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VALIDATING SPIN-ORBIT QMC FOR EQUATION OF STATE CALCULATIONS OF HEAVY ELEMENTS

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OUTLINE

- Overview of the system
- DFT Cold curves.
 - Magnetic state, equation of state, etc.
- QMC Cold Curves
- Compare/Contrast
- Future Work



PURPOSE



- Diffusion Monte Carlo has been used extensively for constructing accurate equations of state, specifically cold curves.
- Spin-Orbit Diffusion Monte Carlo is a generalization of the method to systems with non-collinear wave functions. Most important use case is for systems with spin-orbit coupling, but non-collinear magnetism is also important.
- Implemented in QMCPACK. Compatible with molecules and solids.
- Goal is to understand the systematic biases, practical strengths and limitations of this method.



A community website for pseudopotentials/effective core potentials developed for high accuracy correlated many-body methods such as quantum Monte Carlo and quantum chemistry.

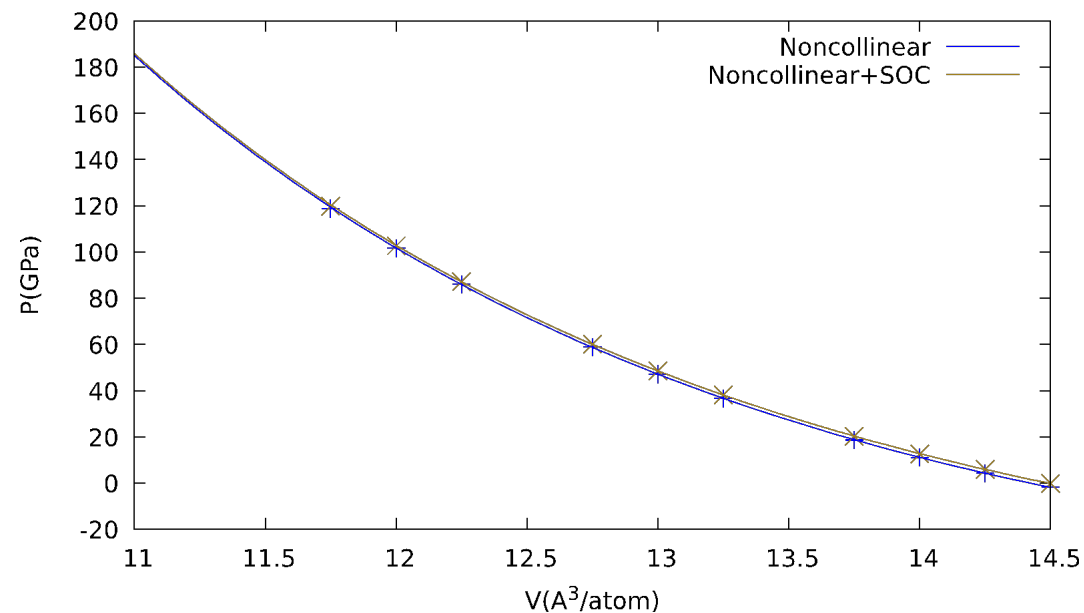
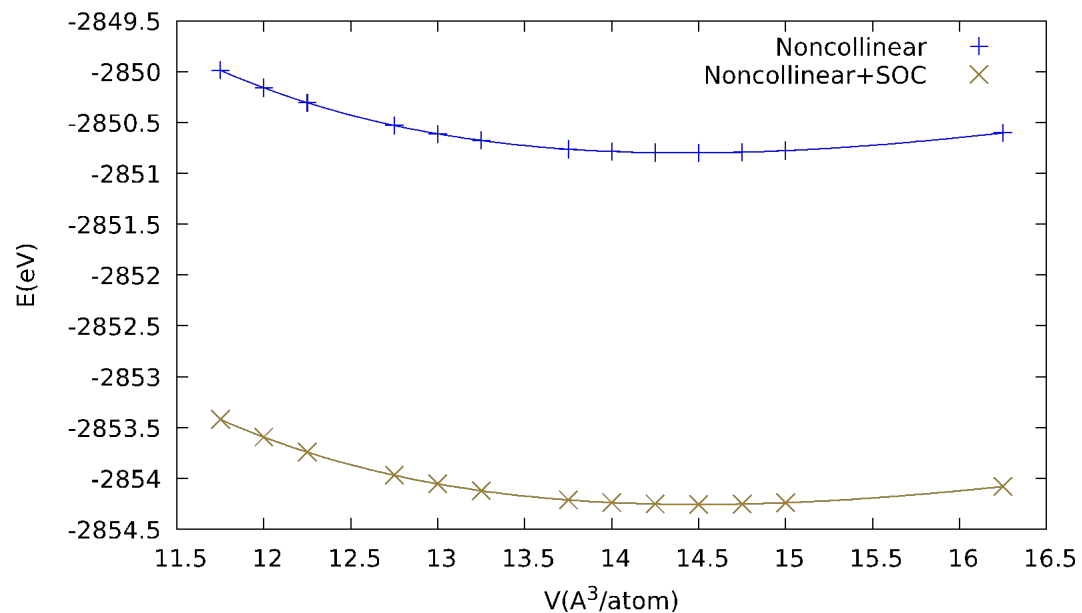
From <https://pseudopotentiallibrary.org>

