Impedance-Aware Graph Convolutional Networks for Voltage Estimation in Active Distribution Networks

Abhijith Ravi, Linquan Bai, Valentina Cecchi
William States Lee College of Engineering
UNC Charlotte
NC, USA
{aravi2,lbai4,vcecchi}@uncc.edu

Jianming Lian, Jin Dong, Teja Kuruganti
Oak Ridge National Laboratory
Grid-interactive Controls Group
TN, USA
{lianj, dongj, kurugantipv}@ornl.gov

***Abstract*—Voltage estimation plays a key role in ensuring the effective control and reliability of distribution networks. However, traditional machine learning methods often fail to capture the details of the distribution network's topology. To overcome this challenge, graph convolutional networks (GCN) have emerged as an alternative. Graph convolutional networks inherently capture the topology of the grid, utilizing correlations to achieve precise voltage estimation. Other machine learning models and conventional GCNs fail to account for the distribution line characteristics found in the real world, limiting their effectiveness. This paper proposes an advanced variant of GCN called the Impedance-Aware Graph Convolutional Network (IA-GCN). The IA-GCN layer incorporates the magnitude of the impedance into the graph convolution mechanism, allowing it to capture topological nuances and provide valuable insights into node interrelationships by considering impedance as an intrinsic dimension. The performance of the IA-GCN layer is then compared with that of GCN and GraphSAGE layers through a surrogate model for voltage estimation. The performance analysis demonstrates that IA-GCN outperforms GCN by reducing the MAE by 87.55% and improving the R-squared value by 98%.**

***Index Terms*—distribution networks, graph convolution networks, impedance-aware graph convolution networks, voltage prediction**

I. Introduction

As the power landscape undergoes a significant transformation, the integration of distributed generation (DG) into the grid presents a multifaceted set of challenges and opportunities. While DG offers the promise of enhanced sustainability, resilience, and grid decentralization, it also introduces complexities in grid management, voltage regulation, and system protection. Herein lies the crucial role of big data. Modern power grids generate enormous amounts of data, including data from substations, smart meters, sensors, and other monitoring devices [1]. This data offers unprecedented insights into system operations, customer behaviors, asset health, etc. Big data techniques can be applied to state estimation, forecasting, and control problems [2]. The machine learning (ML) revolution has revolutionized the way utilities harness this deluge of data, enabling them to gain actionable insights, optimize grid operations, and increase system reliability through the development of actionable insights. We can use ML techniques to predict load, detect anomalies, and integrate renewable energy by transforming raw data into predictive models [3]. In today's increasingly complex and interconnected power systems, leveraging ML's data-driven approaches is essential for making the grid more efficient, resilient, and sustainable. Achieving system reliability and optimal energy delivery requires accurate voltage estimation in distribution networks. Machine learning offers transformative potential in this domain, enabling the capture and analysis of vast, complex datasets to derive real-time, precise voltage profiles. In [4], multiple techniques are combined into one regressor to improve voltage prediction accuracy. The author also proposes a two-step regressor that further refines predictions using local regressors. [5] proposes an artificial neural network (ANN) model for online voltage estimation without power flow calculations, thereby reducing computational burden. [6] establishes a mapping relationship between input features and node voltage using users' active power as input to accurately estimate voltage in distribution stations. In contrast to other black-box methods, graph convolutional networks (GCN) can capture the grid's topology, ensuring accurate voltage estimation by utilizing spatial correlations, reducing data redundancy, and providing better generalizability. [7] uses the GCN model for voltage estimation by utilizing topology data and power flow data of the active distribution network (ADN) as graph data. Despite their remarkable ability to perform various tasks, traditional GCNs ignore the inherent impedance properties of real-world distribution networks.

In this paper, we introduce a novel impedance-aware GCN (IA-GCN) layer that incorporates the magnitude of the impedance of the network into the graph convolution process of the standard GCN layer. By considering impedance as an additional dimension, our approach captures both the topological characteristics of the graph and provides a more nuanced understanding of node relationships. The ability of the IA-GCN layer to integrate the impedance of the network can be used in the accurate modeling of power grid models for ML-based applications. This paper is organized as follows: Section II presents the background and foundation of the GCN and GraphSAGE models. Section III presents the integration of impedance magnitude in the proposed IA-GCN layer and model. Section IV presents the results from the simulation experiments, which demonstrate the improved accuracy of voltage estimation using the IA-GCN model.

