

# Development of a multiphase PIC model for slurry flow modeling



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- MFIX-Exa is a massively-parallel, high-performance multiphase flow code
- Targeted physics: reacting gas-solid flows from dilute particle-laden to dense granular
- CFD+ high-fidelity DEM or low-fidelity PIC



<https://mfix.netl.doe.gov/products/mfix-exa/>

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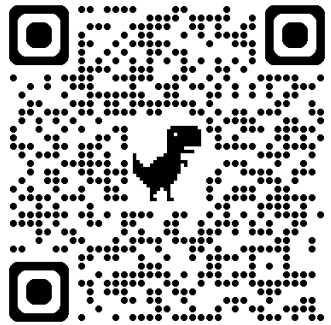


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- Scaled up to 60k GPUs on OLCF's Frontier (65,535 weak scaling, 62078 challenge problem)

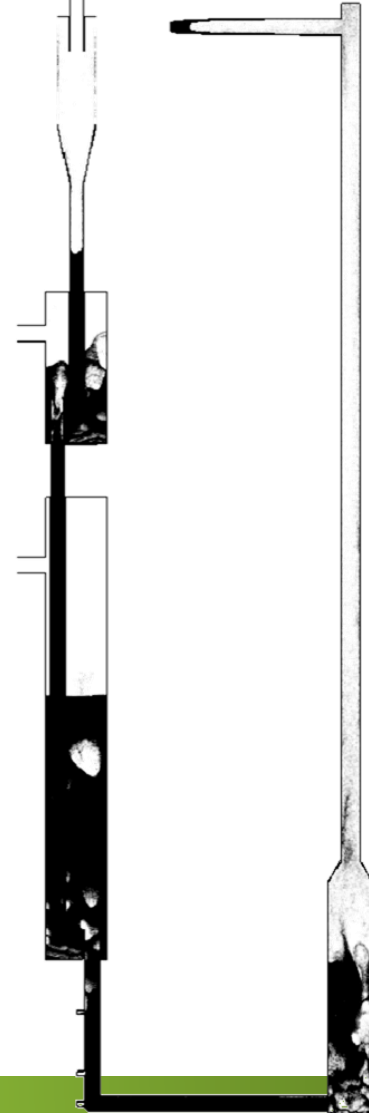


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- Challenge problem: NETL's 50kW CLR



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# problem of interest

[https://www.engineeringtoolbox.com/slurry-transport-velocity-d\\_236.html](https://www.engineeringtoolbox.com/slurry-transport-velocity-d_236.html)



- Liquid fluid and solid particle slurry flow (coal liquefaction driving interest)
- Plethora of data for horizontal pipe flow which is very industrially relevant
- Start with pressure drop (head loss) vs velocity relationship
- Reference: Gillies & Shook (2000) "Modelling High Concentration Settling Slurry Flows" *Canadian J. of Chem. Eng.*, **78**, 709-716.

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Fluid: water

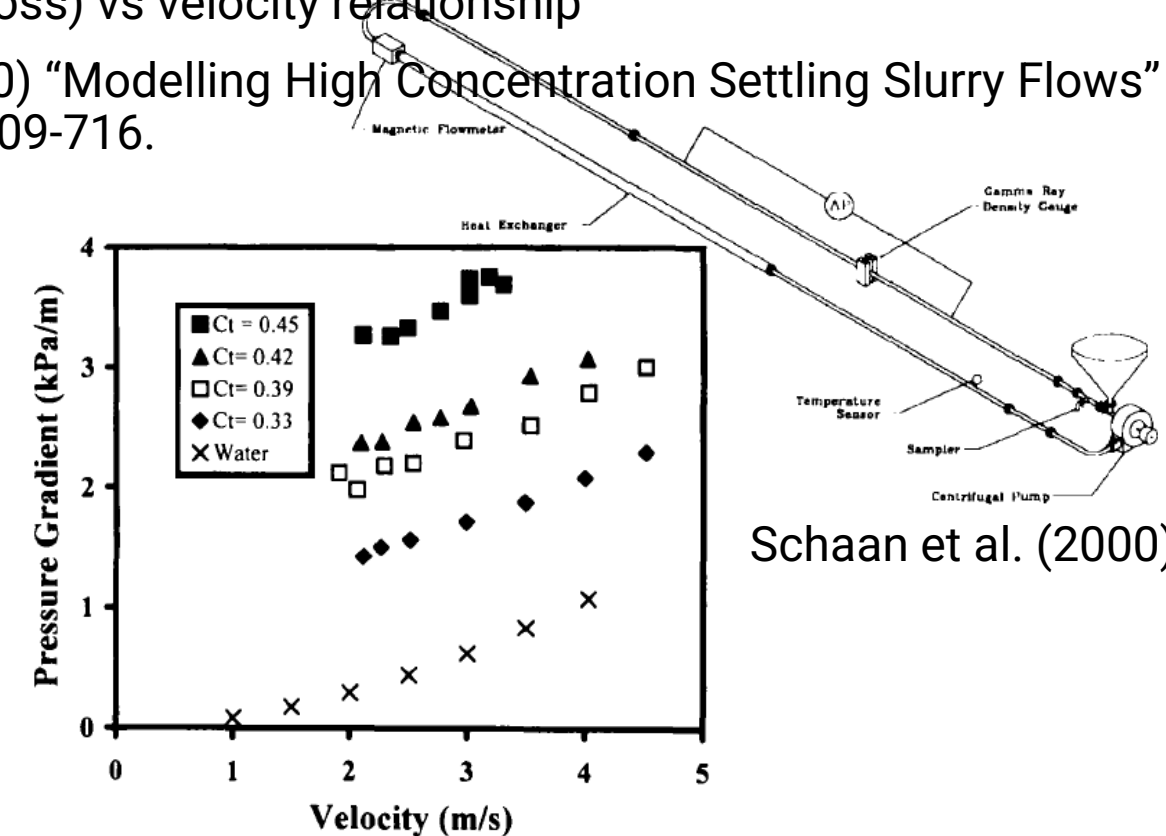
- $T = 25^{\circ}\text{C}$
- assume  $\rho_f = 1000 \text{ kg/m}^3$
- assume  $\mu_f = 0.001 \text{ Pa-s}$

Particles: sand

- $d_p = 420 \text{ micron}$
- $\rho_p = 2655 \text{ kg/m}^3$
- assume monodispersed

Pipe diameter

- $D = 10.5, 26.4 \text{ and } 49.6 \text{ cm}$



Gillies & Shook (2000)

Schaan et al. (2000)



**Table 1**  
Previous numerical studies on slurry pipe flows.

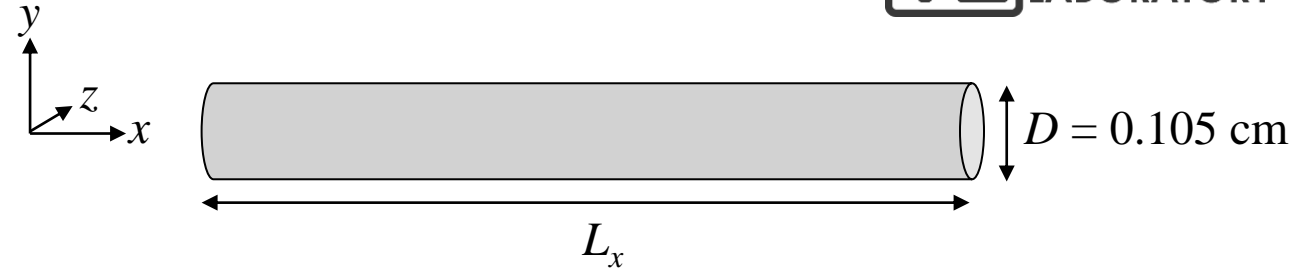
Reference	Approach	Flow regime	Fluid	Particles	$\rho_p$ [kg/m <sup>3</sup> ]	$d_p$ [mm]	$D$ [mm]	$V_m$ [m/s]	$\langle\alpha_s\rangle_A$ [–]
Capecelatro and Desjardins [17]	LES + DPM	SB, HS	Water	Sand	2650	0.165	51.5	0.83, 1.6	0.084
Arolla and Desjardins [18] <sup>a</sup>	LES + DPM	SB, HS	Water	Sand	2650	0.280	69	–	–
Uzi and Levy [12]	RANS+DPM	MB, HS	Brine	NaCl	2150	1.0–4.0	50–100	0.6–4.0	0.05–0.30
Hernández et al. [21]	(KTGF)-TFM	HS, PS	Water	Sand	$\approx 2381$	0.030–0.110	22.1	1.0–3.0	0.0025–0.20
Bossio et al. [22]	(KTGF)-TFM	MB, HS, PS	Laterite	Sand	2381	0.110	26.84	0.671–4.697	0.036–0.083
Ekambara et al. [23]	KTGF-TFM	BF, FS	Water	Sand, glass beads	2470, 2650	0.090–0.500	50–500	1.5–5.5	0.08–0.45
Antaya et al. [24]	KTGF-TFM(MFM)	FS	Water	Sand	2650	0.100–0.370	51.5, 150	2.0–6.0	0.20–0.43
Hashemi et al. [25]	KTGF-TFM	–	Water	Sand	2650	0.370	265	4.0, 6.0	0.20–0.40
Kaushal et al. [26]	KTGF-TFM, MM	FS	Water	Glass beads	2470	0.125	54.9	1.0–5.0	0.30–0.50
Gopaliya and Kaushal [27]	KTGF-TFM	FS	Water	Sand	2650	0.18–2.4	53.2	1.8, 3.1	0.15–0.45
Gopaliya and Kaushal [28]	KTGF-TFM	FS	Water	Sand	2650	0.165–0.55	263	3.5–4.7	0.10–0.34
Kumar et al. [29]	KTGF-TFM	FS	Water	Iron ore	4350	0.012	105	1.35–5.11	0.0263–0.31
Singh et al. [30]	KTGF-TFM	FS	Water	Coal	1560	0.059–0.206	50–150	2–5	0.30–0.60
Roco and Balakrishnam [33]	TFM	BF, FS	Water	Sand, glass beads	–	0.165–0.580	40, 51.5	1.05–4.17	0.07–0.189
Messa et al. [35]	TFM	FS	Water	Sand, glass beads	2440, 2650	0.090–0.370	53–150	1.33–8.0	0.11–0.40
Messa et al. [36]	TFM	FS	Water	Sand, glass beads	2440, 2650	0.090–0.520	50.7–150	1.33–8.0	0.09–0.40
Messa and Malavasi [37]	TFM	MB, FS	Water	sand	$\approx 2650$	0.090–0.520	50.7–150	1.33–6.0	0.09–0.43
Messa and Malavasi [38]	TFM	FS	Water	Sand	$\approx 2650$	0.090–0.640	50–200	2.0–9.0	0.07–0.41
Chen et al. [39]	KTGF-MFM	HS, PS	Water	Coal	1465	0.065 + 0.345	25–50	0.2–5	0.38–0.538
Li et al. [40]	KTGF-MFM	MB, HS, PS	Water	Glass beads	2470	0.125 + 0.440	54.9	2.0–5.0	0.20–0.50
Li et al. [41]	KTGF-MFM	MB, HS, PS	Water	Glass beads	2470	0.125 + 0.440	54.9	2.0–4.0	0.20–0.40
Ling et al. [42]	MM	BF, FS	Water	Silica/zircon sand	2380, 4223	0.11	22.1	1.0–3.0	0.10, 0.20
Lin and Ebadian [43]	MM	BF, FS	Water	Silica/zircon sand	2380, 4223	0.11	22.1	1.0–3.0	0.10, 0.20
Silva et al. [44]	MM	SB, MB, FS	Saline water	Glass beads	2500	0.10–0.60	100	1.0–3.0	0.008–0.11

Messa & Matoušek (2020)

# model setup

## Model overview:

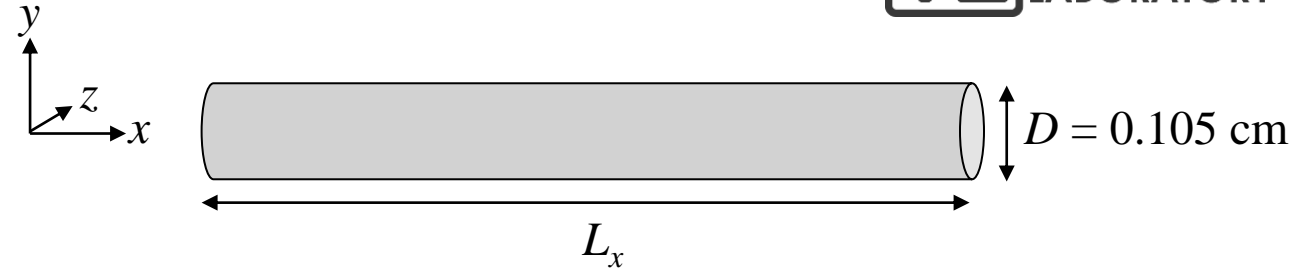
- cylindrical EB:  $L_x = 8D$
- periodic in  $x$  with enforced  $\Delta p_f$
- no slip wall EB
- uniform grid
- boundary adjacent cut-cells
- $dx = D/16 \rightarrow W_{st} = 72.9 \rightarrow$  too coarse for LES
- $dx = D/32 \rightarrow W_{st} = 9.11 \rightarrow$  this work
- $dx = D/64 \rightarrow W_{st} = 1.14 \rightarrow$  why use PIC?



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## Nomenclature:

- “bulk” fluid velocity

$$\langle\langle u_f \rangle\rangle = \frac{\langle \varepsilon_f u_f \rangle}{\langle \varepsilon_f \rangle} = \frac{\frac{1}{\pi R^2 L_x} \int \varepsilon_f u_f dV}{\frac{1}{\pi R^2 L_x} \int \varepsilon_f dV}$$

$$\langle\langle\langle u_f \rangle\rangle\rangle = \frac{1}{\delta t} \int \langle\langle u_f \rangle\rangle dt$$

“base model”

Lagrangian: discrete particles as *parcels*

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{u}_i \quad \text{and} \quad m_i \frac{d\mathbf{u}_i}{dt} = m_i \mathbf{g} + \frac{V_p}{\varepsilon_p} \nabla \tau_p + \mathbf{F}_{fi}$$

Eulerian: continuous fluid

$$\nabla \cdot \varepsilon_f \mathbf{u}_f = 0 \quad \text{and} \quad \rho_f \left( \frac{\partial \varepsilon_f \mathbf{u}_f}{\partial t} + \nabla \cdot \varepsilon_f \mathbf{u}_f \mathbf{u}_f \right) = -\varepsilon_f \nabla \pi_f + \nabla \cdot \boldsymbol{\tau}_f + \mathbf{M}_{pf} + \rho_f \varepsilon_f \mathbf{g}$$



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$$\mu_t = \text{Imaginary}$$

$$\mu_s = \mu(\varepsilon_f) = \begin{cases} \text{Einstein} \\ \text{Bainbridge, Gidlikov} \\ \text{Rouse, Krieger \& Dougherty} \\ \text{Cheng \& Law, etc.} \end{cases}$$

# governing equations

$$\mathbf{F}_{fi} = (-\nabla \varphi_f) \frac{1}{\rho_f}$$

$$\mathbf{F}_D \propto u_R^2, C_D = \text{drag coefficient}$$

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$$\begin{aligned} & \mathbf{F}_D + \mathbf{F}_{vm} + \mathbf{F}_L + \mathbf{F}_{TD} \rightarrow \propto k \nabla \varepsilon_f \\ & \mathbf{F}_D \propto u_R^2, C_D = \text{Gidaspow} \quad \mathbf{F}_{vm} \propto \frac{d\mathbf{u}_i}{dt} - \frac{D\mathbf{u}_f}{Dt} \quad C_{vm} = 0.5 \end{aligned}$$

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$$\rightarrow \propto k \nabla \varepsilon$$

$W_{st} \approx 10$ ,  
probably not.

