quenching of the electron dynamics in the transverse direction. As in the 1D case, application of an electric field parallel to the magnetic field pumps electrons from one chiral branch to the opposite branch. Here we would like to emphasize that the chiral anomaly cancels globally in WSMs, because the Weyl nodes with opposite chirality must appear in pairs as required by the Nielsen-Ninomiya fermion doubling theorem.[30,31] The chiral anomaly leads to an observable effect in topological semimetals, such as negative longitudinal magnetoresistance (NLMR),[4] and the chiral magnetoelectric effect,[3] which have been observed in experiment,[5] see refs. [32–34] for a comprehensive review. However, the attribution of NLMR to the chiral anomaly has been debated theoretically due to current jetting artifacts.[35] Recently, it has also been shown that the chiral anomaly is an underlying mechanism for the observed large phonon magnetic moment in Dirac semimetals.[36,37] This work demonstrated that giant moments can be achieved with high carrier density and mobility, and that the effect is measurable by Raman spectroscopy under magnetic fields in the experiment.

2.2. Parity Anomaly

In a physical system, if the classical action is invariant under a change of parity, but the quantum theory is not invariant, then we have a parity anomaly. It can occur in gauge theories with fermions in even spatial dimensions. Haldane in his 1988 seminal paper[38] presented a 2D lattice model for the quantum Hall effect without an external field (or Landau levels) as a condensed matter realization of parity anomaly.

Let us consider Dirac fermions in 2D, which can be realized in graphene, as an example. Around the Dirac point in the momentum space, the pseudospin associated with the spinor has a vortex structure with winding number equal to one, as depicted in **Figure 5a**. The parity anomaly states that it is impossible to have a single Dirac cone in the 2D Brillouin zone when the time reversal symmetry is present. This can be seen by drawing a small, closed loop around the Dirac point, where there is a winding of the spinor along the loop. Now we enlarge the loop gradually while maintaining the Dirac point in its center, so the winding number does not change. However, since the Brillouin zone is periodic and compact, this means that we can contract the loop eventually. This leads to a contradiction for having a winding number along the loop. Therefore, a single Dirac cone in a 2D Brillouin zone, when the time reversal symmetry is preserved, is not possible due to the parity anomaly. This is why in graphene, there are four Dirac cones due to the spin and the valley degrees of freedom. One signature of the parity anomaly is the appearance of a half-quantized Hall conductance, i.e., $\sigma_{xy} = ½{\cdot}e^2/h$. This half-quantized Hall conductance has been observed in graphene, where $\sigma_{xy} = 2{\cdot}e^2/h$. Here, the factor of 2 is due to the contribution from four Dirac cones under a magnetic field. In condensed matter systems, the low-energy physics is governed by energy near the Fermi energy. One may realize the parity anomaly by keeping one Dirac fermion near the Fermi energy while gapping out the other Dirac fermions. This idea has been discussed by Duncan Haldane in his Nobel Prize-winning paper,[38] where parity anomaly is achieved at the topological phase transition between the Chern insulator and the trivial insulator.

A true realization of a single Dirac cone, or parity anomaly, has only been achieved after the discovery of 3D TIs. At the surface of the TI, there exists a single Dirac cone. Similar to the case of edge state in a quantum Hall system, the parity anomaly at the surface is canceled by the bulk topology, and globally, there is no anomaly. However, sensible physics must be local in space. We can have a subsystem with a parity anomaly if we only probe the surface state of the 3D TI. In this sense, the surface state of the 3D TI represents a new state of matter that cannot exist in any 2D system. The parity anomaly of the surface state gives rise to half-quantized Hall conductance when the surface state is in the proximity of a magnetized material. Recently, experimental signatures of the parity anomaly have been observed in a semi-magnetic TI[21]

