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Fission for Transport Calculations and Reactor Applications and Fission for Integral Experiments and their Use for Applications

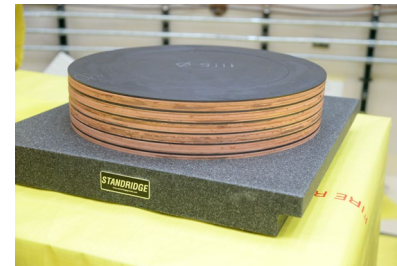
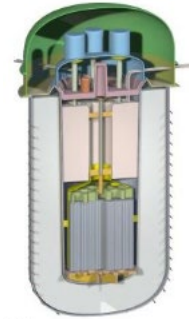
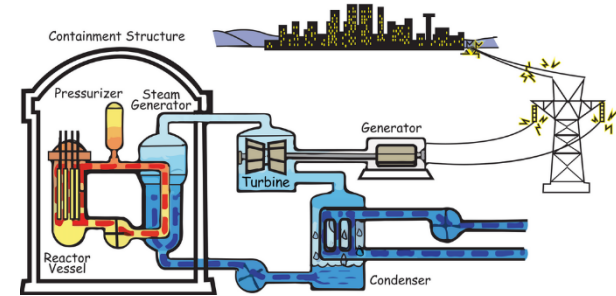
Joetta Goda
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FIESTA 2024

LA-UR-24-32080

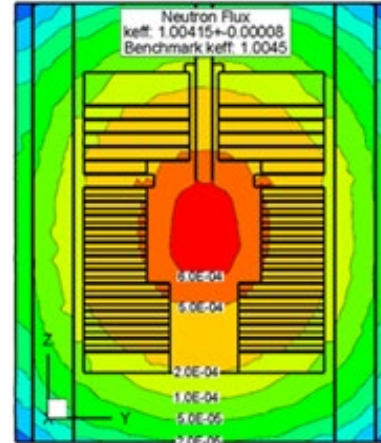
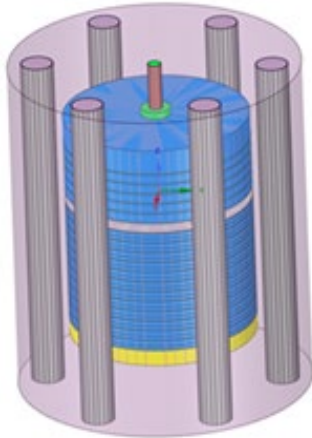
Why do we care about understanding the fission process better?

- Fission is extremely important in any applications involving special nuclear material
 - Current nuclear power plants
 - Advanced reactor design
 - Criticality safety
 - Special nuclear material production
 - Nuclear nonproliferation
 - Safeguards
 - And much more!

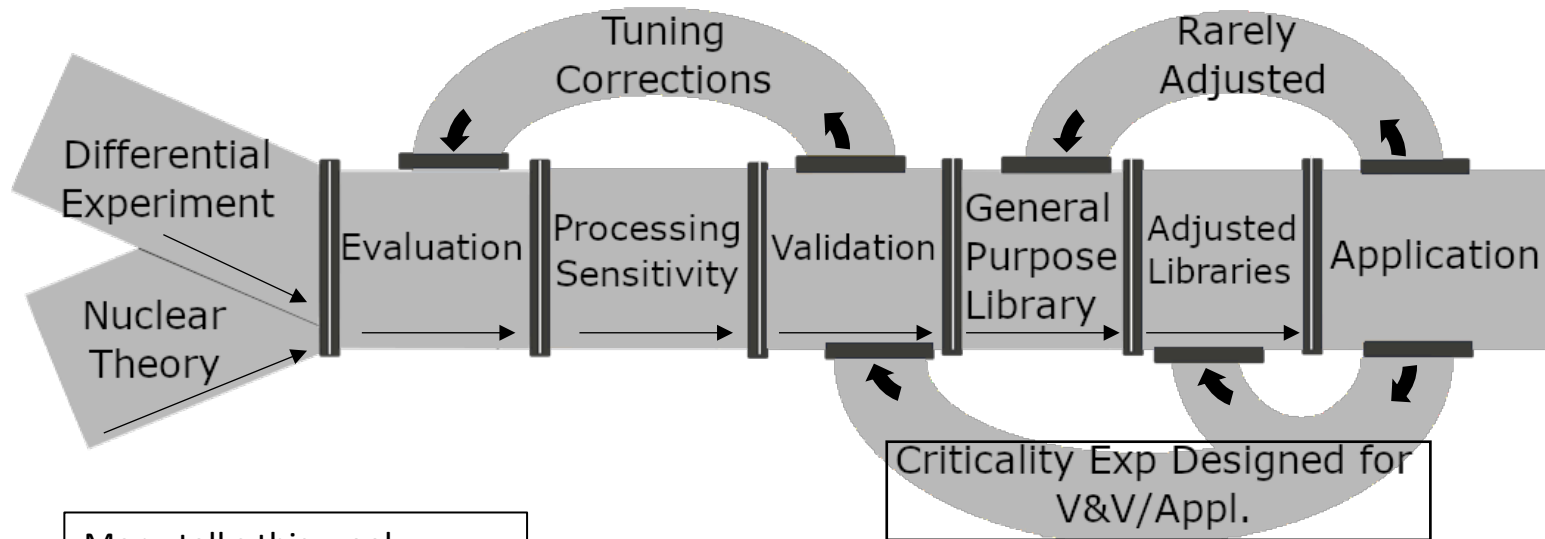


Why do we care about understanding the fission process better?

- Predictive simulations are vital for important data-driven decisions:
 - Performance
 - Safety
 - Cost
- These simulations are only meaningful if we have an accurate and precise understand of the fission process (and other processes too)



The nuclear data pipeline is how we improve our understanding of fission and other reactions



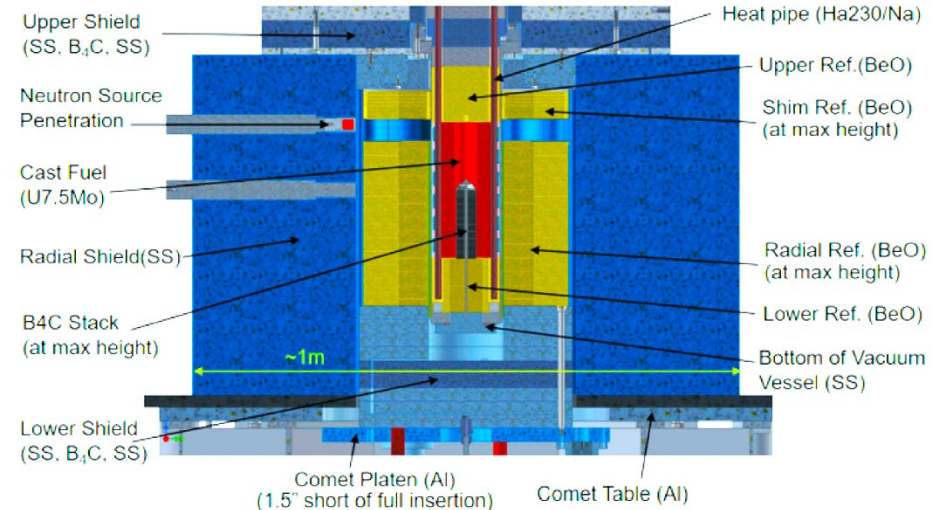
Many talks this week focused on various aspects of the pipeline: differential experiments, theory, evaluation, etc.

This talk will focus primarily on criticality (or integral) experiments

REACTOR MATERIALS

- **Fuel**
 - Material that fissions (releasing energy)
 - Solids (metals, ceramics) or liquids (aqueous)
- **Moderator**
 - Used to moderate (lower) the energy of neutrons through scattering interactions (i.e., fast neutrons from fission to thermal neutrons)
- **Reflector**
 - Used to “reflect” outgoing neutrons back into system
 - Does not have to act as a moderator (but can)
- **Absorber**
 - Used to absorb neutrons (control, safety, detection)
 - Poisons--Unwanted fission fragments (and daughters) that have high absorption cross section

KRUSTY example



David I. Poston , Marc A. Gibson , Thomas Godfroy & Patrick R. McClure (2020)
KRUSTY Reactor Design, Nuclear Technology, 206:sup1, 13-30

FUEL

- **Fissile or fissionable materials that can sustain a fission chain**
- **Typical Materials:**
 - Uranium (U-235)
 - Plutonium (Pu-239)
 - Others:
 - U-233, U-238
 - Pu-241
 - Np-237
- **Material Forms:**
 - Solid (metal, ceramic, oxide)
 - Liquid, Aqueous (uranyl fluoride, uranyl nitrate, uranyl sulfate)

REFLECTORS AND MODERATORS

Moderators:

Used to moderate (lower) the energy of neutrons through scattering interactions (ie, fast neutrons from fission to thermal neutrons)

- Desired Attributes: high scattering cross sections, low absorption cross sections
- Use low mass number elements to increase energy transfer to the moderator ($A < 17$)
- Can be used as a reflector
 - Examples: Anything with hydrogen (water, plastics), Carbon (graphite), Beryllium

Reflectors:

Used to “reflect” outgoing neutrons back into system

- Desired Attributes: high scattering cross sections, low absorption cross sections
- Use high mass number elements if only intending to reflect neutrons (fast or intermediate reactor), in order to limit energy transfer to the reflector
 - Examples: U-238, W

ABSORBER AND POISONS

Neutron Absorber:

Non-fuel material with a high absorption cross section

- Can be used to “control” a system by absorbing excess neutrons (ie, additional neutron losses)
- Can be used for health physics shielding
- Can be used for neutron detection and foil irradiation experiments
- Neutron absorbers have a high absorption cross-section in the thermal region
 - If the neutrons in the system are mostly fast, the neutron absorbers will not absorb many neutrons!
 - Instead the neutron absorber will act as a neutron reflector/moderator
- Examples: Boron-10, Cadmium-113, Helium-3, Lithium-6

ABSORBER AND POISONS (cont'd)

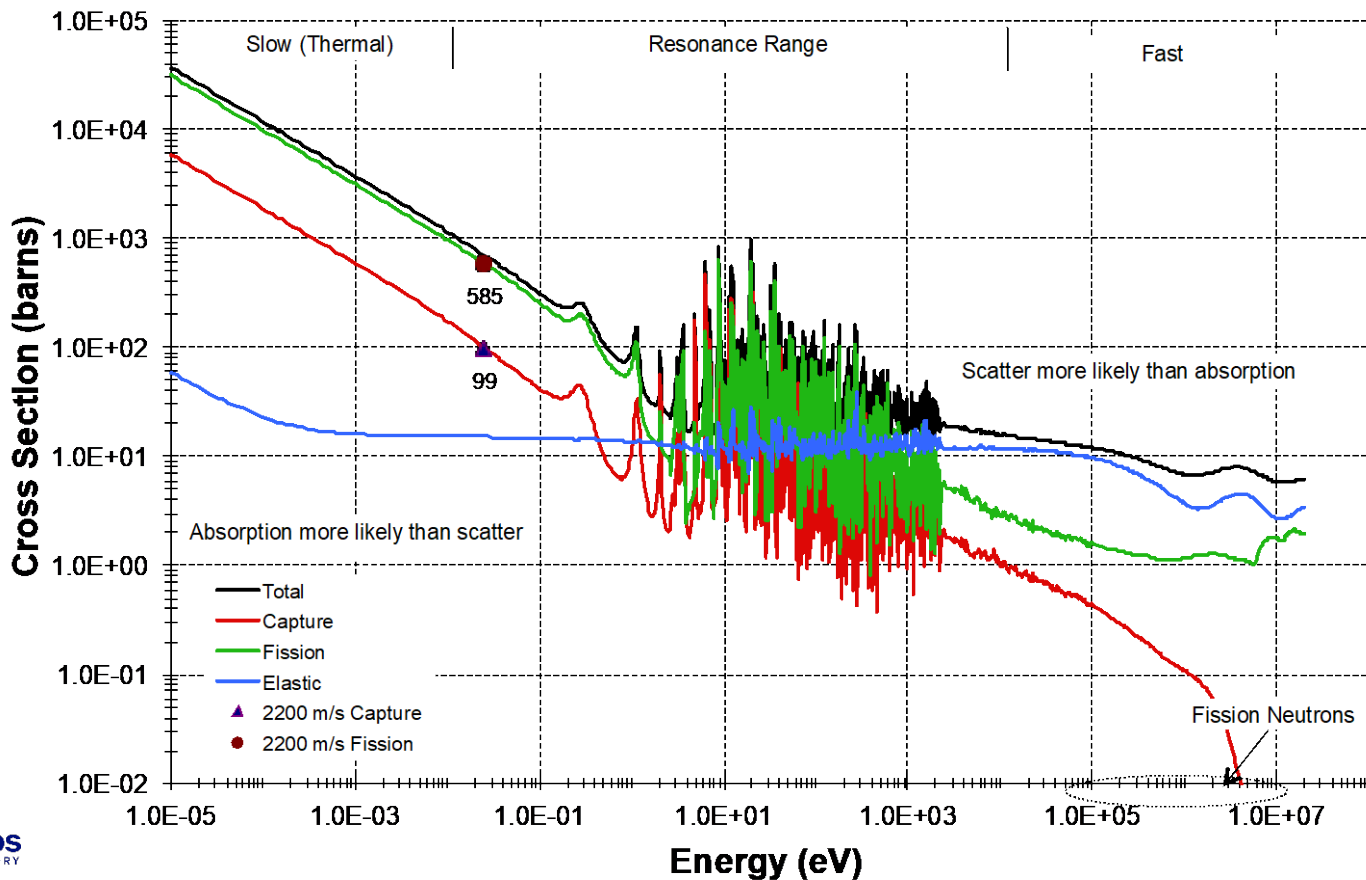
Neutron Poisons:

