

# Accelerated Design of Cost-Effective Thermal/Environmental Barrier Coatings based on High-Entropy Rare Earth Disilicates: A First- Principles Study

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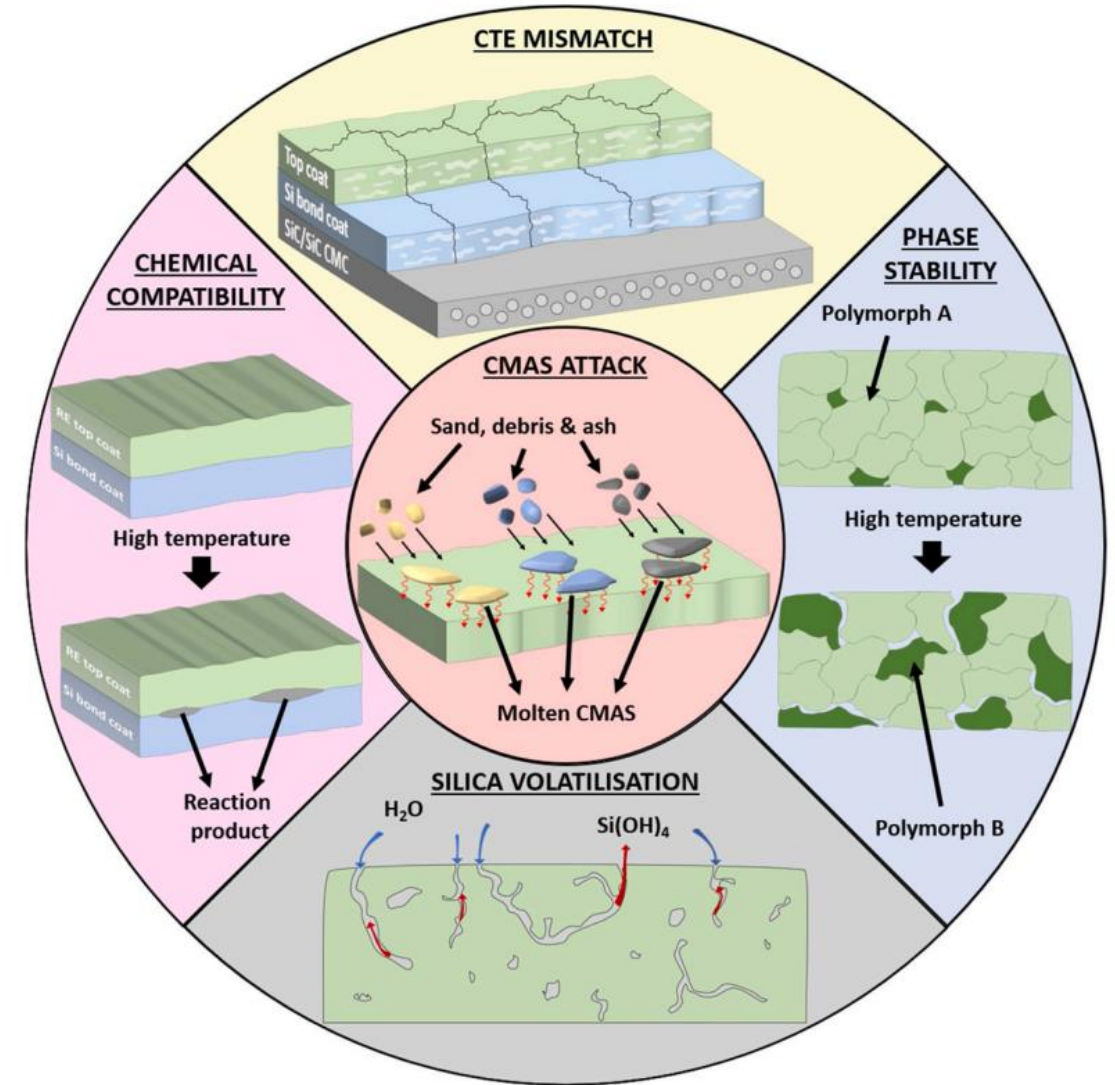
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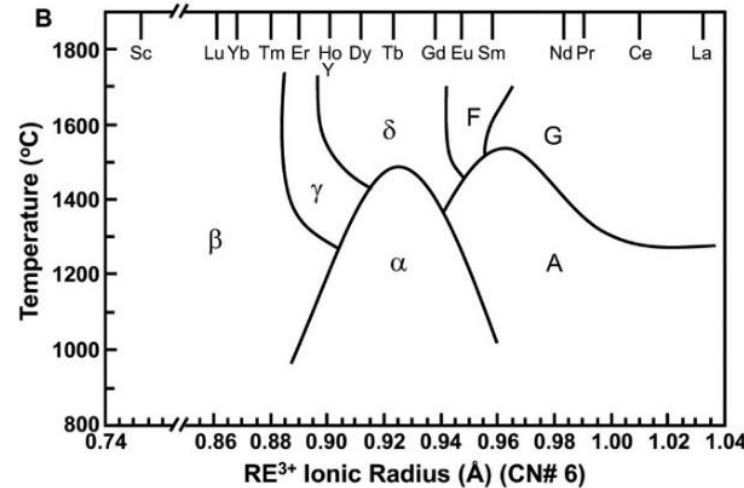
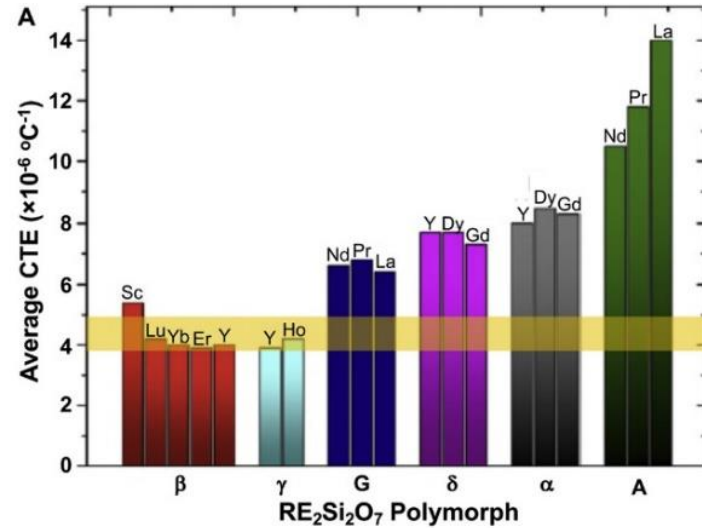
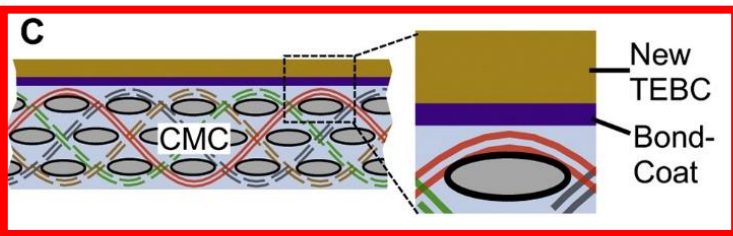
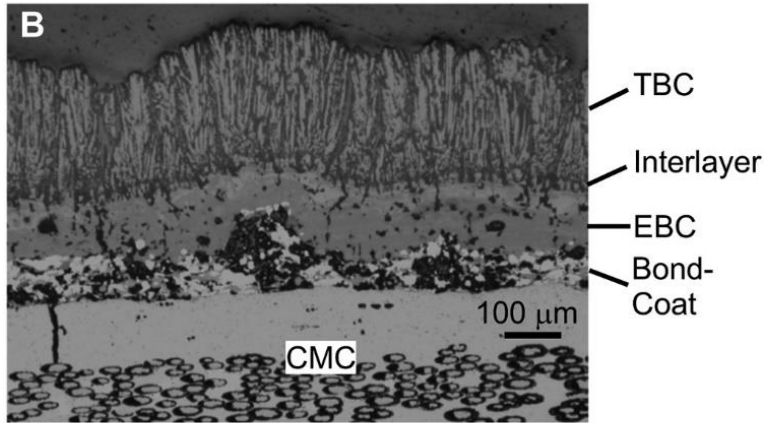
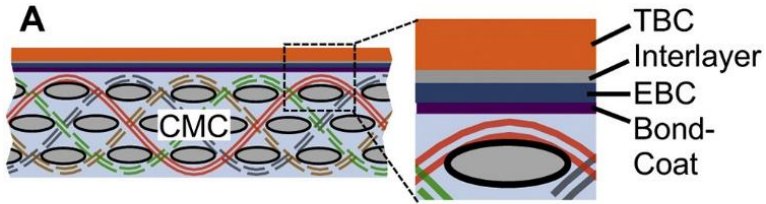


# Introduction

- **Gas turbine engines** have a huge impact on the transportation, energy, and defense sectors of the global economy.
- Experience **extremely high temperatures** (e.g., up to 1800 K) with **corrosive species** in the exhaust gas.
- **SiC-based ceramic matrix composites** can fulfil the stringent requirements to provide an increased service temperature and superior strength at high temperatures.
- Under these harsh conditions, thermal/environmental barrier coatings (**TEBCs**) are required.



# Introduction & Approaches



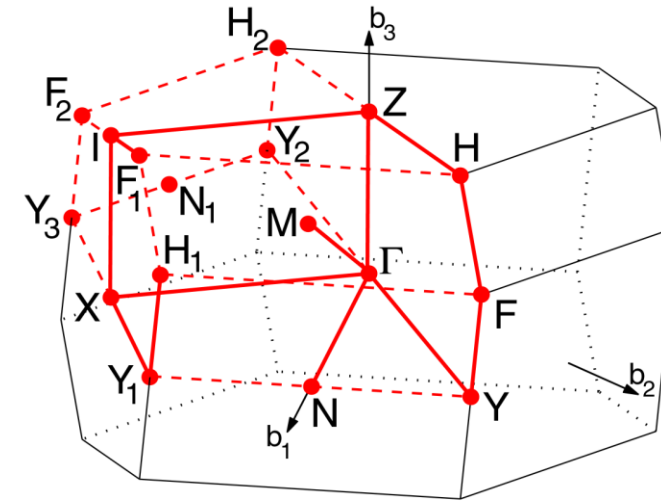
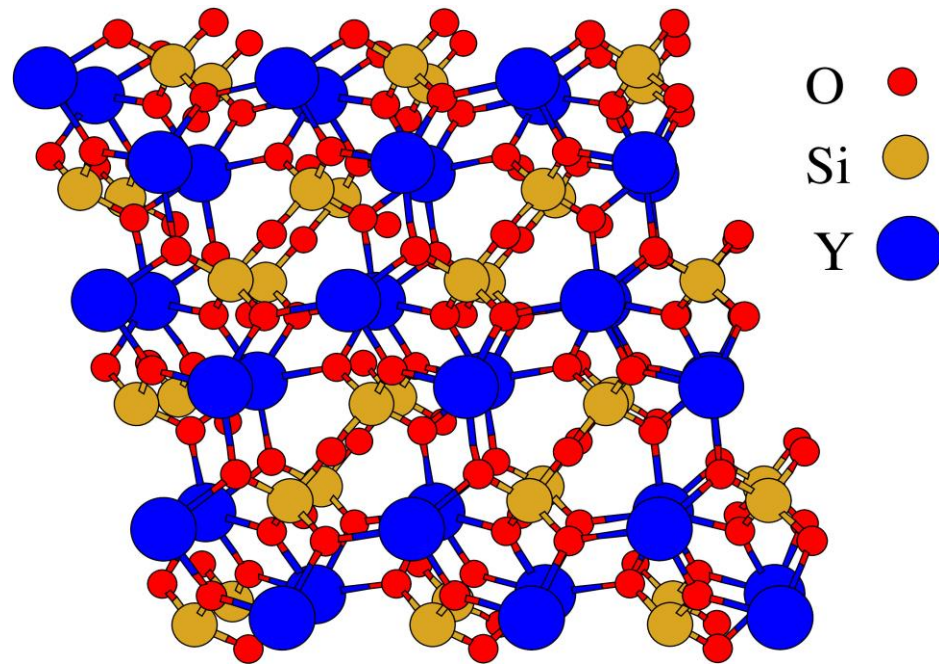
- Corrosion resistance against CMAS attack
- Corrosion resistance against water vapor
- Low thermal conductivity (ideally < 1 Wm<sup>-1</sup>K<sup>-1</sup>)
- CTE match with CMC
- Phase stability at high temperatures
- Fracture toughness

First principles density functional theory calculations to predict:

- ❖ Structure
- ❖ Lattice thermal conductivity
- ❖ Coefficient of thermal expansion
- ❖ Temperature-dependent elastic properties

L.R. Turcer, N.P. Padture, Scripta Mater. 154 (2018) 111-117

# Atomic Structure of $\text{Y}_2\text{Si}_2\text{O}_7$ and $\text{Yb}_2\text{Si}_2\text{O}_7$

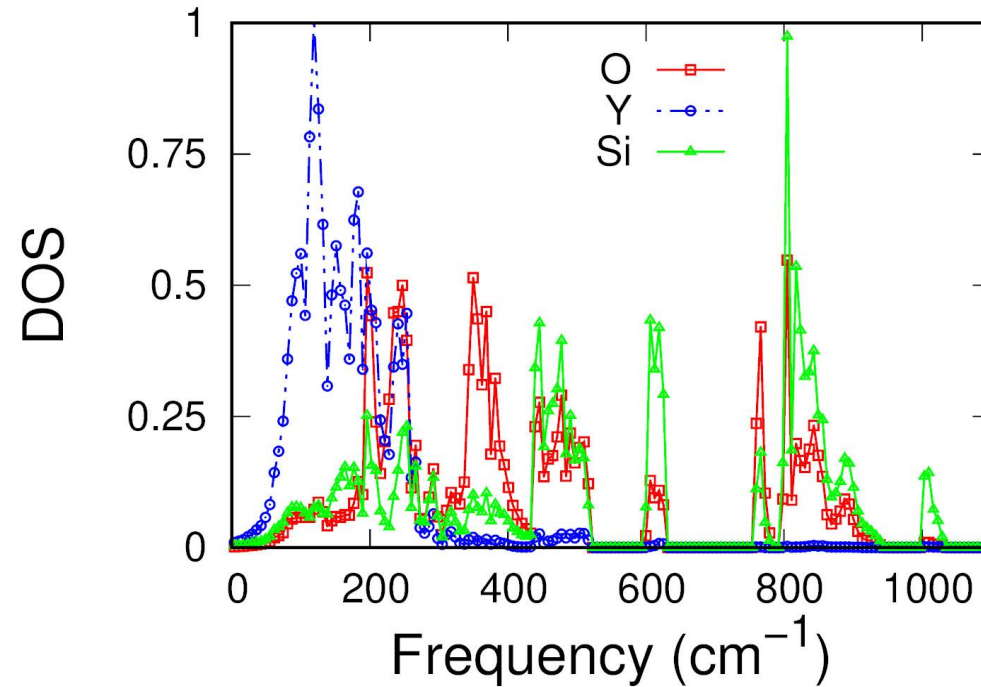
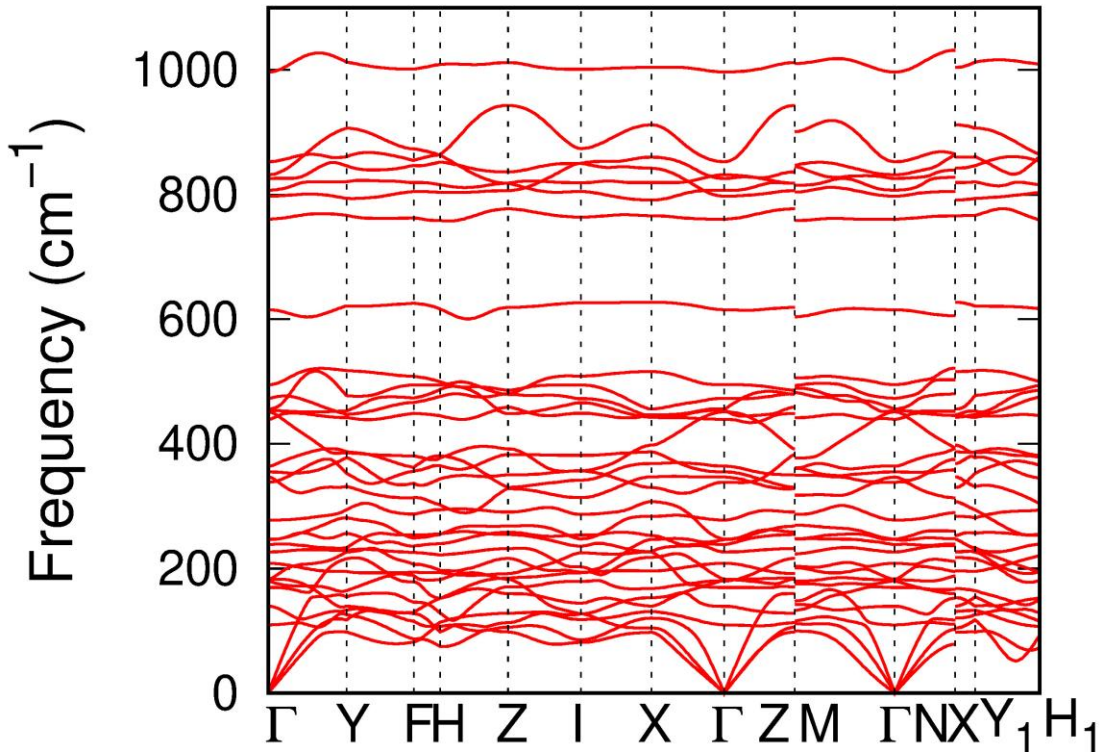


MCLC<sub>3</sub> path:  $\Gamma$ -Y-F-H-Z-I-X- $\Gamma$ -Z|M- $\Gamma$ -N|X-Y<sub>1</sub>-H<sub>1</sub>||I-F<sub>1</sub>

[Setyawan & Curtarolo, DOI: 10.1016/j.commatsci.2010.05.010]

- $\text{Y}_2\text{Si}_2\text{O}_7$   $a=6.88$ ,  $b=8.98$ ,  $c=4.72$ ,
- $\text{Yb}_2\text{Si}_2\text{O}_7$   $a=6.82$ ,  $b=9.01$ ,  $c=4.76$ . **Very close to above values**
- Both in monoclinic system, space group C2/m
- Atomic structure from OQMD.