$W_{st} > 100,000 \dots$   
may be

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$$\mathbf{F}_{fi} = (-\nabla \pi_f) \frac{V_p}{\rho_f} + (\nabla \cdot \boldsymbol{\tau}_f) \frac{V_p}{\rho_f} + \mathbf{F}_D + \mathbf{F}_{vm} + \mathbf{F}_L + \mathbf{F}_{TD}$$

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# governing equations

$$\mathbf{F}_{fi} = (-\nabla \pi_f) \frac{V_p}{\rho_f} + (\nabla \cdot \boldsymbol{\tau}_f) \frac{V_p}{\rho_f} + \mathbf{F}_D + \mathbf{F}_{vm} + \mathbf{F}_L + \mathbf{F}_{TD} \rightarrow \propto k \nabla \epsilon$$

$\mathbf{F}_D \propto u_R^2, C_D = 6 \pi \mu a$   $\mathbf{F}_{vm} \propto \frac{d\mathbf{u}_i}{dt} - \frac{D\mathbf{u}_f}{Dt}$   $C_{vm} = 0.5$   $\mathbf{F}_{TD} \propto u_R \times \nabla \times \mathbf{u}_f$   
 $\eta_{st} \approx 10$ , probably not.  
 $\eta_{st} > 100,000 \dots$  may be

Lagrangian: discrete particles as *parcels*

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{u}_i \quad \text{and} \quad m_i \frac{d\mathbf{u}_i}{dt} = m_i \mathbf{g} + \frac{V_p}{\epsilon_p} \nabla \tau_p + \mathbf{F}_{fi}$$

transfer

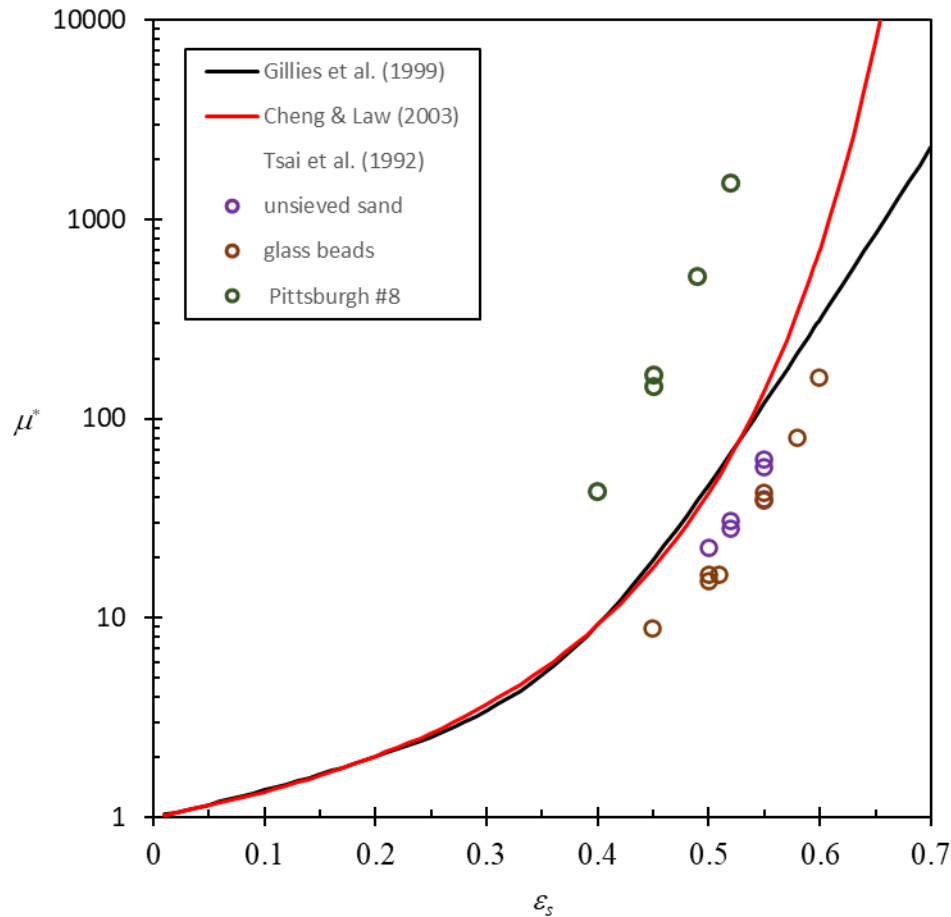
Eulerian: continuous fluid

$$\nabla \cdot \epsilon_f \mathbf{u}_f = 0 \quad \text{and} \quad \rho_f \left( \frac{\partial \epsilon_f \mathbf{u}_f}{\partial t} + \nabla \cdot \epsilon_f \mathbf{u}_f \mathbf{u}_f \right) = -\epsilon_f \nabla \pi_f + \nabla \cdot \boldsymbol{\tau}_f + \mathbf{M}_{pf} + \rho_f \epsilon_f \mathbf{g}$$

$$\mu_f \rightarrow \mu_{eff} = \mu_f + \mu_t + \mu_s$$

$$\mu_t = \text{Maxwellian}$$

$$\mu_s = \mu(\epsilon_f) = \begin{cases} \text{Einstein} \\ \text{Painlevé, Gidalev} \\ \text{Rouse, Krieger & Dougherty} \\ \text{Cheng & Law, etc.} \end{cases}$$



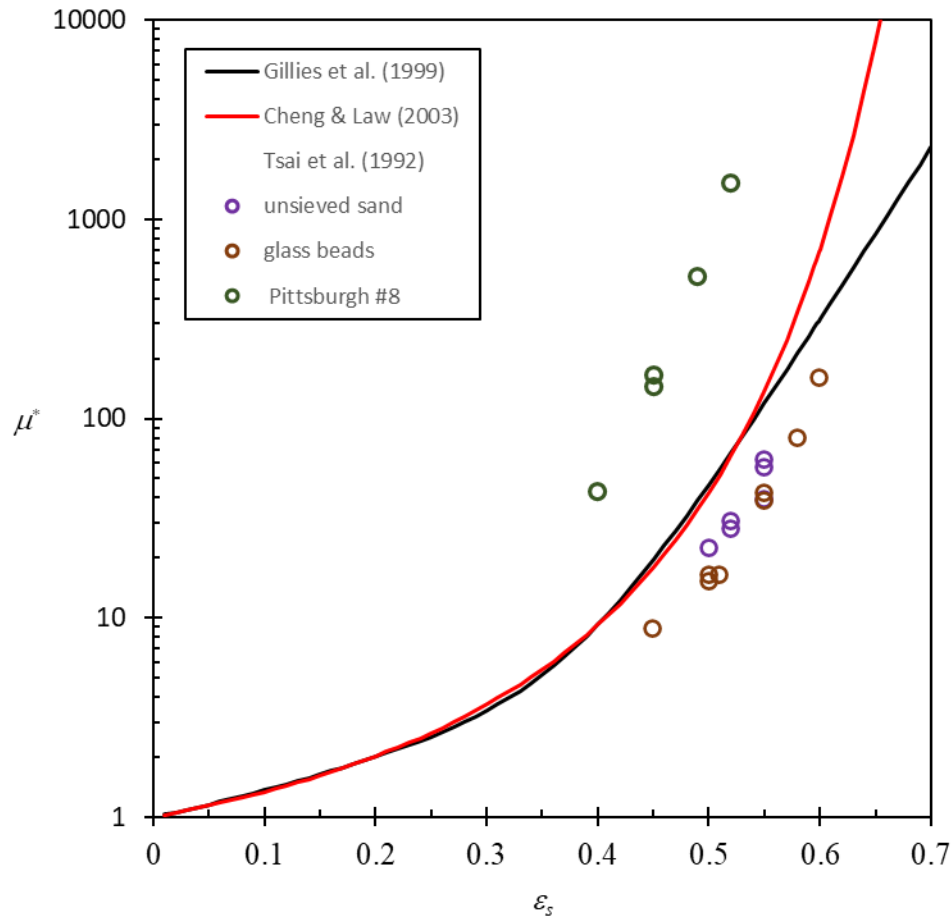
Suspension viscosity model:

- Gillies et al. (1999)

$$\mu^* = 1 + 2.5\varepsilon_f + 10\varepsilon_f^2 + 0.0019e^{20\varepsilon_f}$$

- Cheng & Law (2003)

$$\mu^* = \exp \left[ \frac{2.5}{\beta} \left( \frac{1}{(1 - \varepsilon_f)^\beta} - 1 \right) \right]$$



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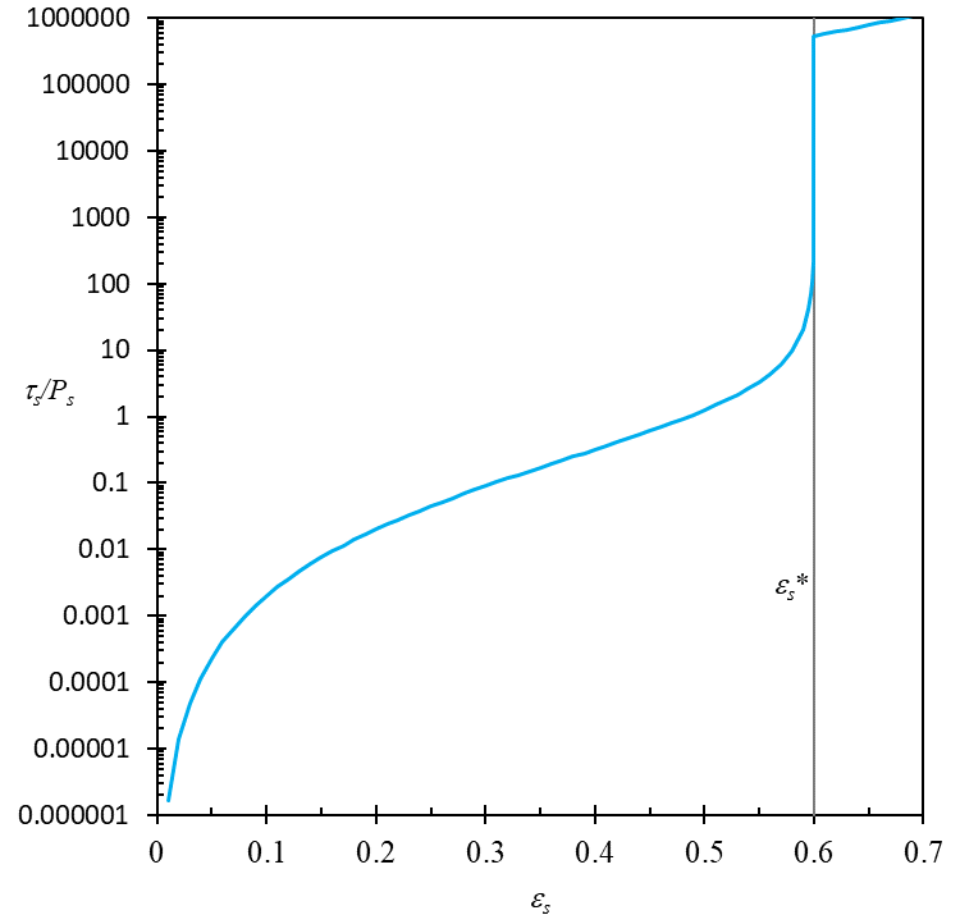
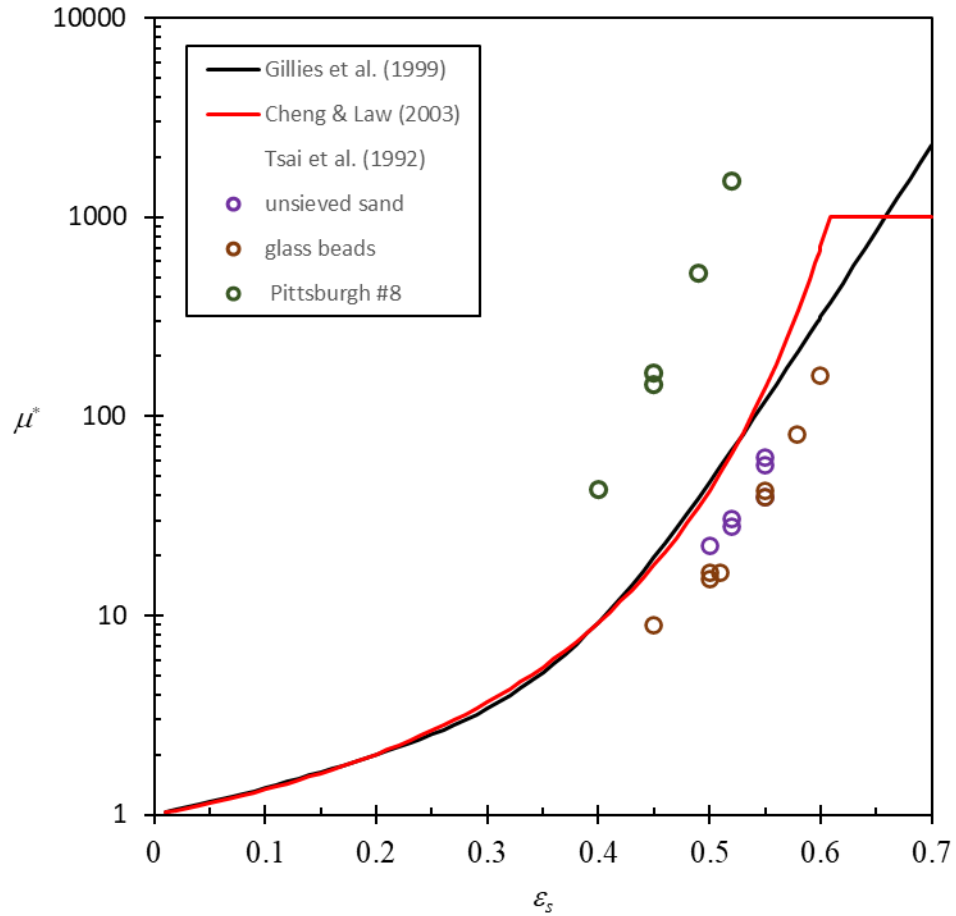
Solids stress model:

