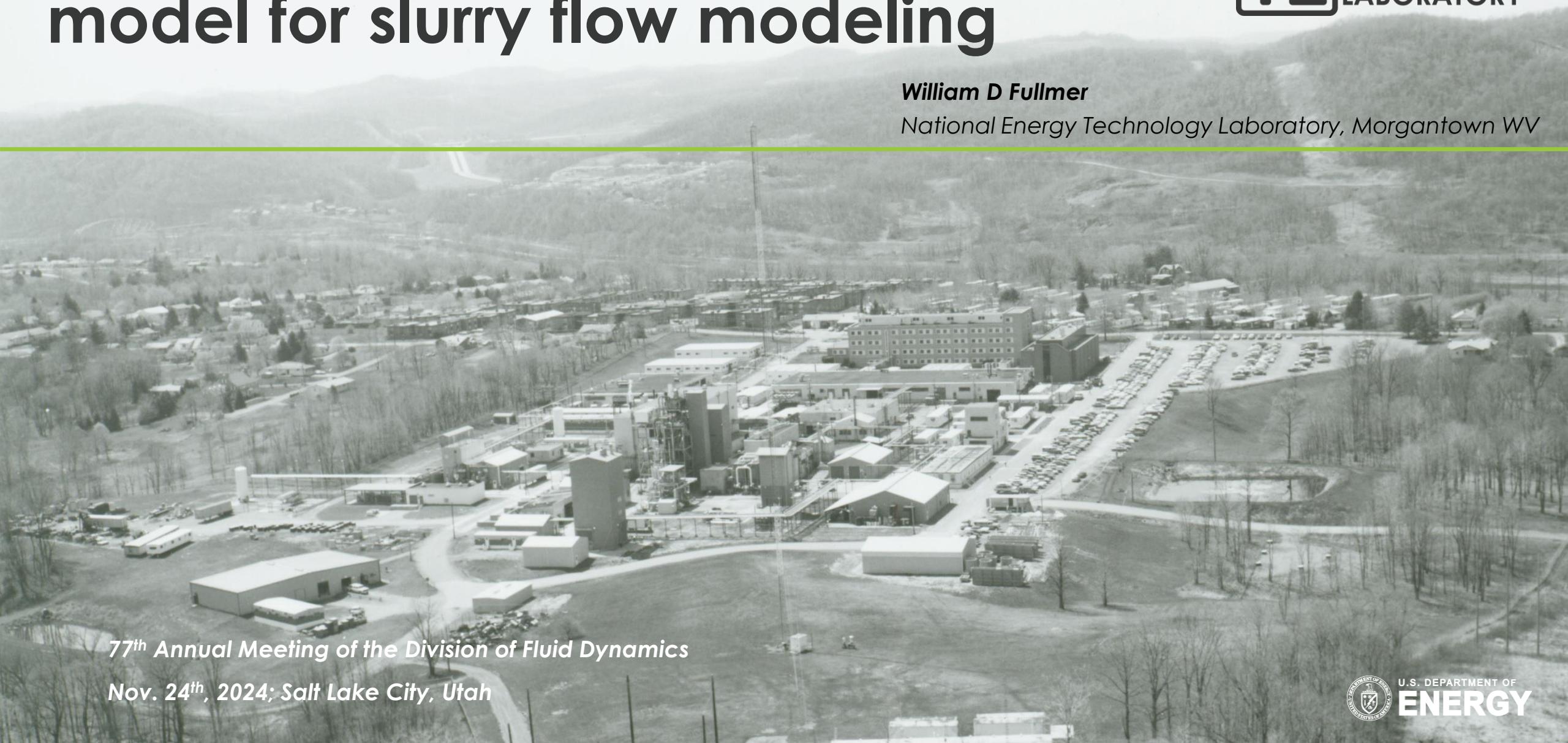


Development of a multiphase PIC model for slurry flow modeling



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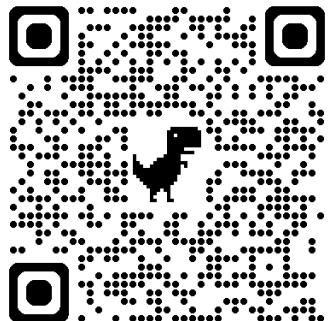


- MFIExa is a massively-parallel, high-performance multiphase flow code
- Targeted physics: reacting gas-solid flows from dilute particle-laden to dense granular
- CFD+ high-fidelity DEM or low-fidelity PIC



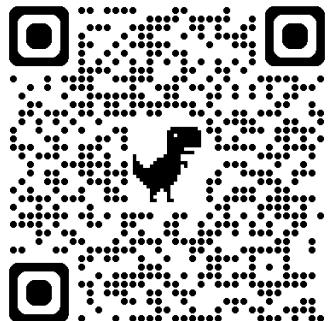
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- Scaled up to 60k GPUs on OLCF's Frontier (65,535 weak scaling, 62078 challenge problem)



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MFIExa overview

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- Challenge problem: NETL's 50kW CLR



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problem of interest

https://www.engineeringtoolbox.com/slurry-transport-velocity-d_236.html



- Liquid fluid and solid particle slurry flow (coal liquefaction driving interest)
- Plethora of data for horizontal pipe flow which is very industrially relevant
- Start with pressure drop (head loss) vs velocity relationship
- Reference: Gillies & Shook (2000) "Modelling High Concentration Settling Slurry Flows" *Canadian J. of Chem. Eng.*, **78**, 709-716.

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Fluid: water

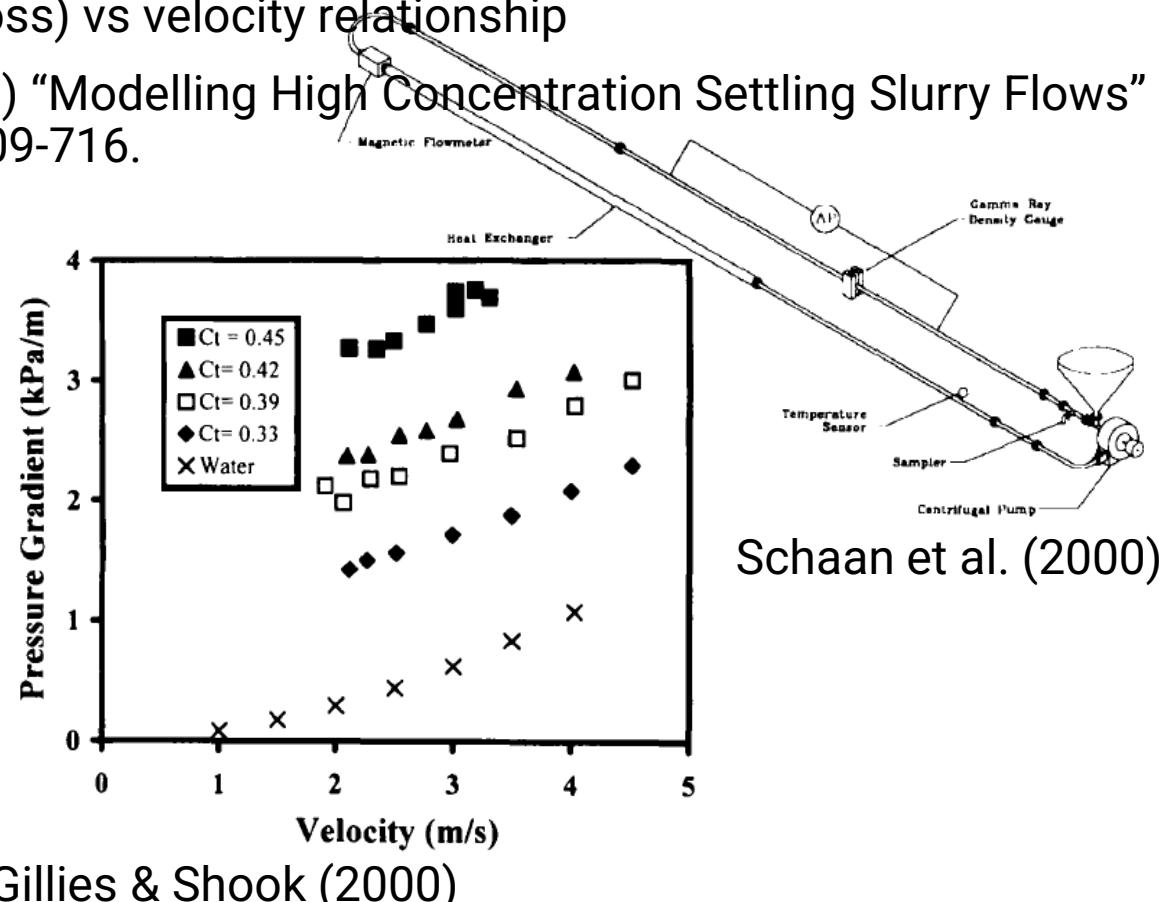
- $T = 25^\circ\text{C}$
- assume $\rho_f = 1000 \text{ kg/m}^3$
- assume $\mu_f = 0.001 \text{ Pa-s}$

Particles: sand

- $d_p = 420 \text{ micron}$
- $\rho_p = 2655 \text{ kg/m}^3$
- assume monodispersed

Pipe diameter

- $D = 10.5, 26.4 \text{ and } 49.6 \text{ cm}$



previous work

Table 1

Previous numerical studies on slurry pipe flows.

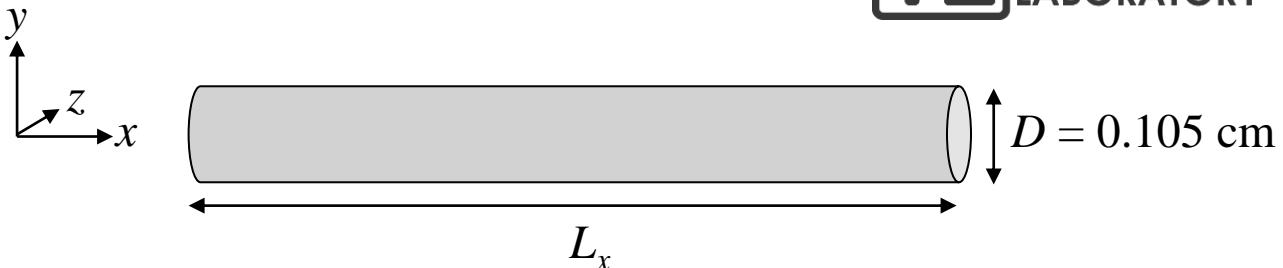
Reference	Approach	Flow regime	Fluid	Particles	ρ_p [kg/m ³]	d_p [mm]	D [mm]	V_m [m/s]	$\langle \alpha_s \rangle_A$ [-]
Capecelatro and Desjardins [17]	LES + DPM	SB, HS	Water	Sand	2650	0.165	51.5	0.83, 1.6	0.084
Arolla and Desjardins [18] ^a	LES + DPM	SB, HS	Water	Sand	2650	0.280	69	-	-
Uzi and Levy [12]	RANS+DPM	MB, HS	Brine	NaCl	2150	1.0–4.0	50–100	0.6–4.0	0.05–0.30
Hernández et al. [21]	(KTGF)-TFM	HS, PS	Water	Sand	≈ 2381	0.030–0.110	22.1	1.0–3.0	0.0025–0.20
Bossio et al. [22]	(KTGF)-TFM	MB, HS, PS	Laterite	Sand	2381	0.110	26.84	0.671–4.697	0.036–0.083
Ekambara et al. [23]	KTGF-TFM	BF, FS	Water	Sand, glass beads	2470, 2650	0.090–0.500	50–500	1.5–5.5	0.08–0.45
Antaya et al. [24]	KTGF-TFM(MFM)	FS	Water	Sand	2650	0.100–0.370	51.5, 150	2.0–6.0	0.20–0.43
Hashemi et al. [25]	KTGF-TFM	-	Water	Sand	2650	0.370	265	4.0, 6.0	0.20–0.40
Kaushal et al. [26]	KTGF-TFM, MM	FS	Water	Glass beads	2470	0.125	54.9	1.0–5.0	0.30–0.50
Gopaliya and Kaushal [27]	KTGF-TFM	FS	Water	Sand	2650	0.18–2.4	53.2	1.8, 3.1	0.15–0.45
Gopaliya and Kaushal [28]	KTGF-TFM	FS	Water	Sand	2650	0.165–0.55	263	3.5–4.7	0.10–0.34
Kumar et al. [29]	KTGF-TFM	FS	Water	Iron ore	4350	0.012	105	1.35–5.11	0.0263–0.31
Singh et al. [30]	KTGF-TFM	FS	Water	Coal	1560	0.059–0.206	50–150	2–5	0.30–0.60
Roco and Balakrishnam [33]	TFM	BF, FS	Water	Sand, glass beads	-	0.165–0.580	40, 51.5	1.05–4.17	0.07–0.189
Messa et al. [35]	TFM	FS	Water	Sand, glass beads	2440, 2650	0.090–0.370	53–150	1.33–8.0	0.11–0.40
Messa et al. [36]	TFM	FS	Water	Sand, glass beads	2440, 2650	0.090–0.520	50.7–150	1.33–8.0	0.09–0.40
Messa and Malavasi [37]	TFM	MB, FS	Water	sand	≈ 2650	0.090–0.520	50.7–150	1.33–6.0	0.09–0.43
Messa and Malavasi [38]	TFM	FS	Water	Sand	≈ 2650	0.090–0.640	50–200	2.0–9.0	0.07–0.41
Chen et al. [39]	KTGF-MFM	HS, PS	Water	Coal	1465	0.065 + 0.345	25–50	0.2–5	0.38–0.538
Li et al. [40]	KTGF-MFM	MB, HS, PS	Water	Glass beads	2470	0.125 + 0.440	54.9	2.0–5.0	0.20–0.50
Li et al. [41]	KTGF-MFM	MB, HS, PS	Water	Glass beads	2470	0.125 + 0.440	54.9	2.0–4.0	0.20–0.40
Ling et al. [42]	MM	BF, FS	Water	Silica/zircon sand	2380, 4223	0.11	22.1	1.0–3.0	0.10, 0.20
Lin and Ebadian [43]	MM	BF, FS	Water	Silica/zircon sand	2380, 4223	0.11	22.1	1.0–3.0	0.10, 0.20
Silva et al. [44]	MM	SB, MB, FS	Saline water	Glass beads	2500	0.10–0.60	100	1.0–3.0	0.008–0.11

Messa & Matoušek (2020)

model setup

Model overview:

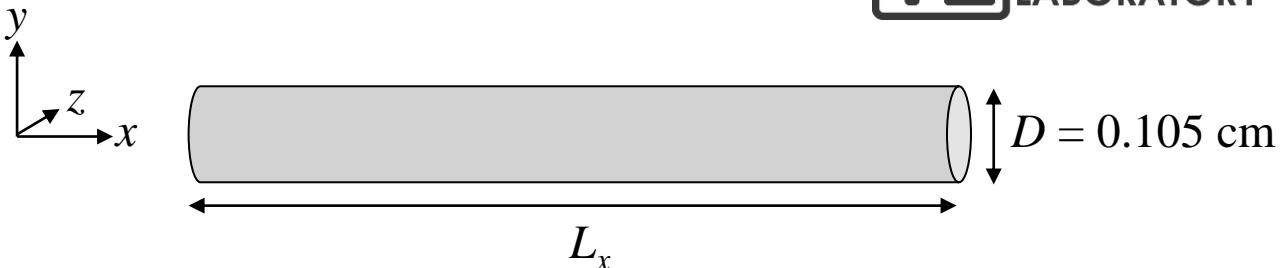
- cylindrical EB: $L_x = 8D$
- periodic in x with enforced Δp_f
- no slip wall EB
- uniform grid
- boundary adjacent cut-cells
- $dx = D/16 \rightarrow W_{st} = 72.9 \rightarrow$ too coarse for LES
- $dx = D/32 \rightarrow W_{st} = 9.11 \rightarrow$ this work
- $dx = D/64 \rightarrow W_{st} = 1.14 \rightarrow$ why use PIC?