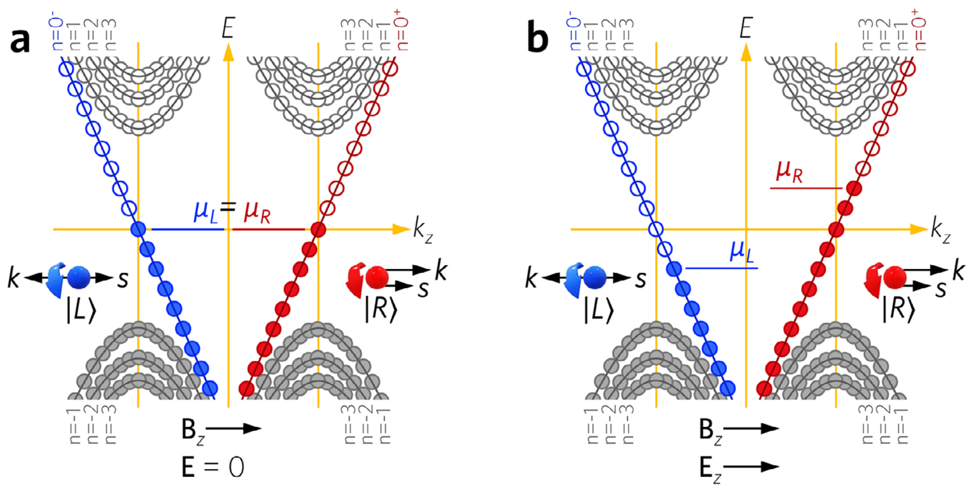

Figure 4. a) Graphical depiction of the chiral anomaly in a 3D Weyl semimetal under an applied external magnetic field, with higher order Landau levels shown in gray and zero-th chiral Landau levels shown in blue (0^-) and red (0^+). b) Zero-th Landau levels assume different chemical potentials (μL, 0^-, blue and μ_R, 0^+, red) when an external electric field is applied. This leads to the chiral anomaly in 3D with experimental consequences related to charge pumping, such as negative longitudinal magnetoresistance.

wherein they demonstrated the half-integer quantization of Hall conductance using a synthetic heterostructure and terahertz magneto-optical spectroscopy. The latter has one surface state gapped and massive due to magnetic doping, whereas the other surface state is gapless, massless, and non-magnetic. Another interesting possibility is the realization of topological superconductivity with Majorana fermions localized in the vortex cores when the surface state is in close contact with a superconductor.[39] There are ongoing experimental efforts to realize this topological superconductivity by fabricating a superconductor-TI hybrid structure. The zero-bias conductance peak observed in scanning tunneling microscopy of certain iron-based superconductors[40] is argued to be due to this mechanism. One major motivation behind these efforts is to realize Majorana fermions, which are promising for robust topological quantum computation (Figure 5b).

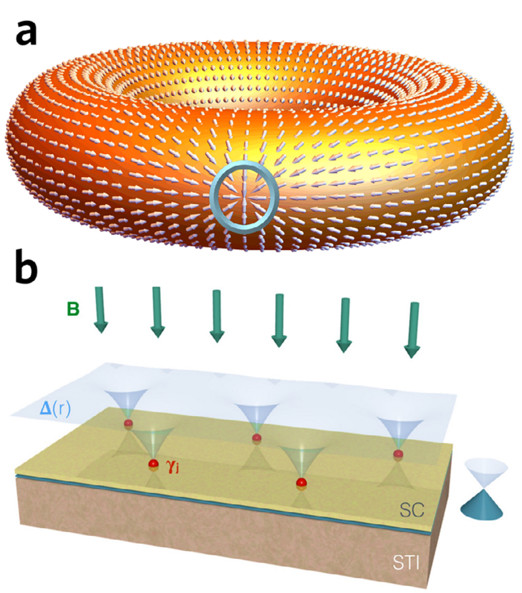

Figure 5. a) Schematic showing the impossibility of putting a single vortex on a torus due to the parity anomaly. The vortex core is highlighted by a circle. b) The combination of the parity anomaly together with superconductivity may lead to a surface that hosts novel and anomalous quantum states, such as Majorana fermions, that have not been reported in 2D systems. Reproduced with permission.[41] Copyright 2015, American Physical Society.

To have the parity anomaly at a surface, it is important to tune the system into a STI phase, which can be achieved through tuning the Fermi mass. Ultrafast optical pulses may offer a new way through which to tune the Fermi mass to allow measurement of signatures of the parity anomaly. Femtosecond laser pulses have been shown to impulsively excite short-lived (sub-ns) single-cycle acoustic wave packets that propagate through the material volume with concomitant dynamic strains of more than 1%. Poisson effects are negligible, and the possibility of sample damage is greatly reduced, thus, dynamic strain can be seen as a powerful tool to drive topological phase transitions in layered topological semimetals.[42–44] Coherently driving the lattice structure using ultrashort pulses of light[45,46] has been demonstrated to result in a topological phase transition through the accompanying lattice distortion.[47] Because symmetry-retaining and symmetry-breaking strain profiles can change the electronic structure, DSMs and WSMs directly excited with acoustic strain waves (thermal expansion from laser heating),[48–50] optical transducers,[51,52] and transient grating techniques[53] all show possibility to enable highly directional strain fields that allow targeting of specific resonances and transparency windows needed to tune the Fermi mass. It can be combined with QAHE transport measurements. The theoretical framework for achieving ultrafast optical control of the parity anomaly, whose signature is the half-integer QAHE,[11] is based on recently developed Floquet theory for controlling a material's Hamiltonian[54,55] via light-matter interactions.[56] In this approach, density functional theory (DFT) calculations must be used to obtain band structures to derive a Hamiltonian whose parameters can be laser-tuned to support Dirac fermions. Then the quantum anomalous current can be calculated using the Boltzmann transport equation and quantum field theory, and the combined approach may allow one to obtain evidence of strain control over the parity anomaly.