# Y<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> Phonon Dispersion and partial DOS

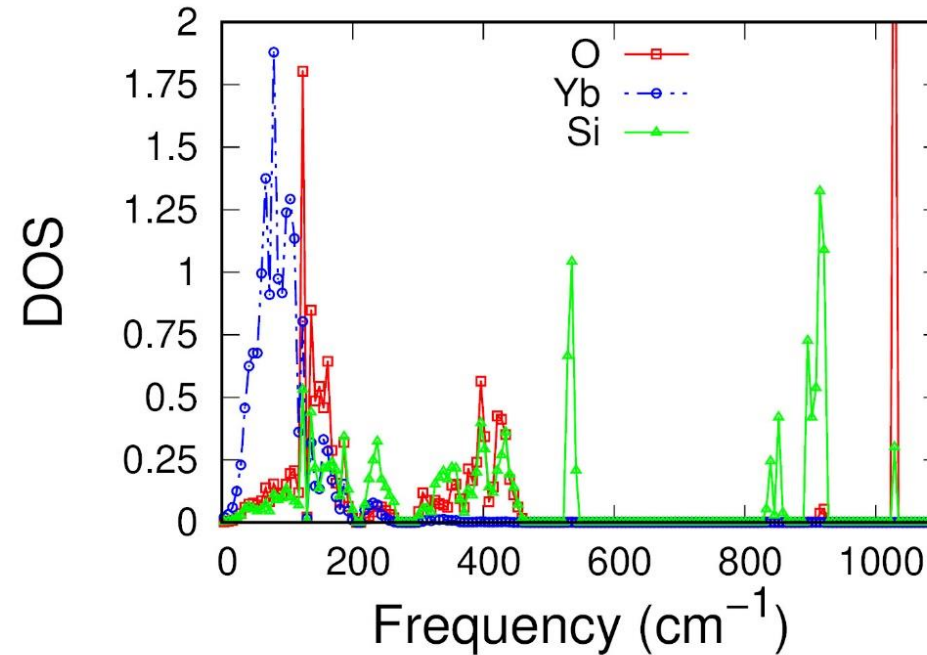
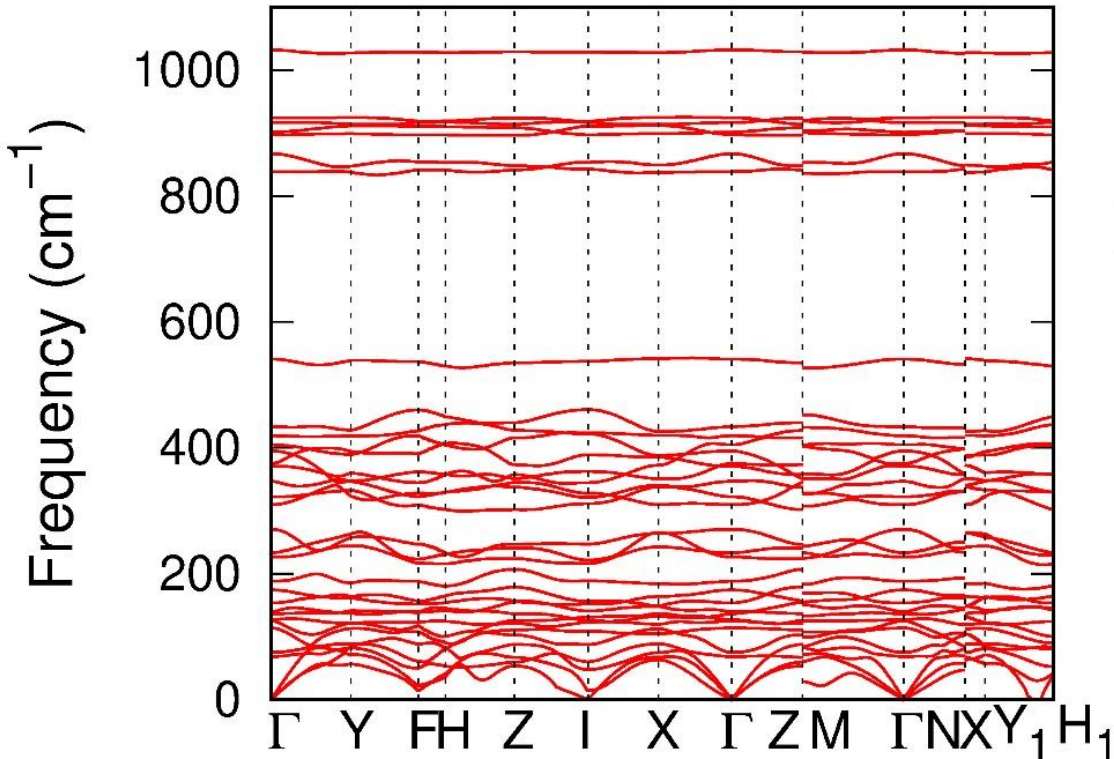


- Phonon dispersion: Y case
- Y contributes to low frequencies.
- O and Si for high frequencies

$$D_{ij}^{\alpha\beta}(\vec{q}) = \frac{1}{\sqrt{m_i m_j}} \sum_{l'l'} \Phi_{li'l'_j}^{\alpha\beta} e^{i\vec{q}\cdot\vec{R}_{l'l'}}$$

[14] L. Chaput, A. Togo, I. Tanaka, and G. Hug, Phonon-phonon interactions in transition metals, Phys. Rev. B 84 (2011) 094302.

# Yb<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> Phonon Dispersion and partial DOS



- Similar shape to Y<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> due to the same atomic structure.
- Yb frequencies are even lower than those of Y due to the heavier atomic mass.
- O and Si for high frequencies: Local vibrational modes.

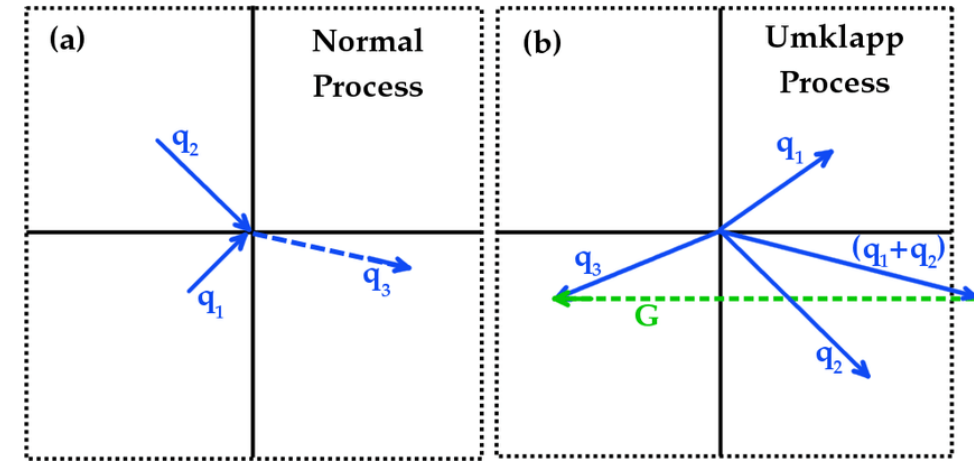
# Lattice Thermal Conductivity: Methods

$$\kappa_i = \frac{1}{3} C_i T^3 \left\{ \int_0^{\Theta_i/T} \frac{\tau_c^i(x) x^4 e^x}{(e^x - 1)^2} dx + \frac{\left[ \int_0^{\Theta_i/T} \frac{\tau_c^i(x) x^4 e^x}{\tau_N^i (e^x - 1)^2} dx \right]^2}{\int_0^{\Theta_i/T} \frac{\tau_c^i(x) x^4 e^x}{\tau_N^i \tau_U^i (e^x - 1)^2} dx} \right\}$$

$$\frac{1}{\tau_N^{\text{LA}}(x)} = \frac{k_B^5 \gamma_{\text{LA}}^2 V}{M \hbar^4 v_{\text{LA}}^5} x^2 T^5$$

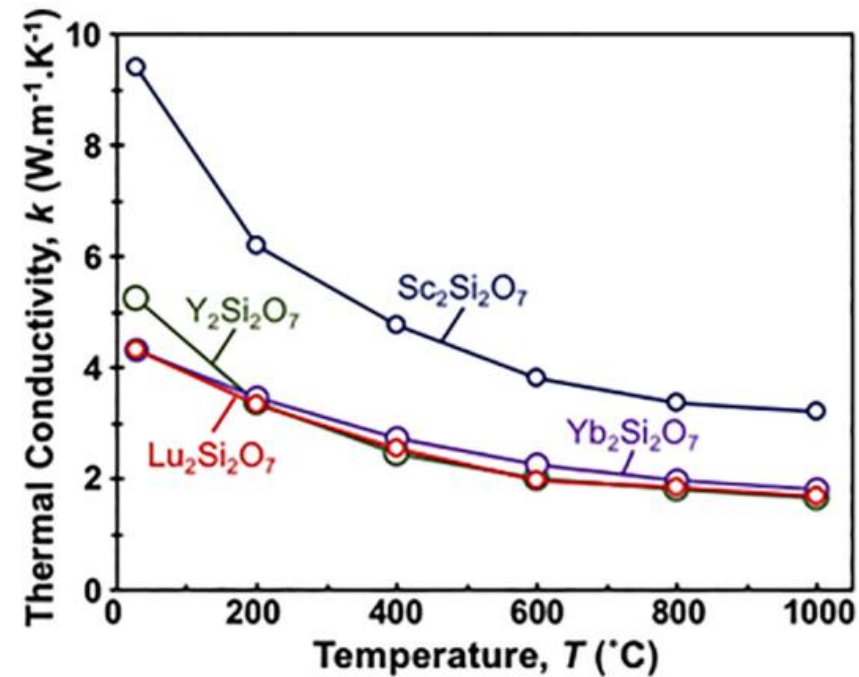
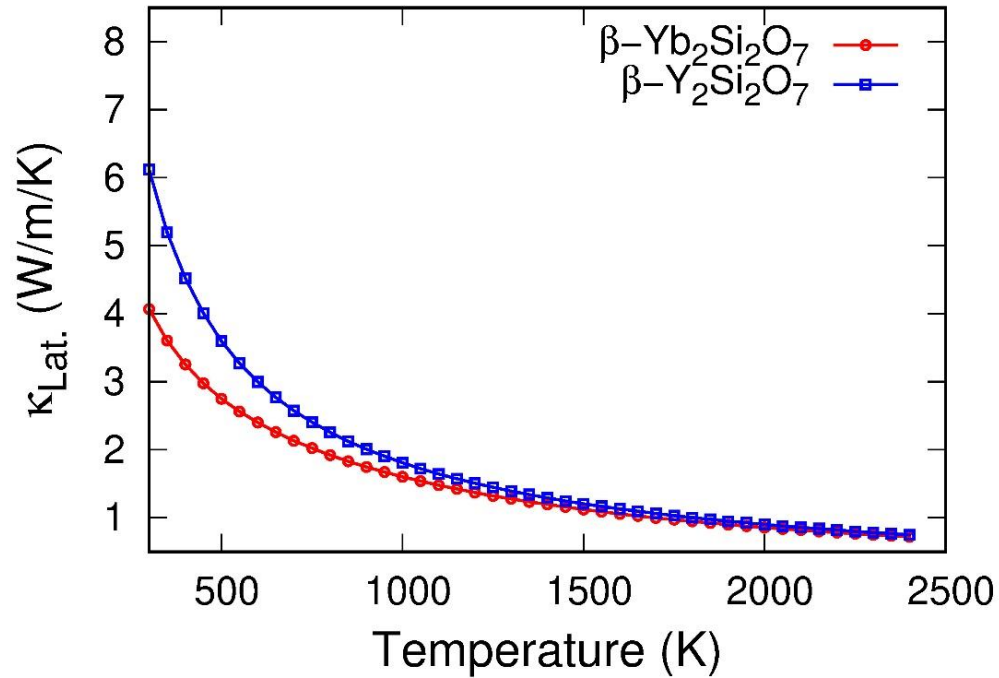
$$\frac{1}{\tau_N^{\text{TA/TA'}}(x)} = \frac{k_B^5 \gamma_{\text{TA/TA'}}^2 V}{M \hbar^4 v_{\text{TA/TA'}}^5} x T^5$$

$$\frac{1}{\tau_U^i(x)} = \frac{k_B^2 \gamma_i^2}{M \hbar v_i^2 \Theta_i} x^2 T^3 e^{-\theta_i/3T}$$



- Debye-Callaway model
- normal phonon scattering
- Umklapp phonon–phonon scattering
- Not considered: impurity, boundary scatterings.

# Lattice Thermal Conductivity: Results

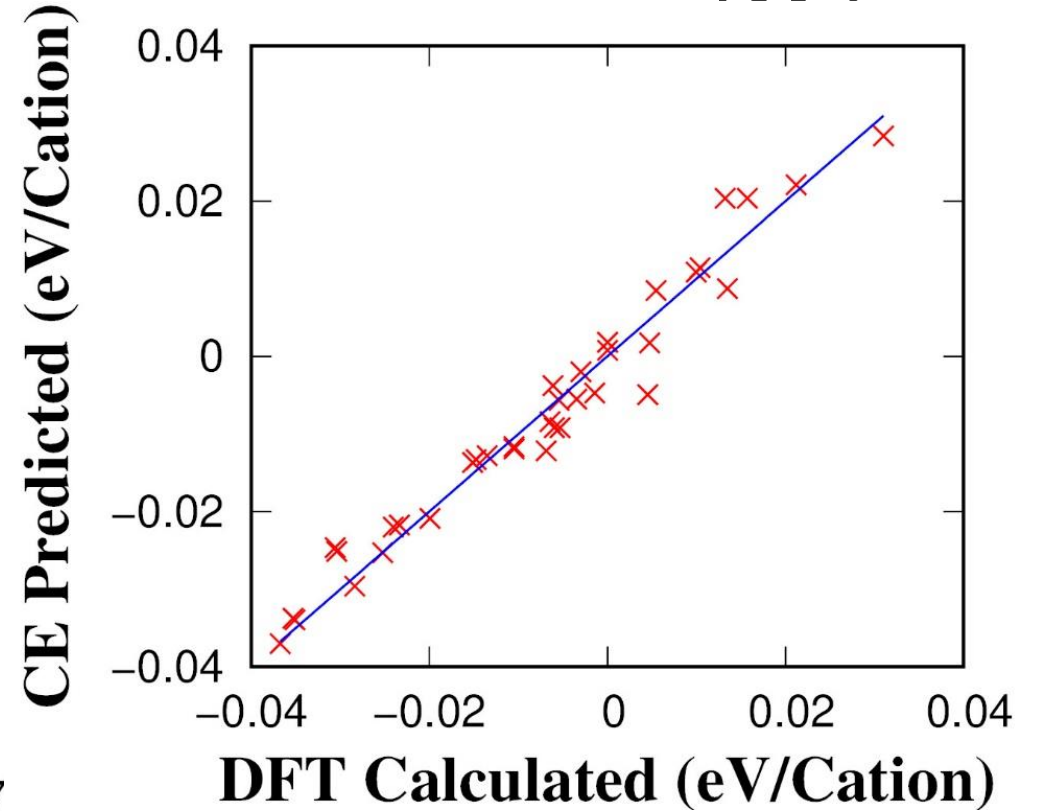
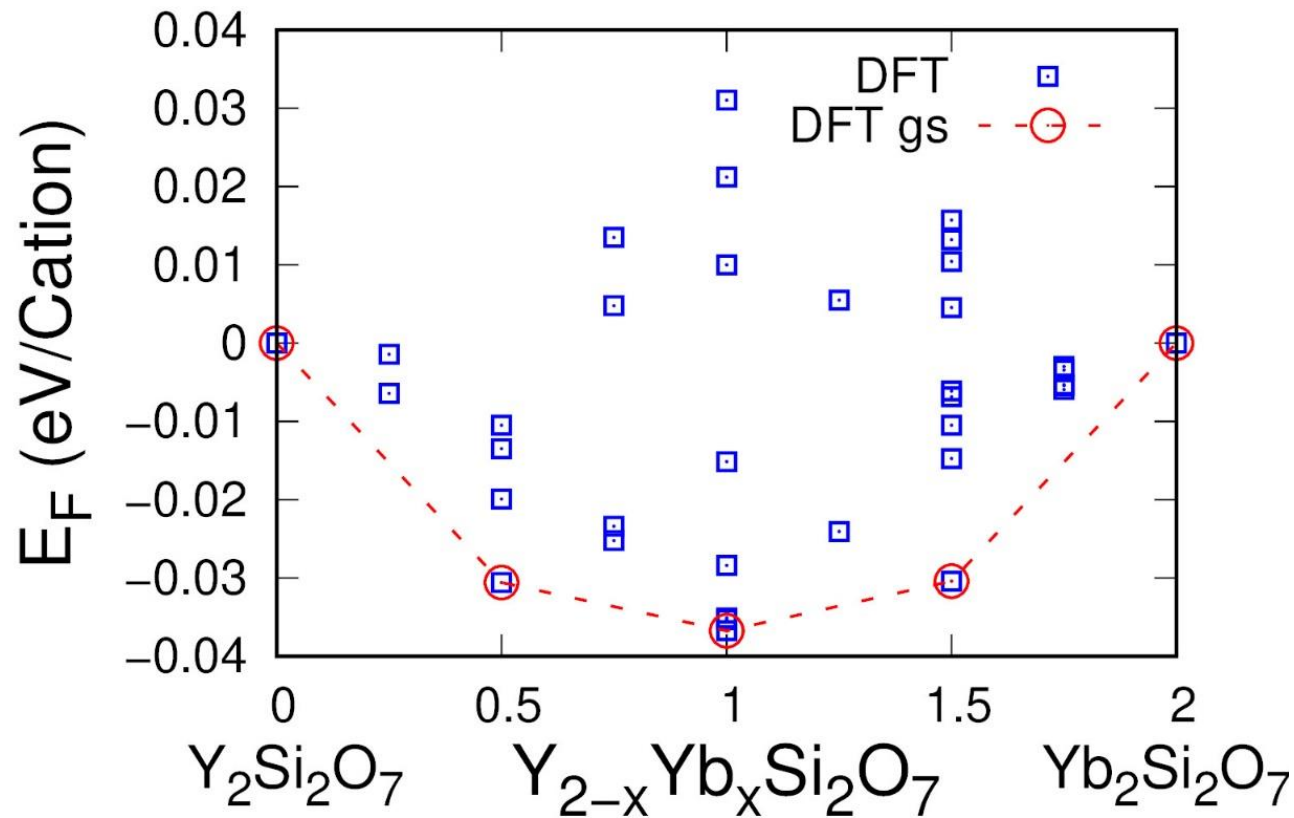


The lattice thermal conductivities of Yb case are lower than those of Y case.

Physical reasons: Heavier Yb atoms induce lower frequencies, thus lower Debye temperatures and slower phonon group velocities. Yb case has larger Gruneisen parameters suggesting stronger anharmonicity than Y case. All these contribute to lowering lattice thermal conductivities.