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Nomenclature:

- “bulk” fluid velocity

$$\langle\langle u_f \rangle\rangle = \frac{\langle \varepsilon_f u_f \rangle}{\langle \varepsilon_f \rangle} = \frac{\frac{1}{\pi R^2 L_x} \int \varepsilon_f u_f dV}{\frac{1}{\pi R^2 L_x} \int \varepsilon_f dV}$$

$$\langle\langle\langle u_f \rangle\rangle\rangle = \frac{1}{\delta t} \int \langle\langle u_f \rangle\rangle dt$$

“base model”

Lagrangian: discrete particles as *parcels*

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{u}_i \quad \text{and} \quad m_i \frac{d\mathbf{u}_i}{dt} = m_i \mathbf{g} + \frac{V_p}{\varepsilon_p} \nabla \tau_p + \mathbf{F}_{fi}$$

Eulerian: continuous fluid

$$\nabla \cdot \varepsilon_f \mathbf{u}_f = 0 \quad \text{and} \quad \rho_f \left(\frac{\partial \varepsilon_f \mathbf{u}_f}{\partial t} + \nabla \cdot \varepsilon_f \mathbf{u}_f \mathbf{u}_f \right) = -\varepsilon_f \nabla \pi_f + \nabla \cdot \boldsymbol{\tau}_f + \mathbf{M}_{pf} + \rho_f \varepsilon_f \mathbf{g}$$

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$$M_f \rightarrow M_{eff} = M_f + M_t + M_s$$

governing equations



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$$\mu_f \rightarrow \mu_{eff} = \mu_f + \mu_t + \mu_s \quad \mu_t = \text{Dissipativity}$$



governing equations

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$$\mu_f \rightarrow \mu_{eff} = \mu_f + \mu_t + \mu_s$$

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$$\mu_s = \mu_s(\varepsilon_f) = \begin{cases} \text{Einstein} \\ \text{Prandtl, Gidko} \\ \text{Reece, Krieger \& Doherty} \\ \text{Cheng \& Law, etc.} \end{cases}$$

governing equations



$$\mathbf{F}_{f_i} = (-\nabla \pi_f) + \mathbf{F}_D$$

$F_D \propto \alpha^2 / C$ (Gidaspow)

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$$\frac{d\mathbf{x}_i}{dt} = \mathbf{u}_i \quad \text{and} \quad m_i \frac{d\mathbf{u}_i}{dt} = m_i \mathbf{g} + \frac{V_p}{\varepsilon_p} \nabla \tau_p + \mathbf{F}_{fi}$$

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governing equations

$$\mathbf{F}_{fi} = (-\nabla \pi_f) + \mathbf{F}_D + \mathbf{F}_{vm}$$

$F_D \propto \mathbf{u}_f^2, C_D = \text{Gidaspow}$ $\mathbf{F}_{vm} \propto \frac{\partial \mathbf{u}_i}{\partial t} - \frac{\mathbf{D}_{uf}}{\mathcal{D}_e}$ $C_{vm} = 0.5$

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governing equations

$$\mathbf{F}_{f_i} = (-\nabla \pi_f) + \mathbf{F}_D + \mathbf{F}_{vm} + \mathbf{F}_L \rightarrow \propto \alpha \mathbf{u}_R \times \nabla x u_f$$

$F_D \propto \alpha R^2 / C_S$ (Gidaspow)

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governing equations

$$\begin{aligned}
 \mathbf{F}_{fi} &= (-\nabla \pi_f) \hat{\mathbf{t}} \\
 &+ \mathbf{F}_D + \mathbf{F}_{vm} \quad \rightarrow \quad \mathbf{F}_L \quad \rightarrow \quad \mathbf{F}_{TD} \\
 &\rightarrow \alpha k \nabla \phi \\
 \mathbf{F}_D &\propto \alpha k^2, \quad C_g = \text{Grashof} \\
 \mathbf{F}_{vm} &\propto \frac{\partial \mathbf{u}_f}{\partial t} - \frac{\nabla \pi_f}{\nabla \epsilon} \quad | \quad C_{vm} = 0.5
 \end{aligned}$$

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$$\begin{aligned}
 \mathbf{F}_{fi} &= (-\nabla \pi_f) \hat{\mathbf{h}} \\
 &+ \mathbf{F}_D + \mathbf{F}_{vm} \\
 &\rightarrow \mathbf{F}_L \rightarrow \alpha \alpha_R \times \nabla x u_f \\
 &\rightarrow \alpha k \nabla \xi \\
 \mathbf{F}_D \propto \alpha_R^2, C_g = \text{Gidaspow} & \mathbf{F}_{vm} \propto \frac{\partial \mathbf{u}_i}{\partial t} - \frac{\nabla u_f}{\nabla \epsilon} & | C_{vm} = 0.5 \\
 & \mathcal{N}_{\delta t} \approx 10, \\
 & \text{probably not.} \\
 & \mathcal{N}_{\delta t} > 100,000 \dots \\
 & \text{may be}
 \end{aligned}$$

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governing equations

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 \mathbf{F}_{fi} &= (-\nabla \pi_f) \hat{\mathbf{h}} \\
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 &\rightarrow \mathbf{F}_L \rightarrow \alpha \alpha_R \times \nabla x u_f \\
 &\rightarrow \alpha k \nabla \xi \\
 \text{For } \alpha_R^2, C_s = \text{Gibson} \\
 \mathbf{F}_{vm} &\propto \frac{\partial \mathbf{u}_i}{\partial t} - \frac{\nabla u_f}{\nabla t} \quad | C_{vm} = 0.5 \\
 &\text{and } \frac{\partial \mathbf{u}_i}{\partial t} \approx 10, \\
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governing equations

$$\mathbf{F}_{fi} = (-\nabla \pi_f)_{fi} + (\nabla \cdot \mathbf{u}_f)_{fi} + \mathbf{F}_D + \mathbf{F}_{vm} + \mathbf{F}_L + \mathbf{F}_{TD}$$

$\mathbf{F}_D \propto \alpha R^2 / C_s$ (Grashof number) $\mathbf{F}_{vm} \propto \frac{\partial \mathbf{u}_i}{\partial t} - \frac{\nabla \pi_f}{\nabla \epsilon}$ $C_{vm} = 0.5$
 $\mathbf{F}_L \propto \alpha \alpha_R \times \nabla x u_f$
 $\mathbf{F}_{TD} \propto \alpha^2$
 $\mathcal{N}_{8t} \approx 10,$
 probably not.

$\rightarrow \alpha \propto R^2 \nabla \epsilon$
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governing equations

$$\mathbf{F}_{fi} = (-\nabla \pi_f)_{\mathbf{f}} + (\nabla \cdot \mathbf{u}_f)_{\mathbf{f}} + \mathbf{F}_D + \mathbf{F}_{vm} + \mathbf{F}_L + \mathbf{F}_{TD}$$

$\mathbf{F}_D \propto \alpha^2, C_f = \text{Gidaspow}$
 $\mathbf{F}_{vm} \propto \frac{\partial \mathbf{u}_i}{\partial t} - \frac{\nabla \pi_f}{\Delta t}, C_{vm} = 0.5$
 $\mathbf{F}_L \propto \alpha \alpha_R \times \Delta x \mathbf{u}_f$
 $\mathbf{F}_{TD} \propto \alpha^2$
 $\rightarrow \alpha \propto k \Delta \xi$
 $\mathcal{N}_{\Delta t} \approx 10,$
 probably not.

Lagrangian: discrete particles as parcels

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{u}_i \quad \text{and} \quad m_i \frac{d\mathbf{u}_i}{dt} = m_i \mathbf{g} + \frac{V_p}{\varepsilon_p} \nabla \tau_p + \mathbf{F}_{fi}$$

transfer

Eulerian: continuous fluid

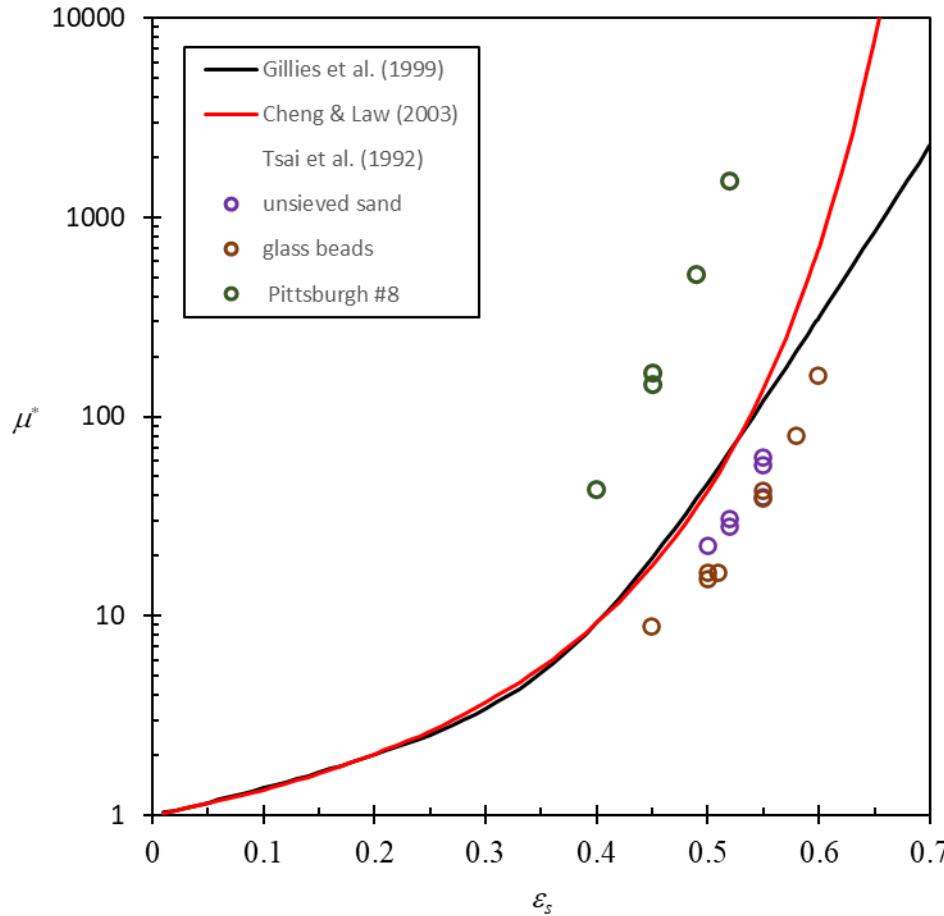
$$\nabla \cdot \varepsilon_f \mathbf{u}_f = 0 \quad \text{and} \quad \rho_f \left(\frac{\partial \varepsilon_f \mathbf{u}_f}{\partial t} + \nabla \cdot \varepsilon_f \mathbf{u}_f \mathbf{u}_f \right) = -\varepsilon_f \nabla \pi_f + \nabla \cdot \boldsymbol{\tau}_f + \mathbf{M}_{pf} + \rho_f \varepsilon_f \mathbf{g}$$

$$\mu_f \rightarrow \mu_{eff} = \mu_f + \mu_t + \mu_s$$

$$\mu_t = \text{Smagorinsky}$$

$$\mu_s = \mu_s(\varepsilon_f) = \begin{cases} \text{Einstein} \\ \text{Prandtl, Gidaspow} \\ \text{Rouse, Krieger \& Doehring} \\ \text{Cheung \& Lau, etc.} \end{cases}$$

suspension viscosity



Suspension viscosity model:

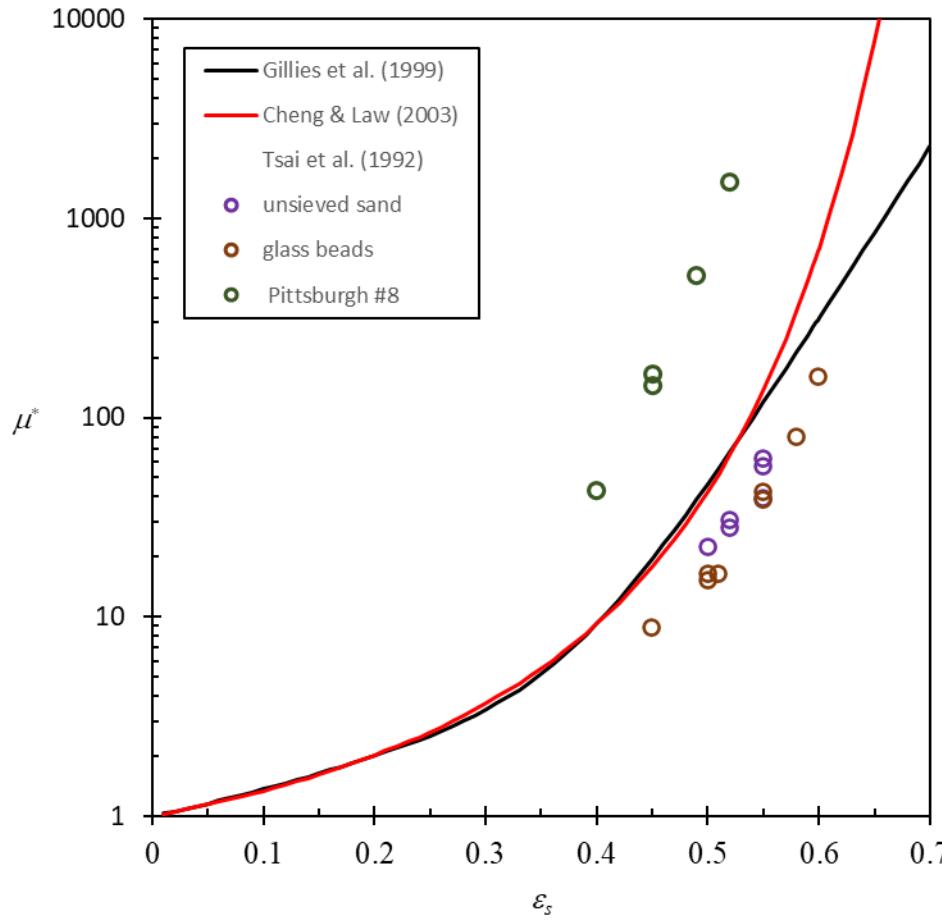
- Gillies et al. (1999)

$$\mu^* = 1 + 2.5\varepsilon_f + 10\varepsilon_f^2 + 0.0019e^{20\varepsilon_f}$$

- Cheng & Law (2003)

$$\mu^* = \exp \left[\frac{2.5}{\beta} \left(\frac{1}{(1 - \varepsilon_f)^\beta} - 1 \right) \right]$$