2.3. Scale Anomaly

The scale (or conformal or trace or Weyl) anomaly breaks the conformal symmetry of the classical theory. It relates to the generation of mass by quantum mechanics. It also means that a measurement depends on the scale of energy and/or distance at which it is measured. The Lagrangian for massless Dirac fermions is scale invariant and invariant under a rescaling transformation. With quantum fluctuations (particle-hole excitation screening), the effective charge is modified at different energies or distances, which leads to a scale anomaly. For the case of graphene, this leads to a renormalization of velocity,[57] which has been shown experimentally.[58] In topological materials, the scale anomaly can manifest as an edge current under an external magnetic field,[8,9] and as charge accumulation under an external electric field.[59]

One approach to observing the scale anomaly is grounded in Luttinger's[60] demonstration that statistical thermal diffusion constants equate to certain zero-momentum correlation functions of the energy-momentum current (i.e., the constants that couple to the gravitational field), which is now made actionable due to the recent discovery of DSMs,[55,61] which are also topologically invariant over a significant momentum and energy range.[60,62] Thus, if a temperature gradient ∇T drives a DSM system out of equilibrium, a gravitational potential Φ can compensate, such that $\nabla T/T = -\nabla\Phi/c^2$, where c is the speed of light. Theoretically, measurable thermal and thermoelectric effects[63] in DSMs are related to quantum anomalies, even in the complete absence of gravitational fields or space-time curvature. **Figure 6a** illustrates the scale anomaly, where a simple and concise theory predicts that in a DSM, a temperature-gradient-driven anomalous electric current will arise in an external magnetic field $\mathbf{B}$ as $\boldsymbol{J} = e^2 v_F \mathbf{B}\times\nabla T/(18\pi^2 T\hbar)$,[12] where v_F is the Fermi velocity at the Dirac nodes (Figure 6b, **inset**). Although real materials have impurities that complicate the measurement and interpretation of anomalous currents, we note that strain in DSMs can be used to precisely tune through a topological phase transition and enforce a purely Dirac linear $E(k)$ dispersion by controlling the electronic band gap.[42,43,64–66] Additionally, a recent report has demonstrated the observation of Luttinger liquids in twin boundaries of monolayer MoS_2 thin films.[67]

Theoretical calculations predict that the signature of the scale anomaly in the DSM Cd_3As_2 will be as high as ≈2.7 µA in 60 T high pulsed magnetic field experiments (Figure 6b), and that a similar effect will be observable in another DSM $HfTe_5$ at ≈16 nA. Upper and lower limits are based on a typical range of temperature gradients reached in the experiment (0.01–0.3 K µm^{-1}), with a base temperature range of 0.75–4 K for Cd_3As_2 and 65 K for $HfTe_5$. For clarity, we note that the quantum anomalies discussed here are unrelated to general reports of transport signatures that deviate from condensed matter theory (colloquially termed "anomalies").

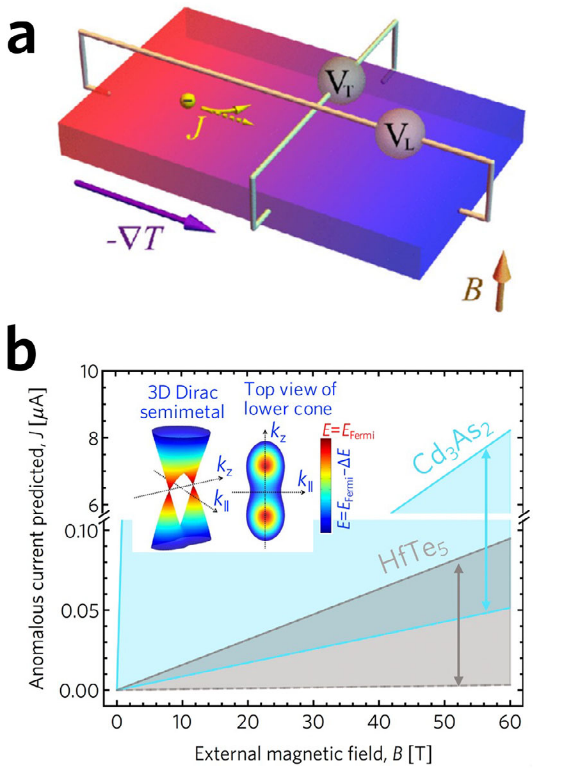

Figure 6. a) Schematic of the scale anomaly measurement. Reproduced with permission.[12] Copyright 2018, American Physical Society. b) Predicted scale anomaly signature in the DSMs Cd_3As_2 and $HfTe_5$ supports the viability of an experimental approach in high magnetic fields. (Inset) Schematic of a DSM band structure with Dirac nodes.

