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## Sierra/Aria Verification Manual – Version 5.22

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# **ABSTRACT**

Presented in this document is a portion of the tests that exist in the Sierra Thermal/Fluids verification test suite. Each of these tests is run nightly with the Sierra/TF code suite and the results of the test checked under mesh refinement against the correct analytic result. For each of the tests presented in this document the test setup, derivation of the analytic solution, and comparison of the code results to the analytic solution is provided. This document can be used to confirm that a given code capability is verified or referenced as a compilation of example problems.

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# 1. INTRODUCTION

The Sierra/TF Verification Manual is divided into chapters based on related capabilities. Each section of a chapter represents a distinct verification test. Some problems that are not yet fully documented are listed at the end of each chapter.

All of these verification tests are run nightly by the development team to continually verify code accuracy under mesh refinement. The graphics and charts in this document are automatically generated by the nightly test runs.

The test files for these problems may be found in the Sierra regression test repository. Most are in the sub-directory called “verification.”

`aria_rtest/verification`

All tests are assigned the keyword “verification”. Those that appear in this document also have the keyword “self-documenting”.

For each test, the approximate finite element solution  $T_h$  is compared to the exact solution  $T$  using several global norms, and in some cases using response quantities of interest. This is repeated over a series of uniformly refined meshes (not necessarily nested) with mesh sizes  $\{h_i\}$ , giving a sequence of errors  $\{E_i\}$ . For each pair of meshes, a convergence rate is estimated using the formula

$$r_i \equiv \log(E_i/E_{i-1})/\log(h_i/h_{i-1}). \quad (1.1)$$

The convergence of  $r_i$  to the expected rate is monitored as the mesh is refined. A test passes if all of the estimated convergence rates on the finest pair of meshes are within a given tolerance of the expected rates.

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## 2. BASIC THERMAL TESTS

### 2.1. STEADY HEAT CONDUCTION: HEX8 MESHES

This problem tests basic steady state heat conduction in a 3D domain. The geometry consists of a unit cube. A variety of different source terms and boundary conditions are simultaneously applied. The exact solution is a manufactured solution.

#### 2.1.1. Features Tested

Basic heat conduction on Hex8 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

#### 2.1.2. Boundary Conditions

At surfaces 4 and 6, the temperature is prescribed as a constant value. On surfaces 3 and 5, a heat flux condition is prescribed using a sum of a constant heat flux and a heat flux from an Encore function (user subroutine). On surfaces 1 and 2, heat flux condition is prescribed using a sum of a convective flux boundary condition (with constant flux and convective coefficient) and a heat flux from an Encore function (user subroutine). Within the domain a source term is prescribed using a sum of a constant source and an Encore function (user subroutine).

#### 2.1.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

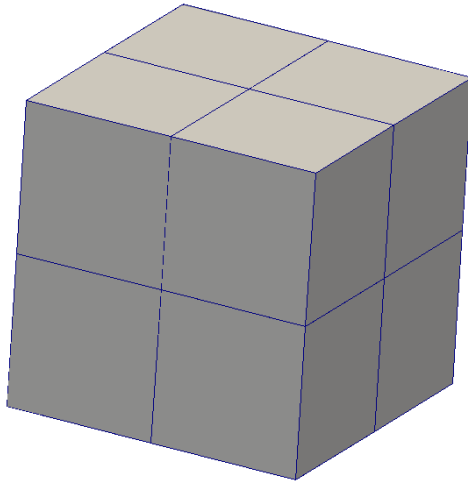
#### 2.1.4. Verification of Solution

A manufactured solution is chosen as

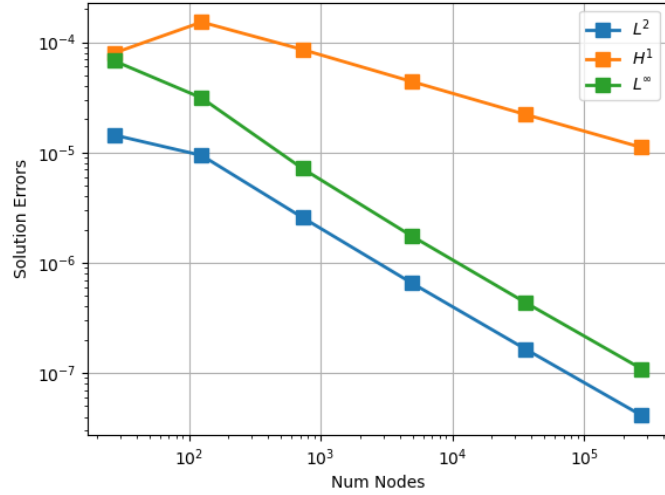
$$T(x, y, z) = 1 + (x - x^2)^2(y - y^2)^2(z - z^2)^2.$$

The source and heat flux user subroutines are chosen so that the solution satisfies the heat equation with the correct boundary conditions.

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, and 1, respectively (within a tolerance).



Coarse Mesh



Error Norms

**Figure 2.1-1.. Steady Heat Conduction: Hex8 Meshes**

**Table 2.1-1.. Steady Heat Conduction: Convergence Rates for Hex8 Meshes**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 125      | 0.83  | -1.27 | 1.52       |
| 729      | 2.20  | 0.98  | 2.51       |
| 4913     | 2.15  | 1.05  | 2.21       |
| 35937    | 2.08  | 1.04  | 2.10       |
| 274625   | 2.04  | 1.02  | 2.05       |

For input decks see Appendix [12.1.1](#).

## 2.2. STEADY HEAT CONDUCTION: HEX20 MESHES

This problem tests basic steady state heat conduction in a 3D domain. The geometry consists of a unit cube. A variety of different source terms and boundary conditions are simultaneously applied. The exact solution is a manufactured solution.

### 2.2.1. Features Tested

Basic heat conduction on Hex20 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 2.2.2. Boundary Conditions

At surfaces 4 and 6, the temperature is prescribed as a constant value. On surfaces 3 and 5, a heat flux condition is prescribed using a sum of a constant heat flux and a heat flux from an Encore function (user subroutine). On surfaces 1 and 2, heat flux condition is prescribed using a sum of a convective flux boundary condition (with constant flux and convective coefficient) and a heat flux from an Encore function (user subroutine). Within the domain a source term is prescribed using a sum of a constant source and an Encore function (user subroutine).

### 2.2.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 2.2.4. Verification of Solution

A manufactured solution is chosen as

$$T(x, y, z) = 1 + (x - x^2)^2(y - y^2)^2(z - z^2)^2.$$

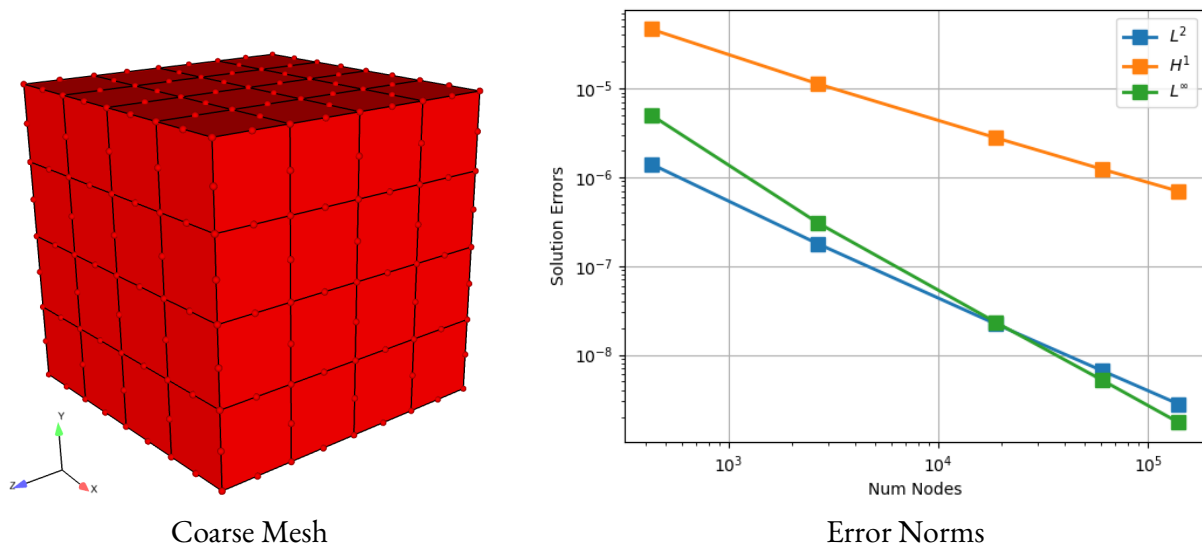
The source and heat flux user subroutines are chosen so that the solution satisfies the heat equation with the correct boundary conditions.

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, and 1, respectively (within a tolerance).

**Table 2.2-1.. Steady Heat Conduction: Convergence Rates for Hex20 Meshes**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 2673     | 3.39  | 2.32  | 4.58       |
| 18785    | 3.19  | 2.15  | 3.95       |
| 60625    | 3.11  | 2.08  | 3.85       |
| 140481   | 3.08  | 2.06  | 3.91       |

For input decks see Appendix [12.1.2](#).



**Figure 2.2-1.. Steady Heat Conduction: Hex20 Meshes**

## 2.3. STEADY HEAT CONDUCTION: HEX27 MESHES

This problem tests basic steady state heat conduction in a 3D domain. The geometry consists of a unit cube. A variety of different source terms and boundary conditions are simultaneously applied. The exact solution is a manufactured solution.

### 2.3.1. Features Tested

Basic heat conduction on Hex27 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 2.3.2. Boundary Conditions

At surfaces 4 and 6, the temperature is prescribed as a constant value. On surfaces 3 and 5, a heat flux condition is prescribed using a sum of a constant heat flux and a heat flux from an Encore function (user subroutine). On surfaces 1 and 2, heat flux condition is prescribed using a sum of a convective flux boundary condition (with constant flux and convective coefficient) and a heat flux from an Encore function (user subroutine). Within the domain a source term is prescribed using a sum of a constant source and an Encore function (user subroutine).

2.3.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

2.3.4. Verification of Solution

A manufactured solution is chosen as

$$T(x,y,z) = 1 + (x - x^2)^2(y - y^2)^2(z - z^2)^2.$$

The source and heat flux user subroutines are chosen so that the solution satisfies the heat equation with the correct boundary conditions.

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, and 1, respectively (within a tolerance).

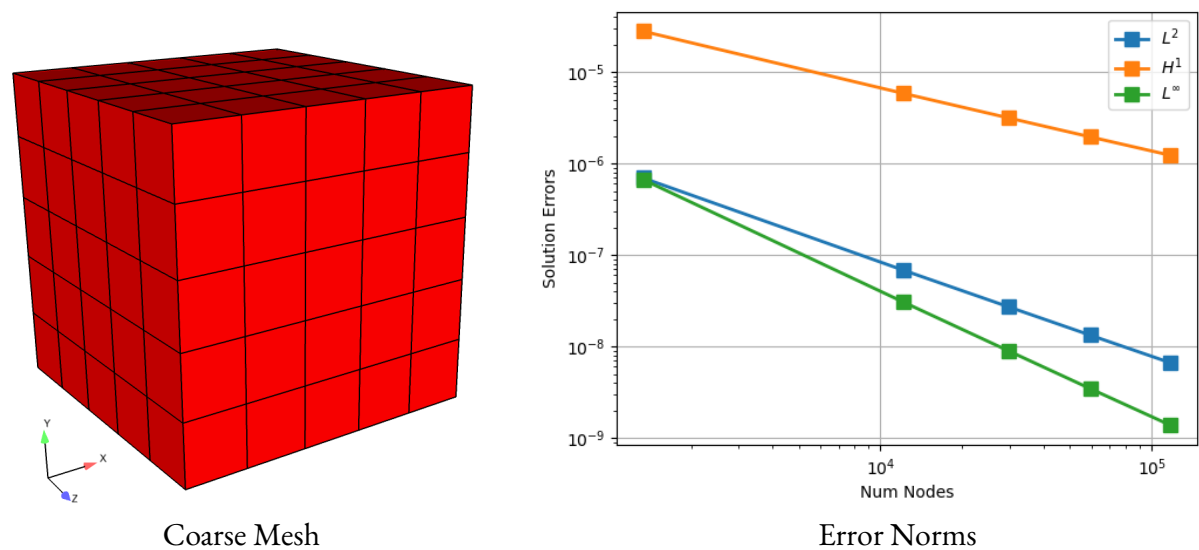


Figure 2.3-1.. Steady Heat Conduction: Hex27 Meshes

Table 2.3-1.. Steady Heat Conduction: Convergence Rates for Hex27 Meshes

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 12167    | 3.15  | 2.12  | 4.18       |
| 29791    | 3.10  | 2.07  | 4.13       |
| 59319    | 3.08  | 2.06  | 4.10       |
| 117649   | 3.06  | 2.05  | 4.08       |

For input decks see Appendix [12.1.3](#).

## 2.4. STEADY HEAT CONDUCTION: TET4 MESHES

This problem is identical to the one in Section [2.1](#) except that unstructured Tet4 meshes are used instead. The meshes are obtained from Cubit.

### 2.4.1. Features Tested

Basic heat conduction on Tet4 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 2.4.2. Boundary Conditions

Same as in Section [2.1](#).

### 2.4.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 2.4.4. Verification of Solution

Same as in Section [2.1](#).

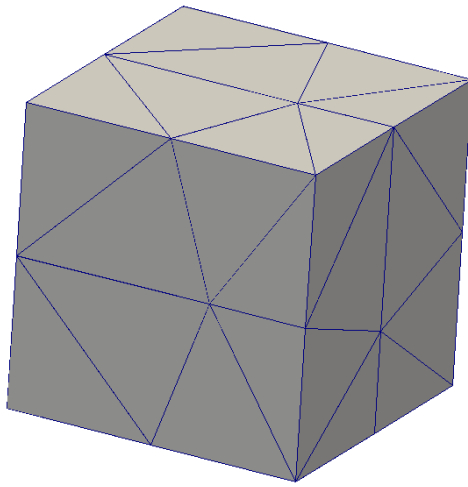
Unlike the Hex8 case, we have observed in many cases, and in this test, that the convergence rate for the temperature in the  $L^\infty$  norm is somewhat less than 2, in this case about 1.9. The exact reason for this behavior is unclear.

**Table 2.4-1.. Steady Heat Conduction: Convergence Rates for Tet4 Meshes**

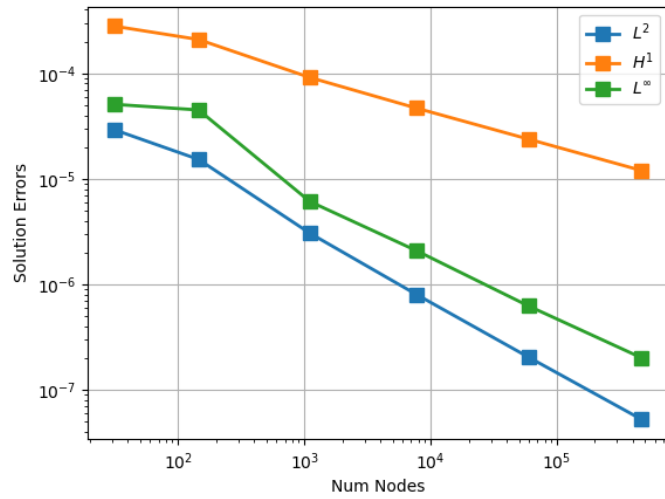
| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 145      | 1.27  | 0.57  | 0.24       |
| 1104     | 2.37  | 1.22  | 2.95       |
| 7725     | 2.06  | 1.03  | 1.65       |
| 59636    | 2.00  | 0.99  | 1.77       |
| 469317   | 1.98  | 0.99  | 1.65       |

For input decks see Appendix [12.1.4](#).





Coarse Mesh



Error Norms

**Figure 2.4-1.. Steady Heat Conduction: Tet4 Meshes**

## 2.5. STEADY HEAT CONDUCTION: TET4TET10 MESHES

This problem is identical to the one in Section 2.1 with the exception of constant thermal conductivity and use of unstructured Tet10 meshes. The meshes are obtained from Cubit.

### 2.5.1. Features Tested

Basic heat conduction with Tet4 solution on Tet10 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 2.5.2. Boundary Conditions

Same as in Section 2.1.

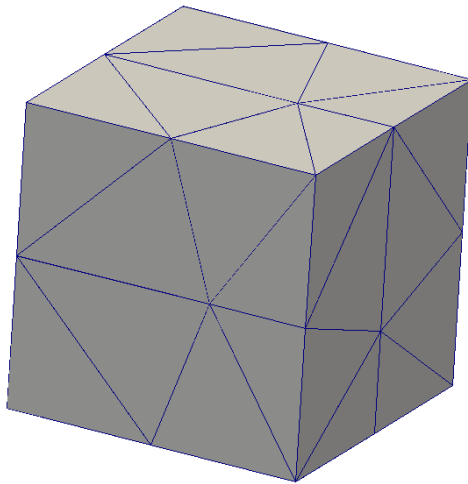
### 2.5.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

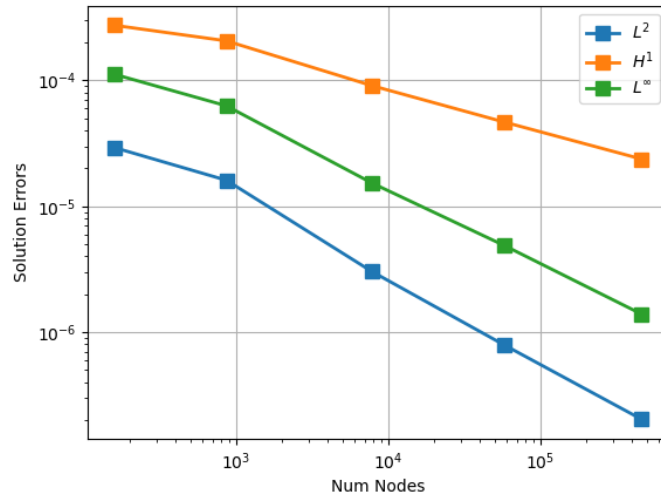
## 2.5.4. Verification of Solution

Same as in Section 2.1.

Unlike the Hex8 case, we have observed in many cases, and in this test, that the convergence rate for the temperature in the  $L^\infty$  norm is somewhat less than 2, in this case about 1.9. The exact reason for this behavior is unclear.



Coarse Mesh



Error Norms

**Figure 2.5-1.. Steady Heat Conduction: Tet4 Solutions on Tet10 Meshes**

**Table 2.5-1.. Steady Heat Conduction: Convergence Rates for Tet4Tet10 Meshes**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 865      | 1.06  | 0.50  | 1.02       |
| 7831     | 2.27  | 1.12  | 1.92       |
| 58211    | 2.01  | 1.00  | 1.71       |
| 464414   | 1.96  | 0.98  | 1.81       |

For input decks see Appendix 12.1.5.

## 2.6. STEADY HEAT CONDUCTION: TET10 MESHES

This problem is identical to the one in Section 2.1 except that unstructured Tet10 meshes are used instead. The meshes are obtained from Cubit.

## 2.6.1. Features Tested

Basic heat conduction on Tet10 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

## 2.6.2. Boundary Conditions

Same as in Section 2.1.

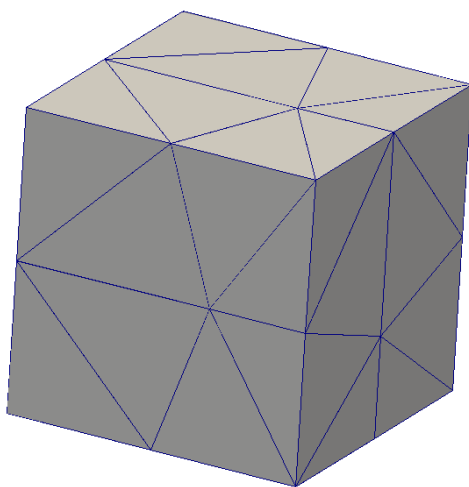
## 2.6.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

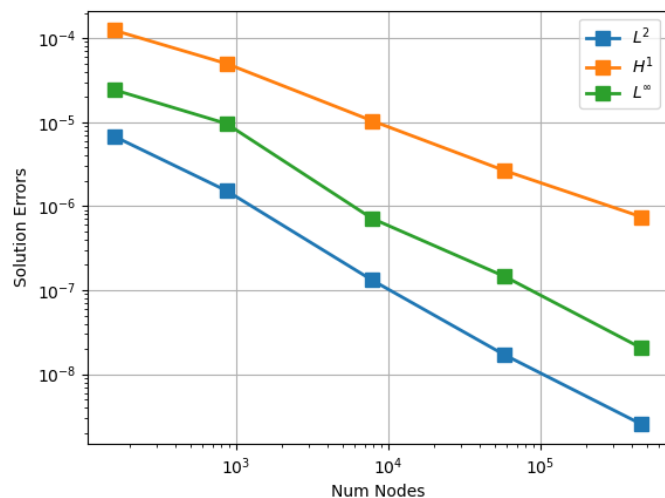
## 2.6.4. Verification of Solution

Same as in Section 2.1.

Unlike the Hex8 case, we have observed in many cases, and in this test, that the convergence rate for the temperature in the  $L^\infty$  norm is somewhat less than 3, in this case about 2.7. The exact reason for this behavior is unclear.



Coarse Mesh



Error Norms

**Figure 2.6-1.. Steady Heat Conduction: Tet10 Meshes**

For input decks see Appendix 12.1.6.

**Table 2.6-1.. Steady Heat Conduction: Convergence Rates for Tet10 Meshes**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 865      | 2.63  | 1.61  | 1.64       |
| 7831     | 3.34  | 2.13  | 3.54       |
| 58211    | 3.06  | 2.05  | 2.38       |
| 464414   | 2.75  | 1.84  | 2.84       |

## 2.7. TRANSIENT HEAT CONDUCTION: HEX8 MESHES

This problem tests basic transient heat conduction in a 3D domain. The geometry consists of a unit cube.

### 2.7.1. Features Tested

Basic transient heat conduction on Hex8 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 2.7.2. Boundary Conditions

At surfaces 4 and 6, the temperature is prescribed as a constant value. On surfaces 3 and 5, a heat flux condition is prescribed using a sum of a constant heat flux and a heat flux from an Encore function (user subroutine). On surfaces 1 and 2, heat flux condition is prescribed using a sum of a convective flux boundary condition (with constant flux and convective coefficient) and a heat flux from an Encore function (user subroutine). Within the domain a source term is prescribed using a sum of a constant source and an Encore function (user subroutine).

### 2.7.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 2.7.4. Verification of Solution

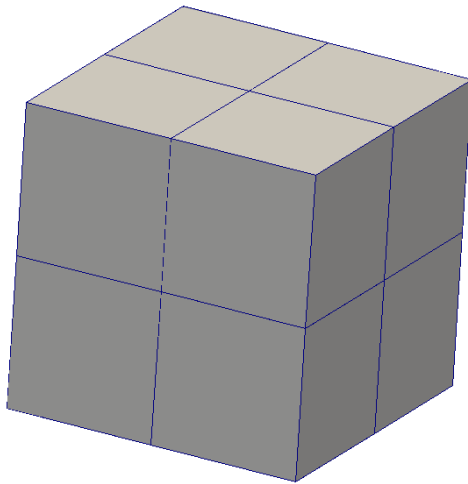
A manufactured solution is chosen as

$$T(x, y, z, t) = (x - x^2)^2 (y - y^2)^2 (z - z^2)^2 m(t) + 1,$$

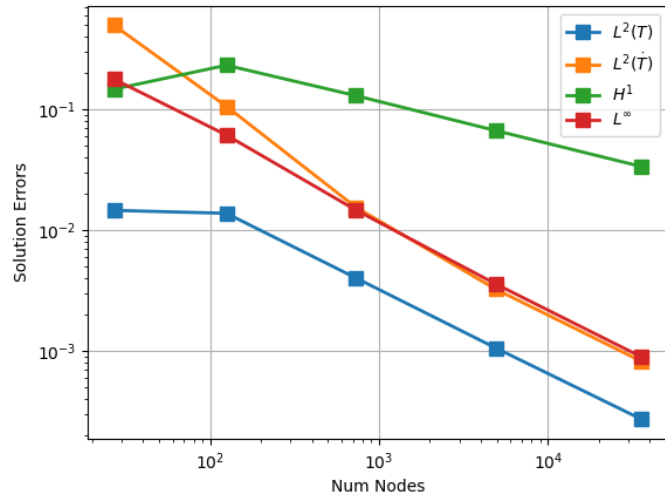
$$m(t) = 10^4 (1 - \exp(-t) + t \exp(-(t - 1)^2))$$

The source and heat flux user subroutines are chosen so that the solution satisfies the heat equation with the correct boundary conditions.

For each mesh, the errors in the temperature solution at final time are computed in the  $L^2$  norm of  $T$  and  $\dot{T}$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, 2 and 1, respectively (within a tolerance).



Coarse Mesh



Error Norms

**Figure 2.7-1.. Transient Heat Conduction: Hex8 Meshes**

**Table 2.7-1.. Transient Heat Conduction: Convergence Rates for Hex8 Meshes**

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ |
|----------|----------|----------------|-------|------------|
| 125      | 0.12     | 3.02           | -0.89 | 2.08       |
| 729      | 2.09     | 3.28           | 0.98  | 2.42       |
| 4913     | 2.09     | 2.46           | 1.05  | 2.22       |
| 35937    | 2.06     | 2.07           | 1.04  | 2.10       |

For input decks see Appendix [12.1.7](#).

## 2.8. TRANSIENT HEAT CONDUCTION: TET4 MESHES

This problem tests basic transient heat conduction in a 3D domain as in Section [2.7](#). The geometry consists of a unit cube and a single bulk fluid element.

## 2.8.1. Features Tested

Basic transient heat conduction on Tet4 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; bulk fluid element; heat flux and source term from Encore user subroutines.

## 2.8.2. Boundary Conditions

Identical to Section 2.7 except one convective flux boundary condition is now connected to a bulk fluid element.

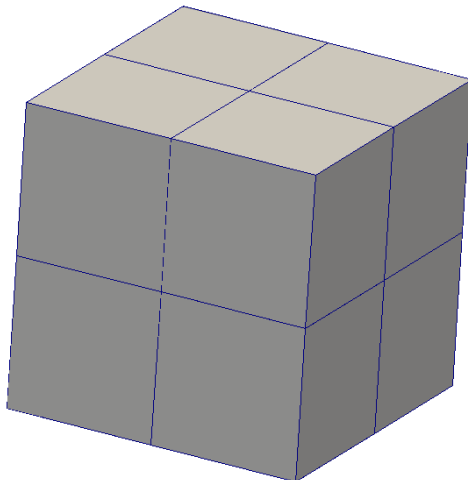
## 2.8.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

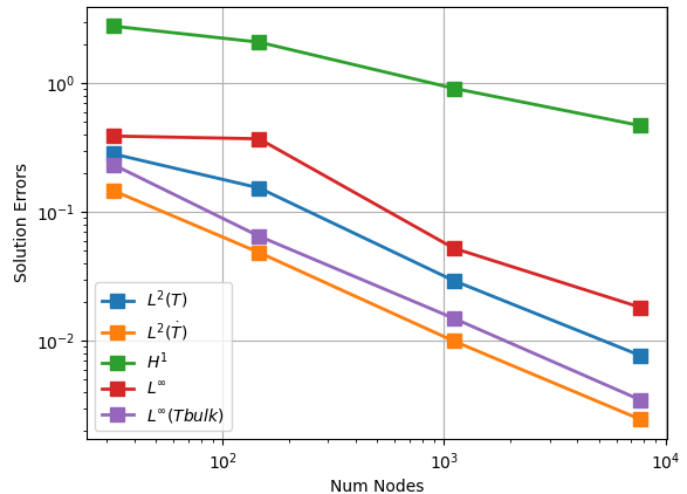
## 2.8.4. Verification of Solution

A manufactured solution is chosen as in Section 2.7.

For each mesh, the errors in the temperature solution at final time are computed in the  $L^2$  norm of  $T$  and  $\dot{T}$ ,  $L^\infty$  and  $H^1$  norms. As in Section 2.4, we see convergence rates for  $L^\infty$  that are slightly less than 2.



Coarse Mesh



Error Norms

Figure 2.8-1.. Transient Heat Conduction: Tet4 Meshes

For input decks see Appendix 12.1.8.

**Table 2.8-1.. Transient Heat Conduction: Convergence Rates for Tet4 Meshes**

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ | $L^\infty(T_{bulk})$ |
|----------|----------|----------------|-------|------------|----------------------|
| 146      | 1.20     | 2.19           | 0.57  | 0.10       | 2.53                 |
| 1105     | 2.45     | 2.34           | 1.22  | 2.89       | 2.17                 |
| 7726     | 2.07     | 2.16           | 1.03  | 1.64       | 2.26                 |

## 2.9. TRANSIENT HEAT CONDUCTION: TET4TET10 MESHES

This problem tests basic transient heat conduction in a 3D domain as in Section 2.8. The geometry consists of a unit cube.

### 2.9.1. Features Tested

Basic transient heat conduction Tet4 analysis on Tet10 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 2.9.2. Boundary Conditions

Identical to Section 2.8

### 2.9.3. Material Parameters

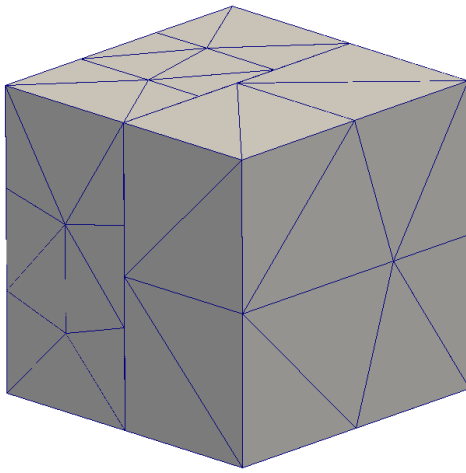
The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 2.9.4. Verification of Solution

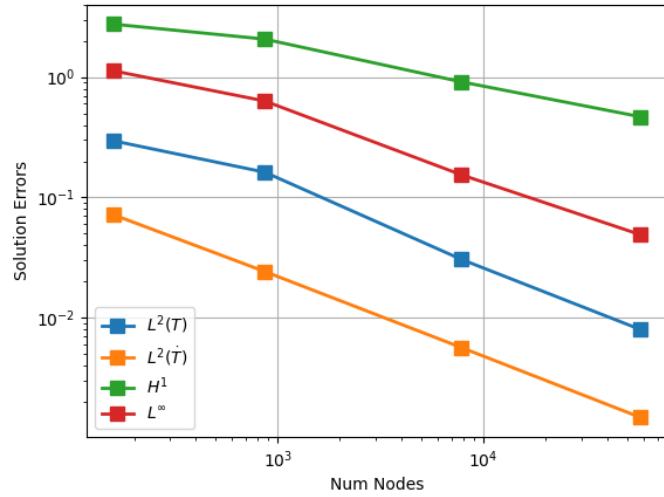
A manufactured solution is chosen as in Section 2.8.

For each mesh, the errors in the temperature solution at final time are computed in the  $L^2$  norm of  $T$  and  $\dot{T}$ ,  $L^\infty$  and  $H^1$  norms. As in Section 2.8, we see convergence rates for  $L^\infty$  that are slightly less than 2.

For input decks see Appendix 12.1.9.



Coarse Mesh



Error Norms

**Figure 2.9-1.. Transient Heat Conduction: Tet4 Solution on Tet10 Meshes**

**Table 2.9-1.. Transient Heat Conduction: Convergence Rates for Tet4 Solution on Tet10 Meshes**

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ |
|----------|----------|----------------|-------|------------|
| 865      | 1.06     | 1.93           | 0.50  | 1.02       |
| 7831     | 2.27     | 1.98           | 1.12  | 1.92       |
| 58211    | 2.01     | 2.00           | 1.00  | 1.72       |

## 2.10. TRANSIENT HEAT CONDUCTION: TET10 MESHES

This problem tests basic transient heat conduction in a 3D domain as in Section 2.7. The geometry consists of a unit cube.

### 2.10.1. Features Tested

Basic transient heat conduction on Tet10 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 2.10.2. Boundary Conditions

Identical to Section 2.7



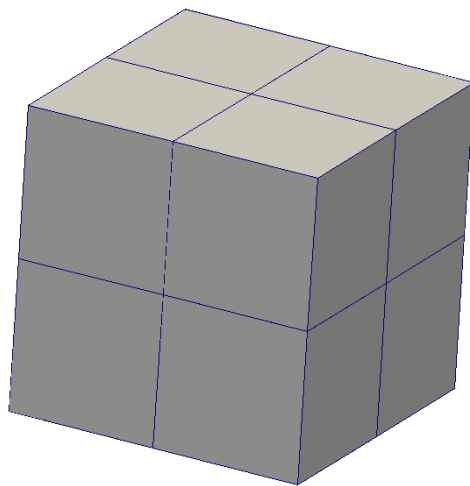
### 2.10.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

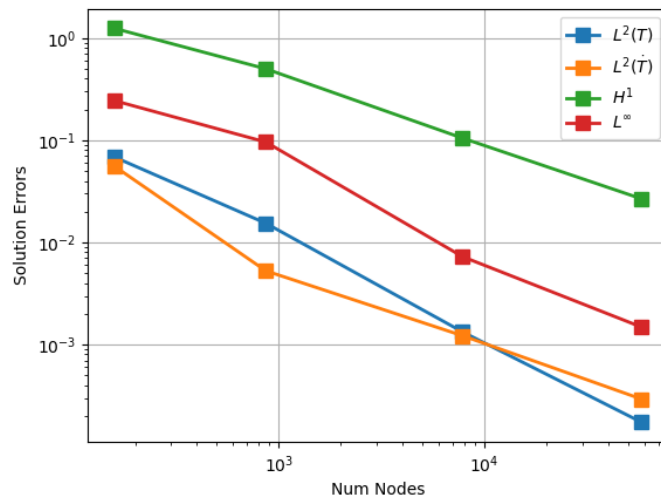
### 2.10.4. Verification of Solution

A manufactured solution is chosen as in Section 2.7.

For each mesh, the errors in the temperature solution at final time are computed in the  $L^2$  norm of  $T$  and  $\dot{T}$ ,  $L^\infty$  and  $H^1$  norms. As in Section 2.6, we see convergence rates for  $L^\infty$  that are slightly less than 2.



Coarse Mesh



Error Norms

**Figure 2.10-1.. Transient Heat Conduction: Tet10 Meshes**

**Table 2.10-1.. Transient Heat Conduction: Convergence Rates for Tet10 Meshes**

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ |
|----------|----------|----------------|-------|------------|
| 865      | 2.64     | 4.17           | 1.61  | 1.65       |
| 7831     | 3.33     | 1.99           | 2.13  | 3.52       |
| 58211    | 3.06     | 2.14           | 2.05  | 2.38       |

For input decks see Appendix 12.1.10.

## 2.11. POSTPROCESS MIN/MAX

### 2.11.1. Problem Description

This problem tests the min/max postprocessors in Aria.

### 2.11.2. Features Tested

min max postprocessors

### 2.11.3. Boundary Conditions

Dirichlet BCs are specified using the exact solution on surfaces 1-4.

A source term is applied within all blocks based on substituting the exact solution into the heat conduction operator.

### 2.11.4. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant.

### 2.11.5. Verification of Solution

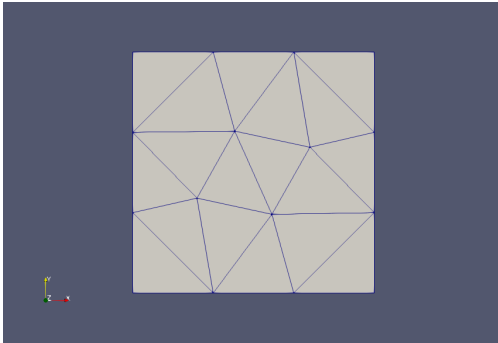
The manufactured solution is

$$\sin(7x) \sin(8y).$$

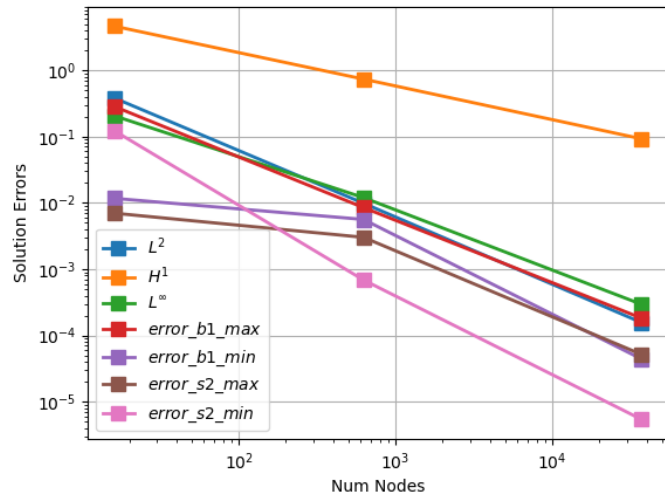
For each uniformly refined mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $H^1$ , and  $L^\infty$  norms and for various postprocessors. Additionally, the nodal maximum and minimum values on both block 1 and surface 2 are computed using Encore postprocessors and the convergence of these values is compared as well. Since the maximum and minimums are nodal, the location of the nodes will reflect the max/min values produced for a given mesh. Provided that the mesh is uniformly refined (without smoothing that may shift the nodal locations), every mesh refinement will produce a better result, dependent on how much closer to the maximum/minimum true solution the new nodes are.

**Table 2.11-1.. Min Max Postprocess: Convergence Rates**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ | $error\_b1\_max$ | $error\_b1\_min$ | $error\_s2\_max$ | $error\_s2\_min$ |
|----------|-------|-------|------------|------------------|------------------|------------------|------------------|
| 625      | 2.00  | 1.00  | 1.55       | 1.92             | 0.40             | 0.46             | 2.83             |
| 37249    | 2.03  | 1.02  | 1.81       | 1.88             | 2.37             | 1.99             | 2.37             |



Coarse Mesh



Error Norms

Figure 2.11-1.. Min Max Postprocess

## 2.12. ADAPTIVITY

This problem is identical to the one in Section 2.4 except that we use adaptive mesh refinement to refine from a coarse base mesh obtained from Cubit.

### 2.12.1. Features Tested

Basic heat conduction on Tet4 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines; adaptive mesh refinement; local error indicators based on jump in heat flux.

### 2.12.2. Boundary Conditions

Same as in Section 2.1.

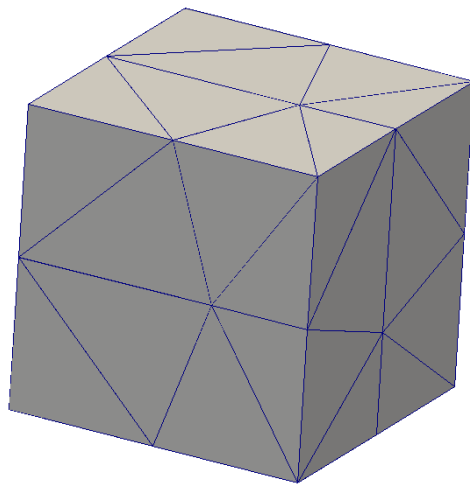
### 2.12.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

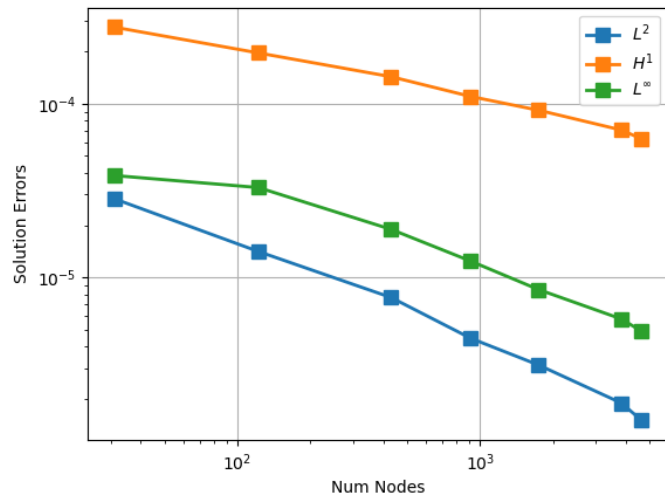
## 2.12.4. Verification of Solution

The mesh is adapted using code from Sierra/Percept that refines tetrahedral meshes without any hanging nodes (conformal meshes only). The element error indicator is computed using a residual-based error indicator in Encore, that computes the integrated jump in the normal heat flux across inter-element faces. The input file is configured to refine elements so that the sum of the error in the refined elements is approximately 75% of the total error in all elements.

Because of variability in the meshes, we expect the error reduction to be noisy. In this case, we use linear least squares to estimate the slope of the error on a log-log plot against mesh size. Since the solution is smooth we also expect the meshes to eventually refine everywhere. We estimate convergence in the usual error norms and observe rates close to the theoretical ones (second order convergence for the  $L^2$  and  $L^\infty$  norms and first order convergence for the  $H^1$  norm). Mesh size is estimated using the formula  $h \approx N^{-1/3}$ , where  $N$  is the number of nodes in the mesh.