Z. Tian, L. Zheng, Z. Li, J. Li, J. Wang, J. Eur. Ceram. Soc. 36 (2016) 2813.

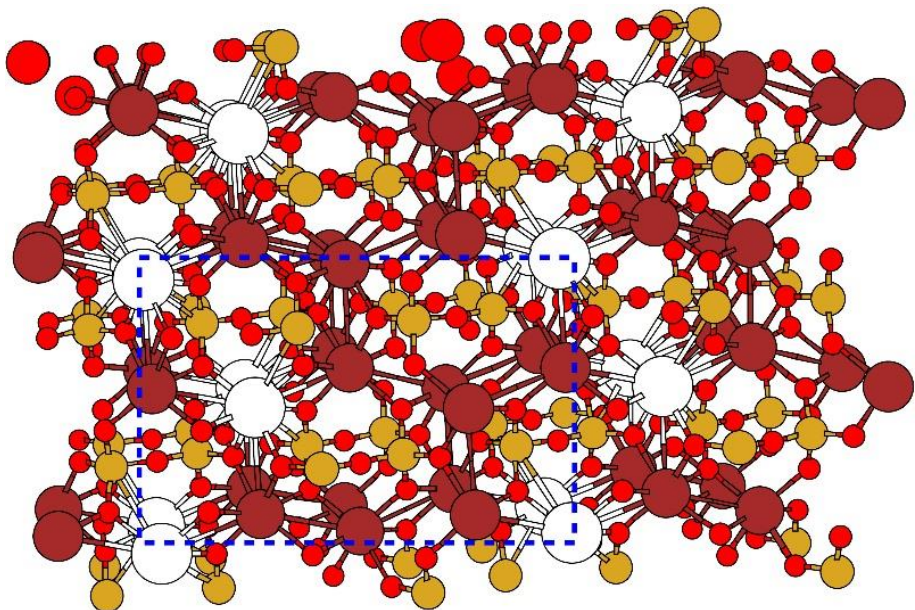
# Cluster Expansion for mixing of $\text{Yb}_2\text{Si}_2\text{O}_7$ and $\text{Y}_2\text{Si}_2\text{O}_7$



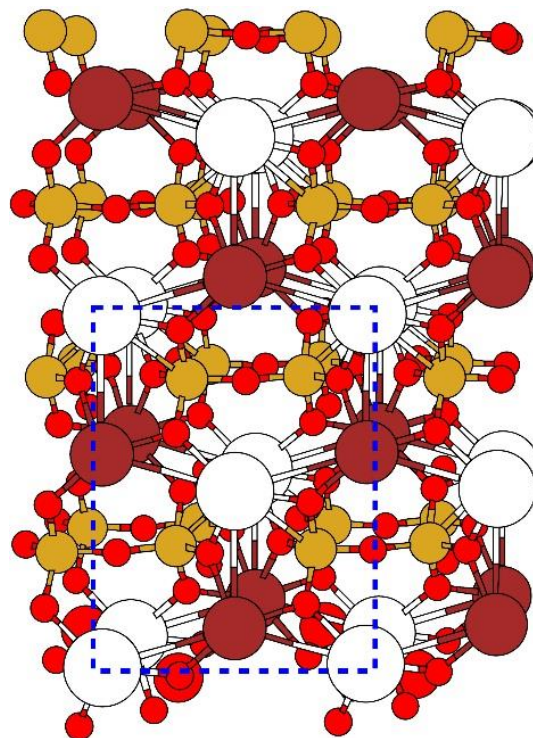
$$E(\sigma) = \sum_{\alpha} m_{\alpha} J_{\alpha} \langle \Pi \sigma_i \rangle$$

Accurate cluster expansion predicts three stable ordered compounds,  $\text{Y}_{1.5}\text{Yb}_{0.5}\text{Si}_2\text{O}_7$ ,  $\text{Y}_1\text{Yb}_1\text{Si}_2\text{O}_7$ ,  $\text{Y}_{0.5}\text{Yb}_{1.5}\text{Si}_2\text{O}_7$ .

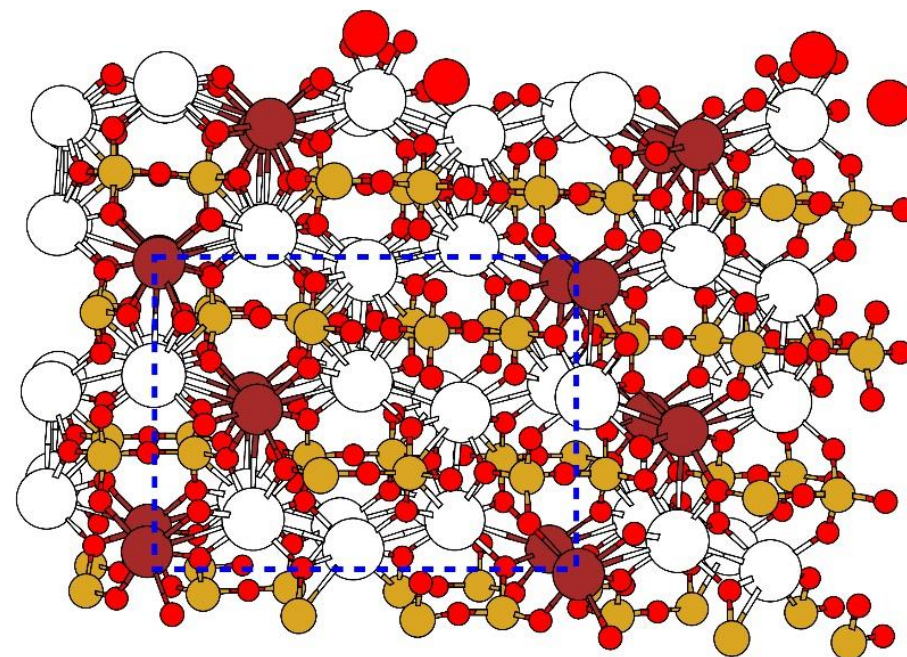
# Predicted New Ordered Structures



$Y_{1.5}Yb_{0.5}Si_2O_7$   
Space group :  $P2_1$

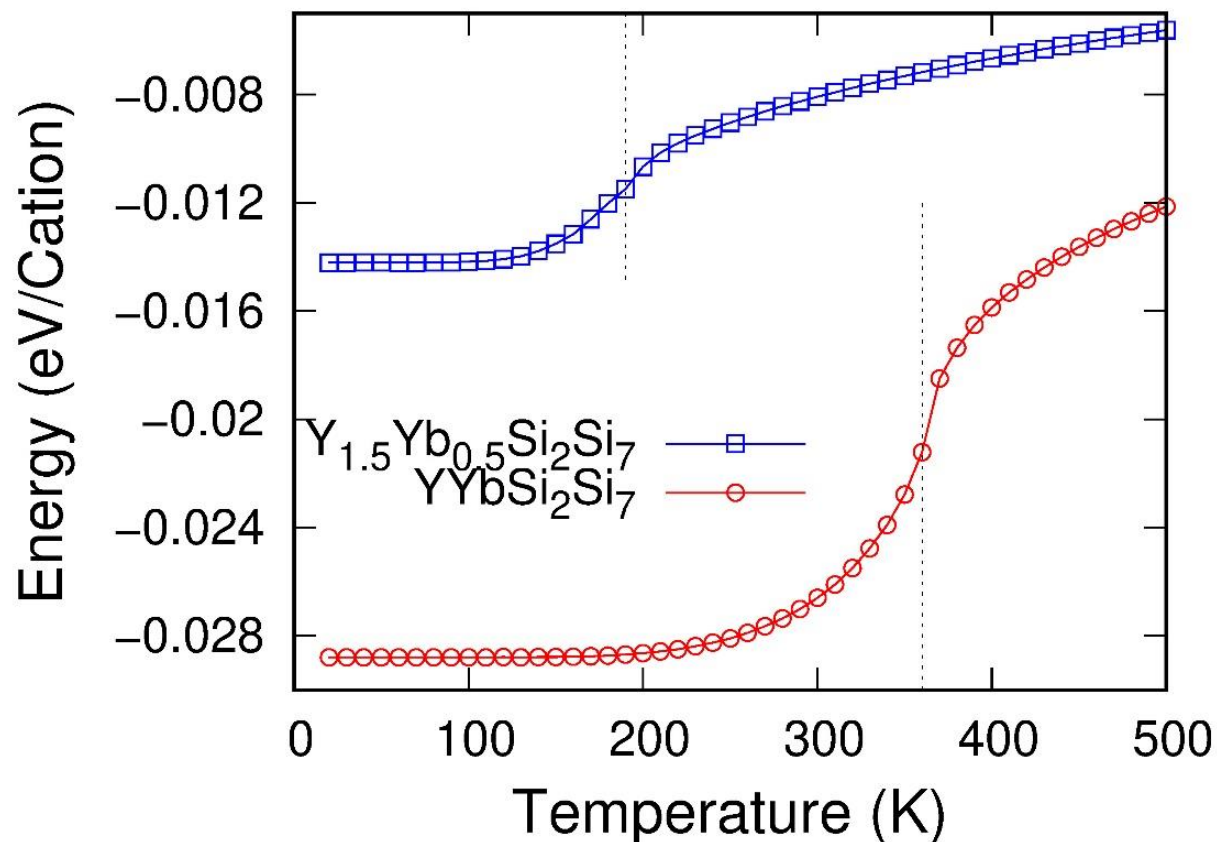


$Y_1Yb_1Si_2O_7$   
Space group :  $C2$



$Y_{0.5}Yb_{1.5}Si_2O_7$   
Space group :  $P2_1$

# Order Disorder Transition Temperatures



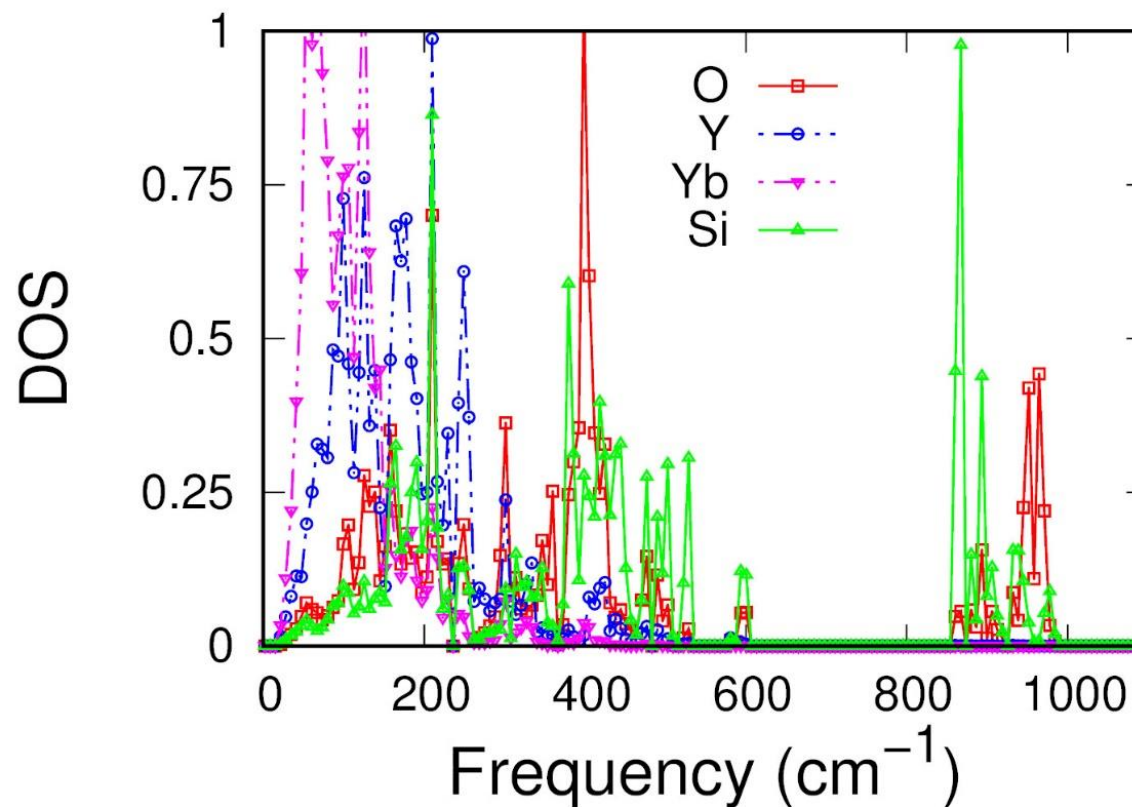
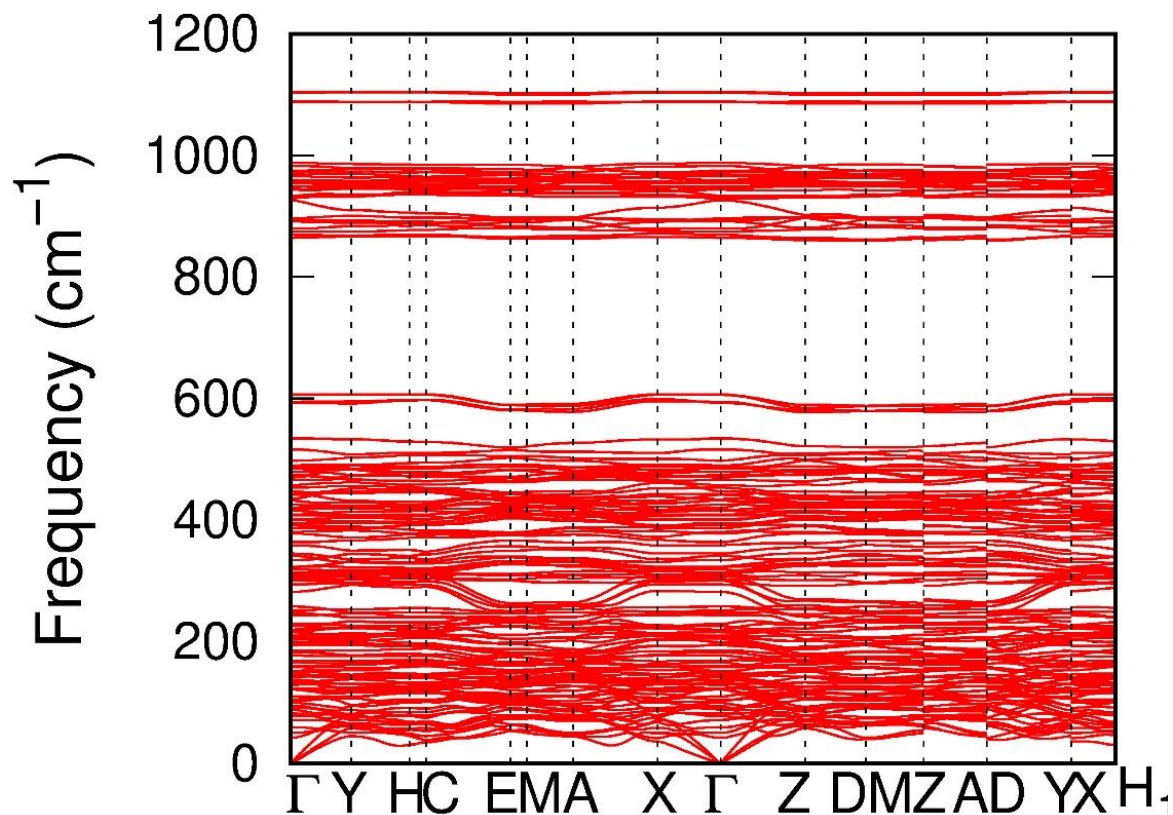
Canonical Monte Carlo simulation for  $YYbSi_2O_7$  based on the spin-polarized cluster expansion.

Order disorder transition temperature: 360 K.

**Novel ordered stable structure prediction !!**

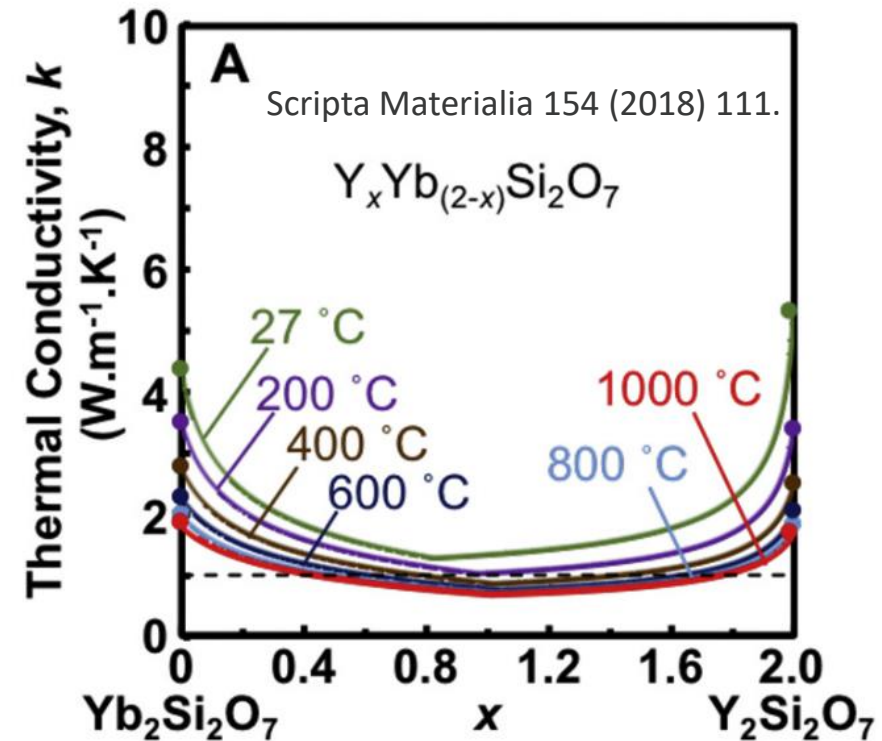
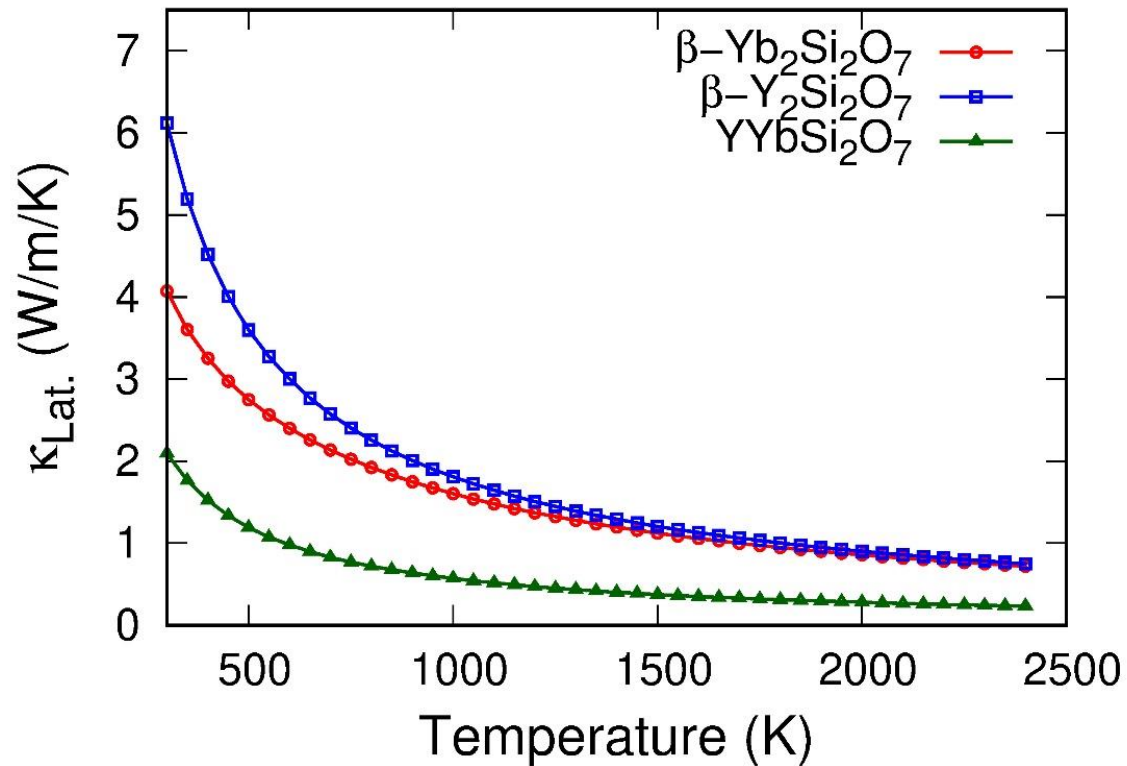
Suggest experimental confirmation.