2.4. Distinguishing Between Trivial and Non-Trivial Electronic Bands

To realize the physics of such bands, experimentalists must be able to drive the material into the right phase to observe quantum anomalies. For observing quantum anomalies, we must have truly Dirac (massless) fermions. One viable approach to achieve Dirac fermions is to tune the system from a STI to a weak topological insulator (WTI) through strain, where the massless Dirac fermion appears at the critical point between these topological phases. Topological bands have a distinct and additional contribution to electronic transport, optical, and magnetic properties, and it is possible that there could be a coexistence of trivial and non-trivial bands in any given topological material. The presence of other trivial bands at the Fermi surface will complicate the observation of the QAHE, and other transport signatures. Thus, distinguishing between topologically trivial and non-trivial bands in the search for quantum anomalies must be performed through integrated theory and experiment. Theoretically, analytical calculations are used to determine whether a band is topological by calculating its Chern number; for example, if the Chern number is zero, it is trivial, and if it is non-zero, it is non-trivial.

Figure 7 illustrates one very promising way of realizing Dirac cones by strain tuning between WTI and STI states.[68] A Ti is an insulator by definition, and thus it is certain that other trivial bands are not playing any role in transport. By tuning between

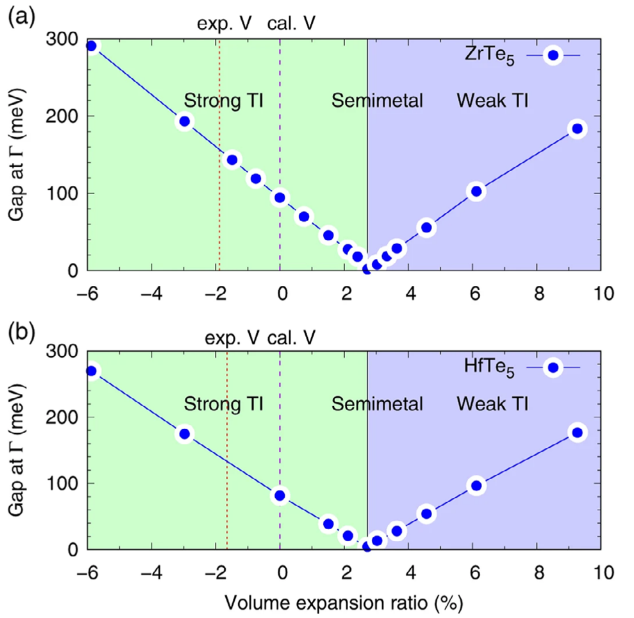

Figure 7. Strain tuning between the STI and WTI phases in a) $ZrTe_5$ and b) $HfTe_5$ based on DFT calculations. At the critical expansion, the DSM electronic phase can be realized. Reproduced under terms of the CC-BY license.[68] Copyright 2017, The Authors, published by Springer Nature.

these two TI states, it is guaranteed that an isolated Dirac cone will appear near the Fermi surface. In angle-resolved photoemission spectroscopy (ARPES), one can actually visualize these linear bands and determine what is topological and what is trivial, as well as the separation of the cones in energy.[69] ARPES has recently been used to validate the experimental band structure shift from a STI to a WTI in $HfTe_5$ with mechanical strain tuning, even in the presence of non-negligible tellurium vacancies.[70] Theoretically, tellurium vacancy defects have been shown to account for the difference in reported experimental spectroscopic measurements, transport properties, and topological phases of $ZrTe_5$ and $HfTe_5$ which have obfuscated attribution of anomalous transport to broken chiral symmetry,[71] owing in part to the strong strain sensitivity of these materials and the local strain fields generated by the vacancies. Additionally, theorists have shown that strain leads to a topological phase transition in Cd_3As_2 using DFT.[72]