Coarse Mesh



Error Norms

**Figure 2.12-1.. Steady Heat Conduction: Tet4 Meshes (Adaptive Mesh Refinement)**

Documentation for the following tests is in progress:

```
1 nlin_verify1/1dnonlin_verify1.test|np8
2 o_2d/aniso_2d.test|np8
3 o_3d/aniso_3d.test|np8
4 shell_2d/cyl_shell_2d.test|np8
5 shell_3d/cyl_shell_3d.test|np8
6 in_C_fi/nonlin_C_fi.test|np1
7 in_C_trap/nonlin_C_trap.test|np1
8 ce_parab/source_parab.test|np1
9 ce_parab_2d/source_parab_2d.test|np1
```

```
10 shell_axi/sph_shell_axi.test|np1  
11 rical_shell/spherical_shell.test|np4  
12 11_nonlin/x11b11_nonlin.test|np1
```

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## 3. THERMAL BOUNDARY CONDITIONS

### 3.1. RADIATIVE HEAT FLUX

This problem tests the radiative flux boundary condition under steady state heat conduction in a 2D domain. The geometry consists of a unit square.

#### 3.1.1. Features Tested

Basic heat conduction on Quad4 meshes; radiative flux boundary conditions with constant emissivity and reference temperature; radiation form factor from C-style user subroutine; temperature boundary conditions from C-style user subroutine and constant values.

#### 3.1.2. Boundary Conditions

At surface 3, the temperature is prescribed from a C-style user subroutine. On surfaces 2 and 4, a constant temperature boundary condition is used. On surface 1, a radiative heat flux condition is prescribed. No source term is needed.

#### 3.1.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant.

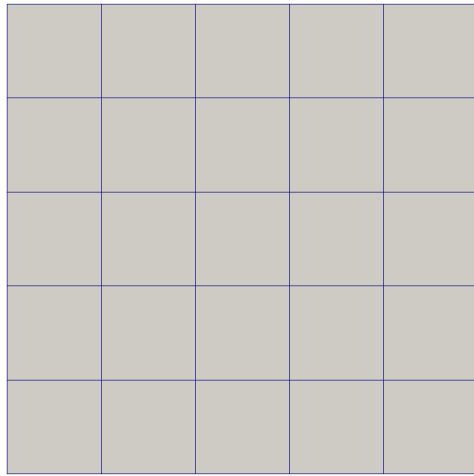
#### 3.1.4. Verification of Solution

A manufactured solution is chosen as

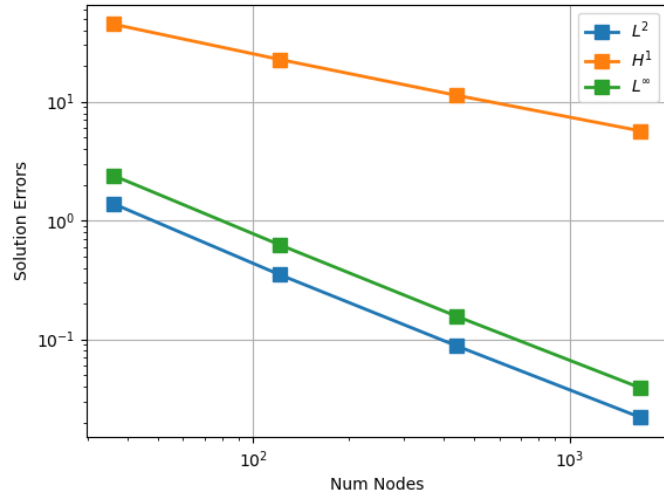
$$T(x, y) = 200 \exp(-\pi y) \sin(\pi x) + 600$$

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $H^1$  and  $L^\infty$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 1, and 2, respectively (within a tolerance).

For input decks see Appendix [12.2.1](#).



Coarse Mesh



Error Norms

**Figure 3.1-1.. Radiative Heat Flux**

**Table 3.1-1.. Radiative Heat Flux: Convergence Rates for Hex8 Meshes**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 121      | 2.27  | 1.13  | 2.21       |
| 441      | 2.14  | 1.07  | 2.15       |
| 1681     | 2.07  | 1.04  | 2.07       |

## 3.2. RADIATIVE HEAT FLUX FROM FORTRAN USER SUBROUTINE

This test verifies that a user-supplied subroutine for convective coefficient and reference temperature (restricted to a surface patch) produces the same results as the equivalent input syntax with constant values. The user subroutine is applied to the entire exterior surface, while the case using constant values must be applied only to specific sidesets that span a portion of the exterior surface.

### 3.2.1. Features Tested

Basic heat conduction on a Hex8 mesh; convective and radiative flux BCs, Fortran user subroutines.

### 3.2.2. Boundary Conditions

Convective and radiative flux BCs are applied to the exterior boundary.



### **3.2.3. Material Parameters**

The values of density, thermal conductivity, emissivity and specific heat are all constant.

### **3.2.4. Verification of Solution**

The test compares Exodus output between two input files. The first does not use any user subroutines and instead relies on sidesets to apply the correct convective and radiative boundary conditions with constant coefficients. The second uses a single convective boundary condition with user subroutines for both the convective coefficient and reference temperature. The two input files produce results that agree to the default tolerances in the exodiff script.

For input decks see Appendix [12.2.2.](#)

## **3.3. CONVECTIVE HEAT FLUX**

This problem tests the convective flux boundary condition under transient heat conduction in a 2D domain. The geometry consists of a unit square.

### **3.3.1. Features Tested**

Transient heat conduction on Quad4 meshes; convective flux boundary conditions with user subroutines for convective coefficient and reference temperature; temperature boundary conditions from C-style user subroutine and constant values.

### **3.3.2. Boundary Conditions**

At surface 3, the temperature is prescribed from a C-style user subroutine. On surfaces 2 and 4, a constant temperature boundary condition is used. On surface 1, a convective heat flux condition is prescribed. No source term is needed. The initial condition is provided by a C-style user subroutine

### **3.3.3. Material Parameters**

The values of density, thermal conductivity, and specific heat are all constant.

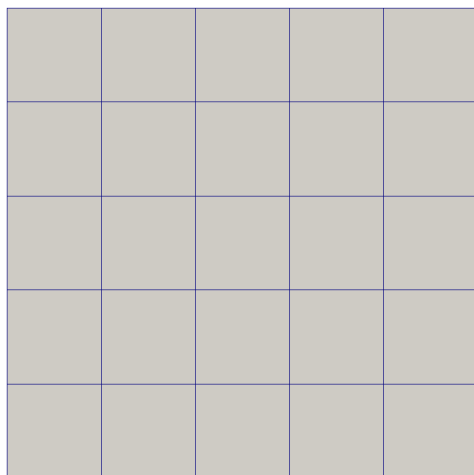
### 3.3.4. Verification of Solution

A manufactured solution is chosen as

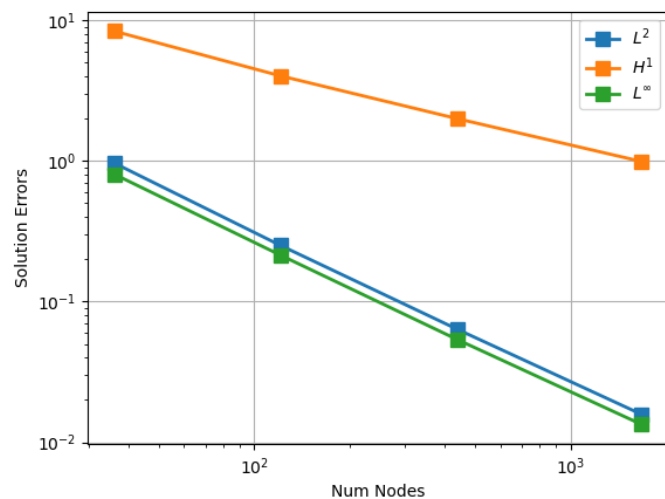
$$T(x, y, t) = 100 \exp(-2\pi^2 t) \sin(\pi x) (\cos(\pi y) + \sin(\pi y))$$

Because the solution is based on eigenfunctions, it satisfies the heat equation with no source term.

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, and 1, respectively (within a tolerance).



Coarse Mesh



Error Norms

Figure 3.3-1.. Convective Heat Flux

Table 3.3-1.. Convective Heat Flux: Convergence Rates for Hex8 Meshes

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 121      | 2.22  | 1.21  | 2.19       |
| 441      | 2.13  | 1.09  | 2.15       |
| 1681     | 2.07  | 1.04  | 2.07       |

For input decks see Appendix [12.2.3](#).

## 3.4. THERMAL CONVECTIVE FLUX (FORTRAN SUB-ROUTINE)

### 3.4.1. Problem Description

This problem tests the convective flux boundary condition with a convective coefficient Fortran subroutine for a steady thermal problem in a 3D domain whose geometry consists of a unit-sized cube.

### 3.4.2. Features Tested

Convective Flux BC, Convective Coefficient Fortran Subroutine, user subroutine, integrated flux, integrated power

### 3.4.3. Boundary Conditions

Convective flux boundary conditions are imposed on surfaces 1 and 2. Dirichlet BCs are specified using the exact solution on surfaces 3-6. A source term is applied within all blocks based on substituting the exact solution into the heat conduction operator.

### 3.4.4. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant.

### 3.4.5. Verification of Solution

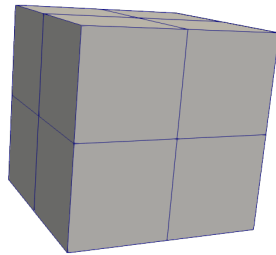
The manufactured solution is

$$T(x, y, z) = (x - x^2)^2(y - y^2)^2(z - z^2)^2 + (z + z^2).$$

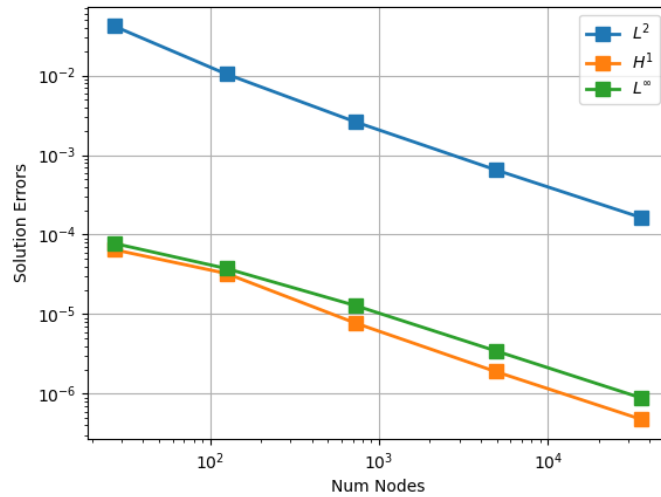
For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $H^1$ , and  $L^\infty$  norms.

**Table 3.4-1.. Thermal Convective BC: Convergence Rates**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 125      | 2.71  | 1.35  | 1.43       |
| 729      | 2.36  | 2.45  | 1.83       |
| 4913     | 2.18  | 2.20  | 2.06       |
| 35937    | 2.09  | 2.09  | 2.06       |



Coarse Mesh



Error Norms

**Figure 3.4-1.. Convergence for 3D thermal steady convective flux BCs.**

## 3.5. THERMAL CONVECTIVE FLUX (USER FIELD FROM EXODUS READ-IN)

### 3.5.1. Problem Description

This problem evaluates a convective flux boundary condition with a convective coefficient and a reference temperature from an exodus file for a steady thermal problem in a 3D domain whose geometry consists of a unit-sized cube.

### 3.5.2. Features Tested

Convective Flux BC, Convective Coefficient, transfers, user subroutine, integrated flux, integrated power

### 3.5.3. Boundary Conditions

Convective flux boundary conditions are imposed on surfaces 1 and 2. Dirichlet BCs are specified using the exact solution on surfaces 3-6. A source term is applied within all blocks based on substituting the exact solution into the heat conduction operator.

### 3.5.4. Material Parameters

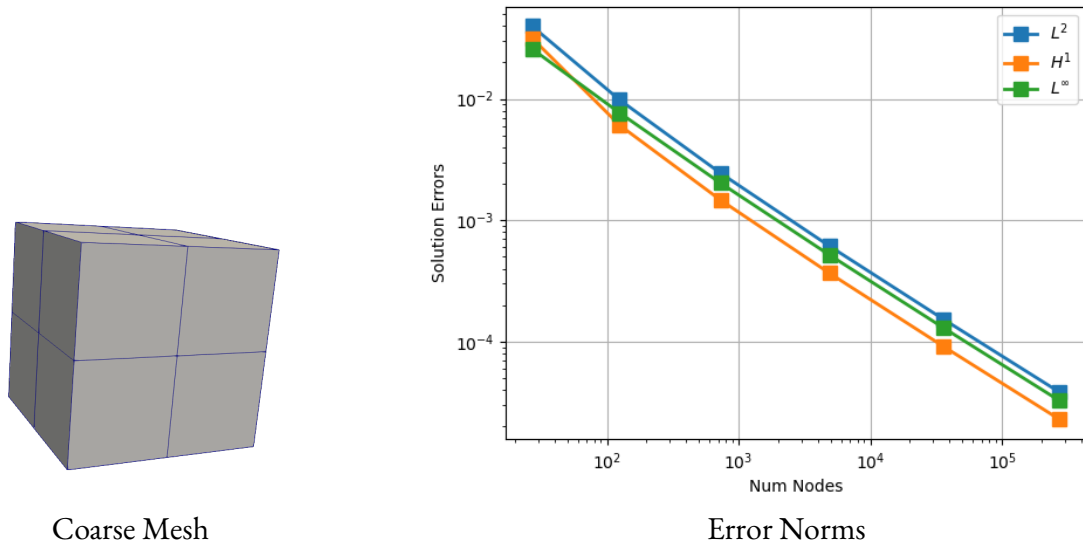
The values of density, thermal conductivity, and specific heat are all constant.

### 3.5.5. Verification of Solution

The manufactured solution is

$$T(x, y, z) = (x - x^2)^2(y - y^2)^2(z - z^2)^2 + (z^2 + z).$$

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $H^1$ , and  $L^\infty$  norms.



**Figure 3.5-1.. Convergence for 3D thermal steady convective flux BCs.**

**Table 3.5-1.. Thermal Convective BC: Convergence Rates**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 125      | 2.74  | 3.24  | 2.38       |
| 729      | 2.36  | 2.41  | 2.26       |
| 4913     | 2.18  | 2.19  | 2.15       |
| 35937    | 2.09  | 2.09  | 2.08       |
| 274625   | 2.05  | 2.05  | 2.04       |

## 3.6. THERMAL RADIATIVE HEAT FLUX

### 3.6.1. Basic Calore-Style BC

#### 3.6.1.1. Problem Description

This problem evaluates a steady thermal solution with radiative heat flux boundary conditions on a 3D unit cube domain.

#### 3.6.1.2. Features Tested

Basic heat conduction, Calore style radiative heat flux BCs, Integrated Flux Output, Integrated Power Output, Hex8 meshes.

#### 3.6.1.3. Boundary Conditions

Dirichlet BCs are specified using the exact solution on surfaces 2-6. On surface 1, a radiative heat flux BC is specified with constant emissivity and a radiation form factor of 0.2. A source term is applied within all blocks based on substituting the exact solution into the heat conduction operator.

#### 3.6.1.4. Material Parameters

The values of density, thermal conductivity, specific heat, and emissivity are all constant values.

#### 3.6.1.5. Verification of Solution

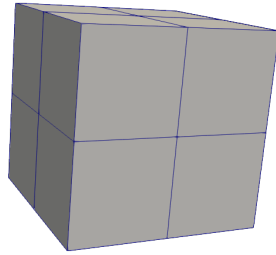
The manufactured solution is

$$T(x, y, z) = (x - x^2)^2(y - y^2)^2(z - z^2)^2 + T - \frac{\partial T}{\partial n}(z^2 - z).$$

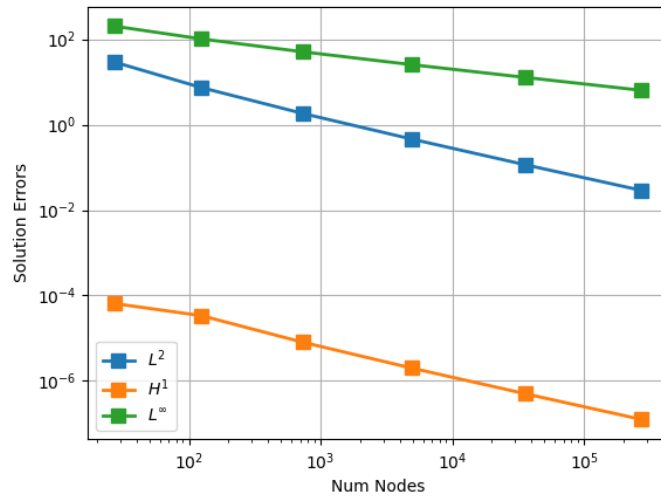
For each discretization, the errors in the temperature solution are computed in the  $L^2$ ,  $H^1$ , and  $L^\infty$  norms. The observed rates of convergence are 2 (except for the  $L^\infty$  norm, with convergence order 1).

**Table 3.6-1.. Thermal Radiative Flux BC: Convergence Rates**

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 125      | 2.71  | 1.29       | 1.36  |
| 729      | 2.36  | 2.43       | 1.18  |
| 4913     | 2.18  | 2.19       | 1.09  |
| 35937    | 2.09  | 2.09       | 1.05  |
| 274625   | 2.05  | 2.05       | 1.02  |



Coarse Mesh



Error Norms

**Figure 3.6-1.. Thermal Radiative Flux**

## 3.6.2. With Fortran Subroutines

### 3.6.2.1. Problem Description

This problem evaluates a steady thermal solution with radiative heat flux boundary conditions using Fortran subroutines on a 3D unit cube domain.

### 3.6.2.2. Features Tested

Basic heat conduction, Calore style radiative heat flux BCs, Integrated Flux Output, Integrated Power Output, Hex8 meshes, Fortran subroutines.

### 3.6.2.3. Boundary Conditions

Dirichlet BCs are specified using the exact solution on surfaces 2-6. On surface 1, a radiative heat flux BC is specified with emissivity, reference temperature, and radiation form factor of provided by Fortran subroutines. A source term is applied within all blocks based on substituting the exact solution into the heat conduction operator.

### 3.6.2.4. Material Parameters

The values of density, thermal conductivity, specific heat, and emissivity are all constant values.

### 3.6.2.5. Verification of Solution

The manufactured solution is

$$T(x, y, z) = (x - x^2)^2(y - y^2)^2(z - z^2)^2 + T - \frac{\partial T}{\partial n}(z^2 - z).$$

For each discretization, the errors in the temperature solution are computed in the  $L^2$ ,  $H^1$ , and  $L^\infty$  norms. The observed rates of convergence are 2 (except for the  $L^\infty$  norm, with convergence order 1).

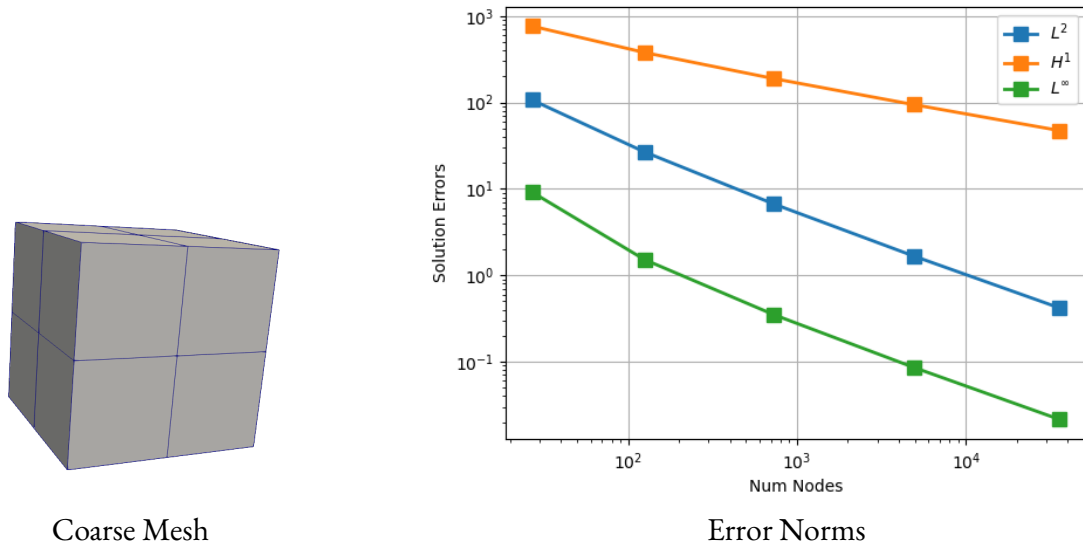


Figure 3.6-2.. Thermal Radiative Flux

Table 3.6-2.. Thermal Radiative Flux BC: Convergence Rates

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 125      | 2.72  | 1.38       | 3.52  |
| 729      | 2.36  | 1.18       | 2.50  |
| 4913     | 2.18  | 1.09       | 2.21  |
| 35937    | 2.09  | 1.05       | 2.10  |

### 3.6.3. With User Subroutines

#### 3.6.3.1. Problem Description

This problem evaluates a steady thermal solution with radiative heat flux boundary conditions with user subroutines on a 3D unit cube domain.



### 3.6.3.2. *Features Tested*

Basic heat conduction, Calore style radiative heat flux BCs, Integrated Flux Output, Integrated Power Output, Hex8 meshes, user subroutines.

### 3.6.3.3. *Boundary Conditions*

Dirichlet BCs are specified using the exact solution on surfaces 2-6. On surface 1, a radiative heat flux BC is specified with emissivity, reference temperature and radiation form factor provided by user subroutines. A source term is applied within all blocks based on substituting the exact solution into the heat conduction operator.

### 3.6.3.4. *Material Parameters*

The values of density, thermal conductivity, specific heat, and emissivity are all constant values.

### 3.6.3.5. *Verification of Solution*

The manufactured solution is

$$T(x, y, z) = (x - x^2)^2(y - y^2)^2(z - z^2)^2 + T - \frac{\partial T}{\partial n}(z^2 - z).$$

For each discretization, the errors in the temperature solution are computed in the  $L^2$ ,  $H^1$ , and  $L^\infty$  norms. The observed rates of convergence are 2 (except for the  $L^\infty$  norm, with convergence order 1).

**Table 3.6-3.. Thermal Radiative Flux BC: Convergence Rates**

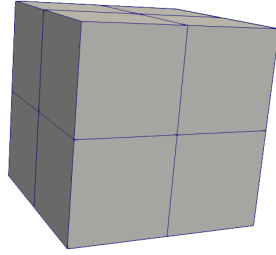
| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 125      | 2.72  | 1.38       | 3.52  |
| 729      | 2.36  | 1.18       | 2.50  |
| 4913     | 2.18  | 1.09       | 2.21  |
| 35937    | 2.09  | 1.05       | 2.10  |

## 3.7. **ADVECTIVE BAR**

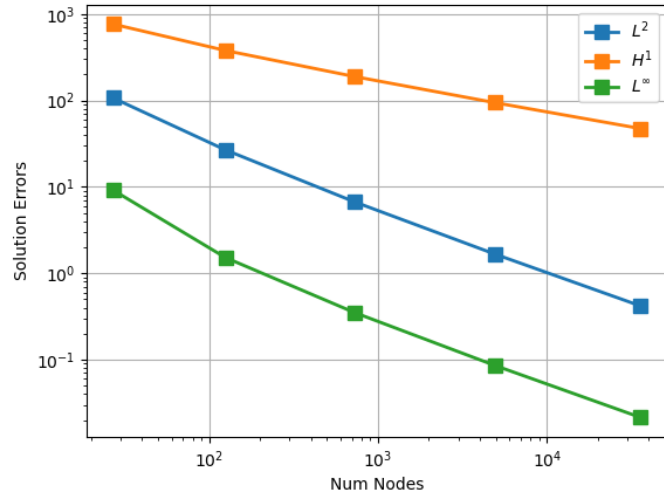
Advective bar model verification tests.

### 3.7.1. **Steady Advection-Diffusion**

The three dimensional Bar2 meshes of one element block are generated in Cubit.



Coarse Mesh



Error Norms

**Figure 3.6-3.. Thermal Radiative Flux**

### 3.7.2. Features Tested

Steady heat conduction on 3D Bar2 meshes, Dirichlet boundary conditions, constant source term, advection and SUPG stabilization.

### 3.7.3. Boundary Conditions

$$T(0) = T(1) = 0$$

### 3.7.4. Material Parameters

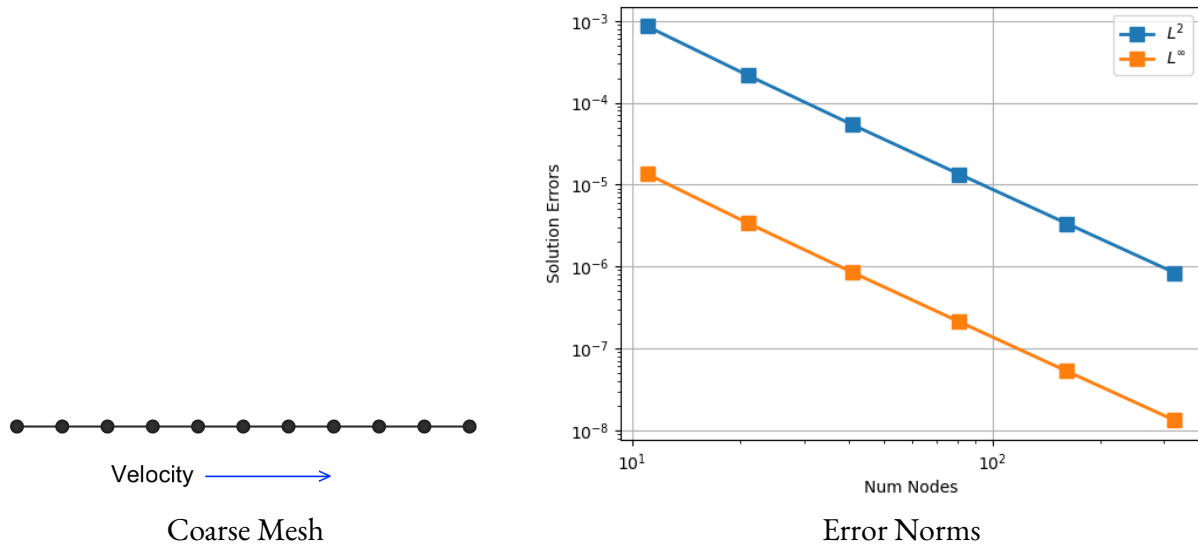
The values of density, thermal conductivity, and specific heat are all constant, equal to one in the block.

### 3.7.5. Verification of Solution

Solution verification is carried out by computing the error in the numerical solution based upon comparison with the analytic solution.

$$T(x) = \frac{1}{\rho CV} \left[ x - \frac{1 - \exp(x\gamma)}{1 - \exp(\gamma)} \right]$$

where  $\gamma = \rho CV/k$  where  $\rho$  is the density,  $C$  is specific heat,  $k$  is the thermal conductivity and  $V$  is the advection velocity. In this test, we find that the convergence rate for the temperature in the  $L^\infty$  and  $L^2$  norms are 2.



**Figure 3.7-1.. Steady Advective Conduction: 3D Bar2 Meshes**

**Table 3.7-1.. Steady Advective Conduction: Convergence Rates for 3D Bar2 Meshes**

| Num Dofs | $L^2$ | $L^\infty$ |
|----------|-------|------------|
| 21       | 2.14  | 2.14       |
| 41       | 2.07  | 2.07       |
| 81       | 2.04  | 2.04       |
| 161      | 2.02  | 2.02       |
| 321      | 2.01  | 2.01       |

### 3.7.6. Transient Advection-Diffusion

The three dimensional Bar2 meshes of one element block are generated in Cubit.

### 3.7.7. Features Tested

Transient heat conduction on 3D Bar2 meshes, Dirichlet boundary conditions and Encore function source term.

### 3.7.8. Boundary Conditions

Dirichlet boundary conditions on the bar ends based upon the manufactured solution  $T(x)$

$$T(0) = T(1) = T_i$$

### 3.7.9. Material Parameters

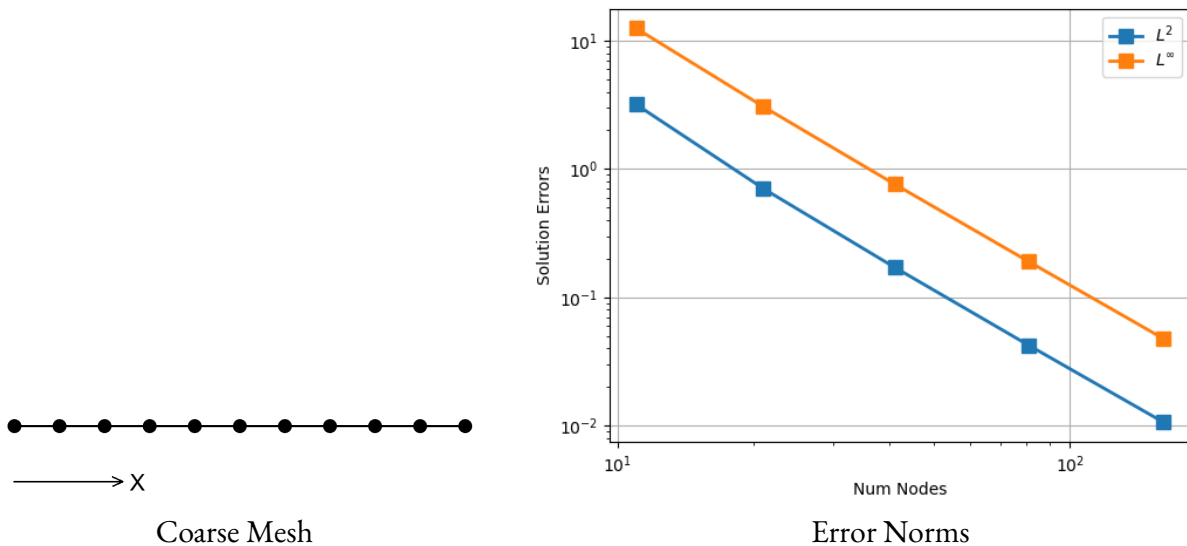
The values of density, thermal conductivity, and specific heat are all constant, equal to one in the bar block.

### 3.7.10. Verification of Solution

Solution verification is carried out by computing the error in the numerical solution based upon comparison with the analytic solution.

$$T(x) = T_i + Atx(x - 1) \exp(-Bt) \exp(-Bx)$$

In this test, we find that the convergence rate for the temperature in the  $L^\infty$  and  $L^2$  norms are 2.



**Figure 3.7-2.. Transient Heat Conduction: 3D Bar2 Meshes**

### 3.7.11. Transient Advection-Diffusion in 2D

The two dimensional Bar2 meshes of one element block are generated in Cubit.

**Table 3.7-2.. Transient Heat Conduction: Convergence Rates for 3D Bar2 Meshes**

| Num Dofs | $L^2$ | $L^\infty$ |
|----------|-------|------------|
| 21       | 2.33  | 2.17       |
| 41       | 2.12  | 2.08       |
| 81       | 2.05  | 2.04       |
| 161      | 2.02  | 2.02       |

### 3.7.12. Features Tested

Transient heat conduction on 2D Bar2 meshes, Dirichlet boundary conditions and Encore function source term.

### 3.7.13. Boundary Conditions

Dirichlet boundary conditions on the bar ends

$$T(0) = T(1) = T_i$$

### 3.7.14. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in the bar block.

### 3.7.15. Verification of Solution

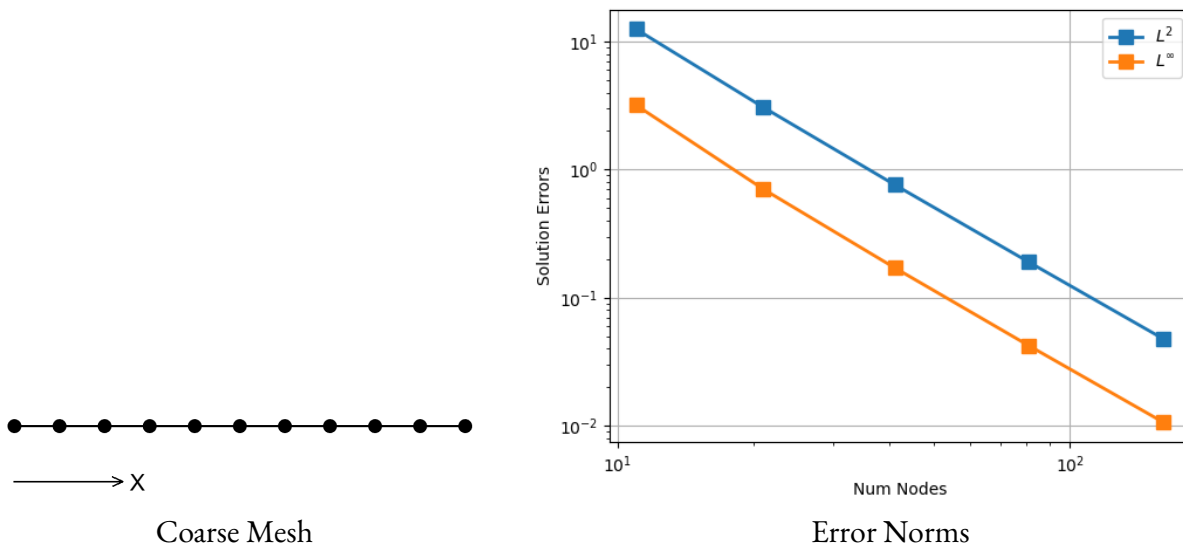
Solution verification is carried out by computing the error in the numerical solution based upon comparison with the analytic solution.

$$T(x) = T_i + Atx(x-1)\exp(-Bt)\exp(-Bx)$$

In this test, we find that the convergence rate for the temperature in the  $L^\infty$  and  $L^2$  norms are 2.

**Table 3.7-3.. Transient Heat Conduction: Convergence Rates for 2D Bar2 Meshes**

| Num Dofs | $L^2$ | $L^\infty$ |
|----------|-------|------------|
| 21       | 2.17  | 2.33       |
| 41       | 2.08  | 2.12       |
| 81       | 2.04  | 2.05       |
| 161      | 2.02  | 2.02       |



**Figure 3.7-3.. Transient Heat Conduction: Bar2 Meshes**

## 3.8. SOLUTION VERIFICATION

This test is for a Mock AFF (including a metal case, foam, mock components, and temperature-dependent properties) that uses extrapolation to determine an approximation to the exact solution as a function of the results from three levels of meshes.

### 3.8.1. Features Tested

Extrapolation, Radiative flux boundary condition

### 3.8.2. Material Parameters

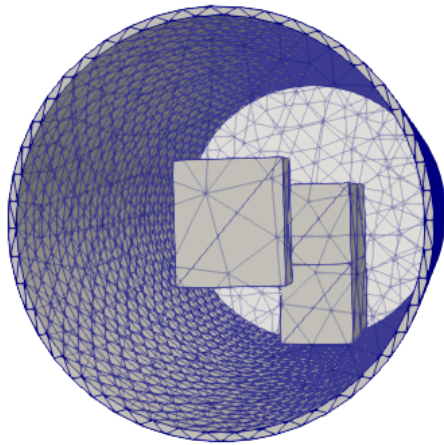
Constant density, emissivity. Temperature dependent user functions for specific heat and thermal conductivity.

### 3.8.3. Verification of Solution

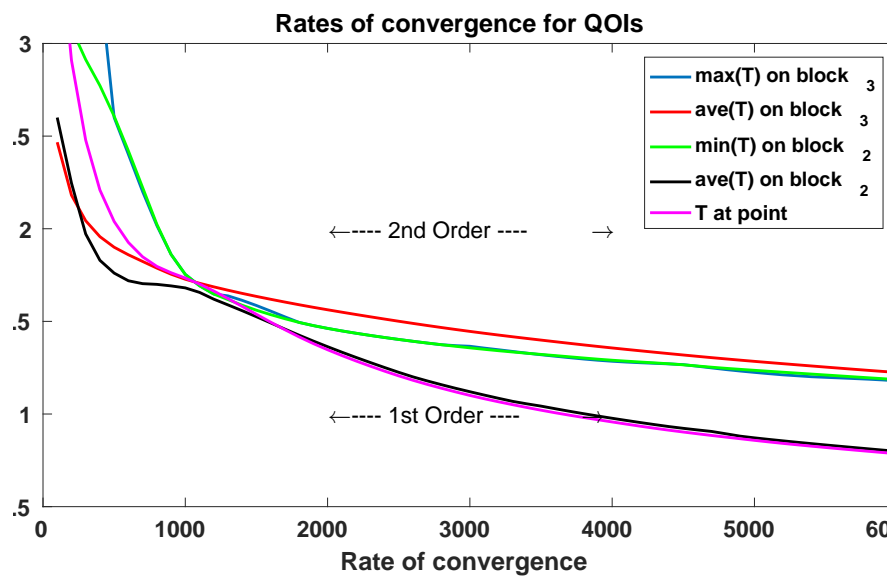
Quantities of interest are the maximum, minimum, and average temperatures on both blocks and points. There is no manufactured solution in this case, instead an extrapolated solution is calculated and used to measure convergence and approximate the absolute error for a given mesh resolution.

Documentation for the following tests is in progress:

```
nic_material_decomposition/organic_material_decomposition.test|np4
```



**Figure 3.8-1.. Mock AFF Solution Verification**



**Figure 3.8-2.. The convergence rates can vary over time and between QOIs**



## 4. THERMAL CONTACT

### 4.1. 1D FLAT CONTACT

This problem tests thermal contact along a flat surface using 3D domains. The geometry consists of two thick blocks, which are in contact along a common flat surface. The mesh nodes on either side of the contact surface are not aligned in general.

In this problem we observe sub-optimal convergence rates in the  $L^\infty$  norm when using Tet elements. This is a known issue with unknown cause.

The contact search tolerances are fixed for all meshes, with a zero tangential and normal tolerances.

#### 4.1.1. Features Tested

Basic heat conduction, tied and resistance thermal contact between non-matching meshes (Hex-Hex, Tet-Tet, Hex-Tet).

#### 4.1.2. Boundary Conditions

The interface between the two blocks is a thermal contact boundary condition. Both tied contact and resistance contact (with finite contact resistance) are tested. The left and right boundary conditions are prescribed using constant values. The remaining boundary conditions are adiabatic. A constant source term is applied in each block (with different signs).

#### 4.1.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

#### 4.1.4. Verification of Solution

A manufactured solution is chosen based on the contact interface at  $x = 0$ :

$$T(x, y, z) = \begin{cases} \frac{1}{2}(1+x)(\gamma+x), & x < 0, \\ 1 + \frac{1}{2}(1-x)(-\gamma+x), & x > 0 \end{cases}$$

where  $\gamma = (2 - R)/(2 + R)$  is a constant depending on the thermal contact resistance  $R$ . Here  $R$  is the inverse of the contact conductance that is provided as a code input. In the case of tied contact,  $R = 0$  and therefore  $\gamma = 1$ . We note that when  $R > 0$ , this exact solution exhibits a jump in temperature across the contact interface.

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, and 1, respectively (within a tolerance).

These rates are observed for the Hex-Hex case; however, both of the cases involving Tet meshes exhibit a reduced order of convergence in the  $L^\infty$  norms (convergence rate about 1.7).

For input decks see Appendix 12.3.1.