DFT SIMULATIONS



- PBE functional.
- Ir ccECP pseudopotentials for scalar relativistic and spin-orbit.
- No evidence of ghost-states.
- Does NOT support a magnetic ground state (even in supercells).
- 28x28x28 gamma-shifted Monkhorst-Pack grid.
- 800Ry planewave cutoff.
- Cross-checked against ELK and VASP. Some differences in lattice constant, but very consistent results.

DFT RESULTS



Quantity	No-SOC	SOC	Δ
E_0 (eV)	-2850.799	-2854.255	-3.456
V_0 (\AA^3)	14.420	14.492	0.071
B_0 (GPa)	349.590	338.254	-11.336

SPIN SAMPLING

Rather than sample discrete spin-flips, we shift to an overcomplete continuous representation of spin degrees of freedom.

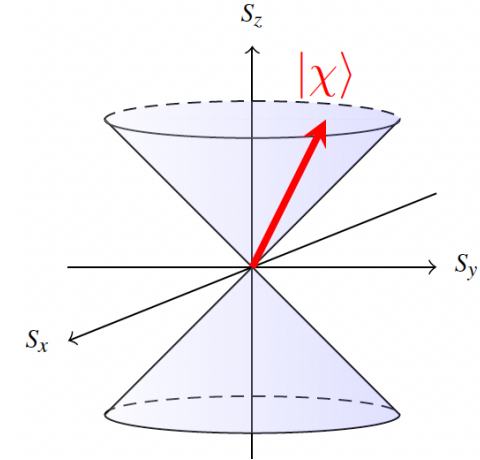
$$\langle \mathbf{x} | \psi \rangle = \phi^\uparrow(\mathbf{r}) \chi^\uparrow(s) + \phi^\downarrow(\mathbf{r}) \chi^\downarrow(s)$$

$$|\mathbf{x}\rangle = |\mathbf{r}\rangle |s\rangle$$

New continuous representation

$$\begin{aligned} \langle s | \chi^\uparrow \rangle &= \chi^\uparrow(s) = e^{is} \\ \langle s | \chi^\downarrow \rangle &= \chi^\downarrow(s) = e^{-is} \end{aligned} \quad s \in [0, 2\pi)$$

$$\langle \chi^\beta | \chi^\alpha \rangle = \int_0^{2\pi} \frac{ds}{2\pi} \langle \chi^\beta | s \rangle \langle s | \chi^\alpha \rangle = \delta_{\beta, \alpha}$$





With a continuous representation, the spin variable can be sampled in the same way the spatial variables are. Monte Carlo sampling is efficient, simply changes $\mathbf{R} \in \mathbb{R}^{3N} \Rightarrow \mathbf{X} \in \mathbb{R}^{4N}$

DIFFUSION MONTE CARLO WITH DYNAMIC SPINS



$$|\Psi_0\rangle \propto \lim_{\tau \rightarrow \infty} \exp[-\tau \mathcal{H}] |\Psi_T\rangle \quad E_0 = \lim_{\tau \rightarrow \infty} \frac{\langle \Psi(\tau) | \mathcal{H} | \Psi_T \rangle}{\langle \Psi(\tau) | \Psi_T \rangle}$$

