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EIC Physics from Lattice QCD: investigations beyond leading twist

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Contents

1 Description of research achievements from current award

The current DOE Early Career Award (ECA) has supported three graduate students (Jack Dodson, Joey Delmar, Josh Miller), a postdoctoral fellow (Aurora Scapellato), and two undergraduate students (Connor Bekaert, Sarah Lampreich). Presently, it supports Delmar and Miller, and starting in May of 2024 will support Marcos Morales; Delmar will graduate in the summer of 2024. This proposal seeks funding for Miller and Morales; the former will graduate by the end of the requested funding.

The research of the current award resulted, within 4 years, in **27 publications** in peer-reviewed high-impact journals (Nature Communications, PRL, PRD, EPJA), 3 invited reviews, 7 whitepapers/community papers, and 24 conference proceedings most of which are peer-reviewed. In addition, the funded participants have presented results from this project in 18 invited talks, 23 contributed talks, 15 colloquia and seminars, and 2 posters. Notably, the undergraduate student Sarah Lambreich gave a talk at the 2023 Annual Meeting of the APS Mid-Atlantic Section. Her summer project on pion GPDs was accepted for the REU poster session at the 2023 APS DNP & JPS meeting.

The ECA is centered around understanding the internal structure of proton, pion, and kaon. We employ the twisted mass lattice QCD formulation, a variant of Wilson fermions with the major advantage of an automatic $\mathcal{O}(a)$ improvement in physical observables. It is achieved by tuning the untwisted bare quark mass to its critical value, with no further operator improvement; the formulation has proven very successful for hadron structure studies. Our research addresses open questions via calculations of PDFs, GPDs, and TMDs using two main directions:

A. *Mellin moments:* Distribution functions are light-cone correlation functions, and, as such, it is not straightforward to calculate them directly on a Euclidean lattice. Instead, one may calculate their Mellin moments, expressed as a tower of hadron matrix elements of local operators. Partial information is obtained through such Mellin moments, and, in principle, one can reconstruct the distributions using an operator product expansion (OPE). Practically, an exact reconstruction is a difficult task due to the high computational cost required to obtain reliably high moments: the signal-to-noise rapidly decreases, and a power-law mixing occurs beyond the third non-trivial moment. Nevertheless, Mellin moments are extracted from phenomenological analyses of experimental data and can be directly compared to lattice results.

B. *Large Momentum Effective Theory and Short-Distance Factorization:* Alternative methods have been proposed over the years to access distribution functions, such as hadronic tensor [1], fictitious heavy quark [2], higher moments [3], the quasi-PDFs [4], pseudo-PDFs [5, 6], good lattice cross sections [7, 8], and Compton amplitude [9]. For further details, see the PI's review article [10]. The most widely used methods are the quasi- and pseudo-distributions, for which it was demonstrated that matrix elements of momentum-boosted hadrons coupled with non-local operators can be related to light-cone distributions because both share the same infrared physics; their differences lie in the ultraviolet regime. This allows the matching of lattice data and light-cone distributions using perturbation theory.

Here, we summarize our ECA's highlights for methodologies **A** and **B**. Due to space limitation, we do not discuss our work on TMDs [11, 12], as it is not our focus for the proposed research (Sec. **??**).

1. Nucleon charges, form factors and generalized form factors: The charges are key quantities for understanding the nucleon structure at the most basic level. For example, the nucleon axial charge is a fundamental quantity within the Standard Model (SM). It determines the rate of the weak decay of neutrons into protons and provides a quantitative measure of spontaneous chiral symmetry breaking in hadronic physics. It enters into the analysis of neutrinoless double-beta decay and the unitarity tests of the Cabibbo-Kobayashi-Maskawa matrix. Experimentally, it is known precisely from neutron beta decay measurements using polarized ultracold neutrons. Thus, it is essential for lattice QCD calculations to reproduce its experimental value or, if a deviation is observed, to understand its origin. Furthermore, the scalar and tensor charges may put limits on the existence of beyond SM interactions with scalar and tensor structures; these charges are not well known experimentally. A determination within lattice QCD can provide essential input for precision measurements, probing the existence of novel scalar and tensor interactions aiding experimental searches.

Calculations of the proton charges have been among the research investigations of the current project. There has been remarkable progress in recent years, mainly due to algorithmic improvements, theoretical developments, and computational resources. The synergy of the above enabled simulations to be carried out with quark masses tuned to their physical values (*physical point*). Being able to perform simulations at the physical point allows us to address several challenging questions and resolve discrepancies between experimental data and theoretical results. For instance, in Refs. [13–16] we showed that the long-standing disagreement between lattice data and measurements of the axial charge is resolved with simulations at the physical point and when excited states are eliminated. These publications are now well-known in the particle and nuclear physics communities and are among the most quoted internationally.