#### 4.1.5. Results: Hex8 Tied

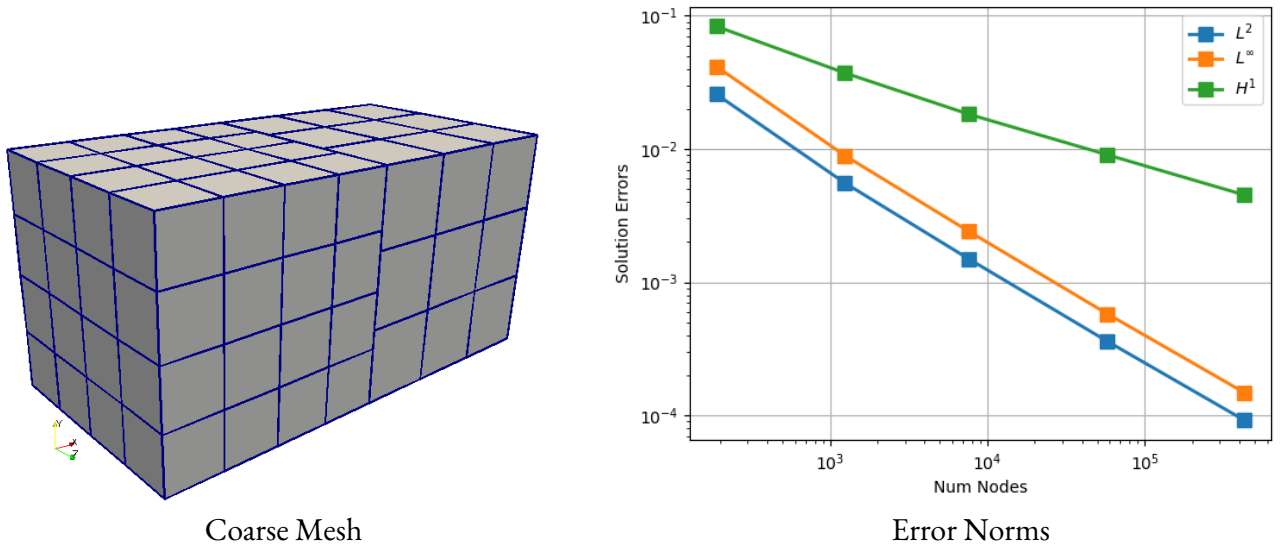
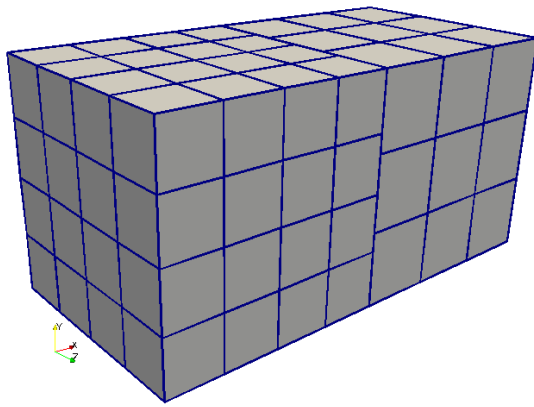


Figure 4.1-1.. 1D Flat Contact: Hex8 Tied

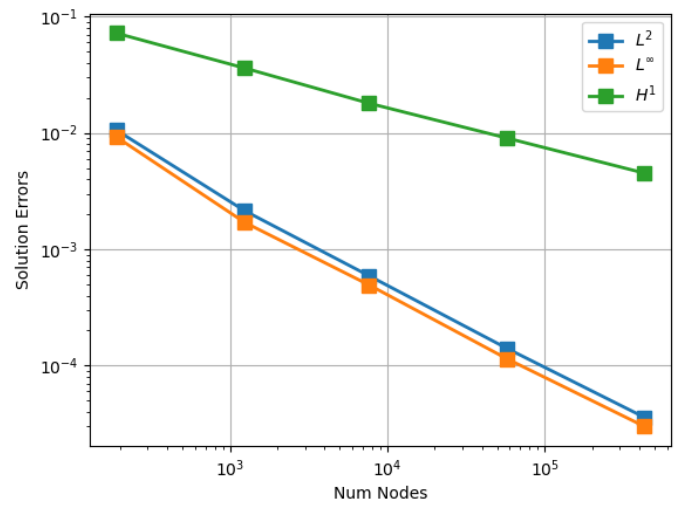
#### 4.1.6. Results: Hex8 Resistance

**Table 4.1-1.. 1D Flat Contact: Convergence Rates for Hex8 Tied**

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 1241     | 2.45  | 2.46       | 1.29  |
| 7657     | 2.17  | 2.16       | 1.18  |
| 57889    | 2.11  | 2.11       | 1.04  |
| 432089   | 2.04  | 2.04       | 1.04  |



Coarse Mesh



Error Norms

**Figure 4.1-2.. 1D Flat Contact: Hex8 Resistance**

**Table 4.1-2.. 1D Flat Contact: Convergence Rates for Hex8 Resistance**

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 1241     | 2.55  | 2.70       | 1.10  |
| 7657     | 2.12  | 2.04       | 1.14  |
| 57889    | 2.13  | 2.17       | 1.03  |
| 432089   | 2.04  | 2.00       | 1.03  |

#### 4.1.7. Results: Tet4 Tied

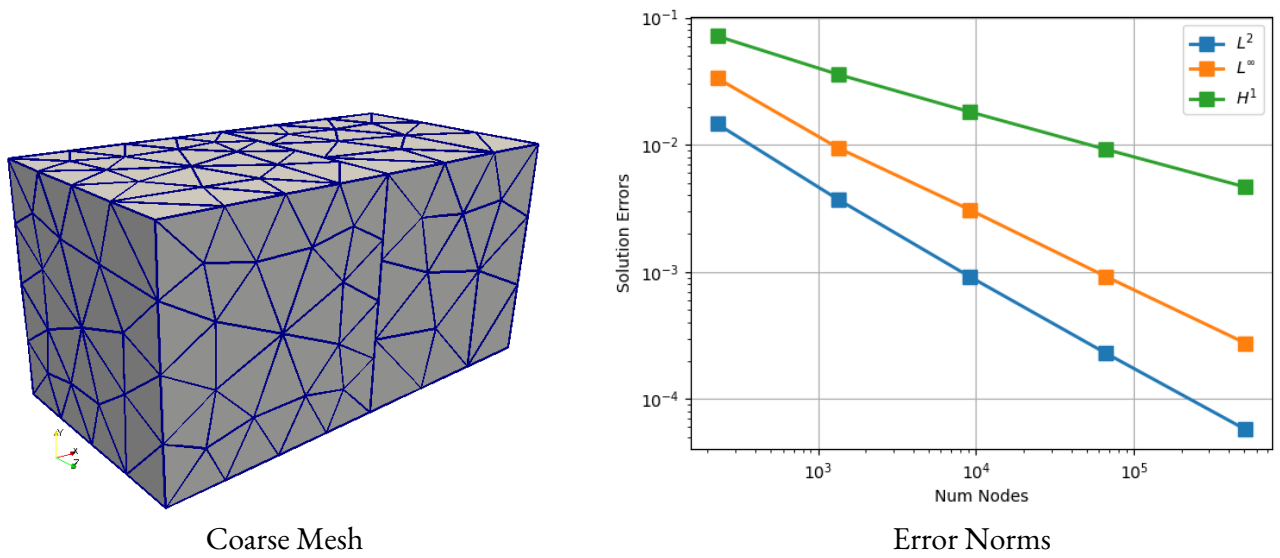
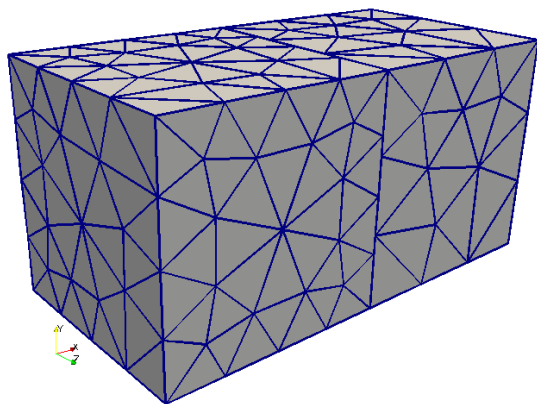


Figure 4.1-3.. 1D Flat Contact: Tet4 Tied

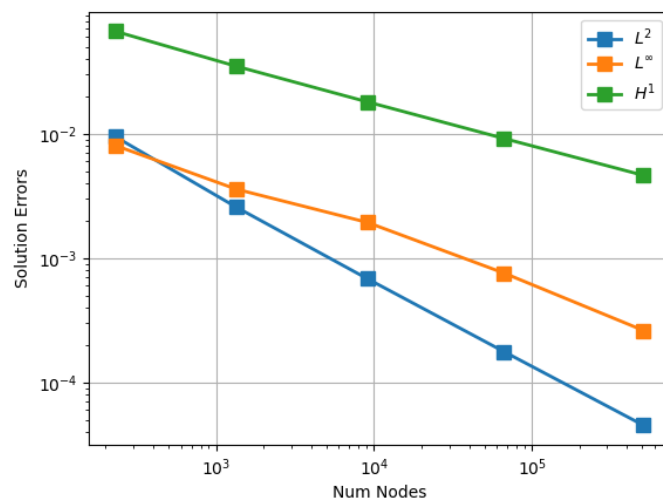
Table 4.1-3.. 1D Flat Contact: Convergence Rates for Tet4 Tied

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 1348     | 2.33  | 2.15       | 1.18  |
| 9102     | 2.19  | 1.75       | 1.06  |
| 66618    | 2.09  | 1.83       | 1.02  |
| 509170   | 2.04  | 1.78       | 1.01  |

#### 4.1.8. Results: Tet4 Resistance



Coarse Mesh



Error Norms

**Figure 4.1-4.. 1D Flat Contact: Tet4 Resistance**

**Table 4.1-4.. 1D Flat Contact: Convergence Rates for Tet4 Resistance**

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 1348     | 2.22  | 1.37       | 1.10  |
| 9102     | 2.08  | 0.96       | 1.04  |
| 66618    | 2.04  | 1.42       | 1.02  |
| 509170   | 2.01  | 1.57       | 1.01  |

#### 4.1.9. Results: Hex8-Tet4 Tied

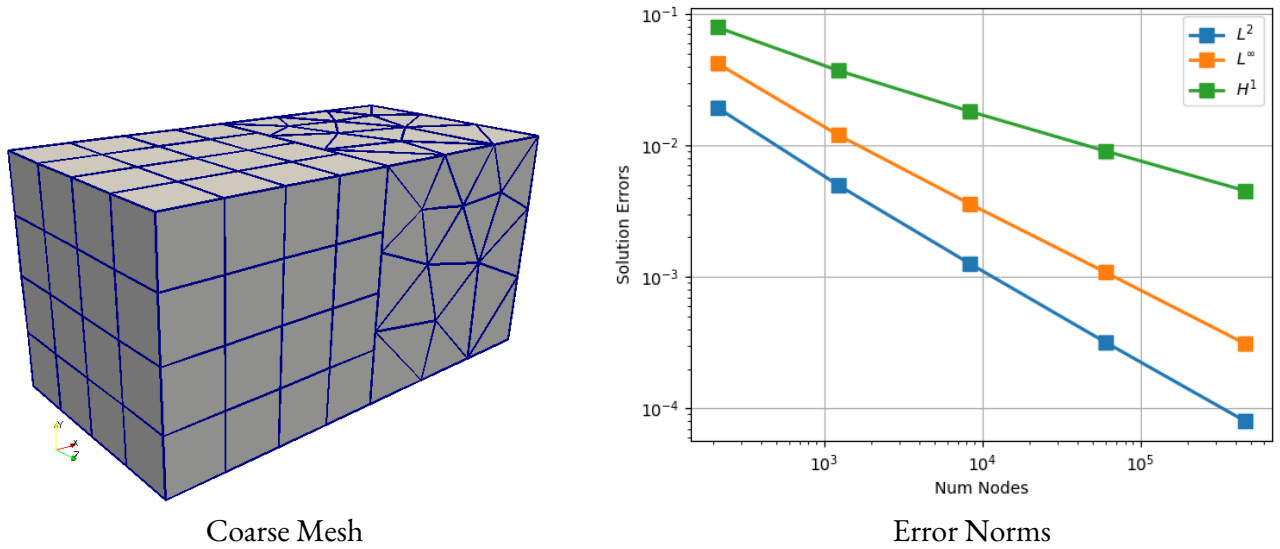


Figure 4.1-5.. 1D Flat Contact: Hex8-Tet4 Tied

Table 4.1-5.. 1D Flat Contact: Convergence Rates for Hex8-Tet4 Tied

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 1231     | 2.32  | 2.15       | 1.29  |
| 8284     | 2.16  | 1.89       | 1.12  |
| 60566    | 2.09  | 1.82       | 1.05  |
| 462762   | 2.04  | 1.84       | 1.02  |

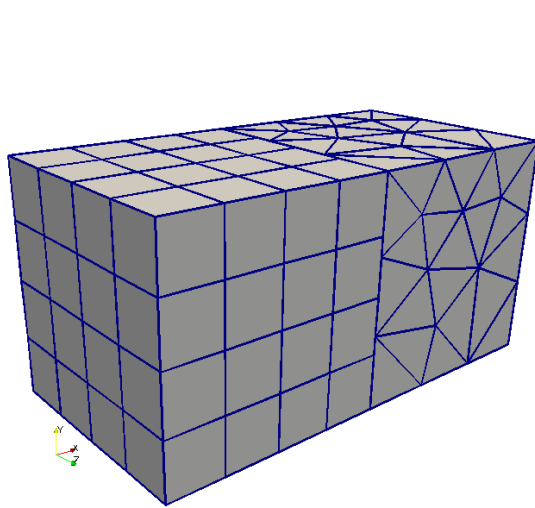
#### 4.1.10. Results: Hex8-Tet4 Resistance

### 4.2. 3D CURVED CONTACT

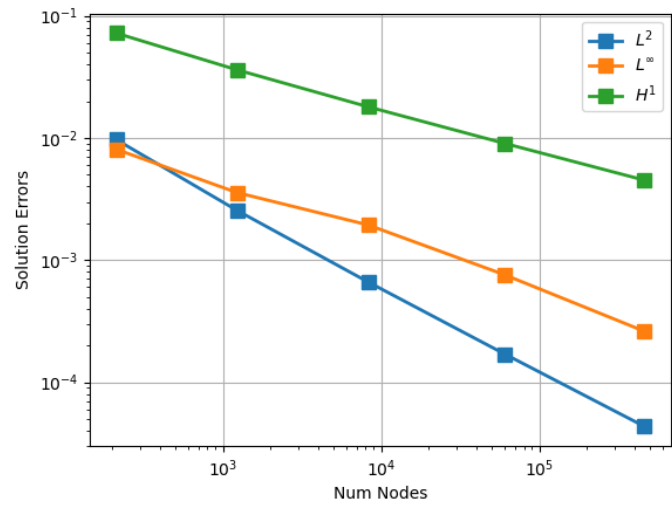
This problem tests thermal contact along a curved surface in 3D. The geometry consists of two thick spherical shells, which are in contact along a shared surface. The mesh nodes on either side of the contact surface are not aligned in general.

In this problem we observe sub-optimal convergence rates in the  $L^\infty$  norm when using tet elements. This is a known issue with unknown cause.

The contact search tolerances are fixed for all meshes, with a zero tangential tolerance and a normal tolerance large enough to insure a proper contact search on the coarsest mesh.



Coarse Mesh



Error Norms

**Figure 4.1-6.. 1D Flat Contact: Hex8-Tet4 Resistance**

**Table 4.1-6.. 1D Flat Contact: Convergence Rates for Hex8-Tet4 Resistance**

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 1231     | 2.28  | 1.39       | 1.18  |
| 8284     | 2.12  | 0.96       | 1.09  |
| 60566    | 2.05  | 1.42       | 1.05  |
| 462762   | 2.01  | 1.57       | 1.02  |

### 4.2.1. Features Tested

Basic heat conduction, tied thermal contact between non-matching meshes (hex-hex, tet-tet, hex-tet).

### 4.2.2. Boundary Conditions

The interface between the two blocks is a tied thermal contact boundary condition. The outer and inner boundary conditions are prescribed at the nodes using the analytic solution.

### 4.2.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

#### 4.2.4. Verification of Solution

A manufactured solution is chosen as

$$T(x, y, z) = -3x^2z - 3y^2z + 2z^3$$

This solution is harmonic, implying that no source term is needed for the steady state heat equation with constant conductivity.

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, and 1, respectively (within a tolerance).

These rates are observed for the hex-hex case; however, both of the cases involving tet meshes exhibit a reduced order of convergence in the  $L^\infty$  norms (convergence rate about 1.7).

For input decks see Appendix 12.3.2.

#### 4.2.5. Results: Hex8-Hex8 Contact

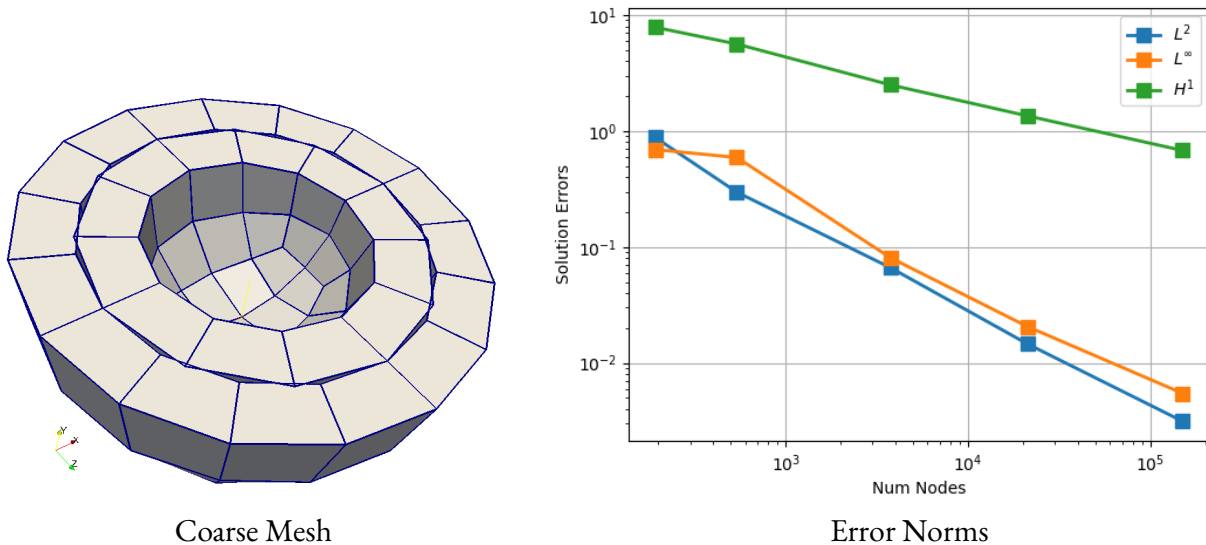


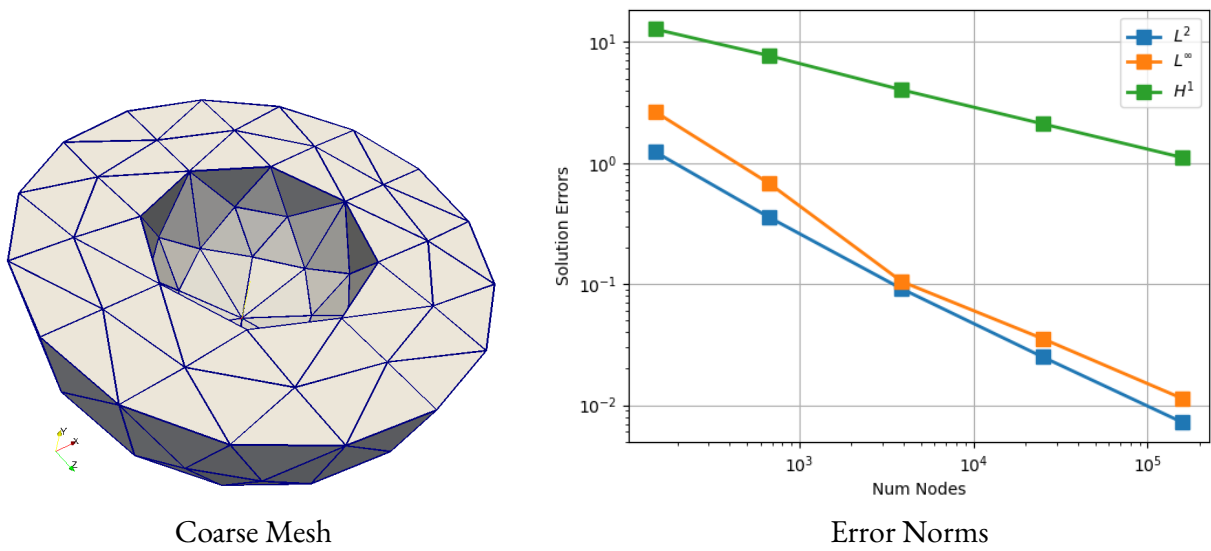
Figure 4.2-1.. 3D Curved Contact: Hex8-Hex8 Case



**Table 4.2-1.. 3D Curved Contact: Convergence Rates for Hex8-Hex8**

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 540      | 3.16  | 0.46       | 0.96  |
| 3752     | 2.32  | 3.08       | 1.25  |
| 21216    | 2.62  | 2.37       | 1.07  |
| 150744   | 2.35  | 2.02       | 1.06  |

### 4.2.6. Results: Tet4-Tet4 Contact



**Figure 4.2-2.. 3D Curved Contact: Tet4-Tet4 Case**

**Table 4.2-2.. 3D Curved Contact: Convergence Rates for Tet4-Tet4**

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 674      | 2.47  | 2.71       | 1.00  |
| 3881     | 2.33  | 3.20       | 1.11  |
| 25008    | 2.09  | 1.75       | 1.04  |
| 159147   | 2.02  | 1.85       | 1.04  |

### 4.2.7. Results: Hex8-Tet4 Contact

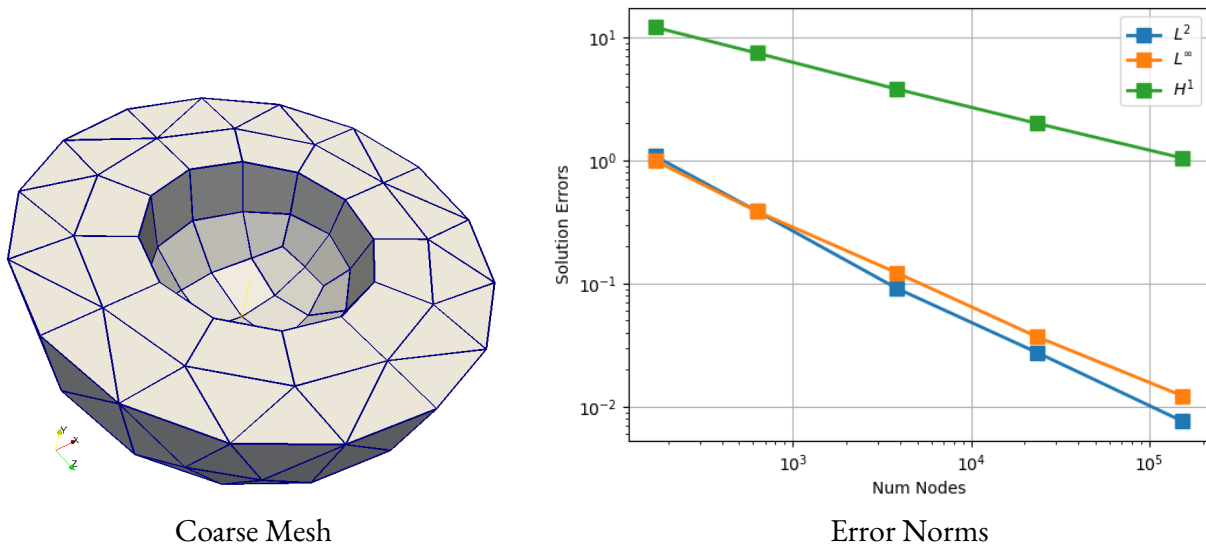


Figure 4.2-3.. 3D Curved Contact: Hex8-Tet4 Case

Table 4.2-3.. 3D Curved Contact: Convergence Rates for Hex8-Tet4

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 630      | 2.34  | 2.16       | 1.11  |
| 3830     | 2.40  | 1.91       | 1.12  |
| 23416    | 1.98  | 1.98       | 1.06  |
| 153671   | 2.06  | 1.77       | 1.04  |

## 4.3. STEADY HEX8 CONTACT

This problem tests basic steady state heat conduction in a 3D domain. The geometry consists of a unit cube.

### 4.3.1. Features Tested

Basic heat conduction on Hex8 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 4.3.2. Boundary Conditions

At surfaces 4 and 6, the temperature is prescribed as a constant value. On surfaces 3 and 5, a heat flux condition is prescribed using a sum of a constant heat flux and a heat flux from an Encore function (user subroutine). On surfaces 1 and 2, heat flux condition is prescribed using a sum of a convective flux boundary condition (with constant flux and convective coefficient) and a heat flux from an Encore function (user subroutine). Within the domain a source term is prescribed using a sum of a constant source and an Encore function (user subroutine).

### 4.3.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 4.3.4. Verification of Solution

A manufactured solution is chosen as

$$T(x, y, z) = 1 + (x - x^2)^2(y - y^2)^2(z - z^2)^2.$$

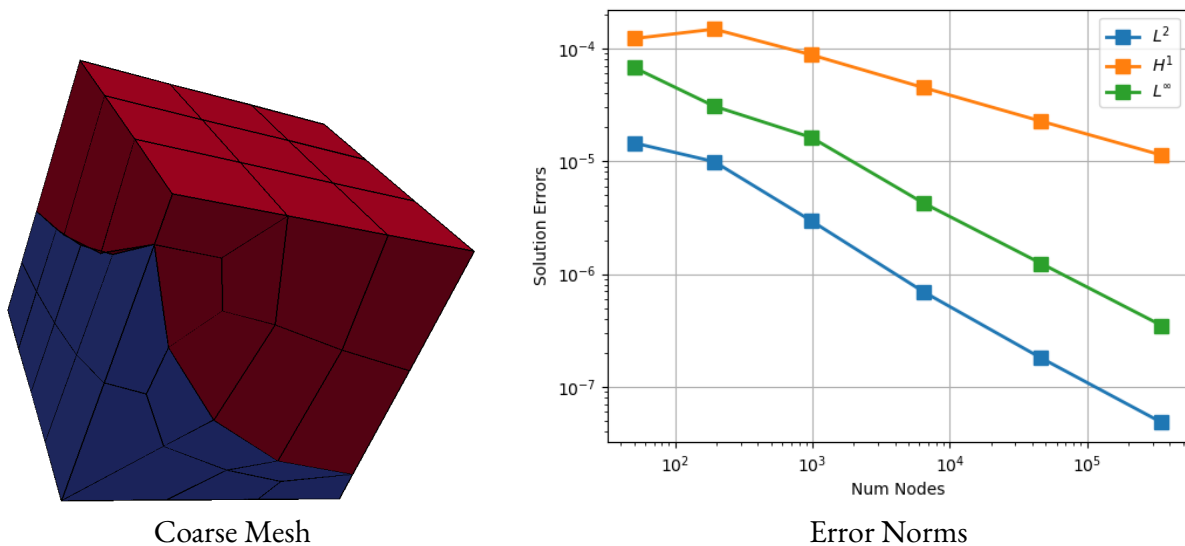
The source and heat flux user subroutines are chosen so that the solution satisfies the heat equation with the correct boundary conditions.

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, and 1, respectively (within a tolerance).

**Table 4.3-1.. Steady Tied Contact: Convergence Rates for Hex8 Meshes**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 192      | 0.86  | -0.44 | 1.76       |
| 982      | 2.22  | 0.97  | 1.18       |
| 6419     | 2.31  | 1.07  | 2.13       |
| 46277    | 2.06  | 1.04  | 1.87       |
| 350649   | 1.95  | 1.02  | 1.88       |

For input decks see Appendix [12.3.3](#).



**Figure 4.3-1.. Steady Tied Contact: Hex8 Meshes**

## 4.4. STEADY HEX20 CONTACT

This problem tests basic steady state heat conduction in a 3D domain. The geometry consists of a unit cube. A variety of different source terms and boundary conditions are simultaneously applied. The exact solution is a manufactured solution.

### 4.4.1. Features Tested

Basic heat conduction on Hex20 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 4.4.2. Boundary Conditions

At surfaces 4 and 6, the temperature is prescribed as a constant value. On surfaces 3 and 5, a heat flux condition is prescribed using a sum of a constant heat flux and a heat flux from an Encore function (user subroutine). On surfaces 1 and 2, heat flux condition is prescribed using a sum of a convective flux boundary condition (with constant flux and convective coefficient) and a heat flux from an Encore function (user subroutine). Within the domain a source term is prescribed using a sum of a constant source and an Encore function (user subroutine).

### 4.4.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

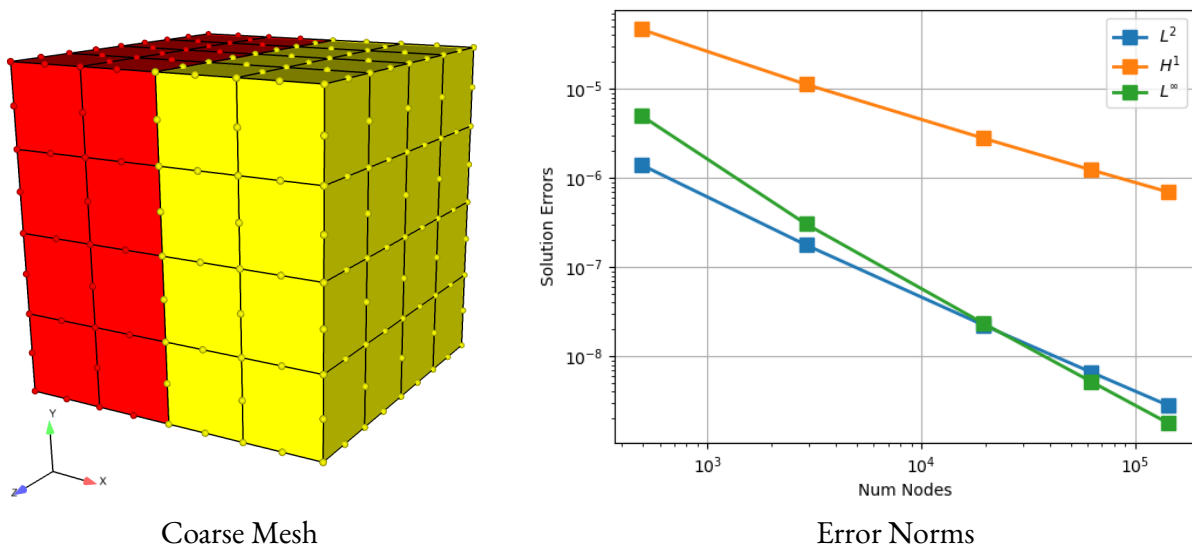
### 4.4.4. Verification of Solution

A manufactured solution is chosen as

$$T(x, y, z) = 1 + (x - x^2)^2(y - y^2)^2(z - z^2)^2.$$

The source and heat flux user subroutines are chosen so that the solution satisfies the heat equation with the correct boundary conditions.

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, and 1, respectively (within a tolerance).



**Figure 4.4-1.. Steady Heat Conduction: Hex20 Meshes**

**Table 4.4-1.. Steady Heat Conduction: Convergence Rates for Hex20 Meshes**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 2898     | 3.50  | 2.40  | 4.74       |
| 19618    | 3.25  | 2.19  | 4.03       |
| 62450    | 3.15  | 2.10  | 3.89       |
| 143682   | 3.10  | 2.07  | 3.87       |

For input decks see Appendix [12.3.4](#).

## 4.5. STEADY HEX27 CONTACT

This problem tests basic steady state heat conduction in a 3D domain. The geometry consists of a unit cube. A variety of different source terms and boundary conditions are simultaneously applied. The exact solution is a manufactured solution.

### 4.5.1. Features Tested

Basic heat conduction on Hex27 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 4.5.2. Boundary Conditions

At surfaces 4 and 6, the temperature is prescribed as a constant value. On surfaces 3 and 5, a heat flux condition is prescribed using a sum of a constant heat flux and a heat flux from an Encore function (user subroutine). On surfaces 1 and 2, heat flux condition is prescribed using a sum of a convective flux boundary condition (with constant flux and convective coefficient) and a heat flux from an Encore function (user subroutine). Within the domain a source term is prescribed using a sum of a constant source and an Encore function (user subroutine).

### 4.5.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 4.5.4. Verification of Solution

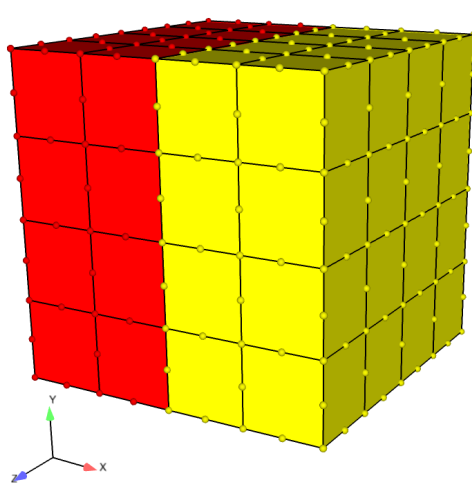
A manufactured solution is chosen as

$$T(x, y, z) = 1 + (x - x^2)^2(y - y^2)^2(z - z^2)^2.$$

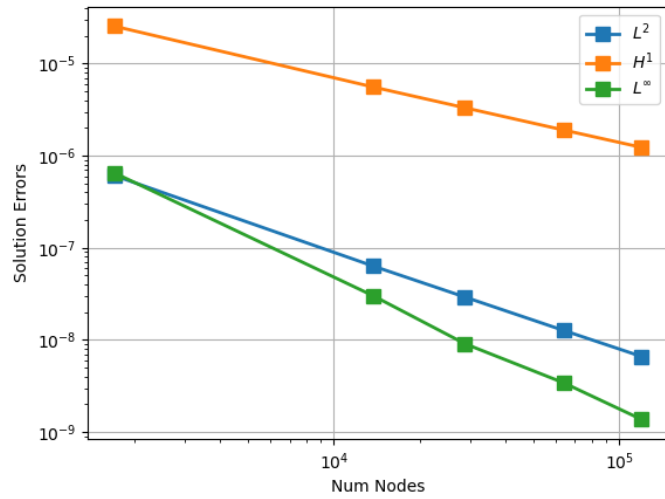
The source and heat flux user subroutines are chosen so that the solution satisfies the heat equation with the correct boundary conditions.

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, and 1, respectively (within a tolerance).

For input decks see Appendix [12.3.5](#).



Coarse Mesh



Error Norms

**Figure 4.5-1.. Steady Heat Conduction: Hex27 Meshes**

**Table 4.5-1.. Steady Heat Conduction: Convergence Rates for Hex27 Meshes**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 13754    | 3.25  | 2.18  | 4.41       |
| 28830    | 3.13  | 2.10  | 4.85       |
| 63882    | 3.14  | 2.09  | 3.68       |
| 120050   | 3.11  | 2.07  | 4.38       |

## 4.6. STEADY TET4 CONTACT

This problem is identical to the one in Section 2.1 except that unstructured Tet4 meshes are used instead.

### 4.6.1. Features Tested

Basic heat conduction on Tet4 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 4.6.2. Boundary Conditions

Same as in Section 2.1.

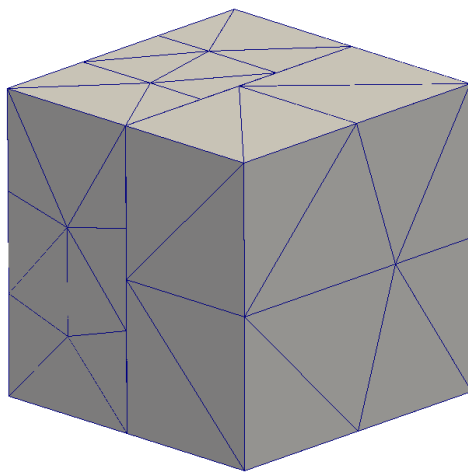


### 4.6.3. Material Parameters

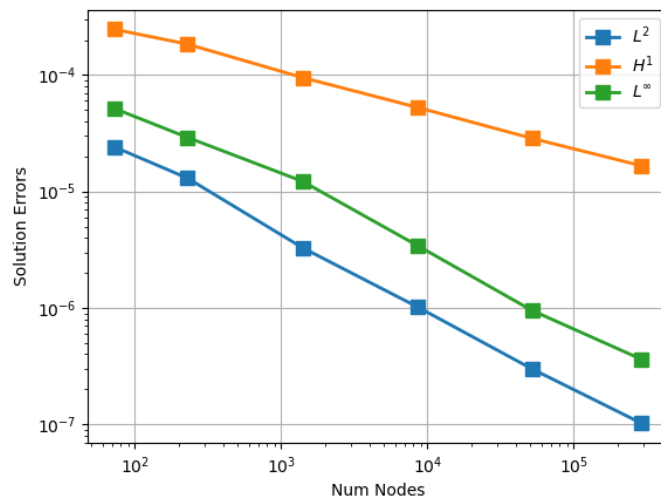
The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 4.6.4. Verification of Solution

Same as in Section 2.1.



Coarse Mesh



Error Norms

Figure 4.6-1.. Steady Tied Contact: Tet4 Meshes

Table 4.6-1.. Steady Tied Contact: Convergence Rates for Tet4 Meshes

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 229      | 1.61  | 0.76  | 1.49       |
| 1402     | 2.29  | 1.10  | 1.44       |
| 8535     | 1.93  | 0.98  | 2.11       |
| 51622    | 2.05  | 1.02  | 2.13       |
| 291153   | 1.88  | 0.94  | 1.68       |

For input decks see Appendix 12.3.6.

## 4.7. STEADY TET4TET10 CONTACT

This problem is identical to the one in Section 2.1 except that unstructured Tet4 meshes are used instead.

### 4.7.1. Features Tested

Basic heat conduction on Tet4 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 4.7.2. Boundary Conditions

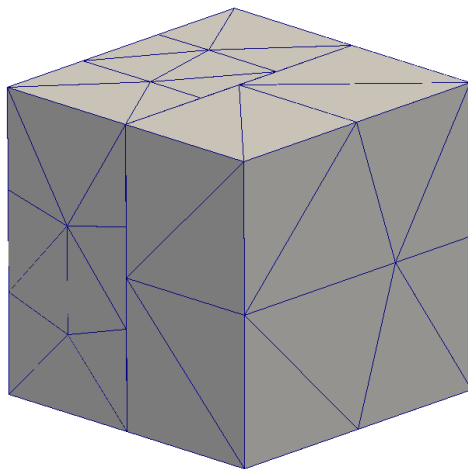
Same as in Section 2.1.

### 4.7.3. Material Parameters

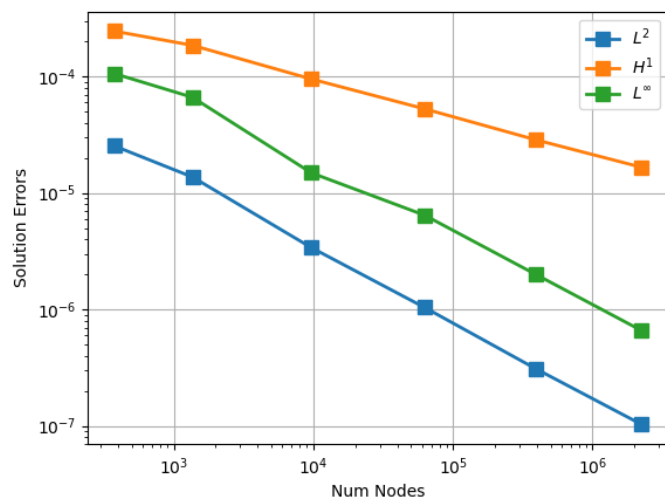
The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 4.7.4. Verification of Solution

Same as in Section 2.1.



Coarse Mesh



Error Norms

**Figure 4.7-1.. Steady Tied Contact: Tet4 Meshes**

For input decks see Appendix 12.3.7.

**Table 4.7-1.. Steady Tied Contact: Convergence Rates for Tet4 Meshes**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 1364     | 1.43  | 0.65  | 1.10       |
| 9663     | 2.13  | 1.02  | 2.28       |
| 62724    | 1.90  | 0.95  | 1.35       |
| 392046   | 1.98  | 1.00  | 1.92       |
| 2249757  | 1.89  | 0.93  | 1.90       |

## 4.8. STEADY TET10 CONTACT

This problem is identical to the one in Section 2.1 except that unstructured Tet10 meshes are used instead.

### 4.8.1. Features Tested

Basic heat conduction on Tet10 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 4.8.2. Boundary Conditions

Same as in Section 2.1.

### 4.8.3. Material Parameters

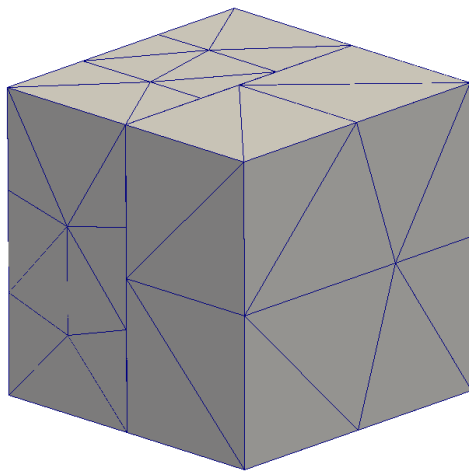
The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 4.8.4. Verification of Solution

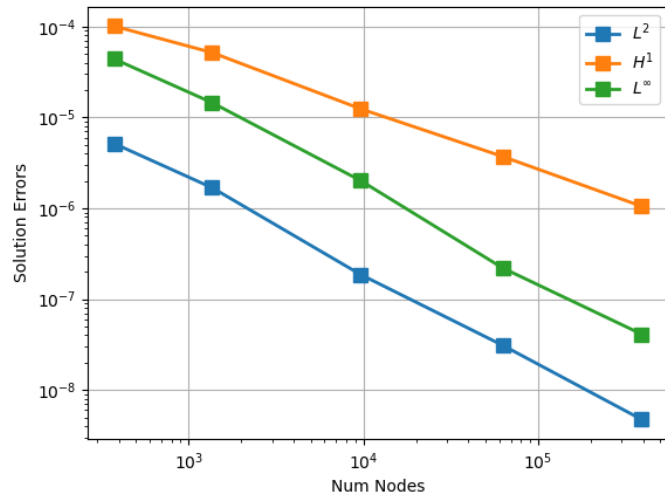
Same as in Section 2.1.