3. Outlook: Potential Devices Enabled by Quantum Anomalies

Several potential device applications may be enabled by the new physics of the quantum anomalies upon their reproducible discovery, centering on applications in microelectronics and quantum computing (Table 1). For example, the scale anomaly will enable ultra-sensitive micro-bolometric particle detection for thermal imaging. In micro-bolometry, the noise equivalent thermal detection (NETD) limit is governed by the thermal coefficient of resistance (TCR), where NETD $\propto$ TCR^{-1}. In the DSM $HfTe_5$, TCR peaks at ≈4% K^{-1} at 90 K[27,66] (two times higher than the industry-leading amorphous silicon). However, in a future scale-anomaly-based detection scheme, NETD will be governed by the change in anomalous current with temperature $(\partial J/\partial T)/J$, and because the Fermi velocity of DSMs can be tuned by ≈5 times[73] when the electronic band structure changes between gapless and gapped,[74] sensitivity will increase by more than two orders of magnitude even at low static magnetic fields – one order of magnitude higher than the most sensitive TCR-based process of electron tunneling. Likewise, the parity anomaly has the potential to accelerate quantum information science, as it can exist as a surface state in a Dirac material. Thus, when a DSM is put in contact with a superconductor, it forms a topological superconductor that can host Majorana fermions,[41] which may enable a decoherence-resilient topological quantum computing platform. In addition, the chiral (axial) anomaly will allow a new type of terahertz (THz)-based qubit with optical readout[75,76] and enable novel high-performance electromagnetic devices. For example, in a WSM, effects closely related to the axial anomaly have enabled control over the THz polarization state.[77] Theoretical prediction also hints that device applications in far infrared modulators could be enabled by two unique axial anomaly effects: current at a distance and the resonant transparency of electromagnetic wave propagation in WSMs and DSMs.[78,79] Lastly, we note a recent report that claims to observe the braiding of Majorana fermions in hybrid superconductor-semiconductor nanowire devices.[80] These topologically protected boundary states can be compared to Weyl fermions in 3D systems and support the possibility that quantum anomalies may play a fundamental role in ensuring the robustness of quantum information. Any one of these potential applications would be a breakthrough for next-generation precision quantum sensing and detection.

4. Conclusion

A classical symmetry can be broken in the quantum world, which gives rise to an anomaly. Because quantum anomalies need to be canceled globally, they are closely related to topology, which enables new physical manifestations in both quantum field theory and condensed matter. Thus, quantum anomalies are an important organizing principle for the physical world, specifically in the classification of quantum phases by anomalies in a variety of quantum materials. In this way, quantum anomalies act as design principles for novel quantum states, which may enable new device technologies centering on quantum information science and microelectronics applications. It is an emerging frontier of materials science and condensed matter physics with possible analogs in photonics.

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Conflict of Interest

The authors declare no conflict of interest.

Keywords

axial (chiral) anomaly, parity anomaly, scale anomaly, strain engineering, topological quantum materials