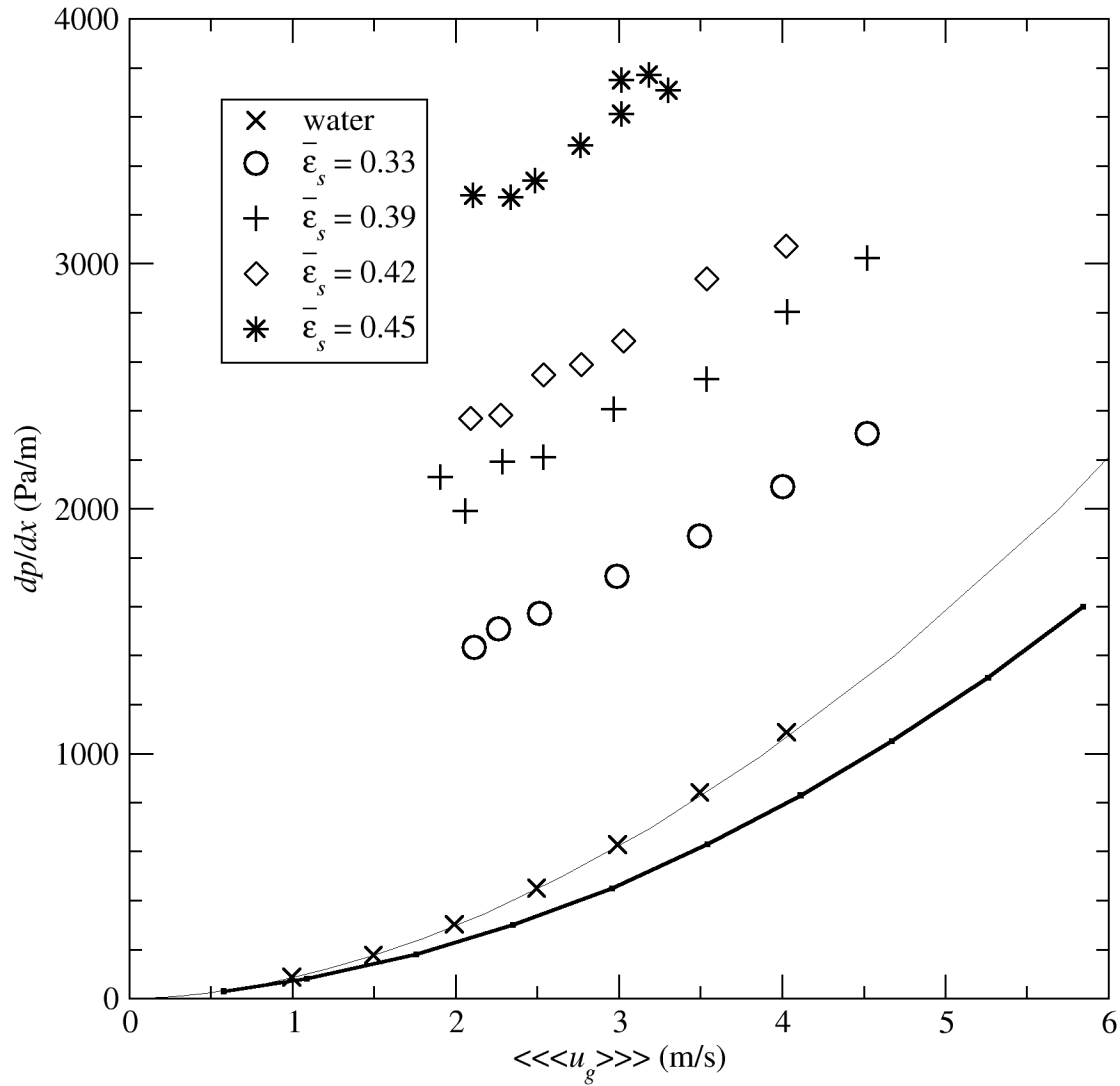
- Snider (2001), Harris & Crighton (1994)

$$\tau_s = \frac{P_s \varepsilon_s^\beta}{\max [(\varepsilon_s^* - \varepsilon_s), \epsilon (1 - \varepsilon_s)]}$$

$$P_s = 100, \beta = 3, \epsilon = 10^{-6}$$

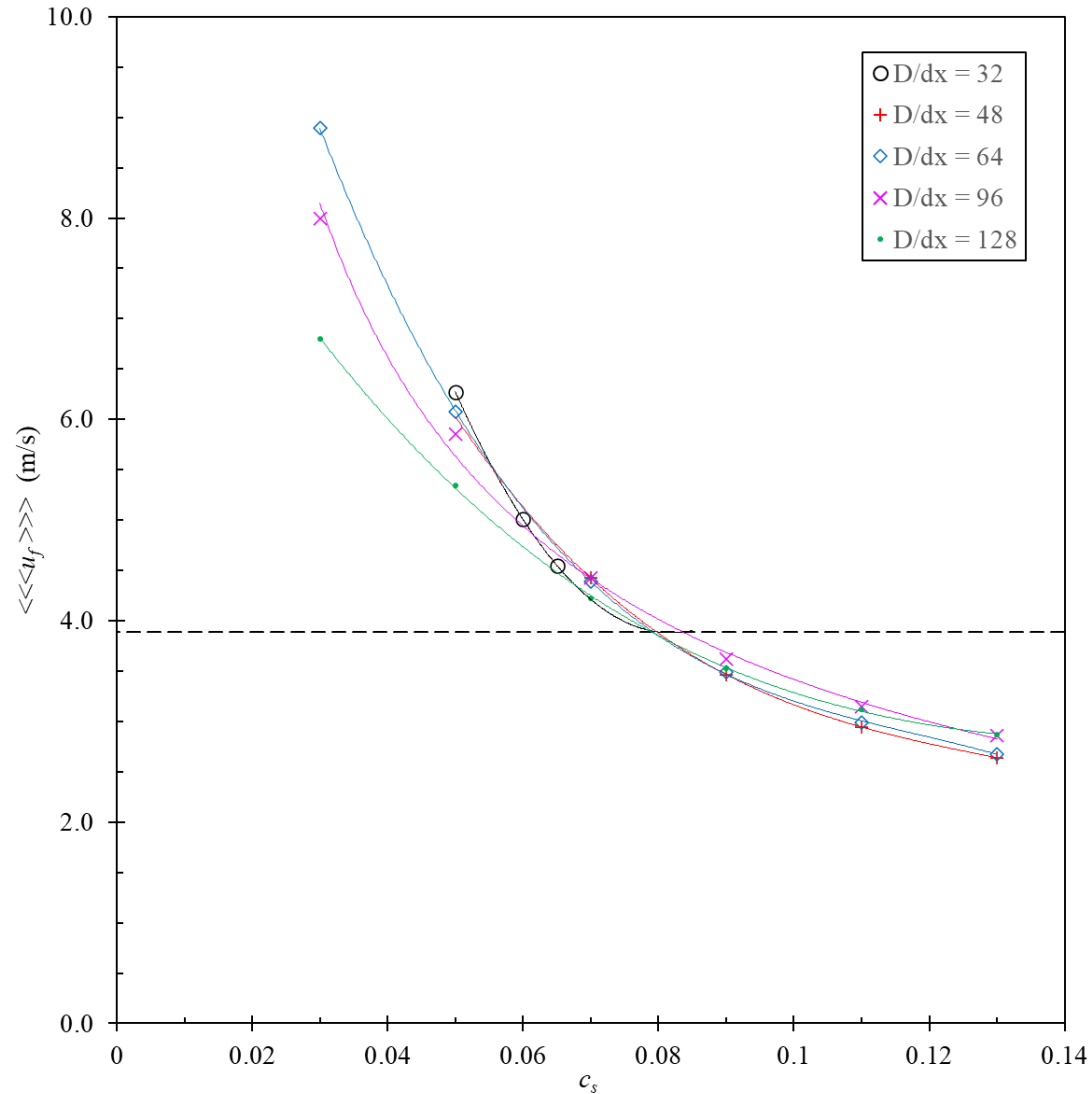
# suspension viscosity



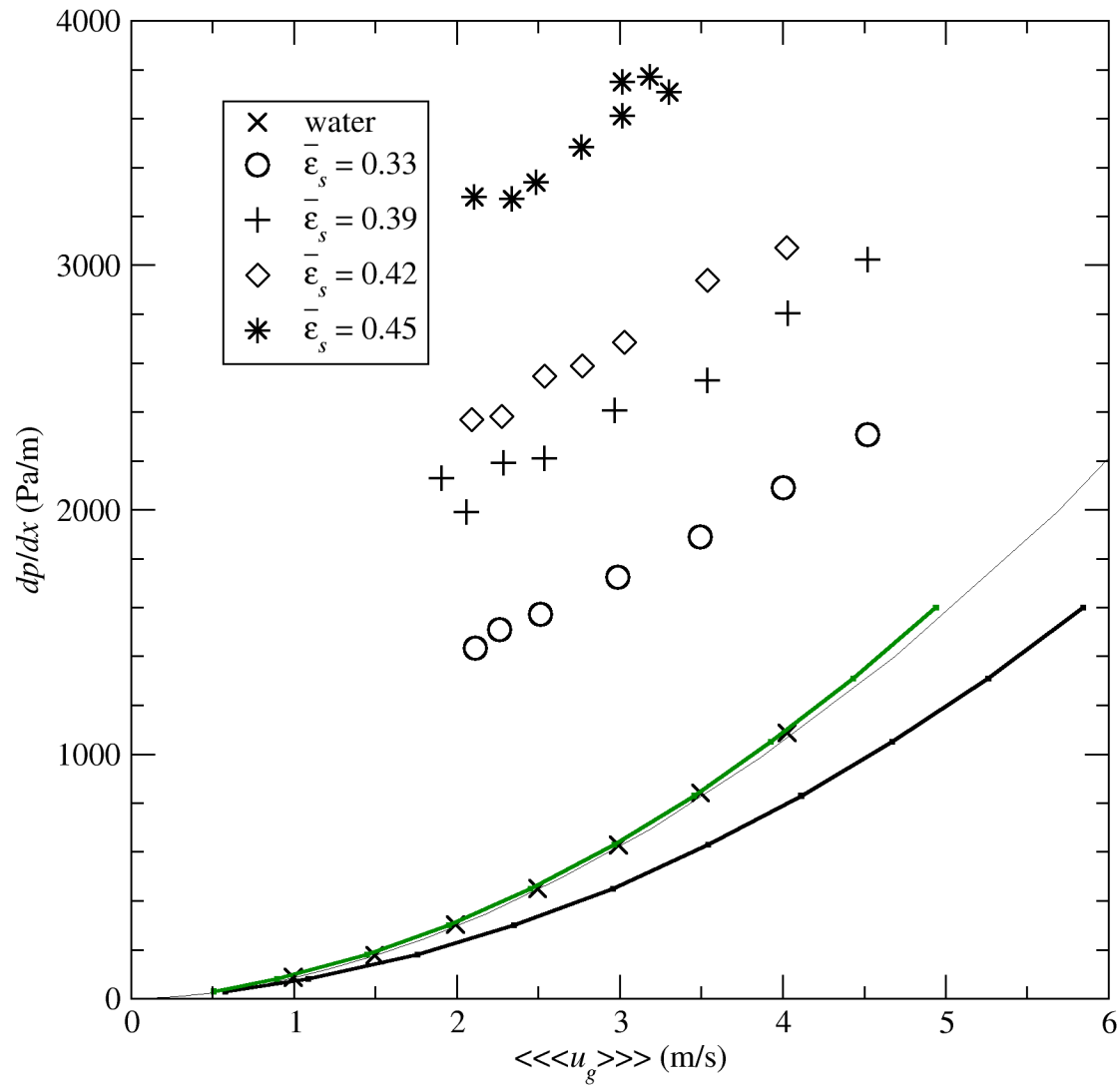




# Smagorinsky constant

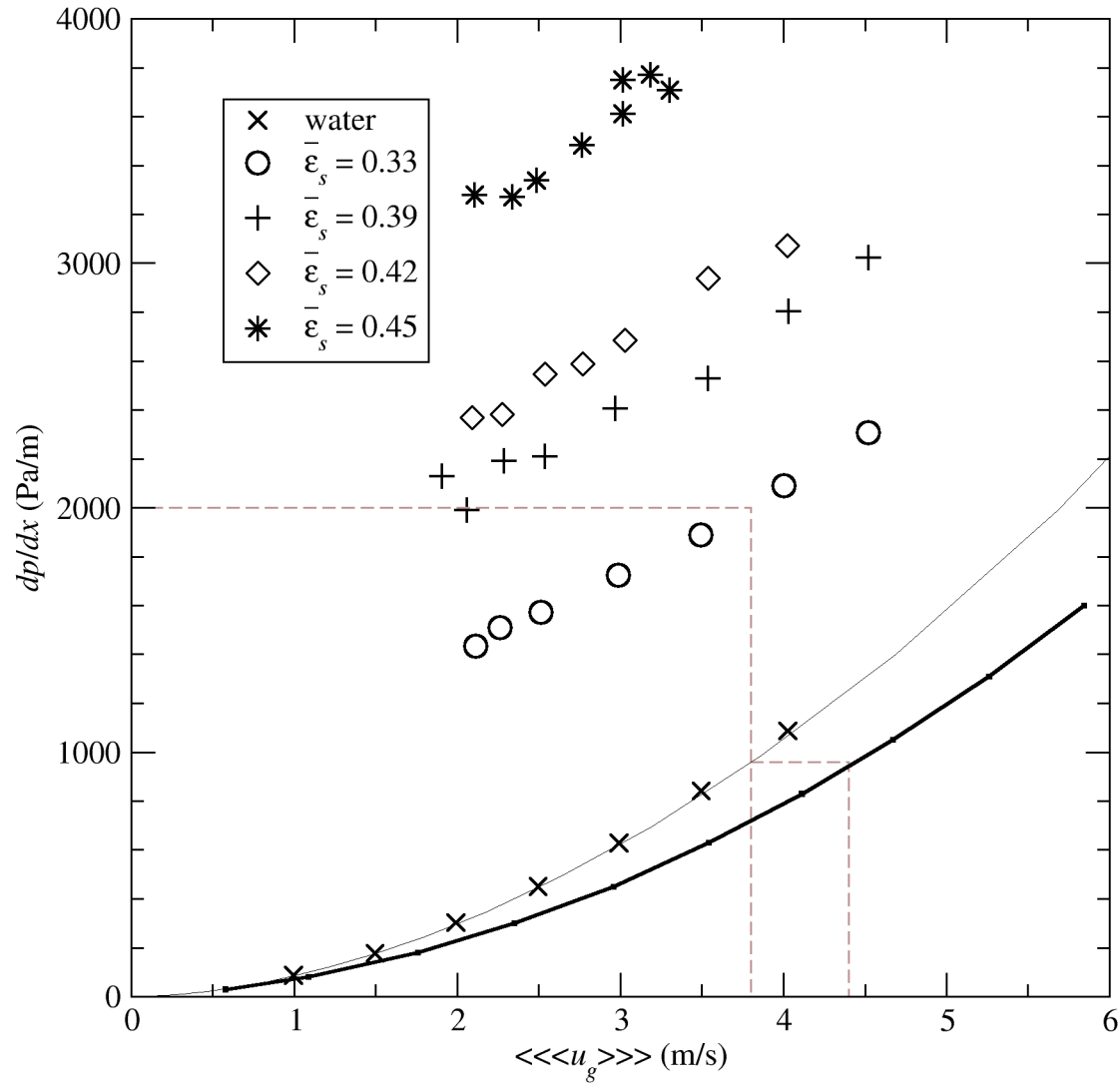


For a wide range of resolutions, we find that a constant of  $c_s = 0.08$  is ideal, but this is too diffusive for this coarse grid