**Table 4.8-1.. Steady Tied Contact: Convergence Rates for Tet10 Meshes**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 1364     | 2.60  | 1.55  | 2.57       |
| 9663     | 3.37  | 2.19  | 3.03       |
| 62724    | 2.87  | 1.93  | 3.56       |
| 392046   | 3.08  | 2.06  | 2.74       |



Coarse Mesh



Error Norms

**Figure 4.8-1.. Steady Tied Contact: Tet10 Meshes**

For input decks see Appendix [12.3.8](#).

## 4.9. STEADY TET10 DASH CONTACT

This problem is identical to the one in Section [2.1](#) except that unstructured Tet10 meshes are used instead.

### 4.9.1. Features Tested

Basic heat conduction on Tet10 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 4.9.2. Boundary Conditions

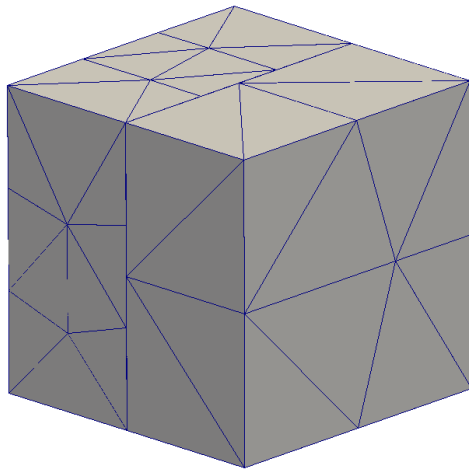
Same as in Section [2.1](#).

### 4.9.3. Material Parameters

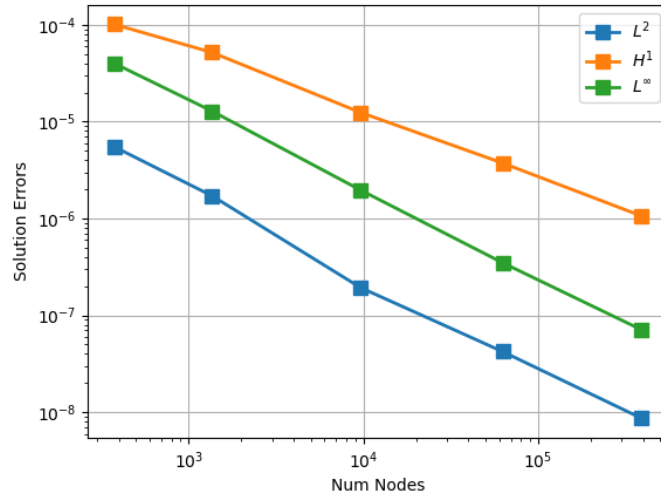
The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

## 4.9.4. Verification of Solution

Same as in Section 2.1.



Coarse Mesh



Error Norms

Figure 4.9-1.. Steady Tied Dash Contact: Tet10 Meshes

Table 4.9-1.. Steady Tied DASH Contact: Convergence Rates for Tet10 Meshes

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 1364     | 2.70  | 1.57  | 2.63       |
| 9663     | 3.35  | 2.19  | 2.88       |
| 62724    | 2.43  | 1.92  | 2.76       |
| 392046   | 2.60  | 2.06  | 2.61       |

For input decks see Appendix 12.3.9.

## 4.10. TRANSIENT TET4TET10 CONTACT

This problem tests basic transient heat conduction with contact in a 3D domain. The geometry consists of two halves of a unit cube meshed with Tet10 elements. The problem is solved using Tet4 interpolation and applying thermal contact at the common interface between the two domains.

### 4.10.1. Features Tested

Basic heat conduction on Tet10 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

## 4.10.2. Boundary Conditions

Using a manufactured solution, Dirichlet boundary conditions are applied on all the non-contact exposed faces.

## 4.10.3. Material Parameters

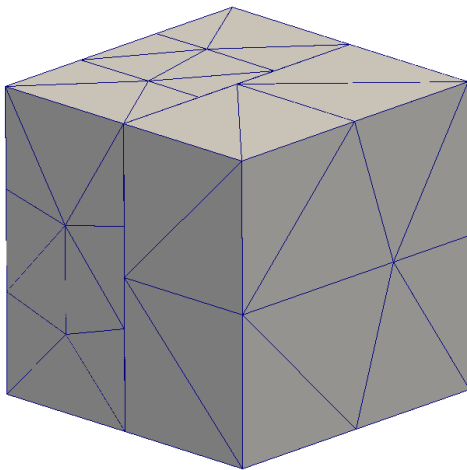
The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

## 4.10.4. Verification of Solution

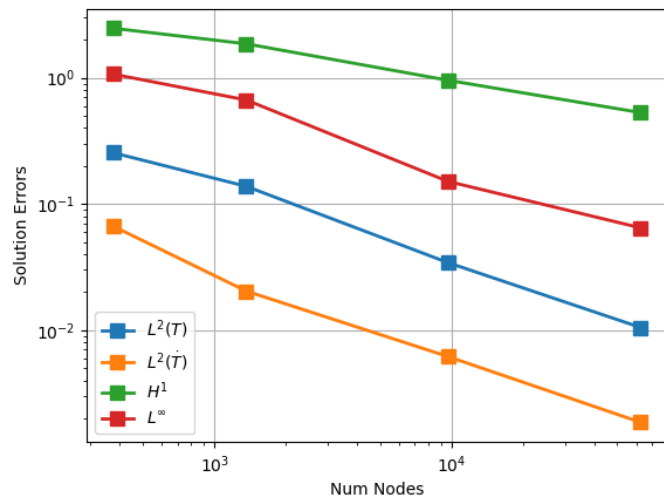
A manufactured solution is chosen as

$$T(x, y, z, t) = (x - x^2)^2 (y - y^2)^2 (z - z^2)^2 m(t) + 1,$$
$$m(t) = 10^4 [1. - \exp(-t) + t * \exp(-(t - 1.0) * (t - 1.0))];$$

The source and heat flux user subroutines are chosen so that the solution satisfies the heat equation with the correct boundary conditions.



Coarse Mesh



Error Norms

**Figure 4.10-1.. Transient Tied Contact: Tet10 Meshes**

For input decks see Appendix [12.3.10](#).

**Table 4.10-1.. Transient Tied Contact: Convergence Rates for Tet10 Meshes**

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ |
|----------|----------|----------------|-------|------------|
| 1364     | 1.43     | 2.75           | 0.65  | 1.10       |
| 9663     | 2.13     | 1.83           | 1.02  | 2.28       |
| 62724    | 1.90     | 1.93           | 0.95  | 1.35       |

## 4.11. TRANSIENT TET10 CONTACT

This problem tests basic transient heat conduction with contact in a 3D domain. The geometry consists of two halves of a unit cube meshed with Tet10 elements. The problem is solved by applying thermal contact at the common interface between the two domains.

### 4.11.1. Features Tested

Basic heat conduction on Tet10 meshes; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 4.11.2. Boundary Conditions

Using a manufactured solution, Dirichlet boundary conditions are applied on all the non-contact exposed faces.

### 4.11.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 4.11.4. Verification of Solution

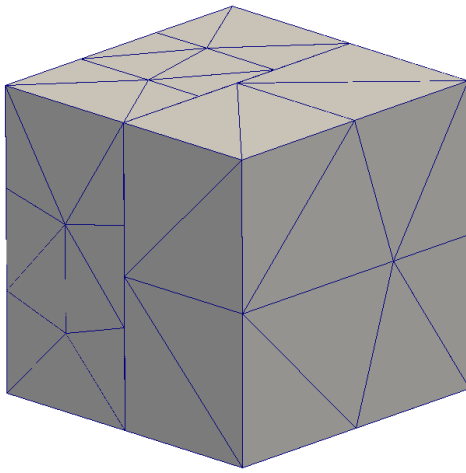
A manufactured solution is chosen as

$$T(x, y, z, t) = (x - x^2)^2 (y - y^2)^2 (z - z^2)^2 m(t) + 1,$$

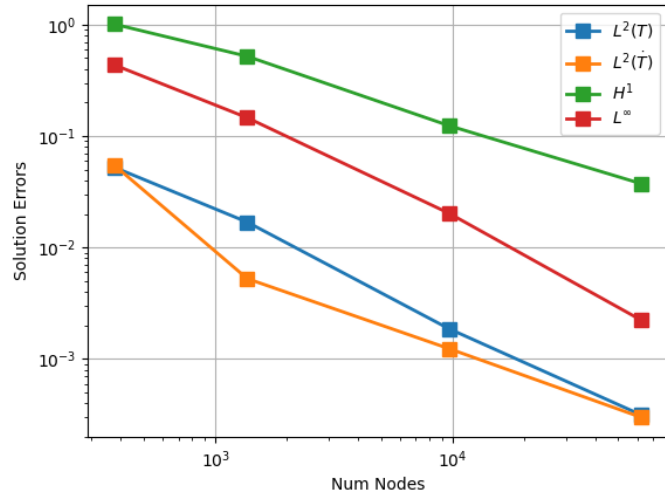
$$m(t) = 10^4 [1. - \exp(-t) + t * \exp(-(t - 1.0) * (t - 1.0))];$$

The source and heat flux user subroutines are chosen so that the solution satisfies the heat equation with the correct boundary conditions.

For input decks see Appendix [12.3.11](#).



Coarse Mesh



Error Norms

**Figure 4.11-1.. Transient Tied Contact: Tet10 Meshes**

**Table 4.11-1.. Transient Tied Contact: Convergence Rates for Tet10 Meshes**

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ |
|----------|----------|----------------|-------|------------|
| 1364     | 2.63     | 5.44           | 1.55  | 2.53       |
| 9663     | 3.37     | 2.21           | 2.19  | 3.02       |
| 62724    | 2.86     | 2.28           | 1.93  | 3.55       |

## 4.12. TRANSIENT HEX8 TIED CONTACT

This problem tests transient heat conduction on a 3D domains with a nonconformal mesh between two blocks. Tied temperature (generalized contact) is used for matching the energy equation between nonconformal blocks. The geometry consists of a unit cube.

### 4.12.1. Features Tested

Transient heat conduction on Hex8 meshes; dirichlet, heat flux, and convective flux boundary conditions, Tied Contact, Nonconformal; constant source terms; heat flux and source term from Encore user subroutines.

### 4.12.2. Boundary Conditions

At surfaces 4 and 6, the temperature is prescribed as a constant value. On surfaces 3 and 5, a heat flux condition is prescribed using a sum of a constant heat flux and a heat flux from an Encore function



(user subroutine). On surfaces 1 and 2, heat flux condition is prescribed using a sum of a convective flux boundary condition (with constant flux and convective coefficient) and a heat flux from an Encore function (user subroutine). Within the domain a source term is prescribed using a sum of a constant source and an Encore function (user subroutine). On the two interior surfaces connecting the nonconformal blocks (surfaces 7 and 8), a contact definition is defined as tied temperature.

### 4.12.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 4.12.4. Verification of Solution

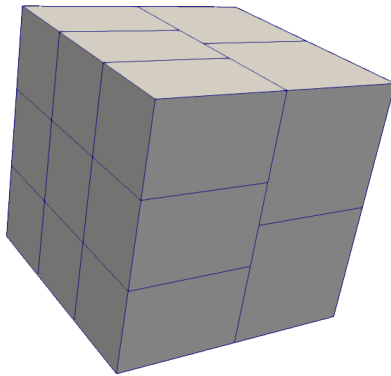
A manufactured solution is chosen as

$$T(x, y, z, t) = (x - x^2)^2 (y - y^2)^2 (z - z^2)^2 m(t) + 1,$$

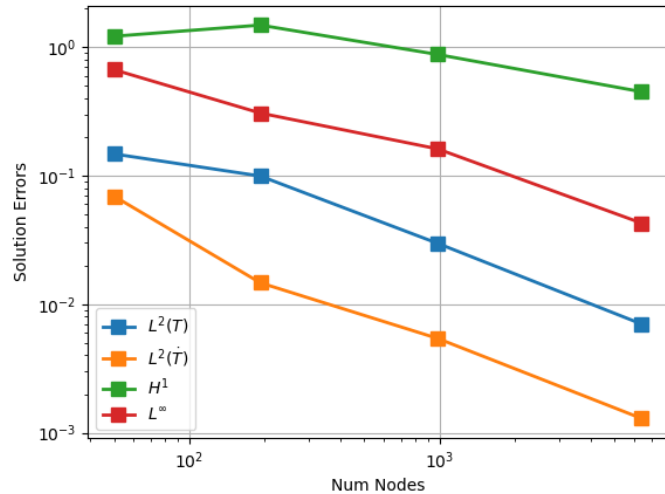
$$m(t) = 10^4 (1 - \exp(-t) + t \exp(-(t - 1)^2))$$

The source and heat flux user subroutines are chosen so that the solution satisfies the heat equation with the correct boundary conditions.

For each mesh, the errors in the temperature solution at final time are computed in the  $L^2$  norm of  $T$  and  $\dot{T}$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, 2 and 1, respectively (within a tolerance).



Coarse Mesh



Error Norms

**Figure 4.12-1.. Tied Contact Transient Heat Conduction: Hex8 Meshes**

**Table 4.12-1.. Tied Contact Transient Heat Conduction: Convergence Rates for Hex8 Meshes**

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ |
|----------|----------|----------------|-------|------------|
| 192      | 0.88     | 3.46           | -0.44 | 1.73       |
| 982      | 2.22     | 1.84           | 0.97  | 1.18       |
| 6419     | 2.31     | 2.29           | 1.07  | 2.13       |

## 4.13. TRANSIENT TET4 TIED CONTACT

This problem tests transient heat conduction and tied thermal contact in a 3D domain as in Section 2.7. The geometry consists of a unit cube that is split along the plane at  $x = 0.5$ .

### 4.13.1. Features Tested

Basic transient heat conduction on Tet4 meshes; non-conformal tied thermal contact; dirichlet, heat flux, and convective flux boundary conditions; constant source terms; heat flux and source term from Encore user subroutines.

### 4.13.2. Boundary Conditions

Identical to Section 2.7.

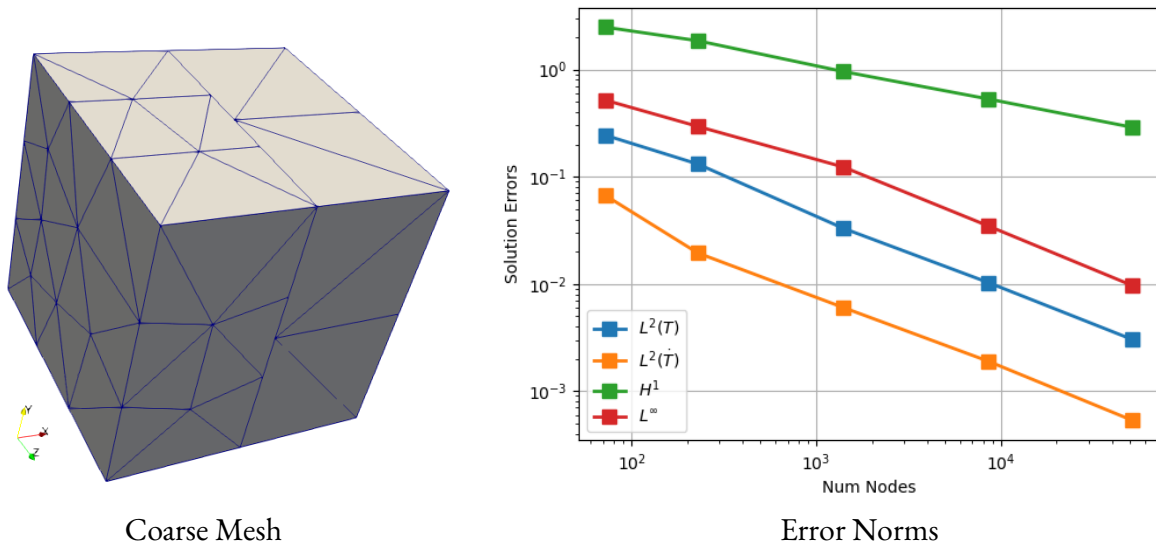
### 4.13.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 4.13.4. Verification of Solution

A manufactured solution is chosen as in Section 2.7.

For each mesh, the errors in the temperature solution at final time are computed in the  $L^2$  norm of  $T$  and  $\dot{T}$ ,  $L^\infty$  and  $H^1$  norms. We see convergence rates for  $\dot{T}$  that are slightly greater than two.



**Figure 4.13-1.. Transient Heat Conduction with Tied Contact:  
Tet4 Meshes**

**Table 4.13-1.. Transient Heat Conduction with Tied Contact:  
Convergence Rates for Tet4 Meshes**

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ |
|----------|----------|----------------|-------|------------|
| 229      | 1.61     | 3.22           | 0.76  | 1.47       |
| 1402     | 2.29     | 1.94           | 1.10  | 1.44       |
| 8535     | 1.93     | 1.91           | 0.98  | 2.11       |
| 51622    | 2.05     | 2.12           | 1.02  | 2.14       |

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## 5. ELEMENT DEATH

### 5.1. CDFEM ELEMENT DEATH (HEAT FLUX)

This problem tests transient conduction and CDFEM element death using 2D and 3D domains. The geometry consists of a thick 1/4 cylindrical or 1/8 spherical shell.

#### 5.1.1. Features Tested

Transient heat conduction, adaptive second order time integration (BDF2), CDFEM element death, temperature and heat flux boundary conditions, Tri3 and Tet4 meshes.

#### 5.1.2. Boundary Conditions

On one surface, the exact solution is used to specify a time-varying temperature. At the other surface, an analytic heat flux is applied using the exact solution. The erosion of the volume from CDFEM element death causes the surface with the heat flux BC to gradually recede as the material is removed.

#### 5.1.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

#### 5.1.4. Verification of Solution

A manufactured solution  $T$  and exact source term  $S$  are chosen in 2D to be:

$$T(r, t) = \frac{\ln(r)}{\ln(2-t)}, \quad S(r, t) = \frac{\ln(r)}{(\ln(2-t))^2(2-t)}$$

and in 3D to be:

$$T(r, t) = (1+t)/r, \quad S(r, t) = 1/r.$$

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms over the volume, and in the  $L^2$  and  $L^\infty$  norms over the outer surface. The test passes, only if the observed

rates of convergence in these norms are one (within a tolerance). First order convergence is expected in this case, due to the nature of the coupling of the CDFEM mesh decomposition and the heat conduction solve.

### 5.1.5. Results: Tri3

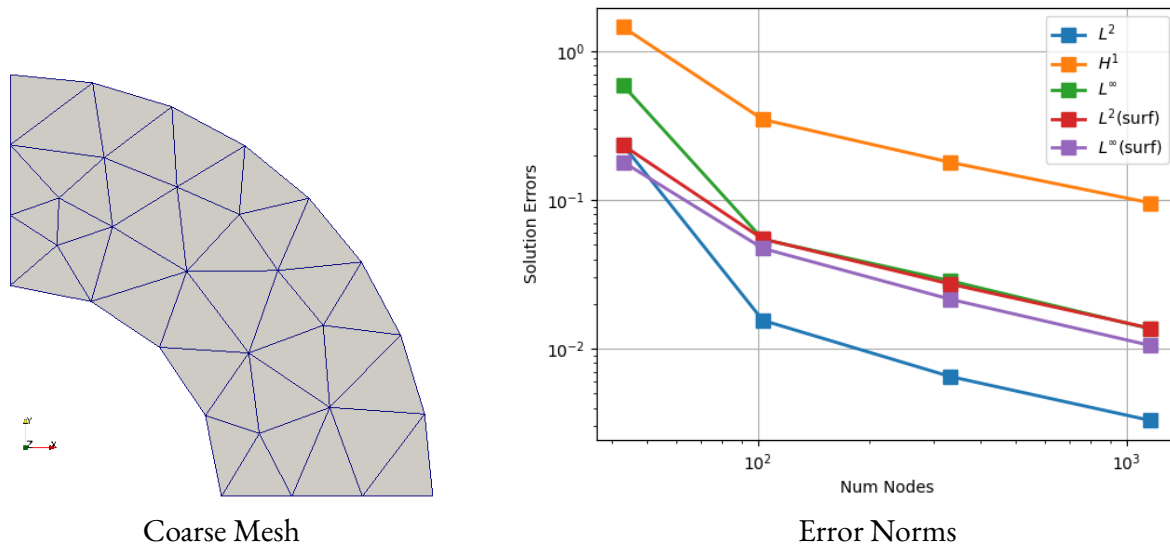


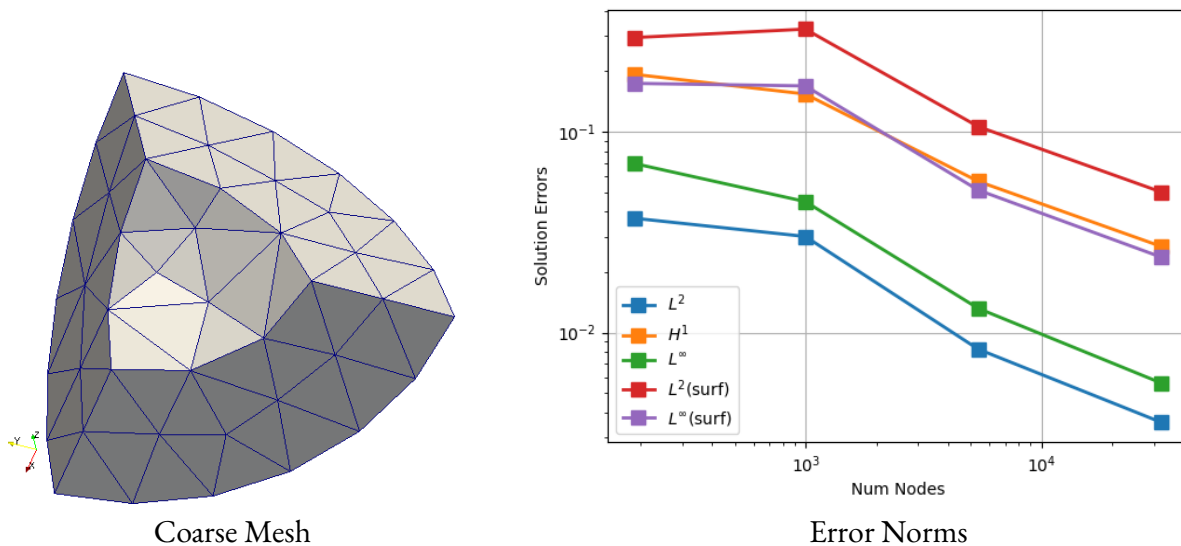
Figure 5.1-1.. CDFEM Element Death (Heat Flux): Tri3

Table 5.1-1.. CDFEM Element Death (Heat Flux): Convergence Rates for Tri3

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ | $L^2(\text{surf})$ | $L^\infty(\text{surf})$ |
|----------|-------|-------|------------|--------------------|-------------------------|
| 103      | 3.92  | 2.06  | 3.44       | 2.10               | 1.93                    |
| 332      | 1.25  | 0.96  | 0.93       | 1.01               | 1.13                    |
| 1162     | 0.98  | 0.91  | 1.07       | 0.99               | 1.03                    |

### 5.1.6. Results: Tet4

For input decks see Appendix [12.4.1](#).



**Figure 5.1-2.. CDFEM Element Death (Heat Flux): Tet4**

**Table 5.1-2.. CDFEM Element Death (Heat Flux): Convergence Rates for Tet4**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ | $L^2(\text{surf})$ | $L^\infty(\text{surf})$ |
|----------|-------|-------|------------|--------------------|-------------------------|
| 996      | 0.30  | 0.32  | 0.63       | -0.14              | 0.04                    |
| 5395     | 1.86  | 1.44  | 1.76       | 1.62               | 1.72                    |
| 32358    | 1.21  | 1.08  | 1.24       | 1.08               | 1.11                    |

## 5.2. 3D SPHERICAL SHELL ENCLOSURE

### 5.2.1. Problem Description

This problem tests transient conduction, enclosure radiation, and CDFEM element death. The initial geometry of this problem is a hollow sphere (block 2) inside and in contact with a second hollow sphere (block 1). The geometry is such that the solution maintains radial symmetry. The inner sphere decomposes at a specific failure temperature, resulting in a changing enclosure geometry.

### 5.2.2. Features Tested

Transient heat conduction, enclosure radiation, CDFEM element death, Tet4 meshes.

### 5.2.3. Boundary and Initial Conditions

The initial condition is a piecewise steady state temperature distribution defined below in (5.1). The boundary conditions specify the temperature  $T_4$  at the outer surface (4) of the outer sphere and  $T_1$  at the inner surface (1) of the inner sphere. The inner temperature  $T_1$  will be gradually increased, while  $T_4$  remains constant in time.

An enclosure is defined initially using the outer surface of the inner volume (surface 2 of block 2) and the inner surface of the outer volume (surface 3 of block 1). The erosion of the inner volume (block 2) from CDFEM element death causes surface 2 to gradually recede as the material within block 2 is removed.

Dimensions are defined in Table 5.2-1.

**Table 5.2-1.. Dimensions of problem**

|                     |       |      |
|---------------------|-------|------|
| radius of surface_1 | $r_1$ | 0.01 |
| radius of surface_2 | $r_2$ | 0.02 |
| radius of surface_3 | $r_3$ | 0.03 |
| radius of surface_4 | $r_4$ | 0.04 |

### 5.2.4. Material Parameters

Material properties are shown in Table 5.2-2.

**Table 5.2-2.. Material properties**

|                               |              |                  |
|-------------------------------|--------------|------------------|
| Thermal conductivity          | $\kappa$     | 1.0              |
| Density                       | $\rho$       | 7682.0           |
| Specific heat                 | $C_p$        | 10.0             |
| emissivity (inner)            | $\epsilon_2$ | 0.6              |
| emissivity (outer)            | $\epsilon_3$ | 0.7              |
| Stefan-Boltzmann constant     | $\sigma$     | 5.6704e-8        |
| failure temperature (block 2) | $T_c$        | 867.011674920813 |

### 5.2.5. Verification of Solution

The solution after failure occurs is specified using inner and outer temperature solutions of the form:

$$T_i(r) \equiv T_1 + (T_c - T_1) \frac{1/r - 1/r_1}{1/r_2 - 1/r_1}, \quad r_1 \leq r \leq r_2, \quad (5.1)$$

$$T_o(r) \equiv C_o + (T_4 - C_o) \frac{1/r - 1/r_3}{1/r_4 - 1/r_3}, \quad r_3 \leq r \leq r_4 \quad (5.2)$$



Here all parameters are known except  $r_2$  and  $C_o$ , which will vary with time. The initial value of  $r_2$  is given in Table 5.2-1; the initial value of  $C_o$  is chosen to satisfy the enclosure radiation equilibrium equations below.

To complete the solution, we now derive a system of two nonlinear equations to solve for  $r_2$  and  $C_o$ . These are the energy balances on the outer and inner enclosure surfaces, given by

$$R_2 \equiv q_2 - \sigma \epsilon_2 T_2^4 + \epsilon_2 (F_{22} J_2 + F_{23} J_3) \quad (5.3)$$

$$R_3 \equiv -q_3 - \sigma \epsilon_3 T_3^4 + \epsilon_3 (F_{32} J_2 + F_{33} J_3) \quad (5.4)$$

where the three terms in each equation represent fluxes from conduction, radiative emission, and radiative reflection. The conductive fluxes are defined by Fourier's law as

$$q_2 = -\kappa_2 \frac{\partial T_i}{\partial r} \Big|_{r=r_2} = \kappa_2 \frac{T_c - T_1}{r_1^2 (1/r_2 - 1/r_1)} \quad (5.5)$$

$$q_3 = -\kappa_3 \frac{\partial T_o}{\partial r} \Big|_{r=r_3} = \kappa_3 \frac{T_4 - C_o}{r_3^2 (1/r_4 - 1/r_3)} \quad (5.6)$$

The surface temperatures are

$$T_2 \equiv T_i|_{r=r_2} = T_c, \quad T_3 \equiv T_o|_{r=r_3} = C_o$$

The radiosities are obtained by solving the linear system for enclosure radiation

$$\begin{bmatrix} 1 - (1 - \epsilon_2)F_{22} & -(1 - \epsilon_2)F_{23} \\ -(1 - \epsilon_3)F_{32} & 1 - (1 - \epsilon_3)F_{33} \end{bmatrix} \begin{bmatrix} J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} \sigma \epsilon_2 T_2^4 \\ \sigma \epsilon_3 T_3^4 \end{bmatrix}$$

to obtain

$$\begin{bmatrix} J_2 \\ J_3 \end{bmatrix} = \frac{1}{a} \begin{bmatrix} 1 - (1 - \epsilon_3)F_{33} & (1 - \epsilon_2)F_{23} \\ (1 - \epsilon_3)F_{32} & 1 - (1 - \epsilon_2)F_{22} \end{bmatrix} \begin{bmatrix} \sigma \epsilon_2 T_2^4 \\ \sigma \epsilon_3 T_3^4 \end{bmatrix}$$

where  $a$  is the determinant

$$a = (1 - (1 - \epsilon_2)F_{22})(1 - (1 - \epsilon_3)F_{33}) - (1 - \epsilon_2)F_{23}(1 - \epsilon_3)F_{32}$$

The viewfactor coefficients  $F_{ij}$  are given by

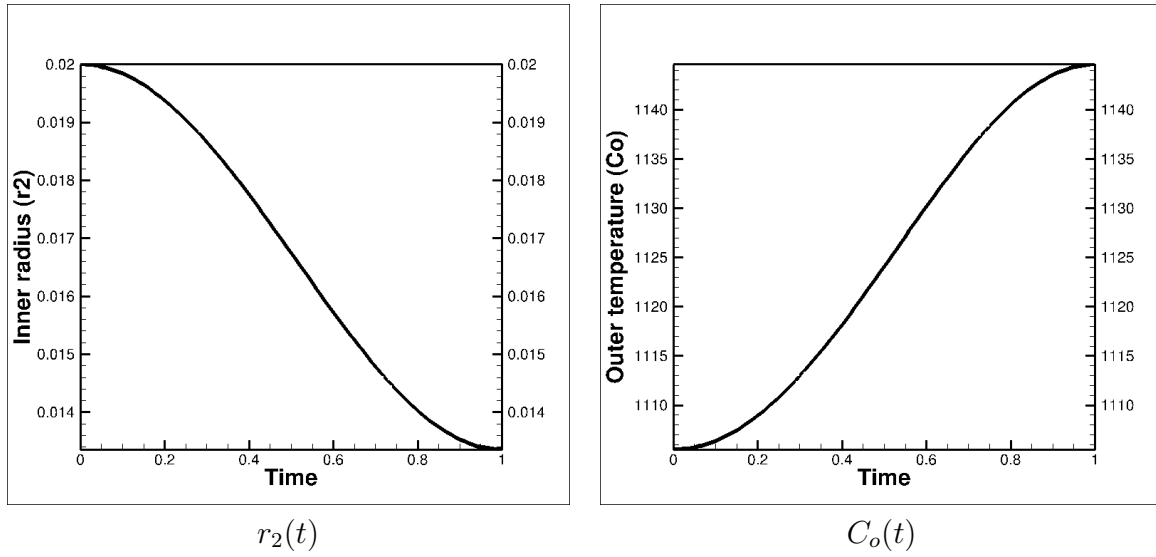
$$F_{22} = 0, \quad F_{23} = 1, \quad F_{32} = (r_2/r_3)^2, \quad F_{33} = 1 - F_{32}$$

The specific function we choose for  $T_1(t)$  is

$$T_1(t) \equiv T_1 + 400(1 - \cos(\pi t))/2$$

The time histories of  $r_2$  and  $C_o$  are shown in Figure 5.2-1.

In order to derive the source term, the time derivatives of  $r_2$  and  $C_o$  are computed once the pair of nonlinear equations is solved using Newton's method. Since the spatial part of the piecewise solution is



**Figure 5.2-1.. Evolution of parameters  $r_2$  and  $C_o$ .**

harmonic, the source terms become just  $\rho c_p \partial_t T$ , where

$$\partial_t T_i \equiv \dot{T}_1 + (T_c - \dot{T}_1) \frac{1/r - 1/r_1}{1/r_2 - 1/r_1} + (T_c - T_1) \frac{\dot{r}_2(1/r - 1/r_1)}{r_2^2(1/r_2 - 1/r_1)^2}, \quad r_1 \leq r \leq r_2, \quad (5.7)$$

$$\partial_t T_o \equiv \dot{C}_o \left(1 - \frac{1/r - 1/r_3}{1/r_4 - 1/r_3}\right), \quad r_3 \leq r \leq r_4 \quad (5.8)$$

## 5.2.6. Results

Results are presented running the problem on three meshes up to time  $t = 0.9$ .

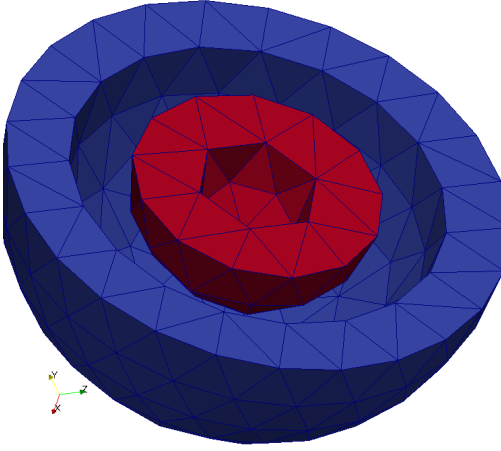
**Table 5.2-3.. Convergence Rates at  $t = 0.9$**

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $L^\infty$ | $H^1$ |
|----------|----------|----------------|------------|-------|
| 4289     | 2.09     | 1.93           | 0.96       | 1.06  |
| 25590    | 2.26     | 1.61           | 1.21       | 1.05  |

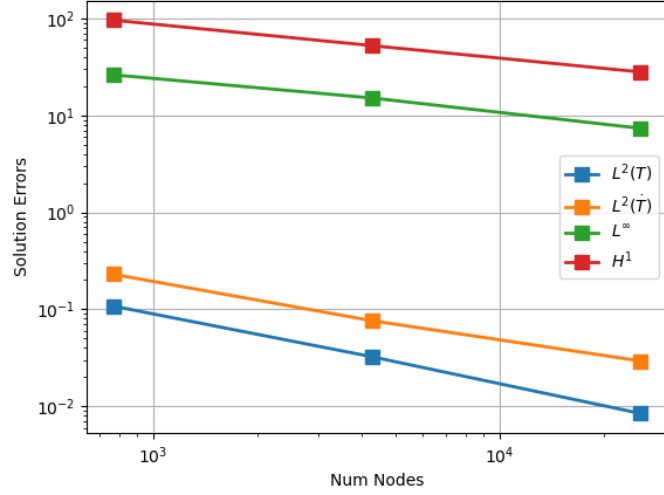
For input decks see Appendix [12.4.2](#).

## 5.3. STANDARD ELEMENT DEATH (HEAT FLUX)

This problem tests transient conduction with standard element death on a 2D square domain than is essentially a 1D problem.



Coarse Mesh



Error Norms ( $t = 0.9$ )

### 5.3.1. Features Tested

Transient heat conduction, adaptive second order time integration (BDF2), standard element death, temperature and heat flux boundary conditions, Tri3, Hex8 and Quad4 meshes.

### 5.3.2. Boundary Conditions

On one surface, the exact solution is used to specify a time-varying temperature. At the other surface, an analytic heat flux is applied using the exact solution. The erosion of the volume from element death causes the surface with the heat flux BC to recede element by element as the material is removed.

### 5.3.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 5.3.4. Verification of Solution

A manufactured solution  $T$  and exact source term  $S$  are chosen to be:

$$T(r, t) = \exp(t - x).$$

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms over the area. The test passes, only if the observed rates of convergence in these norms are one (within a tolerance). First order convergence is expected in this case.

5.3.5. Results: 1D Hex8

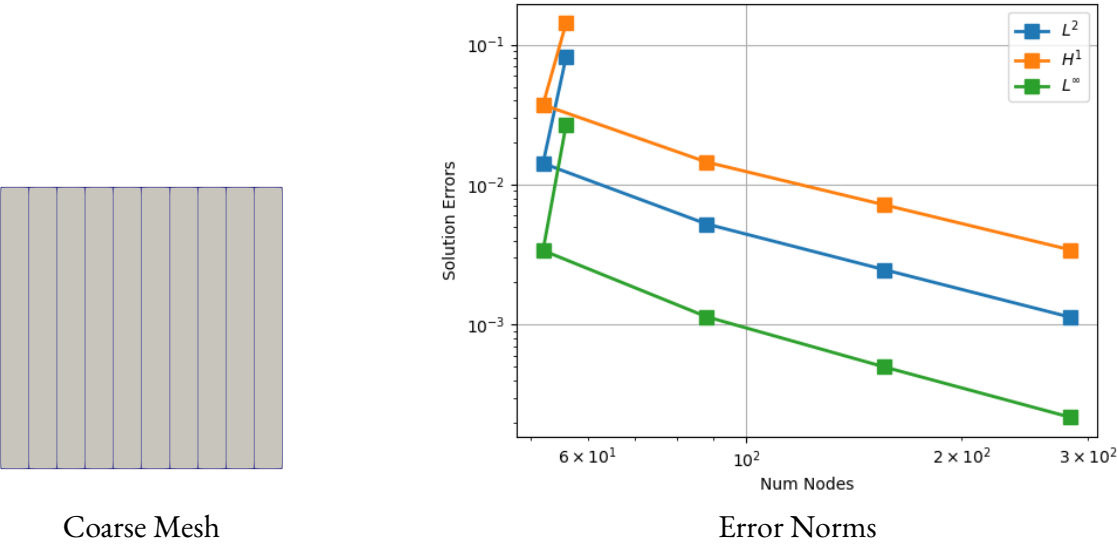


Figure 5.3-1.. Element Death (Heat Flux): Hex8

Table 5.3-1.. Element Death (Heat Flux): Convergence Rates for Hex8

| Num Dofs | Var1 | Var2 | Var3 |
|----------|------|------|------|
| 52       | nan  | nan  | nan  |
| 88       | 1.44 | 1.36 | 1.58 |
| 156      | 1.09 | 1.02 | 1.19 |
| 284      | 1.13 | 1.07 | 1.19 |

5.3.6. Results: 1D Quad4

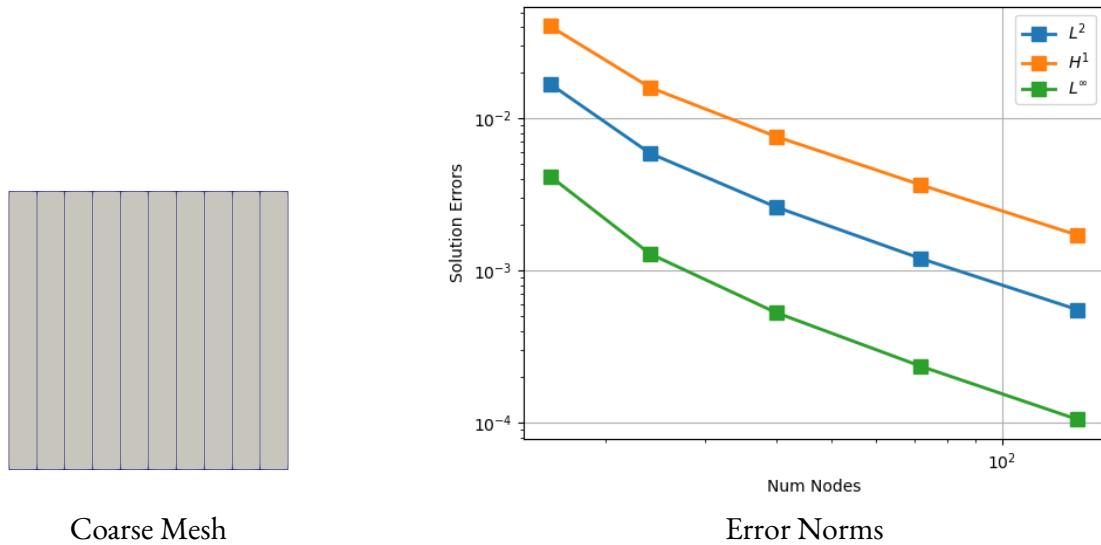
5.3.7. Results: 1D Tri3

5.3.8. Results: 2D Quad4

This problem tests transient conduction with standard element death on a 2D quarter slice of an annulus.

5.3.9. Features Tested

Transient heat conduction, fixed first order time integration, standard element death, Quad4 mesh.