Non-fuel material with a high absorption cross section

- Main difference from Neutron Absorbers: **Poisons are (generally!) undesirable and are a result of the fission process**
 - Burnable poisons are an exception (B-10)
- Some fission fragments (or fission fragment daughters) have very high neutron absorption cross-sections
 - As more of these fission fragments build up in the system, the effectiveness of the system is decreased (ie, poisoning the system)
- Examples: Xe-135, Sm-149

CROSS SECTION VERSUS ENERGY

U-235 Cross Sections



MICROSCOPIC CROSS SECTION σ

- **Neutron beam falling on “thin” target**

- $R \left(\frac{\# \text{ of reactions}}{\text{cm}^2\text{-s}} \right) = I \left(\frac{\# \text{ of incident neutrons}}{\text{cm}^2\text{-s}} \right) N_A \left(\frac{\# \text{ of nuclei}}{\text{cm}^2} \right) \sigma$

- $\sigma = \frac{R}{IN_A} \left(\frac{\# \text{ of reactions per nuclei}}{\# \text{ of incident neutrons/cm}^2} \right)$

- $\frac{\sigma}{A}$: Probability of interaction per nucleus

- $\sigma_{total} = \sum_i \sigma_i$

- $\sigma_s = \text{scatter}$ $\sigma_a = \text{absorption}$ $\sigma_f = \text{fission}$

- **Unit: 1 barn = $1.0 \times 10^{-24} \text{ cm}^2$**

MACROSCOPIC CROSS SECTION Σ_i

- **Neutron beam falling on “thick” target**

- $R = -dI = -(I(x + dx) - I(x)) = I\sigma Ndx \rightarrow I = I_0 e^{-N\sigma x}$

- **Mean Free Path (MFP)**

- $-\frac{dI}{I dx} = \frac{R}{I dx} = N\sigma \rightarrow \frac{I dx}{R} = MFP = \lambda = \frac{1}{N\sigma}$

- **Probability of interaction $P(x)$**

- $\frac{R}{I} = -\frac{dI}{I} = \frac{dx}{\lambda} \rightarrow -dP = \frac{dx}{\lambda} P(x) \rightarrow P(x) = e^{-\frac{x}{\lambda}}$

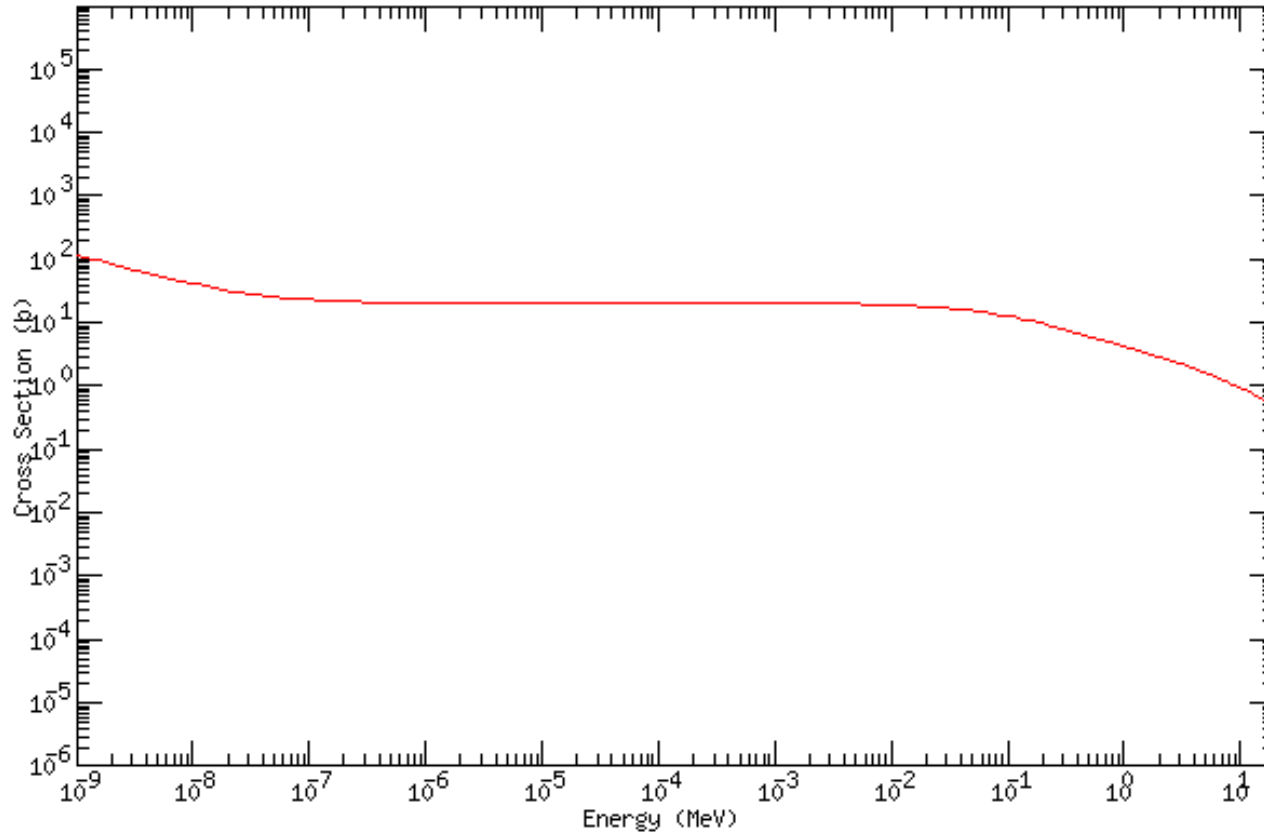
- $\Sigma_i = N\sigma_i$: Number of interactions per tracklength

- $n \left(\frac{\# \text{ of neutrons}}{\text{cm}^3} \right) v \left(\frac{\text{cm}}{\text{s}} \right) = \frac{\text{total tracklength}}{\text{cm}^3 \cdot \text{s}} \rightarrow nv\Sigma_i = \frac{\# \text{ of reactions}}{\text{cm}^3 \cdot \text{s}}$

MACROSCOPIC CROSS SECTION

- **Scattering interactions:** $\Sigma_s = \Sigma_{si} + \Sigma_{se}$
- **Absorption interactions:** $\Sigma_a = \Sigma_f + \Sigma_c + \Sigma_{n,alpha} \dots$
- **Total interactions:** $\Sigma_T = \Sigma_s + \Sigma_a$
- **Added reactions:** $\Sigma_f^{fuel} = \Sigma_f^{U-235} + \Sigma_f^{U-238}$

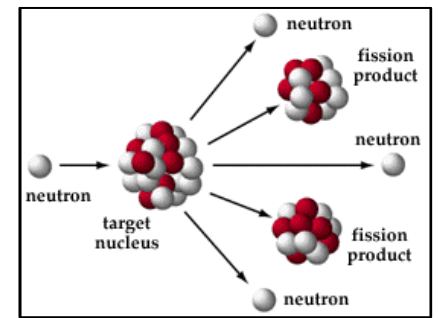
SCATTERING CROSS SECTION VERSUS ENERGY - *HYDROGEN*



Theory

ENERGY BALANCE FROM FISSION

- **Roughly 200 MeV is released per fission**
 - Most of the energy is distributed to the fission fragments
- **Typical energy distribution (U-235) Note: $3.1\text{E}+10$ fissions = 1 Joule**



| Where The Energy is Divvyed Up | Energy (MeV) |
|---------------------------------|--------------|
| Fission Fragment Kinetic Energy | 168 |
| Neutron Kinetic Energy | 5 |
| Fission Gamma Ray Energy | 11 |
| Beta and Neutrino Energy | 18 |
| Total | 202 |

NEUTRONS FROM FISSION

- 0-10 neutrons released per fission event
- Statistical process, we are more concerned about the average number of neutrons per fission event
- Average number of neutrons released per fission event is denoted as: $\bar{\nu}$
- Average depends upon:
 - Energy of the incident neutron
 - Nuclide that fissions
 - Higher energy incident neutron leads to larger average number of neutrons released per fission

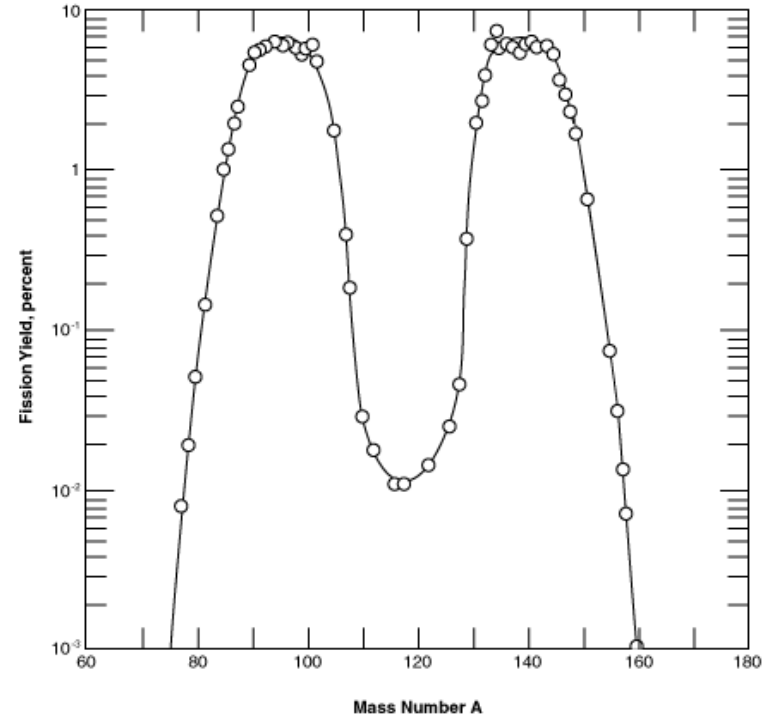
| $\bar{\nu}_c$ | U-233 | U-235 | U-238 | Pu-239 |
|---------------|-------|-------|-------|--------|
| 0.0235 eV | 2.50 | 2.43 | - | 2.87 |
| 1 MeV | 2.62 | 2.58 | - | 3.00 |
| 2 MeV | 2.73 | 2.70 | 2.69 | 3.11 |

- Recall that ~5 MeV of energy from fission process ends up in the neutrons
 - Average neutron released during fission event has ~1-2 MeV of energy

FISSION FRAGMENTS

- **Usually fission produces two fission fragments** (binary fission)
 - Small chance (<1%) of three fission fragments (ternary fission)
- **Fission fragment mass numbers between $A=70$ and $A=160$**
 - Most probable mass numbers are ~ 95 and ~ 135
- **Fission fragments are heavy charged particles** (positive charged) **with large amount of energy from the fission event** (kinetic energy)
- **Fission fragments are unstable** (radioactive) **beta and gamma emitters**

Thermal Neutron Fission of U-235



DELAYED NEUTRONS

- Fission fragments are neutron-rich and emit neutrons
 - Called delayed neutrons as their appearance after a fission event is delayed in time
 - Time to appear based upon half-lives of the fission fragments
- Delayed neutron precursor groups
 - Large number of possible fission fragments that could release a delayed neutron
 - For simplicity, group fission fragment isotopes into similar half-life groups
 - Keepin's six group:
 - 1: Br-87
 - 2: I-137, Br-88, Se-86
 - 3: I-138, Br-90, Br-89, Cs-144
 - 4: I-139, Br-91, Kr-93, Xe-141, Xe-142
 - 5: I-140, Kr-95, Br-92, Cs-145
 - 6: Br-93

DELAYED NEUTRONS

- About 99% of fission neutrons appear instantly (prompt neutrons)
- Less than 1% of all neutrons released in a fission event are delayed neutrons
 - 0.65% (650 pcm) for U-235 (thermal) **But they are extremely important!**
 - 0.21% (210 pcm) for Pu-239 (thermal) **They are what allow control of a reactor.**
- Delayed Neutron Fraction: β , fraction of neutrons released from fission that are delayed

$$\beta = \frac{\bar{v}_d}{\bar{v}}$$

- Delayed neutrons have less energy than prompt neutrons, roughly 500 keV
This makes them more likely to cause fission, increasing their “effectiveness”