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[1] S. L. Adler, *Phys. Rev.* **1969**, *177*, 2426.
[2] J. S. Bell, R. Jackiw, *Nuovo Cimento. A* **1969**, *60*, 47.
[3] K. Fukushima, D. E. Kharzeev, H. J. Warringa, *Phys. Rev. D* **2008**, *78*, 074033.
[4] D. T. Son, B. Z. Spivak, *Phys. Rev. B* **2013**, *88*, 104412.
[5] Q. Li, D. E. Kharzeev, C. Zhang, Y. Huang, I. Pletikosić, A. V. Fedorov, R. D. Zhong, J. A. Schneeloch, G. D. Gu, T. Valla, *Nat. Phys.* **2016**, *12*, 550.
[6] D. Schmeltzer, A. Saxena, *Phys. Rev. B* **2021**, *103*, 235113.
[7] C. Corianò, M. Cretì, S. Lionetti, D. Melle, R. Tommasi, *Adv. Phys. Res.* **2024**, 2400043.
[8] C.-S. Chu, R.-X. Miao, *Phys. Rev. Lett.* **2018**, *121*, 251602.
[9] M. N. Chernodub, V. A. Goy, A. V. Molochkov, *Phys. Lett. B* **2019**, *789*, 556.
[10] M. F. Lapa, *Phys. Rev. B* **2019**, *99*, 235144.
[11] D. Dudal, F. Matusalem, A. J. Mizher, A. R. Rocha, C. Villavicencio, *Sci. Rep.* **2022**, *12*, 5439.
[12] M. N. Chernodub, A. Cortijo, M. A. H. Vozmediano, *Phys. Rev. Lett.* **2018**, *120*, 206601.
[13] M. Chernodub, V. Goy, A. Molochkov, *Adv. Phys. Res.* **2023**, 2300058.
[14] E. Noether, *Kgl. Ges. d. Wiss. Nachrichten. Math.-phys. Klasse* **1918**, *2*, 235.
[15] M. N. Chernodub, Y. Ferreiros, A. G. Grushin, K. Landsteiner, M. A. H. Vozmediano, *Phys. Rep.* **2022**, *977*, 1.
[16] J. C. Ward, *Phys. Rev.* **1950**, *78*, 182.
[17] H. B. Nielsen, M. Ninomiya, *Phys. Lett. B* **1983**, *130*, 389.
[18] J. Xiong, K. K Satya, T. Liang, W. Krizan Jason, M. Hirschberger, W. Wang, R. J. Cava, N. P. Ong, *Science* **2015**, *350*, 413.
[19] J. Gooth, A. C. Niemann, T. Meng, A. G. Grushin, K. Landsteiner, B. Gotsmann, F. Menges, M. Schmidt, C. Shekhar, V. Süß, R. Hühne, B. Rellinghaus, C. Felser, B. Yan, K. Nielsch, *Nature* **2017**, *547*, 324.
[20] A. M Puneet, N. Defenu, L. Bayha, M. Holten, M. Preiss Philipp, T. Enss, S. Jochim, *Science* **2019**, *365*, 268.
[21] M. Mogi, Y. Okamura, M. Kawamura, R. Yoshimi, Y. Yasuda, A. Tsukazaki, K. S. Takahashi, T. Morimoto, N. Nagaosa, M. Kawasaki, Y. Takahashi, Y. Tokura, *Nat. Phys.* **2022**, *18*, 390.
[22] C.-Z. Chang, J. Zhang, X. Feng, J. Shen, Z. Zhang, M. Guo, K. Li, Y. Ou, P. Wei, L.-L. Wang, Z.-Q. Ji, Y. Feng, S. Ji, X. Chen, J. Jia, X. Dai, Z. Fang, S.-C. Zhang, K. He, Y. Wang, L. Lu, X.-C. Ma, Q.-K. Xue, *Science* **2013**, *340*, 167.
[23] S. Oh, *Science* **2013**, *340*, 153.
[24] T. Liang, J. Lin, Q. Gibson, T. Gao, M. Hirschberger, M. Liu, R. J. Cava, N. P. Ong, *Phys. Rev. Lett.* **2017**, *118*, 136601.
[25] S. J. Watzman, T. M. McCormick, C. Shekhar, S.-C. Wu, Y. Sun, A. Prakash, C. Felser, N. Trivedi, J. P. Heremans, *Phys. Rev. B* **2018**, *97*, 161404.
[26] J. Hu, M. Caputo, E. B. Guedes, S. Tu, E. Martino, A. Magrez, H. Berger, J. H. Dil, H. Yu, J.-P. Ansermet, *Phys. Rev. B* **2019**, *100*, 115201.
[27] S. Galeski, X. Zhao, R. Wawrzyńczak, T. Meng, T. Förster, P. M. Lozano, S. Honnali, N. Lamba, T. Ehmcke, A. Markou, Q. Li, G. Gu, W. Zhu, J. Wosnitza, C. Felser, G. F. Chen, J. Gooth, *Nat. Commun.* **2020**, *11*, 5926.
[28] M. Jaime, R. Movshovich, G. R. Stewart, W. P. Beyermann, M. G. Berisso, M. F. Hundley, P. C. Canfield, J. L. Sarrao, *Nature* **2000**, *405*, 160.
[29] T. Nomoto, C. Zhong, H. Kageyama, Y. Suzuki, M. Jaime, Y. Hashimoto, S. Katsumoto, N. Matsuyama, C. Dong, A. Matsuo, K. Kindo, K. Izawa, Y. Kohama, *Rev. Sci. Instrum.* **2023**, *94*, 054901.