suspension viscosity



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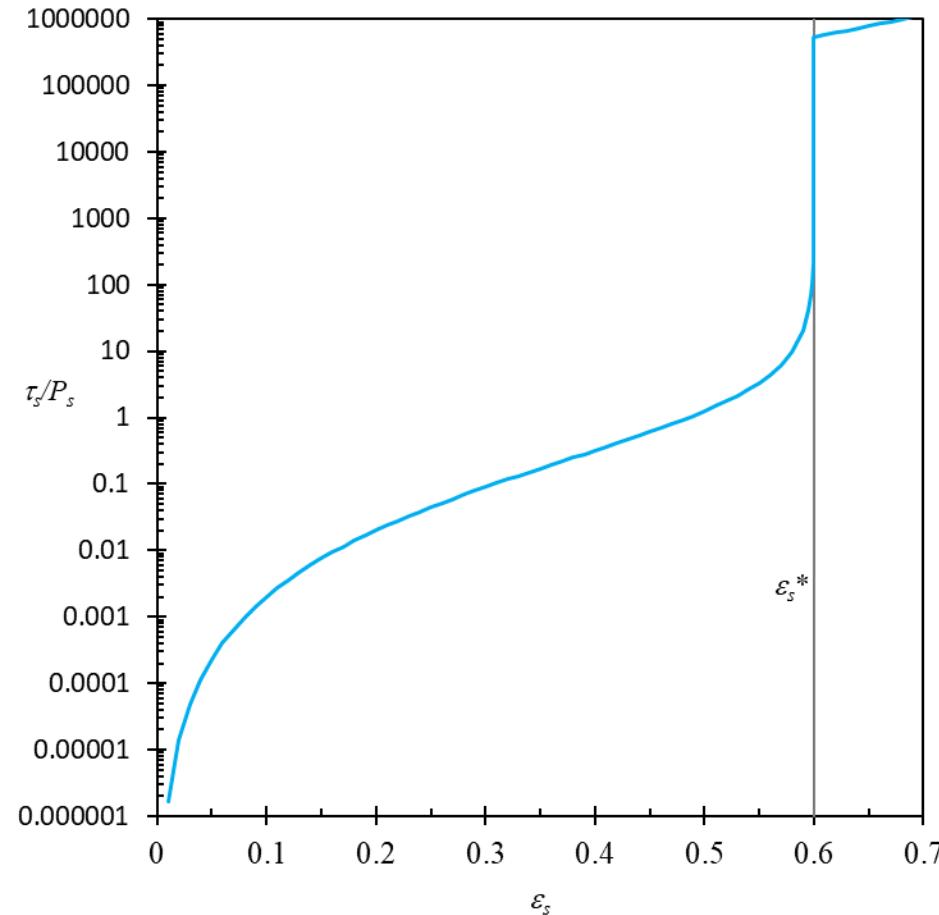
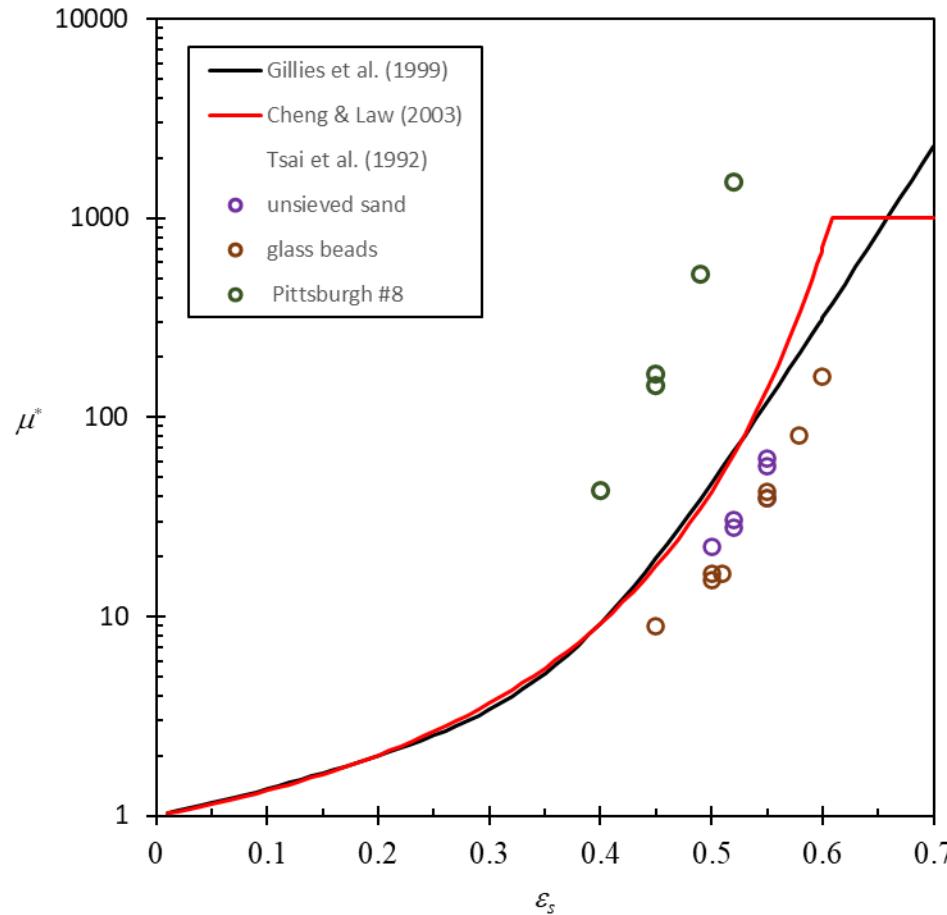
Solids stress model:

- Snider (2001), Harris & Crighton (1994)

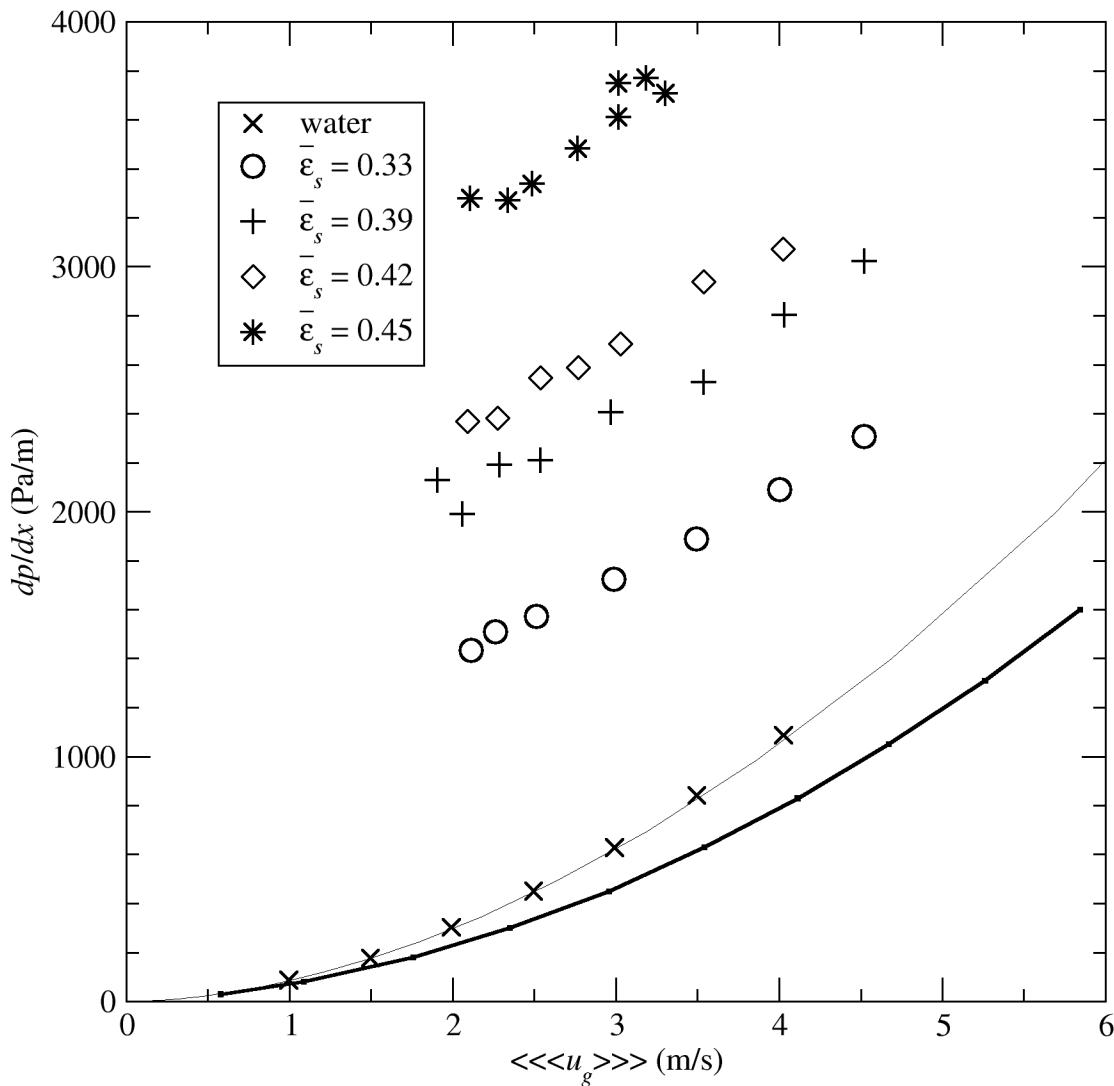
$$\tau_s = \frac{P_s \epsilon_s^\beta}{\max [(\epsilon_s^* - \epsilon_s), \epsilon (1 - \epsilon_s)]}$$

$$P_s = 100, \beta = 3, \epsilon = 10^{-6}$$

suspension viscosity



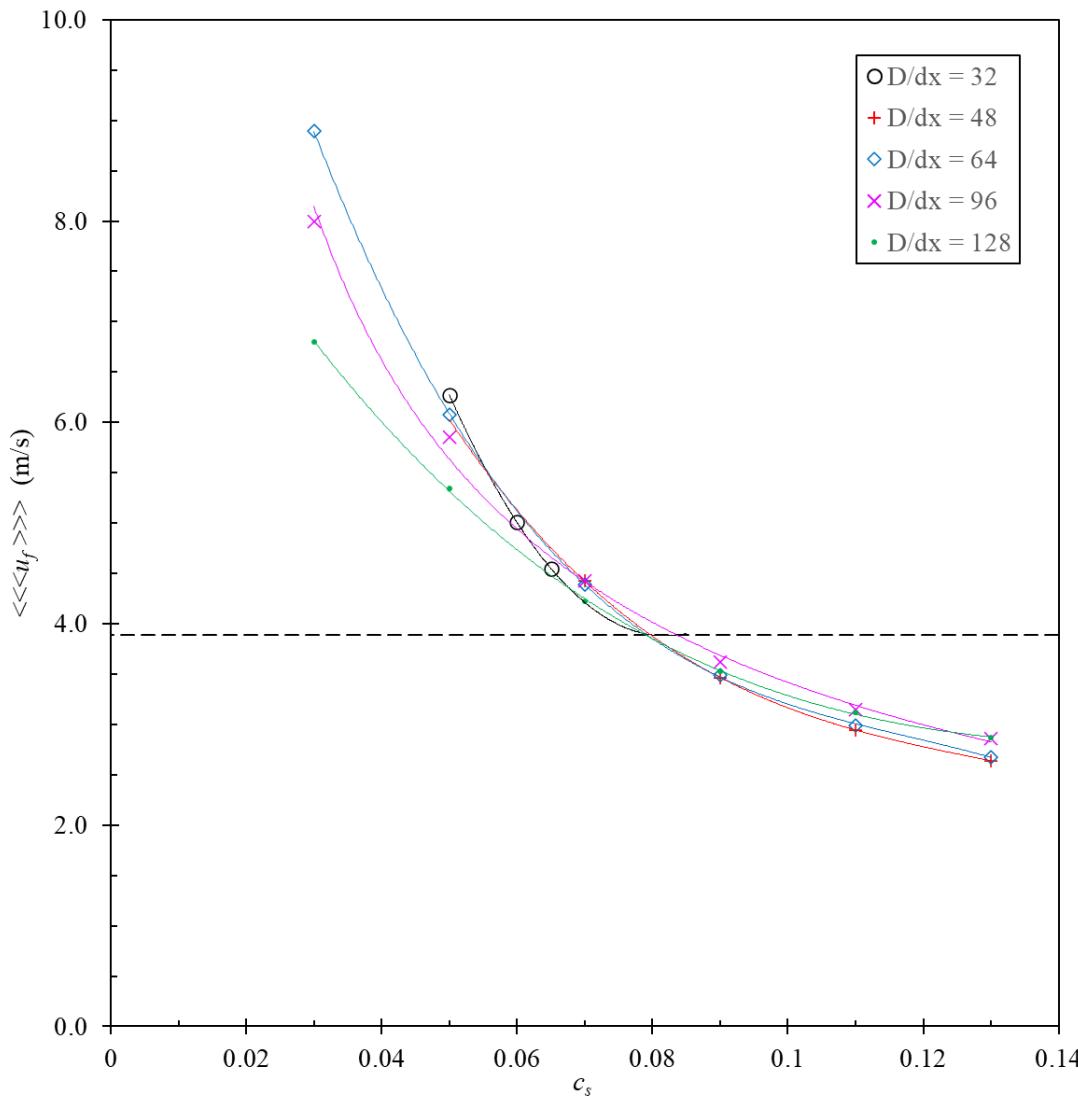
results



Darcy's law (Superpipe fit)