# YYbSi<sub>2</sub>O<sub>7</sub> Phonon Dispersion and partial DOS



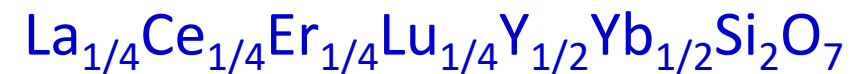
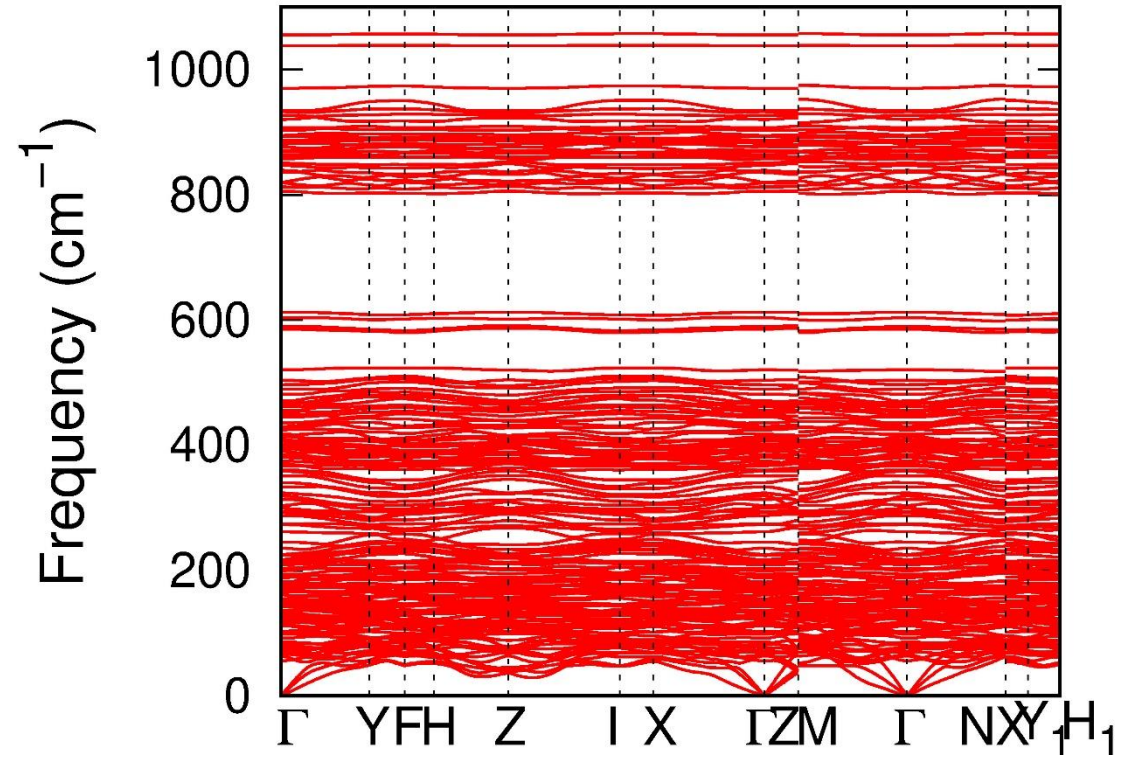
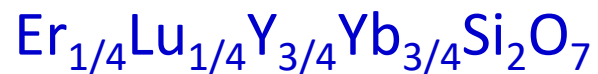
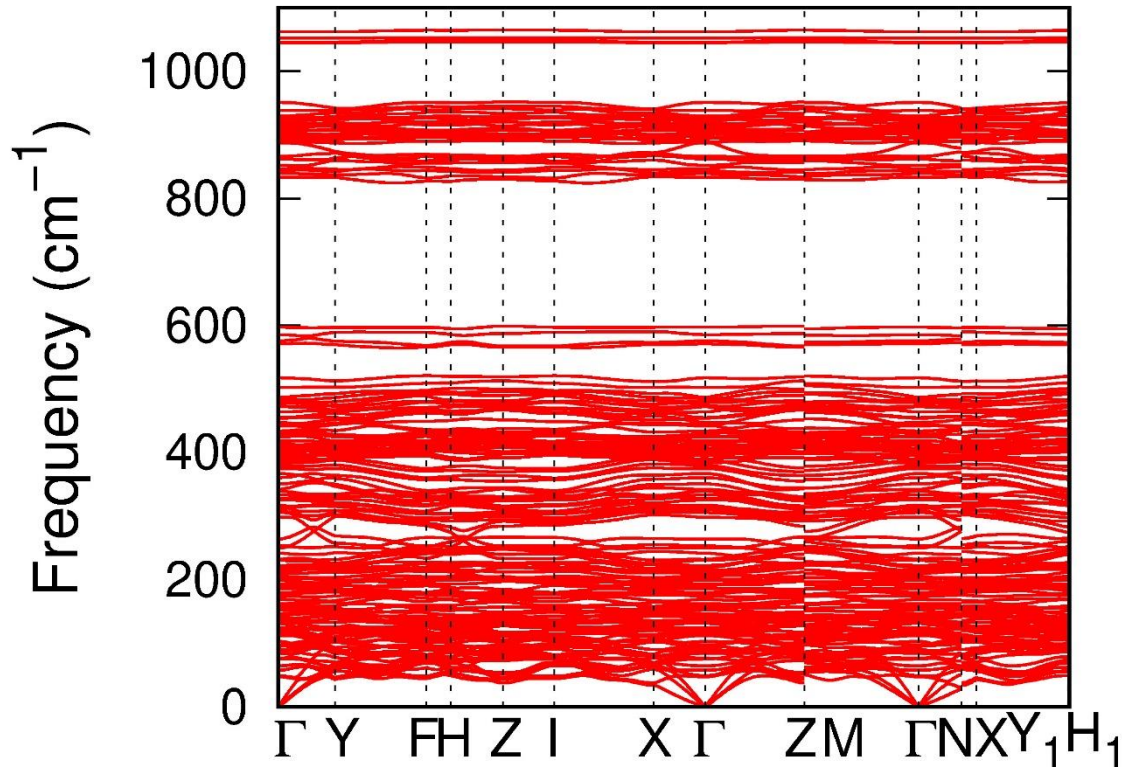
Special quasi-random structure for the solid solution of Y<sub>1</sub>Yb<sub>1</sub>Si<sub>2</sub>O<sub>7</sub>

# YYbSi<sub>2</sub>O<sub>7</sub> Lattice Thermal Conductivity



- $\text{Y}_1\text{Yb}_1\text{Si}_2\text{O}_7$  has **even lower lattice thermal conductivity than both Y and Yb cases**. Calculated results are slightly higher than literature.
- Physical reasons: disordered Y and Yb sublattice forming mass fluctuation and lattice distortion induce strong phonon scatterings.

# More Solid Solutions: Phonon Dispersions

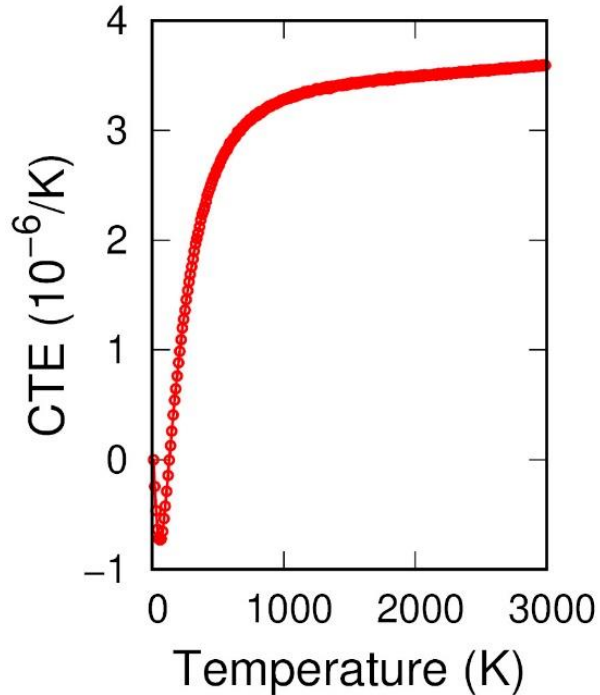
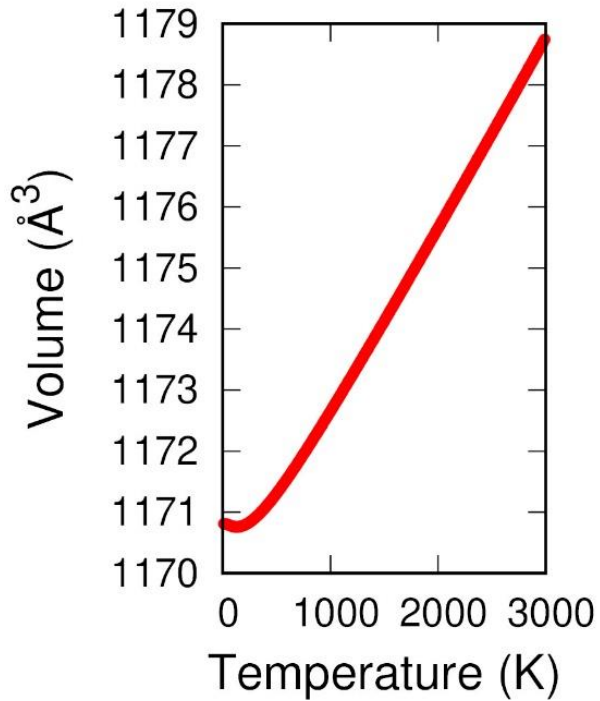
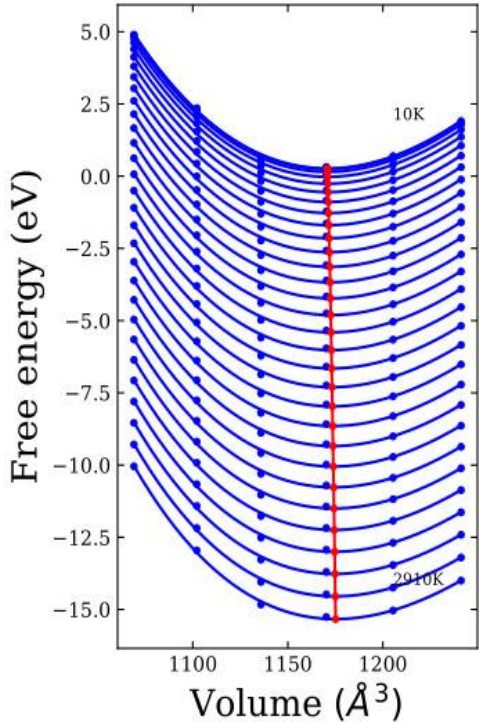


# Thermal Expansion: Quasi-harmonic Approximation

$$F_{total}(T, V) = E_{DFT}(V) + F_{vib}(T, V).$$

$$F_{vib} = k_B T \int dv g(v) \ln \left[ 2 \sinh \frac{hv}{2k_B T} \right],$$

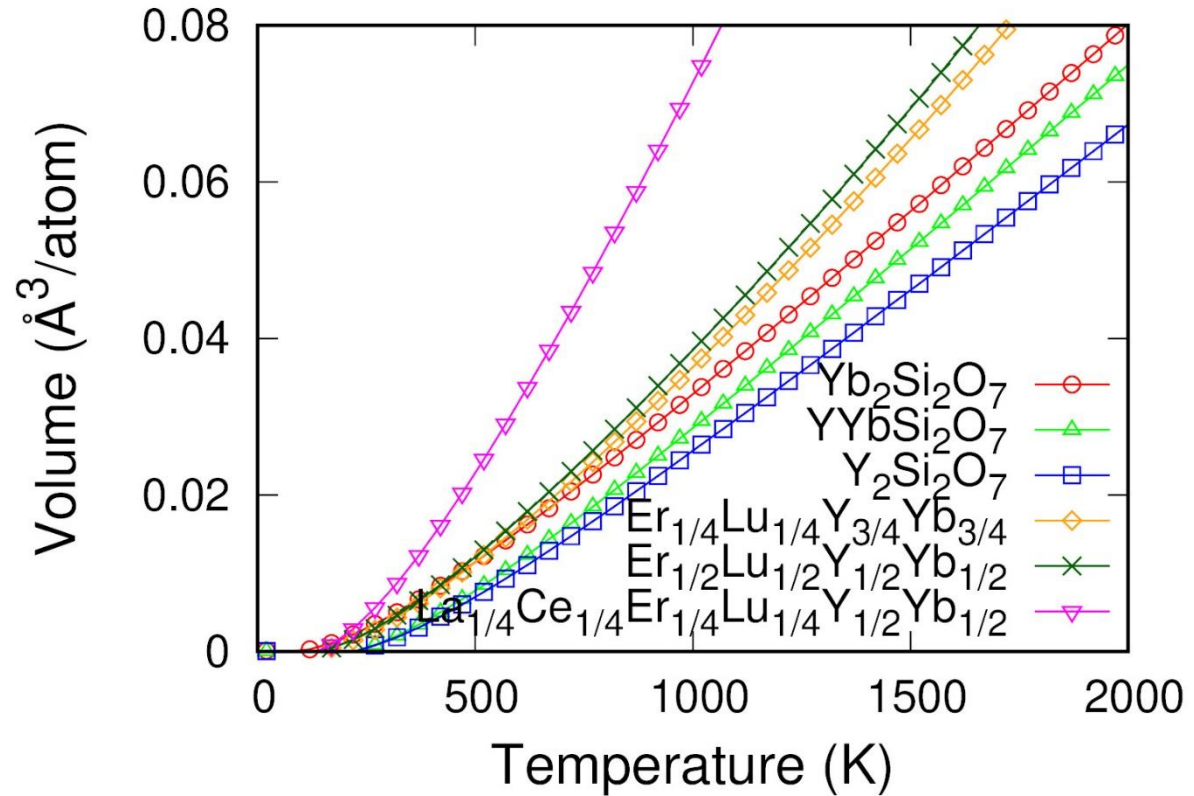
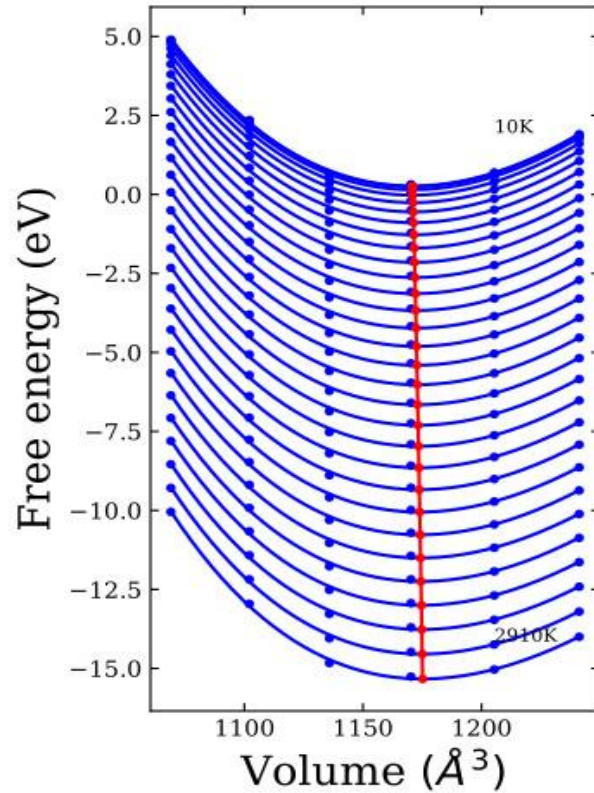
$$\beta = \frac{1}{V} \frac{\partial V}{\partial T}$$



CTE  $Y_2Si_2O_7$ :

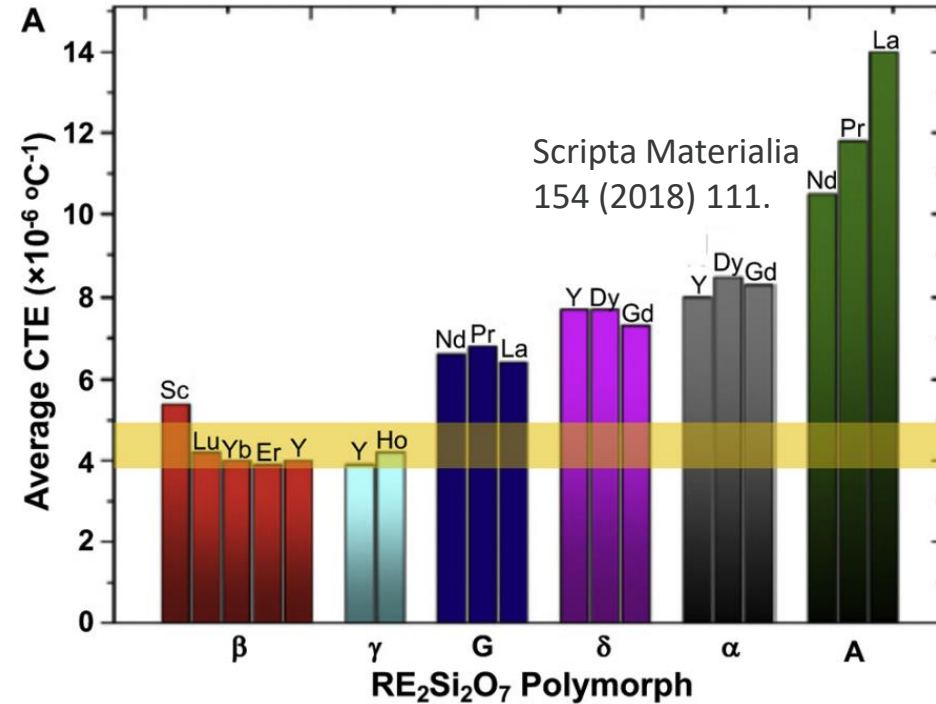
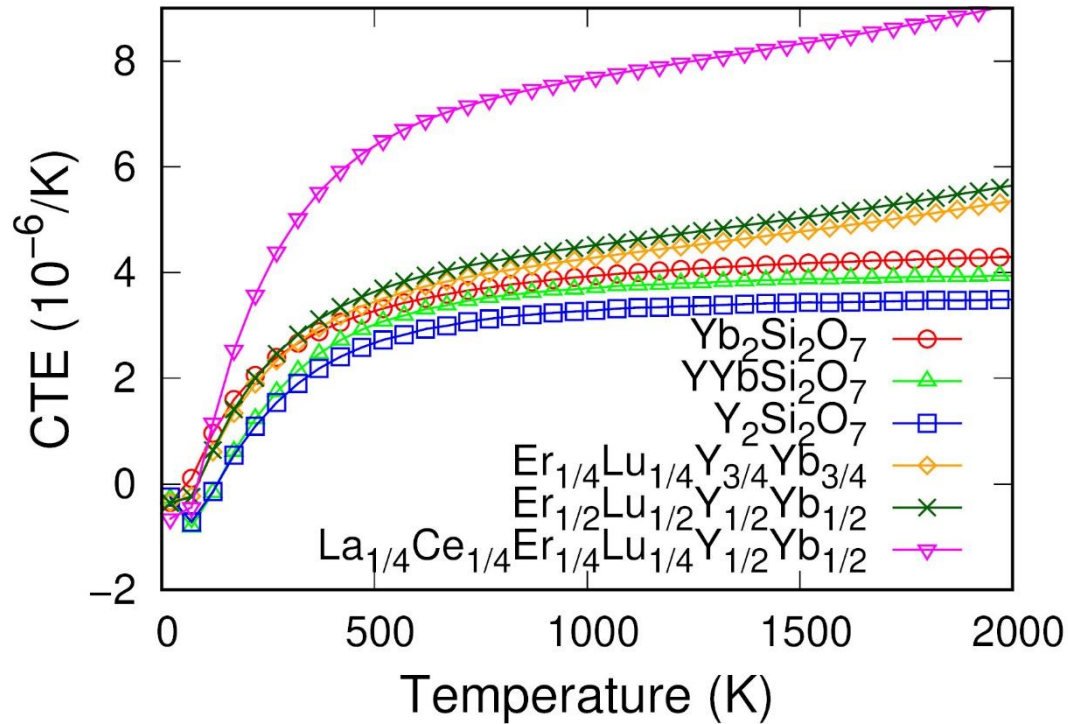
$3.4 \times 10^{-6} \text{ K}^{-1}$  at high T.