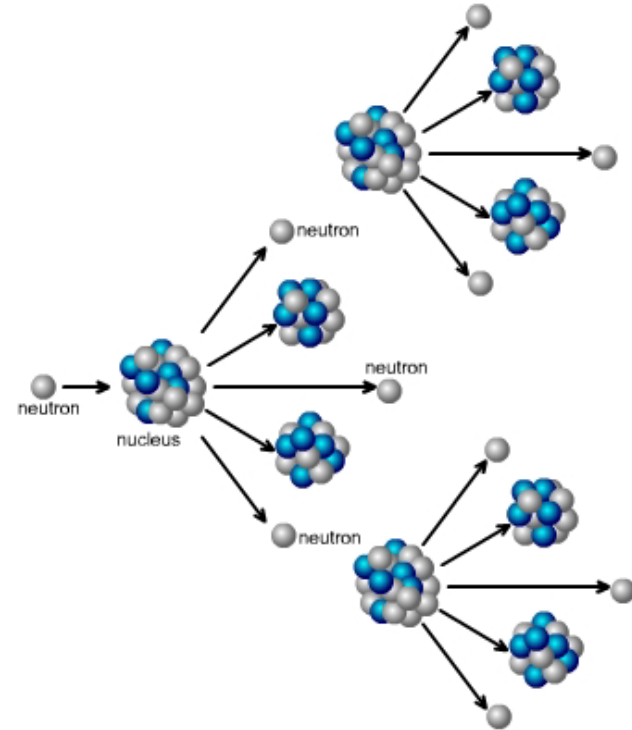
DELAYED NEUTRON DATA (*U-235 FOR THERMAL INCIDENT NEUTRON*)

| Group | Half-life (sec) | Decay constant (1/sec) | Relative Yield | Yield (neutrons per fission) |
|-------|-----------------|------------------------|----------------|------------------------------|
| 1 | 55.72 | 0.0124 | 0.033 | 0.00052 |
| 2 | 22.72 | 0.0305 | 0.219 | 0.00346 |
| 3 | 6.22 | 0.111 | 0.196 | 0.00310 |
| 4 | 2.3 | 0.301 | 0.395 | 0.00624 |
| 5 | 0.61 | 1.14 | 0.115 | 0.00182 |
| 6 | 0.23 | 3.01 | 0.042 | 0.00066 |
| Total | | | 1.000 | 0.0158 |

- $\bar{\nu}$ for U-235 (thermal incident neutron energy) ~ 2.43
- $\beta = 0.0158 / 2.43 = 0.0065$

FISSION CHAIN

- **Neutron fission chain uses neutrons produced in fission to induce other fissions, which in turn induces additional fissions, etc.**
 - Statistical process due to very large number of neutrons and fission chains at one time
 - Neutrons that do not cause a fission are either absorbed (and don't cause a fission) or leak out of the reactor
- **A reactor is critical when it maintains a steady-state neutron chain - the total number of neutrons (power) in the reactor is constant**
- **If a reactor is subcritical, the total number of neutrons (power) decreases**
- **If a reactor is supercritical, the total number of neutrons (power) increases**



MULTIPLICATION FACTOR

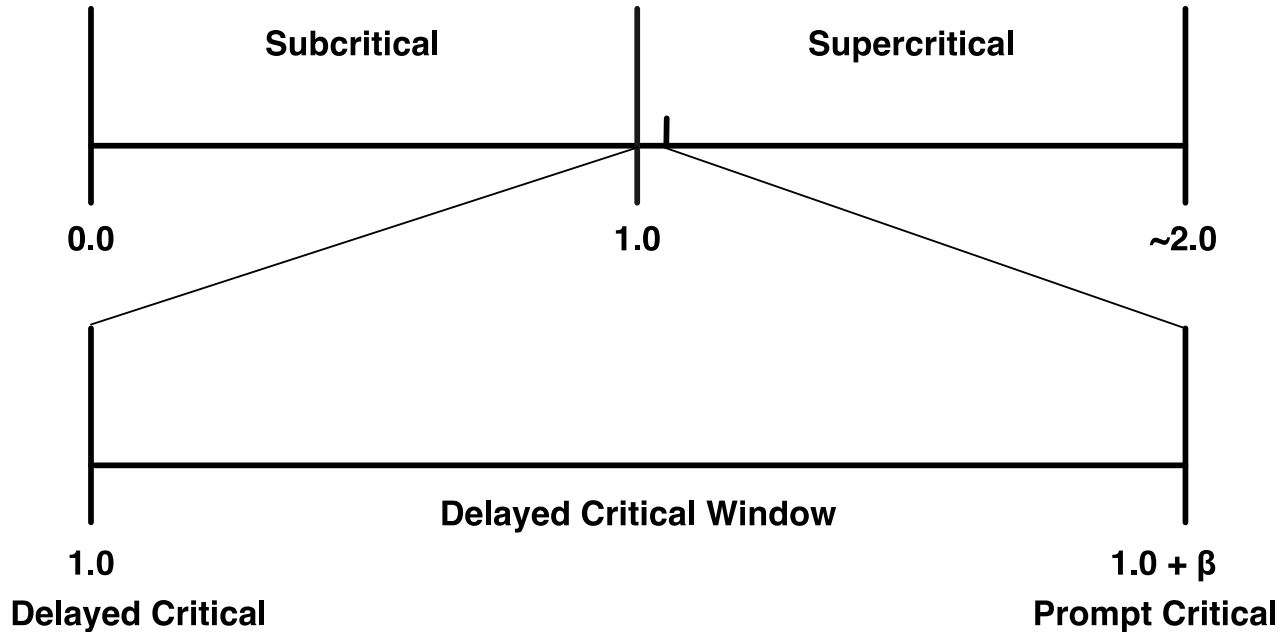
- k_{eff} (k-effective) is the multiplication factor
- k_{eff} is a measure of the number of fission neutrons in one generation compared to the previous generation

$$k_{\text{eff}} = \frac{\text{fission neutrons in a generation}}{\text{fission neutrons in preceding generation}}$$

$$k_{\text{eff}} = \frac{\text{fissions in a generation caused by fission neutrons}}{\text{fissions in preceding generation caused by fission neutrons}}$$

- **Three possible ranges for k_{eff} :**
 - $k_{\text{eff}} < 1$, system is **subcritical**, neutron population **drops** from generation to generation
 - $k_{\text{eff}} = 1$, system is **critical**, neutron population is **constant**
 - $k_{\text{eff}} > 1$, system is **supercritical**, neutron population **grows** with each generation

MULTIPLICATION FACTOR



- The Delayed Critical Window
 - β is the delayed neutron fraction ($\beta < 1\%$)
 - Delayed Critical Window is where delayed neutrons are required to maintain a critical (or slightly supercritical) system
 - $1.0 \leq k_{\text{eff}} < 1.0 + \beta$

Neutron Transport Equation

- Neutron interactions (scattering, capture, fission, etc.) lead to neutron losses or gains
- Overall balance of neutron gains and losses in a system can be modeled using the neutron transport equation (time independent equation below)

Movement
and Leakage
(loss)

$$\Omega \cdot \nabla \psi(\mathbf{r}, E, \Omega) + \Sigma_t(\mathbf{r}, E, \Omega) \psi(\mathbf{r}, E, \Omega) \quad \leftarrow \text{Removal (loss)}$$

$$= \int_0^\infty \int_{4\pi} \Sigma_s(\mathbf{r}, E' \rightarrow E, \Omega' \rightarrow \Omega) \psi(\mathbf{r}, E', \Omega') d\Omega' dE' \quad \leftarrow \text{Scattering}$$

$$+ \frac{1}{k_{eff}} \frac{\chi_f(E)}{4\pi} \int_0^\infty \int_{4\pi} \bar{v}_t(\mathbf{r}, E') \Sigma_f(\mathbf{r}, E', \Omega') \psi(\mathbf{r}, E', \Omega') d\Omega' dE' + S(\mathbf{r}, E, \Omega)$$

Fission (gain)

External Source (gain)

- A very complex equation, generally cannot solve by hand
 - Numerical solutions (deterministic and Monte Carlo methods)
 - Simplified equations (diffusion theory, point reactor kinetics)

Neutron Diffusion Equation

- Simplified form of transport equation using assumptions (Fick's Law, etc.)
- As with neutron transport equation, an overall balance of neutron gains and losses in a system (time and energy independent equation below)

Movement and Leakage (loss)

$$\nabla \cdot D \nabla \varphi(\mathbf{r}) - \Sigma_A \varphi(\mathbf{r}) + \frac{v \Sigma_f}{k_{eff}} \varphi(\mathbf{r}) + S(\mathbf{r}) = 0$$

Removal (loss)

Fission (gain)

External Source (gain)

- Gives approximate solutions to the transport equation, assuming assumptions hold
- Can be solved by hand for limited systems (generally solved numerically)
- Can be as simple or as complicated as desired for a system (multiple regions, multiple energy groups, down scattering, up scattering, etc.)
- While still an approximation, neutron diffusion theory is very powerful and extensively used
 - Corrections can be applied to diffusion theory to improve results for specific applications

Point Reactor Kinetics Model

- Spatially independent kinetics equation that accounts for time dependent behavior of a system
- Accounts for changes in criticality/reactivity of a system (i.e., a control rod is changing the criticality/reactivity of a reactor) as well as delayed neutrons being born into the system
- Coupled set of equations with neutron population (i.e., power) and delayed neutron precursors

$$\frac{dn(t)}{dt} = \frac{k(1 - \bar{\gamma}\beta) - 1}{l} n(t) + \sum_{i=1}^N \lambda_i C_i(t)$$
$$\frac{dC_i(t)}{dt} = \frac{\bar{\gamma}\beta_i}{l} n(t) - \lambda_i C_i(t) \quad i=1, \dots, N$$

$n(t)$: neutron population

$C_i(t)$: delayed neutron population for precursor group i

λ_i, β_i : delayed neutron precursor decay constant and abundance for group i

$\bar{\gamma}\beta$: total effective delayed neutron fraction

l : prompt neutron lifetime

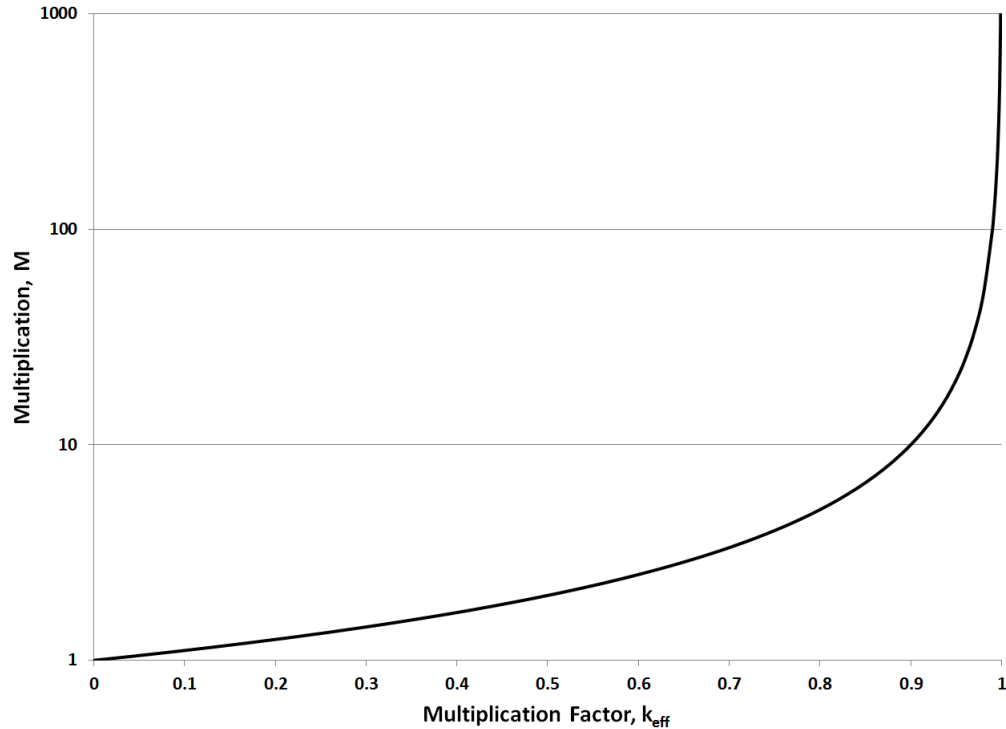
- “Simple” set of equations, but incredibly useful and valuable for reactor control and operation
- Inhour relation can be derived from these set of equations that relate reactor period (ie, how quickly the power is increasing) to criticality/reactivity

Point Reactor Kinetics Model and Neutron Multiplication

- Point reactor kinetics applications (reactor operations)
 - Reactor startup
 - Steady state operations (i.e., equilibrium conditions)
 - Power level changes
 - Transient operations (free runs, prompt bursts, insertion of control rods, withdrawal of control rods)
 - Reactor shutdown
- Thought Exercise: If a neutron is introduced to a **subcritical** system, on average how many neutrons are generated from this single neutron
 - Subcritical systems are still multiplying systems, so some fission events will occur!
 - What about a critical or supercritical system?
 - This is called Neutron Multiplication (M) (can be derived from the Point Reactor Kinetics equations)

$$M = \frac{1}{1 - k_{eff}} \text{ for } k_{eff} < 1$$

NEUTRON MULTIPLICATION



| k_{eff} | M |
|------------------|------|
| 0 | 1 |
| 0.5 | 2 |
| 0.9 | 10 |
| 0.95 | 20 |
| 0.99 | 100 |
| 0.999 | 1000 |

Neutron Multiplication

- Neutron Multiplication can be measured and useful
- How close to critical is this system (safeguards, criticality safety, emergency response)
- If we want to take a system from subcritical to critical, neutron multiplication measurements can assist in an approach to critical