II. Background and Foundations

A. Basics of GCN

The GCN learns a robust representation of the graph by using the topology from the adjacency matrix to leverage the local neighborhood information of every node in the graph. The principle behind GCNs is that data from neighboring nodes is transformed and aggregated to capture the relationship between the input and output data from the neighborhood. [8]. Equation.(1) presents the primary operation in a GCN layer.

$$H^{(l+1)} = \sigma(\tilde{D}^{-\frac{1}{2}}\tilde{A}\tilde{D}^{-\frac{1}{2}}H^{(l)}W^{(l)}) \tag{1}$$

where $H^{(l)}$ is the matrix of node features at layer l, $\tilde{A}$ is the adjacency matrix with added self-loops, $\tilde{D}$ is the diagonal node degree matrix, $W^{(l)}$ is the weight matrix at layer l and σ is the activation function.

B. Basics of GraphSAGE

GraphSAGE extends the principles of GCNs by introducing a scalable inductive framework. GraphSAGE enhances GCNs by selectively sampling and aggregating local neighborhood information to represent nodes. With this sampling mechanism, GraphSAGE can handle large graphs efficiently and adapt to dynamic graph structures. In contrast to traditional GCN models, GraphSAGE generates embeddings for nodes it hasn't seen during training inductively. Due to its inductive nature, it is particularly suitable for graphs that evolve over time, where new nodes and edges might appear.

III. Impedance-Aware Graph Convolutional Network

A. Motivation

Power distribution networks are characterized by their tree-like radial architecture. The distribution network voltages and power flows are inherently interrelated with the impedance of the distribution lines. A standard GCN does not consider the impedance of the network. In this paper, an advanced variant of GCN called the IA-GCN was created to integrate the impedance magnitude data of the distribution network into a standard GCN's message-passing framework. The difference between message-passing in a standard GCN and IA-GCN is explained in the following subsection.

B. Integrating Impedance in GCN's Message-Passing Framework

This paper models the IA-GCN to incorporate impedance into the standard GCN layer. The operation of a standard GCN layer for estimating voltage using active and reactive power injection at a node can be shown using the Fig. 1. From the perspective of node 1, the process begins with a linear transformation of the input features P_1 and Q_1. In subsequent steps, the transformed features are propagated

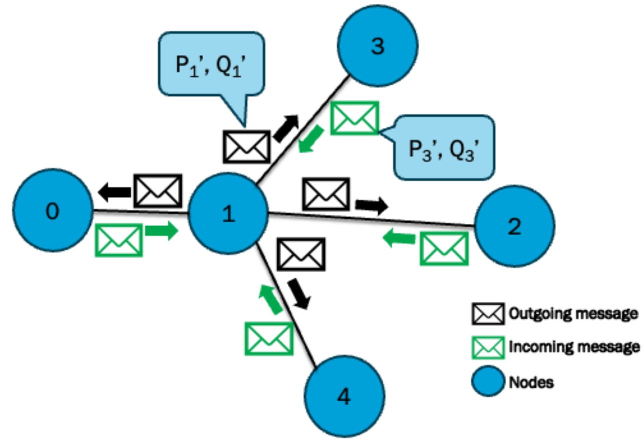

Fig. 1. Message Passing within a Standard GCN Layer

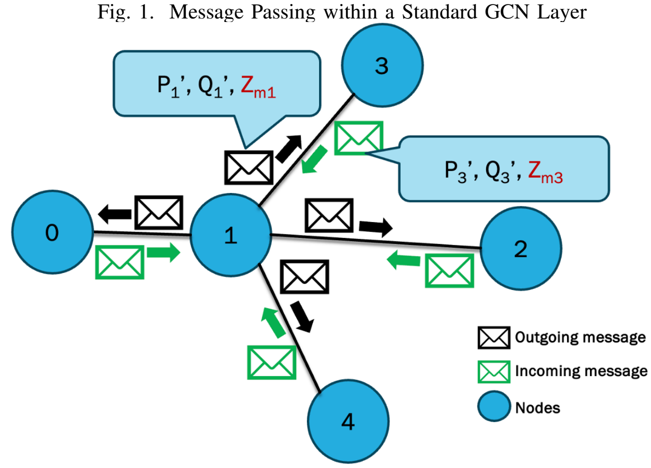

Fig. 2. Message Passing within an Impedance-Aware GCN Layer

across the graph to neighboring nodes 0, 2, 3, and 4 using the adjacency matrix. During this phase, node 1 collects transformed input features from its neighbors, thereby capturing the local graph structure. The data from all the neighboring nodes is aggregated by the node. The aggregated data from the neighboring nodes combined with transformed data from node 1 is passed through a non-linear activation function such as ReLU to capture complex relationships. Using another transformation, the activated features are used to predict the voltage at node 1. For integrating the impedance values in the standard GCN layer, the novel IA-GCN layer employs the magnitude-only impedance matrix of the distribution network. The magnitude of the impedance matrix was employed to create impedance-based modulation. As depicted in Fig.2, the IA-GCN integrates an impedance modulation Z_m during message passing. While node 1 sends the transformed features and the modulation to the neighboring nodes 0, 2, 3, and 4, the modulation specific to node 1, Z_{m1}, is influenced by the impedance values from the impedance matrix corresponding to node 1's connections.The neighboring nodes also send their transformed features with their respective impedance modulations to node 1, enhancing the exchange of information with physical network characteristics. The aggregated incoming messages, which include both these transformed features and the respective impedance modulations, are combined to form the pre-activation data. Similar to the standard GCN, the IA-GCN goes through activation and then a transformation before being used for voltage estimation. This integration of node-specific impedance modulation is critical in differentiating the IA-GCN layer from the standard GCN layer, adding a level of complexity and adaptability.