[30] H. B. Nielsen, M. Ninomiya, *Nucl. Phys. B* **1981**, *185*, 20.
[31] H. B. Nielsen, M. Ninomiya, *Nucl. Phys. B* **1981**, *193*, 173.
[32] B. Yan, C. Felser, *Annu. Rev. Condens. Matter Phys.* **2017**, *8*, 337.
[33] N. P. Armitage, E. J. Mele, A. Vishwanath, *Rev. Mod. Phys.* **2018**, *90*, 015001.
[34] B. Q. Lv, T. Qian, H. Ding, *Rev. Mod. Phys.* **2021**, *93*, 025002.
[35] N. P. Ong, S. Liang, *Nat. Rev. Phys.* **2021**, *3*, 394.
[36] B. Cheng, T. Schumann, Y. Wang, X. Zhang, D. Barbalas, S. Stemmer, N. P. Armitage, *Nano Lett.* **2020**, *20*, 5991.
[37] W. Chen, X.-W. Zhang, Y. Su, T. Cao, D. Xiao, S.-Z. Lin, arXiv **2024**, arXiv:2405.10318.
[38] F. D. M. Haldane, *Phys. Rev. Lett.* **1988**, *61*, 2015.
[39] L. Fu, C. L. Kane, *Phys. Rev. Lett.* **2008**, *100*, 096407.
[40] D. Wang, L. Kong, P. Fan, H. Chen, S. Zhu, W. Liu, L. Cao, Y. Sun, S. Du, J. Schneeloch, R. Zhong, G. Gu, L. Fu, H. Ding, H.-J. Gao, *Science* **2018**, *362*, 333.
[41] C.-K. Chiu, D. I. Pikulin, M. Franz, *Phys. Rev. B* **2015**, *91*, 165402.
[42] J. Mutch, W.-C. Chen, P. Went, T. Qian, Z. W Ilham, A. Andreev, C.-C. Chen, J.-H. Chu, *Sci. Adv.* **2019**, *5*, eaav9771.
[43] P. Zhang, R. Noguchi, K. Kuroda, C. Lin, K. Kawaguchi, K. Yaji, A. Harasawa, M. Lippmaa, S. Nie, H. Weng, V. Kandyba, A. Giampietri, A. Barinov, Q. Li, G. D. Gu, S. Shin, T. Kondo, *Nat. Commun.* **2021**, *12*, 406.
[44] P. P. Ferreira, A. L. R. Manesco, T. T. Dorini, L. E. Correa, G. Weber, A. J. S. Machado, L. T. F. Eleno, *Phys. Rev. B* **2021**, *103*, 125134.
[45] P. Padmanabhan, S. M. Young, M. Henstridge, S. Bhowmick, P. K. Bhattacharya, R. Merlin, *Phys. Rev. Lett.* **2014**, *113*, 027402.
[46] P. Padmanabhan, F. L. Buessen, R. Tutchton, K. W. C. Kwock, S. Gilinsky, M. C. Lee, M. A. McGuire, S. R. Singamaneni, D. A. Yarotski, A. Paramekanti, J.-X. Zhu, R. P. Prasankumar, *Nat. Commun.* **2022**, *13*, 4473.
[47] C. Vaswani, L. L. Wang, D. H. Mudiyanselage, Q. Li, P. M. Lozano, G. D. Gu, D. Cheng, B. Song, L. Luo, R. H. J. Kim, C. Huang, Z. Liu, M. Mootz, I. E. Perakis, Y. Yao, K. M. Ho, J. Wang, *Phys. Rev. X* **2020**, *10*, 021013.
[48] E. J. Sie, C. M. Nyby, C. D. Pemmaraju, S. J. Park, X. Shen, J. Yang, M. C. Hoffmann, B. K. Ofori-Okai, R. Li, A. H. Reid, S. Weathersby, E. Mannebach, N. Finney, D. Rhodes, D. Chenet, A. Antony, L. Balicas, J. Hone, T. P. Devereaux, T. F. Heinz, X. Wang, A. M. Lindenberg, *Nature* **2019**, *565*, 61.
[49] J. R. Hortensius, D. Afanasiev, A. Sasani, E. Bousquet, A. D. Caviglia, *npj Quantum Mater.* **2020**, *5*, 95.
[50] M.-C. Lee, N. Sirica, S. W. Teitelbaum, A. Maznev, T. Pezeril, R. Tutchton, V. Krapivin, G. A. de la Pena, Y. Huang, L. X. Zhao, G. F. Chen, B. Xu, R. Yang, J. Shi, J.-X. Zhu, D. A. Yarotski, X. G. Qiu, K. A. Nelson, M. Trigo, D. A. Reis, R. P. Prasankumar, *Phys. Rev. Lett.* **2022**, *128*, 155301.
[51] C. v. Korff Schmising, M. Bargheer, M. Kiel, N. Zhavoronkov, M. Woerner, T. Elsaesser, I. Vrejoiu, D. Hesse, M. Alexe, *Phys. Rev. Lett.* **2007**, *98*, 257601.
[52] T. Pezeril, P. Ruello, S. Gougeon, N. Chigarev, D. Mounier, J. M. Breteau, P. Picart, V. Gusev, *Phys. Rev. B* **2007**, *75*, 174307.
[53] G. de Haan, T. J. van den Hooven, P. C. M. Planken, *Opt. Express* **2021**, *29*, 32051.