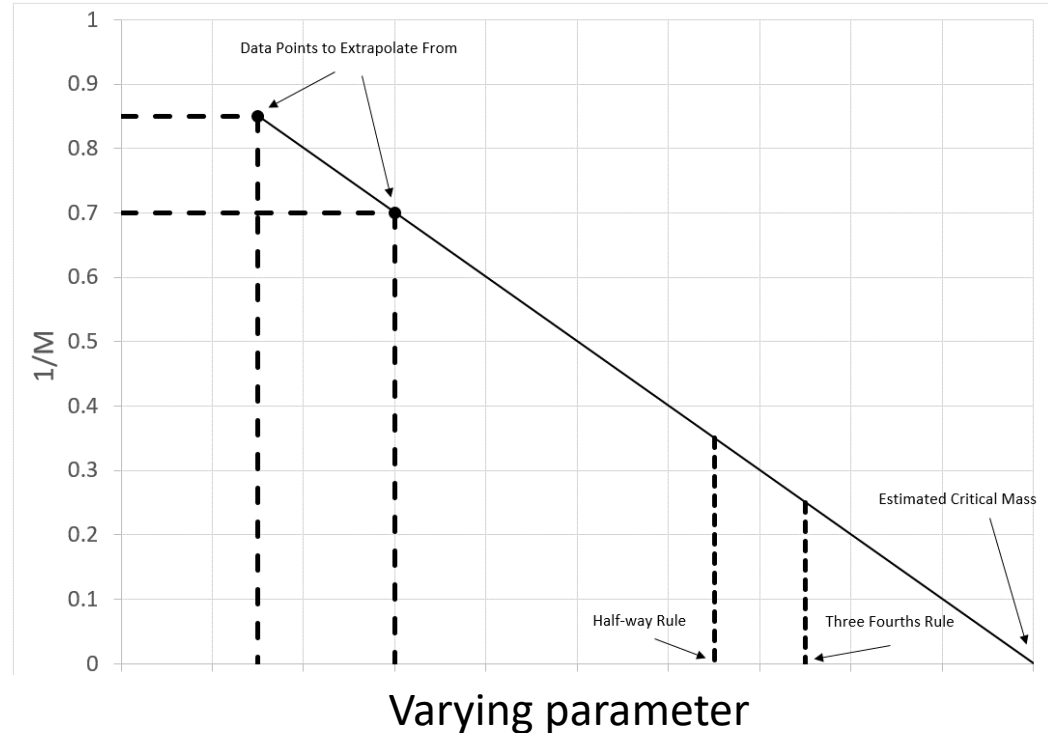
$$M = \frac{1}{1 - k}$$
$$\frac{1}{M} = 1 - k$$
$$\frac{1}{\left(\frac{M}{M_o}\right)} = \frac{CR_o}{CR} = \frac{1 - k}{1 - k_o}$$

CR_o : Count rate for an initial configuration
 CR : Count rate for a perturbed system

- Count rate ratios can give relative changes in neutron multiplication and can be used to approach a critical system safely
- Called a 1/M (or One-Over-M) approach to critical

APPROACH-TO-CRITICAL

- Due to small span of the delayed neutron window, criticality must be approached in a safe manner
- Generally done by varying a single parameter
 - Fuel mass, reflector thickness, separation distance, control rod position, etc.
- Ratio of neutron count rates can be used to determine $1/M$
- Extrapolate to critical ($1/M=0$)
- Rules govern how large the following change can be



APPROACH-TO-CRITICAL

- Actual approach-to-critical example to the right (EUCLID experiment)
- X-axis is the number of layers (directly proportional to fuel mass)
- Note that assumptions are very important during approach-to-critical

- $\frac{C_0}{C_1} = \frac{\epsilon S M_0 \Omega}{\epsilon S M_1 \Omega} = ? \frac{M_0}{M_1}$

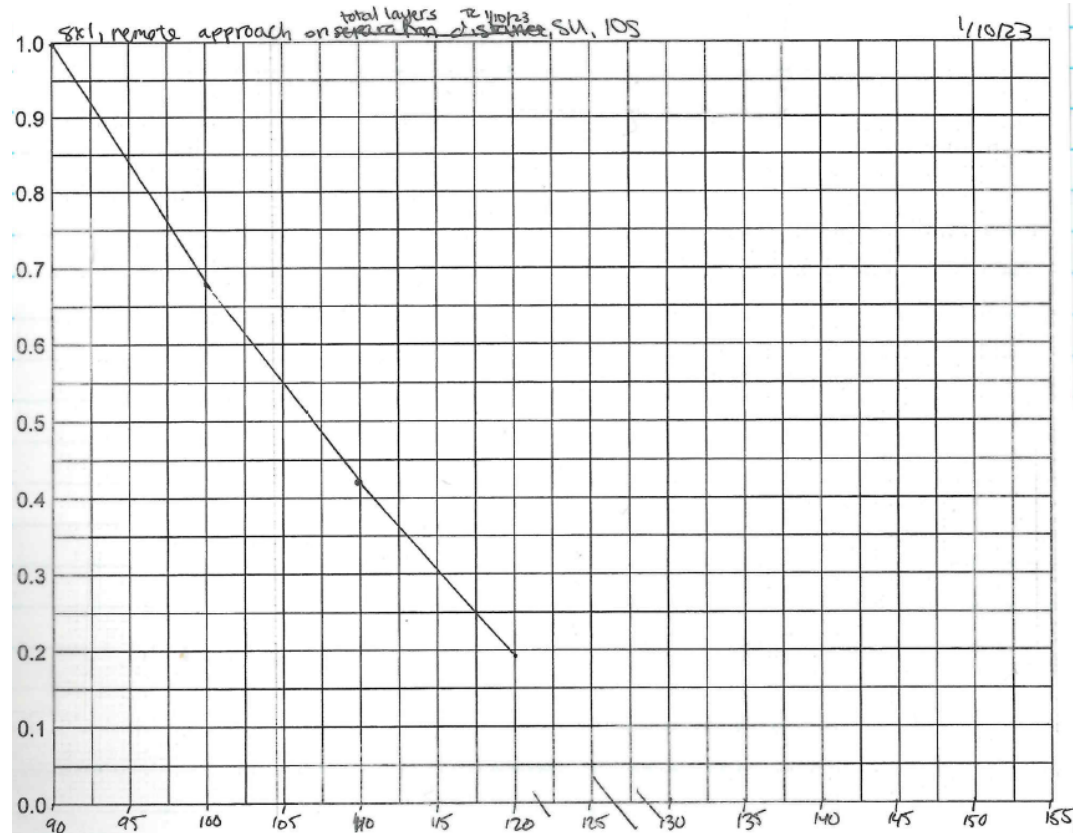
C is count rate

ϵ is the efficiency of the detector

S is the neutron source (n/sec)

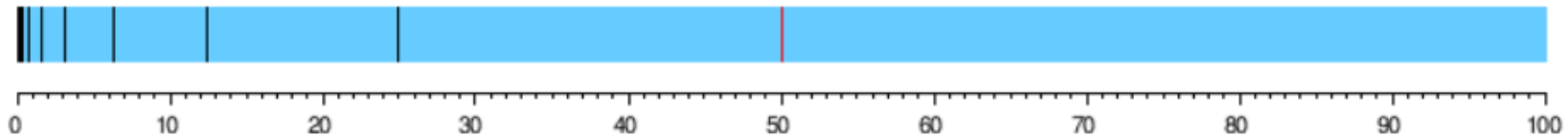
M is the relative multiplication

Ω is the solid angle



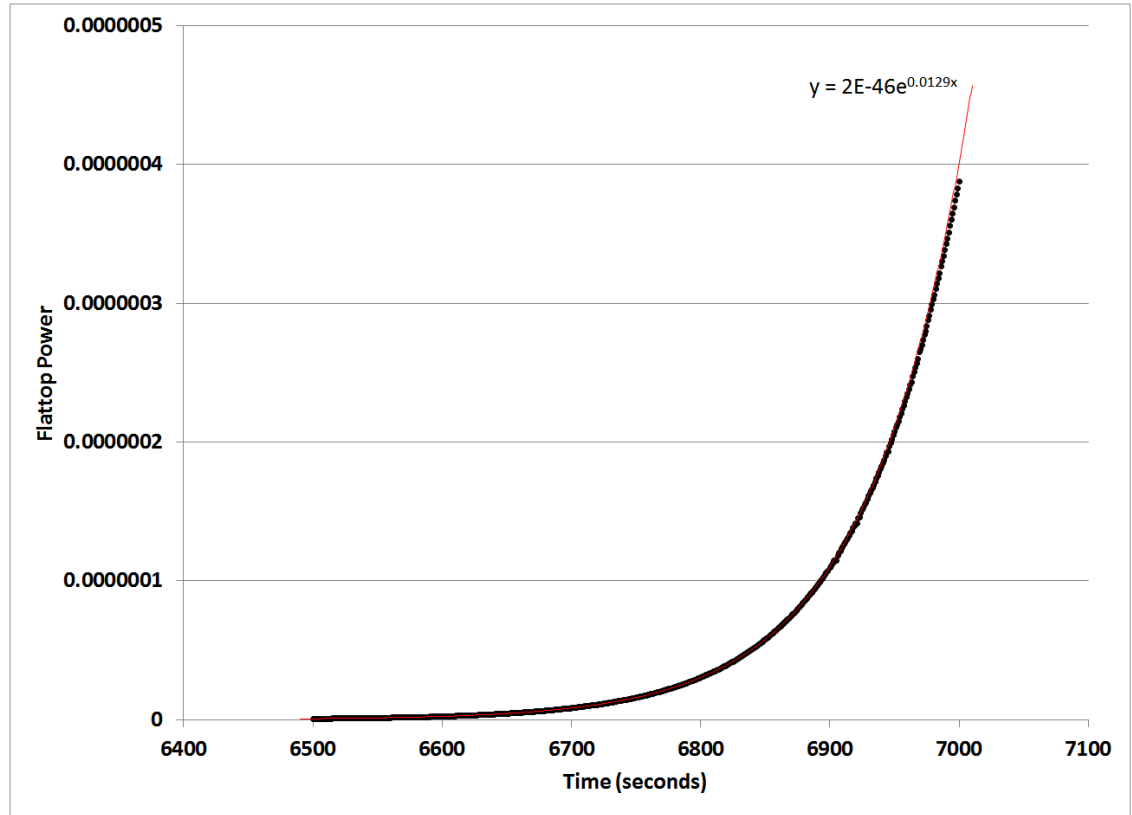
APPROACH-TO-CRITICAL

- 1/2-way rule: can only make a change that is up to half-way between current configuration and predicted critical configuration
- If we follow this forever...we never get critical!
 - Related to Dichotomy paradox and other math problems
- So we must break this rule.
- But, we shouldn't break it until
 - We are very close to critical
 - We understand the system very well
 - We are confident that we can break it and be inside the delayed critical window



MEASURING REACTOR PERIOD

- Reactor Period: $\tau = \frac{1}{\omega}$
- The fit gives an $\omega = 0.0129 \text{ s}^{-1}$, and the reactor period is then:
- $\tau = \frac{1}{\omega} = 77.5 \text{ s}$



UNITS OF REACTIVITY

- **Reactivity itself is unitless**
- **However, it can be quoted in units of:**
 - Dollars and cents
 - pcm (per cent millirho)
- **Take the Delayed Critical Window ($0 < \rho < \beta_{\text{eff}}$) and split that interval into 100 units (called cents)**
 - One dollar is 100 cents
- **1 cent of reactivity: $\rho = \$0.01 = \beta_{\text{eff}} / 100$**
- **1 dollar of reactivity: $\rho = \$1.00 = \beta_{\text{eff}}$**
- **By definition, \$1.00 of reactivity is prompt critical**

INHOUR EQUATION

$$\rho(\$) = \frac{l}{\beta_{eff}\tau} + \sum_{i=1}^6 \frac{\frac{\beta_i}{\beta_{eff}}}{1 + \lambda_i\tau}$$

where

$\rho(\$)$ is the reactivity in dollars

β_{eff}/l is the Rossi- α at delayed critical

β_{eff} is the effective delayed neutron fraction for the system

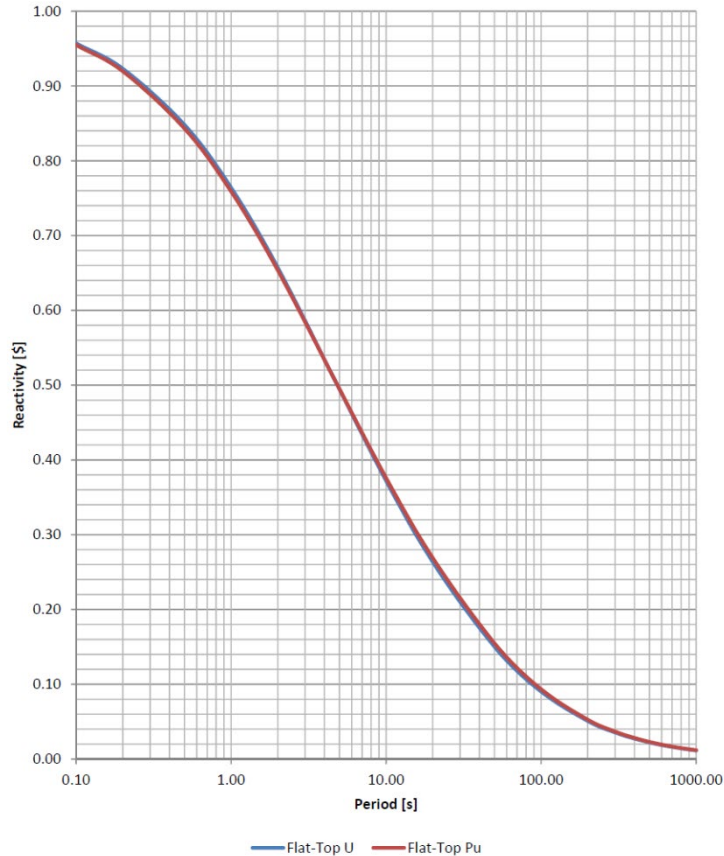
T is the reactor period

β_i/β_{eff} is the relative abundance for ^{235}U for each of the six groups from thermal fission