C. Mathematical Representation of the IA-GCN Layer

The IA-GCN layer can be represented by the equation (2).

$$H^{(l+1)} = \sigma(A(\text{propagate}(L, M, Z))) \tag{2}$$

where $H^{(l)}$ is the matrix of node features at layer l, and A is the adjacency matrix without self-loops. The function propagate represents the process of propagating the linearly transformed and the impedance magnitude-modulated features through the graph structure. The linear transformation and impedance magnitude modulation are represented by equations (3) and (4), respectively:

$$L = H^{(l)} W_1 \tag{3}$$

$$M = H^{(l)} W_Z \tag{4}$$

In these equations, L represents the linear transformation of the node features at layer l with the weight matrix W_1, and M represents the modulation of the node features by the magnitude of impedance at layer l with the weight matrix W_Z. The element-wise modulation, determined by the magnitude of impedance Z, is implicitly incorporated in the propagation process. σ denotes a non-linear activation function.

D. IA-GCN based Voltage Estimation Model

The IA-GCN layer is defined to incorporate impedance into the process of graph convolution. The data flow within the IA-GCN model is presented in algorithm 1. The structure of the model consists of three convolution layers and one fully connected layer. Leaky ReLU activation functions are applied to the input data. The model is trained using the Adam optimizer, mean squared error loss, and a learning rate decay strategy. The input features considered are nodal active power injection P, reactive power injection Q, and DER status D_{status}. These features, along with the label data representing the nodal data V, were normalized using zero-mean normalization. The nature and dimension of the feature matrix X, and adjacency matrix A are presented in [7]. The magnitude-only impedance matrix, denoted as Z, was constructed by extracting the magnitudes from the complex impedance values in the full impedance matrix [9]. For each time-period in the dataset, nodal features are stacked and the corresponding impedance data is added. The adjacency matrix is used to generate the edge index for each node. Each graph instance consists of the input and output feature data and the edge index of the graph. This data is used to train the IA-GCN model by iteratively processing batches of data. In each epoch, the model performs a forward pass, computes loss against the observed voltage values, and updates its parameters through backpropagation. The model tracks validation losses during the training process to avoid overfitting. The training process is halted when the validation loss has been reduced to a significantly low value. Hyperparameters were tuned to optimize the performance of the model.

Algorithm 1 ImpedanceAwareGCN Model Data Flow

Input: Graph data (node features x, edge indices $edge\_index$, edge impedance $impedance$)
Output: Final model output
1: **Initialise** GCN layers ($conv1$, $conv2$, $conv3$) and fully connected layer fc
2: **Forward Pass:**
3: $x \leftarrow conv1(x, edge\_index, impedance)$ {Apply first GCN layer}
4: $x \leftarrow$ **F.leaky_relu**$(x, negative\_slope)$ {Apply non-linear activation}
5: $x \leftarrow conv2(x, edge\_index, impedance)$ {Apply second GCN layer}
6: $x \leftarrow$ **F.leaky_relu**$(x, negative\_slope)$ {Apply non-linear activation}
7: $x \leftarrow conv3(x, edge\_index, impedance)$ {Apply third GCN layer}
8: $final\_output \leftarrow fc(x)$ {Pass through fully connected layer for output}
9: **return** $final\_output$

IV. RESULTS

The proposed local voltage estimation model for the ADN is tested using the modified IEEE 33-node distribution system in this section. The proposed method was implemented in Visual Studio Code using PyTorch. The numerical experiments were conducted on a computer with an Apple M2 Pro processor and 32 GB of RAM.