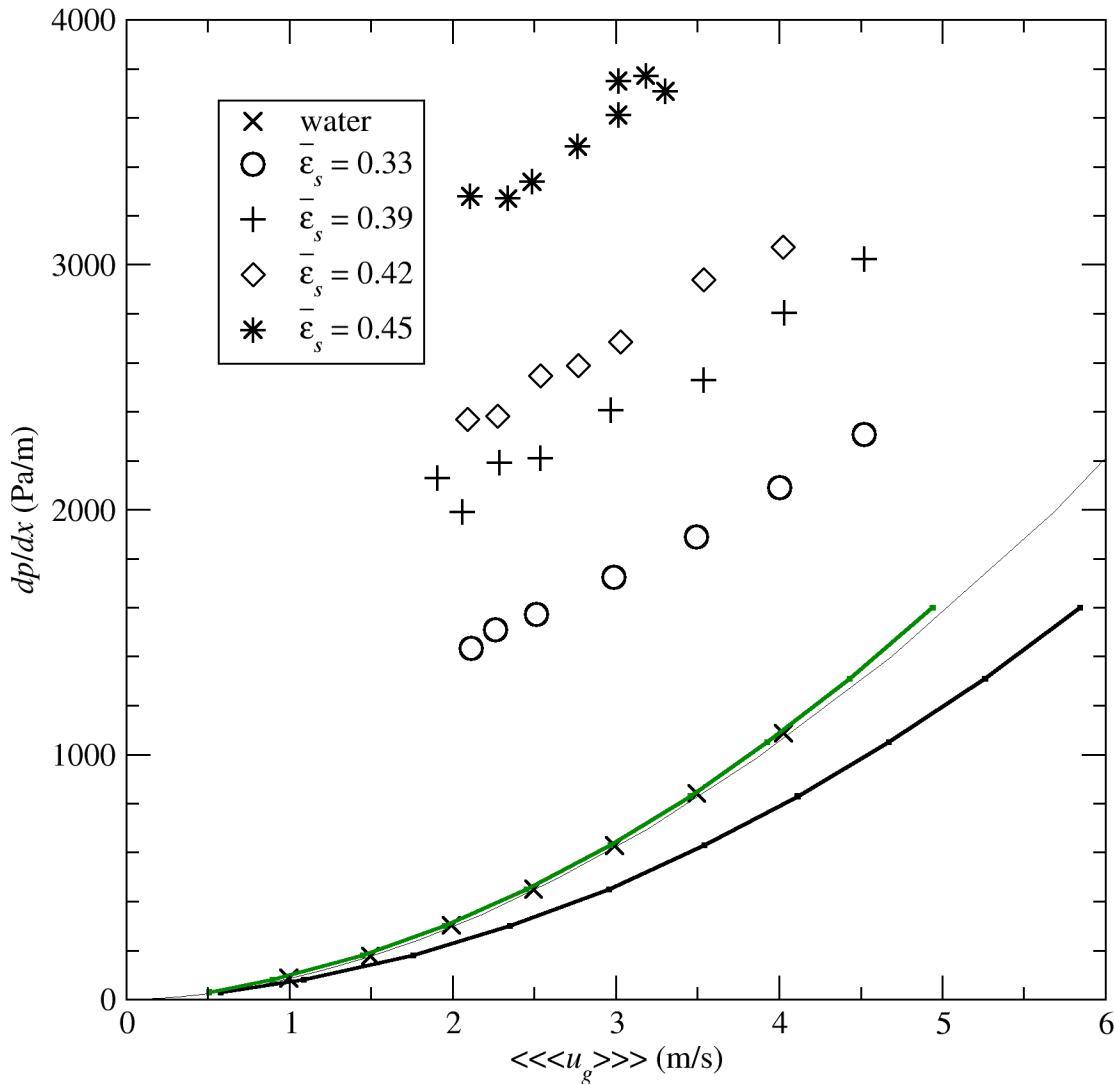
$$c_s = 0.065$$

Smagorinsky constant



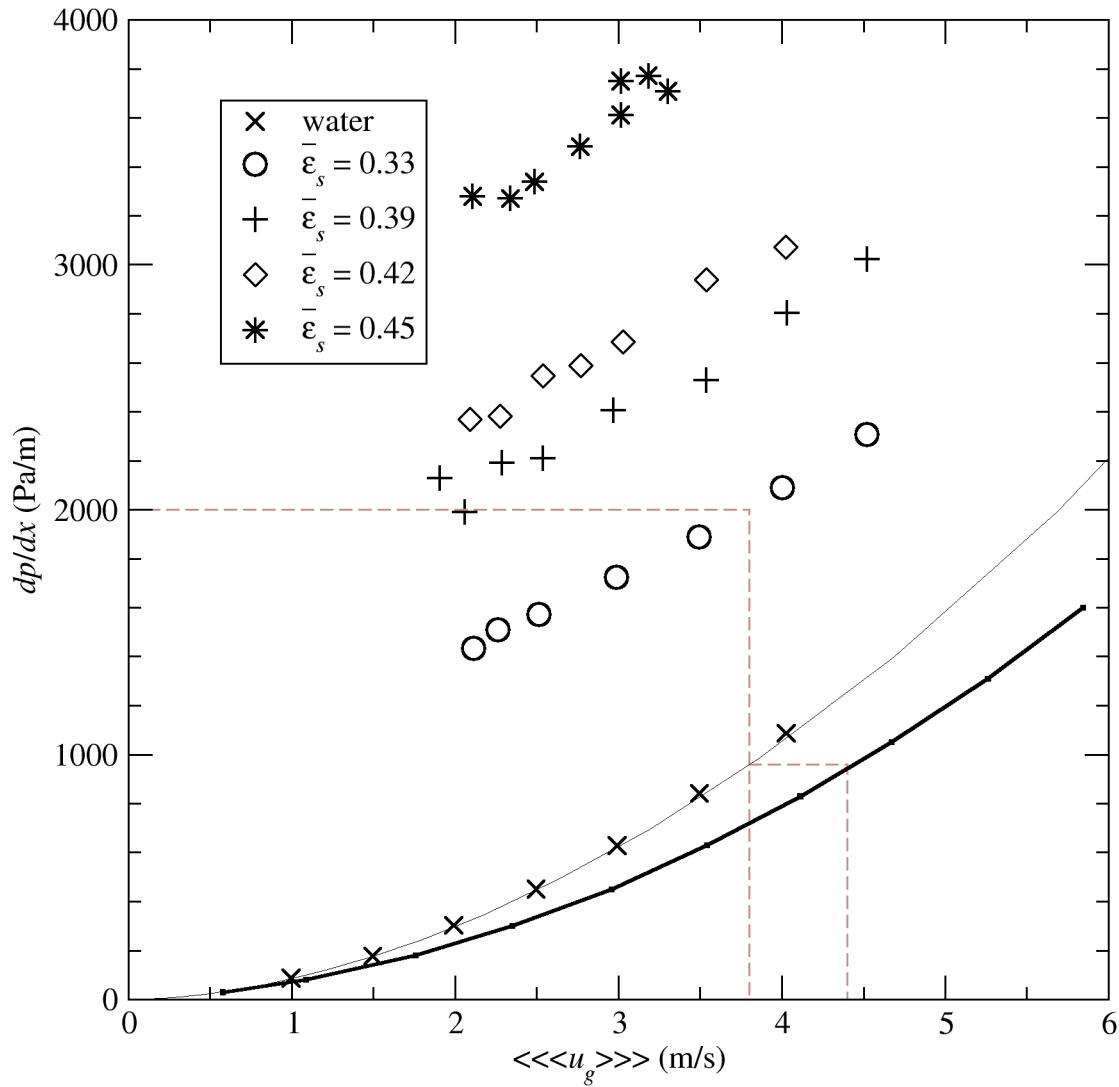
For a wide range of resolutions, we find that a constant of $c_s = 0.08$ is ideal, but this is too diffusive for this coarse grid

results



$dx = D/64$ and $c_s = 0.08$

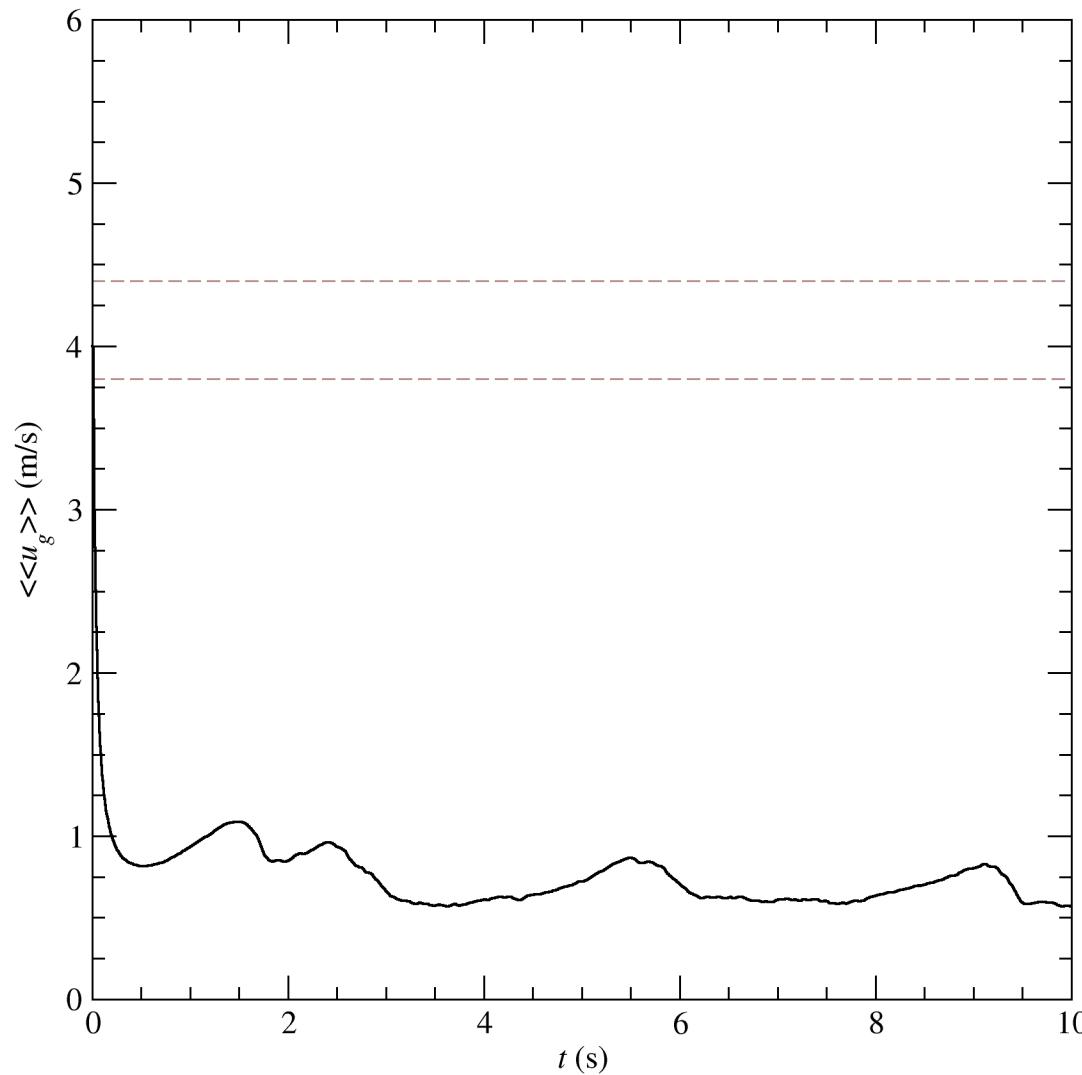
results



set: $dp/dx = 2000$ Pa/s

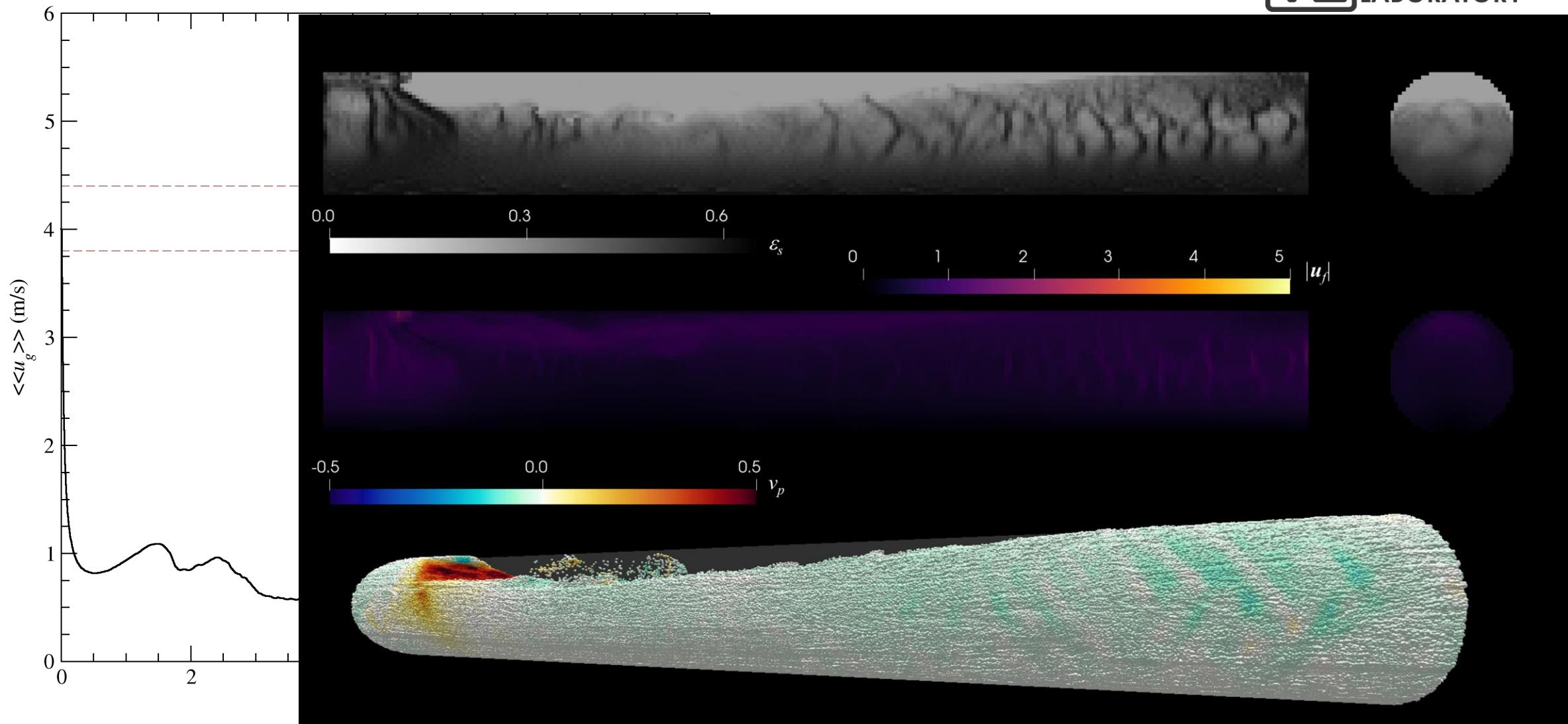
target bulk velocity: $3.8 \sim 5$ m/s

results

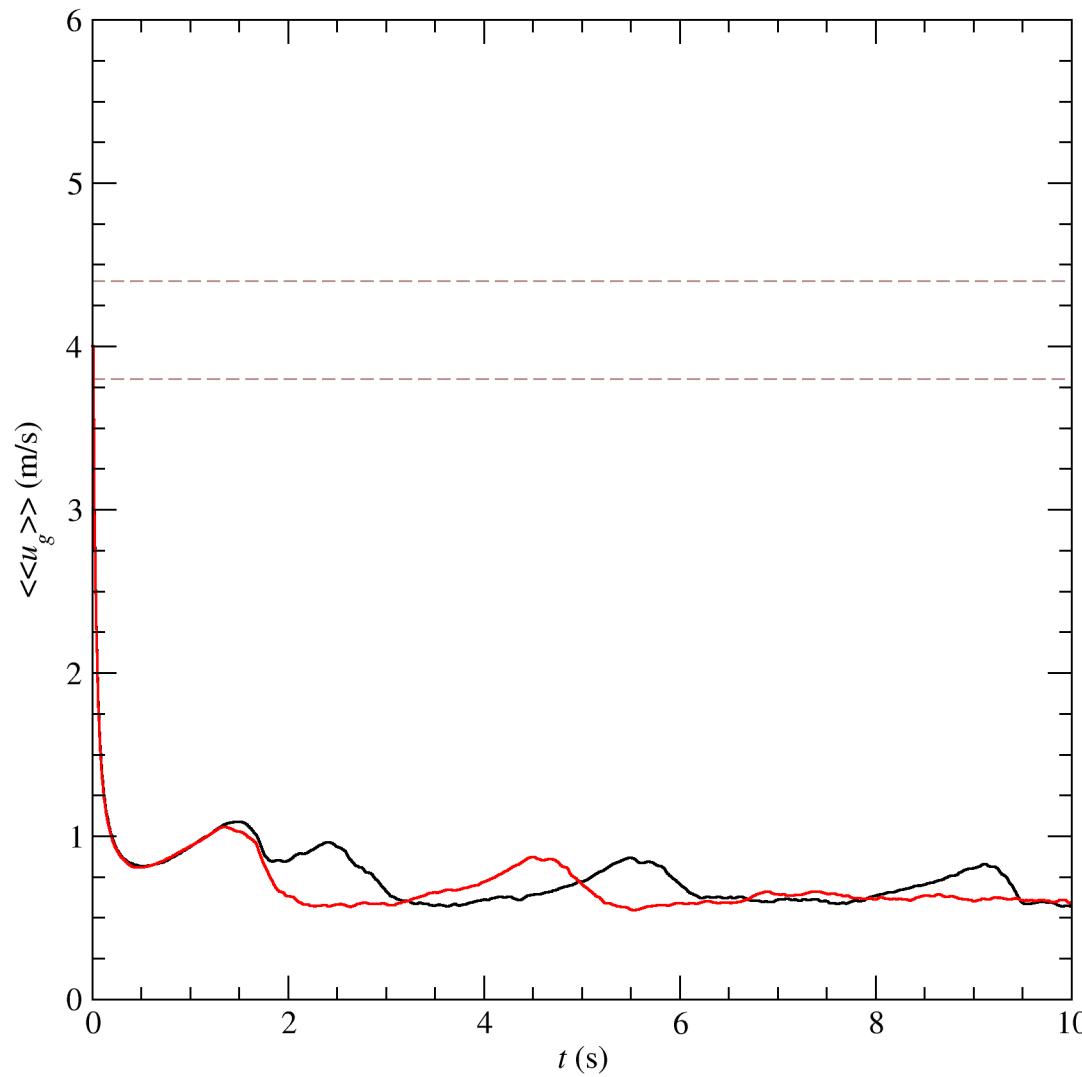


"base model"