**Figure 5.3-2.. Element Death (Heat Flux): Quad4**

**Table 5.3-2.. Element Death (Heat Flux): Convergence Rates for Quad4**

| Num Dofs | Var1 | Var2 | Var3 |
|----------|------|------|------|
| 24       | 1.51 | 1.34 | 1.70 |
| 40       | 1.18 | 1.07 | 1.28 |
| 72       | 1.12 | 1.06 | 1.17 |
| 136      | 1.12 | 1.09 | 1.16 |

### 5.3.10. Boundary Conditions

On one surface, the exact solution is used to specify a time-varying temperature. On the other surfaces, the exact source solution is provided as the flux boundary condition. The erosion of the volume from element death is caused by having a minimum nodal value of temperature less than 1.

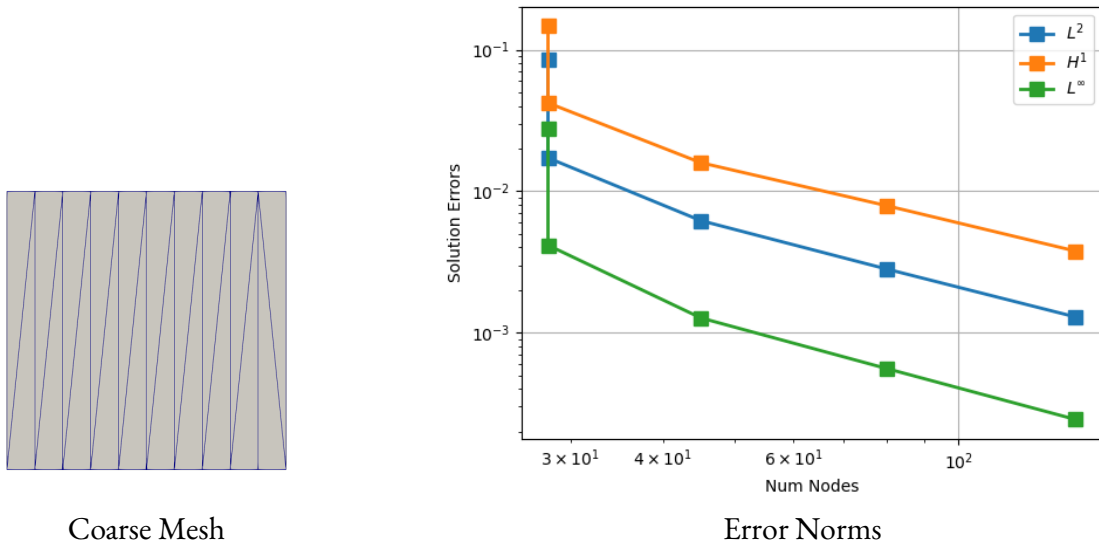
### 5.3.11. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

### 5.3.12. Verification of Solution

A manufactured solution  $T$  and exact source term  $S$  are chosen to be:

$$T(r, t) = \ln(\sqrt{x^2 + y^2})(1/\ln(2 - t)).$$



**Figure 5.3-3.. Element Death (Heat Flux): Tri3**

**Table 5.3-3.. Element Death (Heat Flux): Convergence Rates for Tri3**

| Num Dofs | Var1 | Var2 | Var3 |
|----------|------|------|------|
| 28       | nan  | nan  | nan  |
| 45       | 1.48 | 1.41 | 1.70 |
| 80       | 1.13 | 1.01 | 1.19 |
| 144      | 1.13 | 1.07 | 1.20 |

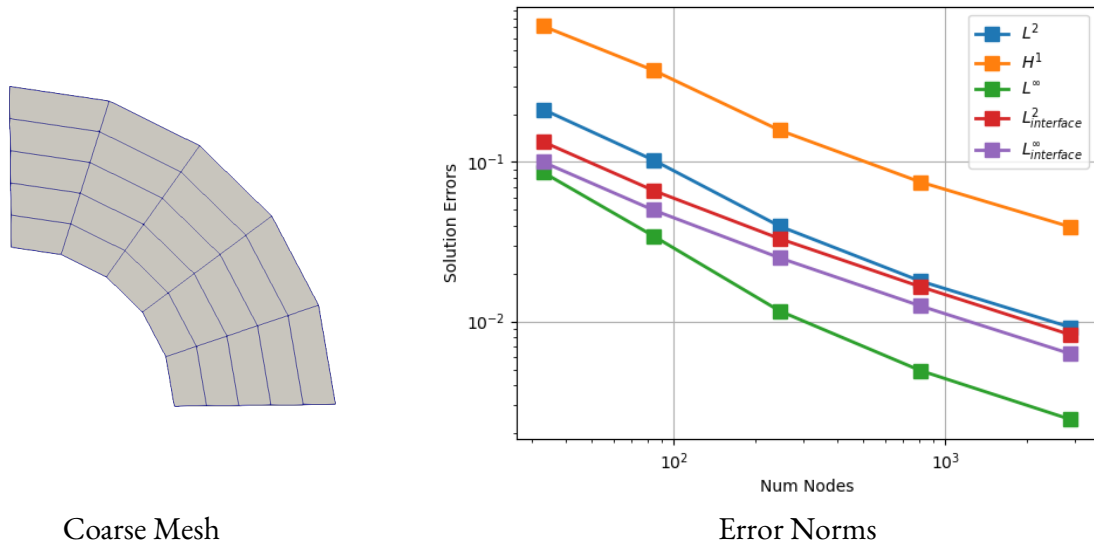
For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms over the area. The test passes, only if the observed rates of convergence in these norms are one (within a tolerance). First order convergence is expected in this case.

### 5.3.13. Results: 3D Hex8

This problem evaluates transient conduction with standard element death on a 3D quarter of a hollow sphere geometry.

### 5.3.14. Features Tested

Transient heat conduction, fixed first order time integration, standard element death, Hex8 mesh.



**Figure 5.3-4.. Element Death (Heat Flux): Quad4**

**Table 5.3-4.. 2D Element Death (Heat Flux): Convergence Rates for Quad4**

| Num Dofs | Var1 | Var2 | Var3 | Var4 | Var5 |
|----------|------|------|------|------|------|
| 84       | 1.56 | 1.37 | 1.97 | 1.50 | 1.48 |
| 246      | 1.79 | 1.62 | 2.02 | 1.30 | 1.29 |
| 810      | 1.33 | 1.25 | 1.45 | 1.17 | 1.16 |
| 2898     | 1.05 | 1.02 | 1.11 | 1.09 | 1.09 |

### 5.3.15. Boundary Conditions

On surface 2, the exact solution is used to specify a time-varying temperature. On all remaining surfaces, a heat flux boundary condition is imposed with a flux time function specified. The erosion of the volume from element death is caused by having a maximum nodal value of temperature greater than 1.

### 5.3.16. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks.

5.3.17. Verification of Solution

A manufactured solution  $T$  and exact source term  $S$  are chosen to be:

$$T(r,t) = \frac{1+t}{\sqrt{x^2+y^2+z^2}}.$$

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms over the area. The observed rates of convergence in these norms are one (within a tolerance). First order convergence is expected in this case.

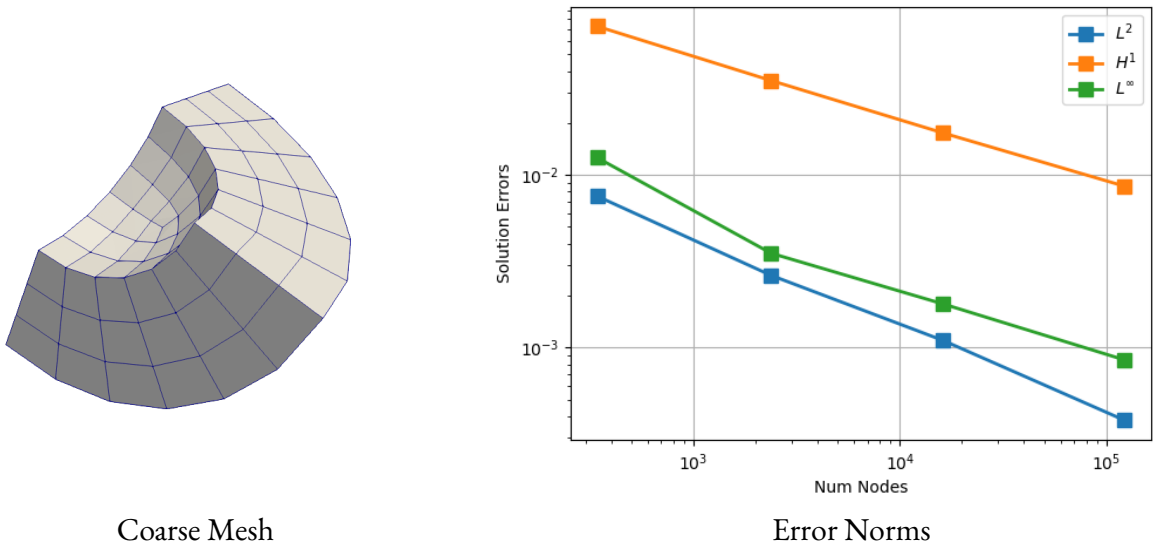


Figure 5.3-5.. Element Death (Heat Flux): Hex8

Table 5.3-5.. Element Death (Heat Flux): Convergence Rates for Hex8

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ |
|----------|-------|-------|------------|
| 2382     | 1.64  | 1.12  | 1.97       |
| 16214    | 1.36  | 1.10  | 1.06       |
| 122892   | 1.57  | 1.05  | 1.11       |



# 6. TIME INTEGRATION

## 6.1. ADAPTIVE TIME INTEGRATION

This problem tests the various implicit time integrators using both fixed and adaptive time stepping. The integrators are first order (Backward Euler), second order (Crank-Nicolson) and BDF2. The geometry is a 2D square.

### 6.1.1. Features Tested

Transient heat conduction, time integrators, adaptive time stepping, polynomial temperature dependence of density and thermal conductivity.

### 6.1.2. Boundary Conditions

The boundary conditions are prescribed at the nodes using the analytic solution. The initial condition is specified using an Encore function evaluated at the nodes.

### 6.1.3. Material Parameters

The specific heat is constant. The density and thermal conductivity are linear polynomials in the temperature.

### 6.1.4. Verification of Solution

A manufactured solution is chosen as

$$T(x, y, t) = \sin(C_1 t) + 2x \cos(C_2 t) + 3y \sin(C_3 t) + 4xy \cos(C_4 t) + 5x^2 \sin(C_5 t) + 6y^2 \cos(C_6 t)$$

which requires a source term. This solution is designed to have a non-trivial time-dependence using constants:

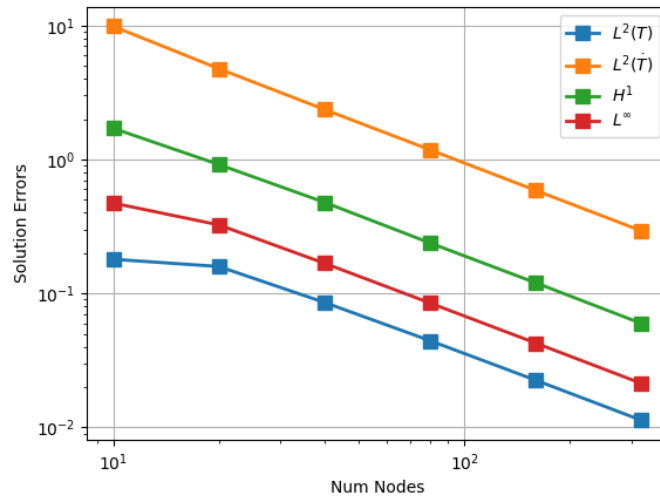
$$C_1 = \pi, \quad C_2 = 2\pi, \quad C_3 = 3\pi, \quad C_4 = \pi, \quad C_5 = 2.5\pi, \quad C_6 = 0.5\pi$$

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The  $L^2$  error in the temperature time derivative is also computed. The test passes, only if the observed rates of convergence in these norms are 1 for  $H^1$  and 2 for all other norms (within a tolerance).

Because the adaptive meshes use less time steps, we use time step size instead of mesh size for estimation of the convergence rates. We also include the  $L^2$  error in the time derivative of the temperature.

For input decks see Appendix [12.5.1](#).

### 6.1.5. Results: First Order Fixed

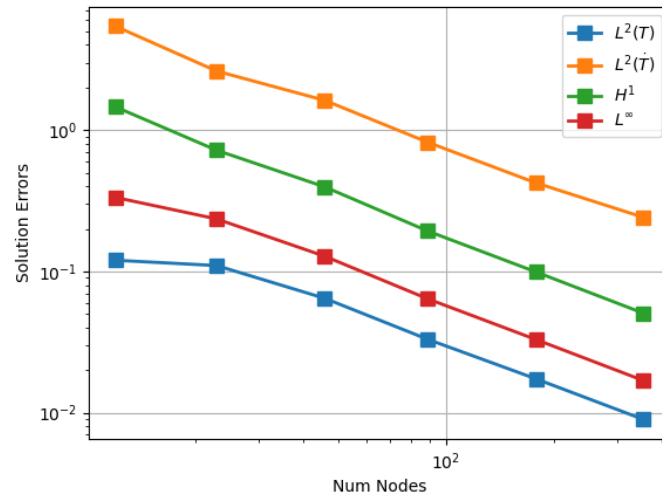


**Figure 6.1-1.. Adaptive Time Integration: Errors for First Order Fixed**

**Table 6.1-1.. Adaptive Time Integration: Convergence Rates for First Order Fixed**

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ |
|----------|----------|----------------|-------|------------|
| 20       | 0.18     | 1.05           | 0.89  | 0.54       |
| 40       | 0.89     | 1.01           | 0.94  | 0.95       |
| 80       | 0.95     | 1.01           | 1.01  | 0.99       |
| 160      | 0.98     | 1.00           | 0.99  | 0.99       |
| 320      | 0.99     | 1.00           | 1.00  | 1.00       |

### 6.1.6. Results: First Order Adaptive



**Figure 6.1-2.. Adaptive Time Integration: Errors for First Order Adaptive**

**Table 6.1-2.. Adaptive Time Integration: Convergence Rates for First Order Adaptive**

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ |
|----------|----------|----------------|-------|------------|
| 23       | 0.13     | 1.13           | 1.09  | 0.54       |
| 46       | 0.77     | 0.68           | 0.86  | 0.87       |
| 89       | 1.01     | 1.04           | 1.09  | 1.05       |
| 178      | 0.93     | 0.95           | 0.96  | 0.95       |
| 355      | 0.96     | 0.81           | 0.98  | 0.97       |

### 6.1.7. Results: Second Order Fixed

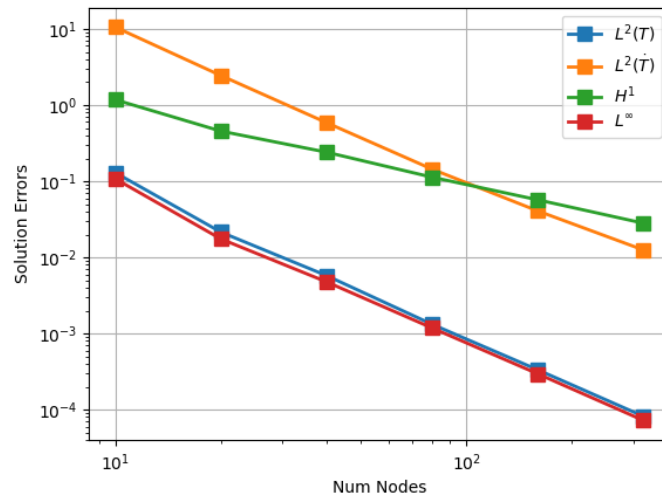
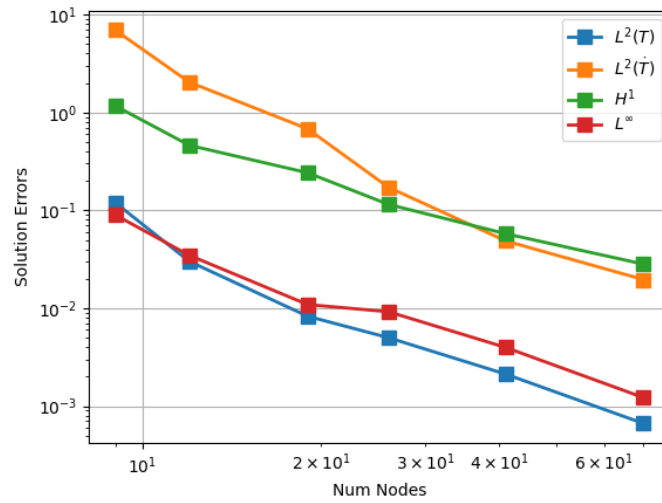


Figure 6.1-3.. Adaptive Time Integration: Errors for Second Order Fixed

Table 6.1-3.. Adaptive Time Integration: Convergence Rates for Second Order Fixed

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ |
|----------|----------|----------------|-------|------------|
| 20       | 2.59     | 2.11           | 1.37  | 2.62       |
| 40       | 1.90     | 2.06           | 0.92  | 1.88       |
| 80       | 2.13     | 2.03           | 1.10  | 2.02       |
| 160      | 1.98     | 1.82           | 0.99  | 2.00       |
| 320      | 2.03     | 1.71           | 1.02  | 2.04       |

### 6.1.8. Results: Second Order Adaptive



**Figure 6.1-4.. Adaptive Time Integration: Errors for Second Order Adaptive**

**Table 6.1-4.. Adaptive Time Integration: Convergence Rates for Second Order Adaptive**

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ |
|----------|----------|----------------|-------|------------|
| 12       | 4.81     | 4.26           | 3.24  | 3.31       |
| 19       | 2.81     | 2.39           | 1.41  | 2.52       |
| 26       | 1.61     | 4.38           | 2.38  | 0.54       |
| 41       | 1.88     | 2.77           | 1.51  | 1.85       |
| 70       | 2.17     | 1.70           | 1.34  | 2.21       |

### 6.1.9. Results: BDF2 Fixed

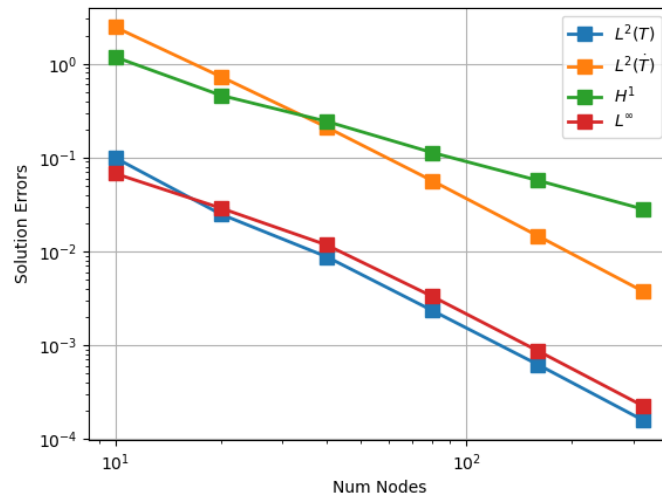
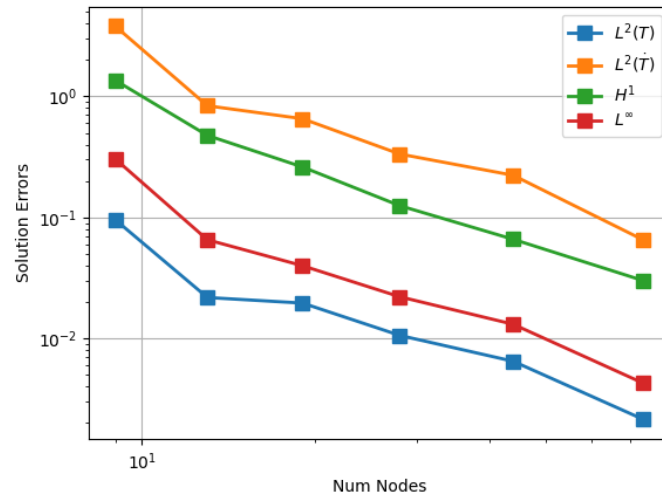


Figure 6.1-5.. Adaptive Time Integration: Errors for BDF2 Fixed

Table 6.1-5.. Adaptive Time Integration: Convergence Rates for BDF2 Fixed

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ |
|----------|----------|----------------|-------|------------|
| 20       | 2.00     | 1.75           | 1.36  | 1.22       |
| 40       | 1.51     | 1.79           | 0.92  | 1.30       |
| 80       | 1.90     | 1.90           | 1.10  | 1.82       |
| 160      | 1.92     | 1.95           | 0.99  | 1.93       |
| 320      | 1.98     | 1.98           | 1.02  | 1.96       |

### 6.1.10. Results: BDF2 Adaptive



**Figure 6.1-6.. Adaptive Time Integration: Errors for BDF2 Adaptive**

**Table 6.1-6.. Adaptive Time Integration: Convergence Rates for BDF2 Adaptive**

| Num Dofs | $L^2(T)$ | $L^2(\dot{T})$ | $H^1$ | $L^\infty$ |
|----------|----------|----------------|-------|------------|
| 13       | 4.03     | 4.12           | 2.86  | 4.19       |
| 19       | 0.27     | 0.65           | 1.58  | 1.30       |
| 28       | 1.59     | 1.73           | 1.90  | 1.52       |
| 44       | 1.09     | 0.90           | 1.42  | 1.16       |
| 74       | 2.13     | 2.37           | 1.51  | 2.16       |

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# 7. ENCLOSURE RADIATION

## 7.1. 2D CYLINDRICAL SHELL ENCLOSURE

### 7.1.1. Problem Description

This problem tests steady state coupled conduction and enclosure radiation. The geometry of this problem is a hollow cylinder (block 2) inside a second hollow cylinder (block 1), which is a radially symmetric problem.

### 7.1.2. Features Tested

Basic heat conduction, enclosure radiation, Quad4/Tri3 meshes.

### 7.1.3. Boundary Conditions

The boundary conditions specify the temperature of the outer surface of the outer sphere ( $T(r_4) = T_4$ ) and the inner surface of the inner sphere ( $T(r_1) = T_1$ ).

The problem is steady state but is initialized with a constant temperature of 300 in both blocks. The inner surface temperature  $T_1$  is set to 300. The outer surface temperature  $T_4$  is set to 1300.

Dimensions are defined in Table 7.1-1.

**Table 7.1-1.. Dimensions of problem**

|                     |       |      |
|---------------------|-------|------|
| radius of surface_1 | $r_1$ | 0.01 |
| radius of surface_2 | $r_2$ | 0.02 |
| radius of surface_3 | $r_3$ | 0.03 |
| radius of surface_4 | $r_4$ | 0.04 |

### 7.1.4. Material Parameters

Material properties are shown in Table 7.1-2.

**Table 7.1-2.. Material properties**

|                                |              |           |
|--------------------------------|--------------|-----------|
| Thermal conductivity (block_1) | $\kappa_1$   | 2.0       |
| Thermal conductivity (block_2) | $\kappa_2$   | 0.35      |
| Density                        | $\rho$       | 1.0       |
| Specific heat                  | $C_p$        | 1.0       |
| emissivity (surface_2)         | $\epsilon_2$ | 0.50      |
| emissivity (surface_3)         | $\epsilon_3$ | 0.55      |
| Stefan-Boltzmann constant      | $\sigma$     | 5.6704e-8 |

### 7.1.5. Verification of Solution

In cylindrical coordinates, the temperature is independent of  $\theta$  and  $z$ . Integrating this equation twice with respect to the radius  $r$ , we obtain the general solution in either hollow cylinder to be

$$T(r) = C_1 \log(r) + C_2,$$

for arbitrary constants  $C_1$  and  $C_2$ . We will use  $r_i, i = 1, \dots, 4$  to denote the location of the four surfaces of constant  $r$ , numbered from inside to outside. Unless specified otherwise, we will use these subscripts for other quantities which are evaluated at one of the four surfaces.

Including the boundary conditions into the solution allows us to eliminate two constants and gives

$$T_{inner}(r) = T_1 + c_I \log(r/r_1) \text{ for } r_1 < r < r_2 \quad (7.1)$$

$$T_{outer}(r) = T_4 + c_O \log(r/r_4) \text{ for } r_3 < r < r_4 \quad (7.2)$$

To solve for  $c_I$  and  $c_O$  we compute the temperatures at the enclosure surfaces  $r_2$  and  $r_3$ , defined as  $T_2 = T_{inner}(r_2)$  and  $T_3 = T_{outer}(r_3)$ :

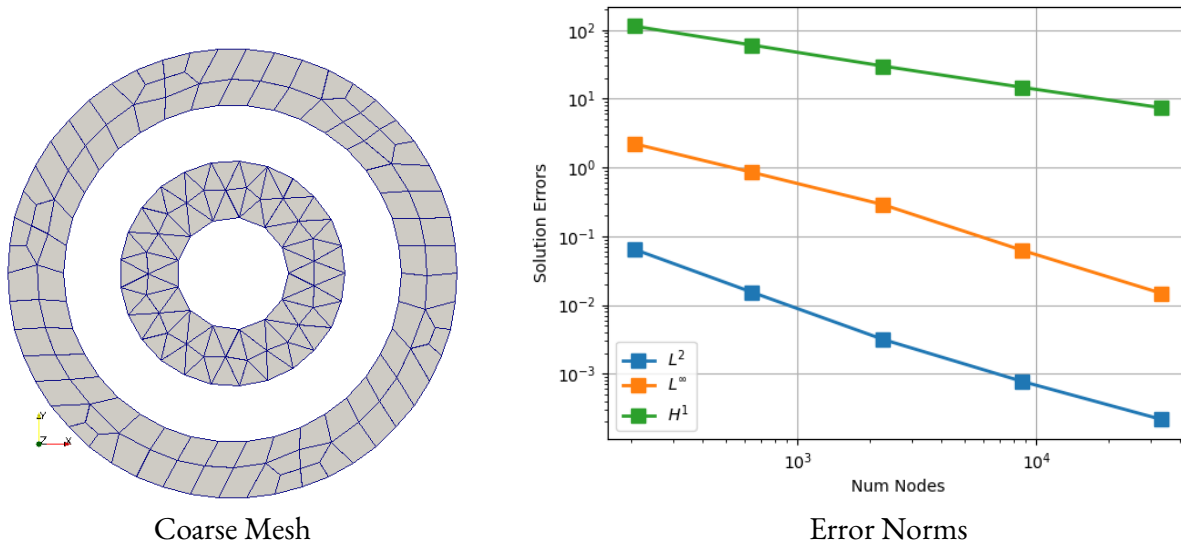
### 7.1.6. Results

The exact temperatures at the enclosure surfaces (to six digits precision) are  $T_2 = 444.7977$  and  $T_3 = 956.5915$ . From these values we can compute the values of  $c_O$  and  $c_I$  and thus the exact solution.

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, and 1, respectively (within a tolerance).

These optimal rates are observed in this test.

For input decks see Appendix [12.6.1](#).



**Figure 7.1-1.. Enclosure Radiation 2D**

**Table 7.1-3.. Enclosure Radiation 2D: Convergence Rates**

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 640      | 2.54  | 1.68       | 1.11  |
| 2276     | 2.50  | 1.72       | 1.11  |
| 8673     | 2.10  | 2.28       | 1.07  |
| 33500    | 1.90  | 2.14       | 1.02  |

## 7.2. 2D ANNULAR ENCLOSURE

### 7.2.1. Problem Description

This problem tests steady state coupled conduction and enclosure radiation. The geometry is an annulus with a crack.

### 7.2.2. Features Tested

Basic heat conduction, enclosure radiation, Tri3 mesh.

### 7.2.3. Boundary Conditions

The outer and crack boundary conditions are prescribed at the nodes using the analytic solution. The inner boundary uses an enclosure boundary condition.

## 7.2.4. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant.

## 7.2.5. Verification of Solution

The manufactured solution is

$$\begin{aligned}
 J(\theta) &= k_1 + k_2 \sqrt{\varepsilon} \cos\left(\frac{\sqrt{\varepsilon}\theta}{2}\right) / \sin\left(\sqrt{\frac{\varepsilon}{\pi}} 2\right), \\
 H(\theta) &= k_1 + k_2 \left(\frac{\sqrt{\varepsilon}}{1 - \varepsilon}\right) \left(\cos\left(\frac{\sqrt{\varepsilon}\theta}{2}\right) / \sin\left(\frac{\sqrt{\varepsilon}\pi}{2}\right) - \sqrt{\varepsilon} \cos\left(\frac{\theta}{2}\right)\right), \\
 q(\theta) &= J(\theta) - H(\theta), \\
 \beta(\theta) &= \left(\frac{k_1 + k_2 \cos\left(\frac{\theta}{2}\right)}{\sigma}\right)^{1/4}, \\
 T(r, \theta) &= r\beta(\theta) + (r - r_{\text{cyl}}) \left(\frac{q(\theta)}{\kappa} - \beta(\theta)\right),
 \end{aligned}$$

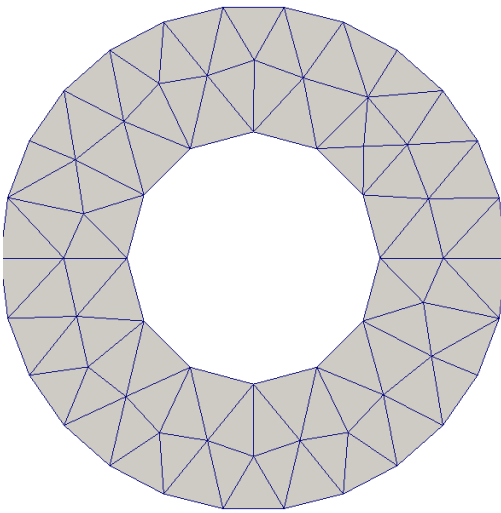
where  $J$  is the radiosity,  $H$  is the irradiance,  $q$  is the flux, and

$$\begin{aligned}
 \sigma &= 5.6704 \times 10^{-8}, \\
 \kappa &= 1, \\
 r_{\text{cyl}} &= 1, \\
 \varepsilon &= 0.9, \\
 k_1 &= 8000, \\
 k_2 &= 400.
 \end{aligned}$$

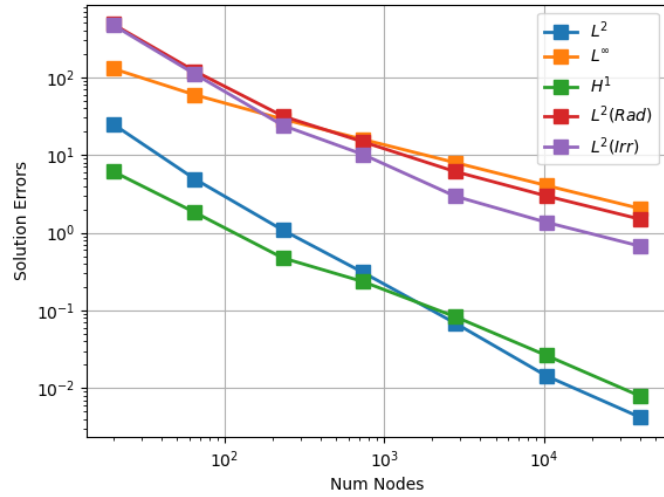
For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, and 1, respectively (within a tolerance). Additionally, the errors in the radiosity and irradiance are computed in the  $L^2$  norms and be 1 (within a tolerance).

These optimal rates are observed in this test.

For input decks see Appendix [12.6.2](#).



Coarse Mesh



Error Norms

**Figure 7.2-1.. 2D Full Enclosure Radiation**

**Table 7.2-1.. 2D Full Enclosure Radiation: Convergence Rates**

| Num Dofs | $L^2$ | $H^1$ | $L^\infty$ | $L^2(Rad)$ | $L^2(Irr)$ |
|----------|-------|-------|------------|------------|------------|
| 65       | 2.76  | 1.31  | 2.05       | 2.36       | 2.46       |
| 232      | 2.38  | 1.13  | 2.13       | 2.10       | 2.40       |
| 734      | 2.18  | 1.04  | 1.21       | 1.32       | 1.49       |
| 2788     | 2.26  | 1.03  | 1.56       | 1.32       | 1.85       |
| 10415    | 2.37  | 1.03  | 1.74       | 1.10       | 1.18       |
| 40529    | 1.81  | 1.01  | 1.78       | 1.01       | 1.04       |

## 7.3. 3D SPHERICAL SHELL ENCLOSURE

### 7.3.1. Problem Description

This problem tests steady state coupled conduction and enclosure radiation. The geometry of this problem is a hollow sphere (block 2) inside a second hollow sphere (block 1), which is a radially symmetric problem.

### 7.3.2. Features Tested

Basic heat conduction, enclosure radiation, Hex8 meshes.

### 7.3.3. Boundary Conditions

The boundary conditions specify the temperature of the outer surface of the outer sphere ( $T(r_4) = T_4$ ) and the inner surface of the inner sphere ( $T(r_1) = T_1$ ).

The problem is steady state but is initialized with a constant temperature of 300 in both blocks. The inner surface temperature  $T_1$  is set to 300. The outer surface temperature  $T_4$  is set to 1300.

Dimensions are defined in Table 7.3-1.

**Table 7.3-1.. Dimensions of problem**

|                     |       |      |
|---------------------|-------|------|
| radius of surface_1 | $r_1$ | 0.01 |
| radius of surface_2 | $r_2$ | 0.02 |
| radius of surface_3 | $r_3$ | 0.03 |
| radius of surface_4 | $r_4$ | 0.04 |

### 7.3.4. Material Parameters

Material properties are shown in Table 7.3-2.

**Table 7.3-2.. Material properties**

|                                |              |           |
|--------------------------------|--------------|-----------|
| Thermal conductivity (block_1) | $\kappa_1$   | 2.0       |
| Thermal conductivity (block_2) | $\kappa_2$   | 0.35      |
| Density                        | $\rho$       | 1.0       |
| Specific heat                  | $C_p$        | 1.0       |
| emissivity (surface_2)         | $\epsilon_2$ | 0.50      |
| emissivity (surface_3)         | $\epsilon_3$ | 0.55      |
| Stefan-Boltzmann constant      | $\sigma$     | 5.6704e-8 |

### 7.3.5. Verification of Solution

In spherical coordinates, the temperature is independent of  $\theta$  and  $\phi$ . Integrating this equation twice with respect to the radius  $r$ , we obtain the general solution in either hollow sphere to be

$$T(r) = C_1 r^{-1} + C_2,$$

for arbitrary constants  $C_1$  and  $C_2$ . We will use  $r_i, i = 1, \dots, 4$  to denote the location of the four surfaces of constant  $r$ , numbered from inside to outside. Unless specified otherwise, we will use these subscripts for other quantities which are evaluated at one of the four surfaces.

Including the boundary conditions into the solution allows us to eliminate two constants and gives

$$T_{inner}(r) = T_1 + c_I \left( \frac{1}{r} - \frac{1}{r_4} \right) \text{ for } r_1 < r < r_2 \quad (7.3)$$

$$T_{outer}(r) = T_4 + c_O \left( \frac{1}{r} - \frac{1}{r_1} \right) \text{ for } r_3 < r < r_4 \quad (7.4)$$

To solve for  $c_I$  and  $c_O$  we compute the temperatures at the enclosure surfaces  $r_2$  and  $r_3$ , defined as  $T_2 = T_{inner}(r_2)$  and  $T_3 = T_{outer}(r_3)$ :

$$T_2 = T_1 + c_I \left( \frac{1}{r_2} - \frac{1}{r_4} \right) \quad (7.5)$$

$$T_3 = T_4 + c_O \left( \frac{1}{r_3} - \frac{1}{r_1} \right) \quad (7.6)$$

The fluxes at the surfaces between the two hollow spheres are

$$q_2 = \left( -\kappa \frac{\partial T}{\partial r} \Big|_{r=r_3} \right) \cdot \mathbf{n} = \frac{\kappa_1 c_I}{r_2^2}$$

$$q_3 = \left( -\kappa \frac{\partial T}{\partial r} \Big|_{r=r_2} \right) \cdot \mathbf{n} = \frac{\kappa_2 c_O}{r_3^2}$$

Here we have used  $\kappa_1$  and  $\kappa_2$  to denote the thermal conductivity of the inner and outer blocks, respectively.

These normal conductive fluxes are included in the total energy balance at the enclosure surfaces using the radiative transport equations (for grey diffuse surfaces):

$$q_2 = \sigma \epsilon_2 T_2^4 - \epsilon_2 \sum_j F_{2j} J_j$$

$$q_3 = \sigma \epsilon_3 T_3^4 - \epsilon_3 \sum_j F_{3j} J_j$$

where  $\sigma$  is the Stefan Boltzmann constant,  $\epsilon$  is the emissivity,  $F_{ij}$  is the geometric viewfactor of surface  $i$  with respect to surface  $j$  and  $J_j$  is the radiosity for surface  $j$ .

The viewfactor coefficient  $F_{ij}$  is the fraction of energy that leaves surface  $i$  and arrives at surface  $j$ . For this geometric setup, no point on the inner surface at  $r_2$  can “see” itself (no straight line can be drawn from a point on its surface onto itself) and so  $F_{22} = 0$ . By viewfactor reciprocity

$$\sum_j F_{ij} = 1$$

we must have  $F_{23} = 1$ . The outer-to-inner view factor  $F_{32}$  can be computed analytically to be

$$F_{32} = \frac{r_2^2}{r_3^2}$$

and again by viewfactor reciprocity

$$F_{33} = 1 - F_{32} = 1 - \frac{r_2^2}{r_3^2}$$

The system of equations that must be solved for the radiosities at the inner and outer surfaces is given by

$$J_2 = \epsilon_2 \sigma T_2^4 + (1 - \epsilon_2)[F_{22}J_2 + F_{23}J_3]$$

$$J_3 = \epsilon_3 \sigma T_3^4 + (1 - \epsilon_3)[F_{32}J_2 + F_{33}J_3]$$

Solving this system of equations, we can write  $J_2$  and  $J_3$  in terms of temperature, and plug this back into the equation for the surface flux. We then get a system of two nonlinear equations to solve for  $T_2$  and  $T_3$ , the temperatures of the adjacent surfaces without Dirichlet boundary conditions. For our given set of parameters, these equations are solved iteratively in Matlab using the fsolve function.

### 7.3.6. Results

The exact temperatures at the enclosure surfaces (to six digits precision) are  $T_2 = 564.783$  and  $T_3 = 1047.825$ . From these values we can compute the values of  $c_O$  and  $c_I$  and thus the exact solution.

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, and 1, respectively (within a tolerance).

These optimal rates are observed in this test.

**Table 7.3-3.. Enclosure Radiation: Convergence Rates**

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 15588    | 2.14  | 1.90       | 1.06  |
| 117572   | 2.05  | 2.11       | 1.03  |

For input decks see Appendix [12.6.3](#).



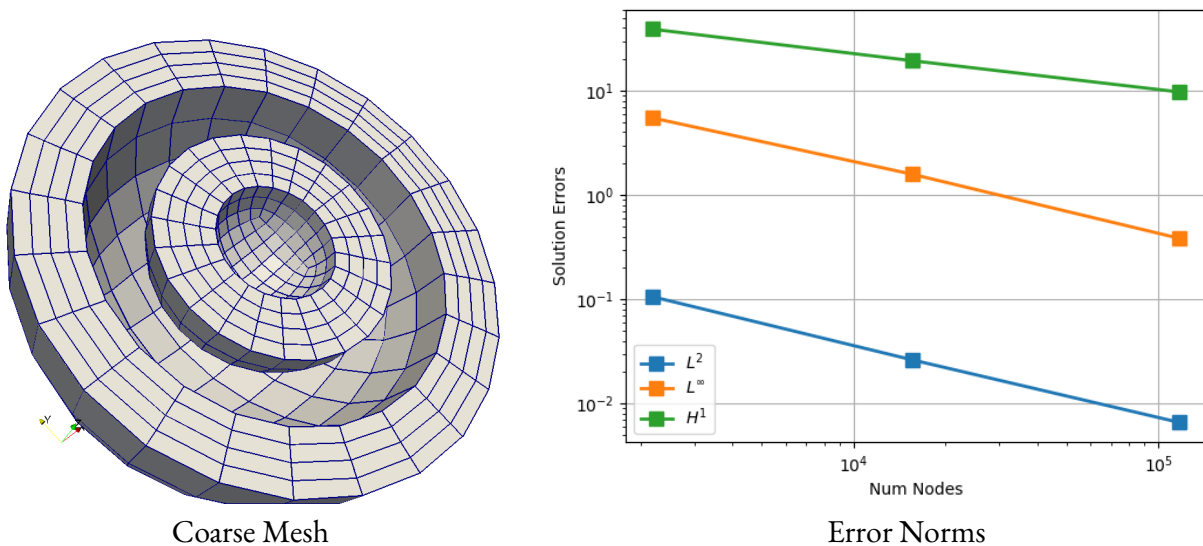


Figure 7.3-1.. Enclosure Radiation

## 7.4. 3D SPHERICAL SHELL PARTIAL ENCLOSURE

### 7.4.1. Problem Description

This problem tests coupled conduction and enclosure radiation with a partial enclosure. The geometry consists of two thick spherical shells separated by a gap. The outer shell has a section removed so that the enclosure is only partial.