The calculation of proton charges was extended by including momentum transfer between the initial and final state. This allows the extraction of form factors (FFs) and generalized form factors (GFFs), which are of great interest. For example, the electromagnetic (EM) FFs probe the internal structure of hadrons mapping their charge and magnetic distributions, and their momentum-transfer dependence defines their radii. The axial FFs are relevant to experiments searching neutrino oscillations, and their momentum-transfer dependence gives the axial mass. The latter is needed for the antineutrino charged-current quasi-elastic double-differential cross-section. Notably, extracting the axial mass from experimental data (e.g, pion electroproduction, charged current muon-neutrino scattering, neutrino-nucleus cross-sections using deuterium target) leads to tensions in the axial mass. Therefore, lattice QCD results are valuable, as they can provide first-principle estimates.

In the reporting period, several advances were made that led to the analysis of the complete up, down, and strange quark contributions to the nucleon form factors. We have been pursuing calculations directly at the physical point for the EM [17] and axial [15, 16, 18] FFs. We have also obtained the unpolarized and helicity GFFs [19] that have implications on the total proton spin. Our results on the EM FFs and vector GFFs, have been combined with the transversity FFs and GFFs to extract the first two moments of the nucleon transverse quark spin densities [17].

2. Proton spin decomposition: Experimentally, there are still open questions on the proton spin decomposition. In 2017, we completed a pioneering calculation of all quark and gluon contributions to the proton spin, and we obtained, for the first time, the individual components for the up, down, strange, and charm quarks [20]. These were notoriously difficult to extract and have been neglected in many prior studies. Since then, we have devoted significant efforts to investigating and applying techniques developed to run on GPUs to calculate such contributions at the physical point. In Ref. [21], we have improved the calculation with state-of-the-art ensembles at the physical point and addressed challenges related to the renormalization of the gluon operator, which includes unavoidable mixing with the singlet quark momentum fraction. We computed its non-perturbative renormalization and used the mixing coefficients from perturbation theory [22]. The latter is highly complex, with millions of algebraic expressions requiring intensive human effort and computational resources. Our results for the proton spin [21] reproduce the $\frac{1}{2}$ value, as can be seen in Fig. 1. Understanding the spin decomposition from first principles sheds light on the long-standing proton spin puzzle.

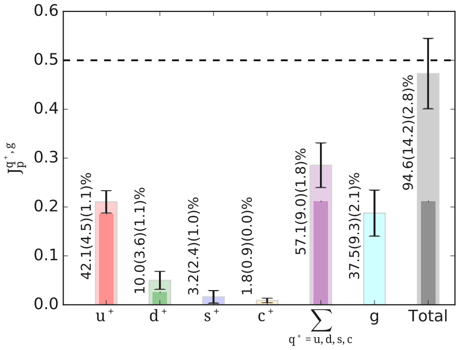

Figure 1: The decomposition of the proton spin from Ref. [21]. We show the contribution of the up (red bar), down (green bar), strange (blue bar), charm (orange bar), quarks and their sum (purple bar), the gluon (cyan bar) and the total sum (grey bar). Whenever two overlapping bars appear, the inner bar denotes the purely connected contribution while the outer one is the total contribution, which includes disconnected, taking into account the mixing. Results are given in $\overline{\text{MS}}$ scheme at 2 GeV.

3. Pion and kaon structure through higher Mellin moments: QCD is characterized by several emergent phenomena not apparent in the QCD Lagrangian, such as the dynamical chiral symmetry breaking (DCSB). In QCD, DCSB leads to an octet of Nambu-Goldstone bosons, which includes the triplet of pions and the four kaons. Kaons are particularly interesting because of the valence strange quark, which has a much larger mass than the up and down quarks that form the pions. Thus, a comparison between pion and kaon observables provides a unique window into the interplay between QCD dynamics and quark mass effects. Significant SU(3) flavor-breaking effects have already been observed in the pion and kaon, and studying their structure is essential for qualitatively understanding these features. While the proton has been extensively studied in lattice QCD, there are limited pion and kaon structure calculations. Given the small number of experimental data and the increasing interest due to the EIC, obtaining results from first principles is crucial.

Within the ECA, we led a research program on the pion and kaon Mellin moments using operators containing up to three covariant derivatives. We studied the moments of the PDF, $\langle x^n \rangle$ with $n \leq 3$, as well as FFs and GFFs. Some of the quantities have never been explored before, particularly for the kaon. One of the most notable aspects is reconstructing the x dependence of their PDFs using the moments calculated in our work [23, 24]. We find the reconstruction feasible and draw qualitative conclusions on the large-x behavior. Taking integrals of the obtained PDF allows us to extract even higher moments with good signal and without mixing under renormalization.