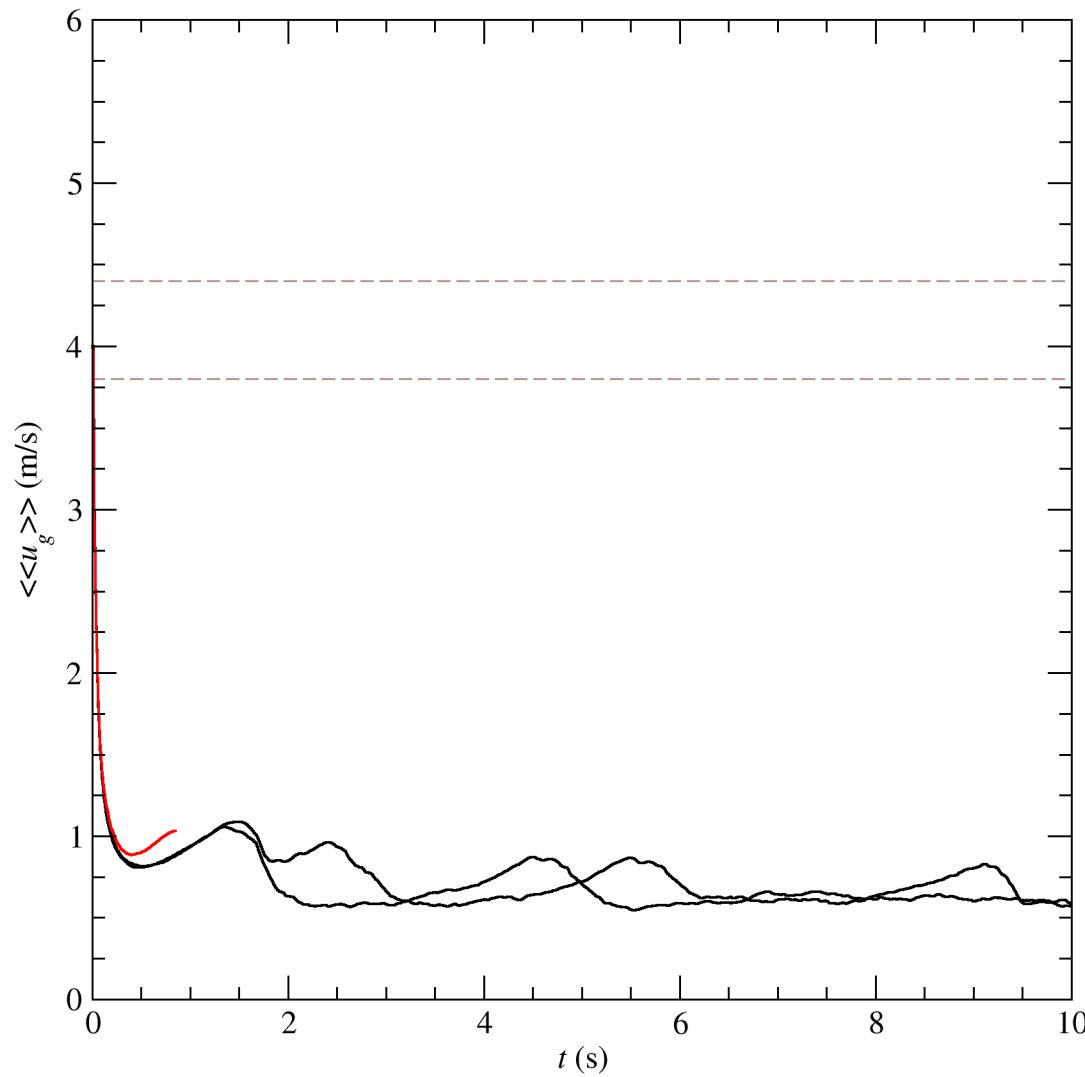
results



results

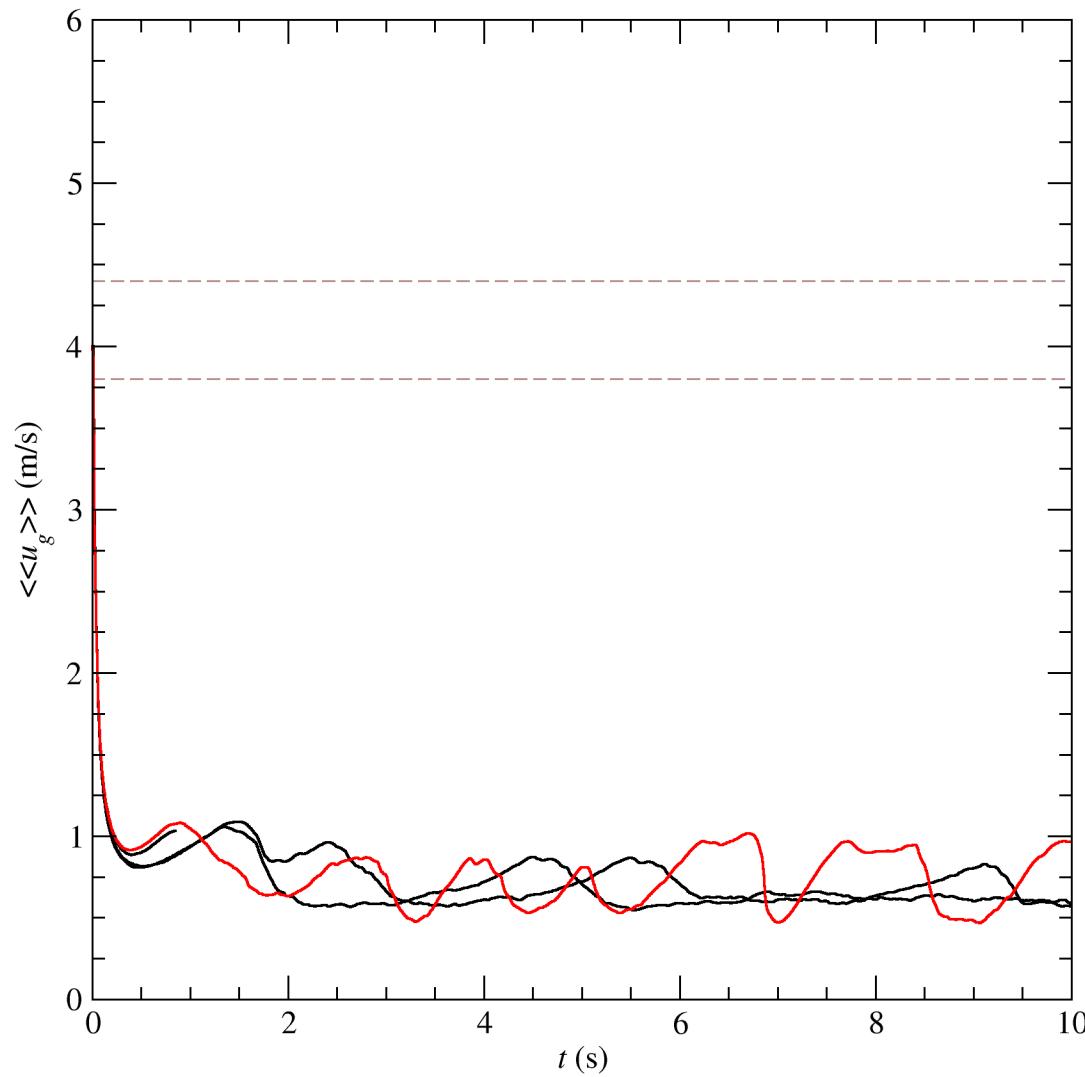


results



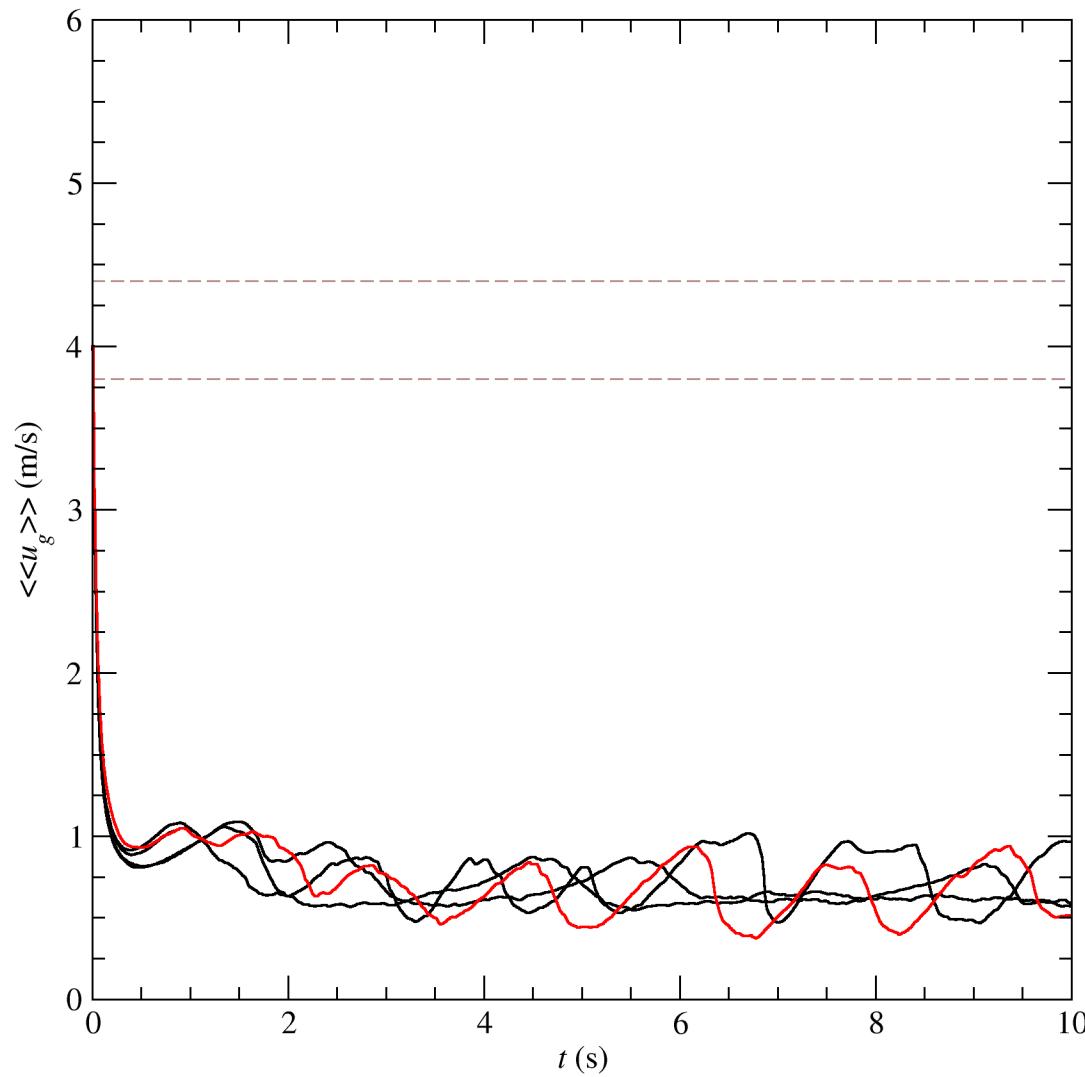
w/ Smoothing
+
 μ_s = Chay & Law

results



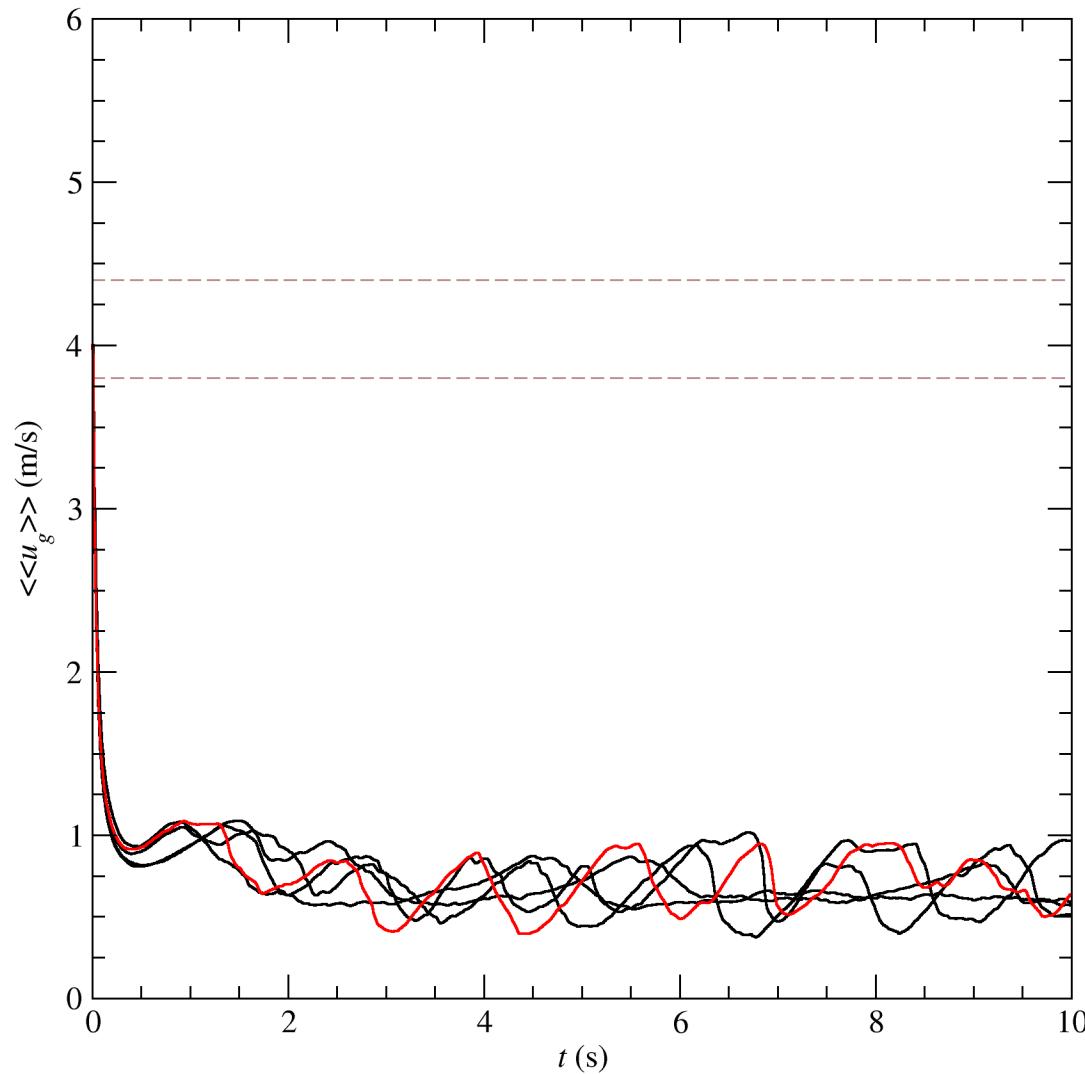
w/ Smoothing
+
 μ_s = Chay & Lau
w/ limiter