### 7.4.2. Features Tested

Basic heat conduction, enclosure radiation with partial enclosure, Hex8 meshes.

### 7.4.3. Boundary Conditions

The outer and inner boundary conditions are prescribed at the nodes using the analytic solution. The analytic solution is used to set the boundary conditions on the cutaway face near the opening in the outer shell.

### 7.4.4. Material Parameters

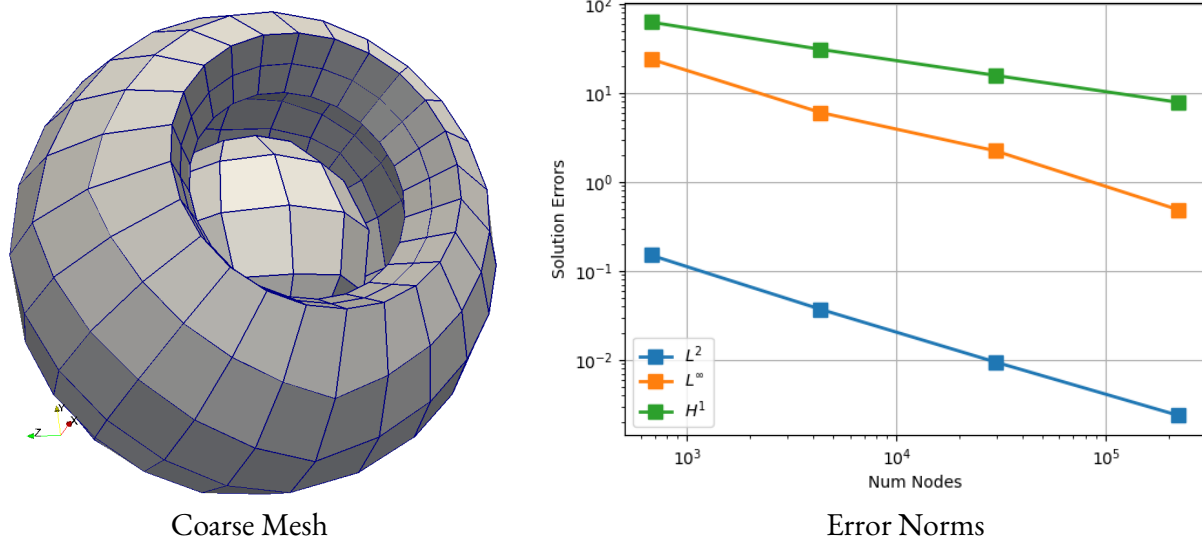
The values of density, thermal conductivity, and specific heat are all constant within each element block; however, the values differ between blocks.

## 7.4.5. Verification of Solution

The analytic solution is identical to Section 7.3. The area for the partial enclosure is computed analytically.

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 2, 2, and 1, respectively (within a tolerance).

These optimal rates are observed in this test.



**Figure 7.4-1.. Partial Enclosure Radiation**

**Table 7.4-1.. Partial Enclosure Radiation: Convergence Rates**

| Num Dofs | $L^2$ | $L^\infty$ | $H^1$ |
|----------|-------|------------|-------|
| 4338     | 2.26  | 2.22       | 1.13  |
| 29694    | 2.13  | 1.54       | 1.06  |
| 223162   | 2.06  | 2.27       | 1.03  |

For input decks see Appendix 12.6.4.

## 8. CHEMISTRY

### 8.1. FIRST ORDER REACTION (SPATIALLY VARYING TEMPERATURE)

This problem tests the interface to the CHEMEQ solver under the assumption that the temperature remains variable in space but remains constant in time. The geometry consists of a unit cube meshed with Hex8 elements refined only in one direction ( $x$ ).

#### 8.1.1. Features Tested

CHEMEQ solver; source term from chemistry; nonlinear solver; second order time integrator with fixed time steps; constant initial temperature; constant temperature boundary condition.

#### 8.1.2. Boundary Conditions

A constant temperature is applied on surface 1. The initial temperature is provided by an Encore user subroutine and the initial species values are  $A = 1$  and  $B = 0$ .

#### 8.1.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks. The CHEMEQ parameters are chosen to model a single first order reaction  $A \rightarrow B$  with constant values of pre-exponential factor and activation energy.

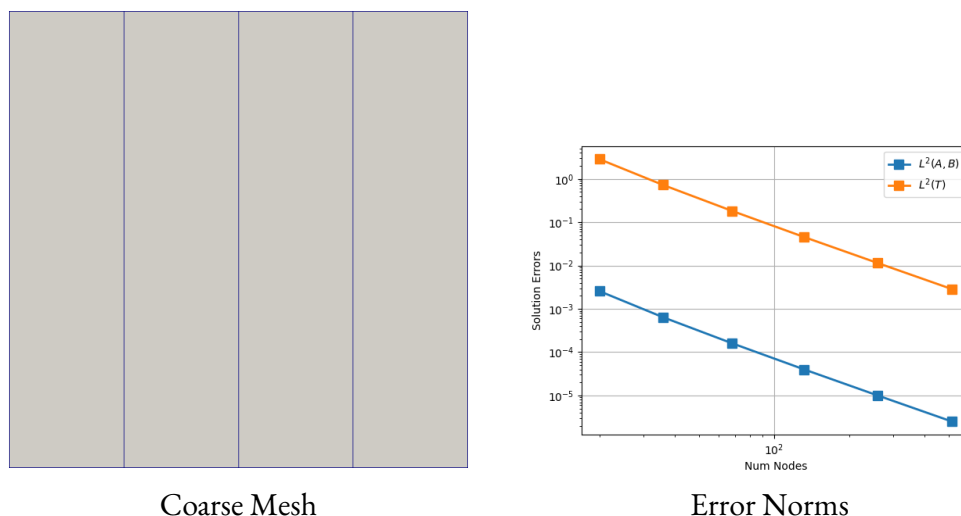
#### 8.1.4. Verification of Solution

A manufactured solution is chosen as

$$\begin{aligned}T(x) &= 400 (1 + 0.2 \cos(\pi x)), \\A(x, t) &= \exp \left\{ -\exp(5) \exp\left(-\frac{1000}{RT(x)}\right) t \right\}, \\B(x, t) &= 1 - A(x, t)\end{aligned}$$

where  $R = 1.9872$  is the ideal gas constant. A source term is used to insure that the temperature does not vary in time.

For each mesh, the errors in the temperature and species  $A$  and  $B$  are computed in the  $L^2$  norm. The test passes, only if the observed rates of convergence in these norms are 2 (within a tolerance).



**Figure 8.1-1.. First Order Reaction (Spatially Varying Temperature)**

**Table 8.1-1.. First Order Reaction (Spatially Varying Temperature): Convergence Rates for Hex8 Meshes**

| Num Dofs | $L^2(A, B)$ | $L^2(T)$ |
|----------|-------------|----------|
| 36       | 2.37        | 2.34     |
| 68       | 2.18        | 2.18     |
| 132      | 2.09        | 2.09     |
| 260      | 2.05        | 2.04     |
| 516      | 2.02        | 2.02     |

For input decks see Appendix [12.7.2](#).

## 8.2. FIRST ORDER REACTION

This problem tests the interface to the CHEMEQ solver under a temperature field that is variable in space and time. The geometry consists of a unit cube meshed with Hex8 elements refined only in one direction.

### 8.2.1. Features Tested

CHEMEQ solver; source term from chemistry; nonlinear solver; second order time integrator with fixed time steps; initial temperature from user sub; constant temperature boundary condition.

### 8.2.2. Boundary Conditions

The initial temperature and the temperature boundary condition on surface 1 are provided by an Encore user subroutine and the initial species values are  $A = 1$  and  $B = 0$ .

### 8.2.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant, equal to one in both blocks. The CHEMEQ parameters are chosen to model a single first order reaction  $A \rightarrow B$  with constant values of pre-exponential factor and activation energy.

### 8.2.4. Verification of Solution

A manufactured solution is chosen as

$$\begin{aligned}\phi(x, t) &= \exp(a - E/(RT_0))(1 + 0.1 \sin(x)) \exp(t), \\ \Phi(x, t) &= \exp(a - E/(RT_0))(1 + 0.1 \sin(x))(\exp(t) - 1), \\ T(x, t) &= (E/R)/(a - \ln(\phi(x, t))), \\ A(x, t) &= \exp(-\Phi(x, t)), \\ B(x, t) &= 1 - A(x, t)\end{aligned}$$

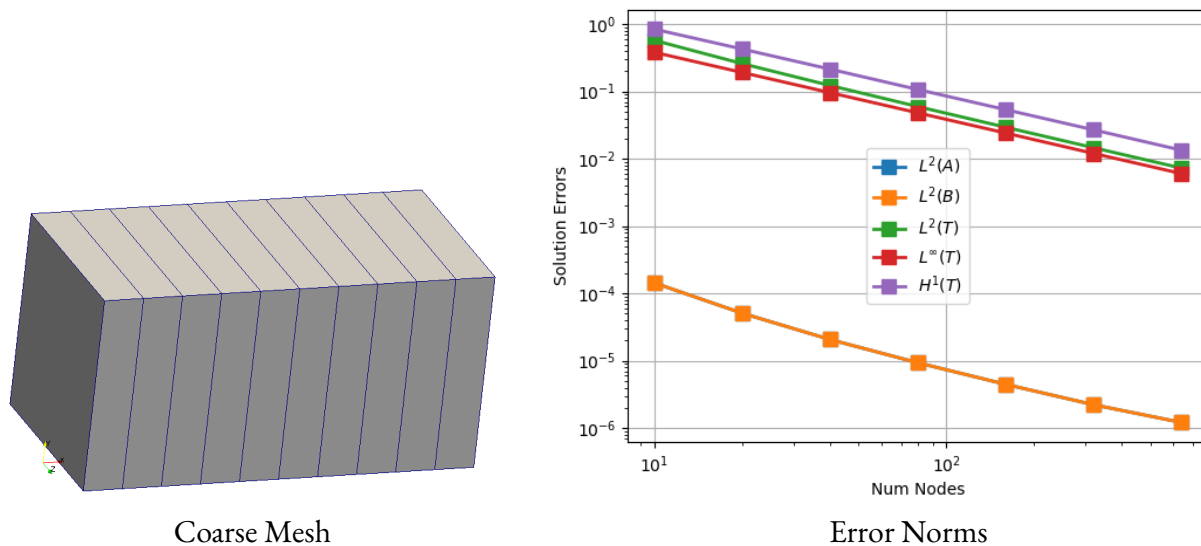
where  $a$  is the log pre-exponential factor,  $R$  is the ideal gas constant,  $E$  is the activation energy, and  $T_0$  is a reference temperature value. The form of the solution is contrived so that

$$\begin{aligned}\partial_t A(x, t) &= -\partial_t B(x, t) = -\phi(x, t) A(x, t) \\ \phi(x, t) &= \exp(a) \exp\left(-\frac{E}{RT(x, t)}\right)\end{aligned}$$

This allows the chemistry ODEs to be satisfied exactly, but a source term is needed in the energy equation.

For each mesh, the errors in the temperature and species  $A$  and  $B$  are computed in the  $L^2$  norm. The test passes, only if the observed rates of convergence in these norms are 1 (within a tolerance). Currently it is not clear why the convergence rates are only first order.

For input decks see Appendix [12.7.3](#).



**Figure 8.2-1.. First Order Reaction**

**Table 8.2-1.. First Order Reaction: Convergence Rates for Hex8 Meshes**

| Num Dofs | $L^2(A)$ | $L^2(B)$ | $L^2(T)$ | $L^\infty(T)$ | $H^1(T)$ |
|----------|----------|----------|----------|---------------|----------|
| 20       | 1.50     | 1.50     | 1.15     | 1.00          | 0.98     |
| 40       | 1.30     | 1.30     | 1.08     | 1.00          | 1.00     |
| 80       | 1.15     | 1.15     | 1.04     | 1.00          | 1.00     |
| 160      | 1.07     | 1.07     | 1.02     | 1.00          | 1.00     |
| 320      | 1.00     | 1.00     | 1.01     | 1.00          | 1.00     |
| 640      | 0.88     | 0.88     | 1.01     | 1.00          | 1.00     |

## 8.3. DAE AND PRESSURE TEST

This test runs CHEMEQ with a kinetics model that includes both pressure dependence and distributed activation energy for a single element mesh with uniform temperature and pressure.

### 8.3.1. Features Tested

Basic heat conduction on a Hex8 mesh; CHEMEQ solver with pressure dependence and distributed activation energy.

### 8.3.2. Boundary Conditions

No boundary conditions are prescribed, resulting in an adiabatic flux BC.

### 8.3.3. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant.

### 8.3.4. Verification of Solution

The analytic solution for the concentration of species A as a function of time for the constant values used in this test case is

$$A(t) = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{6} \left( \sqrt{2} - 6 \operatorname{erf}^{-1}\left(1 - 45t / \left(2 \exp\left(\frac{61}{18}\right)\right)\right)\right)\right)$$

The test compares the temperature errors against a gold file of the error at each time step. The exact solution for the concentration of A is also output to the exodus file and a comparison plotting that and the solved for concentration as a function of time has them lying on top of one another.

For input decks see Appendix [12.7.4](#).

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## 9. MISCELLANEOUS

### 9.1. THERMAL POSTPROCESSING

#### 9.1.1. Problem Description

This problem tests basic thermal postprocessors in Aria.

#### 9.1.2. Features Tested

Basic heat conduction, thermal postprocessors, Hex8 meshes.

#### 9.1.3. Boundary Conditions

Dirichlet BCs are specified using the exact solution on surface 1. On surface 2, a natural convection BC is specified, using the exact solution as the reference temperature and a constant heat transfer coefficient. Similarly, a radiative flux BC is applied on surface 3, with constant values of emissivity and radiation form factor. A source term is applied within all blocks based on substituting the exact solution into the heat conduction operator.

#### 9.1.4. Material Parameters

The values of density, thermal conductivity, and specific heat are all constant with the same value for both blocks.

#### 9.1.5. Verification of Solution

The manufactured solution is

$$T_0 + \exp(C_0(x^2 - 1) + C_1(y^2 - 0.25) + C_2(z^2 - 0.25) + C_3t).$$

Postprocessors are computed for the integrated power output for convective and radiative BCs (cf\_bc\_ipo, rf\_bc\_ipo), the integrated flux output for convective and radiative BCs (cf\_bc\_ifo, rf\_bc\_ifo), the integrated power output for volume source terms (src\_ipo), and several point evaluations (eval\_b1, eval\_b1b2, eval\_s2).

For each mesh, the errors in the temperature solution are computed in the  $L^2$  norm and for various postprocessors. The test passes, only if the observed rates of convergence are 2 (except for the integrated power output for source terms, which convergences with order 4).

These optimal rates are observed in this test clearly in most cases. However, for the point evaluation cases, a large amount of variability exists in the convergence rates.

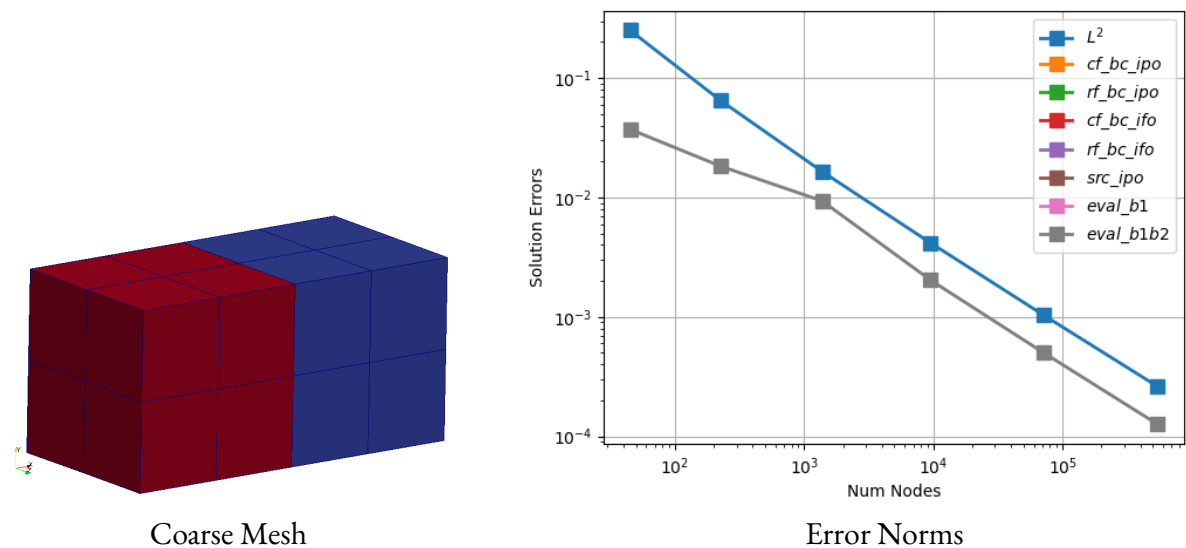


Figure 9.1-1.. Thermal Postprocess

| Table 9.1-1.. Thermal Postprocess: Convergence Rates |       |                  |                  |                  |                  |                |                  |                |
|--|-------|------------------|------------------|------------------|------------------|----------------|------------------|----------------|
| Num Dofs   | $L^2$ | <i>cf_bc_ipo</i> | <i>rf_bc_ipo</i> | <i>cf_bc_ifo</i> | <i>rf_bc_ifo</i> | <i>eval_b1</i> | <i>eval_b1b2</i> | <i>eval_s2</i> |
| 225  | 2.52  | 2.87             | 2.82             | 2.87             | 2.82             | 1.16           | -2.02            | 1.33           |
| 1377   | 2.27  | 2.52             | 2.48             | 2.52             | 2.48             | 1.77           | 1.66             | 1.11           |
| 9537   | 2.14  | 2.27             | 2.25             | 2.27             | 2.25             | 2.11           | 2.15             | 2.37           |
| 70785  | 2.07  | 2.13             | 2.12             | 2.13             | 2.12             | 2.06           | 2.07             | 2.09           |
| 545025   | 2.04  | 2.06             | 2.05             | 2.06             | 2.05             | 2.03           | 2.04             | 2.04           |

For input decks see [Appendix 12.8.1](#).

## 9.2. LOCAL COORDINATES: CARTESIAN

This problem tests the use of a local Cartesian coordinate system in a material model. The geometry is a 3D cube that has been rotated.

### 9.2.1. Features Tested

Steady heat conduction, time integrators, tensor thermal conductivity, local Cartesian coordinates in a material model.

### 9.2.2. Boundary Conditions

The boundary conditions are prescribed at the nodes using the analytic solution. The initial condition is specified using an Encore function evaluated at the nodes.

### 9.2.3. Material Parameters

The specific heat is constant. The density and thermal conductivity are constant, with a diagonal (tensor) thermal conductivity in the local coordinate space of the material.

### 9.2.4. Verification of Solution

A manufactured solution is chosen as

$$T(X, Y, Z) = T_0 + T_1 \cos(x_k X) \cos(y_k Y) \cos(z_k Z)$$

where  $(X, Y, Z)$  are the local material coordinates, which are related to the Cartesian coordinates  $(x, y, z)$  by a rotation matrix consisting of a product of rotations (22.5 deg around the  $z$ -axis and 45 deg around the  $x$ -axis).

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 1 for  $H^1$  and 2 for all other norms (within a tolerance).

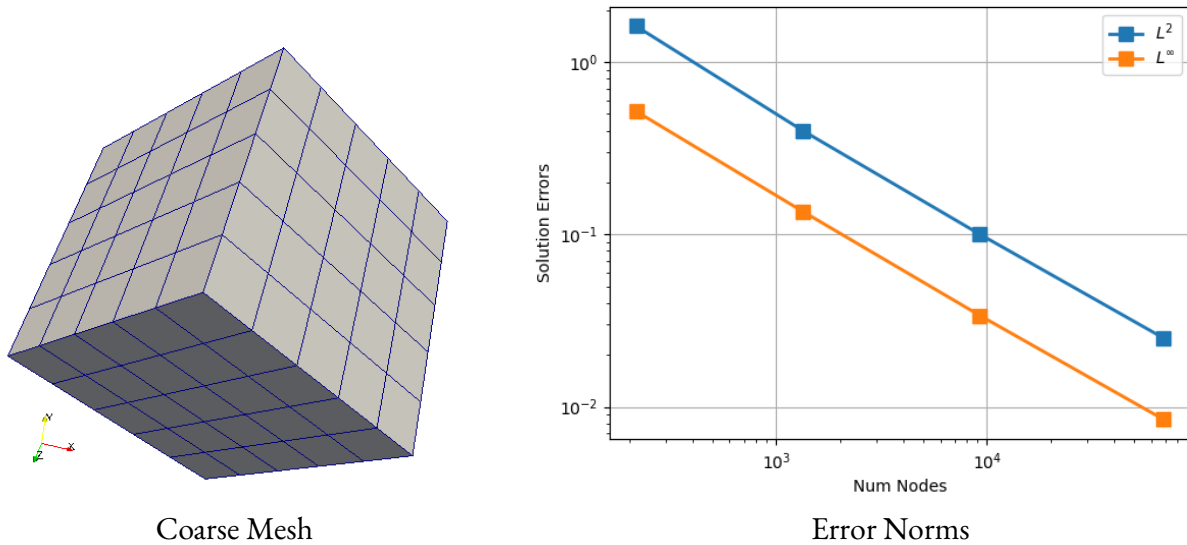
**Table 9.2-1.. Local Cartesian Coordinate System: Convergence Rates**

| Num Dofs | $L^2$ | $L^\infty$ |
|----------|-------|------------|
| 1331     | 2.29  | 2.20       |
| 9261     | 2.15  | 2.15       |
| 68921    | 2.07  | 2.07       |

For input decks see Appendix [12.8.3](#).

## 9.3. LOCAL COORDINATES: CYLINDRICAL

This problem tests the use of a local cylindrical coordinate system in a material model. The geometry is a 3D cube that has been rotated.



**Figure 9.2-1.. Local Cartesian Coordinate System**

### 9.3.1. Features Tested

Steady heat conduction, time integrators, tensor thermal conductivity, local coordinates in a material model.

### 9.3.2. Boundary Conditions

The boundary conditions are prescribed at the nodes using the analytic solution. The initial condition is specified using an Encore function evaluated at the nodes.

### 9.3.3. Material Parameters

The specific heat and density are constant. The diagonal (tensor) components of the thermal conductivity are specified using constant values in the local coordinate space of the material.

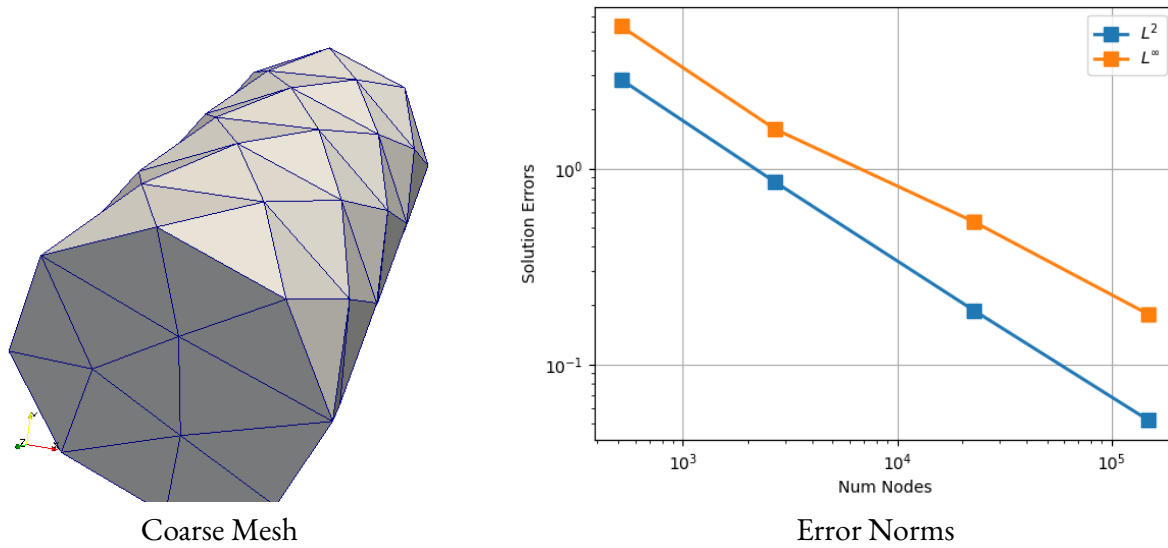
### 9.3.4. Verification of Solution

A manufactured solution is chosen as

$$T(X, Y, Z) = T_0 + T_1(2R)^3 \cos(\theta) \cos(z_k Z)$$

where  $(R, \Theta, Z)$  are the local cylindrical material coordinates, which are related to the standard cylindrical coordinates  $(x, y, z)$  by a rotation matrix consisting of a product of rotations (22.5 deg around the  $z$ -axis and 45 deg around the  $x$ -axis).

For each mesh, the errors in the temperature solution are computed in the  $L^2$ ,  $L^\infty$  and  $H^1$  norms. The test passes, only if the observed rates of convergence in these norms are 1 for  $H^1$  and 2 for all other norms (within a tolerance).



**Figure 9.3-1.. Local Cylindrical Coordinate System**

**Table 9.3-1.. Local Cylindrical Coordinate System: Convergence Rates**

| Num Dofs | $L^2$ | $L^\infty$ |
|----------|-------|------------|
| 2692     | 2.19  | 2.19       |
| 22723    | 2.12  | 1.53       |
| 148839   | 2.07  | 1.74       |

For input decks see Appendix [12.8.4](#).

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# 10. LOW-MACH FLUID FLOW

Documentation for the following tests is in progress:

```
1 _rtest/aria/cvfemConvTaylorVortex/cvfemConvTaylorVortex.test|np4
2 _rtest/aria/gfemConvTaylorVortex/gfemConvTaylorVortex.test|np4
3 _rtest/aria/hfemConvTaylorVortex/hfemConvTaylorVortex.test|np4
4 mConvTaylorVortex/cvfemConvTaylorVortex.test|np8
5 mSteadyTaylorVortex/cvfemSteadyTaylorVortex.test|np8
6 mSteadyTaylorVortexKeps/cvfemSteadyTaylorVortexKeps.test|np8
7 m_couette_flow/cdfem_couette_flow.test|cdfem_couette_flow_tri3
8 m_couette_flow/cdfem_couette_flow.test|cdfem_couette_flow_tri6
9 ConvTaylorVortex/gfemConvTaylorVortex.test|np8
10 SteadyTaylorVortex/gfemSteadyTaylorVortex.test|np8
```

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# 11. HOW TO BUILD THIS DOCUMENT

You need to have Sierra developer access (through WebCars). Then you should clone the Sierra Git repository containing the tests to a location with adequate memory (currently more than 80GB), using a command like this:

```
git clone sierra-git:/git/tests
```

Then you need to assign the verification tests, running the following command from your local tests repository:

```
assign --path aria_rtest/verification
```

This will produce a text file called assigned.tests containing the list of all tests to run. You should edit the second line of this file to indicate the remote location (accessible from the HPC machine where you will run the tests). For example, I might have something like this:

```
# Created by assign at Fri Sep 19 09:52:09 2014
#@ /gscratch1/bcarnes/TESTS
aria_rtest/verification/1dnonlin_verify1/1dnonlin_verify1.test|np8
aria_rtest/verification/cyl_shell_2d/cyl_shell_2d.test|np8
aria_rtest/verification/cyl_shell_3d/cyl_shell_3d.test|np8
...
```

Next you need to copy the test files and the assigned.test file to the remote location (here it is “/gscratch1/bcarnes/TESTS/”):

```
rsync -azv aria_rtest/verification redsky:/gscratch1/bcarnes/TESTS/aria_rtest
scp assigned.tests redsky:/gscratch1/bcarnes/TESTS/
```

Here I am only copying the verification test sub-directory, since I do not want to run any other tests.

On the HPC machine, you will need to load a pre-built version of the code such as the nightly master build:

```
module load sierra/master
```

To see where the executables are located, you can run something like:

```
[bcarnes@redsky-login9 ~]$ which aria
/projects/sierra/redsky/install/master/bin/aria
```

Finally, to run the tests, you use the testrun script, with a few additional arguments. The first locates the source code needed to compile the various user subroutines (which we just found from running “which aria”), the second enables tests to run as long as needed, the third uses the queue, and the fourth saves the results so you can use them in the manual.

```
testrun --user sourcedir=/projects/sierra/tlcc2/install/master/ \
        --allow-multipliers=time \
        --queued \
        --save-all-results
```

It may take 1-2 hours to run all the tests. Note that if the tests start to fail with an error associated with the ACCOUNT not being set, you may need to set it using your WCID:

```
export ACCOUNT=fyXXXXXX
```

To view your available WCIDs, run the following command:

```
mywcid
```

To build this manual, you should clone the Sierra Git repository containing the documentation files using a command like this:

```
git clone sierra-git:/git/docs
```

Then go to the directory within your local repository containing the Aria Verification Manual files:

```
cd aria/doc/verification_manual
```

Once the tests have all ran successfully, you should sync the results from the remote location back to this directory:

```
rsync -azv redsky:/gscratch1/bcarnes/TESTS/results .
```

Then run the a script to execute any local postprocessing needed to create the plots for the tests:

```
python ariaPostprocess.py
```

Finally you can create the manual using pdflatex:

```
pdflatex Aria_Verification_Manual.tex
```

which should create a new PDF output file.

# 12. INPUT DECKS FOR VERIFICATION PROBLEMS

## 12.1. BASIC THERMAL TESTS

### 12.1.1. Steady Heat Conduction: Hex8 Meshes

```
# T_ref_s1 = { T_ref_s1 = 1 }
# h_s1 = { h_s1 = 1 }
# T_ref_s2 = { T_ref_s2 = 2 }
# h_s2 = { h_s2 = 2 }

BEGIN SIERRA myJob
  #{include(mmsMaterial.inc)}

  BEGIN TPETRA EQUATION SOLVER  DIRECT_SOLVER
    BEGIN SUPERLU SOLVER
    END
  END TPETRA EQUATION SOLVER

  BEGIN TPETRA EQUATION SOLVER  ITERATIVE_SOLVER
    BEGIN GMRES SOLVER
      BEGIN DD-ILUT PRECONDITIONER
      END
      MAXIMUM ITERATIONS = 300
      CONVERGENCE TOLERANCE = automatic
    END
  END TPETRA EQUATION SOLVER

  BEGIN FINITE ELEMENT MODEL cube
    database name = cube_h{N}_{TOPO}.e
    coordinate system is cartesian
    decomposition method = rcb

    BEGIN PARAMETERS FOR BLOCK block_1
      material Kryptonite
    END PARAMETERS FOR BLOCK block_1

  END FINITE ELEMENT MODEL cube

  BEGIN PROCEDURE myAriaProcedure

    Begin Solution Control Description
      Use System Main
      Begin System Main
        Begin Sequential The_Time_Block
          Advance myRegion
        End
        Simulation Start Time = 0
```

```

Simulation Termination Time = 1
End
End

BEGIN ARIA REGION myRegion

Nonlinear Solution Strategy = Newton
Maximum Nonlinear Iterations = 10
Nonlinear Residual Tolerance = 1.0e-12
Nonlinear Correction Tolerance = 1.0e-12
Nonlinear Relaxation Factor = 1.0

use finite element model cube
Use Linear Solver Iterative_Solver #Direct_Solver

EQ {EQUATION} for TEMPERATURE on block_1 using Q1 with DIFF SRC

# surface_4: x=0
# surface_6: x=1

# surface_3: y=0
# surface_5: y=1

# surface_1: z=1
# surface_2: z=0

# const Temp BC (x)
BC const dirichlet at surface_4 Temperature = 1.0
BC const dirichlet at surface_6 Temperature = 1.0

# const flux BC (y)
BC FLUX for {EQUATION} on surface_3 = constant flux = 3
BC FLUX for {EQUATION} on surface_3 = scalar_string_function f="exact_flux_y - 3"

BC FLUX for {EQUATION} on surface_5 = constant flux = 5
BC FLUX for {EQUATION} on surface_5 = scalar_string_function f="-exact_flux_y - 5"

# convective flux BC with const Temp and H (z)
BC Flux for {EQUATION} on surface_1 = Nat_Conv T_Ref={T_ref_s1} H={h_s1}
BC FLUX for {EQUATION} on surface_1 = scalar_string_function \$
    f="-exact_flux_z - {h_s1}*(exact_sln - {T_ref_s1})"

BC Flux for {EQUATION} on surface_2 = Nat_Conv T_Ref={T_ref_s2} H={h_s2}
BC FLUX for {EQUATION} on surface_2 = scalar_string_function \$
    f="exact_flux_z - {h_s2}*(exact_sln - {T_ref_s2})"

# const source term
Source For {EQUATION} on block_1 = Constant value=1
Source For {EQUATION} on block_1 = scalar_string_function f="exact_src - 1"

postprocess l_inf_norm of function "exact_sln - temperature" on all_blocks as linf_err
postprocess l2_norm of function "exact_sln - temperature" on all_blocks as l2_err
postprocess l2_norm of function "sqrt((grad_temperature_x-gradExact_sln_x)^2+(grad_temperature_y-gradExact_sln_y)^2+(grad

postprocess value of expression exact_sln on all_blocks as T_exact
postprocess value of function "abs(temperature - exact_sln)" on all_blocks as error

Begin heartbeat testctrl
Stream name = errors_{TOPO}_h{N}.dat
Timestamp Format = ""
Precision = 8
Format = csv
Labels = Off
Legend = On
At step 0, Increment is 1
variable = global time
variable = global number_of_nodes
variable = global l2_err

```

```

        variable = global h1_err
        variable = global linf_err
    end

    BEGIN RESULTS OUTPUT LABEL diffusion output
        database Name = {EQUATION}_{TOPO}_h{N}.e
        at step 1, increment = 1
        title Aria cube test
        nodal variables = nonlinear_solution->TEMPERATURE as T
        nodal variables = error
    END RESULTS OUTPUT LABEL diffusion output

    END ARIA REGION myRegion

    END PROCEDURE myAriaProcedure

END SIERRA myJob

```

## 12.1.2. Steady Heat Conduction: Hex20 Meshes

## 12.1.3. Steady Heat Conduction: Hex27 Meshes

## 12.1.4. Steady Heat Conduction: Tet4 Meshes

```

# T_ref_s1 = { T_ref_s1 = 1 }
# h_s1 = { h_s1 = 1 }
# T_ref_s2 = { T_ref_s2 = 2 }
# h_s2 = { h_s2 = 2 }

BEGIN SIERRA myJob
    #{include(mmsMaterial.inc)}

    BEGIN TPETRA EQUATION SOLVER  DIRECT_SOLVER
        BEGIN SUPERLU SOLVER
        END
    END TPETRA EQUATION SOLVER

    BEGIN TPETRA EQUATION SOLVER  ITERATIVE_SOLVER
        BEGIN GMRES SOLVER
            BEGIN DD-ILUT PRECONDITIONER
            END
            MAXIMUM ITERATIONS = 300
            CONVERGENCE TOLERANCE = automatic
        END
    END TPETRA EQUATION SOLVER

    BEGIN FINITE ELEMENT MODEL cube
        database name = cube_h{N}_{TOPO}.e
        coordinate system is cartesian
        decomposition method = rcb

        BEGIN PARAMETERS FOR BLOCK block_1
            material Kryptonite
        END PARAMETERS FOR BLOCK block_1

    END FINITE ELEMENT MODEL cube

    BEGIN PROCEDURE myAriaProcedure

        Begin Solution Control Description
    
```

```

Use System Main
Begin System Main
  Begin Sequential The_Time_Block
    Advance myRegion
  End
  Simulation Start Time = 0
  Simulation Termination Time = 1
End
End

BEGIN ARIA REGION myRegion

Nonlinear Solution Strategy = Newton
Maximum Nonlinear Iterations = 10
Nonlinear Residual Tolerance = 1.0e-12
Nonlinear Correction Tolerance = 1.0e-12
Nonlinear Relaxation Factor = 1.0

use finite element model cube
Use Linear Solver Iterative_Solver #Direct_Solver

EQ {EQUATION} for TEMPERATURE on block_1 using Q1 with DIFF SRC

# surface_4: x=0
# surface_6: x=1

# surface_3: y=0
# surface_5: y=1

# surface_1: z=1
# surface_2: z=0

# const Temp BC (x)
BC const dirichlet at surface_4 Temperature = 1.0
BC const dirichlet at surface_6 Temperature = 1.0

# const flux BC (y)
BC FLUX for {EQUATION} on surface_3 = constant flux = 3
BC FLUX for {EQUATION} on surface_3 = scalar_string_function f="exact_flux_y - 3"

BC FLUX for {EQUATION} on surface_5 = constant flux = 5
BC FLUX for {EQUATION} on surface_5 = scalar_string_function f="-exact_flux_y - 5"

# convective flux BC with const Temp and H (z)
BC Flux for {EQUATION} on surface_1 = Nat_Conv T_Ref={T_ref_s1} H={h_s1}
BC FLUX for {EQUATION} on surface_1 = scalar_string_Function \$
  f="-exact_flux_z - {h_s1}*(exact_sln - {T_ref_s1})"

BC Flux for {EQUATION} on surface_2 = Nat_Conv T_Ref={T_ref_s2} H={h_s2}
BC FLUX for {EQUATION} on surface_2 = scalar_string_Function \$
  f="exact_flux_z - {h_s2}*(exact_sln - {T_ref_s2})"

# const source term
Source For {EQUATION} on block_1 = Constant value=1
Source For {EQUATION} on block_1 = scalar_string_Function f="exact_src - 1"

postprocess l_inf_norm of function "exact_sln - temperature" on all_blocks as linf_err
postprocess l_2_norm of function "exact_sln - temperature" on all_blocks as l2_err
postprocess l_2_norm of function "sqrt((grad_temperature_x-gradExact_sln_x)^2+(grad_temperature_y-gradExact_sln_y)^2+(grad_temperature_z-gradExact_sln_z)^2)" on all_blocks as l2_err

postprocess value of expression exact_sln on all_blocks as T_exact
postprocess value of function "abs(temperature - exact_sln)" on all_blocks as error

Begin heartbeat testctrl
  Stream name = errors_{TOPO}_h{N}.dat
  Timestamp Format = ""
  Precision = 8
  Format = csv

```

```

Labels = Off
Legend = On
At step 0, Increment is 1
variable = global time
variable = global number_of_nodes
variable = global l2_err
variable = global h1_err
variable = global linf_err
end

BEGIN RESULTS OUTPUT LABEL diffusion output
  database Name = {EQUATION}_{TOPO}_h{N}.e
  at step 1, increment = 1
  title Aria cube test
  nodal variables = nonlinear_solution->TEMPERATURE as T
  nodal variables = error
END RESULTS OUTPUT LABEL diffusion output

END ARIA REGION myRegion

END PROCEDURE myAriaProcedure

END SIERRA myJob

```

## 12.1.5. Steady Heat Conduction: Tet4Tet10 Meshes

```

BEGIN SIERRA myJob

Begin universal aria expressions
  User Expression = Scalar_String_Function \$
    f = "(x-x^2)^2*(y-y^2)^2*(z-z^2)^2 + 1" \$
    user_tag = T_mms

  User Expression = Vector_String_Function \$
    f_x = "2*(x-x^2)*(1-2*x)*(y-y^2)^2*(z-z^2)^2" \$
    f_y = "2*(y-y^2)*(1-2*y)*(x-x^2)^2*(z-z^2)^2" \$
    f_z = "2*(z-z^2)*(1-2*z)*(x-x^2)^2*(y-y^2)^2" \$
    user_tag = gradT

  User Expression = Scalar_String_Function \$
    f = "2*((1-2*x)^2 - 2*(x-x^2))*(y-y^2)^2*(z-z^2)^2" \$
      + "2*((1-2*y)^2 - 2*(y-y^2))*(x-x^2)^2*(z-z^2)^2" \$
      + "2*((1-2*z)^2 - 2*(z-z^2))*(x-x^2)^2*(y-y^2)^2" \$
    user_tag = laplaceT

  User Expression = Scalar_String_Function \$
    f = "-laplaceT - 1.0" \$
    user_tag = mms_src
End

BEGIN ARIA MATERIAL Kryptonite
  Density = Constant rho=1
  Thermal Conductivity = constant k=1
  Specific Heat = Constant cp=1
  heat conduction = basic
END ARIA MATERIAL Kryptonite

BEGIN TPETRA EQUATION SOLVER ITERATIVE_SOLVER
  BEGIN CG SOLVER
    BEGIN JACOBI PRECONDITIONER
      END
    MAXIMUM ITERATIONS = 1000
    RESIDUAL SCALING = NONE
    CONVERGENCE TOLERANCE = 1.000000e-14
  END