λ_i is the decay constant for ^{235}U for each of the six groups from thermal fission

l is the neutron lifetime of the system

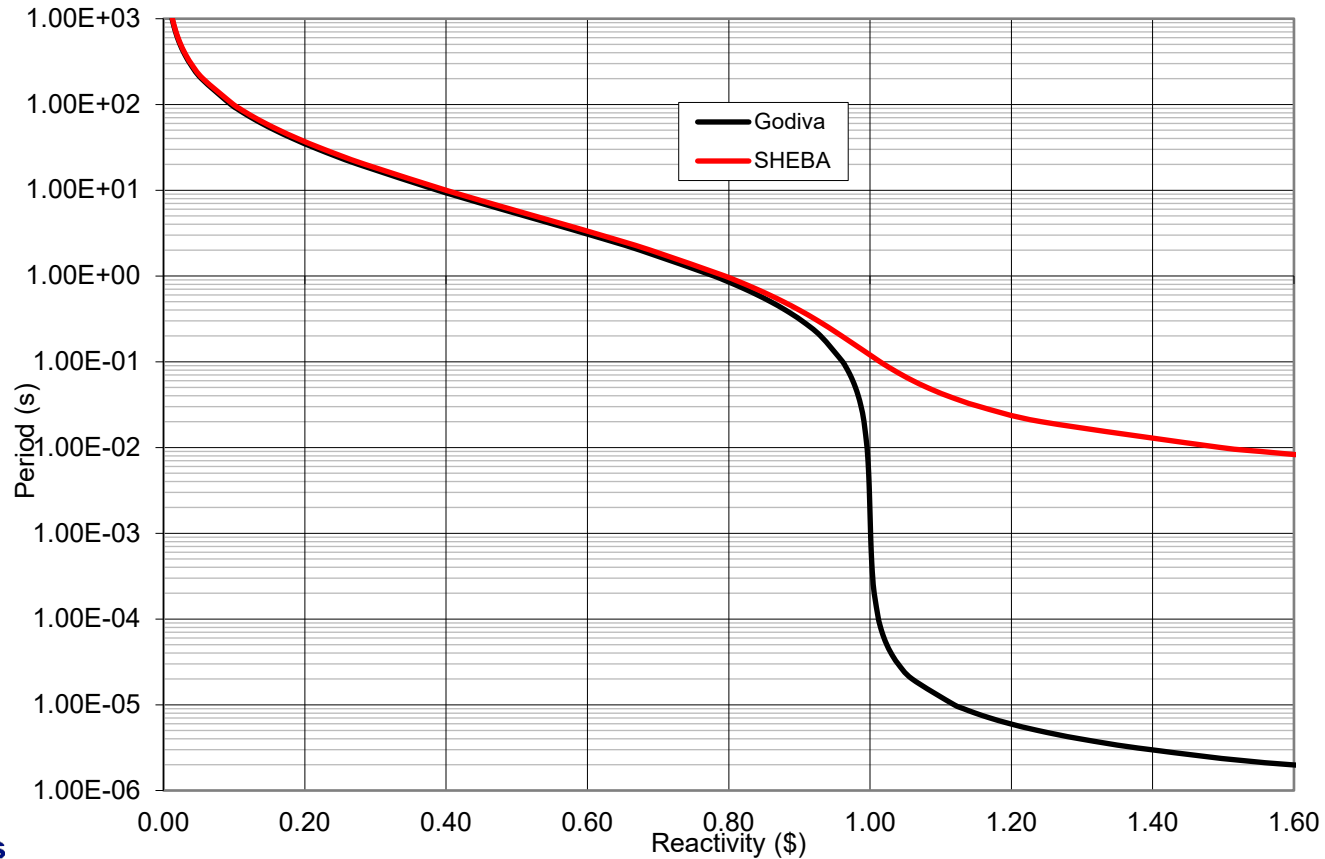
PERIOD AND REACTIVITY



| ρ/β (\$) | τ (sec) |
|-------------------|--------------|
| 0.10 | 100 |
| 0.25 | 20 |
| 0.50 | 5 |
| 0.80 | 1 |

The values in the table above are not exact and vary slightly from system to system but are good rules of thumb

EXAMPLE: INHOUR CURVES



REACTIVITY FEEDBACK

- **As energy is added to a system (i.e., fission energy), the properties of the system can change**
 - Temperature rise -> Volume expansion -> Density decrease
- **As the properties change, the reactivity of the system can change**
- **Energy deposition leads to reactivity feedback**

REACTIVITY FEEDBACK

- **Reactivity feedback is usually (preferably) negative**
 - Increases in energy (or temperature) lead to decreases in reactivity
 - Fast systems: Volume expansion introduces more leakage
 - Thermal systems: Extra leakage, more resonance absorption, and decreasing fission cross section
- **Reactivity feedback modeled with single coefficient,**

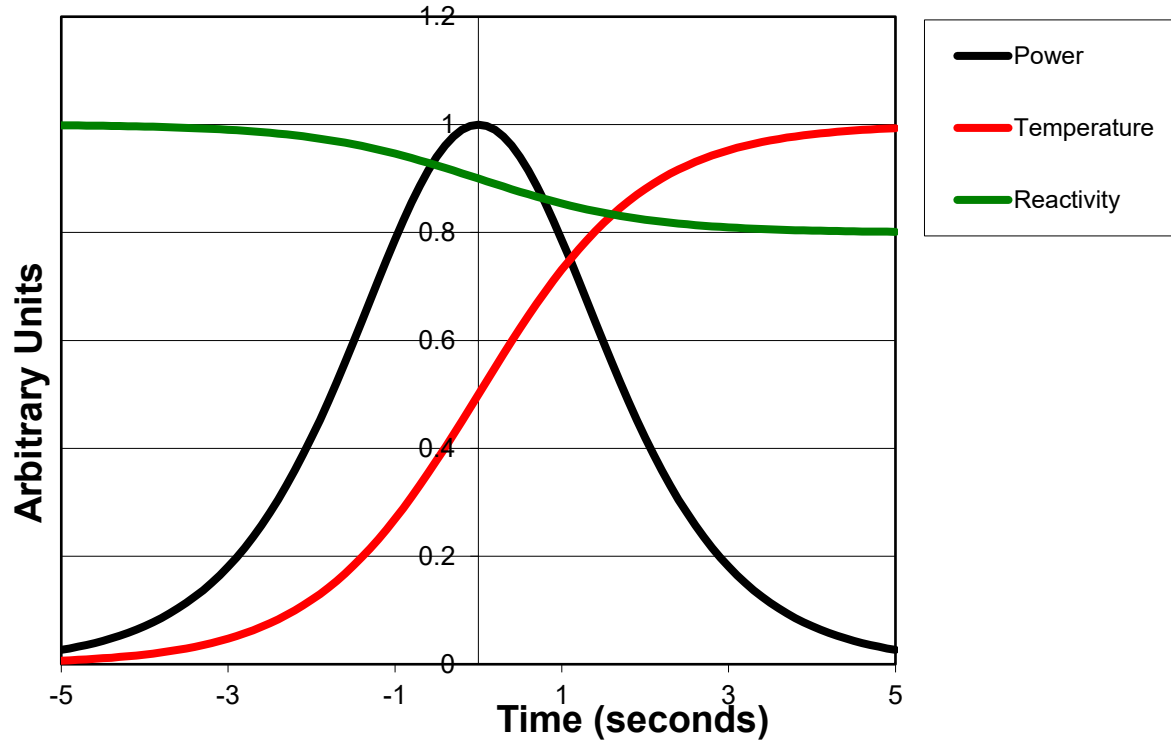
$$\rho = \rho_0 + \alpha T$$

- **Some example values:**

| Assembly | Coefficient (cents/Kelvin) |
|------------------------|----------------------------|
| Godiva IV, Flattop (U) | -0.3 |
| Flattop (Pu) | -0.2 |
| SHEBA | -4.0 -10.0 |

SELF-LIMITING EXCURSION

Increase in reactivity -> power increase -> temperature increase -> reactivity feedback -> decrease in reactivity -> power decrease

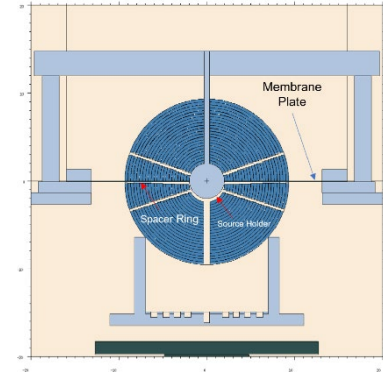


Criticality experiments

Understanding of theory important for design and performance of critical experiments

How are *Integral* experiments used for validation?

- An experiment is designed and built and observables are measured
 - Usually k-effective
 - A configuration above delayed critical
 - A configuration that is exactly critical
 - The predicted critical configuration based on $1/M$
 - Other observables add information
 - Leakage spectrum, etc.
- The experiment is precisely modeled in a Monte Carlo code
 - Geometry, including gaps, etc.
 - Composition of all materials
 - Temperature
- Differences between Experiment and Calculation indicate problems in Nuclear Data



Monte Carlo Particle Transport Method

An approach for simulating random walk of particles through matter on a computer.

“Following each of a large number of particles from the source throughout its life history to its death ... using the elementary probabilities at each stage of its career in determining its fate”¹

1 Geometry

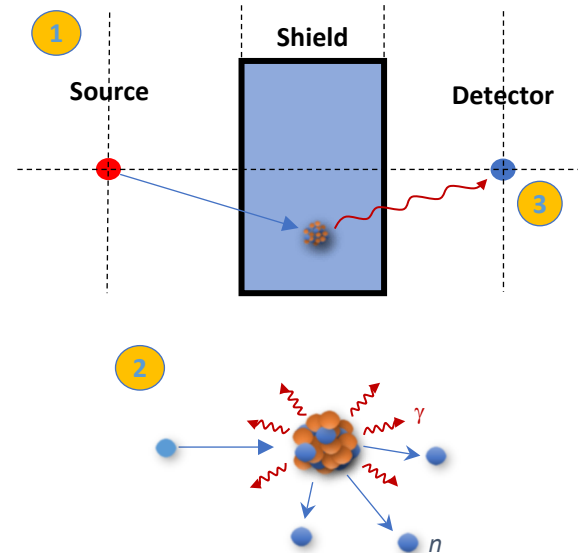
Ray-tracing through "exact" model of problem geometry to determine locations of interactions.

2 Physics

Random sampling using cross-section data, physics models, and source descriptions to simulate interactions.

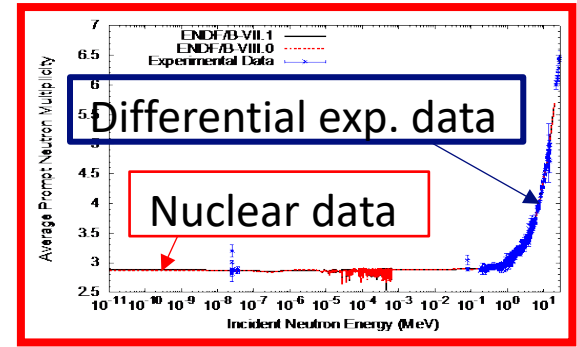
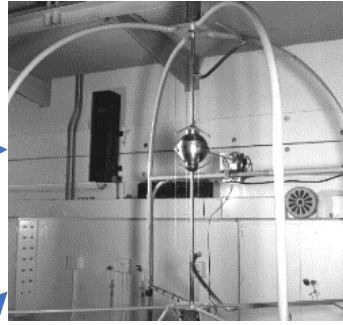
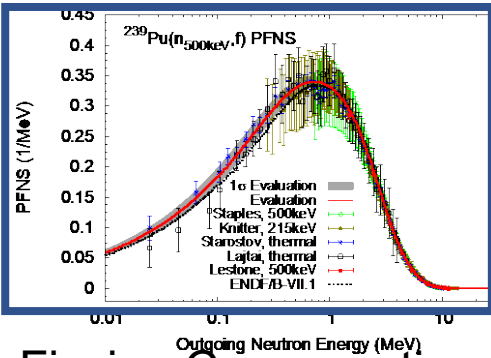
3 Tallies

Bookkeeping to record how often certain events occur during the simulation.

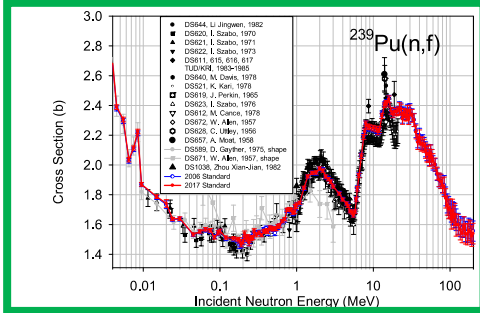


Integral experiments are utilized for ND validation

Prompt Fiss. Neutr. Spectr. Jezebel validation exp.