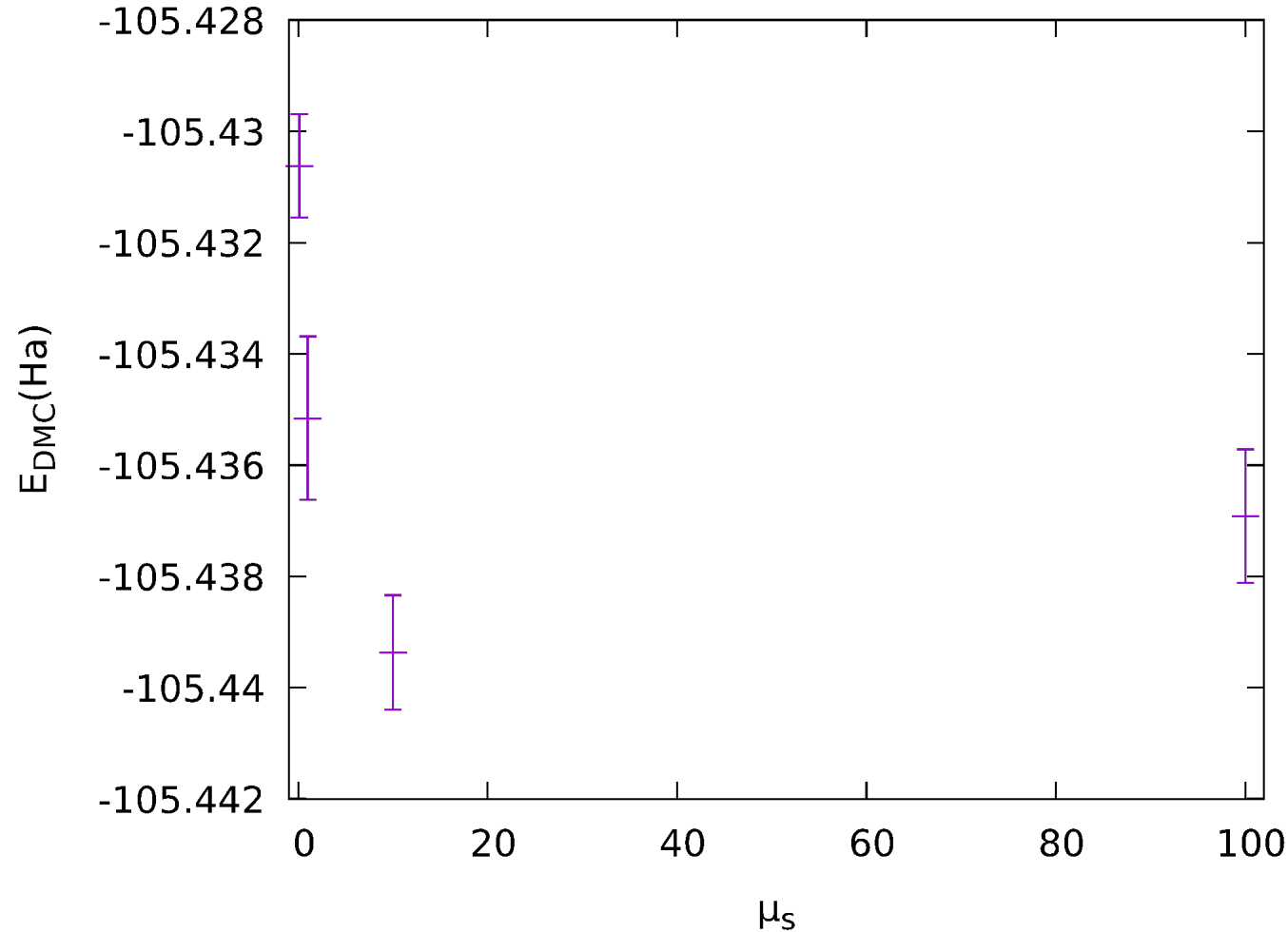
$$\langle \mathbf{X}' = (\mathbf{R}', \mathbf{S}') | \Psi^{(n+1)} \rangle = \int d\mathbf{X} G(\mathbf{X} \rightarrow \mathbf{X}'; \tau) \langle \mathbf{X} | \Psi^{(n)} \rangle$$

$$G(\mathbf{X} \rightarrow \mathbf{X}'; \tau) \propto G(\mathbf{R} \rightarrow \mathbf{R}'; \tau) \exp(-|\mathbf{S}' - \mathbf{S} - \tau_s \mathbf{v}_s(\mathbf{S})|^2 / 2\tau_s)$$


$$H_{\text{DMC}} \rightarrow H_{\text{DMC}} + \sum_i T_i^s \quad T_i^s = -\frac{1}{2\mu_s} \left[\frac{\partial^2}{\partial s_i^2} + 1 \right]$$