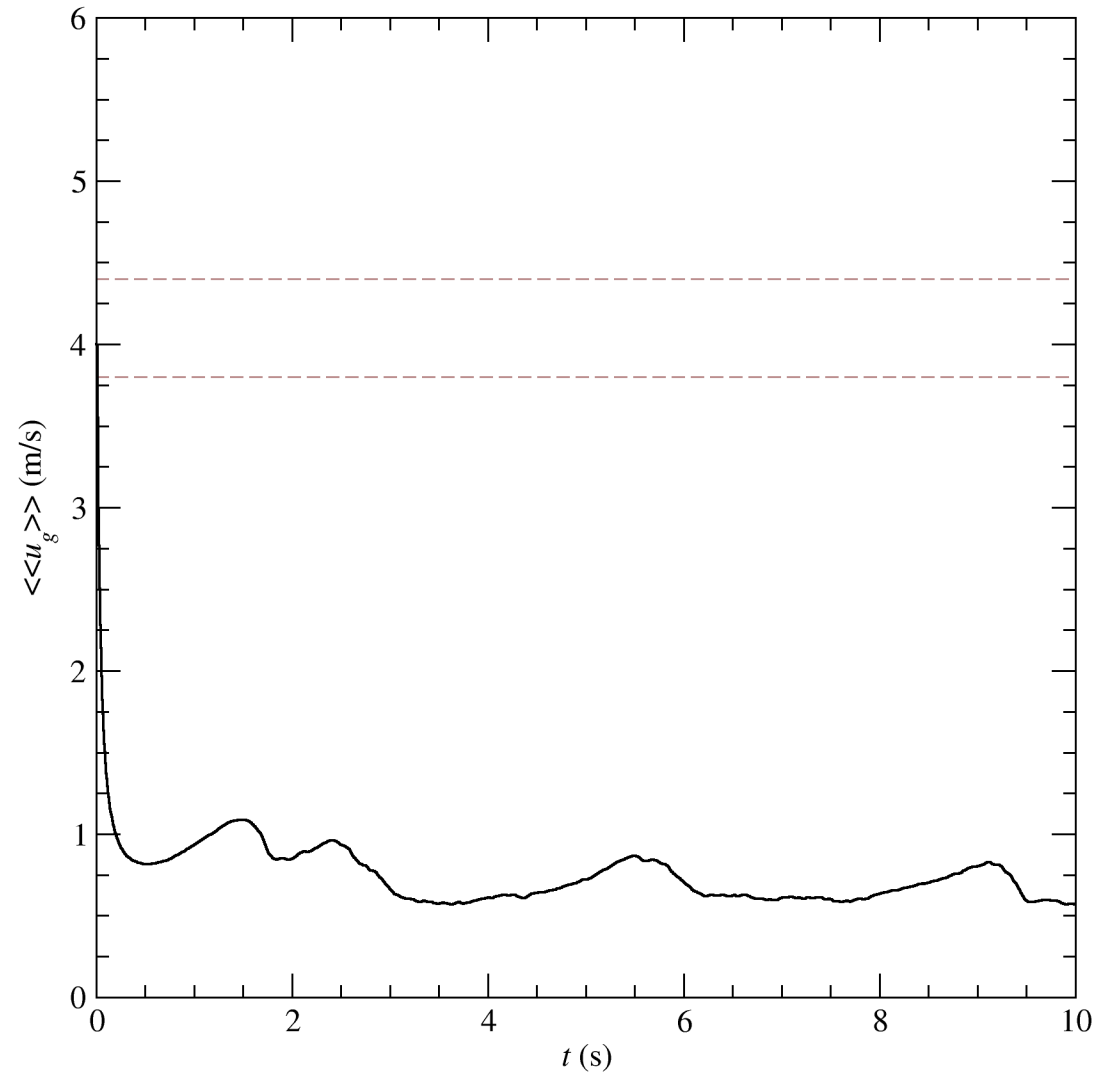


$dx = D/64$  and  $c_s = 0.08$

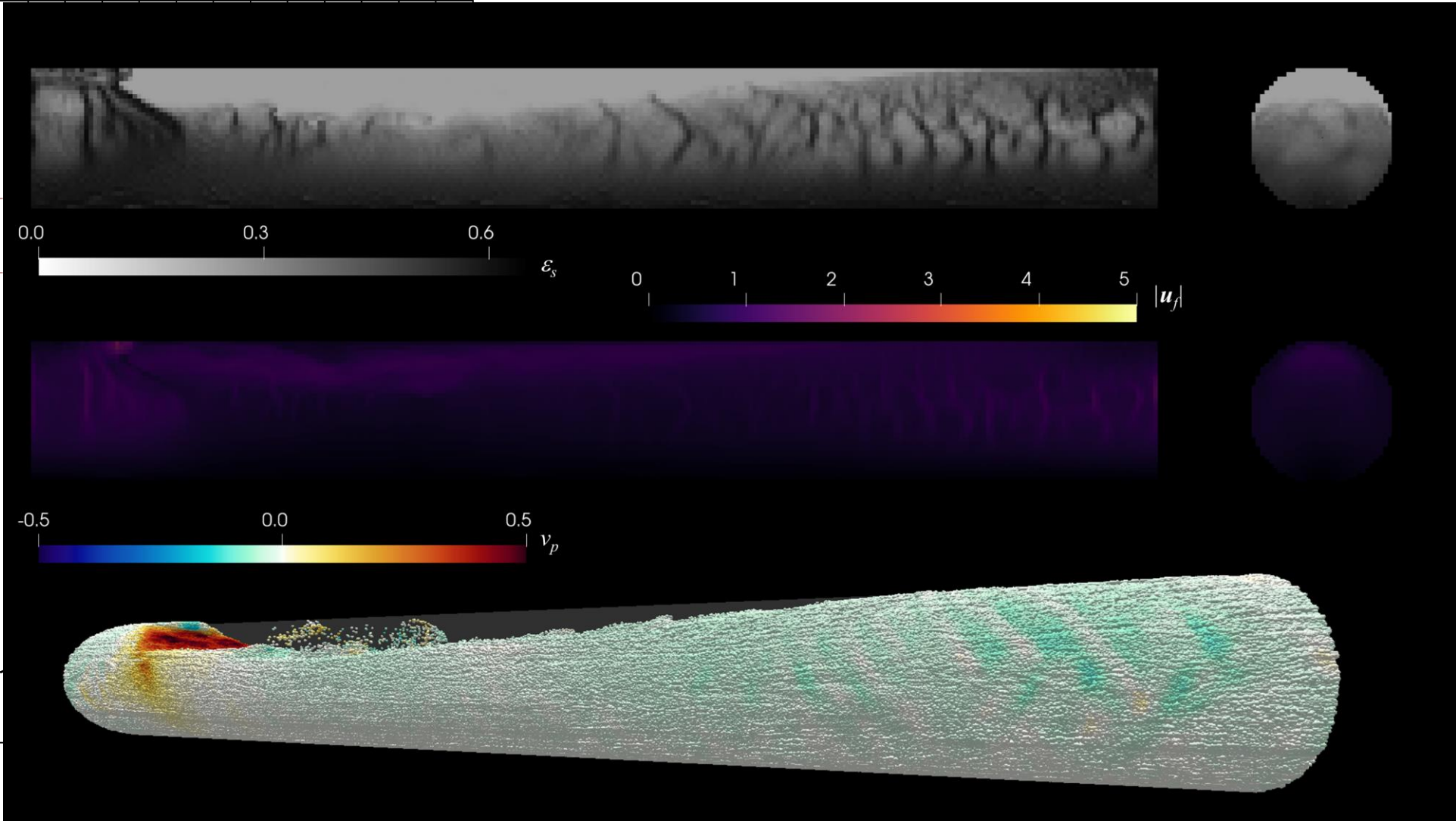
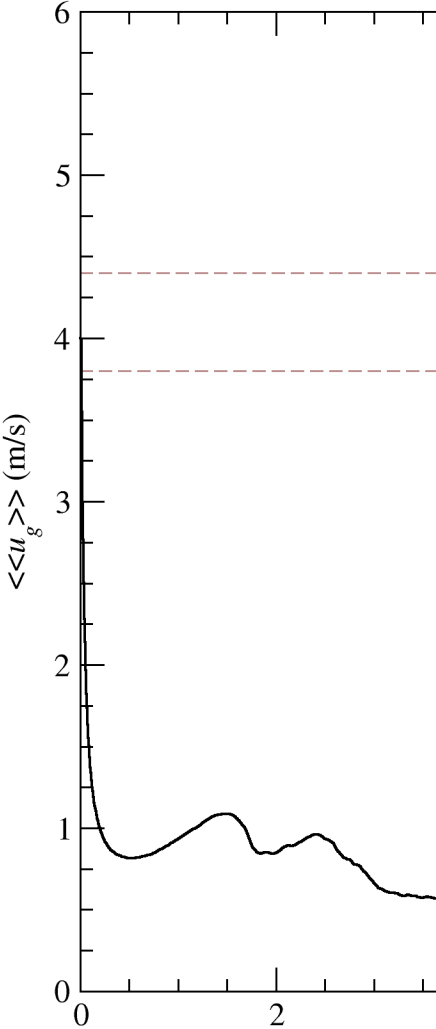
# results



set:  $dp/dx = 2000$  Pa/s  
target bulk velocity: 3.8 ~ 5 m/s

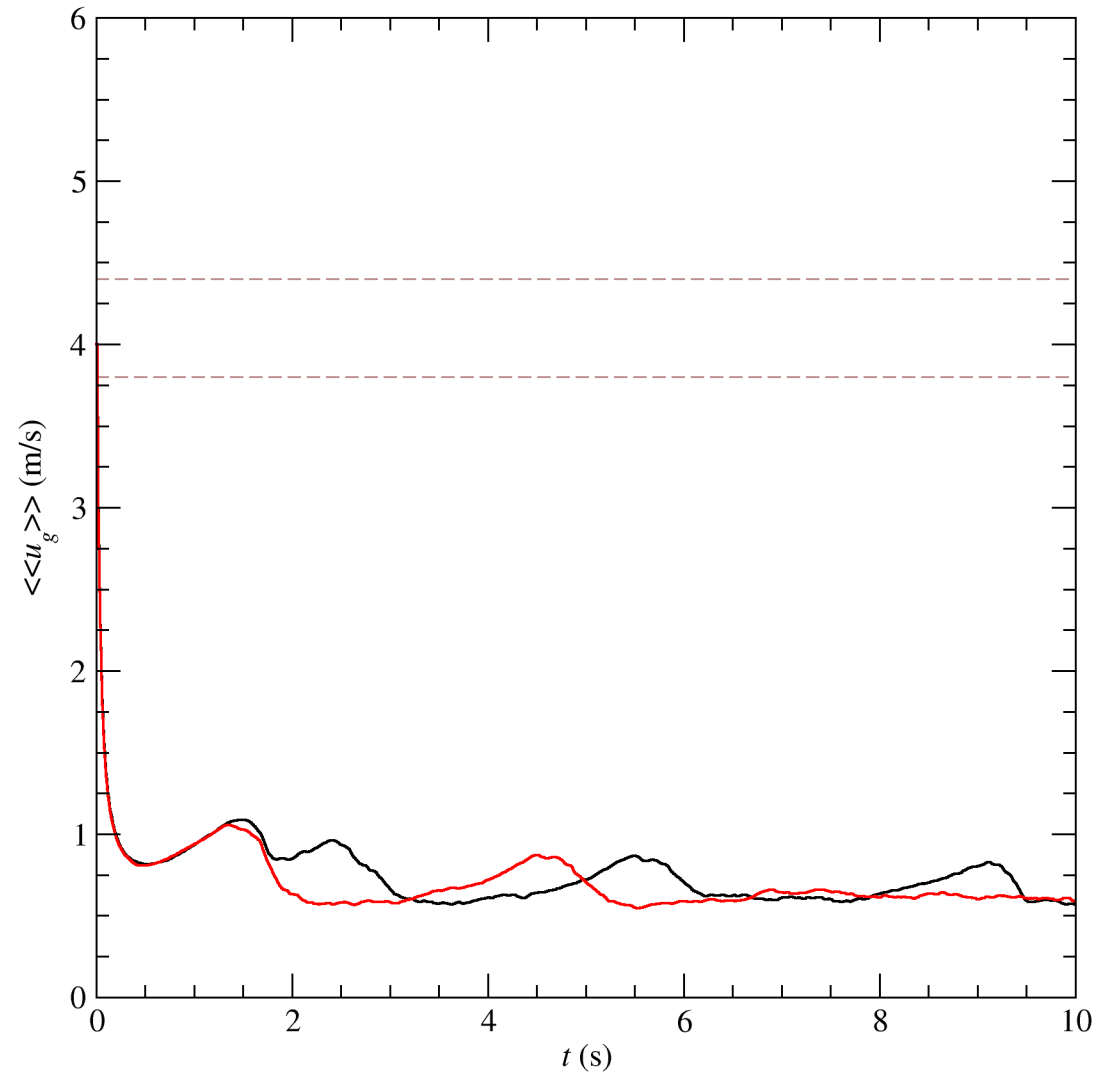


“base model”