Fission Cross-section



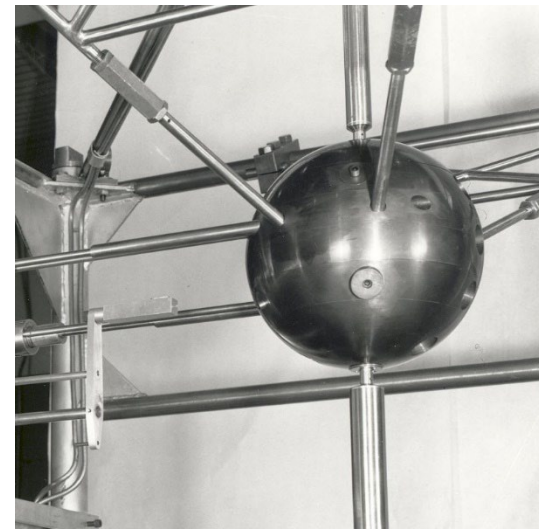
A. Carlson, et. al., "Evaluation of the Neutron Data Standards," Nuclear Data Sheets, 148 (2018).

$$\Omega \cdot \nabla \psi(\mathbf{r}, E, \Omega) + \Sigma_t(\mathbf{r}, E, \Omega) \psi(\mathbf{r}, E, \Omega) = \int_0^\infty \int_{4\pi} \Sigma_s(\mathbf{r}, E' \rightarrow E, \Omega' \rightarrow \Omega) \psi(\mathbf{r}, E', \Omega') d\Omega' dE' + \frac{1}{k} \chi_f(E) \int_0^\infty \int_{4\pi} \bar{\nu}_t(\mathbf{r}, E') \Sigma_f(\mathbf{r}, E', \Omega') \psi(\mathbf{r}, E', \Omega') d\Omega' dE'$$

Integral and differential experiments are complementary and both are needed to improve our understanding of fission

Experiment design

- There are many methods used to design criticality experiments
- Historically, this was done by matching physical characteristics
- In the last 20 years, more sophisticated techniques have been used (nuclear data sensitivity/uncertainty simulation capabilities)

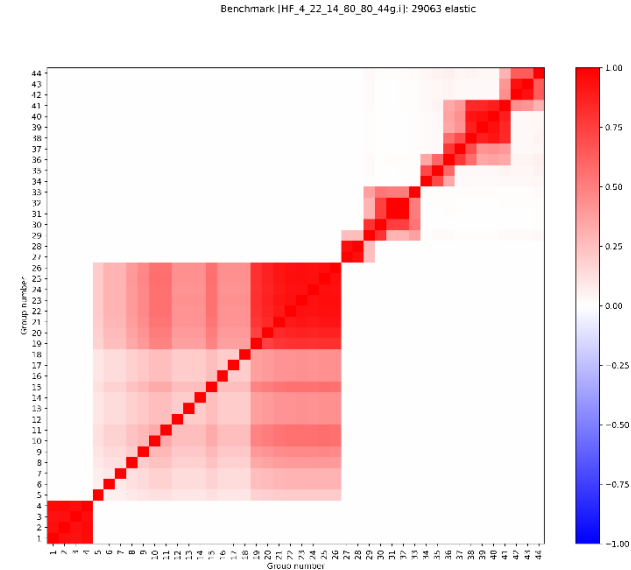
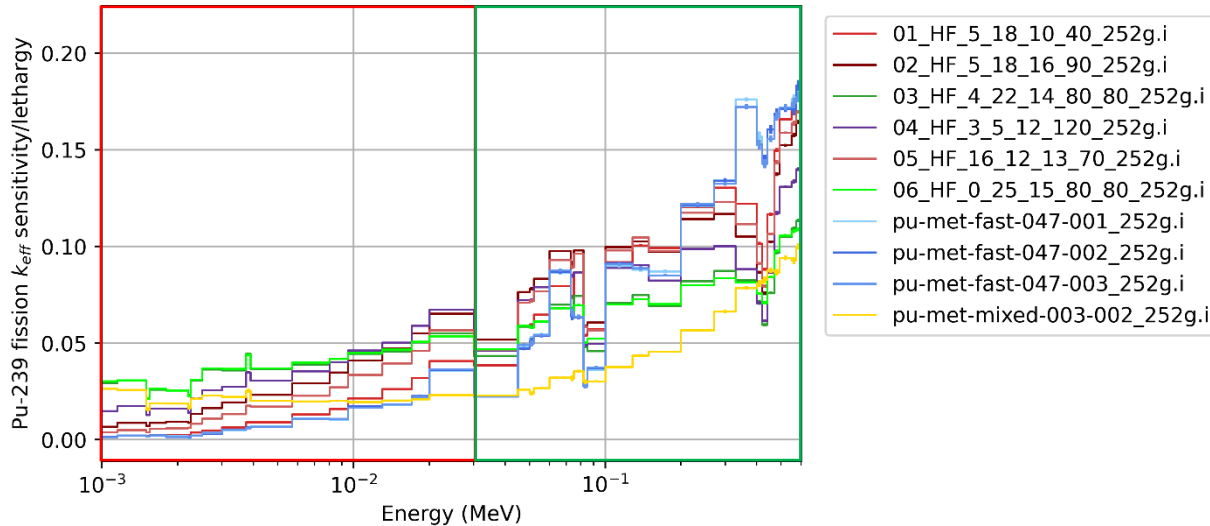


| Metric | ND sensitivities | Covariance | Is there an application? | |
|--------------------------------------|------------------|------------|-----------------------------------|--|
| | | | Yes | No |
| Physical characteristics | No | No | Compare to app | N/A* |
| Spectral characteristics | No | No | Compare to app | Maximize in desired energy range |
| Reactor physics characteristics | No | No | Compare to app | N/A* |
| k_{eff} sensitivity | Yes | No | N/A | Max in desired N/R pair and energy range |
| Similarity: E_{sum} | Yes | No | E_{sum} | N/A* |
| Similarity: partial E_{sum} | Yes | No | Partial/filtered E_{sum} | N/A* |
| Similarity: c_k | Yes | Yes | c_k | N/A* |
| Similarity: partial c_k | Yes | Yes | Partial/filtered c_k | N/A* |
| Other similarity metrics | Yes | No | Multiple | N/A* |
| ND unc | Yes | Yes | Compare to app | Max desired N/R pair |
| Posterior uncertainty reduction | Yes | Yes | N/A | D Opt, A Opt |

*: N/A unless a pseudo-application is used

Experiment design

- Approaches may differ depending on if there is a specific application
- Approaches may choose to utilize nuclear data sensitivities and/or nuclear data covariances



Other responses

- Other responses can help constrain nuclear data in ways that critical experiments alone cannot
- Note that simulating nuclear data sensitivities for some of these other response types can be challenging

| Measurement Method | Observable | | | |
|-------------------------------------|------------|-------|---------|------|
| | σ | ν | β | PFNS |
| Critical experiments | ✓ | ✓ | | ✓ |
| Neutron Multiplication Measurements | ✓ | ✓ | ✓ | |
| Reaction rate ratios | ✓ | ✓ | | ✓ |
| Pulsed Spheres | ✓ | | | |
| Gamma/Neutron Leakage Spectra | ✓ | | | ✓ |
| Delayed Neutron Measurements | | | ✓ | |
| Rossi- α | ✓ | ✓ | ✓ | |
| Reactivity Coefficient | ✓ | | ✓ | |



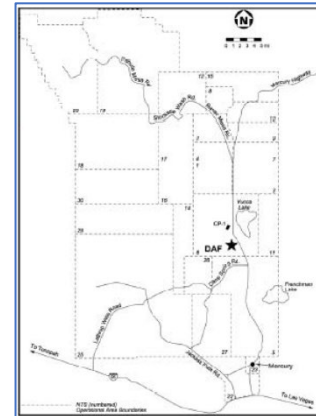
NCERC is our nation's only general-purpose critical experiments facility and is one of only a few that remain operational throughout the world

Location: Device Assembly Facility (DAF) at the Nevada Nuclear Security Site (NNSS)

Operated by: Los Alamos National Laboratory

NCERC Mission Statement:

The mission of the National Criticality Experiments Research Center (NCERC) is to conduct experiments and training with critical assemblies and fissionable material at or near criticality in order to explore reactivity phenomena, and to operate the assemblies in the regions from subcritical through delayed critical. One critical assembly, Godiva-IV, is designed to operate above prompt critical.



Sandia Critical experiment facility

Critical Assemblies at NCERC



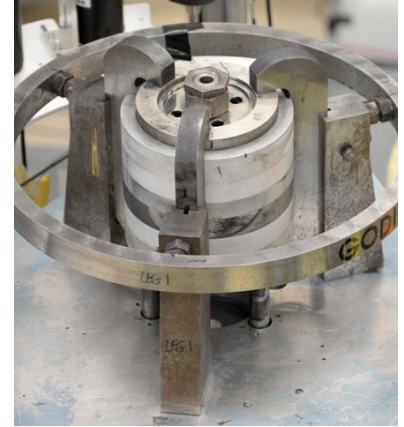
Flattop



Planet



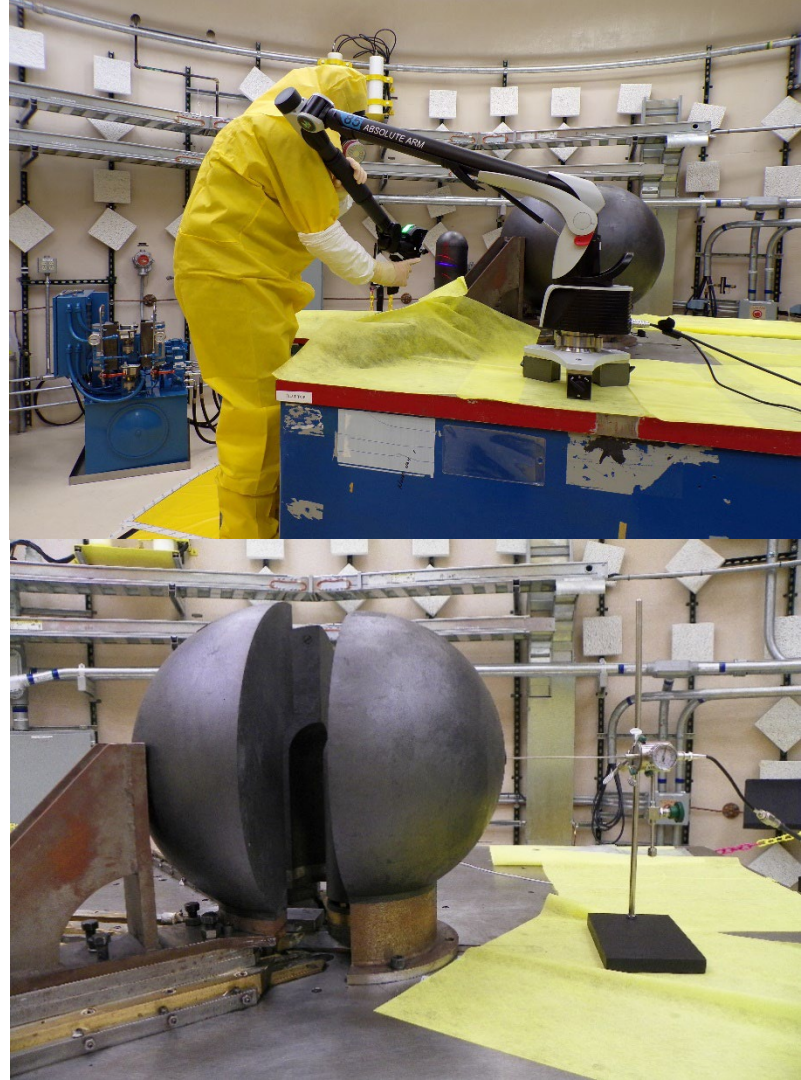
Comet



Godiva-IV

Flattop

- Components
 - 1000 kg natural uranium reflector
 - HEU (~16 kg) or Pu (~6 kg) core
- original application:
 - Simple two region problem
- Current applications:
 - Neutron activation studies
 - Dosimetry measurements
 - Criticality safety training demonstrations



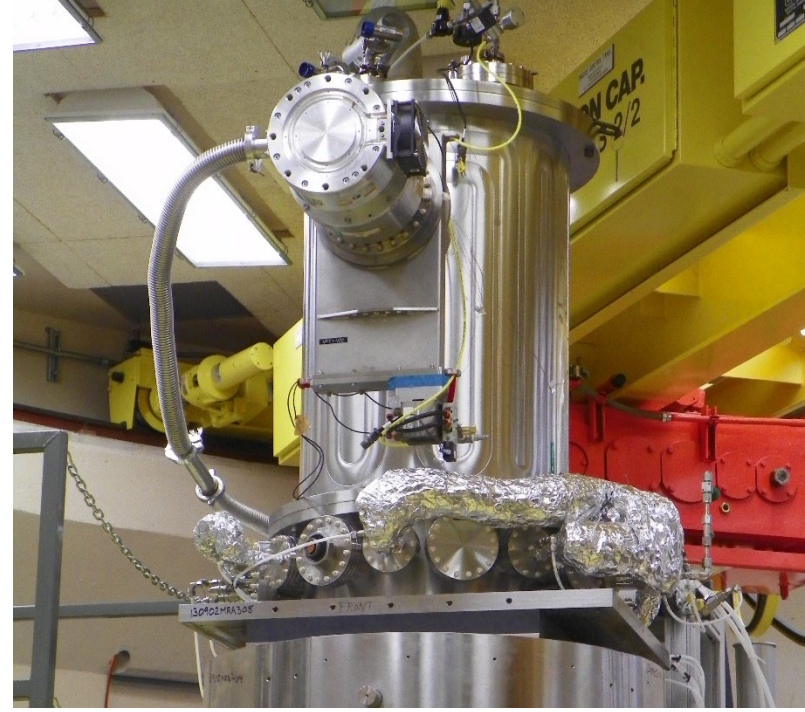
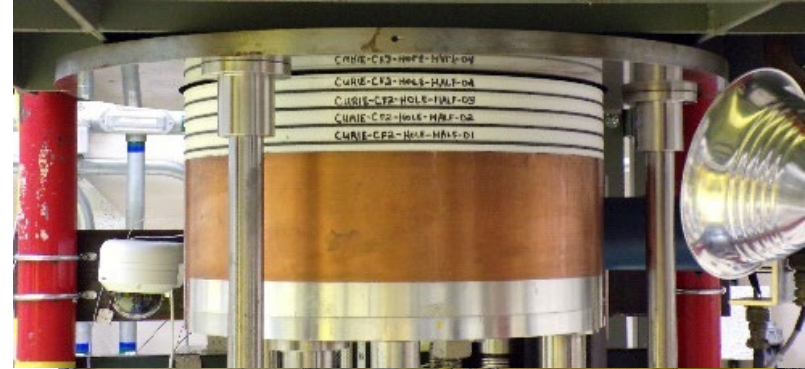
Planet