```

END TPETRA EQUATION SOLVER

```
BEGIN FINITE ELEMENT MODEL cube
  database name = cube_h{N}_tet10.e
  coordinate system is cartesian
  decomposition method = rcb
```

```
BEGIN PARAMETERS FOR BLOCK block_1
  material Kryptonite
END PARAMETERS FOR BLOCK block_1
```

END FINITE ELEMENT MODEL cube

BEGIN PROCEDURE myAriaProcedure

```
  Begin Solution Control Description
    Use System Main
    Begin System Main
      Begin Sequential The_Time_Block
        Advance myRegion
      End
      Simulation Start Time = 0
      Simulation Termination Time = 1
    End
  End
```

BEGIN ARIA REGION myRegion

```
  Nonlinear Solution Strategy = Newton
  Maximum Nonlinear Iterations = 10
  Nonlinear Residual Tolerance = 1.0e-12
  Nonlinear Correction Tolerance = 1.0e-12
  Nonlinear Relaxation Factor = 1.0
  use dof averaged nonlinear residual

  use finite element model cube
  Use Linear Solver Iterative_Solver #Direct_Solver

  EQ ENERGY for TEMPERATURE on block_1 using Q1 with DIFF SRC
```

```
# surface_4: x=0
# surface_6: x=1
```

```
# surface_3: y=0
# surface_5: y=1
```

```
# surface_1: z=1
# surface_2: z=0
```

```
# const Temp BC (x)
BC const dirichlet at surface_4 Temperature = 1.0
BC const dirichlet at surface_6 Temperature = 1.0
```

```
# const flux BC (y)
BC FLUX for Energy on surface_3 = constant flux = 3
BC FLUX for Energy on surface_5 = constant flux = 5
```

```
BC Flux for Energy on surface_3 = Scalar_String_Function f = "-gradT_y - 3.0"
BC Flux for Energy on surface_5 = Scalar_String_Function f = "gradT_y - 5.0"
```

```
# convective flux BC with const Temp and H (z)
BC Flux for Energy on surface_1 = Nat_Conv T_Ref=1 H=1
BC Flux for Energy on surface_2 = Nat_Conv T_Ref=2 H=2
```

```
BC Flux for Energy on surface_1 = Scalar_String_Function f = "gradT_z - (T_mms-1.0)"
BC Flux for Energy on surface_2 = Scalar_String_Function f = "-gradT_z - 2.0*(T_mms-2.0)"
```



```

# const source term
Source For ENERGY on block_1 = Constant value=1
Source for ENERGY on block_1 = Scalar_String_Function f = "mms_src"

postprocess L2_norm of function "temperature-T_mms" on all_blocks as l2
postprocess L_inf_norm of function "temperature-T_mms" on all_blocks as linf
postprocess L2_norm of function "sqrt((grad_temperature_x-gradT_x)^2+(grad_temperature_y-gradT_y)^2+(grad_temperature_z-gradT_z)^2)" on all_blocks as l2_sqrt

Begin Heartbeat hb
  At Step 0 Interval is 1
  Precision = 6
  Stream Name = errors_h{N}.dat
  legend = off
  labels = off
  Timestamp Format = ""
  variable = global time
  variable = global number_of_nodes
  variable = global l2
  variable = global h1
  variable = global linf
End

BEGIN RESULTS OUTPUT LABEL diffusion output
  database Name = thermal_steady_tet4_h{N}.e
  at step 1, increment = 1
  title Aria cube test
  nodal variables = nonlinear_solution->TEMPERATURE as T
END RESULTS OUTPUT LABEL diffusion output

END ARIA REGION myRegion

END PROCEDURE myAriaProcedure

END SIERRA myJob

```

## 12.1.6. Steady Heat Conduction: Tet10 Meshes

## 12.1.7. Transient Heat Conduction: Hex8 Meshes

```

# T_ref_s1 = { T_ref_s1 = 1 }
# h_s1 = { h_s1 = 1 }
# T_ref_s2 = { T_ref_s2 = 2 }
# h_s2 = { h_s2 = 2 }
# C_s = { C_s = 10000.0 }

BEGIN SIERRA myJob
  #{include(exact_transient.inc)}

  BEGIN ARIA MATERIAL Kryptonite
    Density = Constant rho={rho}
    Thermal Conductivity = constant k={k}
    Specific Heat = Constant cp={Cp}
    heat conduction = basic
    latent heat = constant value={L}

    user expression = scalar_string_function user_tag = "m_s" \$
      f = "{C_s}*(exp(-(t) - exp(-(t - 1)*(t - 1)))*(t*(2*t - 2) - 1))"

    user expression = scalar_string_function user_tag = "m_s_dot" \$
      f = "-{C_s}*(exp(-(t) + exp(-(t - 1)*(t - 1)))*(4*t - 2) - exp(-(t - 1)*(t - 1))*(2*t - 2)*(t*(2*t - 2) - 1))"
  END ARIA MATERIAL Kryptonite

  BEGIN TPETRA EQUATION SOLVER DIRECT_SOLVER

```

```

BEGIN SUPERLU SOLVER
END
END TPETRA EQUATION SOLVER

BEGIN TPETRA EQUATION SOLVER  ITERATIVE_SOLVER
BEGIN CG SOLVER
  BEGIN JACOBI PRECONDITIONER
  END
  MAXIMUM ITERATIONS = 1000
  RESIDUAL SCALING = NONE
  CONVERGENCE TOLERANCE = 1.000000e-12
END
END TPETRA EQUATION SOLVER

BEGIN FINITE ELEMENT MODEL cube
  database name = cube_h{N}_hex8.e
  coordinate system is cartesian
  decomposition method = rcb

  BEGIN PARAMETERS FOR BLOCK block_1
    material Kryptonite
  END PARAMETERS FOR BLOCK block_1

END FINITE ELEMENT MODEL cube

BEGIN PROCEDURE myAriaProcedure

  Begin Solution Control Description
    Use System Main
    Begin System Main
      Begin Transient The_Time_Block
        Advance myRegion
      End
      Simulation Start Time = 0
      Simulation Termination Time = 3
    End
    Begin Parameters For Transient The_Time_Block
      Begin Parameters For Aria Region myRegion
        Initial Time Step Size = {0.5/2**N}
        Time Integration Method = Second_Order
        Time Step Variation = fixed
      End
    End
  End

  BEGIN ARIA REGION myRegion

    Nonlinear Solution Strategy = Newton
    Maximum Nonlinear Iterations = 10
    Nonlinear Residual Tolerance = 1.0e-12
    Nonlinear Correction Tolerance = 1.0e-12
    Nonlinear Relaxation Factor = 1.0
    use dof averaged nonlinear residual

    use finite element model cube
    Use Linear Solver Iterative_Solver #Direct_Solver

    EQ ENERGY for TEMPERATURE on block_1 using Q1 with MASS DIFF SRC

    IC for temperature on block_1 = scalar_string_function f="exact_sln"

    # surface_4: x=0
    # surface_6: x=1

    # surface_3: y=0
    # surface_5: y=1

```

```

# surface_1: z=1
# surface_2: z=0

# const Temp BC (x)
BC const dirichlet at surface_4 Temperature = 1.0
BC const dirichlet at surface_6 Temperature = 1.0

# const flux BC (y)
BC FLUX for Energy on surface_3 = constant flux = 3
BC FLUX for Energy on surface_3 = scalar_string_function f="exact_flux_y - 3"

Begin Heat Flux Boundary Condition hfbc2
  Add Surface surface_5
  Flux = -5
End
BC FLUX for Energy on surface_5 = scalar_string_function f="-exact_flux_y - 5"

# convective flux BC with const Temp and H (z)
BC Flux for Energy on surface_1 = Nat_Conv T_Ref={T_ref_s1} H={h_s1}
BC FLUX for Energy on surface_1 = scalar_string_function \$
  f="-exact_flux_z - {h_s1}*(exact_sln - {T_ref_s1})"

Begin Convective Flux Boundary Condition cfbc2
  Add Surface surface_2
  Convective Coefficient = {h_s2}
  Reference Temperature = {T_ref_s2}
End
BC FLUX for Energy on surface_2 = scalar_string_function \$
  f="exact_flux_z - {h_s2}*(exact_sln - {T_ref_s2})"

# const source term
Begin Volume Heating vh1
  Add Volume block_1
  Value = 1
End
Source for Energy on block_1 = melting Ts={Ts} Tl={Tl}
Source For ENERGY on block_1 = scalar_string_function f="exact_src - melting_src - 1"

postprocess max of function "abs(exact_sln - temperature)" on all_blocks as linf_err
postprocess l2_norm of function "exact_sln - temperature" on all_blocks as l2_err
postprocess l2_norm of function "exact_sln_dot - dt_temperature" on all_blocks as l2_dot_err
postprocess integral of function "\$
  (gradExact_sln_x - grad_temperature_x)^2 + \$
  (gradExact_sln_y - grad_temperature_y)^2 + \$
  (gradExact_sln_z - grad_temperature_z)^2" on all_blocks as H1_err_sq
postprocess global_function "sqrt(H1_err_sq)" as H1_err

Begin heartbeat testctrl
  Stream name = errors{N}.dat
  Timestamp Format = ""
  Precision = 8
  Format = csv
  Labels = Off
  Legend = On
  At step 0, Increment is 1
  variable = global time
  variable = global number_of_nodes
  variable = global l2_err
  variable = global l2_dot_err
  variable = global H1_err
  variable = global linf_err
end

BEGIN RESULTS OUTPUT LABEL diffusion output
  database Name = output{N}.e
  at step 0, increment = {2*N}
  #at step 0, increment = 1

```

```

        title Aria cube test
        nodal variables = solution->TEMPERATURE as T
        nodal variables = time_derivative_at_time->TEMPERATURE as TDOT
        END RESULTS OUTPUT LABEL diffusion output

    END ARIA REGION myRegion

END PROCEDURE myAriaProcedure

END SIERRA myJob

```

## 12.1.8. Transient Heat Conduction: Tet4 Meshes

```

# T_ref_s1 = { T_ref_s1 = 1 }
# h_s1 = { h_s1 = 1 }
# h_s2 = { h_s2 = 2 }
# T0 = { T0 = 2 }
# C_s = { C_s = 10000.0 }
# omega = { omega = PI }
# bn_vol = { bn_vol = 0.5 }
# L = { L = 10 }
# rho = { rho = 1 }
# Cp = { Cp = 1 }
# k = { k = 1 }
# Ts = { Ts = 0.5 }
# Tl = { Tl = 1.5 }
# Tm = { Tm = 0.5 * (Ts + Tl) }
# sigma = { sigma = 0.429858 * (Tm - Ts) }
# invsigma = { invsigma = 1. / (sigma * sqrt(2.)) }

BEGIN SIERRA myJob
begin universal aria expressions
    user expression = scalar_string_function user_tag = "m_s" \$
        f = "{C_s}* ( 1. - exp(-t) + t*exp( -(t-1.0)*(t-1.0) ) )"

    user expression = scalar_string_function user_tag = "m_s_dot" \$
        f = "{C_s}* ( exp(-t) + (1.0 + t*( -2.0*(t-1.0) ))*exp( -(t-1.0)*(t-1.0) ) )"

    user expression = scalar_string_function user_tag = "bulk_node_exact_solution" \$
        f = "{T0} * (sin({omega} * t) + 1)"

    user expression = scalar_string_function user_tag = "bulk_node_source" \$
        f = "{rho * Cp * omega * T0} * cos({omega} * t) - ({h_s2} * (1 - bulk_node_exact_solution))/ {bn_vol}"

    user expression = scalar_string_function user_tag = "exact_spatial_var" \$
        f = "(x-x*x)^2*(y-y*y)^2*(z-z*z)^2"

    user expression = scalar_string_function user_tag = "exact_sln" \$
        f = "exact_spatial_var*m_s + 1.0"

    user expression = scalar_string_function user_tag = "exact_sln_dot" \$
        f = "exact_spatial_var*m_s_dot"

    user expression = vector_string_function user_tag = "gradExact_sln" \$
        f_x = "2.0*m_s*(x-x*x)*(1-2*x)*(y-y*y)^2*(z-z*z)^2" \$
        f_y = "2.0*m_s*(y-y*y)*(1-2*y)*(x-x*x)^2*(z-z*z)^2" \$
        f_z = "2.0*m_s*(z-z*z)*(1-2*z)*(x-x*x)^2*(y-y*y)^2"

    user expression = vector_string_function user_tag = "exact_flux" \$
        f_x = "{-k}*gradExact_sln_x" \$
        f_y = "{-k}*gradExact_sln_y" \$
        f_z = "{-k}*gradExact_sln_z"

    user expression = scalar_string_function user_tag = "exact_sln_laplace" \$
        f = "2.0*m_s*( ( (1-2*x)*(1-2*x) - 2*(x-x*x) )*(y-y*y)^2*(z-z*z)^2 \$
            + ( (1-2*y)*(1-2*y) - 2*(y-y*y) )*(x-x*x)^2*(z-z*z)^2 \$

```

```

+ ( (1-2*z)*(1-2*z) - 2*(z-z*z) )*(x-x*x)^2*(y-y*y)^2 )"

user expression = scalar_string_function user_tag = "exact_src" \$
f = "{rho}*{Cp}*exact_sln_dot - {k}*exact_sln_laplace"
end universal aria expressions

BEGIN ARIA MATERIAL Kryptonite
Density = Constant rho={rho}
Thermal Conductivity = constant k={k}
Specific Heat = Constant cp={Cp}
heat conduction = basic
END ARIA MATERIAL Kryptonite

BEGIN ARIA MATERIAL Air
Density = Constant rho={rho}
Thermal Conductivity = constant k={k}
Specific Heat = Constant cp={Cp}
heat conduction = basic
END ARIA MATERIAL

BEGIN TPETRA EQUATION SOLVER DIRECT_SOLVER
BEGIN SUPERLU SOLVER
END
END TPETRA EQUATION SOLVER

BEGIN TPETRA EQUATION SOLVER ITERATIVE_SOLVER
BEGIN CG SOLVER
BEGIN JACOBI PRECONDITIONER
END
MAXIMUM ITERATIONS = 1000
RESIDUAL SCALING = NONE
CONVERGENCE TOLERANCE = 1.000000e-12
END
END TPETRA EQUATION SOLVER

BEGIN FINITE ELEMENT MODEL cube
database name = cube_h{N}_tet4.e
coordinate system is cartesian
decomposition method = rcb

BEGIN PARAMETERS FOR BLOCK block_1
material Kryptonite
END PARAMETERS FOR BLOCK block_1

END FINITE ELEMENT MODEL cube

BEGIN PROCEDURE myAriaProcedure

Begin Solution Control Description
Use System Main
Begin System Main
Begin Transient The_Time_Block
Advance myRegion
End
Simulation Start Time = 0
Simulation Termination Time = 3
End
Begin Parameters For Transient The_Time_Block
Begin Parameters For Aria Region myRegion
Initial Time Step Size = {0.5/2**N}
Time Integration Method = Second_Order
Time Step Variation = fixed
End
End
End
End

```

```

BEGIN ARIA REGION myRegion

Nonlinear Solution Strategy = Newton
Minimum Nonlinear Solves = 1
Maximum Nonlinear Iterations = 10
Nonlinear Residual Tolerance = 1.0e-12
nonlinear residual minimum convergence rate = 0.999 number of steps = 3
Nonlinear Correction Tolerance = 1.0e-12
Nonlinear Relaxation Factor = 1.0
use dof averaged nonlinear residual

use finite element model cube
Use Linear Solver Iterative_Solver #Direct_Solver

EQ ENERGY for TEMPERATURE on block_1 using Q1 with MASS DIFF SRC

IC const on block_1 temperature = 1.0

# surface_4: x=0
# surface_6: x=1

# surface_3: y=0
# surface_5: y=1

# surface_1: z=1
# surface_2: z=0

# const Temp BC (x)
BC const dirichlet at surface_4 Temperature = 1.0
BC const dirichlet at surface_6 Temperature = 1.0

# const flux BC (y)
BC FLUX for Energy on surface_3 = constant flux = 3
BC FLUX for Energy on surface_3 = scalar_string_function f="exact_flux_y - 3"

BC FLUX for Energy on surface_5 = constant flux = 5
BC FLUX for Energy on surface_5 = scalar_string_function f="-exact_flux_y - 5"

# convective flux BC with const Temp and H (z)
BC Flux for Energy on surface_1 = Nat_Conv T_Ref={T_ref_s1} H={h_s1}
BC FLUX for Energy on surface_1 = scalar_string_function \$
f="-exact_flux_z - {h_s1}*(exact_sln - {T_ref_s1})"

Begin Bulk Fluid Element aBulkNode
material = Air
bulk element volume = constant v = {bn_vol}
initial temperature = {T0}
bulk eq energy for temperature using p0 with mass src
bulk source for energy = scalar_string_function f="bulk_node_source"
End
Begin Convective Flux Boundary Condition bulk_flux
add surface surface_2
use bulk element aBulkNode
convective coefficient = {h_s2}
End
BC FLUX for Energy on surface_2 = scalar_string_function \$
f="exact_flux_z - {h_s2}*(exact_sln - bulk_node_exact_solution)"

# const source term
Source For ENERGY on block_1 = Constant value=1
Source For ENERGY on block_1 = scalar_string_function f="exact_src-1"

postprocess l_inf_norm of function "exact_sln - temperature" on block_1 as linf_err
postprocess l_inf_norm of function "bulk_node_exact_solution - temperature" on block_for_abulknode as linf_bulk_node
postprocess l_2_norm of function "exact_sln - temperature" on block_1 as l2_err
postprocess l_2_norm of function "exact_sln_dot - dt_temperature" on block_1 as l2_dot_err
postprocess integral of function "\$
(gradExact_sln_x - grad_temperature_x)^2 + \$

```

```

        (gradExact_sln_y - grad_temperature_y)^2 + \$
        (gradExact_sln_z - grad_temperature_z)^2" on block_1 as H1_err_sq
postprocess global_function "sqrt(H1_err_sq)" as H1_err

Begin heartbeat testctrl
  Stream name = errors_thermal_transient_hex8_tied_contact_h{N}.dat
  Timestamp Format = ""
  Precision = 8
  Format = csv
  Labels = Off
  Legend = On
  At step 0, Increment is 1
  variable = global time
  variable = global number_of_nodes
  variable = global l2_err
  variable = global l2_dot_err
  variable = global H1_err
  variable = global linf_err
  variable = global linf_bulk_node
end

BEGIN RESULTS OUTPUT LABEL diffusion output
  database Name = errors{N}.e
  at step 0, increment = {2**N}
  title Aria cube test
  nodal variables = solution->TEMPERATURE as T
  nodal variables = time_derivative_at_time->TEMPERATURE as TDOT
  global variables = abulknode_T

  nodal variables = err_exact_soln
  nodal variables = err_exact_soln_dot
  nodal variables = err_exact_src
  nodal variables = err_exact_flux
END RESULTS OUTPUT LABEL diffusion output

END ARIA REGION myRegion

END PROCEDURE myAriaProcedure

END SIERRA myJob

```

## 12.1.9. Transient Heat Conduction: Tet4Tet10 Meshes

```

# T_ref_s1 = { T_ref_s1 = 1 }
# h_s1 = { h_s1 = 1 }
# T_ref_s2 = { T_ref_s2 = 2 }
# h_s2 = { h_s2 = 2 }
# C_s = { C_s = 10000.0 }

BEGIN SIERRA myJob
  #{include(exact_transient.inc)}

  # load user plugin file ./exact_transient.so

  BEGIN ARIA MATERIAL Kryptonite
    Density = Constant rho={rho}
    Thermal Conductivity = constant k={k}
    Specific Heat = Constant cp={Cp}
    heat conduction = basic

    user expression = scalar_string_function user_tag = "m_s" \$
      f = "[C_s]*( 1. - exp(-t) + t*exp( -(t-1.0)*(t-1.0) ) )"

    user expression = scalar_string_function user_tag = "m_s_dot" \$

```

```

      f = "{C_s}*( exp(-t) + (1.0 + t*(-2.0*(t-1.0)) )*exp( -(t-1.0)*(t-1.0)) )"
END ARIA MATERIAL Kryptonite

BEGIN TPETRA EQUATION SOLVER  DIRECT_SOLVER
  BEGIN SUPERLU SOLVER
  END
END TPETRA EQUATION SOLVER

BEGIN TPETRA EQUATION SOLVER  ITERATIVE_SOLVER
  BEGIN CG SOLVER
    BEGIN JACOBI PRECONDITIONER
    END
    MAXIMUM ITERATIONS = 1000
    RESIDUAL SCALING = NONE
    CONVERGENCE TOLERANCE = 1.000000e-12
  END
END TPETRA EQUATION SOLVER

BEGIN FINITE ELEMENT MODEL cube
  database name = cube_h{N}_tet10.e
  coordinate system is cartesian
  decomposition method = rcb

  BEGIN PARAMETERS FOR BLOCK block_1
    material Kryptonite
  END PARAMETERS FOR BLOCK block_1

END FINITE ELEMENT MODEL cube

BEGIN PROCEDURE myAriaProcedure

  Begin Solution Control Description
    Use System Main
    Begin System Main
      Begin Transient The_Time_Block
        Advance myRegion
      End
      Simulation Start Time = 0
      Simulation Termination Time = 3
    End
    Begin Parameters For Transient The_Time_Block
      Begin Parameters For Aria Region myRegion
        Initial Time Step Size = {0.5/2**N}
        Time Integration Method = Second_Order
        Time Step Variation = fixed
      End
    End
  End

END

BEGIN ARIA REGION myRegion

  Nonlinear Solution Strategy = Newton
  Maximum Nonlinear Iterations = 10
  Nonlinear Residual Tolerance = 1.0e-12
  Nonlinear Correction Tolerance = 1.0e-12
  Nonlinear Relaxation Factor = 1.0
  use dof averaged nonlinear residual

  use finite element model cube
  Use Linear Solver Iterative_Solver #Direct_Solver

  EQ ENERGY for TEMPERATURE on block_1 using Q1 with MASS DIFF SRC

  IC const on block_1 temperature = 1.0

  # surface_4: x=0

```



```

# surface_6: x=1

# surface_3: y=0
# surface_5: y=1

# surface_1: z=1
# surface_2: z=0

# const Temp BC (x)
BC const dirichlet at surface_4 Temperature = 1.0
BC const dirichlet at surface_6 Temperature = 1.0

# const flux BC (y)
BC FLUX for Energy on surface_3 = constant flux = 3
BC FLUX for Energy on surface_3 = scalar_string_function f="exact_flux_y - 3"

BC FLUX for Energy on surface_5 = constant flux = 5
BC FLUX for Energy on surface_5 = scalar_string_function f="-exact_flux_y - 5"

# convective flux BC with const Temp and H (z)
BC Flux for Energy on surface_1 = Nat_Conv T_Ref={T_ref_s1} H={h_s1}
BC FLUX for Energy on surface_1 = scalar_string_Function \$
  f="-exact_flux_z - {h_s1}*(exact_sln - {T_ref_s1})"

BC Flux for Energy on surface_2 = Nat_Conv T_Ref={T_ref_s2} H={h_s2}
BC FLUX for Energy on surface_2 = scalar_string_Function \$
  f="exact_flux_z - {h_s2}*(exact_sln - {T_ref_s2})"

# const source term
Source For ENERGY on block_1 = Constant value=1
Source For ENERGY on block_1 = scalar_string_Function f="exact_src-1"

postprocess max of function "abs(exact_sln - temperature)" on all_blocks as linf_err
postprocess l2_norm of function "exact_sln - temperature" on all_blocks as l2_err
postprocess l2_norm of function "exact_sln_dot - dt_temperature" on all_blocks as l2_dot_err
postprocess integral of function "\$
  (gradExact_sln_x - grad_temperature_x)^2 + \$
  (gradExact_sln_y - grad_temperature_y)^2 + \$
  (gradExact_sln_z - grad_temperature_z)^2" on all_blocks as H1_err_sq
postprocess global_function "sqrt(H1_err_sq)" as H1_err

Begin heartbeat testctrl
  Stream name = errors{N}.dat
  Timestamp Format = ""
  Precision = 8
  Format = csv
  Labels = Off
  Legend = On
  At step 0, Increment is 1
  variable = global time
  variable = global number_of_nodes
  variable = global l2_err
  variable = global l2_dot_err
  variable = global H1_err
  variable = global linf_err
end

BEGIN RESULTS OUTPUT LABEL diffusion output
  database Name = output{N}.e
  at step 0, increment = {2*N}
  title Aria cube test
  nodal variables = solution->TEMPERATURE as T
  nodal variables = time_derivative_at_time->TEMPERATURE as TDOT
END RESULTS OUTPUT LABEL diffusion output

END ARIA REGION myRegion

END PROCEDURE myAriaProcedure

```

```
END SIERRA myJob
```

## 12.1.10. Transient Heat Conduction: Tet10 Meshes

```
# T_ref_s1 = { T_ref_s1 = 1 }
# h_s1 = { h_s1 = 1 }
# T_ref_s2 = { T_ref_s2 = 2 }
# h_s2 = { h_s2 = 2 }
# C_s = { C_s = 10000.0 }

BEGIN SIERRA myJob
  #{include(exact_transient.inc)}

  BEGIN ARIA MATERIAL Kryptonite
    Density          = Constant rho=1
    Thermal Conductivity = constant k=1
    Specific Heat     = Constant cp=1
    heat conduction   = basic

    user expression = scalar_string_function user_tag = "m_s" \$
      f = "{C_s}*( 1. - exp(-t) + t*exp( -(t-1.0)*(t-1.0) ) )"

    user expression = scalar_string_function user_tag = "m_s_dot" \$
      f = "{C_s}*( exp(-t) + (1.0 + t*( -2.0*(t-1.0) ) )*exp( -(t-1.0)*(t-1.0) ) )"
  END ARIA MATERIAL Kryptonite

  BEGIN TPETRA EQUATION SOLVER  DIRECT_SOLVER
    BEGIN SUPERLU SOLVER
    END
  END TPETRA EQUATION SOLVER

  BEGIN TPETRA EQUATION SOLVER  ITERATIVE_SOLVER
    BEGIN CG SOLVER
      BEGIN JACOBI PRECONDITIONER
      END
      MAXIMUM ITERATIONS = 1000
      RESIDUAL SCALING = NONE
      CONVERGENCE TOLERANCE = 1.000000e-12
    END
  END TPETRA EQUATION SOLVER

  BEGIN FINITE ELEMENT MODEL cube
    database name = cube_h{N}_tet10.e
    coordinate system is cartesian
    decomposition method = rcb

    BEGIN PARAMETERS FOR BLOCK block_1
      material Kryptonite
    END PARAMETERS FOR BLOCK block_1

  END FINITE ELEMENT MODEL cube

  BEGIN PROCEDURE myAriaProcedure

    Begin Solution Control Description
      Use System Main
      Begin System Main
        Begin Transient The_Time_Block
          Advance myRegion
        End
        Simulation Start Time = 0
        Simulation Termination Time = 3
```

```

End
Begin Parameters For Transient The_Time_Block
  Begin Parameters For Aria Region myRegion
    Initial Time Step Size = {0.5/2**N}
    Time Integration Method = Second_Order
    Time Step Variation = fixed
  End
End
End

BEGIN ARIA REGION myRegion

Nonlinear Solution Strategy = Newton
Maximum Nonlinear Iterations = 10
Nonlinear Residual Tolerance = 1.0e-12
Nonlinear Correction Tolerance = 1.0e-12
Nonlinear Relaxation Factor = 1.0
use dof averaged nonlinear residual

use finite element model cube
Use Linear Solver Iterative_Solver #Direct_Solver

EQ ENERGY for TEMPERATURE on block_1 using Q2 with MASS DIFF SRC

IC const on block_1 temperature = 1.0

# surface_4: x=0
# surface_6: x=1

# surface_3: y=0
# surface_5: y=1

# surface_1: z=1
# surface_2: z=0

# const Temp BC (x)
BC const dirichlet at surface_4 Temperature = 1.0
BC const dirichlet at surface_6 Temperature = 1.0

# const flux BC (y)
BC FLUX for Energy on surface_3 = constant flux = 3
BC FLUX for Energy on surface_3 = scalar_string_function f="exact_flux_y - 3"

BC FLUX for Energy on surface_5 = constant flux = 5
BC FLUX for Energy on surface_5 = scalar_string_function f="-exact_flux_y - 5"

# convective flux BC with const Temp and H (z)
BC Flux for Energy on surface_1 = Nat_Conv T_Ref={T_ref_s1} H={h_s1}
BC FLUX for Energy on surface_1 = scalar_string_Function \$
  f="-exact_flux_z - {h_s1}*(exact_sln - {T_ref_s1})"

BC Flux for Energy on surface_2 = Nat_Conv T_Ref={T_ref_s2} H={h_s2}
BC FLUX for Energy on surface_2 = scalar_string_Function \$
  f="exact_flux_z - {h_s2}*(exact_sln - {T_ref_s2})"

# const source term
Source For ENERGY on block_1 = Constant value=1
Source For ENERGY on block_1 = scalar_string_Function f="exact_src-1"

postprocess max of function "abs(exact_sln - temperature)" on all_blocks as linf_err
postprocess l2_norm of function "exact_sln - temperature" on all_blocks as l2_err
postprocess l2_norm of function "exact_sln_dot - dt_temperature" on all_blocks as l2_dot_err
postprocess integral of function "\$
  (gradExact_sln_x - grad_temperature_x)^2 + \$
  (gradExact_sln_y - grad_temperature_y)^2 + \$
  (gradExact_sln_z - grad_temperature_z)^2" on all_blocks as H1_err_sq
postprocess global_function "sqrt(H1_err_sq)" as H1_err

```

```

Begin heartbeat testctrl
  Stream name = errors{N}.dat
  Timestamp Format   = ""
  Precision         = 8
  Format = csv
  Labels = Off
  Legend = On
  At step 0, Increment is 1
  variable = global time
  variable = global number_of_nodes
  variable = global l2_err
  variable = global l2_dot_err
  variable = global H1_err
  variable = global linf_err
end

BEGIN RESULTS OUTPUT LABEL diffusion output
  database Name = output{N}.e
  at step 0, increment = {2*N}
  title Aria cube test
  nodal variables = solution->TEMPERATURE as T
  nodal variables = time_derivative_at_time->TEMPERATURE as TDOT
END RESULTS OUTPUT LABEL diffusion output

END ARIA REGION myRegion

END PROCEDURE myAriaProcedure

END SIERRA myJob

```

## **12.2. THERMAL BOUNDARY CONDITIONS**

### **12.2.1. Radiative Heat Flux [3.1](#)**

### **12.2.2. Radiative Heat Flux From Fortran User Subroutine**

### **12.2.3. Convective Heat Flux [3.3](#)**

## 12.3. THERMAL CONTACT

### 12.3.1. 1D Flat Contact 4.1

#### 12.3.1.1. Hex8 Tied

```
{R=0.0}
begin sierra Aria

    title Adaptive Square

    Begin Universal Aria Expressions
        # gamma = { gamma = (2. - R) / (2. + R) }
        user expression = scalar_string_function user_tag="exactsoln" f="(x < 0.) * (0.5 * (1. + x) * ({gamma} + x)) + (x>=0.) * (0.5 * (1. + x) * ({gamma} + x))"
        user expression = scalar_string_function user_tag="exactsolndx" f="(x < 0.) * 0.5 * ({gamma} + x) + 1. + x + (x>=0.) * 0.5 * ({gamma} + x)"
    End

BEGIN ARIA MATERIAL M1
    Density          = Constant rho=1
    Thermal Conductivity = constant k=1
    Specific Heat     = Constant cp=1
    heat conduction   = basic
END

BEGIN ARIA MATERIAL M2
    Density          = Constant rho=1
    Thermal Conductivity = constant k=1
    Specific Heat     = Constant cp=1
    heat conduction   = basic
END

Begin Finite Element Model bar
    Database Name = 2blocks_contact_unaligned_hex8_h{N}.g
    Begin parameters for block block_1
        material M1
    End
    Begin parameters for block block_2
        material M2
    End
End Finite Element Model bar

BEGIN TPETRA EQUATION SOLVER  DIRECT_SOLVER
    BEGIN UMFPACK SOLVER
    END
END TPETRA EQUATION SOLVER

BEGIN TPETRA EQUATION SOLVER  ITERATIVE_SOLVER
    BEGIN GMRES SOLVER
        BEGIN JACOBI PRECONDITIONER
        END
        MAXIMUM ITERATIONS = 20000
        RESIDUAL SCALING = R0
        CONVERGENCE TOLERANCE = automatic
    END
END TPETRA EQUATION SOLVER

begin procedure myProcedure

    Begin Solution Control Description
        Use System Main
        Begin System Main
            Begin Sequential The_Time_Block
                Advance myRegion
            End
            Simulation Start Time = 0
```

```

        Simulation Termination Time = 1
    End
End

begin Aria region myRegion

    Nonlinear Solution Strategy = Newton
    Maximum Nonlinear Iterations = 10
    Nonlinear Residual Tolerance = 1.0e-12
    Nonlinear Correction Tolerance = 1.0e-12
    Nonlinear Relaxation Factor = 1.0
    use dof averaged nonlinear residual

    use finite element model bar
    use linear solver Iterative_Solver #Direct_Solver #

    EQ ENERGY for TEMPERATURE on all_blocks using Q1 with DIFF SRC

    Begin TEMPERATURE BOUNDARY CONDITION xm1
        add surface surface_1
        TEMPERATURE = 0.
    End

    Begin TEMPERATURE BOUNDARY CONDITION xp1
        add surface surface_2
        TEMPERATURE = 1.
    End

    Begin VOLUME HEATING sm
        add volume block_1
        VALUE = -1.
    End

    Begin VOLUME HEATING sp
        add volume block_2
        VALUE = 1.
    End

    begin contact definition res1
        contact surface surf_1 contains surface_3
        contact surface surf_2 contains surface_4

        begin interaction inter_1
            surfaces = surf_1 surf_2
        end interaction inter_1

        begin enforcement enf_1
            Enforcement for Energy = Tied_Temperature
        end enforcement

    end contact definition res1

    postprocess l_2_norm of function "temperature - exactsoln" on all_blocks as l2_err
    postprocess l_inf_norm of function "temperature - exactsoln" on all_blocks as linf_err
    postprocess l_2_norm of scalar_string_function f = "sqrt((exactsolndx - (grad_temperature_x))^2+(0 - (grad_temperatu

    Begin Heartbeat pp_out
        At Step 1 Interval is 1
        Legend = on
        Labels = off
        Format = csv
        Precision = 5
        Stream Name = errors_h{N}.dat
        Global Time as Time
        Global Number_of_Nodes as Num_Nodes
        Global l2_err
        Global linf_err
        Global h1_err

```

```

End

Begin Results Output Label diffusion output
    database Name = 2blocks_tied_h{N}.e
    At Step 1, Increment = 1
    Title Calore Two Blocks
    Nodal Variables = solution->temperature as T
    Element Variables = l2_err linf_err h1_err
End

end

end

end

```

### 12.3.1.2. *Hex8 Resistance*

```

#{R=4.0}
begin sierra Aria

    title Adaptive Square

    Begin Universal Aria Expressions
        # gamma = { gamma = (2. - R) / (2. + R) }
        user expression = scalar_string_function user_tag="exactsoln" f="(x < 0.) * (0.5 * (1. + x) * ({gamma} + x)) + (x>=0.) * (0.5 * (1. + x) * ({gamma} + x))"
        user expression = scalar_string_function user_tag="exactsolndx" f="(x < 0.) * 0.5 * ({gamma} + x) + 1. + x + (x>=0.) * 0.5 * ({gamma} + x)"
    End

    BEGIN ARIA MATERIAL M1
        Density          = Constant rho=1
        Thermal Conductivity = constant k=1
        Specific Heat     = Constant cp=1
        heat conduction   = basic
    END

    BEGIN ARIA MATERIAL M2
        Density          = Constant rho=1
        Thermal Conductivity = constant k=1
        Specific Heat     = Constant cp=1
        heat conduction   = basic
    END

    Begin Finite Element Model bar
        Database Name = 2blocks_contact_unaligned_hex8_h{N}.g
        Begin parameters for block block_1
            material M1
        End
        Begin parameters for block block_2
            material M2
        End
    End Finite Element Model bar

    BEGIN TPETRA EQUATION SOLVER  DIRECT_SOLVER
    BEGIN UMFPACK SOLVER
    END
END TPETRA EQUATION SOLVER

    BEGIN TPETRA EQUATION SOLVER  ITERATIVE_SOLVER
    BEGIN GMRES SOLVER
    BEGIN JACOBI PRECONDITIONER
    END
    MAXIMUM ITERATIONS = 20000
    RESIDUAL SCALING = RO
    CONVERGENCE TOLERANCE = automatic

```



```

END
END TPETRA EQUATION SOLVER

begin procedure myProcedure

  Begin Solution Control Description
    Use System Main
    Begin System Main
      Begin Sequential The_Time_Block
        Advance myRegion
      End
      Simulation Start Time = 0
      Simulation Termination Time = 1
    End
  End

begin Aria region myRegion

  Nonlinear Solution Strategy = Newton
  Maximum Nonlinear Iterations = 10
  Nonlinear Residual Tolerance = 1.0e-12
  Nonlinear Correction Tolerance = 1.0e-12
  Nonlinear Relaxation Factor = 1.0
  use dof averaged nonlinear residual

  use finite element model bar
  use linear solver Iterative_Solver #Direct_Solver #

  EQ ENERGY for TEMPERATURE on all_blocks using Q1 with DIFF SRC

  Begin TEMPERATURE BOUNDARY CONDITION xm1
    add surface surface_1
    TEMPERATURE = 0.
  End

  Begin TEMPERATURE BOUNDARY CONDITION xp1
    add surface surface_2
    TEMPERATURE = 1.
  End

  Begin VOLUME HEATING sm
    add volume block_1
    VALUE = -1.
  End

  Begin VOLUME HEATING sp
    add volume block_2
    VALUE = 1.
  End

  begin contact definition res1
    contact surface surf_1 contains surface_3
    contact surface surf_2 contains surface_4

    begin interaction inter_1
      surfaces = surf_1 surf_2
    end interaction inter_1

    begin enforcement enf_1
      gap conductance coefficient = constant value= {1.0/R}
      Enforcement for Energy = gap_conductance
    end enforcement

  end contact definition res1

  postprocess value of expression exactsoln on all_blocks as exact_soln
  postprocess value of function "temperature - exactsoln" on all_blocks as nodal_l2_err
  postprocess l2_norm of function "temperature - exactsoln" on all_blocks as l2_err

```

```

# The linf on the hex8 mesh is just always 0(1e-11)
postprocess l_1_norm of function "temperature - exactsoln" on block_2 as l1_err
postprocess l_2_norm of scalar_string_function f = "sqrt((exactsolndx - (grad_temperature_x))^2+(0 - (grad_temperatu

Begin Heartbeat pp_out
  At Step 1 Interval is 1
  Legend = on
  Labels = off
  Format = csv
  Precision = 5
  Stream Name = errors_h{N}.dat
  Global Time as Time
  Global Number_of_Nodes as Num_Nodes
  Global l2_err
  Global l1_err
  Global h1_err
End

Begin Results Output Label diffusion output
  database Name = 2blocks_res_h{N}.e
  At Step 1, Increment = 1
  Title Calore Two Blocks
  Nodal Variables = solution->temperature as T
  Nodal Variables = nodal_l2_err
  Nodal Variables = exact_soln
  Element Variables = l2_err linf_err h1_err
End

end

end

end

```

### 12.3.1.3. *Tet4 Tied*

```

#{R=0.0}
begin sierra Aria

  title Adaptive Square

  Begin Universal Aria Expressions
    # gamma = { gamma = (2. - R) / (2. + R) }
    user expression = scalar_string_function user_tag="exactsoln" f="(x < 0.) * (0.5 * (1. + x) * ({gamma} + x)) + (x>=0.) * (
    user expression = scalar_string_function user_tag="exactsolndx" f="(x < 0.) * 0.5 *(({gamma} + x) + 1. + x) + (x>=0.) * 0.
  End

  BEGIN ARIA MATERIAL M1
    Density          = Constant rho=1
    Thermal Conductivity = constant k=1
    Specific Heat     = Constant cp=1
    heat conduction   = basic
  END

  BEGIN ARIA MATERIAL M2
    Density          = Constant rho=1
    Thermal Conductivity = constant k=1
    Specific Heat     = Constant cp=1
    heat conduction   = basic
  END

  Begin Finite Element Model bar
    Database Name = 2blocks_contact_unaligned_tet4_h{N}.g
    Begin parameters for block block_1
      material M1
    End