# Free Energy and Volume as a Function of T



- Phonon energy  $\rightarrow$  Free energy map as a function of V and T  $F(V,T) \rightarrow$  Volume as a function  $V(T)$ .

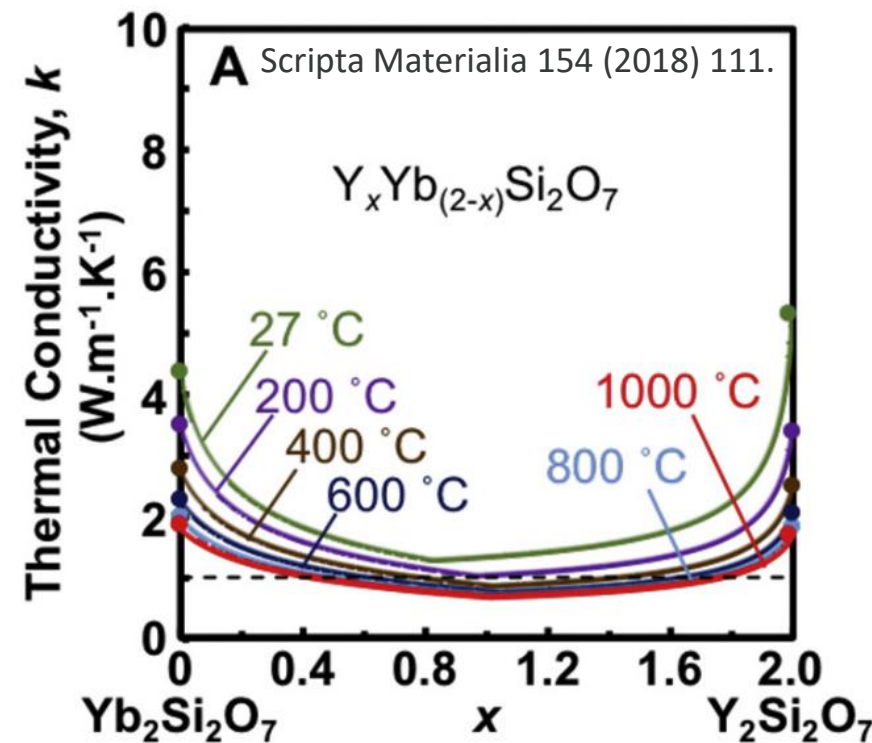
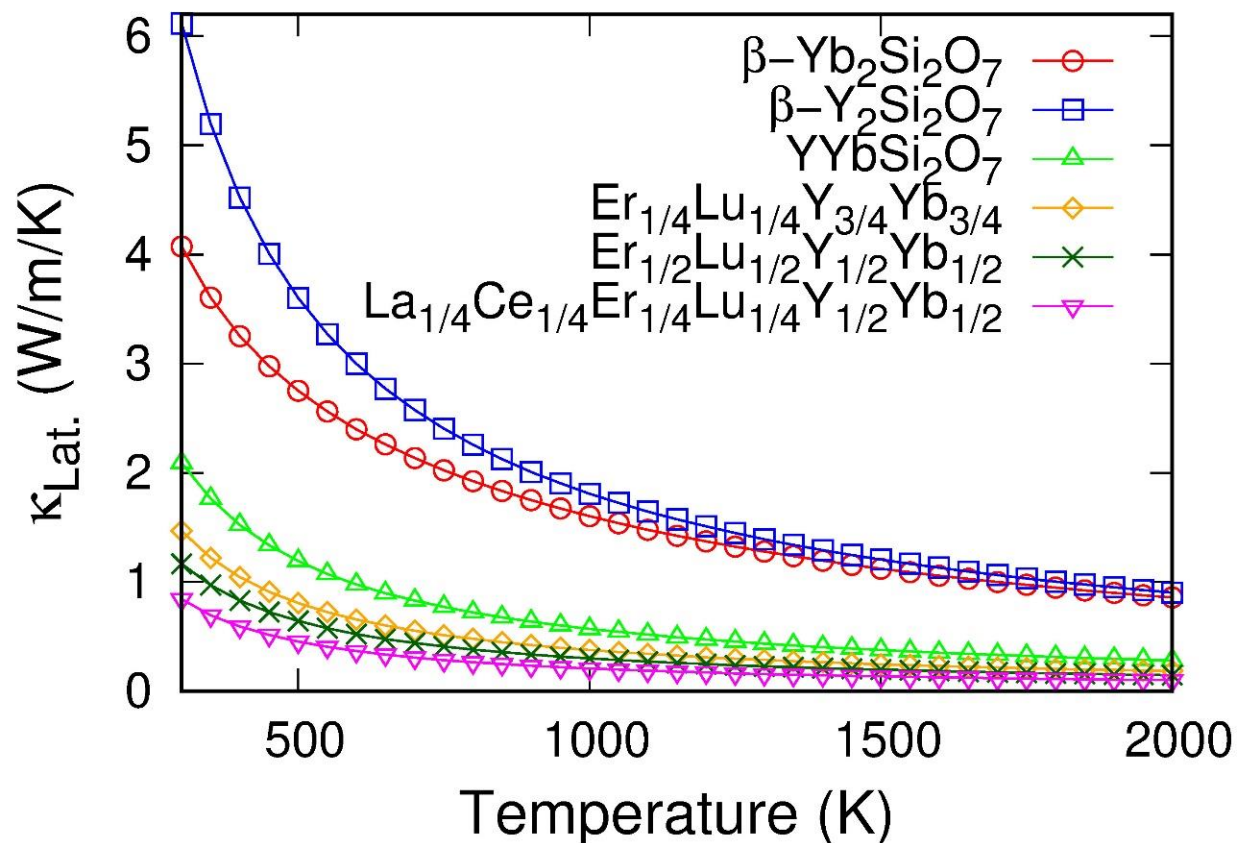
# Comparison of Coefficients of Thermal Expansion



- The coefficients of thermal expansion of  $Y_1Yb_1Si_2O_7$  lie between Y and Yb, slightly higher than the average of Y and Yb.
- $Er_{1/4}Lu_{1/4}Y_{3/4}Yb_{3/4}$  exhibits good match with SiC.
- $La_{1/4}Ce_{1/4}Er_{1/4}Lu_{1/4}Y_{1/2}Yb_{1/2}$  is much higher.
- Beside vibrational entropy, the configuration entropy is also considered due to randomness of Y and Yb in the sublattice.

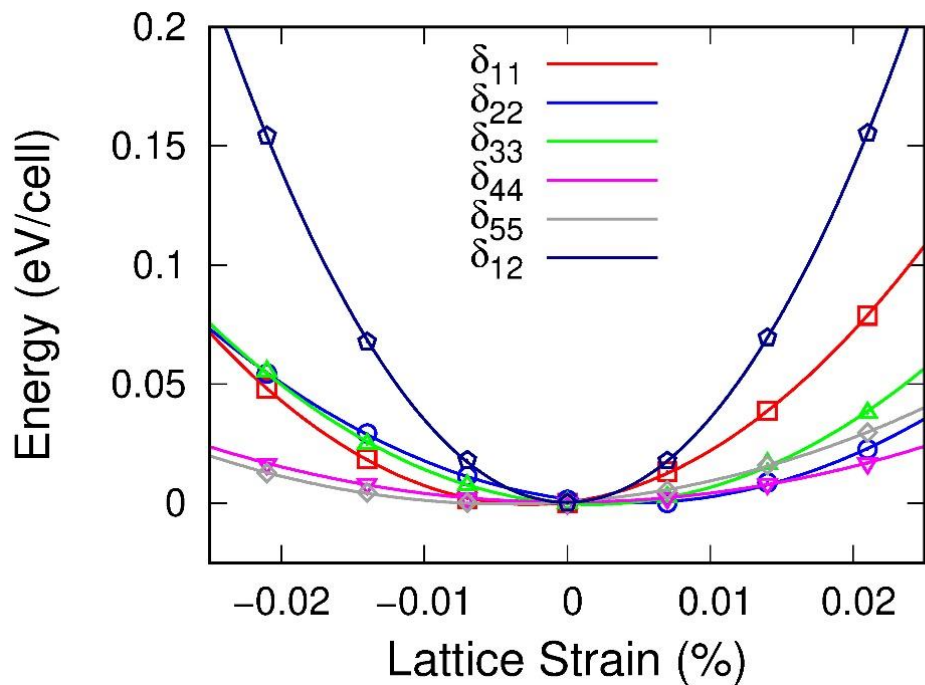
$$S_{Conf} = -R \frac{N_A}{N} \sum_{i_A} \frac{N_{a_i}}{N_A} \ln \frac{N_{a_i}}{N_A}$$

# Comparison of Lattice Thermal Conductivities



- The lattice thermal conductivity of  $\text{Yb}_2\text{Si}_2\text{O}_7$ ,  $\text{Y}_2\text{Si}_2\text{O}_7$ , and  $\text{YYbSi}_2\text{O}_7$  agree well with literature.
- $\text{Er}_{1/4}\text{Lu}_{1/4}\text{Y}_{3/4}\text{Yb}_{3/4}$  and  $\text{La}_{1/4}\text{Ce}_{1/4}\text{Er}_{1/4}\text{Lu}_{1/4}\text{Y}_{1/2}\text{Yb}_{1/2}$  show even lower lattice thermal conductivity.

# Mechanical Properties of $\text{Yb}_2\text{Si}_2\text{O}_7$

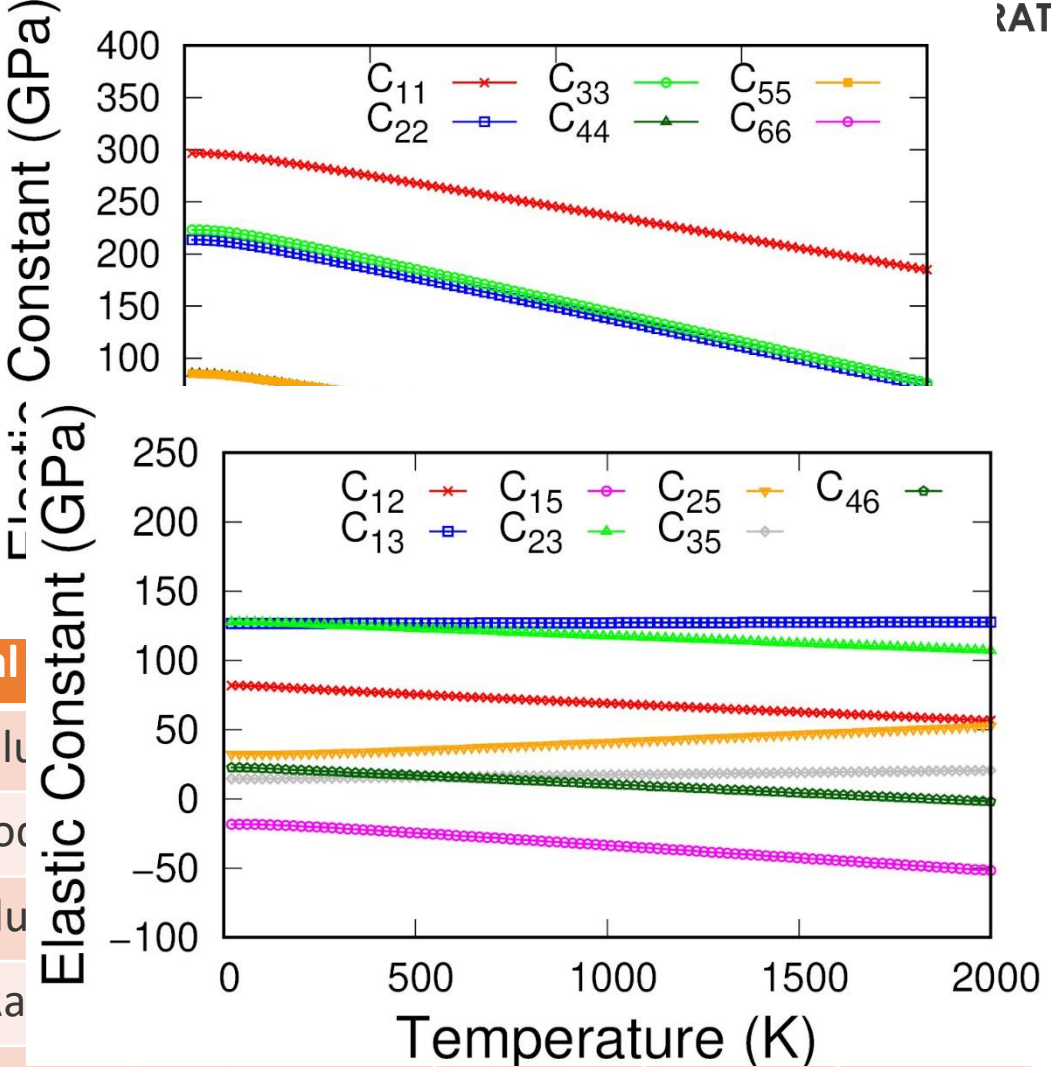


Agree with Literature 162GPa  $\longrightarrow$   
 62GPa  $\longrightarrow$

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$\text{Yb}_2\text{S}$
298.
91.4
110.
0.0
-17.3
0.0

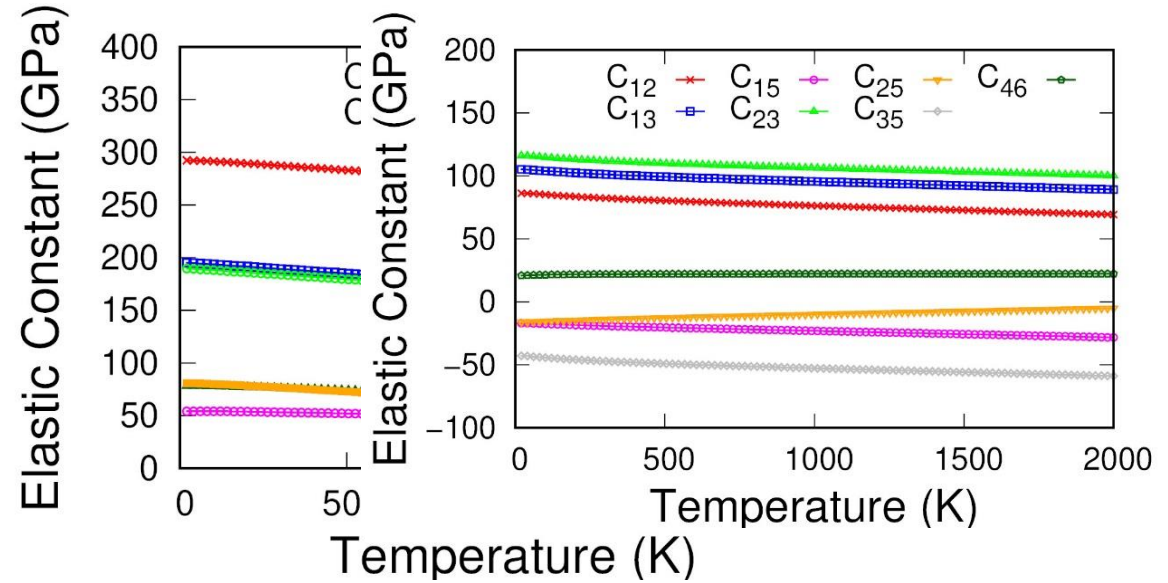
- Mechanical**
- Bulk Modulus
- Young's Modulus
- Shear Modulus
- Poisson's Ratio
- Vickers Hardness (GPa)



7.29	5.04	6.15
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# Mechanical Properties of $Y_2Si_2O_7$

$Y_2Si_2O_7$ Stiffness Tensor (GPa)					
272.6	78.4	96.9	0.0	-15.3	0.0
78.4	190.8	107.4	0.0	32.4	0.0
96.9	107.4	184.9	0.0	13.8	0.0
0.0	0.0	0.0	75.5	0.0	20.2
-15.3	32.4	13.8	0.0	77.2	0.0
0.0	0.0	0.0	20.2	0.0	52.2



Agree with Literature 170GPa  $\longrightarrow$   
65GPa  $\longrightarrow$

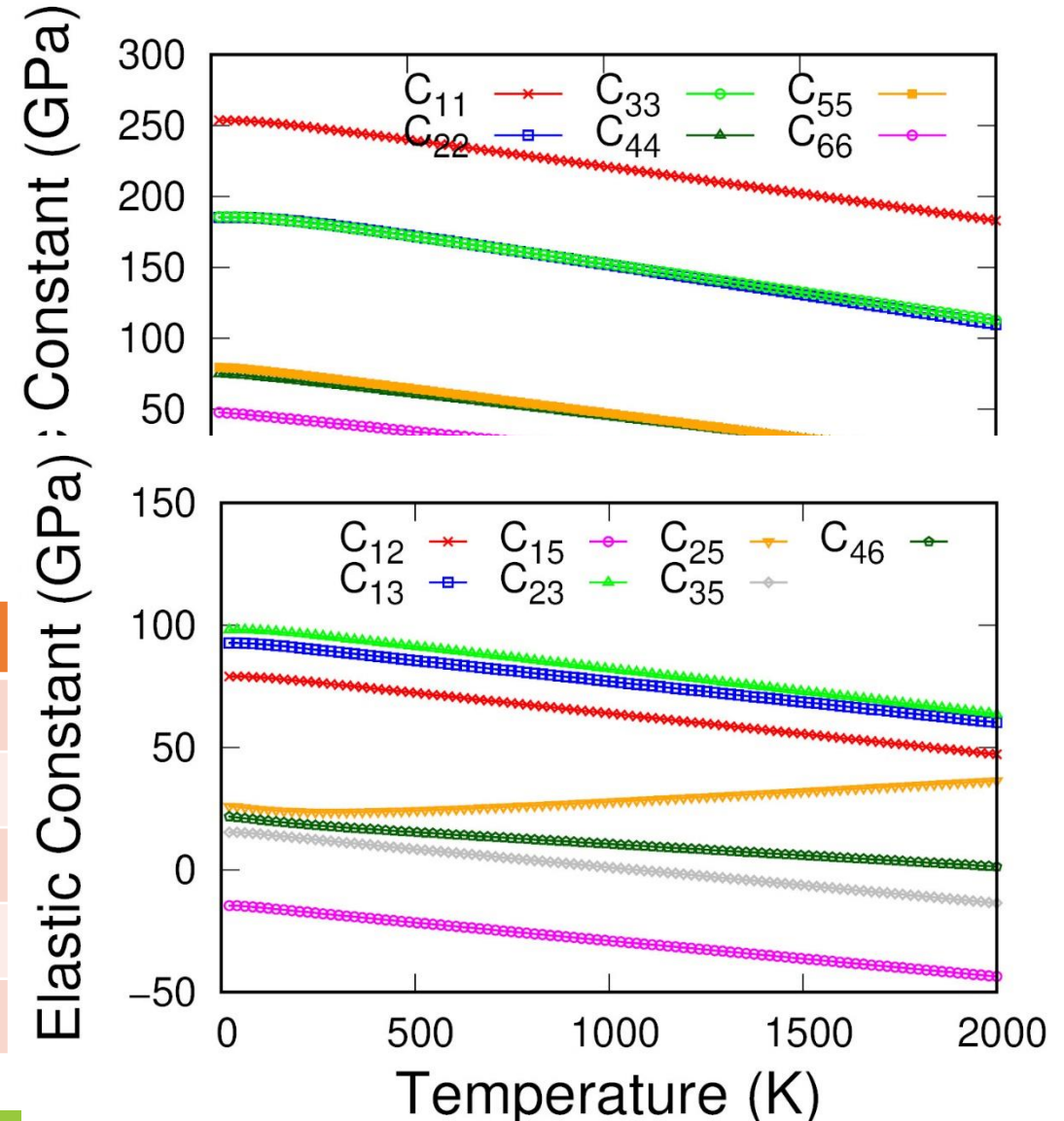
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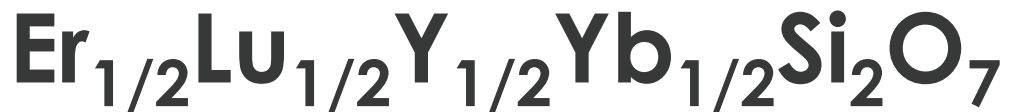
Mechanical Properties	Voigt	Reuss	Hill
Bulk Modulus B (GPa)	134.9	130.2	132.5
Young's Modulus E(GPa)	168.8	145.7	157.3
Shear Modulus G(GPa)	65.4	55.5	60.4
Poisson's Ratio $\nu$	0.29	0.31	0.30
Vickers Hardness (GPa)	6.83	4.69	5.74