- Vertical lift assembly
 - Can accommodate a wide variety of nuclear material and other materials
 - Core is divided into two parts and brought together remotely
- Current applications:
 - Critical mass studies
 - Nuclear data measurements
 - Reactor physics measurements
 - Criticality safety training demonstrations



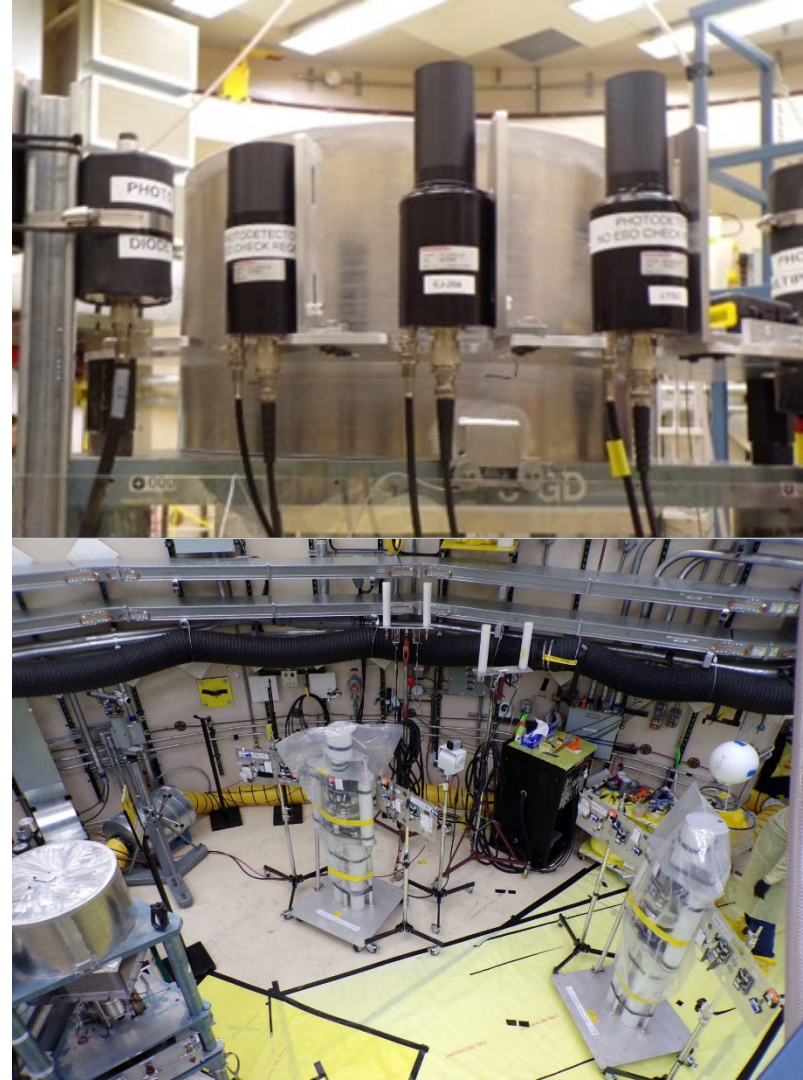
Comet

- Larger vertical lift assembly
 - Can accommodate a wide variety of nuclear material and other materials
 - Core is divided into two parts and brought together remotely
- Current applications:
 - Critical mass studies
 - Reactor demonstrations
 - Nuclear data measurements
 - Reactor physics measurements
 - Criticality safety training demonstrations



Godiva-IV

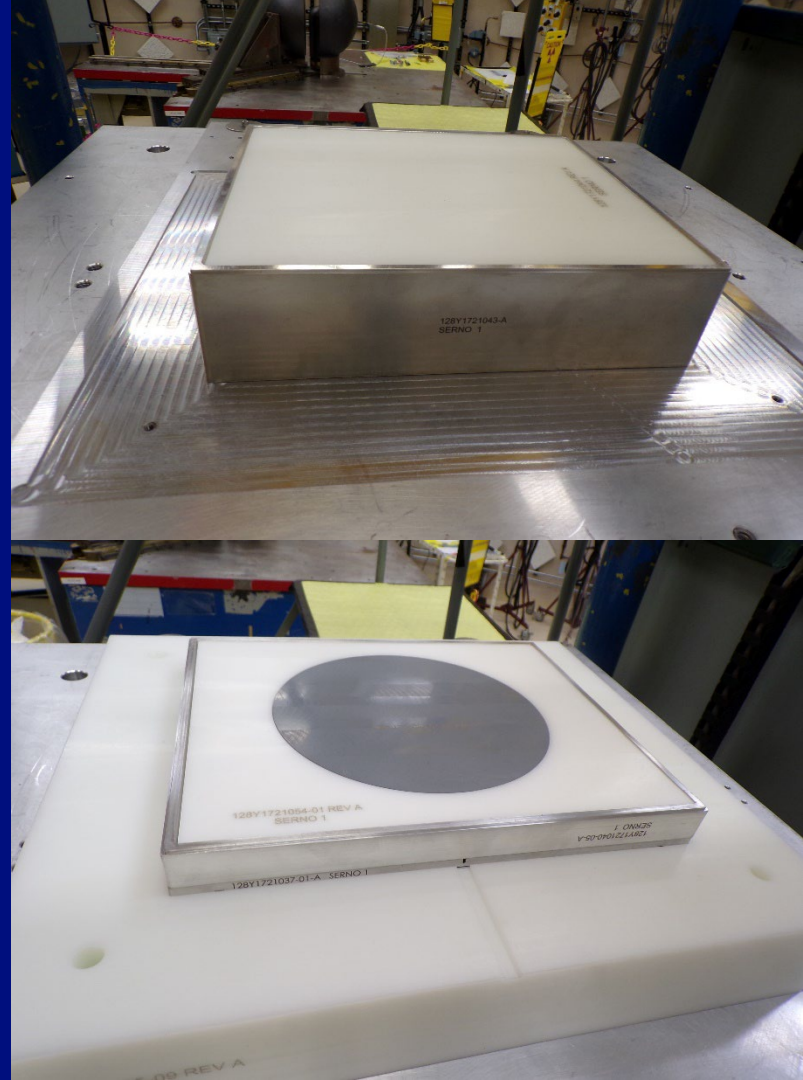
- 66 kg of highly-enriched U (HEU)
- About the size of a coffee can
- Current applications:
 - Neutron activation studies
 - Dosimetry measurements
 - Criticality alarm testing
 - Criticality safety training demonstrations



Recent Experiments

Chlorine Worth Study

- Chlorinated-PVC with Pu fuel
- Supporting the criticality safety program at LANL for waste remediation operations
- Investigating the impact of chlorine absorption



Thermal-Epithermal Experiments (TEX)

- Collaboration between LLNL and LANL
- Designed to benchmark a variety of neutron spectra for both U and Pu



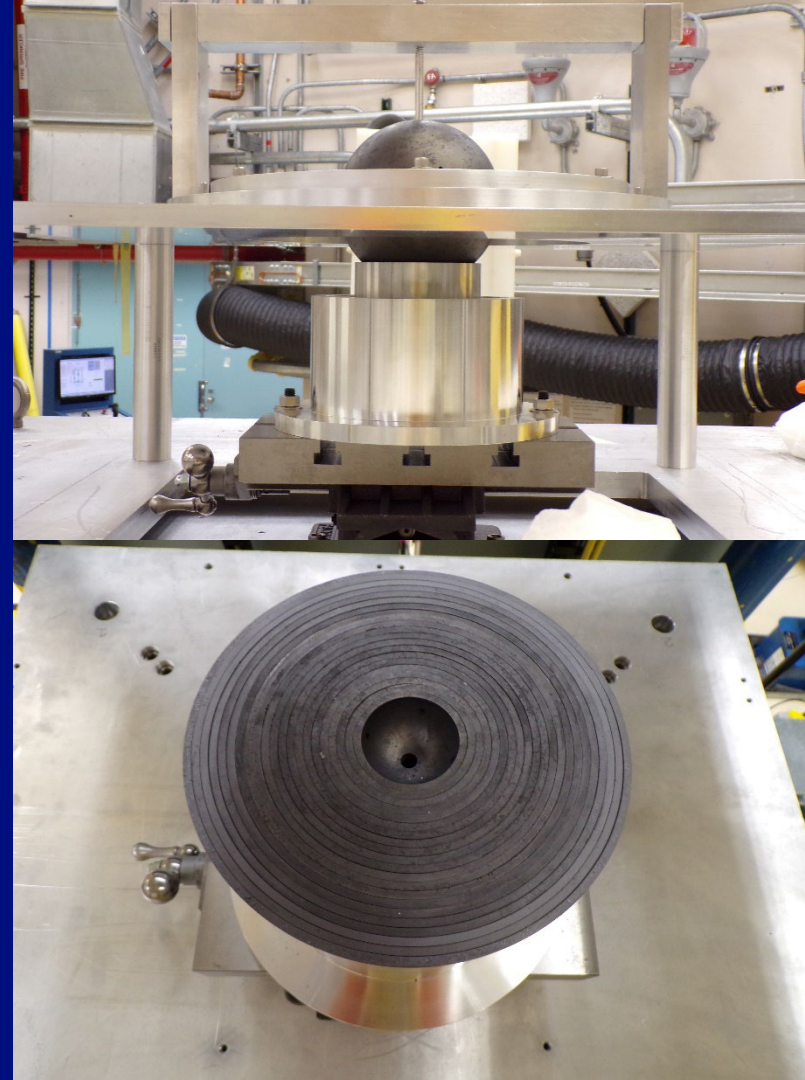
CERBERUS

- Subset of the Zeus series
- Investigating neutron scattering in copper
- Uses Cu as an interstitial material between layers of HEU



Measurements of Uranium Subcritical and Critical (MUSiC)

- Bare HEU system measured to generate a modern spherical HEU benchmark
- Subcritical configurations measured using various diagnostics to determine system kinetic information
- Uses HEU hemi-shells to adjust mass/criticality



EUCLID

- Experiment to reduce compensating error in Pu-239
- Utilized critical (reactivity), reaction rate ratios, reactivity coefficients, Rossi-alpha, neutron leakage spectra, and subcritical experiments



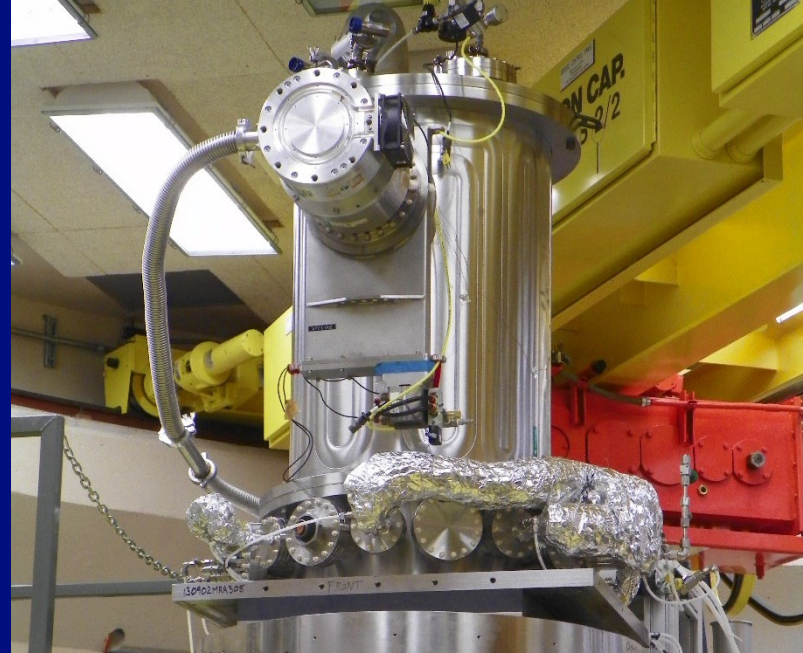
Hypatia

- Microreactor design intended to test properties of YH_x at a variety of temperatures
- Consisted of HEU fuel, Be reflectors, YH_x moderator plates and electric heaters
- Temperatures up to 330 degrees Celsius examined



KRUSTY

- Supporting space reactor applications for NASA
- Uses HEU fuel with BeO reflectors
- Performed a 28-hour operation with multiple reactor transients to test response of stirling engines