A. Case Description

A modified IEEE 33-node network was used to generate the training, testing, and validation data for the models [7]. Four solar DGs of 1 MW each were considered at nodes 18, 22, 25, and 33. The load data for this feeder was varied randomly between 0.8 and 1.2 p.u. of the peak load value of the standard IEEE test case. Then an AC optimal power flow (ACOPF) model used in [10] was employed to generate the voltage and DG status of the network. The voltage was allowed to fluctuate between 0.9 p.u. and 1.1 p.u. The training data generated was assumed to be at a 5-minute frequency for a period of 6 months. The generated nodal voltages are mostly in the range of 1.06 p.u. to 0.98 p.u. except for a couple of nodes with voltages around the lower threshold of the voltage. The temporal characteristics of the load were ignored for data generation.The generated data was split into testing, training, and validation in the percentages of 60, 20, and 20, respectively. The same data was used to train the models GCN, Graph-SAGE, and the novel IA-GCN model. Parameters and hyperparameters of the GCN model were selected from [7]. The same parameters and hyperparameters were used for the

TABLE I
COMPARISON OF PERFORMANCE METRICS

Performance Metrics	IA-GCN	GCN	Graph-SAGE
MAE	5.80E-04	4.66E-03	3.01E-03
MSE	8.21E-07	4.30E-05	2.66E-05
RMSE	9.06E-04	6.55E-03	5.15E-03
R-squared	0.9995	0.9708	0.9809
Explained Variance	0.9995	0.9823	0.9867
MAPE	0.06%	0.46%	0.30%

GraphSAGE model. The IA-GCN used an input layer size of 3, three hidden layers of size 33, and an output layer size of 1. The IA-GCN-based model was trained with a learning rate of 0.01 for 100 epochs. A learning rate decay coefficient of 0.99 was used for the model. $Beta$, the negative slope for leaky-RelU activation, was set at 0.1.

In the development of the IA-GCN model, we prioritized reproducibility and openness. The complete source code, including the custom IA-GCN layer, has been made publicly available for academic use and further development. The code repository can be accessed at [11].

B. Comparison of Performance metrics of IA-GCN, GCN, and Graph-SAGE Models

To identify the most efficient graph-based model for voltage estimation in distribution networks, three models were assessed: GraphSAGE, GCN, and IA-GCN. MAE, MSE, RMSE, R-squared, Explained Variance, and MAPE were used as performance metrics for this evaluation. The superior performance of IA-GCN is clear from the table I. The integration of the physical properties of the network into the standard GCN's convolution process enhances the ability to capture the relationship between the nodal power injection and the resulting nodal voltage. Its near-perfect R-squared value and explained variance show the ability to account for variance in the voltage data completely. Furthermore, the model's low MAPE combined with the other indicators indicate that the surrogate model based on IA-GCN is far superior to the other methods that were used for comparing the novel layer. The graph created by comparing the actual and predicted voltage values was used to visualize the accuracy of the IA-GCN model. The scatter plot in Fig.3 showed that the model predictions closely matched the values, which is understood from the nearly 45-degree line slope. The absence of voltage values between 0.98 pu and 0.96 pu is dependent on the test case and the ACOPF model used for data generation.

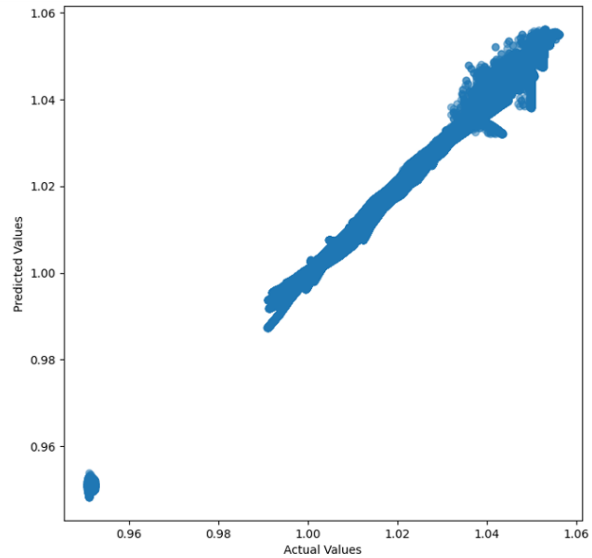

Fig. 3. Actual vs Predicted Voltage

V. CONCLUSION

The standard GCN ignores critical physical properties inherent to real-world distribution systems. To address this deficiency, we introduced IA-GCN, a novel approach that incorporates impedance modulation into the graph convolution mechanism. Our performance analysis demonstrates that IA-GCN is superior to conventional GCN and GraphSAGE models in terms of precision and generalizability. When compared to the standard GCN, IA-GCN reduced the MAE by 87.55%, MSE by 98% and the MAPE by 87.5%. Moreover, the results from IA-GCN indicate that it improved the explained variance by 96.61%, which means that the IA-GCN model's performance is 96.61% closer to the perfect score of 1.0 when compared to the GCN model. By incorporating impedance as an inherent dimension, IA-GCN not only captures the topological nuances of the graph but also provides a refined perspective on node interconnections. As demonstrated, this holistic approach has the potential to significantly improve voltage estimation duties in ADNs, paving the way for more resilient and efficient power distribution systems in the future. This IA-GCN layer can be applied to build surrogate models for both DC and AC power flow for several applications like loss estimation, fault location detection, etc.

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