results



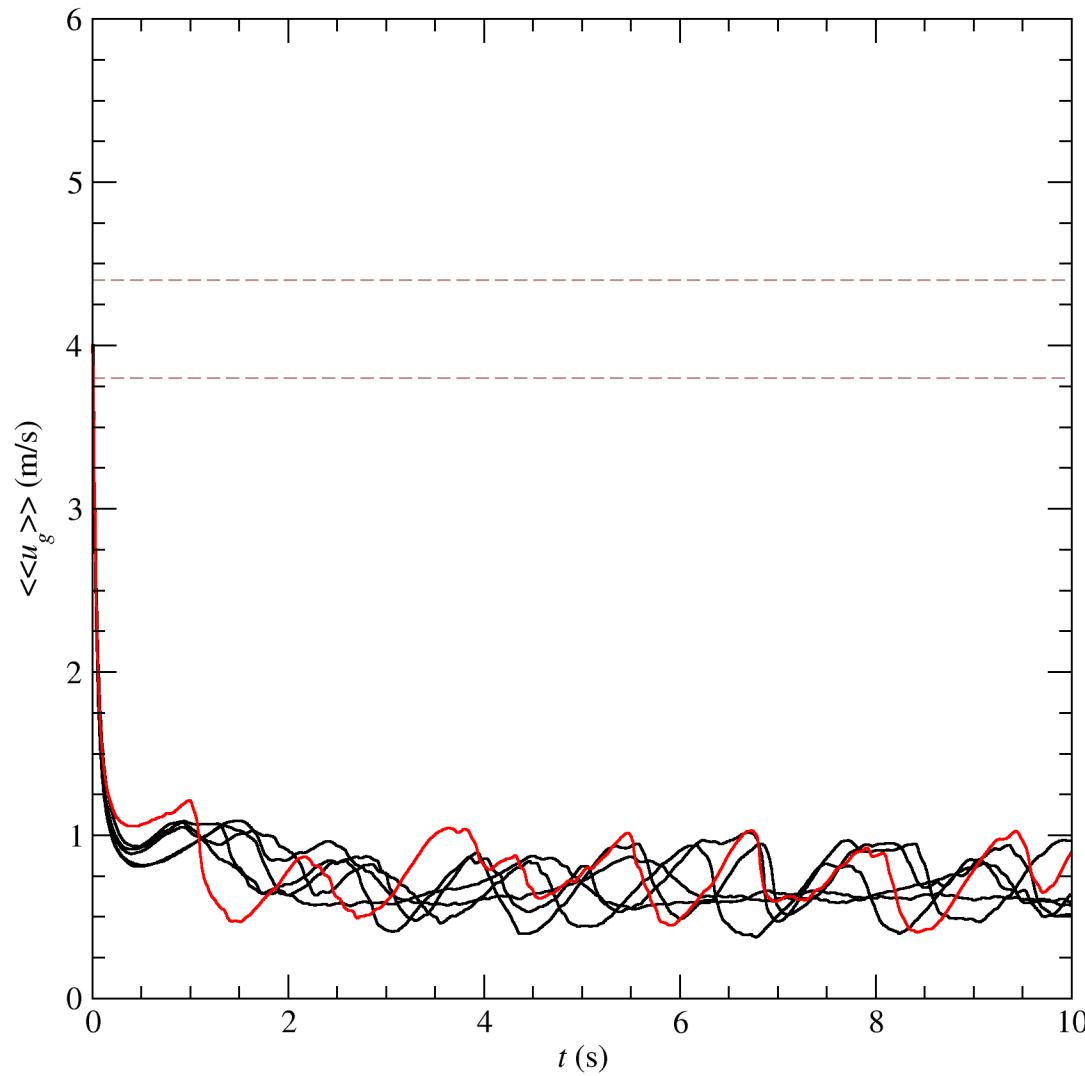
w/ Smagorinsky
+
 μ_S = Chorin & Lai
w/ Lohner
+
DNS drag/law

results



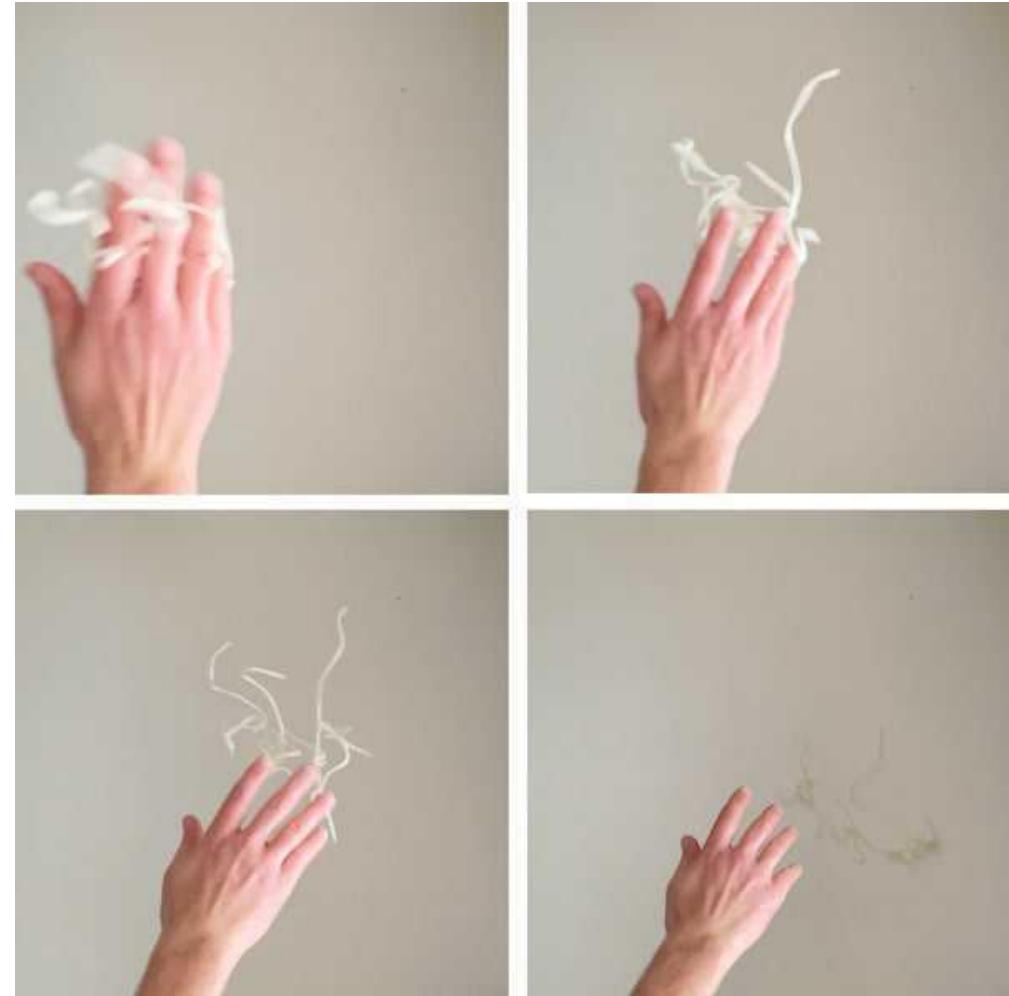
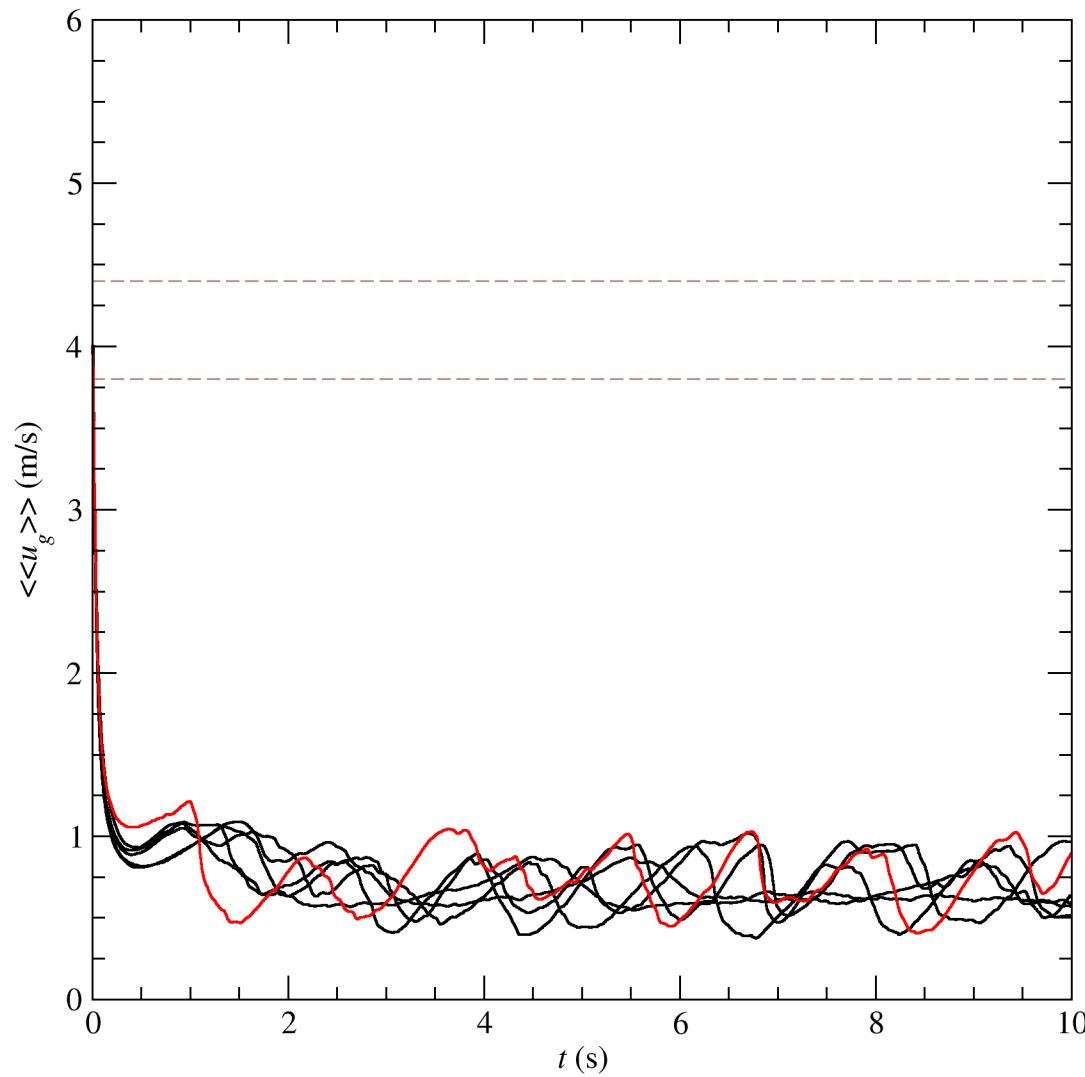
w/ Smagorinsky
+
 $\mu_S = Chou \& Lau$
w/ limiter
+
~~DNS drag law~~
($D - t_f$) $\frac{df}{dt}$

results

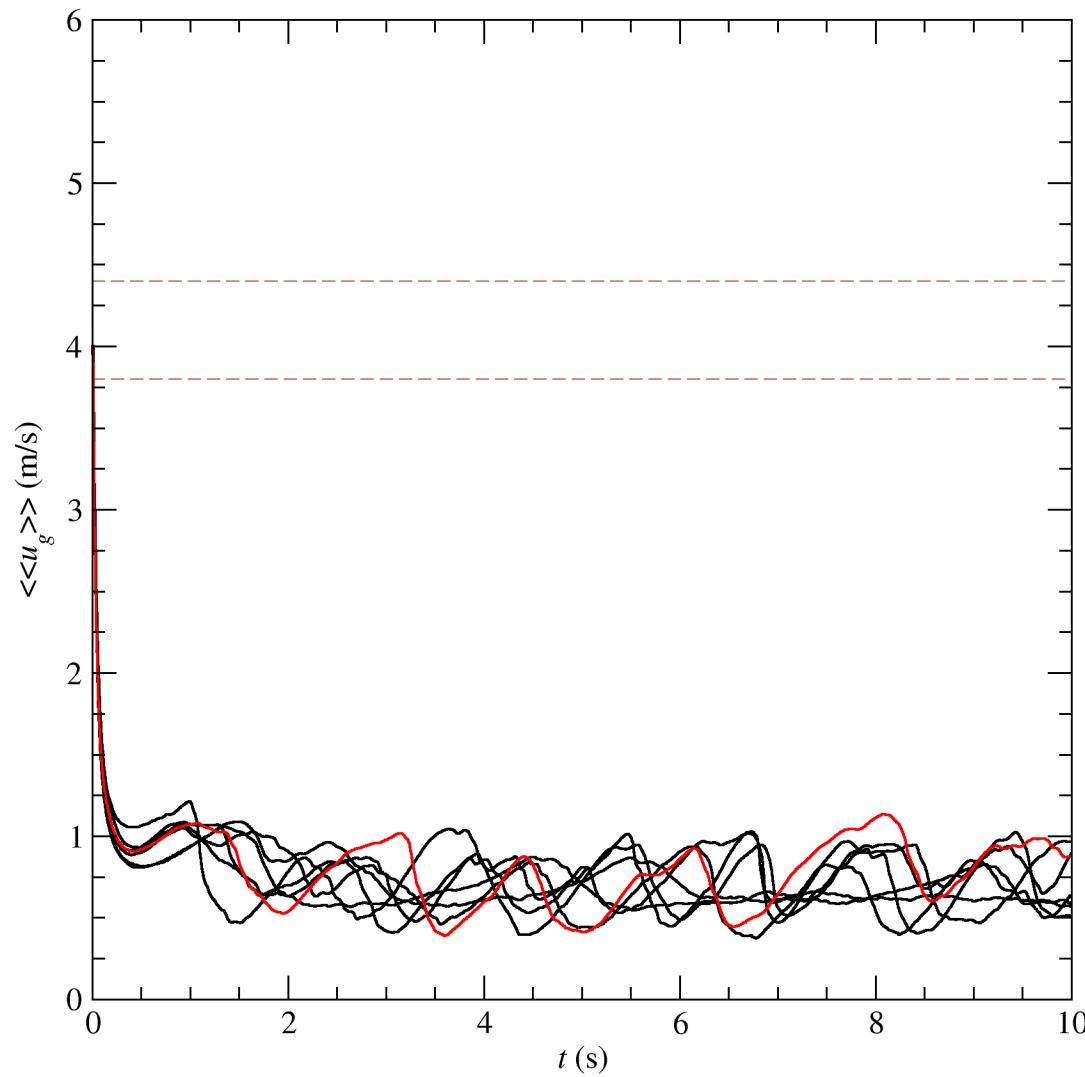


w/ Simeonoff
+
 μ_s = Chap & Lau
w/ limiter
+
~~DNS drag law~~
~~(P-T) f~~
Viability