# Mechanical Properties of $\text{YYbSi}_2\text{O}_7$

$\text{YYbSi}_2\text{O}_7$ Stiffness Tensor (GPa)					
258.4	74.4	91.8	0.0	-14.3	0.0
74.4	184.6	99.9	0.0	25.5	0.0
91.8	99.9	183.6	0.0	8.53	0.0
0.0	0.0	0.0	75.4	0.0	15.5
-14.3	25.5	8.53	0.0	75.1	0.0
0.0	0.0	0.0	15.5	0.0	49.4

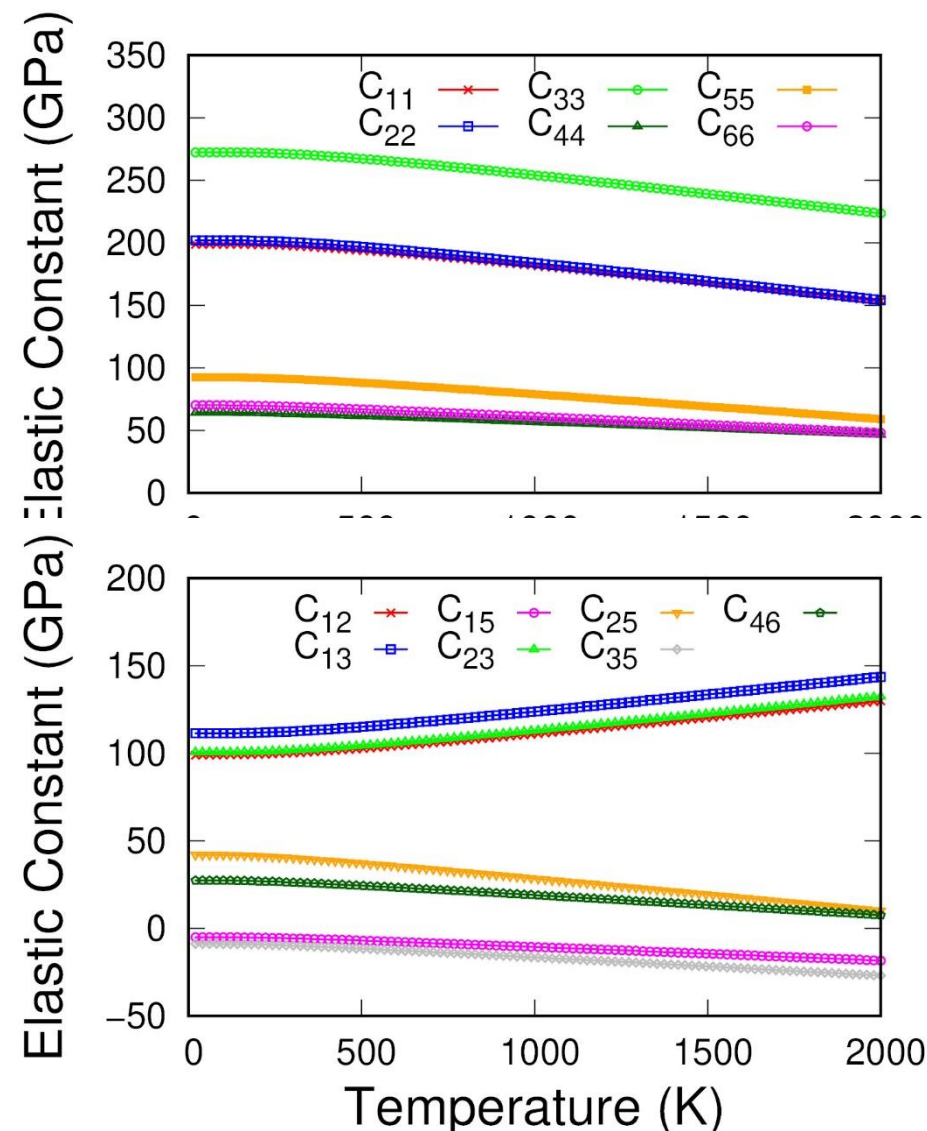
Mechanical Properties	Voigt	Reuss	Hill
Bulk Modulus B (GPa)	128.7	125.8	127.3
Young's Modulus E(GPa)	164.7	147.3	156.1
Shear Modulus G(GPa)	64.0	56.4	60.2
Poisson's Ratio $\nu$	0.29	0.31	0.30
Vickers Hardness (GPa)	7.00	5.24	6.11





$\text{Er}_{1/4}\text{Lu}_{1/4}\text{Y}_{3/4}\text{Yb}_{3/4}\text{Si}_2\text{O}_7$ Stiffness Tensor (GPa)					
201.7	101.7	115.5		-5.1	
101.7	204.1	104.7		39.1	
115.5	104.7	277.1		-16.6	
			65.8		26.1
-5.1	39.1	-16.6		93.4	
			26.1		71.6

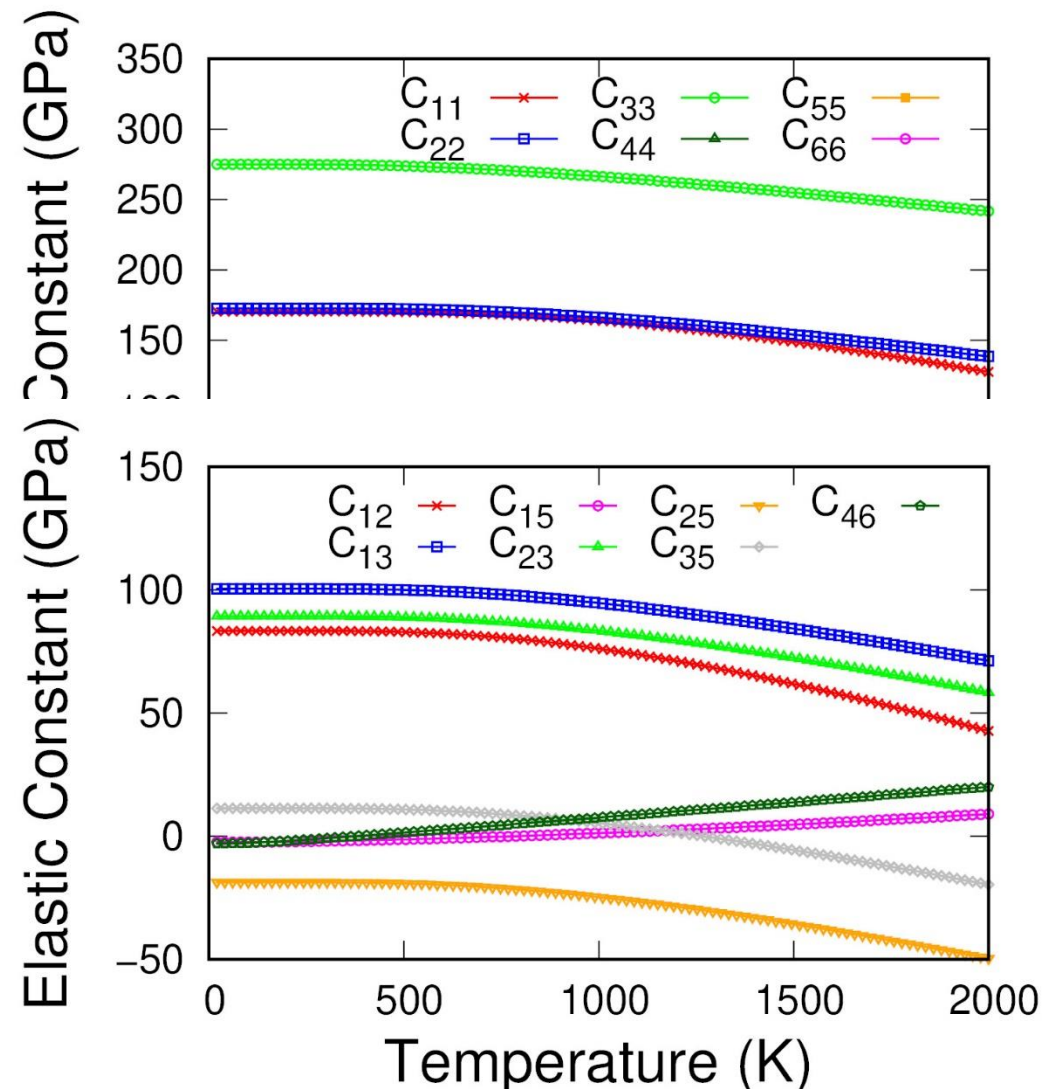
Mechanical Properties	Voigt	Reuss	Hill
Bulk Modulus B (GPa)	147.4	142.4	144.9
Young's Modulus E (GPa)	181.8	155.7	168.9
Shear Modulus G (GPa)	70.2	59.1	64.7
Poisson's Ratio $\nu$	0.29	0.32	0.31
Vickers Hardness (GPa)	7.05	4.73	5.87



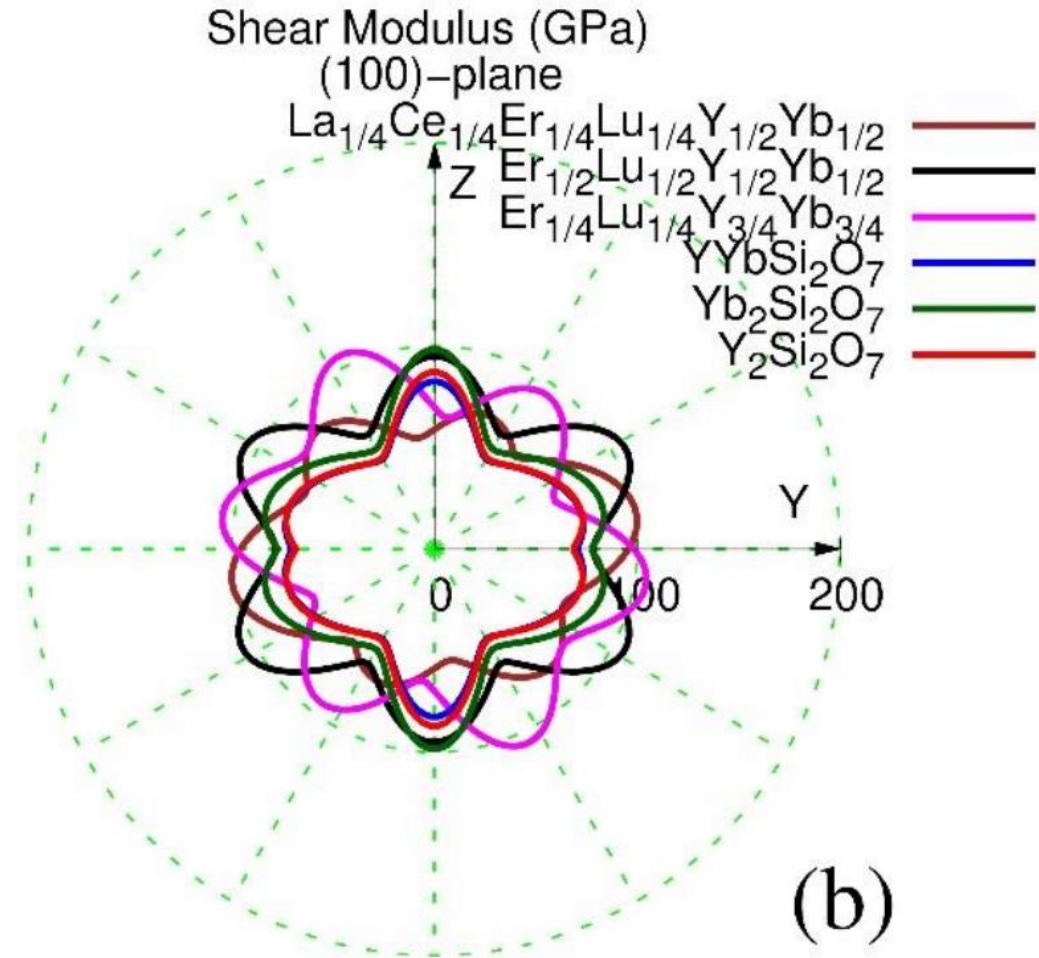
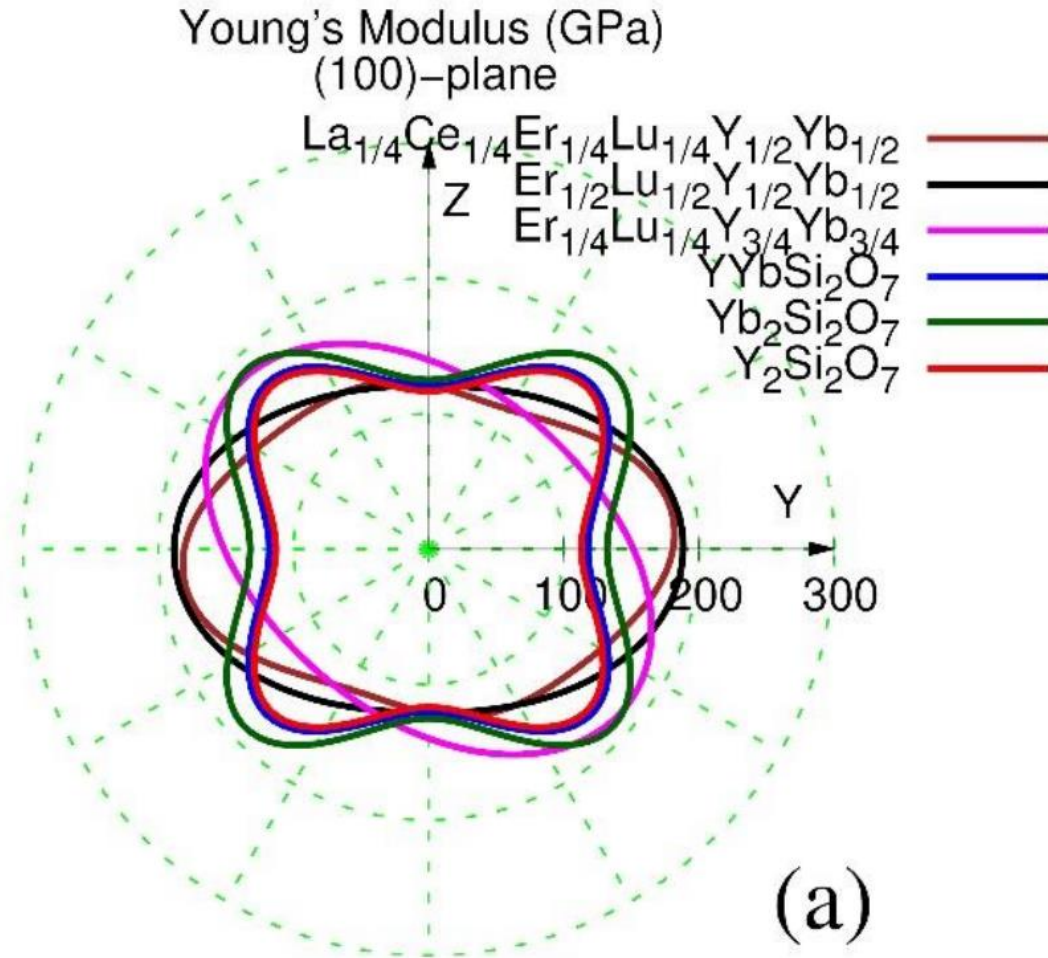
# La<sub>1/4</sub>Ce<sub>1/4</sub>Er<sub>1/4</sub>Lu<sub>1/4</sub>Y<sub>1/2</sub>Yb<sub>1/2</sub>Si<sub>2</sub>O<sub>7</sub>

Er <sub>1/4</sub> Lu <sub>1/4</sub> La <sub>1/4</sub> Ce <sub>1/4</sub> Y <sub>1/2</sub> Yb <sub>1/2</sub> Si <sub>2</sub> O <sub>7</sub> (GPa)					
173.8	86.9	104.4		-1.5	
86.9	178.5	92.9		-18.4	
104.4	92.9	268.8		13.7	
			58.1		-5.74
-1.5	-18.4	13.7	0.0	85.7	
			-5.74		57.8

Mechanical Properties	Voigt	Reuss	Hill
Bulk Modulus B (GPa)	132.2	125.6	128.9
Young's Modulus E(GPa)	162.6	143.1	152.9
Shear Modulus G(GPa)	62.8	54.6	58.7
Poisson's Ratio $\nu$	0.30	0.31	0.30
Vickers Hardness (GPa)	6.38	4.79	5.58