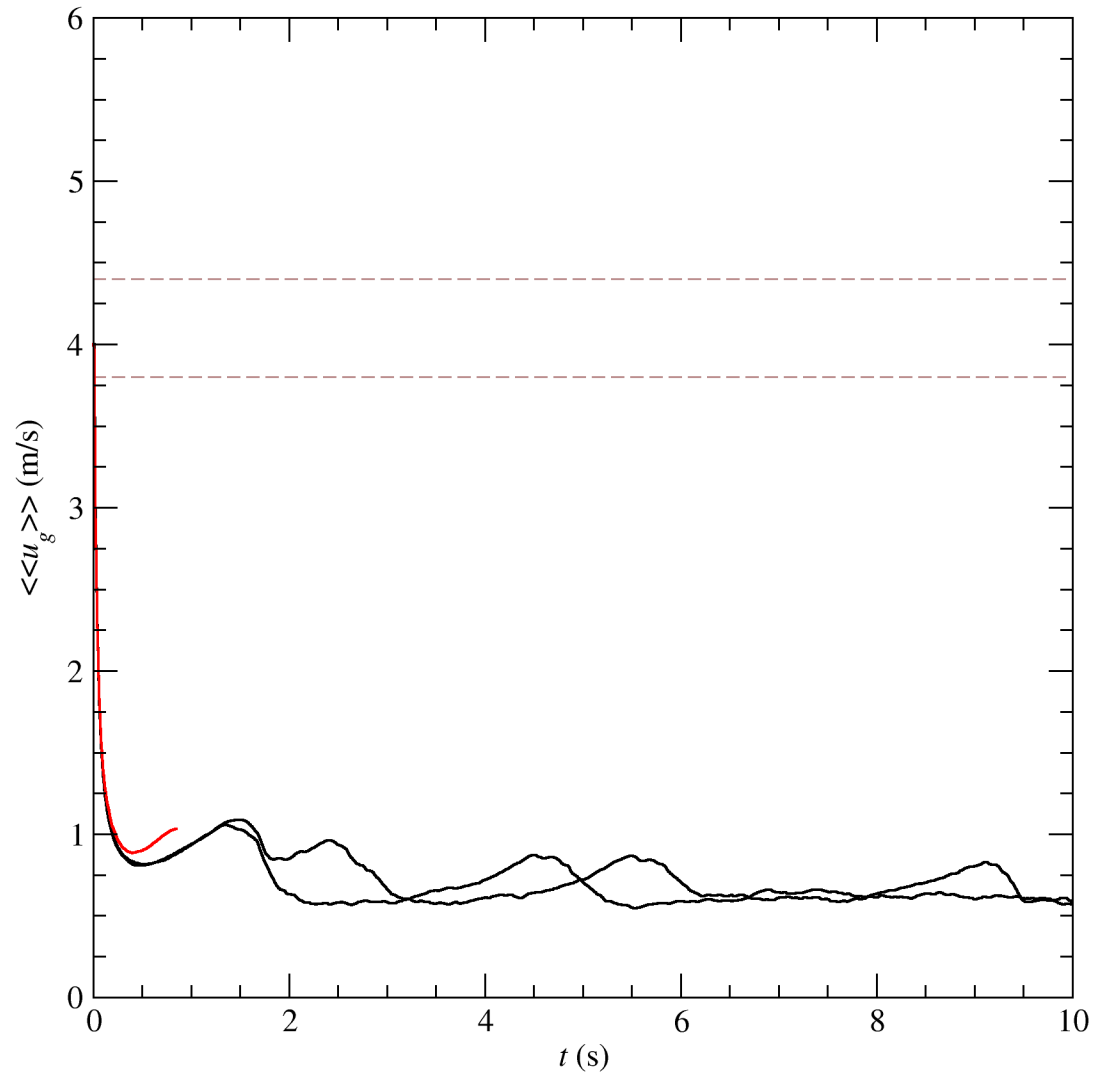


# results

*w/ Smagorinsky*



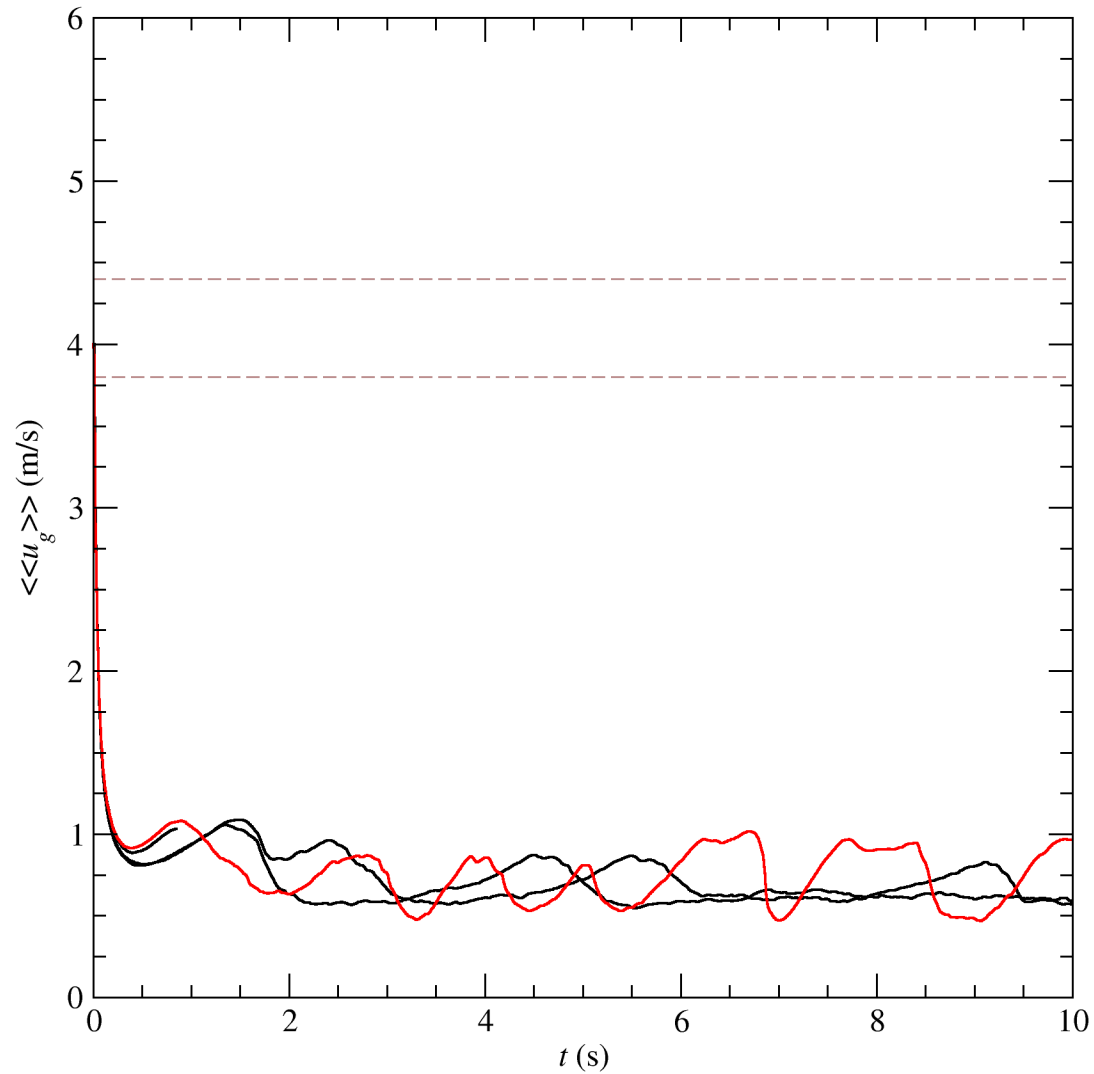
# results



w/ Smagorinsky  
+  
 $\mu_s = \text{Chop \& Low}$

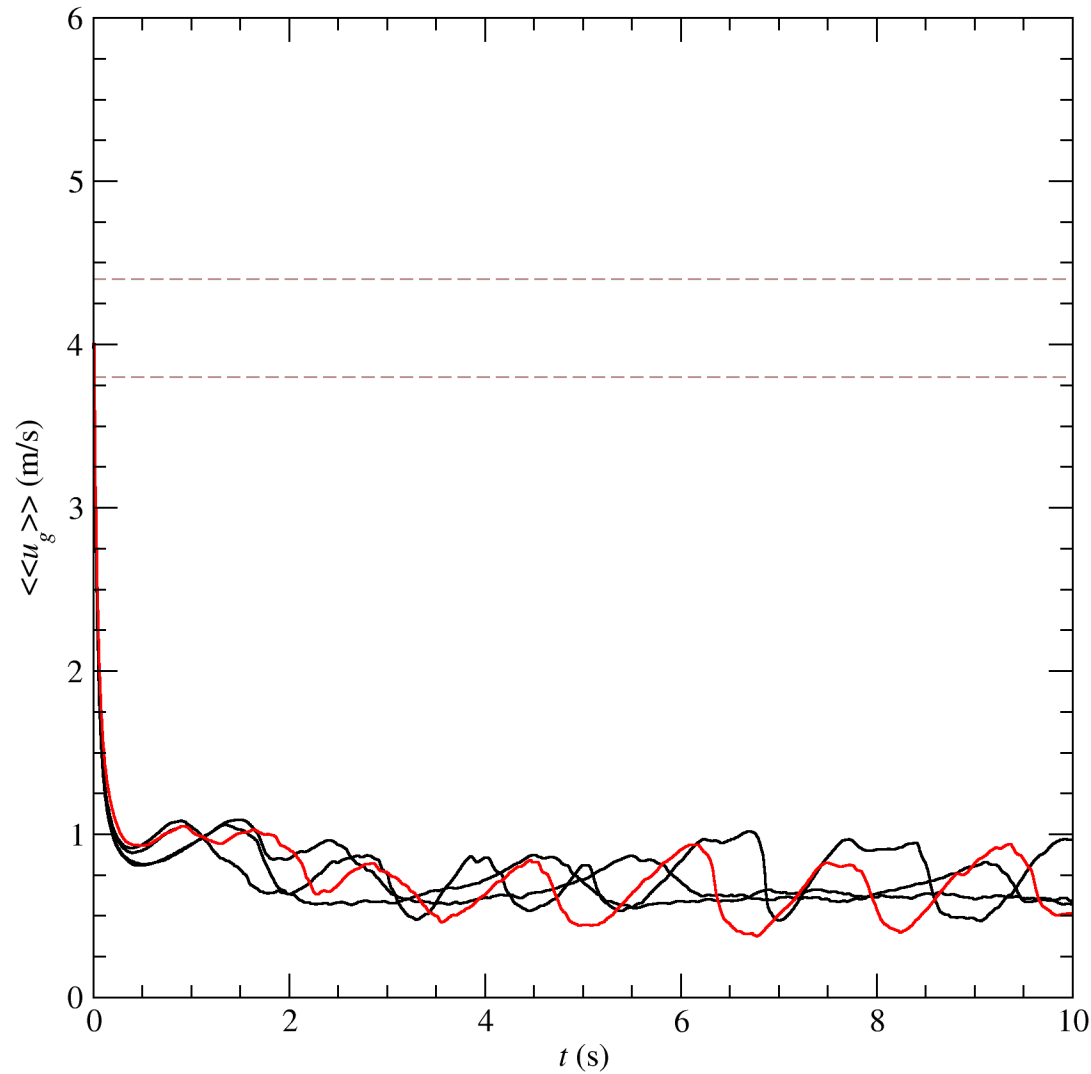


# results



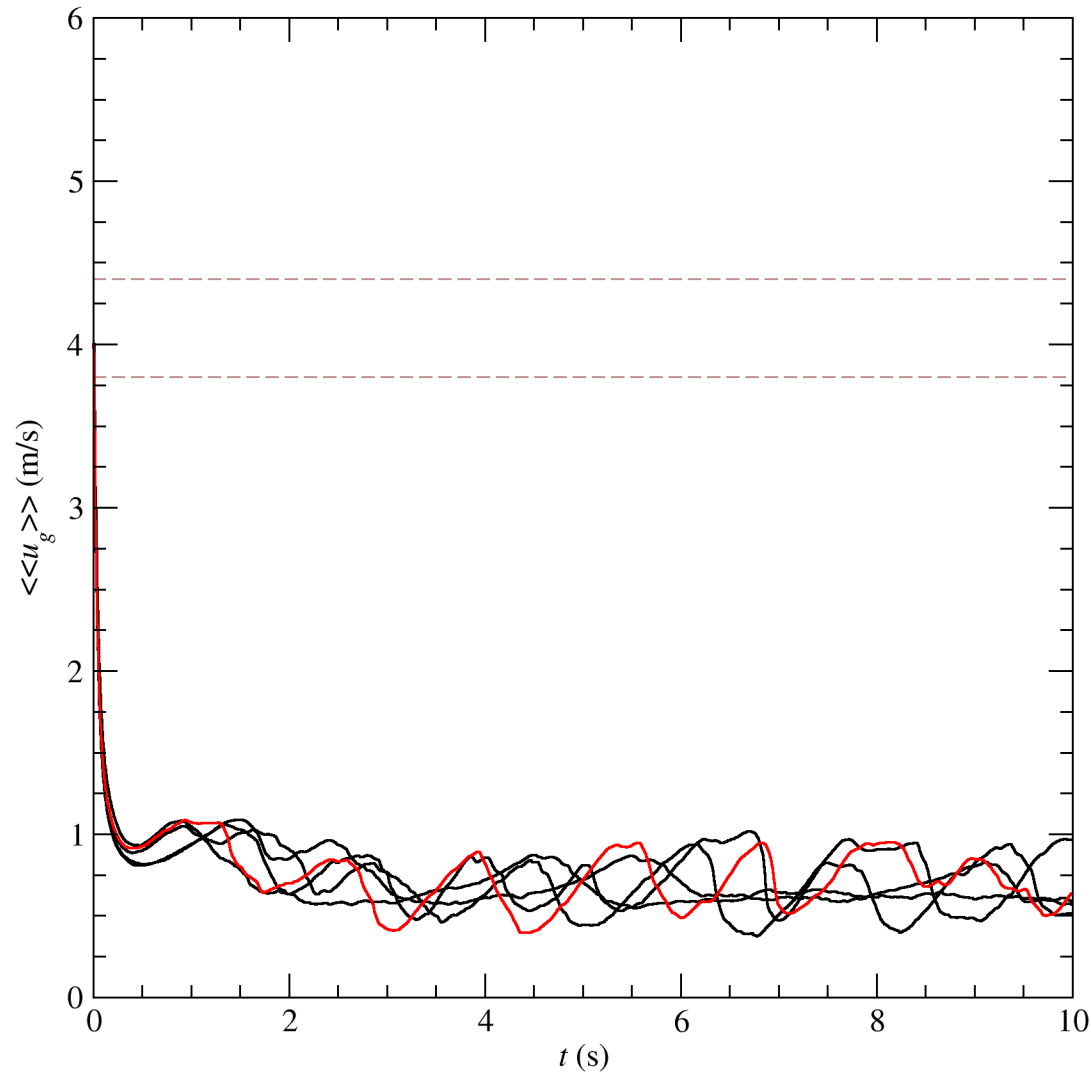
w/ Smagorinsky  
+  
 $\mu_s = \text{Chen \& Lau}$   
w/ limiter