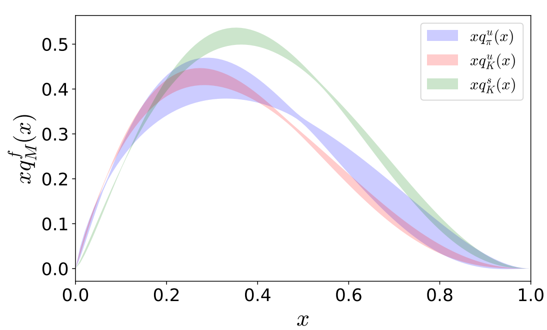

Figure 2: Our results [24] on $xq^u_\pi(x)$, $xq^u_K(x)$ and $xq^s_K(x)$ at 27 GeV^2. The reconstruction uses our lattice data up to $\langle x^3 \rangle$.

Another important aspect is the calculation of the scalar, vector, and tensor FFs that led to a third publication shortly after [25]. We obtained the FFs in both the rest frame and a boosted frame, giving access to $-t = 2.5$ GeV^2 and $-t = 3$ GeV^2 for the pion and kaon, respectively. We applied different parametrizations to describe t-dependence of the FFs to extract the scalar, vector, and tensor radii, as well as the tensor anomalous magnetic moment, k_T. We studied SU(3) flavor symmetry breaking that revealed up to about 20% effect. By combining the data for the vector and tensor FFs, we also obtain the lowest moment of the densities of transversely polarized quarks in the impact parameter space. Finally, we give an estimate for the average transverse shift.

4. x-dependence of twist-2 quark and gluon PDFs: The PI has been involved in several calculations that employ the quasi-PDFs method to obtain their x-dependence, with the first one being in 2016 [26]. In 2017, the PI spearheaded a methodology to renormalize perturbatively and non-perturbatively non-local operators [27, 28], which was until then unknown. Since then, we performed the first calculations at the physical point, including a proper renormalization and matching for the twist-2 unpolarized and helicity PDFs [29], as well as the transversity [30]. In follow-up calculations, we explored several sources of systematic uncertainties, such as excited-states contamination, renormalization, PDF reconstruction, and matching [31]. The PI was invited to prepare a comprehensive review article on novel methods to access x-dependent distributions [10]. Within the ECA, we completed a calculation on the continuum limit for the PDFs [32], as well as an exploratory study for the Δ^+ PDF [33]. Recently, our expertise in calculating disconnected-diagram contributions, matrix elements of non-local operators, and renormalization was combined to perform the first flavor decomposition for the helicity PDF published in Physical Review Letters [34], which was extended to the unpolarized and tansversity cases [35]. These are the only results available in the literature for the light-quark singlet PDFs. In parallel, we performed calculations on PDFs using the pseudo-PDFs approach. The first one was directly at the physical point for the unpolarized valence PDF and its combination with antiquarks [36] (see left panel of Fig. 3). More recently, we performed a continuum limit using ensembles of 350 MeV pion mass [37]. Another direction is an improvement scheme for the renormalization, which we developed recently to eliminate lattice artifacts from the renormalization functions [38]. The PI was invited to prepare another review article [39] (EPJA).

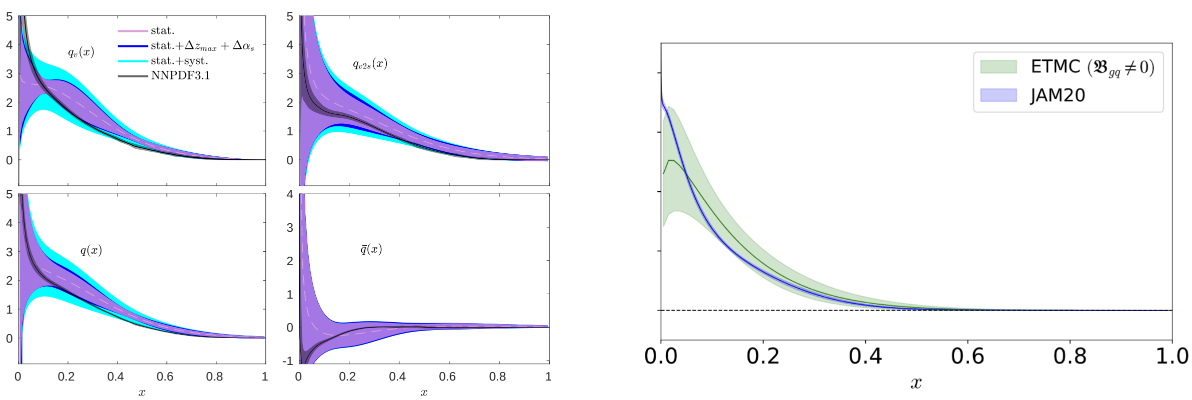

Figure 3: Left: Unpolarized PDFs extracted from our work [36] compared to global fits of NNPDF [40]. Shown distributions: valence (q_v), valence + 2 sea (q_{v2s}), full (q) and sea ($\bar{q}$). Right: Comparison of our final results on the gluon PDF [41] upon elimination of the mixing and the global analysis of JAM20 [42].