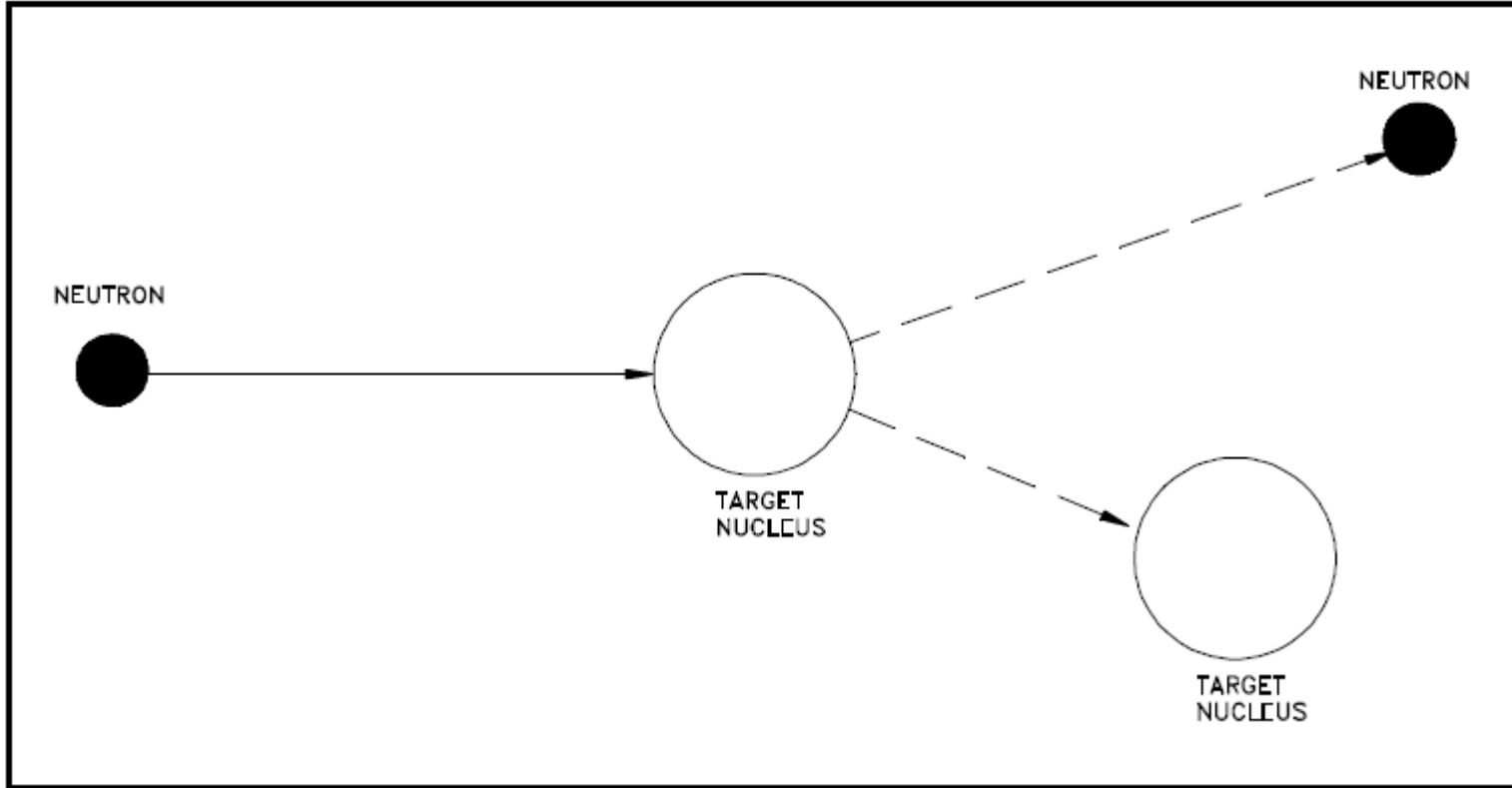
Summary

Big questions

- Criticality experiments are an important part of the nuclear data pipeline
- Better understanding of fission (and better nuclear data) is extremely important for the nuclear industry
 - Criticality experiments play a role in improving this understanding
- Many advances have taken place in the last 80 years, but many challenges remain:
 - Accuracy and precision of many reactions in evaluated nuclear data is still lacking
 - New innovations needed to further constrain nuclear data
 - Capabilities to maximize impact of experiments are still advancing (use of AI/ML)
 - Capabilities to calculate sensitivities for responses other than k_{eff} could be improved
 - Ways to utilize measured responses (beyond k_{eff}) is an area of growth
 - Correlation between experiments is also a topic of great interest
 - And many others!

Backup slides

SCATTERING INTERACTIONS



SCATTERING INTERACTIONS

- As a result of a scattering interaction, neutron direction changes and neutron energy changes
 - Typically, neutron energy decreases due to energy transfer to nucleus
Elastic Scatter: KE is conserved, Inelastic Scatter: KE is not conserved
- More energy can be transferred from a neutron to a lighter nucleus
 - Collisions with a light nucleus not only change neutron direction, but are also good at slowing-down the neutron!
- Less energy can be transferred from a neutron to a heavier nucleus
 - Collisions with a heavy nucleus change the neutron direction, but are poor at slowing-down the neutron!
- Energy transferred to nucleus in an elastic collision:

- $$\frac{KE_f}{KE_o} = \frac{A^2 + 2A \cos \theta + 1}{(A+1)^2}$$

ABSORPTION INTERACTIONS

- In all cases, an initial compound nucleus is formed when the original nucleus absorbs the incident neutron
 - Oftentimes this compound nucleus is unstable (radioactive)
- **Radiative Capture**
 - Nucleus absorbs a neutron, emitting a gamma-ray
 - Neutron loss
- **Particle Emission**
 - Nucleus absorbs a neutron, emits a charged particle (beta, alpha)
 - In some cases, neutrons are also released (n,Xn) reactions, where $X > 1$
 - Usually a neutron loss
- **Fission**
 - Nucleus absorbs a neutron, splitting into fission fragments, neutrons, gamma-rays, etc.

URANIUM FUEL

- Uranium fuel typically contains U-235, U-238, and U-234
- Uranium fuel is typically unclad (for NCERC)
- **Uranium fuel enrichment is measured in weight percentage of U-235**
 - Natural Uranium: 99.27% U-238, 0.72% U-235, 0.0055% U-234
 - NU, Tuballoy, Normal Uranium
 - Depleted Uranium: 99.76% U-238, 0.24% U-235, 0.0018% U-234
 - Commercial Grade Uranium (Light Enriched Uranium): 3-4% U-235
 - Low Enriched Uranium (LEU): <20% U-235
 - High Enriched Uranium (HEU): 20%+ U-235
 - Weapons Grade Uranium: 93% U-235
 - Orallo (Oy)
- Most NCERC uranium fuel is HEU
- Note: *Isotopes* have same Atomic Number Z but different Atomic Mass Number A, example: U-235 and U-238

PLUTONIUM FUEL

- Plutonium fuel is typically clad or double encapsulated
- **Plutonium fuel quality measured by atom percent of Pu-240 in the material**
 - Pu-240 is a spontaneous fissioner (neutron emitter), not good for certain applications
 - Reactor grade: 19%+ Pu-240
 - Fuel grade: 7%-19% Pu-240
 - Weapon grade: 0%-7% Pu-240

Note:

Spontaneous Fission (SF) is a form of radioactive decay in which a heavy nucleus splits in two releasing energy and neutrons. SF is not induced by the absorption of a neutron. SFers may be used as neutron sources.

GODIVA MODEL (*PROMPT BURST*)

- **Point-Reactor Kinetics Model**

- $\frac{dP}{dt} = \frac{\beta}{\Lambda} [(R - 1)P + \sum_i f_i D_i]$

- $f_i = \frac{\beta_i}{\beta}$

- $R = R_0 + \alpha_T T$

- $\frac{dD_i}{dt} = \lambda_i (P - D_i)$

- **Fuel Temperature (Adiabatic Model)**

- $\frac{dT}{dt} = \frac{P}{MC_p}$

NEUTRON LIFECYCLE – SIMPLE THERMAL SYSTEM

- **Neutron Gain Terms**

- Fast Fission factor ε
- Thermal Fission (Reproduction factor η)

- $$\eta = \frac{\nu \Sigma_f^{fuel}}{\Sigma_a^{fuel}}$$

- **Neutron Loss Terms**

- Fast Leakage $P_{NL,f}$
- Resonance Absorption factor p
- Thermal Leakage $P_{NL,t}$
- Thermal Non-Fuel Absorption (Thermal Utilization factor f)

- $$f = \frac{\Sigma_a^{fuel}}{\Sigma_a^{fuel} + \Sigma_a^{nonfuel}}$$

- $k_{eff} = \varepsilon P_{NL,f} p P_{NL,t} f \eta$ *six – factor formula*

NEUTRON LIFECYCLE – SIMPLE FAST SYSTEM

- Neutrons are born fast from fission caused by fast neutrons
- Some fast neutrons will escape out of the system, never to return
- Neutron Gain Terms
 - Fast Fission η
 - $\eta = \frac{v\Sigma_f^{fuel}}{\Sigma_a^{fuel}}$
- Neutron Loss Terms
 - Non-Fuel Absorption f
 - $f = \frac{\Sigma_a^{fuel}}{\Sigma_a^{fuel} + \Sigma_a^{nonfuel}}$
 - Fast Leakage P_{NL}

$$k_{eff} = \eta f P_{NL}$$

TRANSIENT BEHAVIOR AND THE INHOUR EQUATION

• **Constant reactivity solution:** $n(t) = \sum_j A_j e^{\omega_j t} \rightarrow n(t) = A_1 e^{\omega_1 t}$

• For $\rho \ll 0$: $\omega_1 = -\lambda_1$, smallest λ_i For small $\rho < \beta$: $\omega_1 = \frac{\pm\rho}{\Lambda + \sum_i \frac{\beta_i}{\lambda_i}}$

• For $\rho = \beta$: $\omega_1 = \sqrt{\frac{\sum_i \beta_i \lambda_i}{\Lambda}}$ For $\rho > \beta$: $\omega_1 = \frac{(\rho - \beta)}{\Lambda}$

ONE GROUP DIFFUSION THEORY

- **Bare Core**

- $k_{eff} = \frac{v\Sigma_f}{\Sigma_a} P_{NL}$ $P_{NL} = \frac{1}{1+L^2 B_g^2}$ $L \approx \frac{1}{\Sigma_a} = \lambda_a$ $B_g \cong \frac{\pi}{\tilde{r}}$

- **Reflected Core**

- $k_{eff} = \frac{v\Sigma_f}{\Sigma_a} P_{NL} (1 + P_{cr}P_{rc} + (P_{cr}P_{rc})^2 + (P_{cr}P_{rc})^3 + \dots) = \frac{v\Sigma_f}{\Sigma_a} P_{NL} \left(\frac{1}{1-P_{cr}P_{rc}} \right)$

- P_{cr} = Probability of neutron going from core to reflector

- P_{rc} = Probability of neutron going from reflector to core

- P_{NL} = Probability of neutron Leakage

POINT-REACTOR KINETICS MODEL (*DERIVATION*)

- *neutron loss rate* = $-\frac{n}{\tau}$ τ = **neutron lifetime**
- *prompt neutron production rate* = $\frac{(1-\beta)kn}{\tau}$ $\beta = \sum_i \beta_i$
- *delayed neutron production rate* = $\sum_i \lambda_i C_i$
- *production rate of i th precursor* = $\frac{\beta_i kn}{\tau}$
- *loss rate of i th precursor* = $-\lambda_i C_i$
- $\frac{dn}{dt} = \frac{(1-\beta)kn}{\tau} + \sum_i \lambda_i C_i + q - \frac{n}{\tau}$ q = **external source**
- $\frac{dC_i}{dt} = \frac{\beta_i kn}{\tau} - \lambda_i C_i$

POINT-REACTOR KINETICS MODEL

- *reactivity* $= \rho = \frac{k-1}{k}$ *neutron generation time* $= \Lambda = \tau/k$
- $\frac{dn}{dt} = \frac{(\rho-\beta)n}{\Lambda} + \sum_i \lambda_i C_i + q$
- $\frac{dC_i}{dt} = \frac{\beta_i n}{\Lambda} - \lambda_i C_i$
- **Applications (Reactor Operations)**
 - Startup
 - Steady-state operations
 - Power level changes
 - Transient operations (free runs, prompt bursts)
 - Shutdown

EQUILIBRIUM CONDITIONS

- $\frac{d}{dt}(n + \sum_i C_i) = \frac{\rho n}{\Lambda} + q = 0$
- **Subcritical equilibrium:** $\rho < 0$
 - $\rho = -\frac{\Lambda q}{n}$ $n = -\frac{\Lambda q}{\rho}$ $C_i = \frac{\beta_i n}{\Lambda \lambda_i}$
- **Delayed Critical:** $\rho = 0$
 - $q \ll \frac{n}{\Lambda}$ $\rho = -\frac{\Lambda q}{n} \rightarrow 0$ $C_i = \frac{\beta_i n}{\Lambda \lambda_i}$
- **Prompt Critical:** $\rho = \beta$
 - $\frac{dn}{dt} \approx \frac{(\rho - \beta)n}{\Lambda} = 0 \rightarrow \rho = \beta$ $\frac{dn}{dt} = \sum_i \lambda_i C_i$ $\frac{dC_i}{dt} = \frac{\beta_i n}{\Lambda} - \lambda_i C_i$

NEUTRON MULTIPLICATION

- **Subcritical equilibrium**

- $n = -\frac{\Lambda q}{\rho} \rightarrow \frac{n}{\tau} = \frac{q}{1-k} = Mq$

- $M = \frac{1}{1-k} = 1 + k + k^2 + k^3 + \dots$

- $\frac{n_2}{n_1} = \frac{M_2}{M_1} = \frac{1-k_1}{1-k_2}$

- **Approach to critical**

- $\frac{1}{M} = 1 - k$

- $\frac{1}{\left(\frac{M}{M_0}\right)} = \frac{CR_0}{CR} = \frac{1-k}{1-k_0}$