# results



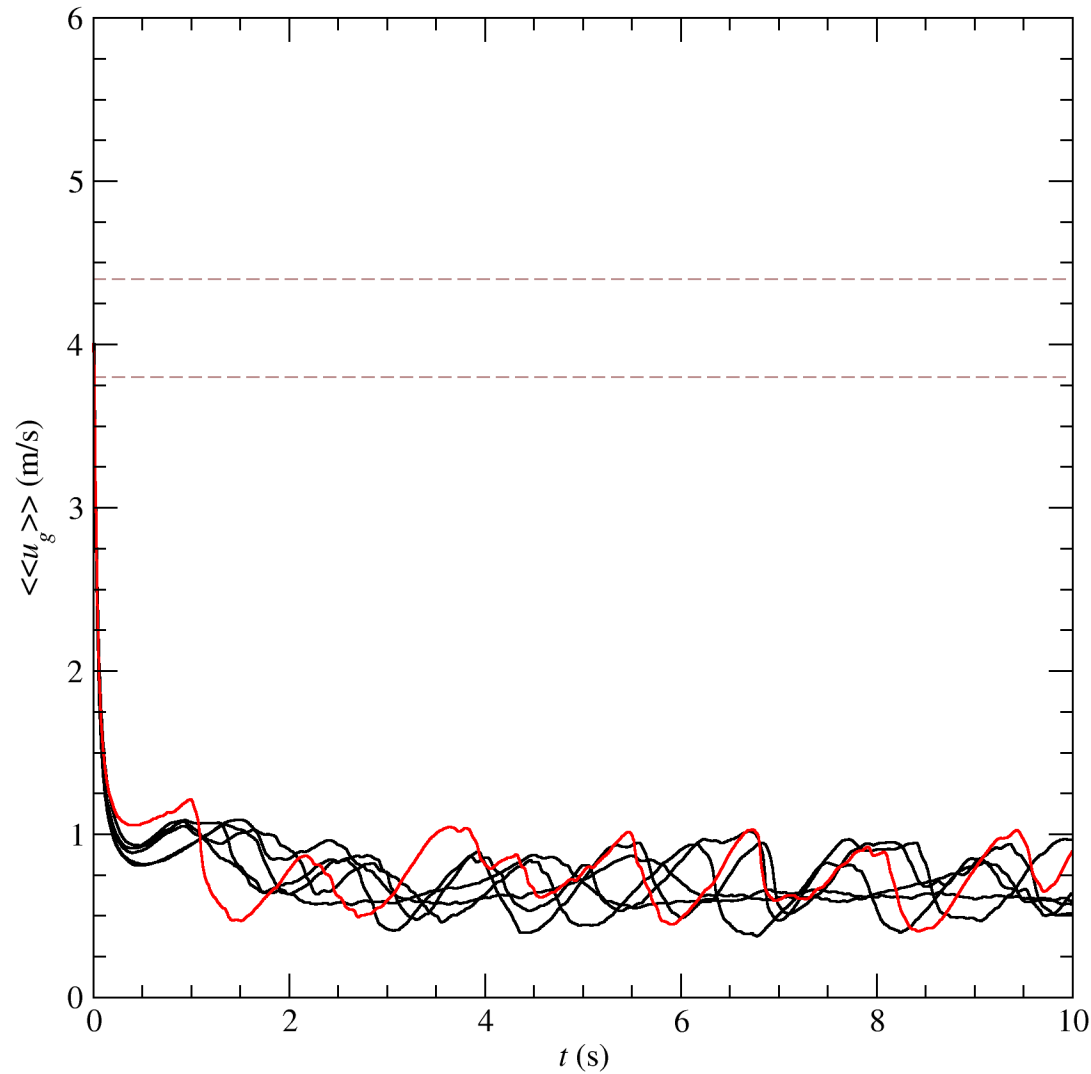
w/ Smagorinsky  
+  
 $\mu_s = \text{Chap \& Lau}$   
w/ limiter  
+  
DNS drag law

# results



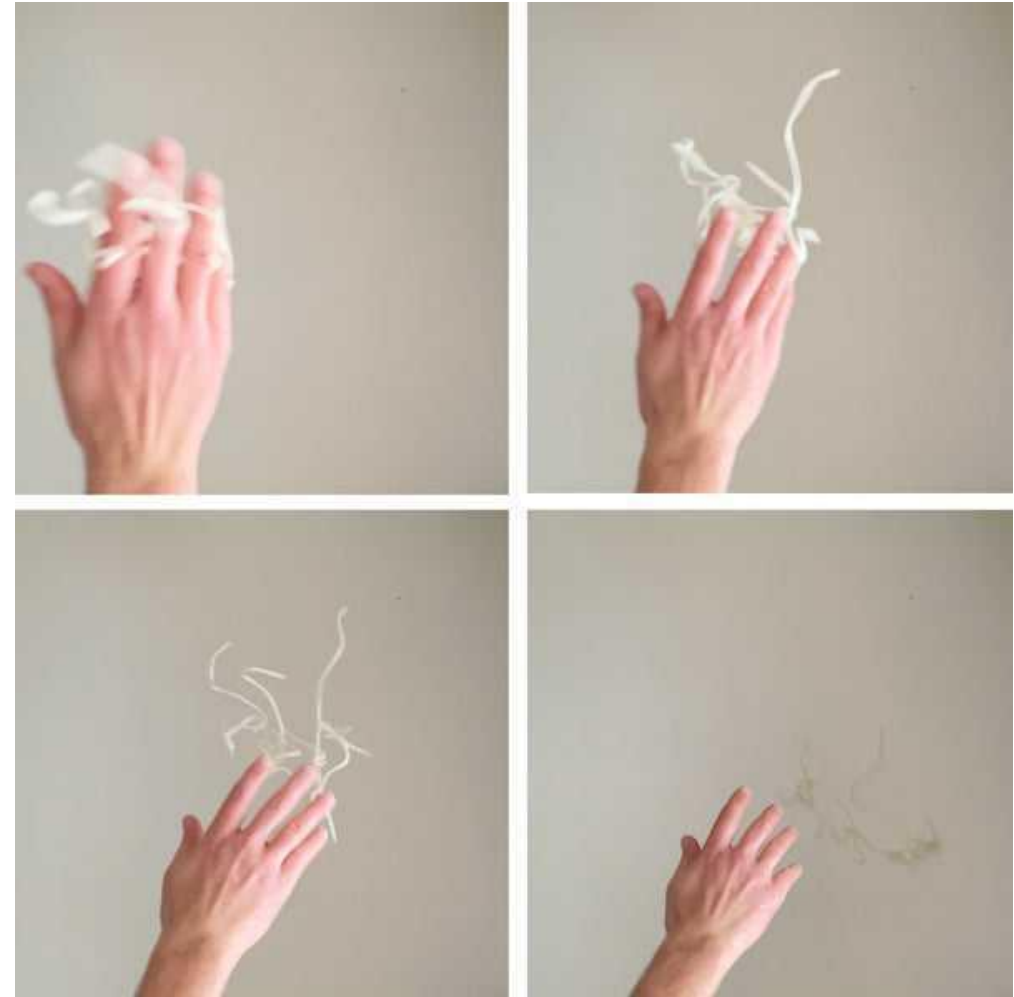
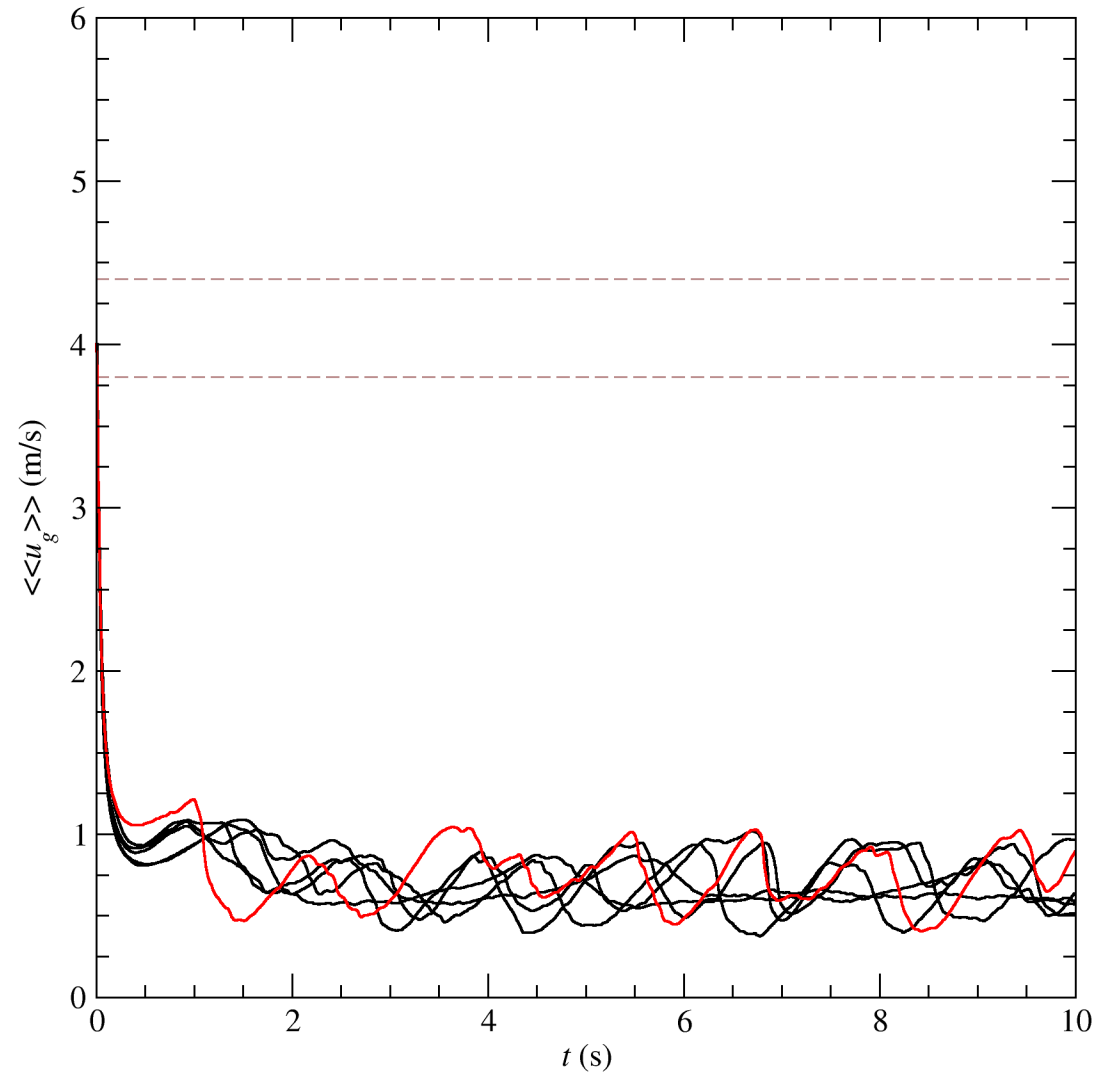
w/ Smagorinsky  
+  
 $\mu_s = \text{Chen \& Lau}$   
w/ limiter  
+  
~~DNS drag law~~  
 $(\tau - \tau_f) H_s$

# results

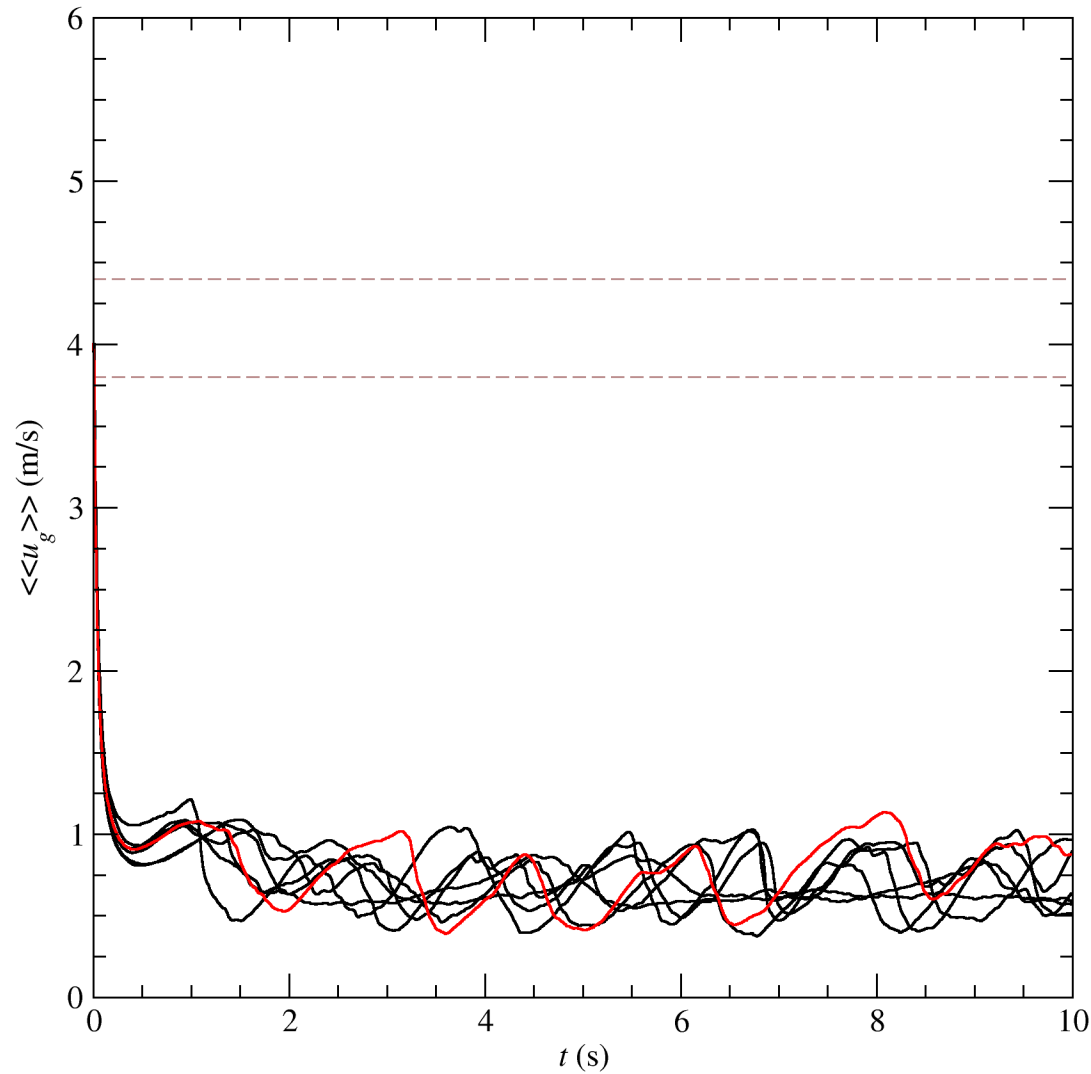


w/ Smagorinsky  
+  
 $\mu_s = \text{Chap \& Lau}$   
w/ limiter  
+  
~~DNS drag law~~  
 ~~$(\tau_s \tau_f)$~~   
vibrated mass