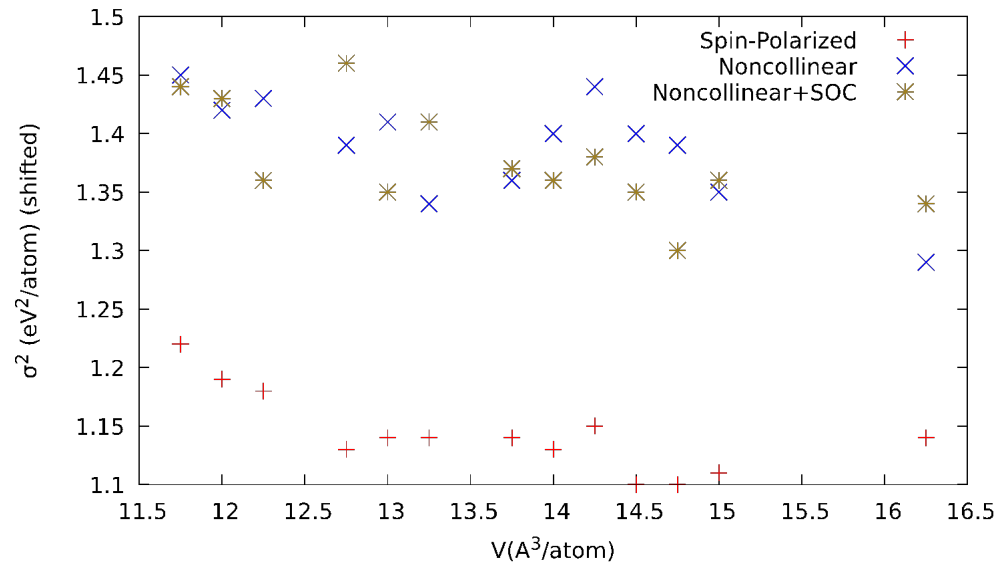
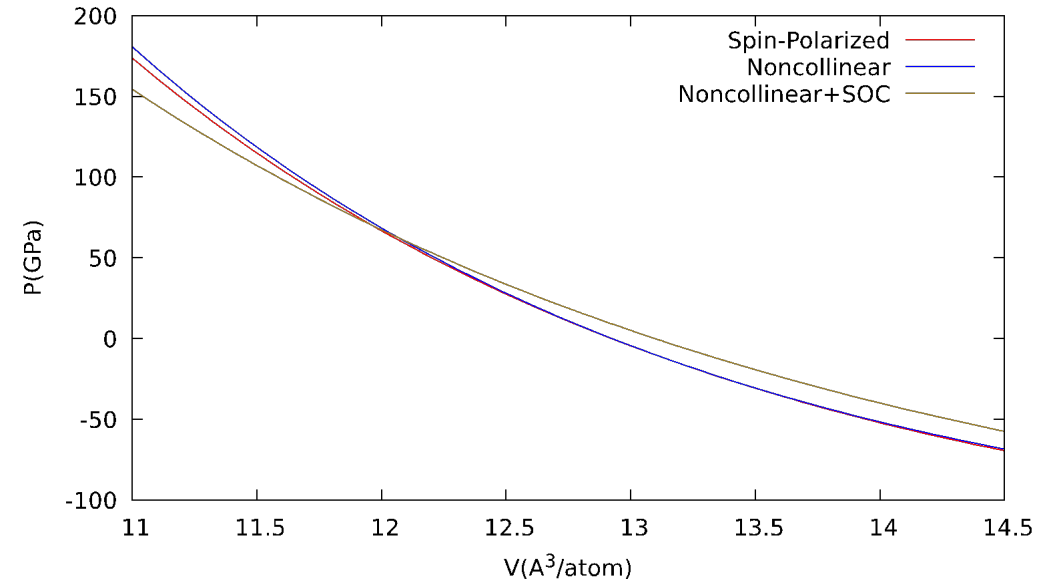
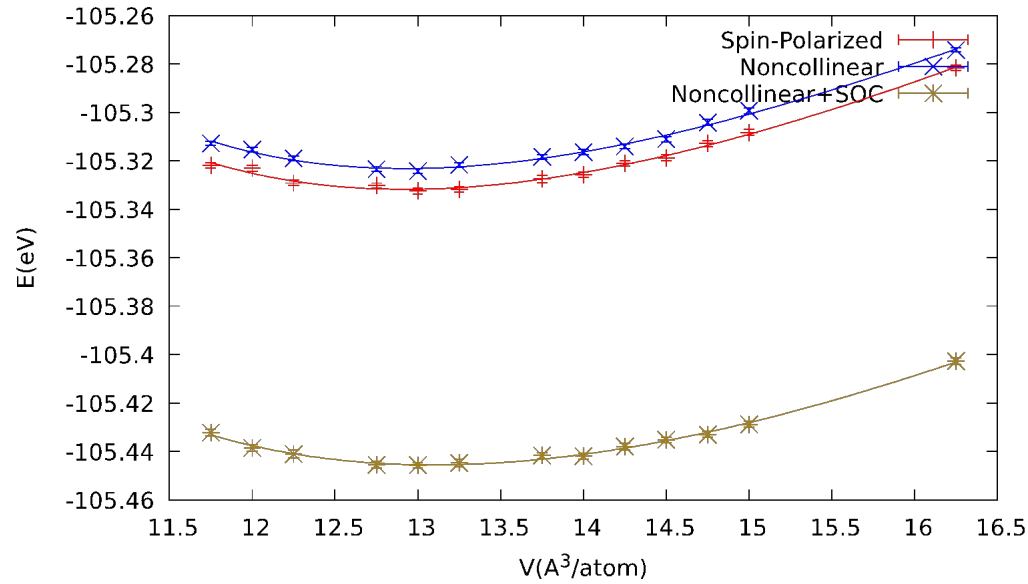
$$\sum_i T_i^s \prod_j \psi_j(\mathbf{r}_i, s_i) = 0$$