We also calculated the gluon PDF using the pseudo-PDFs method [41]. We explored several sources of systematic effects, including the effect of stout smearing, excited states, and the dependence on the maximum value of z entering the fits. Also, for the first time, the mixing between the gluon PDF and the quark singlet PDFs is eliminated, and the final results are shown in the right panel of Fig. 3.

5. x-dependence of twist-2 GPDs: GPDs are generalizations of PDFs to off-forward kinematic, providing information not only on the momentum distribution of partons but also their spatial distribution. In fact, GPDs, together with TMDs, allow for a 3-dimensional mapping of the internal structure of hadrons, which is an important component of the physics investigations of the EIC. GPDs can be accessed in Deeply Virtual Compton Scattering (DVCS) and Deeply Virtual Meson Production (DVMP). However, since the cross-sections are not directly sensitive to GPDs, little information is available on the entire x-dependence of GPDs. Moreover, their extraction is complicated by the exclusive nature of the scattering, the limited kinematic coverage, and their multi-dimensionality. Another process that gives access to GPDs is SDHEP [43], which is still being explored. Given the challenges of extracting GPDs from experiments, lattice QCD is a powerful tool to calculate them and to provide constraints for phenomenological analyses.

Our first calculation of GPDs is published in Physical Review Letters [44]. This includes results on the isovector unpolarized and helicity twist-2 GPDs for zero and nonzero skewness, ξ. The project was extended to the transversity GPD, and the results are the only ones available in the literature [45]. The four tensor GPDs complicate the analysis, but we have developed a computational setup to access them all. The work performs a number of consistency checks that were successful, such as the extraction of the lowest two moments of GPDs and the hierarchy of the moments. These results on GPDs are the first of their kind, and the gained expertise led to the development of a new formulation of extracting GPDs from lattice QCD (see point 7 below).

6. x-dependence of twist-3 PDFs and GPDs: Most of the knowledge is on leading-power PDFs and GPDs. Twist-3 PDFs and GPDs are also essential and sizable. They give information on power corrections in hard exclusive processes, some related to spin physics. Experimentally, it is very challenging to probe twist-3 contributions and isolate them from the leading twist-2 contribution. An exception to this is the twist-3 PDF, $g_T(x)$, which has nevertheless been limitedly studied.

The current ECA supported the first-ever lattice QCD calculation of twist-3 $g_T(x)$ PDF, and the publication was selected as PRD Editors' highlight [46]. The quasi-PDF method has been employed that requires synergy with theorists to match the lattice data and physical quantities. We developed a proposal for the matching for the twist-3 $g_T(x)$, $h_L(x)$, $e(x)$ [47,48]. Our group has also pursued the first calculation for $h_L(x)$ [49]. One of the interesting findings is the implementation of the Wandzura-Wilczek (WW) approximation to assess the effect of genuine twist-3 contributions (see Fig. 4). For $h_L(x)$, we also performed a flavor decomposition for the light quarks because our work of Ref. [35] demonstrated that the disconnected contributions are negligible for the tensor case. By comparing the WW approximation for each quark flavor we assessed the role of the up and down quark in h_L, as shown in Fig. 4. Our lattice results on the twist-3 PDFs are currently the only ones available in the literature and have been very well received by the theoretical and experimental communities.

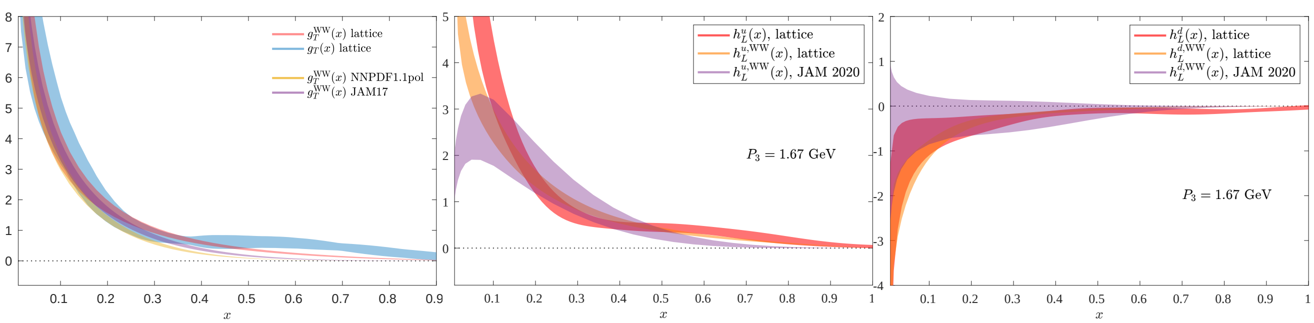

Figure 4: Left: Comparison of our $g_T(x)$ [46] with its WW approximations: lattice-extracted $g_T^{\rm WW}$ and calculated from global fits of NNPDF1.1pol [50] and JAM17 [51]. Center (Right): test of the WW approximation $h_L^{WW,u}$ ($h_L^{WW,d}$) together with h_L^u (h_L^d), at boost $P_3 = 1.67$ GeV. For the separate flavors we show our $h_L(x)$ with $h_L^{\rm WW}(x)$ [49]. We also compare with $h_L^{\rm WW}(x)$ from the JAM collaboration [52].