# results

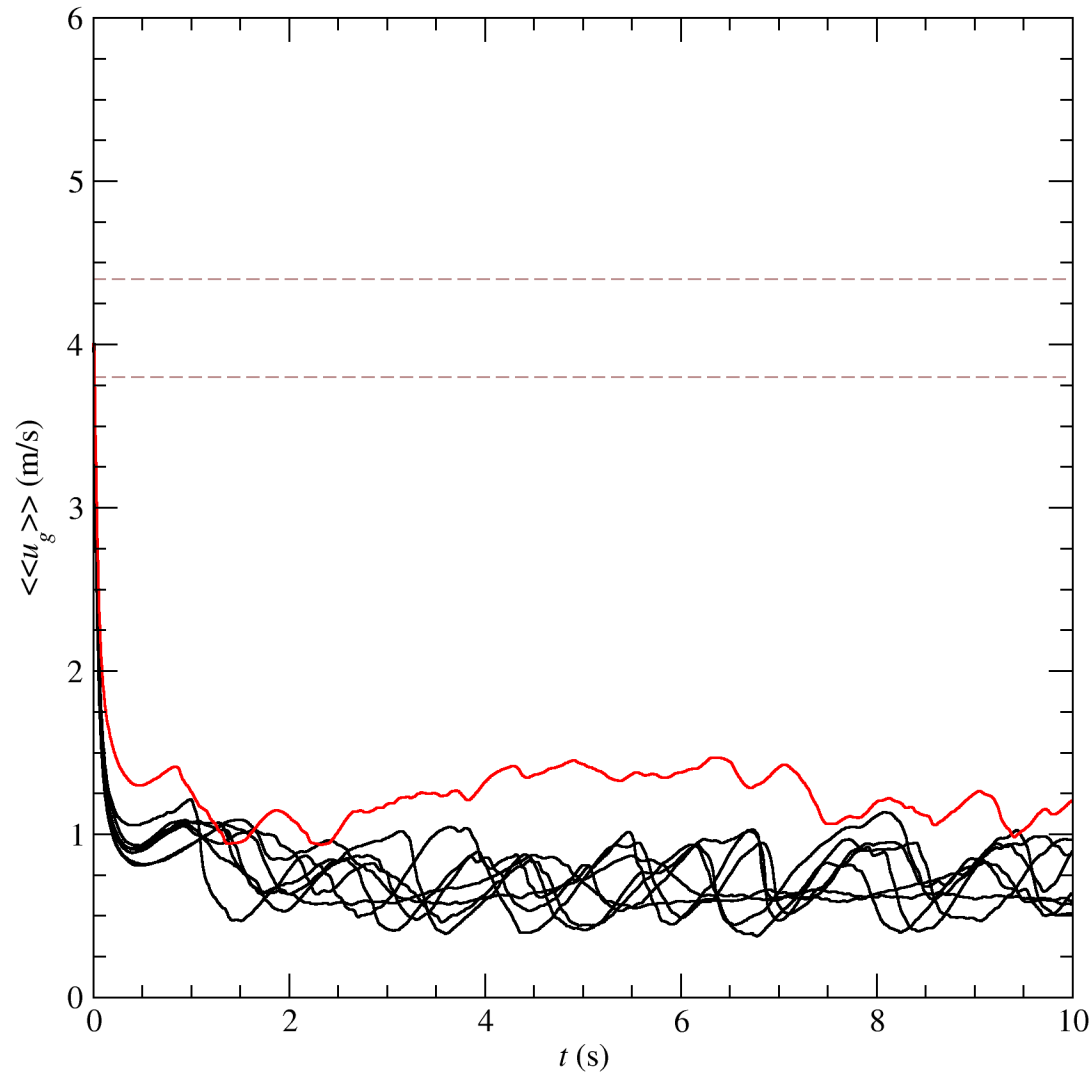


# results



$w/\text{Smagorinsky}$   
 $+$   
 $\mu_s = \text{Chap \& Lau}$   
 $w/\text{limiter}$   
 $+$   
~~DNS drag law~~  
 ~~$(\tau_s \tau_f)_{\text{H}}$~~   
~~viscous stress~~  
 Flux Reflct

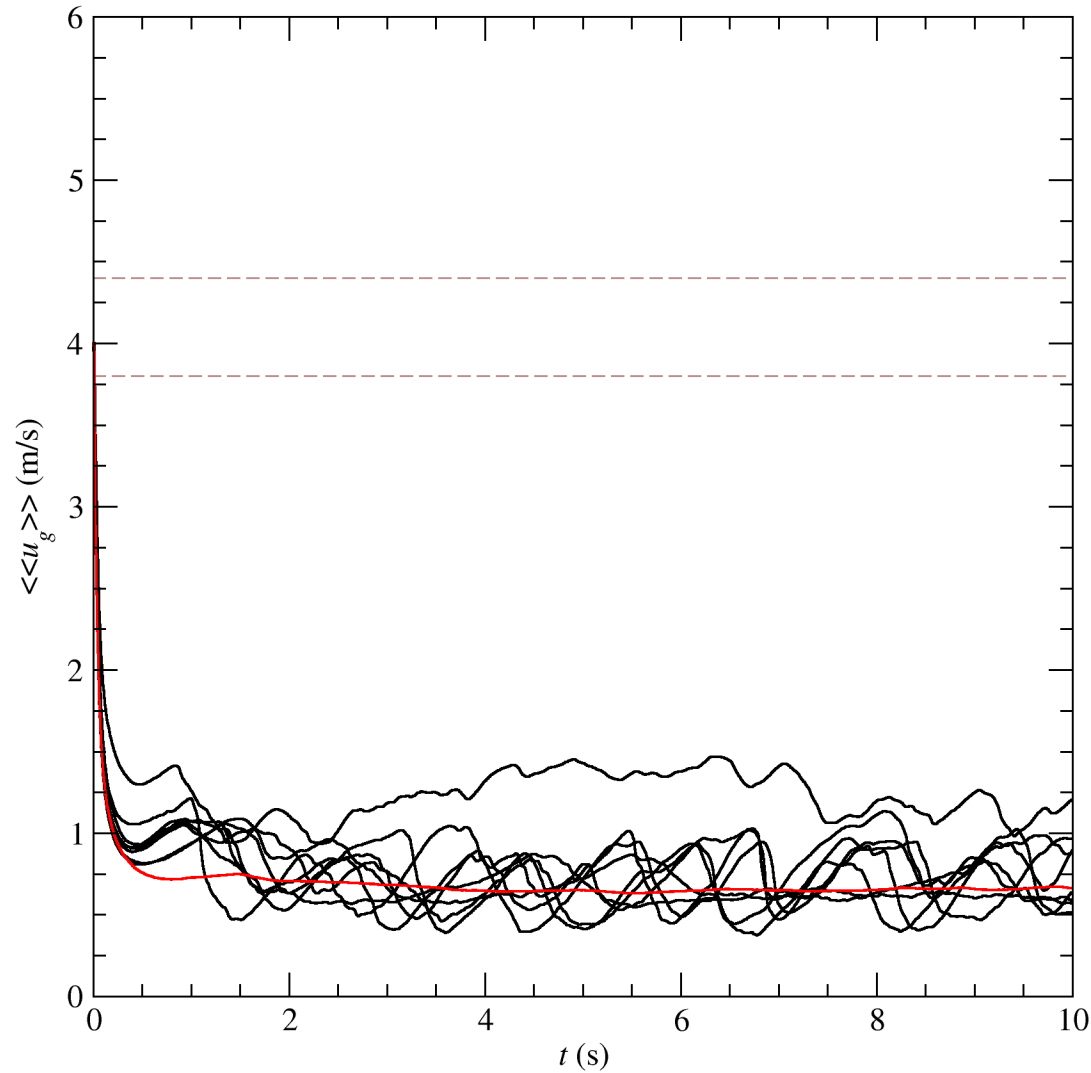
# results



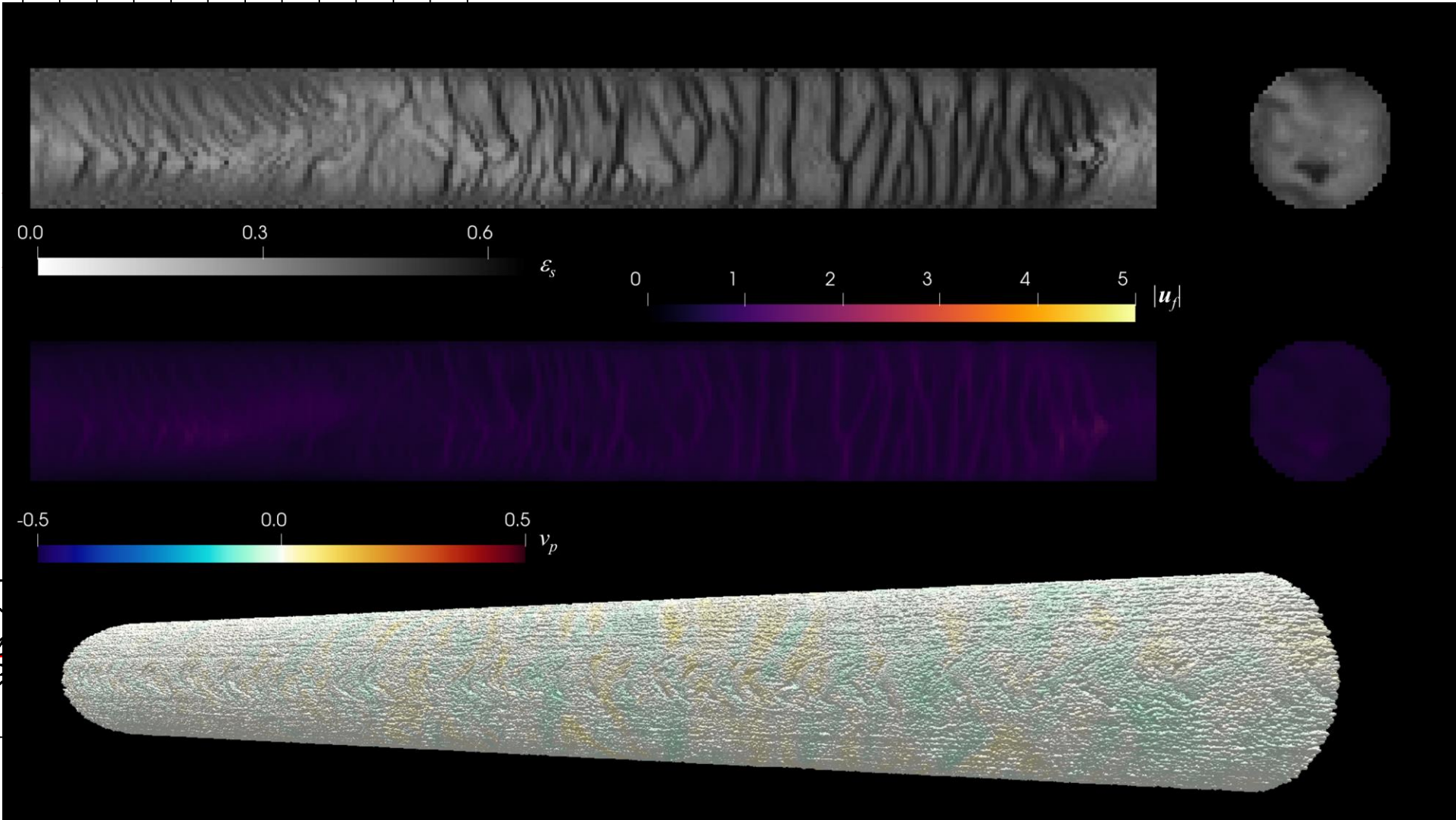
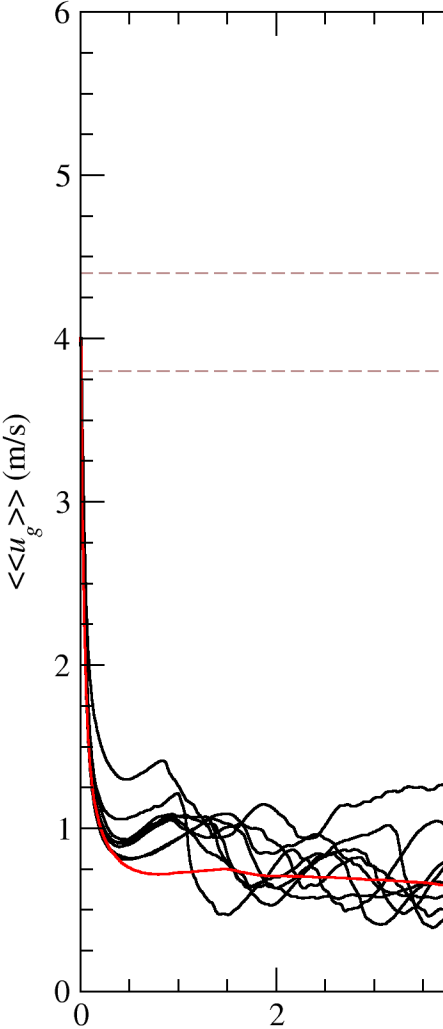
w/ Smagorinsky  
+  
 $\mu_s = \text{Chap \& Lau}$   
w/ limiter  
+  
~~DNS drag law~~  
 ~~$(\tau_s \tau_f)_{\text{th}}$~~   
~~viscous stress~~  
~~Flux Refl~~  
Method of Lines



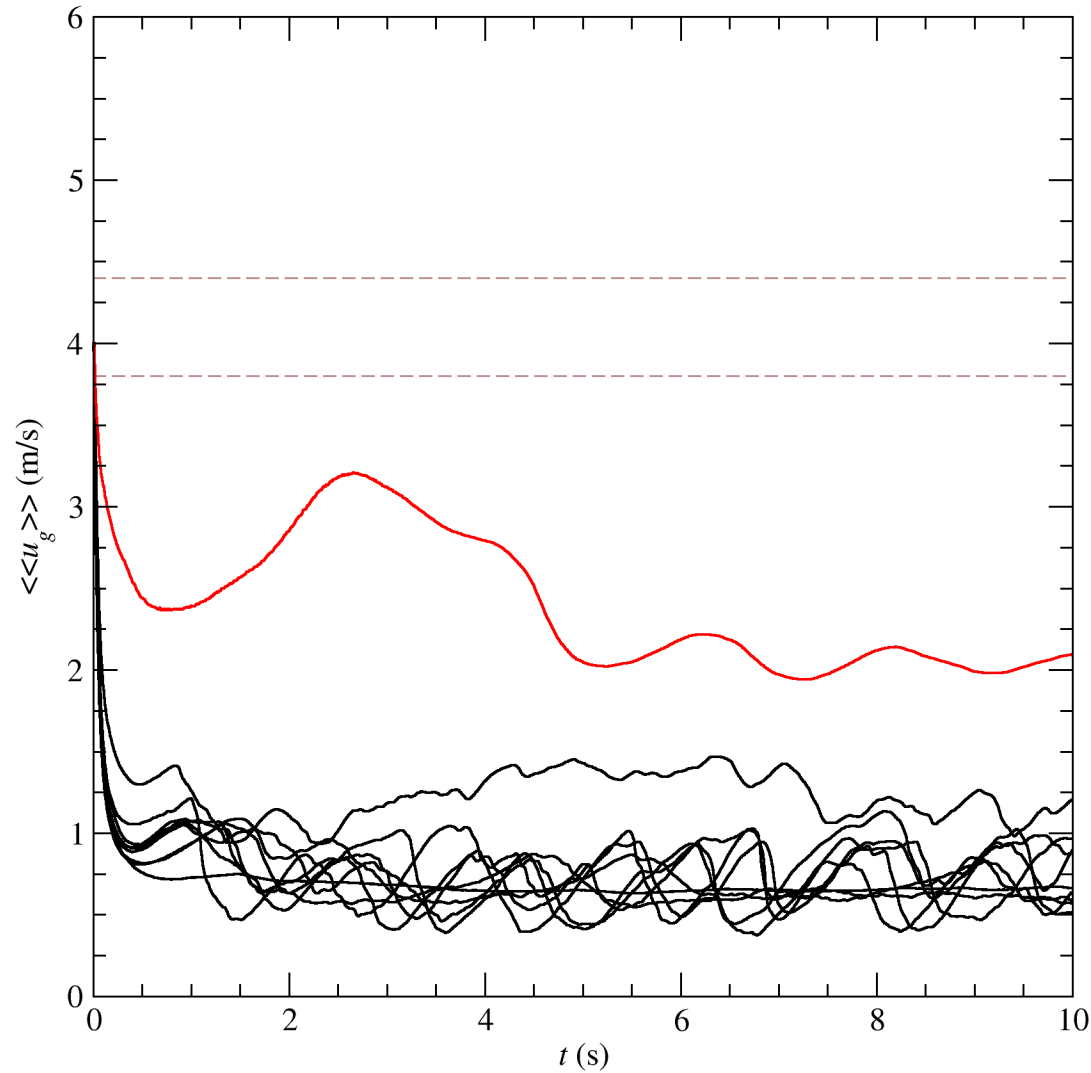
# results



w/ Smagorinsky  
+  
 $\mu_s = \text{Chap \& Lau}$   
w/ limiter  
+  
~~DNS drag law~~  
 ~~$(\tau_s \tau_f)$~~   
~~viscous stress~~  
~~Flux Aetha~~  
~~Method of Lines~~  
no gravity!