SPIN MASS CONVERGENCE



- Spin mass of 10 represents convergence.
- All calculations will use this value.

COMPARISON OF QMC COLD CURVES



(Top Left) DMC Energy vs. volume for a normal Spin-Polarized (no SOC) calculation, versus a noncollinear calculation with and without spin-orbit coupling.

(Top Right) Pressure vs. volume. No noticeable differences between Spin-Polarized and non-collinear DMC, except at higher pressures. Big difference from Noncollinear+SOC.

(Left) Variance vs. volume for all three types of wave functions. Non-collinear calculations have noticeably higher variance.

EFFECT OF SPIN-ORBIT COUPLING IN QMC



Quantity	Value (+SOC)	Value+SOC - Value-NOSOC
$E_0(\text{eV})$	-2869.319	-3.329
$V_0(\text{\AA}^3)$	13.096	0.171
$B_0(\text{GPa})$	669	-110

← DFT predicts
~3.45eV

- All these quantities correspond to the ambient values, obtained from Vinet EOS fits.
- Spin-Orbit energy difference is comparable to DFT (3.45eV), but detectably different.
- Very little change in ambient volume: 1.3%
- Bulk modulus shows a huge difference: 16%
- These are **very different** from DFT, but these are non-twist-averaged, single atom cell results. Important thing is that it gives “sensible” answers right now.

CONCLUSIONS & FUTURE WORK



- Initial stress tests of the method show expected behavior. Orders of magnitudes are correct, numerically stable.
- All results were using a naive algorithm for performing spin integrals. New code 12x faster. Will allow us to hit system sizes comparable to normal DMC calculations.

For the future

- Finish twist-averaging calculations for 8 atom units cells.
- Do finite size extrapolations to get a cold curve in the thermodynamic limit.
- Finish Au and W.
- Perform orbital optimization.
 - Non magnetic ground state with PBE. Its customary to do exact exchange and “+U” scans to improve the starting orbitals, but this might not be necessary with new QMC orbital optimization capability.

ACKNOWLEDGEMENTS



CPSFM

Center for Predictive Simulation
of Functional Materials

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<https://cpsfm.ornl.gov>

QMCPACK

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