results

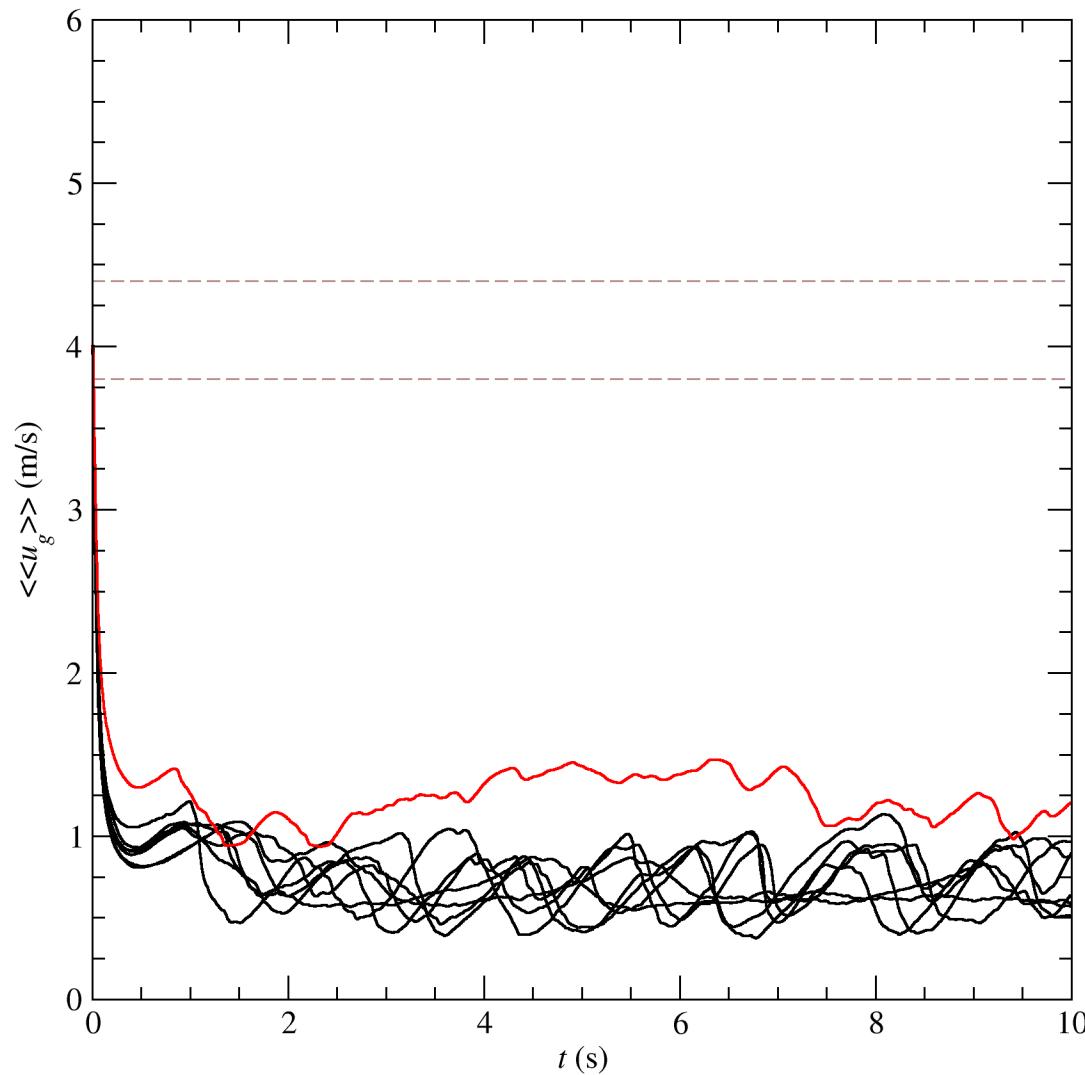


results



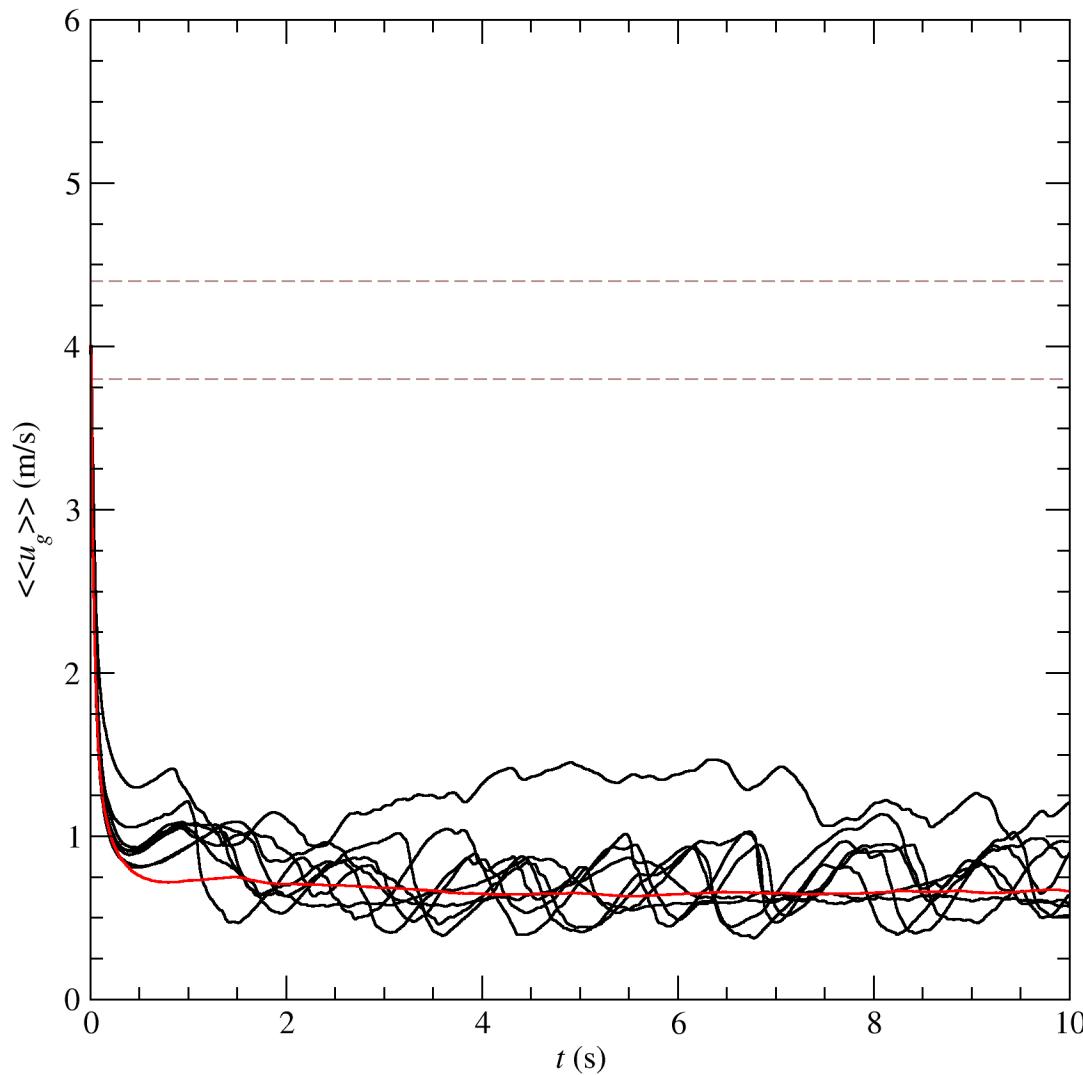
w/ Smoothing
+
 μ_s = Chou & Lau
w/ limiter
+
~~DNS drag law~~
~~(P-T) f~~
~~Virtual mass~~
Flux Rejet

results



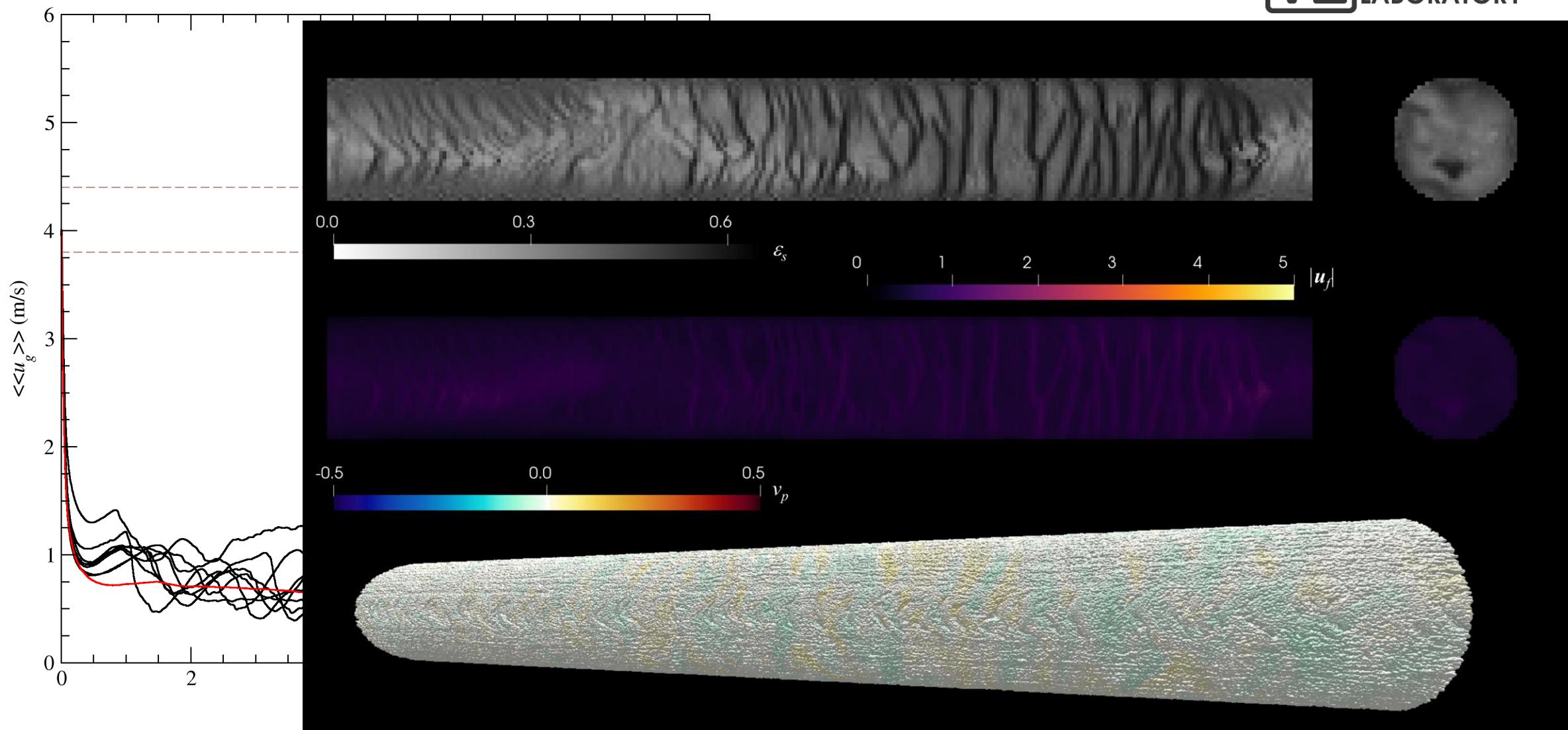
w/ Smoothery
+
 μ_s = Chou & Lau
w/ limiter
+
~~DNS drag law~~
~~(P-T) f~~
~~Virtual mass~~
~~Flux deficit~~
Method of Lines

results

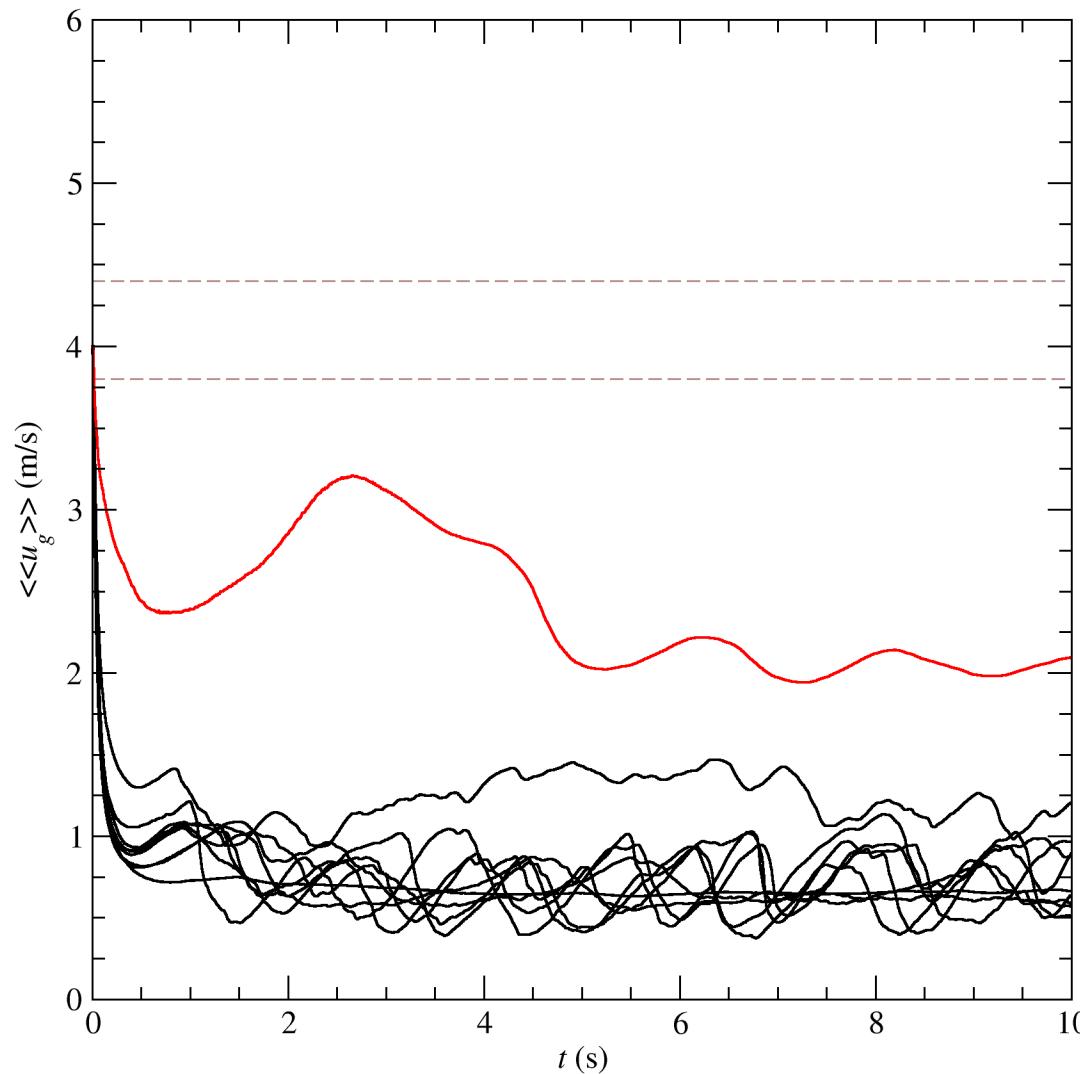


w/ Smagorinsky
+
 $\mu_s = \text{Chap \& Lan}$
w/ limiter
+
~~DNS drag law~~
~~(P-T) f~~
~~viscous~~
~~Flex Dept~~
~~Mixed of Lines~~
no gravity!