[54] S. Banerjee, U. Kumar, S.-Z. Lin, *Phys. Rev. B* **2022**, *105*, L180414.
[55] U. Kumar, S. Banerjee, S.-Z. Lin, *Commun. Phys.* **2022**, *5*, 157.
[56] A. de la Torre, D. M. Kennes, M. Claassen, S. Gerber, J. W. McIver, M. A. Sentef, *Rev. Mod. Phys.* **2021**, *93*, 041002.
[57] J. González, F. Guinea, M. A. H. Vozmediano, *Nucl. Phys. B* **1994**, *424*, 595.
[58] D. C. Elias, R. V. Gorbachev, A. S. Mayorov, S. V. Morozov, A. A. Zhukov, P. Blake, L. A. Ponomarenko, I. V. Grigorieva, K. S. Novoselov, F. Guinea, A. K. Geim, *Nat. Phys.* **2011**, *7*, 701.
[59] M. N. Chernodub, M. A. H. Vozmediano, *Phys. Rev. Research* **2019**, *1*, 032002.
[60] J. M. Luttinger, *Phys. Rev.* **1964**, *135*, A1505.
[61] Y. Su, S.-Z. Lin, *Phys. Rev. Lett.* **2020**, *125*, 226401.
[62] M. Stone, *Phys. Rev. B* **2012**, *85*, 184503.
[63] N. R. Cooper, B. I. Halperin, I. M. Ruzin, *Phys. Rev. B* **1997**, *55*, 2344.
[64] D. Schmeltzer, A. Saxena, *Ann. Phys.* **2017**, *385*, 546.
[65] A. Gaikwad, S. Sun, P. Wang, L. Zhang, J. Cano, X. Dai, X. Du, *Commun. Mater.* **2022**, *3*, 94.
[66] J. Liu, Y. Zhou, S. Y Rodriguez, M. A. Delmont, R. A. Welser, T. Ho, N. Sirica, K. McClure, P. Vilmercati, J. W. Ziller, N. Mannella, J. D. Sanchez-Yamagishi, M. T. Pettes, R. Wu, L. A. Jauregui, *Nat. Commun.* **2024**, *15*, 332.
[67] B. Deng, H. Ahn, J. Wang, G. Moon, N. Dongre, C. Lei, G. Scuri, J. Sung, E. Brutschea, K. Watanabe, T. Taniguchi, F. Zhang, M.-H. Jo, H. Park, *Phys. Rev. Lett.* **2025**, *134*, 046301.
[68] Z. Fan, Q.-F. Liang, Y. B. Chen, S.-H. Yao, J. Zhou, *Sci. Rep.* **2017**, *7*, 45667.
[69] Y. Zhang, C. Wang, L. Yu, G. Liu, A. Liang, J. Huang, S. Nie, X. Sun, Y. Zhang, B. Shen, J. Liu, H. Weng, L. Zhao, G. Chen, X. Jia, C. Hu, Y. Ding, W. Zhao, Q. Gao, C. Li, S. He, L. Zhao, F. Zhang, S. Zhang, F. Yang, Z. Wang, Q. Peng, X. Dai, Z. Fang, Z. Xu, C. Chen, X. J. Zhou, *Nat. Commun.* **2017**, *8*, 15512.
[70] N. H. Jo, O. A. Ashour, Z. Shu, C. Jozwiak, A. Bostwick, Y. Wang, E. Downey, S. H. Ryu, K. Sun, T. Kong, S. M. Griffin, E. Rotenberg, *Phys. Rev. B* **2024**, *109*, 235122.
[71] E. A. Peterson, C. A. Lane, J.-X. Zhu, *Adv. Phys. Res.* **2024**, 2300111.
[72] P. Villar Arribi, J.-X. Zhu, T. Schumann, S. Stemmer, A. A. Burkov, O. Heinonen, *Phys. Rev. B* **2020**, *102*, 155141.
[73] X. Yuan, C. Zhang, Y. Liu, A. Narayan, C. Song, S. Shen, X. Sui, J. Xu, H. Yu, Z. An, J. Zhao, S. Sanvito, H. Yan, F. Xiu, *NPG Asia Mater* **2016**, *8*, e325.
[74] G.-H. Lee, D. K. Efetov, W. Jung, L. Ranzani, E. D. Walsh, T. A. Ohki, T. Taniguchi, K. Watanabe, P. Kim, D. Englund, K. C. Fong, *Nature* **2020**, *586*, 42.
[75] D. E. Kharzeev, Q. Li, arXiv **2014**, arXiv.1903.07133.
[76] D. Kharzeev, Q. Li, Brookhaven Science Associates *10657456*, **2020**.
[77] Y. Gao, S. Kaushik, E. J. Philip, Z. Li, Y. Qin, Y. P. Liu, W. L. Zhang, Y. L. Su, X. Chen, H. Weng, D. E. Kharzeev, M. K. Liu, J. Qi, *Nat. Commun.* **2020**, *11*, 720.
[78] R. Li, J. Wang, X.-L. Qi, S.-C. Zhang, *Nat. Phys.* **2010**, *6*, 284.
[79] Y. Baum, E. Berg, S. A. Parameswaran, A. Stern, *Phys. Rev. X* **2015**, *5*, 041046.
[80] M. A. Quantum, M. Aghaee, A. Alcaraz Ramirez, Z. Alam, R. Ali, M. Andrzejczuk, A. Antipov, M. Astafev, A. Barzegar, B. Bauer, J. Becker, U. K. Bhaskar, A. Bocharov, S. Boddapati, D. Bohn, J. Bommer, L. Bourdet, A. Bousquet, S. Boutin, L. Casparis, B. J. Chapman, S. Chatoor, A. W. Christensen, C. Chua, P. Codd, W. Cole, P. Cooper, F. Corsetti, A. Cui, P. Dalpasso, et al., *Nature* **2025**, *638*, 651.

Michael T. Pettes is currently a scientist and manager at the Center for Integrated Nanotechnologies at Los Alamos National Laboratory. His current research interests focus on developing fundamental physical understanding of strain and defect-related materials physics to impart new functionalities relevant to applications in quantum technologies, energy, optical science, and mechanical performance.

Avadh Saxena is currently a condensed matter theorist at Los Alamos National Laboratory. His current research interests focus on modeling topological quantum materials, including Dirac and Weyl semimetals, quantum field theory, and phase transitions from a materials perspective.