This novel direction is attracting attention and opens new avenues to understanding hadron structure. Our work has already sparked further progress by theorists to understand the matching between lattice data and light-cone PDFs, such as inclusion of the mixing with quark-gluon-quark correlators [53,54]. Another pioneering program we pursued is twist-3 GPDs, an essential component of the ECA. Our first complete calculation on the axial GPDs for zero skewness was published recently [55]. Besides presenting our findings, we performed several consistency checks, including assessing the local limit of the twist-3 GPDs and examining the Burkhardt-Cottingham-type and Efremov-Teryaev-Leader-type sum rules. Furthermore, for the first time, the twist-2 $\widetilde{E}$-GPD made an appearance at zero skewness, a tremendous finding since it is inaccessible at zero skewness from the twist-2 calculation. Our results for the twist-3 GPD containing the $\widetilde{H}$ and $\widetilde{E}$ GPDs are shown in the left and center panels of Fig. 5. Obtaining twist-3 GPDs is a testament to the remarkable progress in lattice QCD.

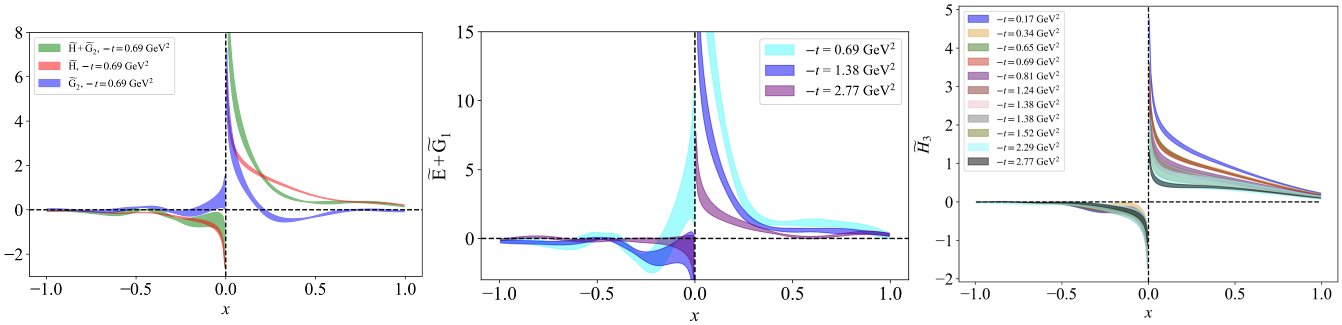

Figure 5: Left: Comparison of $\widetilde{H}$, $\widetilde{H}+\widetilde{G}_2$, and $\widetilde{G}_2$ at $-t = 0.69$ GeV^2. Center: $\widetilde{E}+\widetilde{G}_1$ and momentum boost $P_3 = 1.25$ GeV for various values of $-t$. Right: The t dependence of the light-cone GPD $\widetilde{H}$ at $|P_3| = 1.25$ GeV.

7. Novel methods to access GPDs: Historically, GPDs have been defined in the symmetric kinematic frame, which is, however, computationally very expensive to use in lattice QCD. The main complication is that only one value of the momentum transfer, t, can be accessed in each simulation, as it appears in both the initial and final states. Hence, the status of GPD calculations has remained at the exploratory stage, with a very limited number of values of t and skewness. We recently proposed a new parametrization of off-forward matrix elements relevant to GPDs in terms of Lorentz-invariant amplitudes. The parameterization was developed for the unpolarized [56] and the helicity [57] GPDs. Due to Lorentz invariance, one can extract the amplitudes from any kinematic frame. In fact, asymmetric frames where all the momentum transfer is assigned to the final or initial states are favorable due to a large computational cost reduction. By calculating the amplitudes in two frames, we confirmed their Lorentz invariance and then extracted the GPDs via their unique relation. The use of the asymmetric frame allowed us to obtain the GPDs at several momentum transfer values (see, e.g., right panel of Fig. **??**). A byproduct of the calculation of Ref. [56] is the extraction of the Mellin moments of GPDs, for the first time, up to the fourth order [58]. The impact parameter space interpretation of the GPD moments was discussed, which provides insights into the spatial distribution of unpolarized quarks and their correlations in the transverse plane of an unpolarized or transversely polarized proton. These two publications resulted in a DOE highlight [59].

8. Synergy of lattice QCD and experiments: As lattice QCD advances and provides reliable results with controlled uncertainties, there is an increased interest in utilizing lattice results to advance our knowledge of PDFs and GPDs. An example of a synergistic activity within the ECA is the improved constraint of the neutron's charge radius, $\langle r_{\rm n}^2 \rangle$. We used our lattice data from Ref. [60] on the electric and magnetic neutron FFs to extract the ratio $G_{\rm E}^{\rm n}/G_{\rm M}^{\rm n}$. These lattice data provide further guidance on the Q^2-dependence of the ratio in a region where neutron form factor data are unavailable (see Fig. 6). Aspects of this work are published in EPJA [61] and Nature Communications [62].