results



results



w/ Smagorinsky
+
 μ_s = Chor & Lan
w/ limiter
+
~~DNS drag law~~
~~(P-T) f~~
~~viscous~~
~~flux diff~~
~~Momentum lines~~
~~no gravity!~~
26 + 0
26

governing equations

$$\mathbf{F}_{fi} = (-\nabla \pi_f)_{\mathbb{H}} + (\nabla \cdot \mathbf{u}_f)_{\mathbb{H}} + \mathbf{F}_D + \mathbf{F}_{vm} + \mathbf{F}_L + \mathbf{F}_{TD}$$

$\mathbf{F}_D \propto \alpha^2, C_s = \text{Grashof}$
 $\mathbf{F}_{vm} \propto \frac{\partial \mathbf{u}_i}{\partial t} - \frac{\nabla \pi_f}{\Delta t}, C_{vm} = 0.5$
 $\rightarrow \alpha \propto \Delta x u_f$
 $\mathcal{N}_{\Delta t} \approx 10, \text{ probably not.}$
 $\mathcal{N}_{\Delta t} > 100,000 \dots \text{ may be}$

Lagrangian: discrete particles as parcels

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{u}_i \quad \text{and} \quad m_i \frac{d\mathbf{u}_i}{dt} = m_i \mathbf{g} + \frac{V_p}{\varepsilon_p} \nabla \tau_p + \mathbf{F}_{fi}$$

Eulerian: continuous fluid

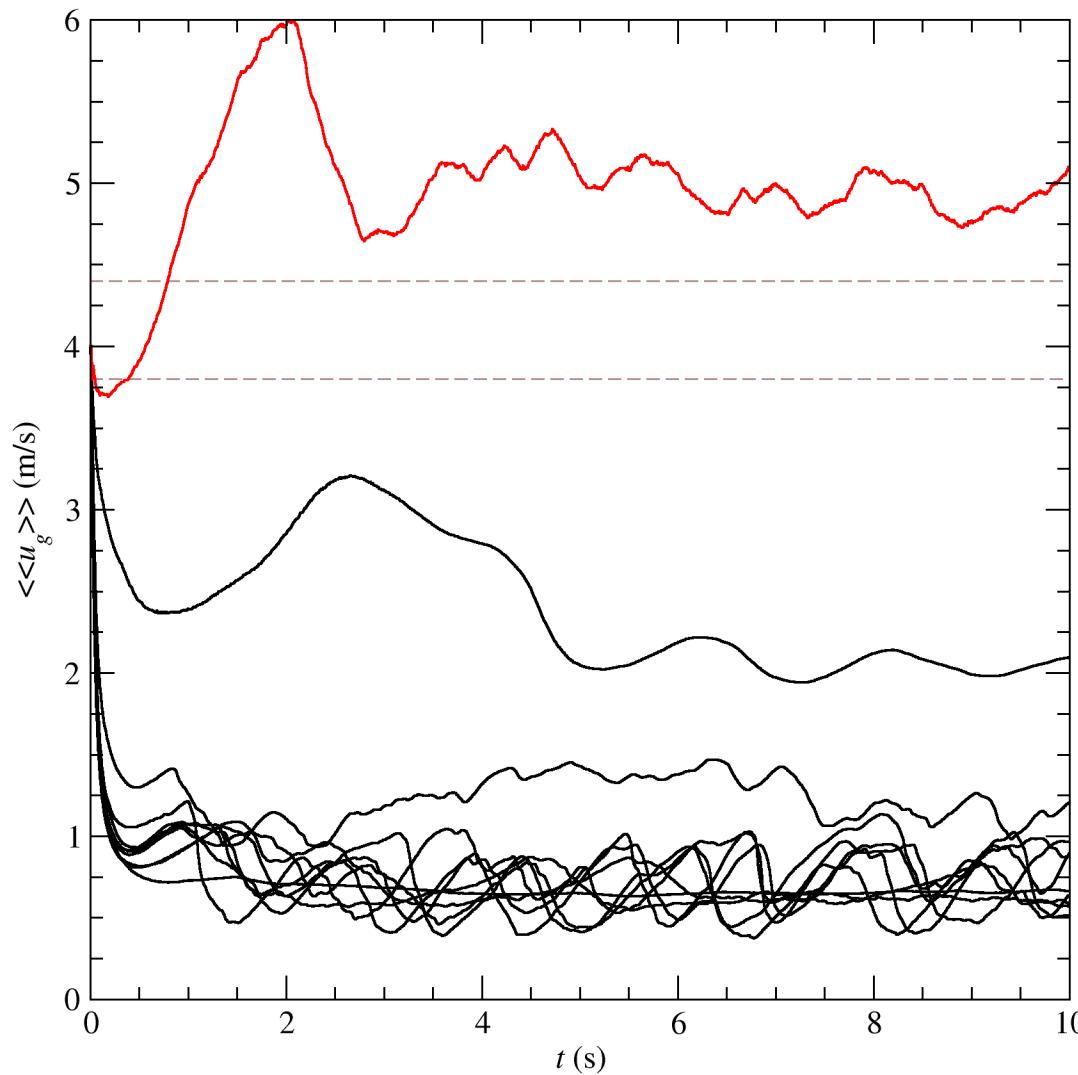
$$\frac{\partial \mathbf{u}_f}{\partial t} + \nabla \cdot \varepsilon_f \mathbf{u}_f = 0 \quad \text{and} \quad \rho_f \left(\frac{\partial \varepsilon_f \mathbf{u}_f}{\partial t} + \nabla \cdot \varepsilon_f \mathbf{u}_f \mathbf{u}_f \right) = -\varepsilon_f \nabla \pi_f + \nabla \cdot \boldsymbol{\tau}_f + \mathbf{M}_{pf} + \rho_f \varepsilon_f \mathbf{g}$$

$$\mu_f \rightarrow \mu_{eff} = \mu_f + \mu_t + \mu_s$$

$$\mu_t = \text{Smagorinsky}$$

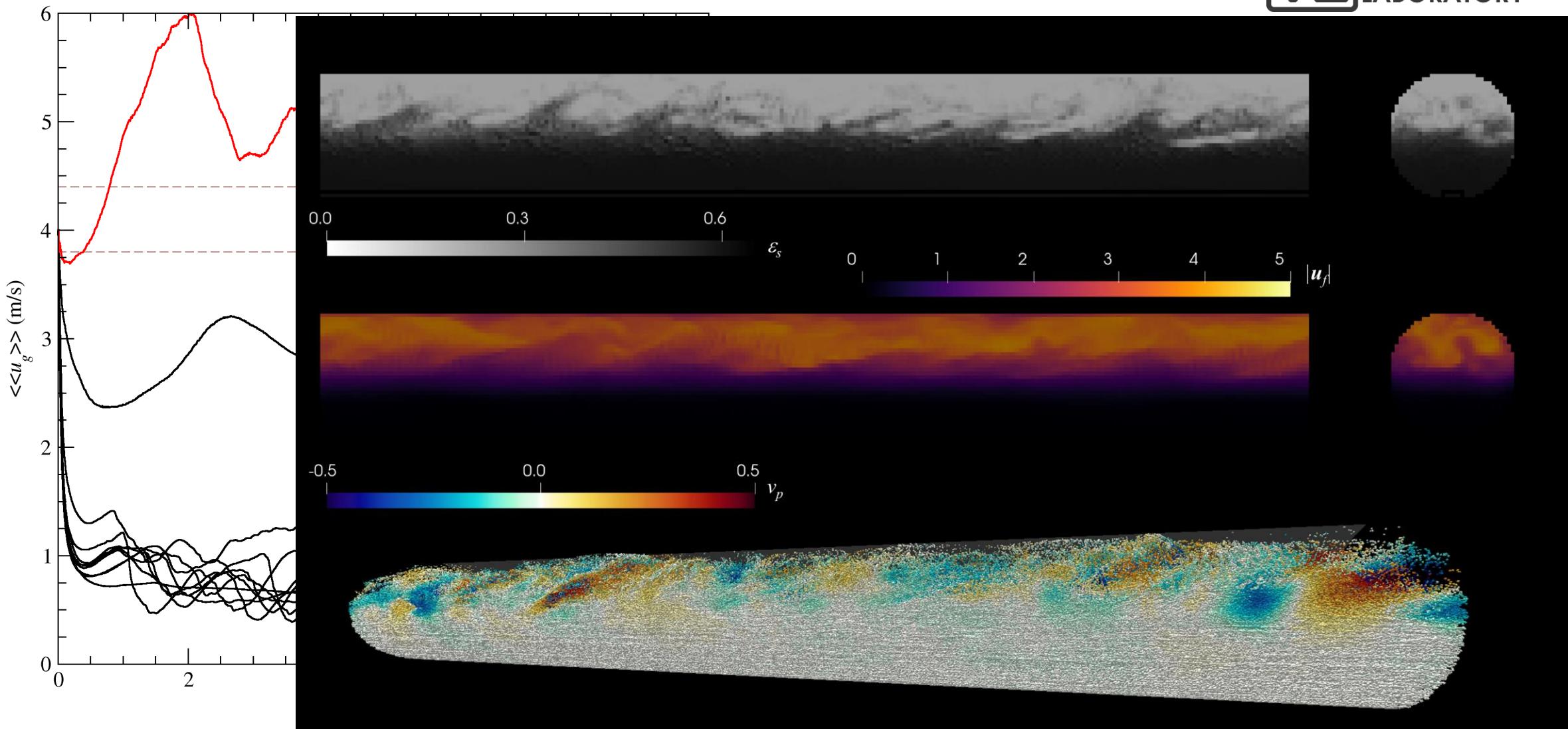
$$\mu_s = \mu_s(\varepsilon_f) = \begin{cases} \text{Einstein} \\ \text{Prandtl, Grashof} \\ \text{Reege, Krieger \& Doherty} \\ \text{Cheng \& Law, etc.} \end{cases}$$

results

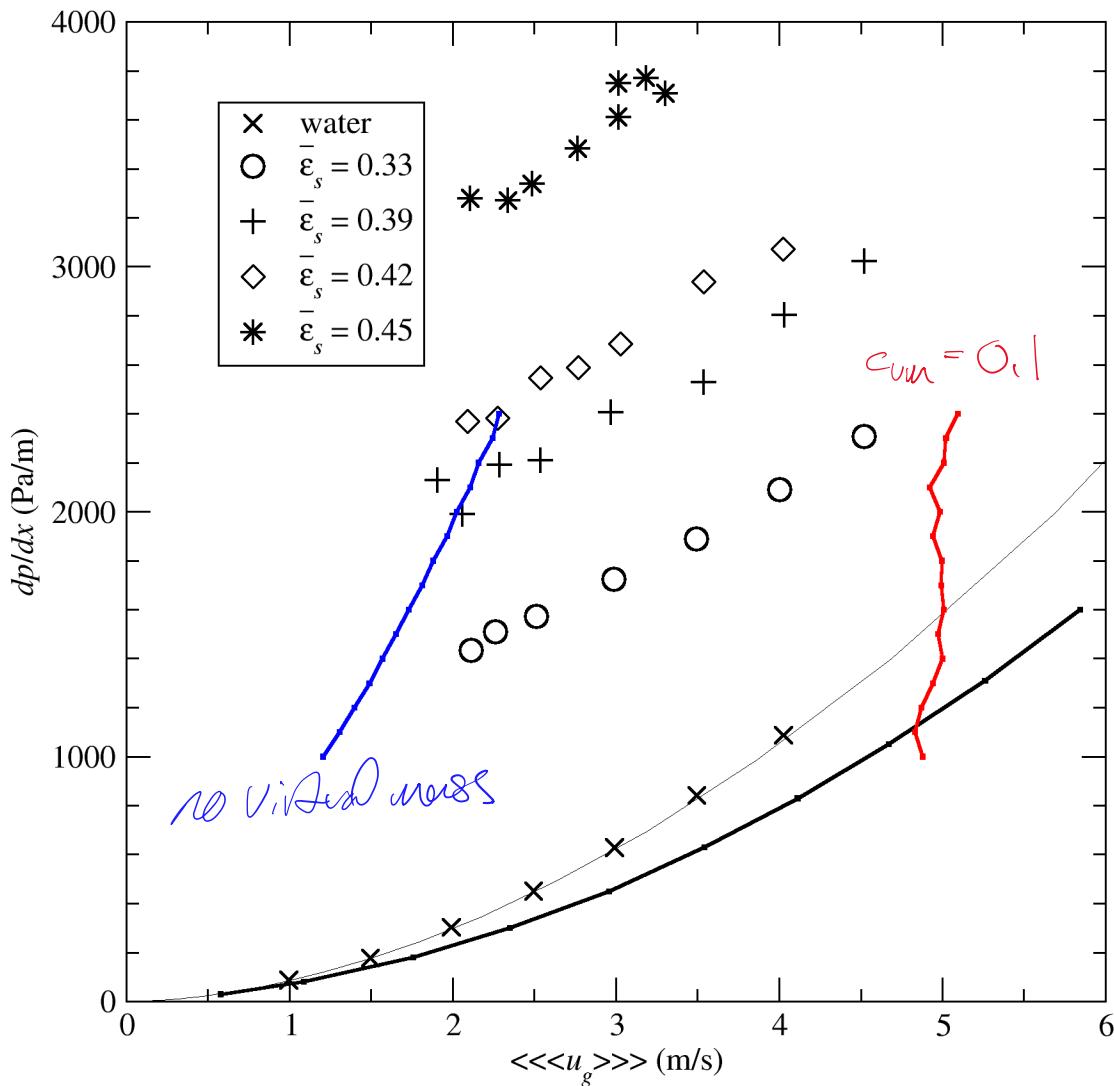


w/ Smoothing
+
 $\mu_S = \text{Chap \& Law}$
w/ limiter
+
viscous mass ($c_{vn} = 0.1$)
+
 $\nabla \cdot \mathbf{f}_f$
+
 $\frac{\partial \mathbf{E}}{\partial t} \neq 0$

results



results



bound the data but we have a lot of work left to do

summary & conclusions^(are there any?)



- Developing a multiphase PIC model for slurry flow with MFIX-Exa
- “Switching” from gas-solid to liquid-solid is non-trivial
- Previously neglected void fraction transient term is important to include in the divergence constraint
- Unclear why virtual mass seems to have such a significant influence on quasi-steady pipe flow (need to validate implementation!)
- Still need to add lift force
- Much more work left to do!

thanks!

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