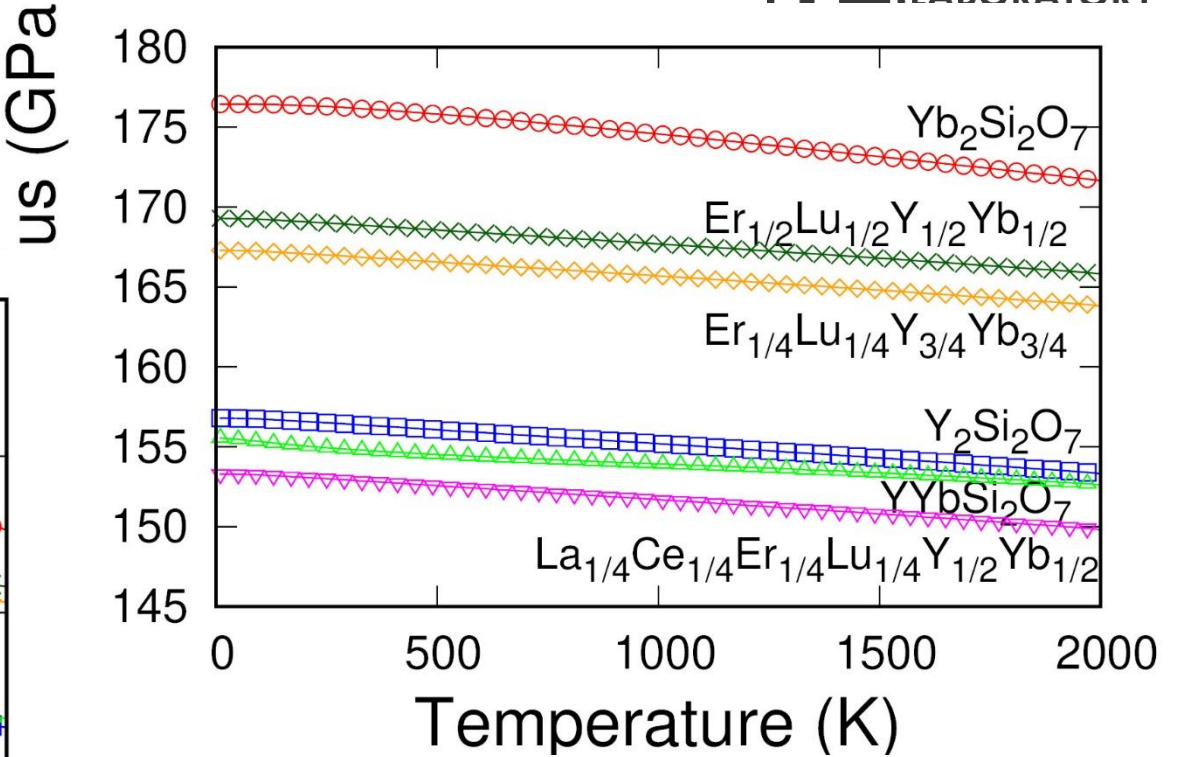
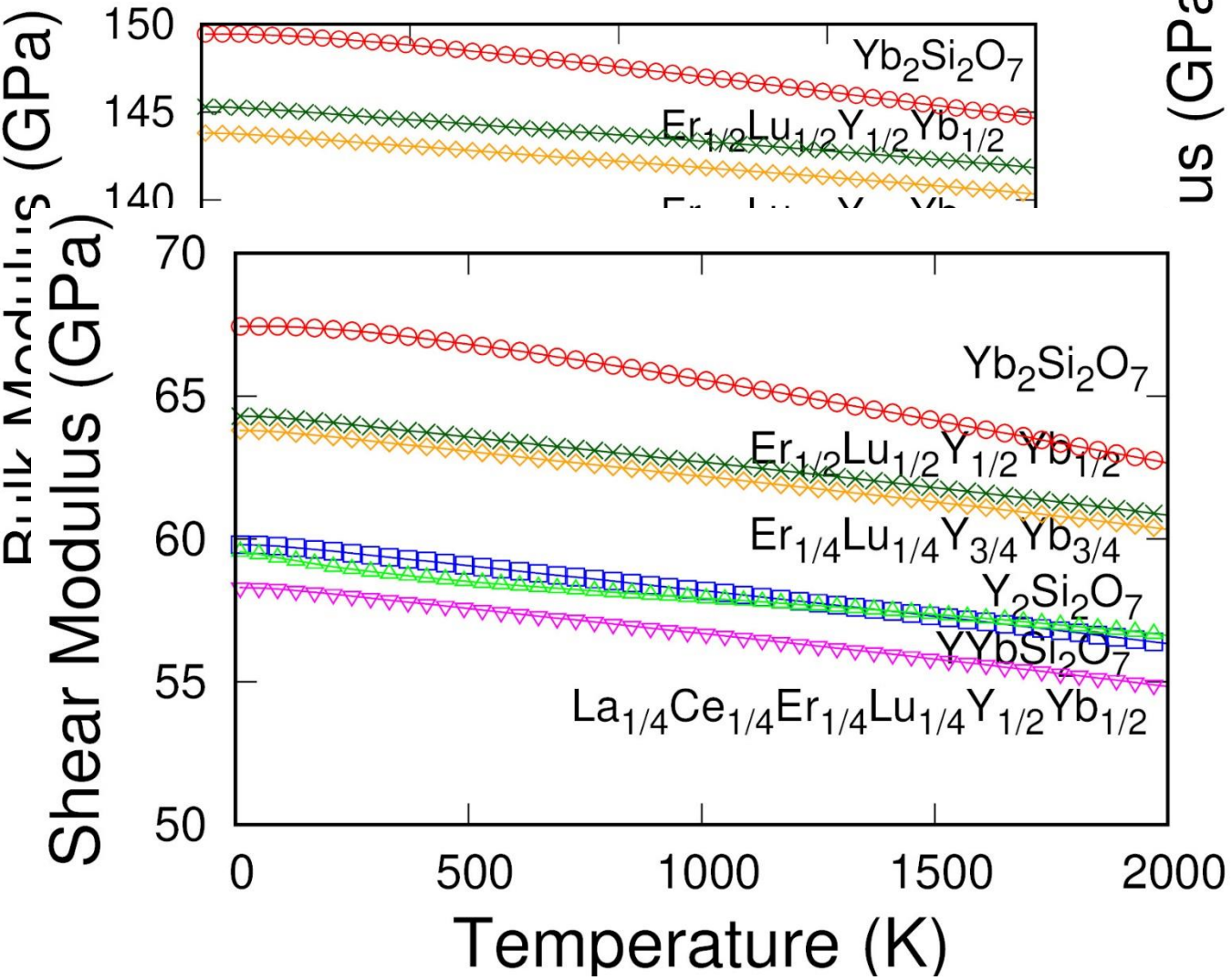


# Elastic Anisotropy



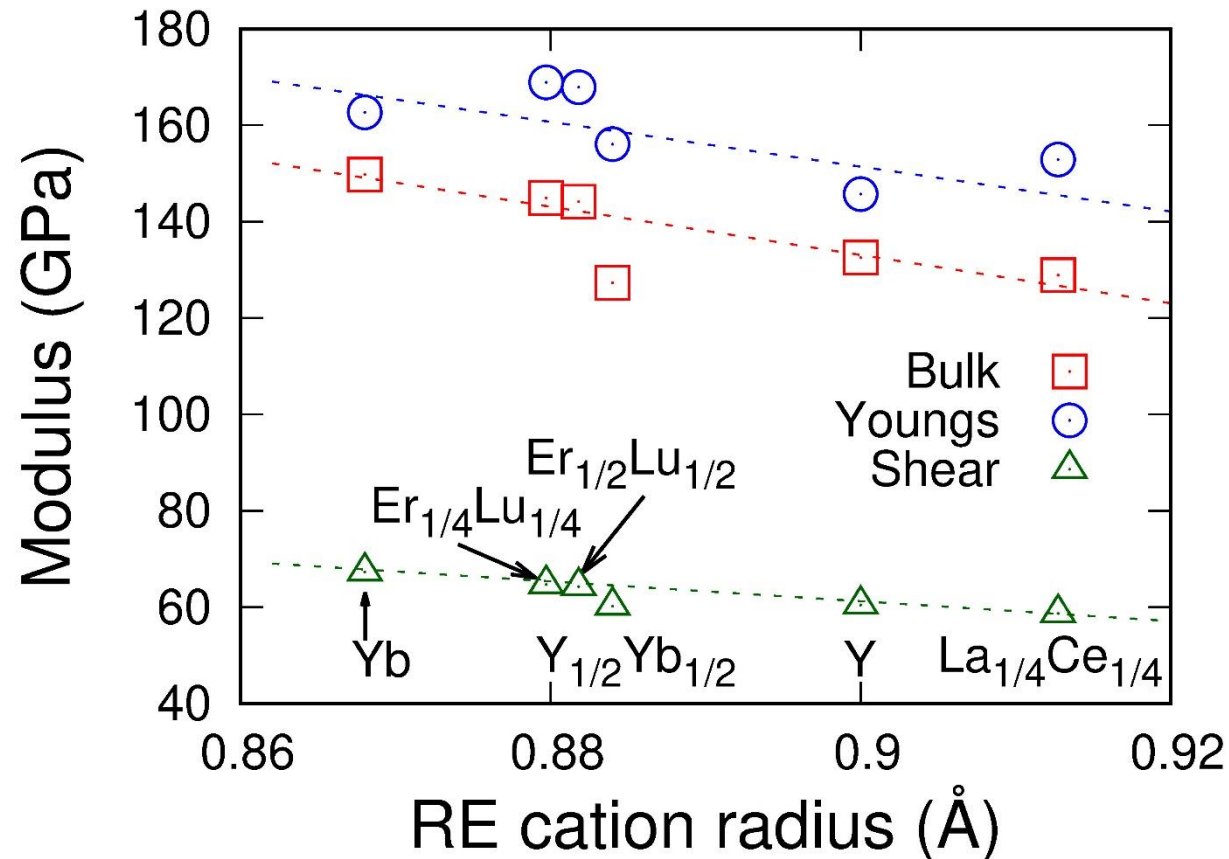
Elastic anisotropy plays a crucial role in understanding the formation of microcracks within coatings. A low modulus corresponds to a direction with low fracture energy.

# Temperature Dependent Moduli



Moduli exhibit decreasing behavior as temperature.

# Correlation Analysis



Relative strong negative Pearson correlations: -0.76, -0.71, -0.89 of bulk, Young's and shear moduli with radius of RE.

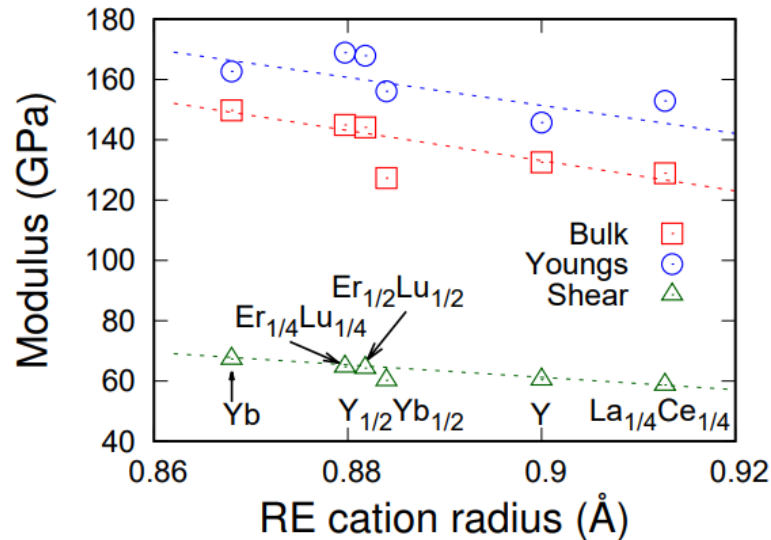
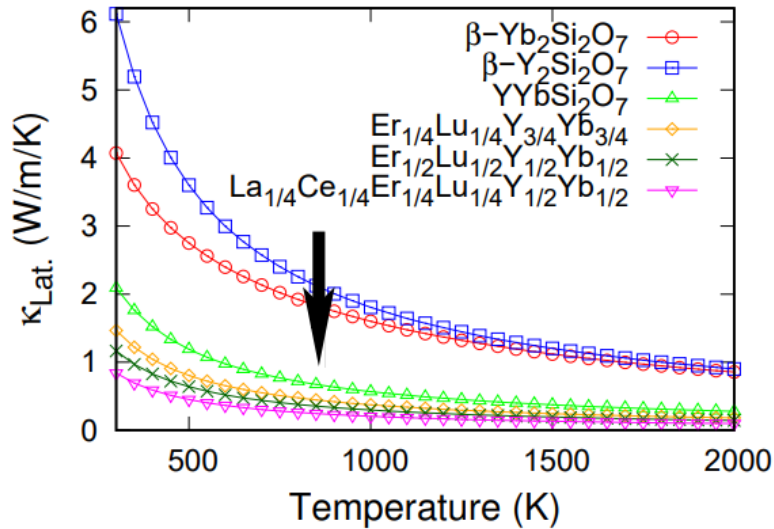
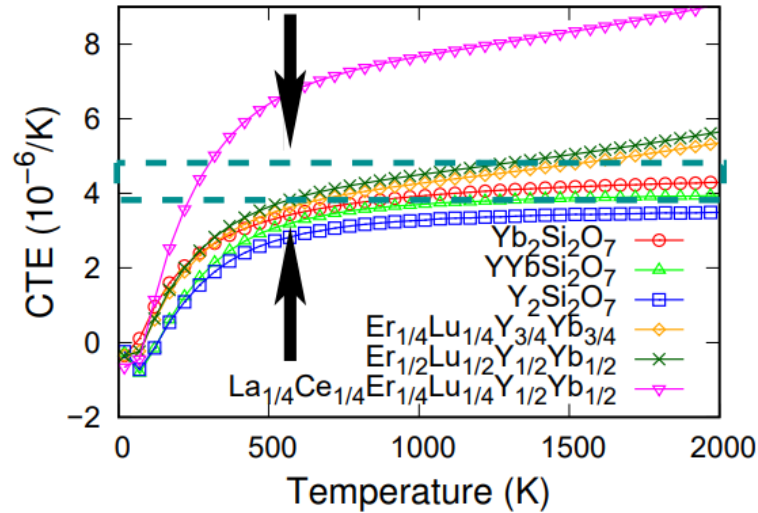
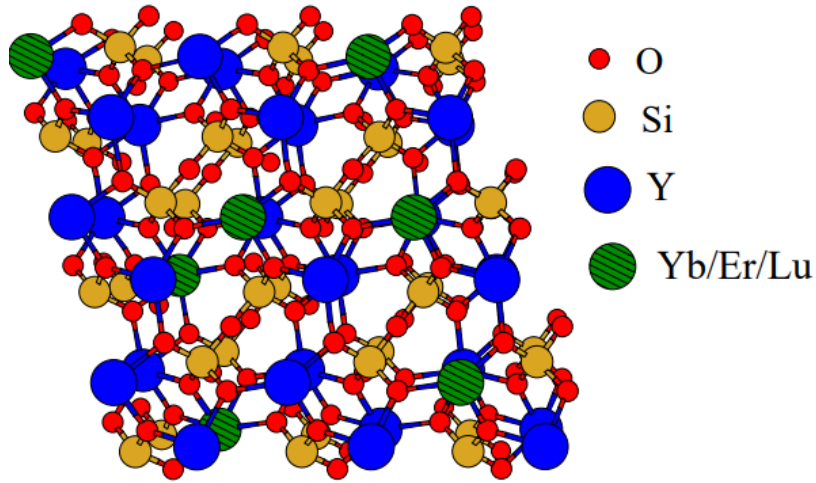
CTE 0.68. LTC -0.21.

$$r = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^n (Y_i - \bar{Y})^2}}$$

$$RMSD = \sqrt{\frac{\sum_{i=1}^n (Y_i - \bar{Y})^2}{(N-1) \sum_{i=1}^n (\bar{Y})^2}}$$

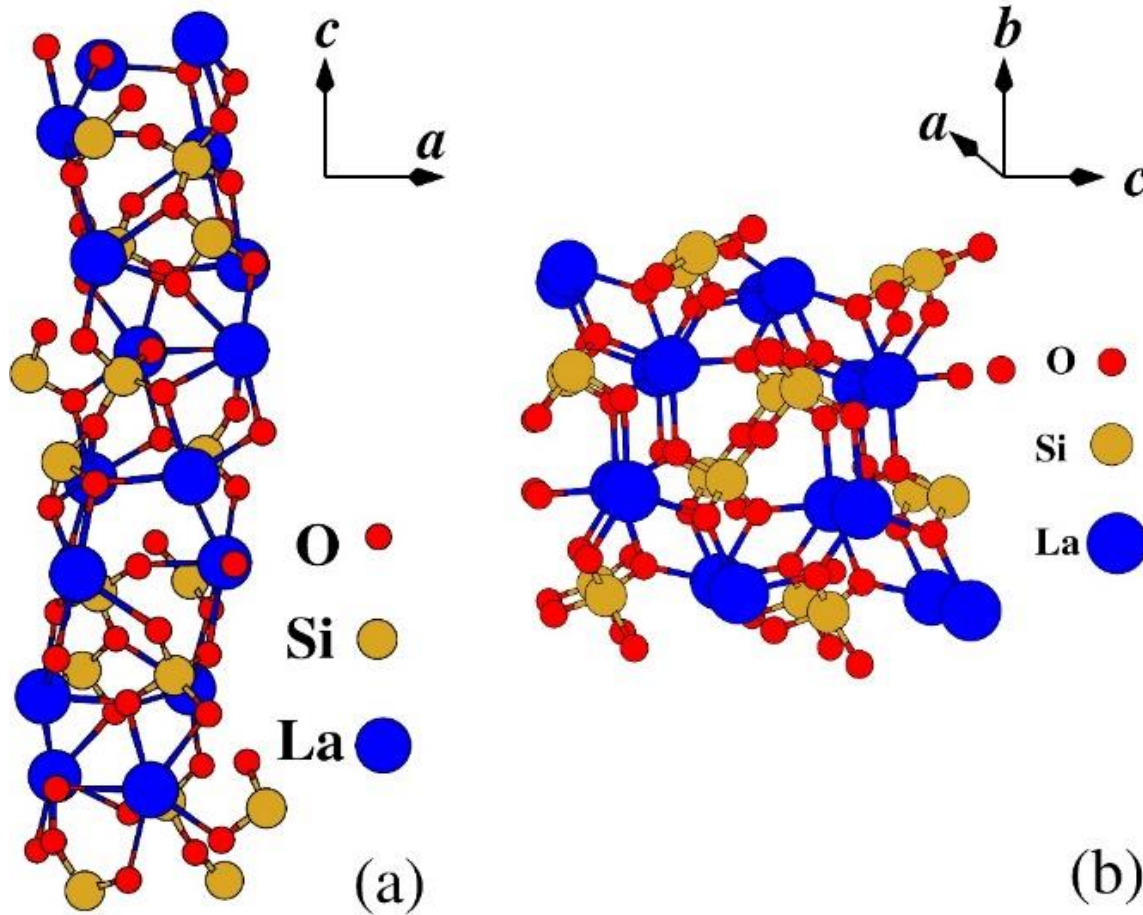
The closer the magnitudes to 1 (or -1), the stronger the correlation, while values closer to zero suggest a weaker correlation. A negative correlation indicates that when one variable increases, the other variable decreases.