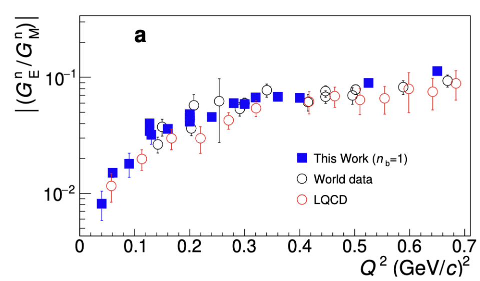

Figure 6: Our lattice results and world data combined analysis for the elastic neutron FFs ratio [62].

The forthcoming section outlines our comprehensive and multi-faceted research program, with several challenges and novel aspects. Our track record of research achievements outlined above underscores our commitment to high productivity and the pursuit of pioneering projects.

2 Bibliography & References Cited

References

[1] K.-F. Liu and S.-J. Dong, "Origin of difference between anti-d and anti-u partons in the nucleon," Phys. Rev. Lett., vol. 72, pp. 1790–1793, 1994.

[2] W. Detmold and C. J. D. Lin, "Deep-inelastic scattering and the operator product expansion in lattice QCD," Phys. Rev., vol. D73, p. 014501, 2006.

[3] Z. Davoudi and M. J. Savage, "Restoration of Rotational Symmetry in the Continuum Limit of Lattice Field Theories," Phys. Rev., vol. D86, p. 054505, 2012.

[4] X. Ji, "Parton Physics on a Euclidean Lattice," Phys.Rev.Lett., vol. 110, p. 262002, 2013.

[5] A. V. Radyushkin, "Quasi-parton distribution functions, momentum distributions, and pseudo-parton distribution functions," Phys. Rev., vol. D96, no. 3, p. 034025, 2017.

[6] A. V. Radyushkin, "Quark pseudodistributions at short distances," Phys. Lett., vol. B781, pp. 433–442, 2018.

[7] Y.-Q. Ma and J.-W. Qiu, "QCD Factorization and PDFs from Lattice QCD Calculation," Int. J. Mod. Phys. Conf. Ser., vol. 37, p. 1560041, 2015.

[8] Y.-Q. Ma and J.-W. Qiu, "Exploring Partonic Structure of Hadrons Using ab initio Lattice QCD Calculations," Phys. Rev. Lett., vol. 120, no. 2, p. 022003, 2018.

[9] A. J. Chambers, R. Horsley, Y. Nakamura, H. Perlt, P. E. L. Rakow, G. Schierholz, A. Schiller, K. Somfleth, R. D. Young, and J. M. Zanotti, "Nucleon Structure Functions from Operator Product Expansion on the Lattice," Phys. Rev. Lett., vol. 118, no. 24, p. 242001, 2017.

[10] K. Cichy and M. Constantinou, "A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results," Adv. High Energy Phys., vol. 2019, p. 3036904, 2019.

[11] Y. Li et al., "Lattice QCD Study of Transverse-Momentum Dependent Soft Function," Phys. Rev. Lett., vol. 128, no. 6, p. 062002, 2022.

[12] C. Alexandrou et al., "Non-perturbative renormalization of asymmetric staple-shaped operators in twisted mass lattice QCD," 5 2023.

[13] C. Alexandrou et al., "Nucleon scalar and tensor charges using lattice QCD simulations at the physical value of the pion mass," Phys. Rev. D, vol. 95, no. 11, p. 114514, 2017. [Erratum: Phys.Rev.D 96, 099906 (2017)].

[14] C. Alexandrou, S. Bacchio, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K. Jansen, G. Koutsou, and A. Vaquero Aviles-Casco, "Nucleon axial, tensor, and scalar charges and σ-terms in lattice QCD," Phys. Rev. D, vol. 102, no. 5, p. 054517, 2020.

[15] C. Alexandrou et al., "Nucleon axial and pseudoscalar form factors from lattice QCD at the physical point," Phys. Rev. D, vol. 103, no. 3, p. 034509, 2021.

[16] C. Alexandrou, S. Bacchio, M. Constantinou, J. Finkenrath, R. Frezzotti, B. Kostrzewa, G. Koutsou, G. Spanoudes, and C. Urbach, "Nucleon axial and pseudoscalar form factors using twisted-mass fermion ensembles at the physical point," 9 2023.

[17] C. Alexandrou et al., "Moments of the nucleon transverse quark spin densities using lattice QCD," Phys. Rev. D, vol. 107, no. 5, p. 054504, 2023.

[18] C. Alexandrou, S. Bacchio, M. Constantinou, K. Hadjiyiannakou, K. Jansen, and G. Koutsou, "Quark flavor decomposition of the nucleon axial form factors," 6 2021.