# results



w/ Smagorinsky  
 +  
 $\mu_s = \text{Chap \& Lau}$   
 w/ limiter  
 +  
~~DNS drag law~~  
 ~~$(\tau_{xy})_f$~~   
~~viscous stress~~  
~~Flux Aetha~~  
~~Method of Lines~~  
~~no gravity!~~  
 $\frac{\partial \varepsilon}{\partial t} \neq 0$

# governing equations

$$\mathbf{F}_{fi} = (-\nabla \pi_f) \frac{V_p}{\rho_f} + (\nabla \cdot \boldsymbol{\varepsilon}_f) \frac{V_p}{\rho_f} + \mathbf{F}_D + \mathbf{F}_{vm} + \mathbf{F}_L + \mathbf{F}_{TD} \rightarrow \propto k \nabla \varepsilon$$

$\mathbf{F}_D \propto u_R^2, C_D = 6 \text{ drag}$      $\mathbf{F}_{vm} \propto \frac{du_i}{dt} - \frac{Du_f}{Dt}$      $C_{vm} = 0.5$      $\mathbf{F}_{TD} \propto u_R \times \nabla \times u_f$

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 $W_{st} > 100,000 \dots$  may be

Lagrangian: discrete particles as *parcels*

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{u}_i \quad \text{and} \quad m_i \frac{d\mathbf{u}_i}{dt} = m_i \mathbf{g} + \frac{V_p}{\varepsilon_p} \nabla \tau_p + \mathbf{F}_{fi}$$

transfer

Eulerian: continuous fluid

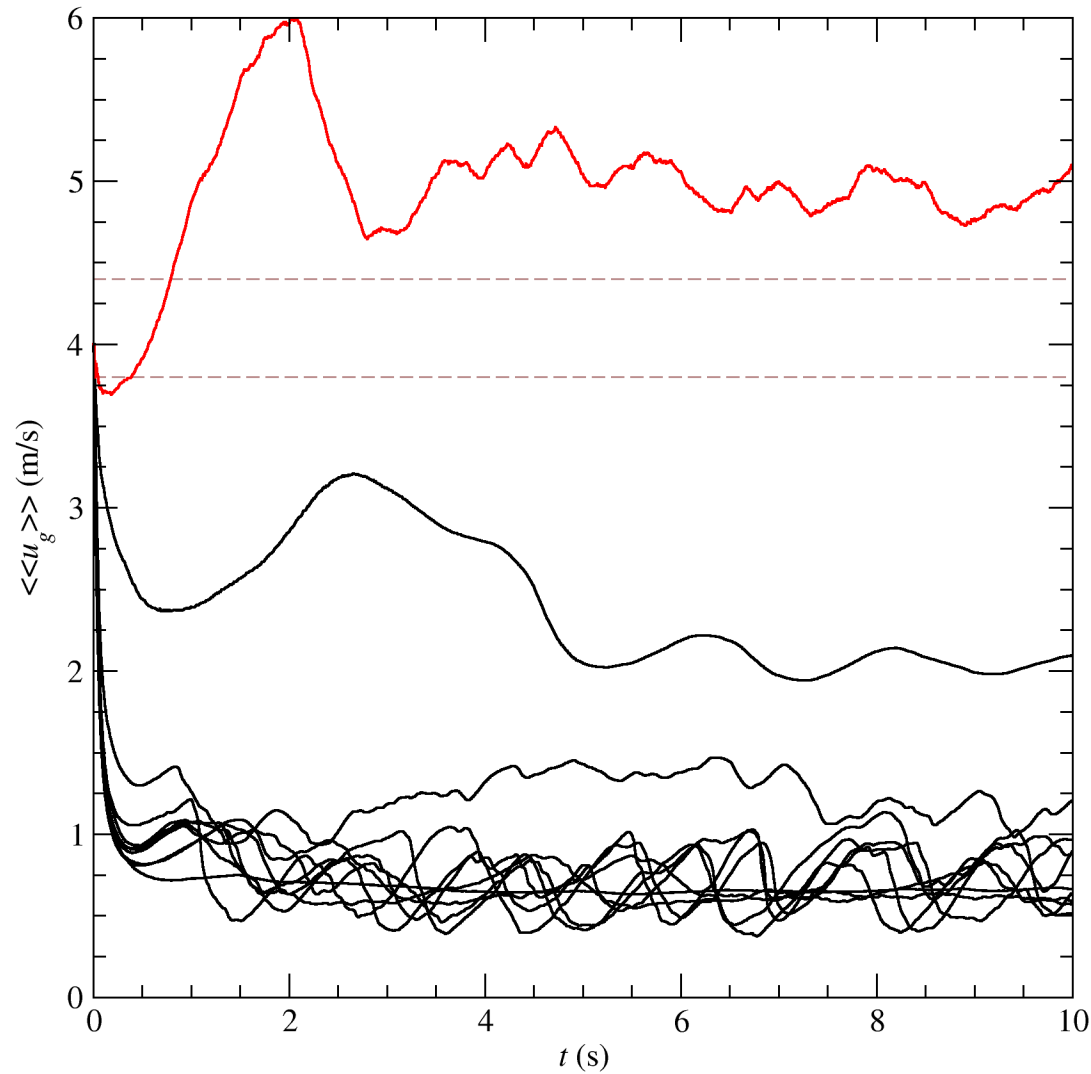
$$\frac{\partial \varepsilon_f}{\partial t} + \nabla \cdot \varepsilon_f \mathbf{u}_f = 0 \quad \text{and} \quad \rho_f \left( \frac{\partial \varepsilon_f \mathbf{u}_f}{\partial t} + \nabla \cdot \varepsilon_f \mathbf{u}_f \mathbf{u}_f \right) = -\varepsilon_f \nabla \pi_f + \nabla \cdot \boldsymbol{\tau}_f + \mathbf{M}_{pf} + \rho_f \varepsilon_f \mathbf{g}$$

$$\mu_f \rightarrow \mu_{eff} = \mu_f + \mu_t + \mu_s$$

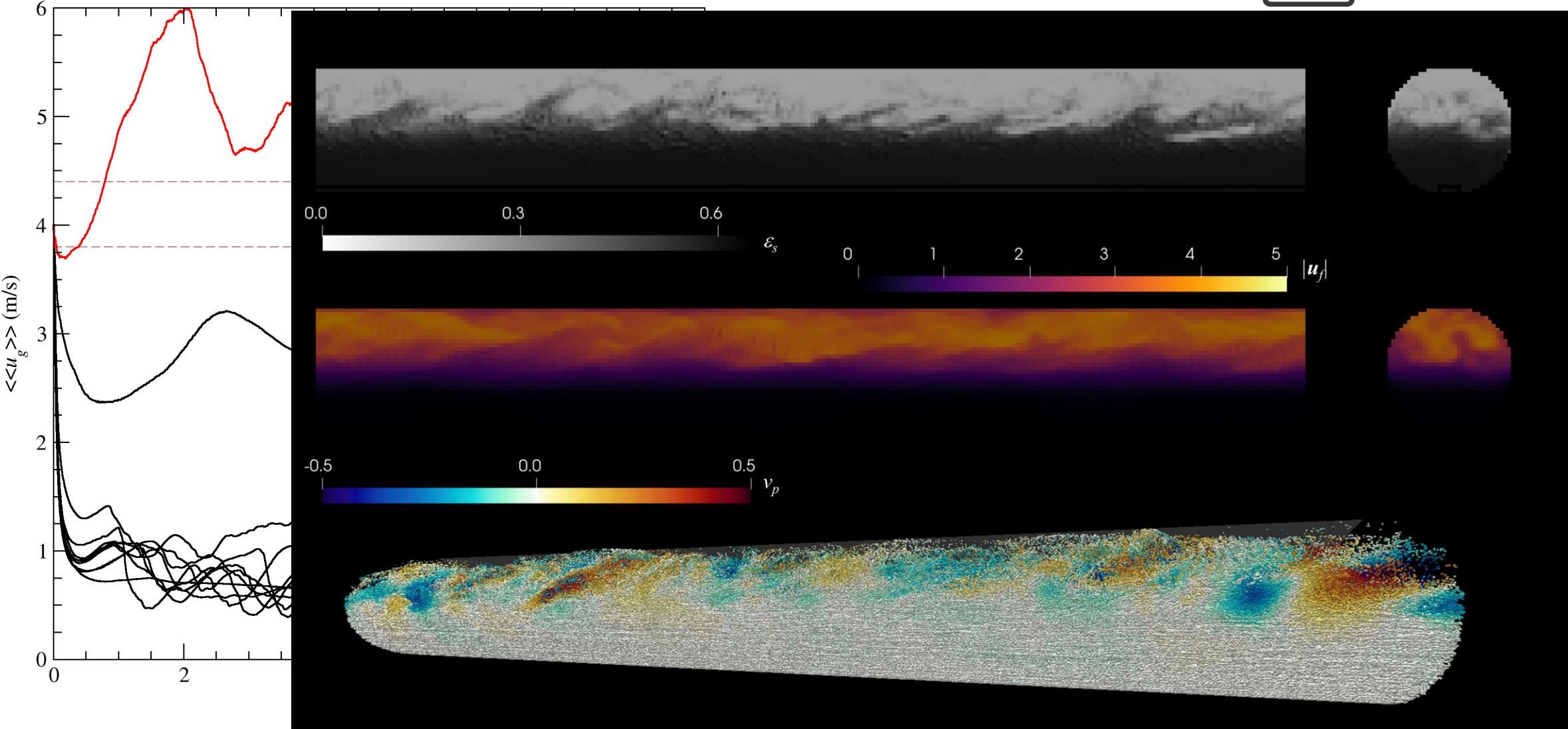
$$\mu_t = \text{Imagorinsky}$$

$$\mu_s = \mu(\varepsilon_f) = \begin{cases} \text{Einstein} \\ \text{Painkman, Gidlikro} \\ \text{Rouse, Krieger \& Dougherty} \\ \text{Cheng \& Law, etc.} \end{cases}$$

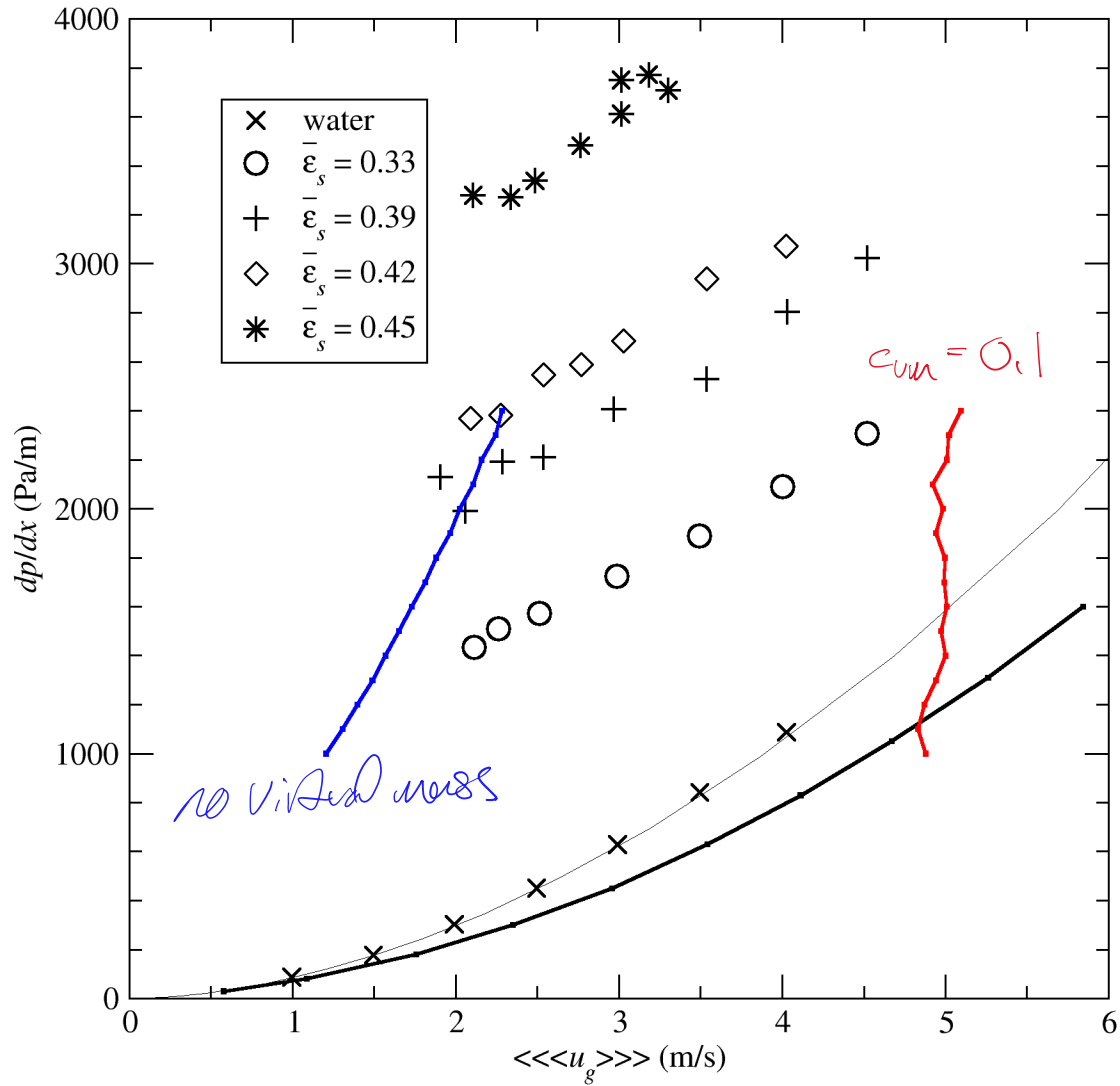
# results



$w/\text{Smagorinsky}$   
 $+$   
 $\mu_s = \text{Chap \& Lau}$   
 $w/\text{limiter}$   
 $+$   
 $\text{vibrated mass } (c_{\text{vib}} = 0.1)$   
 $+(\nabla \cdot \tau_f) H_f$   
 $+$   $\frac{\partial \mathcal{E}}{\partial t} \neq 0$







bound the data but we have a lot of work left to do



# summary & conclusions<sup>(are there any?)</sup>

---

- Developing a multiphase PIC model for slurry flow with MFIX-Exa
- “Switching” from gas-solid to liquid-solid is non-trivial
- Previously neglected void fraction transient term is important to include in the divergence constraint
- Unclear why virtual mass seems to have such a significant influence on quasi-steady pipe flow (need to validate implementation!)
- Still need to add lift force
- Much more work left to do!

# thanks!

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