# Summary on $\beta$ Phase



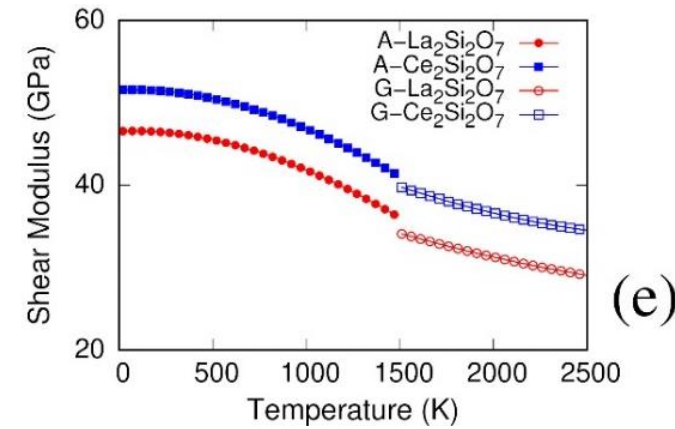
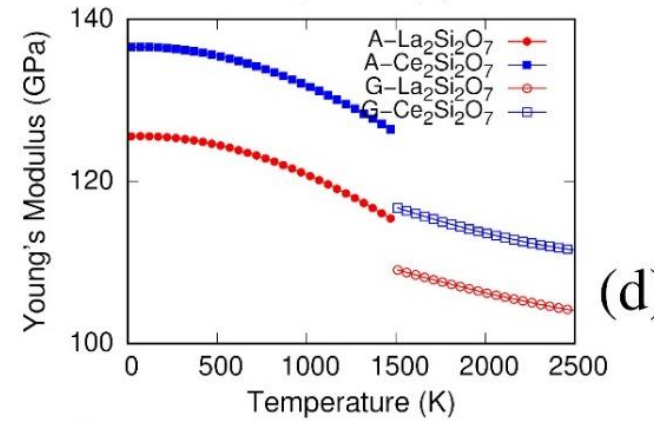
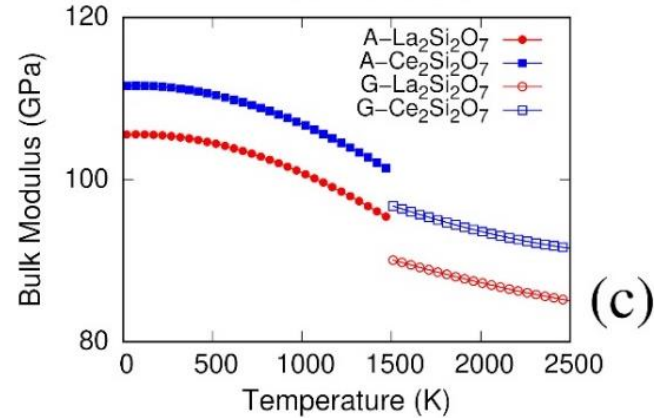
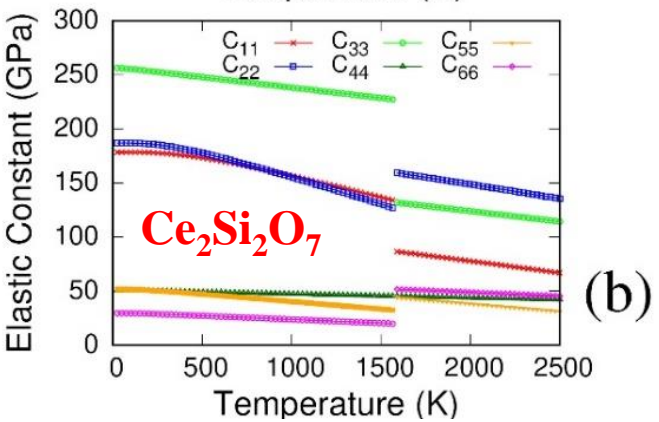
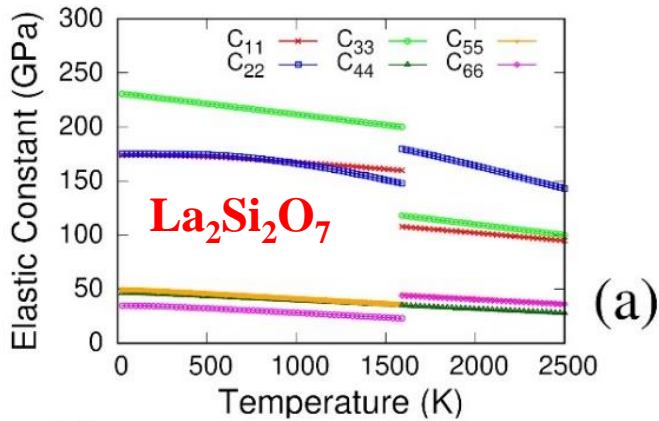
- Using combinatory chemistry and first-principles based methodologies.
- We predict the mechanical properties of solid solutions of  $(\text{ErLuYYb})_2\text{Si}_2\text{O}_7$  are not deteriorated.
- CTE of  $\text{Er}_{1/4}\text{Lu}_{1/4}\text{Y}_{3/4}\text{Yb}_{3/4}\text{Si}_2\text{O}_7$  and  $\text{Er}_{1/2}\text{Lu}_{1/2}\text{Y}_{1/2}\text{Yb}_{1/2}\text{Si}_2\text{O}_7$  exhibits good match with SiC with much decreased lattice thermal conductivity.

# La<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> and Ce<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>



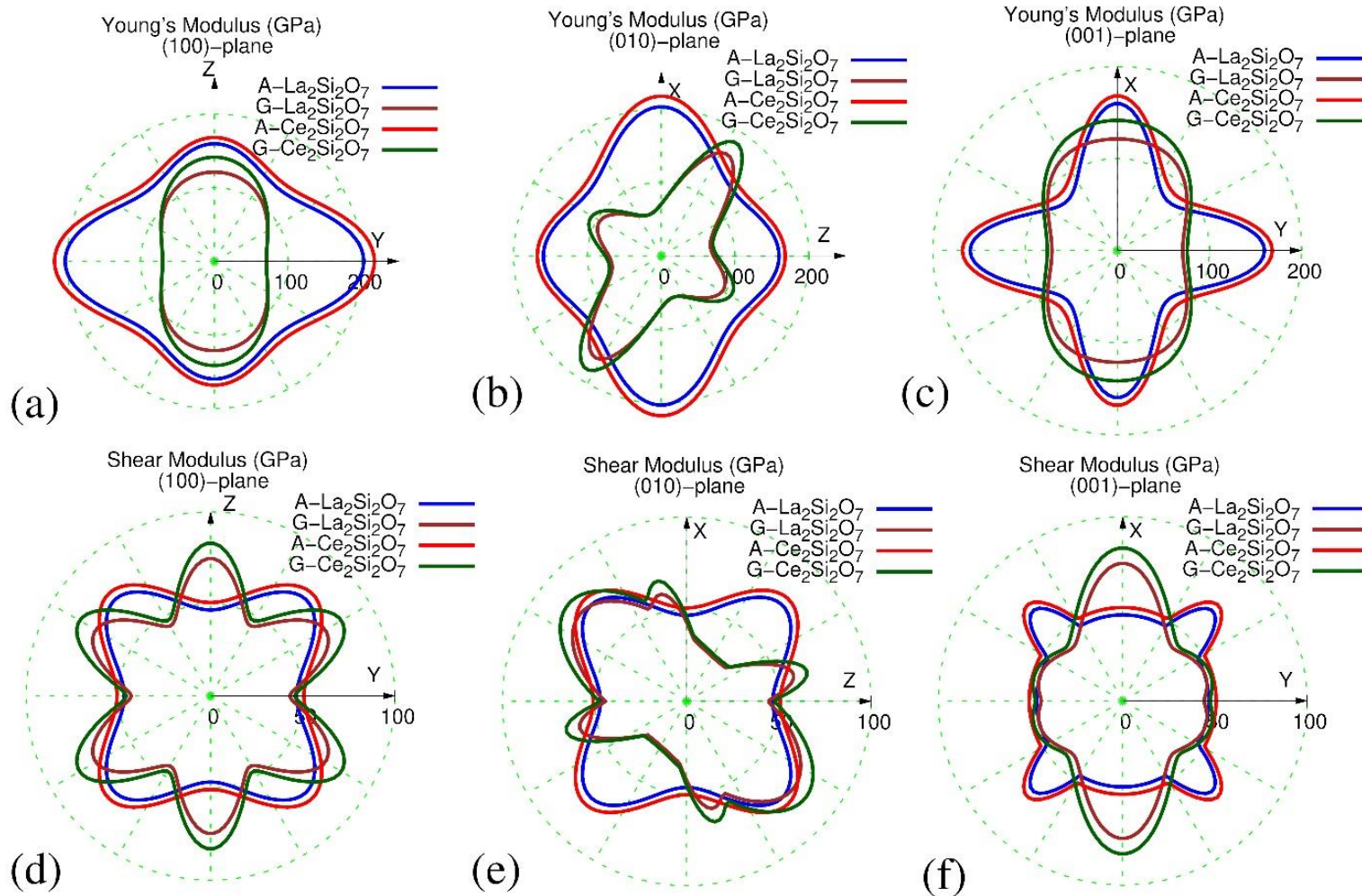
(a) The atomic structure of low temperature A-phase of La<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> with the space group of P4<sub>1</sub>.  
(b) The atomic structure of high temperature G-phase La<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> with space group of P2<sub>1</sub>/C. The corresponding A- and G-phase of Ce<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> are the same by simply replacing La with Ce.

# La<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> and Ce<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>



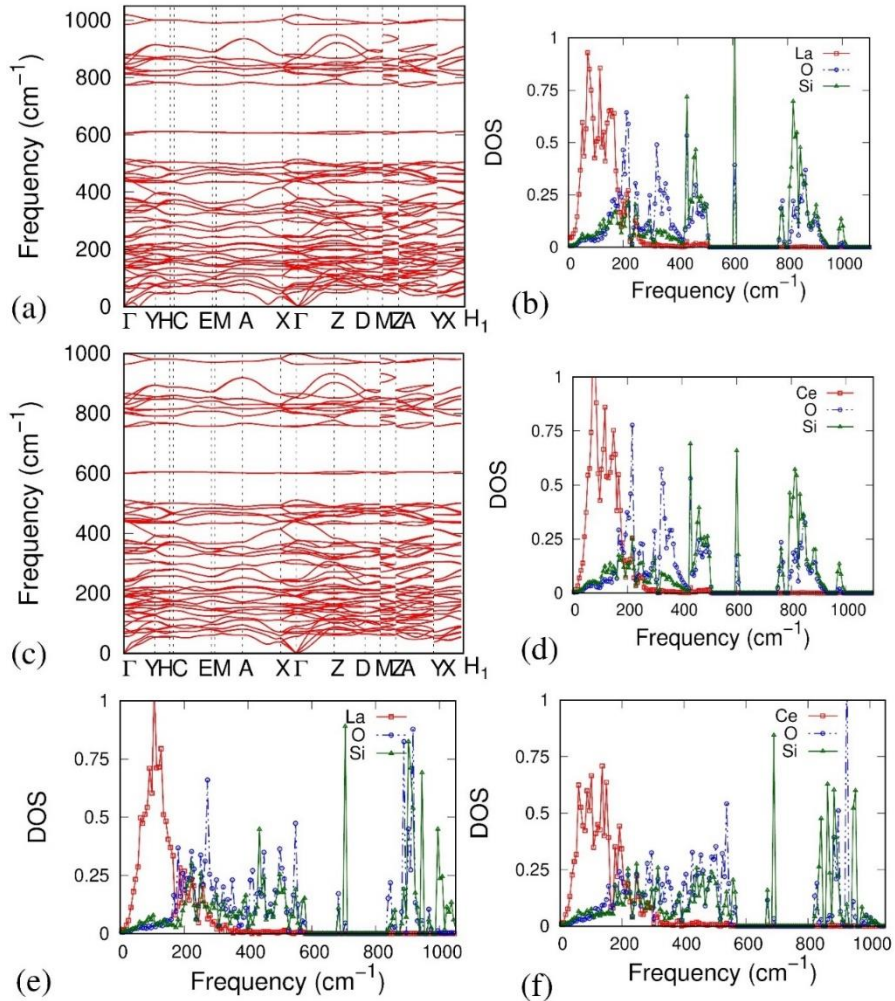
The temperature-dependent elastic constants for (a) La<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> and (b) Ce<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>, and the temperature-dependent (c) bulk modulus, (d) Young's modulus and shear modulus.

# La<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> and Ce<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>



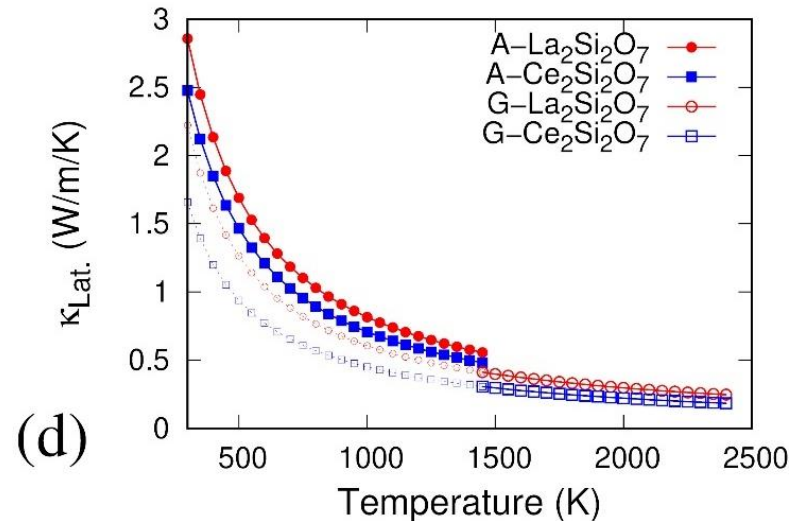
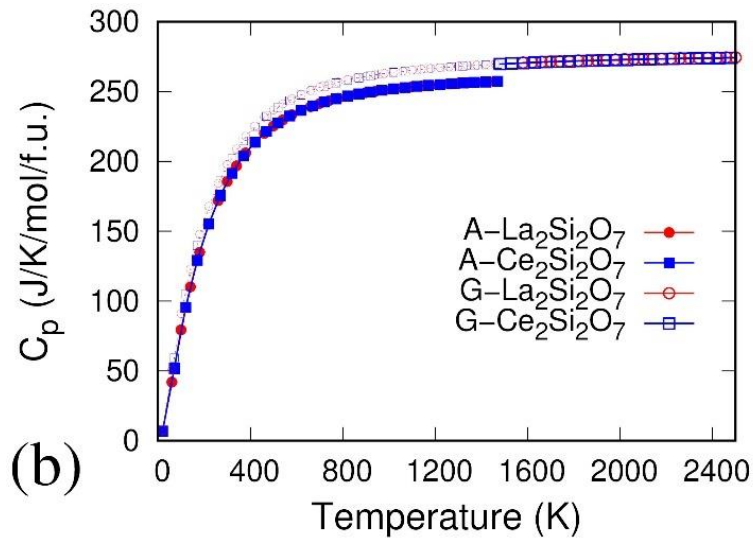
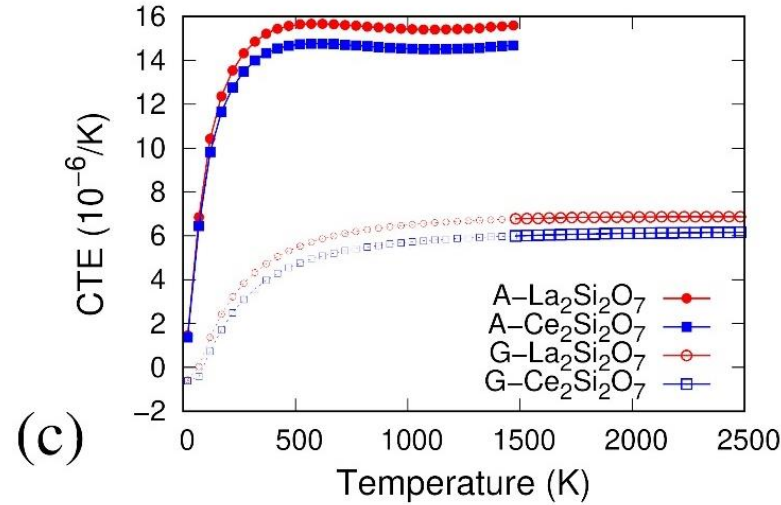
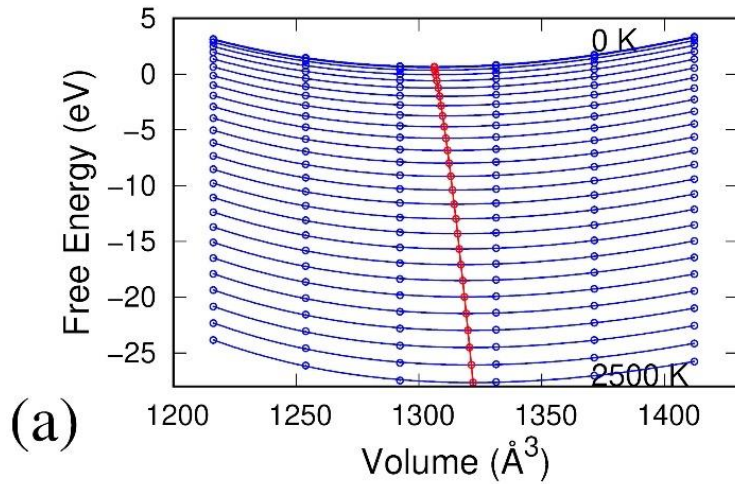
Elastic anisotropy of Young's modulus and shear modulus in different plane of (a,d) (100), (b,e) (010), and (c,f) (001), for all four phases of La<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> and Ce<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>, respectively.

# La<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> and Ce<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>



The phonon dispersion (a, c) and projected phonon density of states (b, d) at corresponding equilibrium volume  $V_0$  for the G-phase La<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> (a, b) and G-phase Ce<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> (c, d). The vibrational projected phonon density of states of A-phase La<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> (e) and A-phase Ce<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> (f).

# La<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> and Ce<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>



(a) An example of free energy as a function of volume and temperature for G-La<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> system. The blue dots and blue lines are respectively DFT calculated values and fitted results. The red dots represent the minimum free energy at corresponding volume and temperatures ranging from 0 to 2500 K with a step of 100 K. (b) Heat capacity (c) coefficients of thermal expansion and (d) lattice thermal conductivity of A- and G-phases of La<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> and Ce<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>. The low temperature A-phase data are plotted in a range of (0-1470 K), the high temperature G phase data are depicted in a range of (1470-2500 K).

# Summary on A-Phase and G-Phase

- Employed density functional theory-based techniques to predict the thermodynamic, mechanical, and thermal properties of the A- and G-phase of abundant rare earth disilicates, specifically  $\text{La}_2\text{Si}_2\text{O}_7$  and  $\text{Ce}_2\text{Si}_2\text{O}_7$ , with the aim of assessing their suitability as low-cost materials as T/EBC.
- Utilized the strain-energy method to derive elastic constants by computing the second derivatives of the total energy with respect to the corresponding lattice strains.
- Verified the mechanical stability of these structures as the calculated elastic constants satisfying established stability criteria. The average polycrystalline elastic properties were estimated using the Voigt-Reuss-Hill's schemes.
- Using phonon calculations at varying volumes under the quasi-harmonic approximation, we computed the system's Helmholtz free energy as a function of both volume and temperature. This enabled us to obtain the values for heat capacity and CTE. Finally, we evaluated the lattice thermal conductivities by using Debye-Callaway model considering three phonon processes.
- The transition from A-phase to G-phase causes notable alterations in crystal symmetry and lattice parameters, and accordingly, CTE and lattice thermal conductivities. These transformations could potentially impose limitations on their suitability for use as T/EBC materials.
- Among all calculated results, the G-phase of  $\text{Ce}_2\text{Si}_2\text{O}_7$  displays characteristics that make it a potentially strong candidate for protecting SiC-based ceramic matrix composites at high temperatures. Specifically, it has an extremely low lattice thermal conductivity of approximately  $0.26 \text{ W/m/K}$  at  $1500 \text{ K}$  and a low CTE (average  $\approx 6.9 \times 10^{-6} \text{ K}^{-1}$ ) that matches well with SiC.
- However, the substantial difference in CTE values between the A-phase and G-phase of  $\text{Ce}_2\text{Si}_2\text{O}_7$  poses obvious challenges during thermal cycling. Methods to stabilize the high-temperature G-phase at lower temperatures, and/or to slow down the kinetics of transformation, may enable the use of these inexpensive and abundant materials as effective T/EBC for protecting SiC-based composites and other high temperature materials.

# Acknowledgments

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