[19] C. Alexandrou et al., "Moments of nucleon generalized parton distributions from lattice QCD simulations at physical pion mass," Phys. Rev. D, vol. 101, no. 3, p. 034519, 2020.

[20] C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, C. Kallidonis, G. Koutsou, A. Vaquero Avilés-Casco, and C. Wiese, "Nucleon Spin and Momentum Decomposition Using Lattice QCD Simulations," Phys. Rev. Lett., vol. 119, no. 14, p. 142002, 2017.

[21] C. Alexandrou, S. Bacchio, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K. Jansen, G. Koutsou, H. Panagopoulos, and G. Spanoudes, "Complete flavor decomposition of the spin and momentum fraction of the proton using lattice QCD simulations at physical pion mass," Phys. Rev. D, vol. 101, no. 9, p. 094513, 2020.

[22] C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, and C. Wiese, "Gluon momentum fraction of the nucleon from lattice QCD," Phys. Rev. D, vol. 96, no. 5, p. 054503, 2017.

[23] C. Alexandrou, S. Bacchio, I. Cloet, M. Constantinou, K. Hadjiyiannakou, G. Koutsou, and C. Lauer, "Mellin moments $\langle x \rangle$ and $\langle x^2 \rangle$ for the pion and kaon from lattice QCD," Phys. Rev. D, vol. 103, no. 1, p. 014508, 2021.

[24] C. Alexandrou, S. Bacchio, I. Cloet, M. Constantinou, K. Hadjiyiannakou, G. Koutsou, and C. Lauer, "The pion and kaon $\langle x^3 \rangle$ from lattice QCD and PDF reconstruction from Mellin moments," 4 2021.

[25] C. Alexandrou, S. Bacchio, I. Cloet, M. Constantinou, J. Delmar, K. Hadjiyiannakou, G. Koutsou, C. Lauer, and A. Vaquero, "Scalar, vector, and tensor form factors for the pion and kaon from lattice QCD," Phys. Rev. D, vol. 105, no. 5, p. 054502, 2022.

[26] C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Steffens, and C. Wiese, "Updated Lattice Results for Parton Distributions," Phys. Rev. D, vol. 96, no. 1, p. 014513, 2017.

[27] M. Constantinou and H. Panagopoulos, "Perturbative renormalization of quasi-parton distribution functions," Phys. Rev. D, vol. 96, no. 5, p. 054506, 2017.

[28] C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, and F. Steffens, "A complete non-perturbative renormalization prescription for quasi-PDFs," Nucl. Phys. B, vol. 923, pp. 394–415, 2017.

[29] C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, and F. Steffens, "Light-Cone Parton Distribution Functions from Lattice QCD," Phys. Rev. Lett., vol. 121, no. 11, p. 112001, 2018.

[30] C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, and F. Steffens, "Transversity parton distribution functions from lattice QCD," Phys. Rev. D, vol. 98, no. 9, p. 091503, 2018.

[31] C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, and F. Steffens, "Systematic uncertainties in parton distribution functions from lattice QCD simulations at the physical point," Phys. Rev. D, vol. 99, no. 11, p. 114504, 2019.

[32] C. Alexandrou, K. Cichy, M. Constantinou, J. R. Green, K. Hadjiyiannakou, K. Jansen, F. Manigrasso, A. Scapellato, and F. Steffens, "Lattice continuum-limit study of nucleon quasi-PDFs," 11 2020.

[33] Y. Chai et al., "Parton distribution functions of Δ^+ on the lattice," Phys. Rev. D, vol. 102, no. 1, p. 014508, 2020.

[34] C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, and F. Manigrasso, "Flavor decomposition for the proton helicity parton distribution functions," Phys. Rev. Lett., vol. 126, no. 10, p. 102003, 2021.

[35] C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, and F. Manigrasso, "Flavor decomposition of the nucleon unpolarized, helicity and transversity parton distribution functions from lattice QCD simulations," 6 2021.

[36] M. Bhat, K. Cichy, M. Constantinou, and A. Scapellato, "Flavor nonsinglet parton distribution functions from lattice QCD at physical quark masses via the pseudodistribution approach," Phys. Rev. D, vol. 103, no. 3, p. 034510, 2021.

[37] M. Bhat, W. Chomicki, K. Cichy, M. Constantinou, J. R. Green, and A. Scapellato, "Continuum limit of parton distribution functions from the pseudodistribution approach on the lattice," Phys. Rev. D, vol. 106, no. 5, p. 054504, 2022.

[38] M. Constantinou and H. Panagopoulos, "Improved renormalization scheme for nonlocal operators," Phys. Rev. D, vol. 107, no. 1, p. 014503, 2023.

[39] M. Constantinou, "The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD," Eur. Phys. J. A, vol. 57, no. 2, p. 77, 2021.

[40] R. D. Ball et al., "Parton distributions from high-precision collider data," Eur. Phys. J., vol. C77, no. 10, p. 663, 2017.

[41] J. Delmar, C. Alexandrou, K. Cichy, M. Constantinou, and K. Hadjiyiannakou, "Gluon PDF of the proton using twisted mass fermions," 10 2023.

[42] E. Moffat, W. Melnitchouk, T. C. Rogers, and N. Sato, "Simultaneous Monte Carlo analysis of parton densities and fragmentation functions," Phys. Rev. D, vol. 104, no. 1, p. 016015, 2021.

[43] J.-W. Qiu and Z. Yu, "Exclusive production of a pair of high transverse momentum photons in pion-nucleon collisions for extracting generalized parton distributions," JHEP, vol. 08, p. 103, 2022.

[44] C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, and F. Steffens, "Unpolarized and helicity generalized parton distributions of the proton within lattice QCD," Phys. Rev. Lett., vol. 125, no. 26, p. 262001, 2020.

[45] C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, and F. Steffens, "Transversity GPDs of the proton from lattice QCD," Phys. Rev. D, vol. 105, no. 3, p. 034501, 2022.

[46] S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, and F. Steffens, "Insights on proton structure from lattice QCD: The twist-3 parton distribution function $g_T(x)$," Phys. Rev. D, vol. 102, no. 11, p. 111501, 2020.

[47] S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, and F. Steffens, "One-loop matching for the twist-3 parton distribution $g_T(x)$," Phys. Rev. D, vol. 102, no. 3, p. 034005, 2020.

[48] S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, and F. Steffens, "The role of zero-mode contributions in the matching for the twist-3 PDFs $e(x)$ and $h_L(x)$," Phys. Rev. D, vol. 102, p. 114025, 2020.

[49] S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, and F. Steffens, "Parton distribution functions beyond leading twist from lattice QCD: The $h_L(x)$ case," 7 2021.

[50] E. R. Nocera, R. D. Ball, S. Forte, G. Ridolfi, and J. Rojo, "A first unbiased global determination of polarized PDFs and their uncertainties," Nucl. Phys., vol. B887, pp. 276–308, 2014.

[51] J. J. Ethier, N. Sato, and W. Melnitchouk, "First simultaneous extraction of spin-dependent parton distributions and fragmentation functions from a global QCD analysis," Phys. Rev. Lett., vol. 119, no. 13, p. 132001, 2017.

[52] J. Cammarota, L. Gamberg, Z.-B. Kang, J. A. Miller, D. Pitonyak, A. Prokudin, T. C. Rogers, and N. Sato, "Origin of single transverse-spin asymmetries in high-energy collisions," Phys. Rev. D, vol. 102, no. 5, p. 054002, 2020.

[53] V. M. Braun, Y. Ji, and A. Vladimirov, "QCD factorization for twist-three axial-vector parton quasidistributions," JHEP, vol. 05, p. 086, 2021.

[54] V. M. Braun, Y. Ji, and A. Vladimirov, "QCD factorization for chiral-odd parton quasi- and pseudo-distributions," JHEP, vol. 10, p. 087, 2021.

[55] S. Bhattacharya, K. Cichy, M. Constantinou, J. Dodson, A. Metz, A. Scapellato, and F. Steffens, "Chiral-even axial twist-3 GPDs of the proton from lattice QCD," Phys. Rev. D, vol. 108, no. 5, p. 054501, 2023.

[56] S. Bhattacharya, K. Cichy, M. Constantinou, J. Dodson, X. Gao, A. Metz, S. Mukherjee, A. Scapellato, F. Steffens, and Y. Zhao, "Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks," Phys. Rev. D, vol. 106, no. 11, p. 114512, 2022.

[57] S. Bhattacharya et al., "Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Axial-vector case," 10 2023.

[58] S. Bhattacharya, K. Cichy, M. Constantinou, X. Gao, A. Metz, J. Miller, S. Mukherjee, P. Petreczky, F. Steffens, and Y. Zhao, "Moments of proton GPDs from the OPE of nonlocal quark bilinears up to NNLO," Phys. Rev. D, vol. 108, no. 1, p. 014507, 2023.

[59] "https://science.osti.gov/np/highlights/2023/np-2023-09-b."

[60] C. Alexandrou, S. Bacchio, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K. Jansen, G. Koutsou, and A. Vaquero Aviles-Casco, "Proton and neutron electromagnetic form factors from lattice QCD," Phys. Rev. D, vol. 100, no. 1, p. 014509, 2019.

[61] H. Atac, M. Constantinou, Z. E. Meziani, M. Paolone, and N. Sparveris, "Charge radii of the nucleon from its flavor dependent Dirac form factors," Eur. Phys. J. A, vol. 57, no. 2, p. 65, 2021.

[62] H. Atac, M. Constantinou, Z. E. Meziani, M. Paolone, and N. Sparveris, "Measurement of the neutron charge radius and the role of its constituents," Nature Commun., vol. 12, no. 1, p. 1759, 2021.