```

```

        Begin parameters for block block_2
            material M2
        End
    End Finite Element Model bar

BEGIN TPETRA EQUATION SOLVER  DIRECT_SOLVER
    BEGIN UMFPACK SOLVER
    END
END TPETRA EQUATION SOLVER

BEGIN TPETRA EQUATION SOLVER  ITERATIVE_SOLVER
    BEGIN GMRES SOLVER
        BEGIN DD-ILU PRECONDITIONER
            subdomain overlap level = 1
        END
        MAXIMUM ITERATIONS = 200
        RESIDUAL SCALING = R0
        CONVERGENCE TOLERANCE = automatic
    END
END TPETRA EQUATION SOLVER

begin procedure myProcedure

    Begin Solution Control Description
        Use System Main
        Begin System Main
            Begin Sequential The_Time_Block
                Advance myRegion
            End
            Simulation Start Time = 0
            Simulation Termination Time = 1
        End
    End

    begin Aria region myRegion

        Nonlinear Solution Strategy = Newton
        Maximum Nonlinear Iterations = 10
        Nonlinear Residual Tolerance = 1.0e-12
        Nonlinear Correction Tolerance = 1.0e-12
        Nonlinear Relaxation Factor = 1.0
        use dof averaged nonlinear residual

        use finite element model bar
        use linear solver Iterative_Solver #Direct_Solver #

        EQ ENERGY for TEMPERATURE on all_blocks using Q1 with DIFF SRC

        Begin TEMPERATURE BOUNDARY CONDITION xm1
            add surface surface_1
            TEMPERATURE = 0.
        End

        Begin TEMPERATURE BOUNDARY CONDITION xp1
            add surface surface_2
            TEMPERATURE = 1.
        End

        Begin VOLUME HEATING sm
            add volume block_1
            VALUE = -1.
        End

        Begin VOLUME HEATING sp
            add volume block_2
            VALUE = 1.
        End
    end
end

```

```

begin contact definition res1
  contact surface surf_1 contains surface_3
  contact surface surf_2 contains surface_4

  begin interaction inter_1
    surfaces = surf_1 surf_2
  end interaction inter_1

  begin enforcement enf_1
    Enforcement for Energy = Tied_Temperature
  end enforcement

end contact definition res1

postprocess l_2_norm of function "temperature - exactsoln" on all_blocks as l2_err
postprocess l_inf_norm of function "temperature - exactsoln" on all_blocks as linf_err
postprocess l_2_norm of scalar_string_function f = "sqrt((exactsolndx - (grad_temperature_x))^2+(0 - (grad_temperatu

Begin Heartbeat pp_out
  At Step 1 Interval is 1
  Legend = on
  Labels = off
  Format = csv
  Precision = 5
  Stream Name = errors_h{N}.dat
  Global Time as Time
  Global Number_of_Nodes as Num_Nodes
  Global l2_err
  Global linf_err
  Global h1_err
End

Begin Results Output Label diffusion output
  database Name = 2blocks_tied_h{N}.e
  At Step 1, Increment = 1
  Title Calore Two Blocks
  Nodal Variables = solution->temperature as T
  Element Variables = l2_err linf_err h1_err
End

end

end

end

```

#### 12.3.1.4. *Tet4 Resistance*

```

#{R=4.0}
begin sierra Aria

  title Adaptive Square

  Begin Universal Aria Expressions
    # gamma = { gamma = (2. - R) / (2. + R) }
    user expression = scalar_string_function user_tag="exactsoln" f="(x < 0.) * (0.5 * (1. + x) * ({gamma} + x)) + (x>=0.) * 0."
    user expression = scalar_string_function user_tag="exactsolndx" f="(x < 0.) * 0.5 *(({gamma} + x) + 1. + x) + (x>=0.) * 0."
  End

  BEGIN ARIA MATERIAL M1
    Density = Constant rho=1
    Thermal Conductivity = constant k=1
    Specific Heat = Constant cp=1
    heat conduction = basic
  END

```

```

BEGIN ARIA MATERIAL M2
  Density          = Constant rho=1
  Thermal Conductivity = constant k=1
  Specific Heat     = Constant cp=1
  heat conduction   = basic
END

Begin Finite Element Model bar
  Database Name = 2blocks_contact_unaligned_tet4_h{N}.g
  Begin parameters for block block_1
    material M1
  End
  Begin parameters for block block_2
    material M2
  End
End Finite Element Model bar

BEGIN TPETRA EQUATION SOLVER  DIRECT_SOLVER
  BEGIN UMFPACK SOLVER
  END
END TPETRA EQUATION SOLVER

BEGIN TPETRA EQUATION SOLVER  ITERATIVE_SOLVER
  BEGIN GMRES SOLVER
    BEGIN DD-ILU PRECONDITIONER
    END
    MAXIMUM ITERATIONS = 200
    RESIDUAL SCALING = R0
    CONVERGENCE TOLERANCE = automatic
  END
END TPETRA EQUATION SOLVER

begin procedure myProcedure

  Begin Solution Control Description
    Use System Main
    Begin System Main
      Begin Sequential The_Time_Block
        Advance myRegion
      End
      Simulation Start Time = 0
      Simulation Termination Time = 1
    End
  End

  begin Aria region myRegion

    Nonlinear Solution Strategy = Newton
    Maximum Nonlinear Iterations = 10
    Nonlinear Residual Tolerance = 1.0e-12
    Nonlinear Correction Tolerance = 1.0e-12
    Nonlinear Relaxation Factor = 1.0
    use dof averaged nonlinear residual

    use finite element model bar
    use linear solver Iterative_Solver #Direct_Solver #

    EQ ENERGY for TEMPERATURE on all_blocks using Q1 with DIFF SRC

    Begin TEMPERATURE BOUNDARY CONDITION xm1
      add surface surface_1
      TEMPERATURE = 0.
    End

    Begin TEMPERATURE BOUNDARY CONDITION xp1
      add surface surface_2

```

```

        TEMPERATURE = 1.
    End

    Begin VOLUME HEATING sm
        add volume block_1
        VALUE = -1.
    End

    Begin VOLUME HEATING sp
        add volume block_2
        VALUE = 1.
    End

    begin contact definition res1
        contact surface surf_1 contains surface_3
        contact surface surf_2 contains surface_4

        begin interaction inter_1
            surfaces = surf_1 surf_2
        end interaction inter_1

        begin enforcement enf_1
            gap conductance coefficient = constant value= {1.0/R}
            Enforcement for Energy = gap_conductance
        end enforcement

    end contact definition res1

    postprocess l_2_norm of function "temperature - exactsoln" on all_blocks as l2_err
    postprocess l_inf_norm of function "temperature - exactsoln" on all_blocks as linf_err
    postprocess l_2_norm of scalar_string_function f = "sqrt((exactsolndx - (grad_temperature_x))^2+(0 - (grad_temperatu

    Begin Heartbeat pp_out
        At Step 1 Interval is 1
        Legend = on
        Labels = off
        Format = csv
        Precision = 5
        Stream Name = errors_h{N}.dat
        Global Time as Time
        Global Number_of_Nodes as Num_Nodes
        Global l2_err
        Global linf_err
        Global h1_err
    End

    Begin Results Output Label diffusion output
        database Name = 2blocks_res_h{N}.e
        At Step 1, Increment = 1
        Title Calore Two Blocks
        Nodal Variables = solution->temperature as T
        Element Variables = l2_err linf_err h1_err
    End

end

end

end

```

#### 12.3.1.5. *Hex8-Tet4 Tied*

```

#{R=0.0}
begin sierra Aria

    title Adaptive Square

```

```

Begin Universal Aria Expressions
  # gamma = { gamma = (2. - R) / (2. + R) }
  user expression = scalar_string_function user_tag="exactsoln" f="(x < 0.) * (0.5 * (1. + x) * ({gamma} + x)) + (x>=0.) * (
  user expression = scalar_string_function user_tag="exactsolndx" f="(x < 0.) * 0.5 *(({gamma} + x) + 1. + x) + (x>=0.) * 0.
End

BEGIN ARIA MATERIAL M1
  Density          = Constant rho=1
  Thermal Conductivity = constant k=1
  Specific Heat     = Constant cp=1
  heat conduction   = basic
END

BEGIN ARIA MATERIAL M2
  Density          = Constant rho=1
  Thermal Conductivity = constant k=1
  Specific Heat     = Constant cp=1
  heat conduction   = basic
END

Begin Finite Element Model bar
  Database Name = 2blocks_contact_unaligned_hex8_tet4_h{N}.g
  Begin parameters for block block_1
    material M1
  End
  Begin parameters for block block_2
    material M2
  End
End Finite Element Model bar

BEGIN TPETRA EQUATION SOLVER  DIRECT_SOLVER
BEGIN UMFPACK SOLVER
END
END TPETRA EQUATION SOLVER

BEGIN TPETRA EQUATION SOLVER  ITERATIVE_SOLVER
BEGIN GMRES SOLVER
  BEGIN DD-ILU PRECONDITIONER
  END
  MAXIMUM ITERATIONS = 200
  RESIDUAL SCALING = R0
  CONVERGENCE TOLERANCE = automatic
END
END TPETRA EQUATION SOLVER

begin procedure myProcedure

  Begin Solution Control Description
    Use System Main
    Begin System Main
      Begin Sequential The_Time_Block
        Advance myRegion
      End
      Simulation Start Time = 0
      Simulation Termination Time = 1
    End
  End

  begin Aria region myRegion

    Nonlinear Solution Strategy = Newton
    Maximum Nonlinear Iterations = 10
    Nonlinear Residual Tolerance = 1.0e-12
    Nonlinear Correction Tolerance = 1.0e-12
    Nonlinear Relaxation Factor = 1.0
    use dof averaged nonlinear residual

```

```

use finite element model bar
use linear solver Iterative_Solver #Direct_Solver #

EQ ENERGY for TEMPERATURE on all_blocks using Q1 with DIFF SRC

Begin TEMPERATURE BOUNDARY CONDITION xm1
  add surface surface_1
  TEMPERATURE = 0.
End

Begin TEMPERATURE BOUNDARY CONDITION xp1
  add surface surface_2
  TEMPERATURE = 1.
End

Begin VOLUME HEATING sm
  add volume block_1
  VALUE = -1.
End

Begin VOLUME HEATING sp
  add volume block_2
  VALUE = 1.
End

begin contact definition res1
  contact surface surf_1 contains surface_3
  contact surface surf_2 contains surface_4

  begin interaction inter_1
    surfaces = surf_1 surf_2
  end interaction inter_1

  begin enforcement enf_1
    Enforcement for Energy = Tied_Temperature
  end enforcement

end contact definition res1

postprocess l2_norm of function "temperature - exactsoln" on all_blocks as l2_err
postprocess l_inf_norm of function "temperature - exactsoln" on all_blocks as linf_err
postprocess l2_norm of scalar_string_function f = "sqrt((exactsolndx - (grad_temperature_x))^2+(0 - (grad_temperatu

Begin Heartbeat pp_out
  At Step 1 Interval is 1
  Legend = on
  Labels = off
  Format = csv
  Precision = 5
  Stream Name = errors_h{N}.dat
  Global Time as Time
  Global Number_of_Nodes as Num_Nodes
  Global l2_err
  Global linf_err
  Global h1_err
End

Begin Results Output Label diffusion output
  database Name = 2blocks_tied_h{N}.e
  At Step 1, Increment = 1
  Title Calore Two Blocks
  Nodal Variables = solution->temperature as T
  Element Variables = l2_err linf_err h1_err
End

end

```



```

end

end

```

### 12.3.1.6. Hex8-Tet4 Resistance

```

#{R=4.0}
begin sierra Aria

    title Adaptive Square

    Begin Universal Aria Expressions
        # gamma = { gamma = (2. - R) / (2. + R) }
        user expression = scalar_string_function user_tag="exactsoln" f="(x < 0.) * (0.5 * (1 + x) * ({gamma} + x)) + (x>=0.) * (1 + x)"
        user expression = scalar_string_function user_tag="exactsolndx" f="(x < 0.) * 0.5 *(({gamma} + x) + 1. + x) + (x>=0.) * 0.5 * ({gamma} + x)"
    End

    BEGIN ARIA MATERIAL M1
        Density          = Constant rho=1
        Thermal Conductivity = constant k=1
        Specific Heat     = Constant cp=1
        heat conduction   = basic
    END

    BEGIN ARIA MATERIAL M2
        Density          = Constant rho=1
        Thermal Conductivity = constant k=1
        Specific Heat     = Constant cp=1
        heat conduction   = basic
    END

    Begin Finite Element Model bar
        Database Name = 2blocks_contact_unaligned_hex8_tet4_h{N}.g
        Begin parameters for block block_1
            material M1
        End
        Begin parameters for block block_2
            material M2
        End
    End Finite Element Model bar

    BEGIN TPETRA EQUATION SOLVER  DIRECT_SOLVER
        BEGIN UMFPACK SOLVER
        END
    END TPETRA EQUATION SOLVER

    BEGIN TPETRA EQUATION SOLVER  ITERATIVE_SOLVER
        BEGIN GMRES SOLVER
        BEGIN DD-ILU PRECONDITIONER
        END
        MAXIMUM ITERATIONS = 200
        RESIDUAL SCALING = R0
        CONVERGENCE TOLERANCE = automatic
    END
    END TPETRA EQUATION SOLVER

    begin procedure myProcedure

        Begin Solution Control Description
            Use System Main
            Begin System Main
                Begin Sequential The_Time_Block
                    Advance myRegion
                End
            End
        End
    end procedure myProcedure

```

```

        Simulation Start Time = 0
        Simulation Termination Time = 1
    End
End

begin Aria region myRegion

    Nonlinear Solution Strategy = Newton
    Maximum Nonlinear Iterations = 10
    Nonlinear Residual Tolerance = 1.0e-12
    Nonlinear Correction Tolerance = 1.0e-12
    Nonlinear Relaxation Factor = 1.0
    use dof averaged nonlinear residual

    use finite element model bar
    use linear solver Iterative_Solver #Direct_Solver #

    EQ ENERGY for TEMPERATURE on all_blocks using Q1 with DIFF SRC

    Begin TEMPERATURE BOUNDARY CONDITION xm1
        add surface surface_1
        TEMPERATURE = 0.
    End

    Begin TEMPERATURE BOUNDARY CONDITION xp1
        add surface surface_2
        TEMPERATURE = 1.
    End

    Begin VOLUME HEATING sm
        add volume block_1
        VALUE = -1.
    End

    Begin VOLUME HEATING sp
        add volume block_2
        VALUE = 1.
    End

    begin contact definition res1
        contact surface surf_1 contains surface_3
        contact surface surf_2 contains surface_4

        begin interaction inter_1
            surfaces = surf_1 surf_2
        end interaction inter_1

        begin enforcement enf_1
            gap conductance coefficient = constant value= {1.0/R}
            Enforcement for Energy = gap_conductance
        end enforcement

    end contact definition res1

    postprocess l_2_norm of function "temperature - exactsoln" on all_blocks as l2_err
    postprocess l_inf_norm of function "temperature - exactsoln" on all_blocks as linf_err
    postprocess l_2_norm of scalar_string_function f = "sqrt((exactsolndx - (grad_temperature_x))^2+(0 - (grad_temperatu

    Begin Heartbeat pp_out
        At Step 1 Interval is 1
        Legend = on
        Labels = off
        Format = csv
        Precision = 5
        Stream Name = errors_h{N}.dat
        Global Time as Time
        Global Number_of_Nodes as Num_Nodes
        Global l2_err

```

```

        Global linf_err
        Global h1_err
    End

    Begin Results Output Label diffusion output
        database Name = 2blocks_res_h{N}.e
        At Step 1, Increment = 1
        Title Calore Two Blocks
        Nodal Variables = solution->temperature as T
        Element Variables = l2_err linf_err h1_err
    End

end

end

end

```

## 12.3.2. 3D Curved Contact 4.2

**12.3.2.1. Hex8-Hex8 Case**

**12.3.2.2. Tet4-Tet4 Case**

**12.3.2.3. Hex8-Tet4 Case**

## 12.3.3. Steady Hex8 Contact

## 12.3.4. Steady Hex20 Contact

## 12.3.5. Steady Hex27 Contact

## 12.3.6. Steady Tet4 Contact

```

#N={N=5}
BEGIN SIERRA myJob

Begin universal aria expressions
    User Expression = Scalar_String_Function \$
        f = "(x-x^2)^2*(y-y^2)^2*(z-z^2)^2 + 1" \$
        user_tag = T_mms

    User Expression = Vector_String_Function \$
        f_x = "2*(x-x^2)*(1-2*x)*(y-y^2)^2*(z-z^2)^2" \$
        f_y = "2*(y-y^2)*(1-2*y)*(x-x^2)^2*(z-z^2)^2" \$
        f_z = "2*(z-z^2)*(1-2*z)*(x-x^2)^2*(y-y^2)^2" \$
        user_tag = gradT

    User Expression = Scalar_String_Function \$
        f = "2*((1-2*x)^2 - 2*(x-x^2))*(y-y^2)^2*(z-z^2)^2" \$
        + "2*((1-2*y)^2 - 2*(y-y^2))*(x-x^2)^2*(z-z^2)^2" \$
        + "2*((1-2*z)^2 - 2*(z-z^2))*(x-x^2)^2*(y-y^2)^2" \$
        user_tag = laplaceT

    User Expression = Scalar_String_Function \$
        f = "-laplaceT - 1.0" \$
        user_tag = mms_src
End

```

```

BEGIN ARIA MATERIAL Kryptonite
  Density          = Constant rho=1
  Thermal Conductivity = constant k=1
  Specific Heat     = Constant cp=1
  heat conduction   = basic
END ARIA MATERIAL Kryptonite

BEGIN TPETRA EQUATION SOLVER  ITERATIVE_SOLVER
  BEGIN GMRES SOLVER
    BEGIN JACOBI PRECONDITIONER
      END
    MAXIMUM ITERATIONS = 1000
    RESIDUAL SCALING = NONE
    CONVERGENCE TOLERANCE = 1.000000e-15
  END
END TPETRA EQUATION SOLVER

BEGIN FINITE ELEMENT MODEL cube
  database name = cube_contact_h{N}_tet4.e
  coordinate system is cartesian
  decomposition method = rcb

  BEGIN PARAMETERS FOR BLOCK block_1
    material Kryptonite
  END PARAMETERS FOR BLOCK block_1

  BEGIN PARAMETERS FOR BLOCK block_2
    material Kryptonite
  END PARAMETERS FOR BLOCK block_2

END FINITE ELEMENT MODEL cube

BEGIN PROCEDURE myAriaProcedure

  Begin Solution Control Description
    Use System Main
    Begin System Main
      Begin Sequential The_Time_Block
        Advance myRegion
      End
      Simulation Start Time = 0
      Simulation Termination Time = 1
    End
  End

  BEGIN ARIA REGION myRegion

    Nonlinear Solution Strategy = Newton
    Maximum Nonlinear Iterations = 10
    Nonlinear Residual Tolerance = 1.0e-12
    Nonlinear Correction Tolerance = 1.0e-12
    Nonlinear Relaxation Factor = 1.0
    use dof averaged nonlinear residual

    use finite element model cube
    Use Linear Solver Iterative_Solver #Direct_Solver

    EQ ENERGY for TEMPERATURE on block_1 using Q1 with DIFF SRC
    EQ ENERGY for TEMPERATURE on block_2 using Q1 with DIFF SRC

    # surface_4: x=0
    # surface_6: x=1

    # surface_3: y=0
    # surface_5: y=1

    # surface_1: z=1

```

```

# surface_2: z=0

# const Temp BC (x)
BC const dirichlet at surface_4 Temperature = 1.0
BC const dirichlet at surface_6 Temperature = 1.0

# const flux BC (y)
BC FLUX for Energy on surface_3 = constant flux = 3
BC FLUX for Energy on surface_5 = constant flux = 5

BC Flux for Energy on surface_3 = Scalar_String_Function f = "-gradT_y - 3.0"
BC Flux for Energy on surface_5 = Scalar_String_Function f = "gradT_y - 5.0"

# convective flux BC with const Temp and H (z)
BC Flux for Energy on surface_1 = Nat_Conv T_Ref=1 H=1
BC Flux for Energy on surface_2 = Nat_Conv T_Ref=2 H=2

BC Flux for Energy on surface_1 = Scalar_String_Function f = "gradT_z - (T_mms-1.0)"
BC Flux for Energy on surface_2 = Scalar_String_Function f = "-gradT_z - 2.0*(T_mms-2.0)"

# const source term
Source For ENERGY on block_1 = Constant value=1
Source For ENERGY on block_2 = Constant value=1

Source for ENERGY on block_1 = Scalar_String_Function f = "mms_src"
Source for ENERGY on block_2 = Scalar_String_Function f = "mms_src"

begin contact definition mpc1
  contact surface surf_1 contains surface_7
  contact surface surf_2 contains surface_8

  begin interaction inter_1
    surfaces = surf_1 surf_2
  end
  begin enforcement enf_1
    Enforcement for Energy = Tied_Temperature
  end
end

postprocess L_2_norm of function "temperature-T_mms" on all_blocks as l2
postprocess L_inf_norm of function "temperature-T_mms" on all_blocks as linf
postprocess L_2_norm of function "sqrt((grad_temperature_x-gradT_x)^2+(grad_temperature_y-gradT_y)^2+(grad_temperature_z-gradT_z)^2)" on all_blocks as l2_sqrt

Begin Heartbeat hb
  At Step 0 Interval is 1
  Precision = 6
  Stream Name = errors_h{N}.dat
  legend = off
  labels = off
  Timestamp Format = ""
  variable = global time
  variable = global number_of_nodes
  variable = global l2
  variable = global h1
  variable = global linf
End

BEGIN RESULTS OUTPUT LABEL diffusion output
  database Name = thermal_steady_tet4_tied_contact_h{N}.e
  at step 1, increment = 1
  title Aria cube test
  nodal variables = nonlinear_solution->TEMPERATURE as T
END RESULTS OUTPUT LABEL diffusion output

END ARIA REGION myRegion

END PROCEDURE myAriaProcedure

```

```
END SIERRA myJob
```

## 12.3.7. Steady Tet4Tet10 Contact

```
#N={N=5}
BEGIN SIERRA myJob

Begin universal aria expressions
  User Expression = Scalar_String_Function \$
    f = "(x-x^2)^2*(y-y^2)^2*(z-z^2)^2 + 1" \$
    user_tag = T_mms

  User Expression = Vector_String_Function \$
    f_x = "2*(x-x^2)*(1-2*x)*(y-y^2)^2*(z-z^2)^2" \$
    f_y = "2*(y-y^2)*(1-2*y)*(x-x^2)^2*(z-z^2)^2" \$
    f_z = "2*(z-z^2)*(1-2*z)*(x-x^2)^2*(y-y^2)^2" \$
    user_tag = gradT

  User Expression = Scalar_String_Function \$
    f = "2*((1-2*x)^2 - 2*(x-x^2))*(y-y^2)^2*(z-z^2)^2 \
      + 2*((1-2*y)^2 - 2*(y-y^2))*(x-x^2)^2*(z-z^2)^2 \
      + 2*((1-2*z)^2 - 2*(z-z^2))*(x-x^2)^2*(y-y^2)^2" \$
    user_tag = laplaceT

  User Expression = Scalar_String_Function \$
    f = "-laplaceT - 1.0" \$
    user_tag = mms_src
End

BEGIN ARIA MATERIAL Kryptonite
  Density = Constant rho=1
  Thermal Conductivity = constant k=1
  Specific Heat = Constant cp=1
  heat conduction = basic
END ARIA MATERIAL Kryptonite

BEGIN TPETRA EQUATION SOLVER ITERATIVE_SOLVER
  BEGIN GMRES SOLVER
    BEGIN JACOBI PRECONDITIONER
    END
    MAXIMUM ITERATIONS = 1000
    RESIDUAL SCALING = NONE
    CONVERGENCE TOLERANCE = 1.000000e-15
  END
END TPETRA EQUATION SOLVER

BEGIN FINITE ELEMENT MODEL cube
  database name = cube_contact_h{N}_tet10.e
  coordinate system is cartesian
  decomposition method = rcb

  BEGIN PARAMETERS FOR BLOCK block_1
    material Kryptonite
  END PARAMETERS FOR BLOCK block_1

  BEGIN PARAMETERS FOR BLOCK block_2
    material Kryptonite
  END PARAMETERS FOR BLOCK block_2

END FINITE ELEMENT MODEL cube

BEGIN PROCEDURE myAriaProcedure
```

```

Begin Solution Control Description
  Use System Main
  Begin System Main
    Begin Sequential The_Time_Block
      Advance myRegion
    End
    Simulation Start Time = 0
    Simulation Termination Time = 1
  End
End

BEGIN ARIA REGION myRegion

  Nonlinear Solution Strategy = Newton
  Maximum Nonlinear Iterations = 10
  Nonlinear Residual Tolerance = 1.0e-12
  Nonlinear Correction Tolerance = 1.0e-12
  Nonlinear Relaxation Factor = 1.0
  use dof averaged nonlinear residual

  use finite element model cube
  Use Linear Solver Iterative_Solver #Direct_Solver

  EQ ENERGY for TEMPERATURE on block_1 using Q1 with DIFF SRC
  EQ ENERGY for TEMPERATURE on block_2 using Q1 with DIFF SRC

  # surface_4: x=0
  # surface_6: x=1

  # surface_3: y=0
  # surface_5: y=1

  # surface_1: z=1
  # surface_2: z=0

  # const Temp BC (x)
  BC const dirichlet at surface_4 Temperature = 1.0
  BC const dirichlet at surface_6 Temperature = 1.0

  # const flux BC (y)
  BC FLUX for Energy on surface_3 = constant flux = 3
  BC FLUX for Energy on surface_5 = constant flux = 5

  BC Flux for Energy on surface_3 = Scalar_String_Function f = "-gradT_y - 3.0"
  BC Flux for Energy on surface_5 = Scalar_String_Function f = "gradT_y - 5.0"

  # convective flux BC with const Temp and H (z)
  BC Flux for Energy on surface_1 = Nat_Conv T_Ref=1 H=1
  BC Flux for Energy on surface_2 = Nat_Conv T_Ref=2 H=2

  BC Flux for Energy on surface_1 = Scalar_String_Function f = "gradT_z - (T_mms-1.0)"
  BC Flux for Energy on surface_2 = Scalar_String_Function f = "-gradT_z - 2.0*(T_mms-2.0)"

  # const source term
  Source For ENERGY on block_1 = Constant value=1
  Source For ENERGY on block_2 = Constant value=1

  Source for ENERGY on block_1 = Scalar_String_Function f = "mms_src"
  Source for ENERGY on block_2 = Scalar_String_Function f = "mms_src"

  begin contact definition mpc1
    contact surface surf_1 contains surface_7
    contact surface surf_2 contains surface_8

    begin interaction inter_1
      surfaces = surf_1 surf_2
    end
    begin enforcement enf_1

```

```

        Enforcement for Energy = Tied_Temperature
    end
end

postprocess L2_norm of function "temperature-T_mms" on all_blocks as l2
postprocess L_inf_norm of function "temperature-T_mms" on all_blocks as linf
postprocess L2_norm of function "sqrt((grad_temperature_x-gradT_x)^2+(grad_temperature_y-gradT_y)^2+(grad_temperature_z-gradT_z)^2)" on all_blocks as l2_sqrt

Begin Heartbeat hb
    At Step 0 Interval is 1
    Precision = 6
    Stream Name = errors_h{N}.dat
    legend = off
    labels = off
    Timestamp Format    = ""
    variable = global time
    variable = global number_of_nodes
    variable = global l2
    variable = global h1
    variable = global linf
End

BEGIN RESULTS OUTPUT LABEL diffusion output
    database Name = thermal_steady_tet4_tied_contact_h{N}.e
    at step 1, increment = 1
    # time interval is 1.0
    title Aria cube test
    nodal variables = nonlinear_solution->TEMPERATURE as T
    element variables = l2_error h1_error linf_error
END RESULTS OUTPUT LABEL diffusion output

END ARIA REGION myRegion

END PROCEDURE myAriaProcedure

END SIERRA myJob

```

## 12.3.8. Steady Tet10 Contact

## 12.3.9. Steady Tet10 Dash Contact

## 12.3.10. Transient Tet4Tet10 Contact

```

#N={N=5}
# T_ref_s1 = { T_ref_s1 = 1 }
# h_s1 = { h_s1 = 1 }
# T_ref_s2 = { T_ref_s2 = 2 }
# h_s2 = { h_s2 = 2 }
# C_s = { C_s = 10000.0 }

BEGIN SIERRA myJob
    #{include(exact_transient.inc)}

    BEGIN ARIA MATERIAL Kryptonite
        Density          = Constant rho={rho}
        Thermal Conductivity = constant k={k}
        Specific Heat     = Constant cp={Cp}
        heat conduction    = basic

        user expression = scalar_string_function user_tag = "m_s" \$
            f = "{C_s}*( 1. - exp(-t) + t*exp( -(t-1.0)*(t-1.0) ) )"
    end
end

```



```

user expression = scalar_string_function user_tag = "m_s_dot" \$
f = "{C_s}*( exp(-t) + (1.0 + t*( -2.0*(t-1.0)) )*exp( -(t-1.0)*(t-1.0)) )"
END ARIA MATERIAL Kryptonite

BEGIN TPETRA EQUATION SOLVER DIRECT_SOLVER
BEGIN SUPERLU SOLVER
END
END TPETRA EQUATION SOLVER

BEGIN TPETRA EQUATION SOLVER ITERATIVE_SOLVER
BEGIN GMRES SOLVER
BEGIN JACOBI PRECONDITIONER
END
MAXIMUM ITERATIONS = 1000
RESIDUAL SCALING = NONE
CONVERGENCE TOLERANCE = 1.000000e-15
END
END TPETRA EQUATION SOLVER

BEGIN FINITE ELEMENT MODEL cube
database name = cube_contact_h{N}_tet10.e
coordinate system is cartesian
decomposition method = rcb

BEGIN PARAMETERS FOR BLOCK block_1
material Kryptonite
END PARAMETERS FOR BLOCK block_1

BEGIN PARAMETERS FOR BLOCK block_2
material Kryptonite
END PARAMETERS FOR BLOCK block_2

END FINITE ELEMENT MODEL cube

BEGIN PROCEDURE myAriaProcedure

Begin Solution Control Description
Use System Main
Begin System Main
Begin Transient The_Time_Block
Advance myRegion
End
Simulation Start Time = 0
Simulation Termination Time = 3
End
Begin Parameters For Transient The_Time_Block
Begin Parameters For Aria Region myRegion
Initial Time Step Size = {0.5/2**N}
Time Integration Method = Second_Order
Time Step Variation = fixed
End
End
End

BEGIN ARIA REGION myRegion

Nonlinear Solution Strategy = Newton
Maximum Nonlinear Iterations = 10
Nonlinear Residual Tolerance = 1.0e-12
Nonlinear Correction Tolerance = 1.0e-12
Nonlinear Relaxation Factor = 1.0
use dof averaged nonlinear residual

use finite element model cube
Use Linear Solver Iterative_Solver #Direct_Solver

```

```

EQ ENERGY for TEMPERATURE on block_1 using Q1 with MASS DIFF SRC
EQ ENERGY for TEMPERATURE on block_2 using Q1 with MASS DIFF SRC

IC const on block_1 temperature = 1.0
IC const on block_2 temperature = 1.0

# surface_4: x=0
# surface_6: x=1

# surface_3: y=0
# surface_5: y=1

# surface_1: z=1
# surface_2: z=0

# const Temp BC (x)
BC const dirichlet at surface_4 Temperature = 1.0
BC const dirichlet at surface_6 Temperature = 1.0

# const flux BC (y)
BC FLUX for Energy on surface_3 = constant flux = 3
BC FLUX for Energy on surface_3 = scalar_string_function f="exact_flux_y - 3"

BC FLUX for Energy on surface_5 = constant flux = 5
BC FLUX for Energy on surface_5 = scalar_string_function f="-exact_flux_y - 5"

# convective flux BC with const Temp and H (z)
BC Flux for Energy on surface_1 = Nat_Conv T_Ref={T_ref_s1} H={h_s1}
BC FLUX for Energy on surface_1 = scalar_string_function \$
f="-exact_flux_z - {h_s1}*(exact_sln - {T_ref_s1})"

BC Flux for Energy on surface_2 = Nat_Conv T_Ref={T_ref_s2} H={h_s2}
BC FLUX for Energy on surface_2 = scalar_string_function \$
f="exact_flux_z - {h_s2}*(exact_sln - {T_ref_s2})"

# const source term
Source For ENERGY on block_1 = Constant value=1
Source For ENERGY on block_1 = scalar_string_function f="exact_src-1"

Source For ENERGY on block_2 = Constant value=1
Source For ENERGY on block_2 = scalar_string_function f="exact_src-1"

begin contact definition mpc1
  contact surface surf_1 contains surface_7
  contact surface surf_2 contains surface_8

  begin interaction inter_1
    surfaces = surf_1 surf_2
  end
  begin enforcement enf_1
    Enforcement for Energy = Tied_Temperature
  end
end

postprocess max of function "abs(exact_sln - temperature)" on all_blocks as l1f_err
postprocess l2_norm of function "exact_sln - temperature" on all_blocks as l2_err
postprocess l2_norm of function "exact_sln_dot - dt_temperature" on all_blocks as l2_dot_err
postprocess integral of function "\$
  (gradExact_sln_x - grad_temperature_x)^2 + \$
  (gradExact_sln_y - grad_temperature_y)^2 + \$
  (gradExact_sln_z - grad_temperature_z)^2" on all_blocks as H1_err_sq
postprocess global_function "sqrt(H1_err_sq)" as H1_err

Begin heartbeat testctrl
  Stream name = errors_h{N}.dat
  Timestamp Format = ""
  Precision = 8
  Format = csv

```

```

Labels = Off
Legend = On
At step 0, Increment is 1
variable = global time
variable = global number_of_nodes
variable = global l2_err
variable = global l2_dot_err
variable = global H1_err
variable = global linf_err
end

BEGIN RESULTS OUTPUT LABEL diffusion output
  database Name = thermal_transient_tet10_tied_contact_h{N}.e
  at step 0, increment = {2*N}
  title Aria cube test
  nodal variables = solution->TEMPERATURE as T
  nodal variables = time_derivative_at_time->TEMPERATURE as TDOT
END RESULTS OUTPUT LABEL diffusion output

END ARIA REGION myRegion

END PROCEDURE myAriaProcedure

END SIERRA myJob

```

## 12.3.11. Transient Tet10 Contact

```

# T_ref_s1 = { T_ref_s1 = 1 }
# h_s1 = { h_s1 = 1 }
# T_ref_s2 = { T_ref_s2 = 2 }
# h_s2 = { h_s2 = 2 }
# C_s = { C_s = 10000.0 }

BEGIN SIERRA myJob
  #{include(exact_transient.inc)}

  BEGIN ARIA MATERIAL Kryptonite
    Density          = Constant rho=1
    Thermal Conductivity = constant k=1
    Specific Heat     = Constant cp=1
    heat conduction   = basic

    user expression = scalar_string_function user_tag = "m_s" \$
      f = "{C_s}*( 1. - exp(-t) + t*exp( -(t-1.0)*(t-1.0) ) )"

    user expression = scalar_string_function user_tag = "m_s_dot" \$
      f = "{C_s}*( exp(-t) + (1.0 + t*( -2.0*(t-1.0) ))*exp( -(t-1.0)*(t-1.0) ) )"
  END ARIA MATERIAL Kryptonite

  BEGIN TPETRA EQUATION SOLVER  DIRECT_SOLVER
    BEGIN SUPERLU SOLVER
    END
  END TPETRA EQUATION SOLVER

  BEGIN TPETRA EQUATION SOLVER  ITERATIVE_SOLVER
    BEGIN GMRES SOLVER
    BEGIN JACOBI PRECONDITIONER
    END
    MAXIMUM ITERATIONS = 1000
    RESIDUAL SCALING = NONE
    CONVERGENCE TOLERANCE = 1.000000e-15
  END
END TPETRA EQUATION SOLVER

```

```

BEGIN FINITE ELEMENT MODEL cube
  database name = cube_contact_h{N}_tet10.e
  coordinate system is cartesian
  decomposition method = rcb

  BEGIN PARAMETERS FOR BLOCK block_1
    material Kryptonite
  END PARAMETERS FOR BLOCK block_1

  BEGIN PARAMETERS FOR BLOCK block_2
    material Kryptonite
  END PARAMETERS FOR BLOCK block_2

END FINITE ELEMENT MODEL cube

BEGIN PROCEDURE myAriaProcedure

  Begin Solution Control Description
    Use System Main
    Begin System Main
      Begin Transient The_Time_Block
        Advance myRegion
      End
      Simulation Start Time = 0
      Simulation Termination Time = 3
    End
    Begin Parameters For Transient The_Time_Block
      Begin Parameters For Aria Region myRegion
        Initial Time Step Size = {0.5/2**N}
        Time Integration Method = Second_Order
        Time Step Variation = fixed
      End
    End
  End

  BEGIN ARIA REGION myRegion

    Nonlinear Solution Strategy = Newton
    Maximum Nonlinear Iterations = 10
    Nonlinear Residual Tolerance = 1.0e-12
    Nonlinear Correction Tolerance = 1.0e-12
    Nonlinear Relaxation Factor = 1.0
    use dof averaged nonlinear residual

    use finite element model cube
    Use Linear Solver Iterative_Solver #Direct_Solver

    EQ ENERGY for TEMPERATURE on block_1 using Q2 with MASS DIFF SRC
    EQ ENERGY for TEMPERATURE on block_2 using Q2 with MASS DIFF SRC

    IC const on block_1 temperature = 1.0
    IC const on block_2 temperature = 1.0

    # surface_4: x=0
    # surface_6: x=1

    # surface_3: y=0
    # surface_5: y=1

    # surface_1: z=1
    # surface_2: z=0

    # const Temp BC (x)
    BC const dirichlet at surface_4 Temperature = 1.0
    BC const dirichlet at surface_6 Temperature = 1.0

    # const flux BC (y)

```

```

BC FLUX for Energy on surface_3 = constant flux = 3
BC FLUX for Energy on surface_3 = scalar_string_function f="exact_flux_y - 3"

BC FLUX for Energy on surface_5 = constant flux = 5
BC FLUX for Energy on surface_5 = scalar_string_function f="-exact_flux_y - 5"

# convective flux BC with const Temp and H (z)
BC Flux for Energy on surface_1 = Nat_Conv T_Ref={T_ref_s1} H={h_s1}
BC FLUX for Energy on surface_1 = scalar_string_Function \$
    f="-exact_flux_z - {h_s1}*(exact_sln - {T_ref_s1})"

BC Flux for Energy on surface_2 = Nat_Conv T_Ref={T_ref_s2} H={h_s2}
BC FLUX for Energy on surface_2 = scalar_string_Function \$
    f="exact_flux_z - {h_s2}*(exact_sln - {T_ref_s2})"

# const source term
Source For ENERGY on block_1 = Constant value=1
Source For ENERGY on block_1 = scalar_string_Function f="exact_src-1"

Source For ENERGY on block_2 = Constant value=1
Source For ENERGY on block_2 = scalar_string_Function f="exact_src-1"

begin contact definition mpc1
    contact surface surf_1 contains surface_7
    contact surface surf_2 contains surface_8

    begin interaction inter_1
        surfaces = surf_1 surf_2
    end
    begin enforcement enf_1
        Enforcement for Energy = Tied_Temperature
    end
end

postprocess max of function "abs(exact_sln - temperature)" on all_blocks as linf_err
postprocess l2_norm of function "exact_sln - temperature" on all_blocks as l2_err
postprocess l2_norm of function "exact_sln_dot - dt_temperature" on all_blocks as l2_dot_err
postprocess integral of function "\$
    (gradExact_sln_x - grad_temperature_x)^2 + \$
    (gradExact_sln_y - grad_temperature_y)^2 + \$
    (gradExact_sln_z - grad_temperature_z)^2" on all_blocks as H1_err_sq
postprocess global_function "sqrt(H1_err_sq)" as H1_err

Begin heartbeat testctrl
    Stream name = errors_h{N}.dat
    Timestamp Format = ""
    Precision = 8
    Format = csv
    Labels = Off
    Legend = On
    At step 0, Increment is 1
    variable = global time
    variable = global number_of_nodes
    variable = global l2_err
    variable = global l2_dot_err
    variable = global H1_err
    variable = global linf_err
end

BEGIN RESULTS OUTPUT LABEL diffusion output
    database Name = thermal_transient_tet10_tied_contact_h{N}.e
    at step 0, increment = {2**N}
    title Aria cube test
    nodal variables = solution->TEMPERATURE as T
    nodal variables = time_derivative_at_time->TEMPERATURE as TDOT
END RESULTS OUTPUT LABEL diffusion output

END ARIA REGION myRegion

```

```
END PROCEDURE myAriaProcedure  
END SIERRA myJob
```

## 12.4. ELEMENT DEATH

### 12.4.1. CDFEM Element Death (Heat Flux)

#### 12.4.1.1. *Tri3*

```
BEGIN SIERRA Aria
```

```
Title \$
```

```
1-d standard conduction problem, Carslaw and Jaeger P. 292\$
```

```
BEGIN TPETRA EQUATION SOLVER SOLVE_TEMPERATURE
```

```
  BEGIN CG SOLVER
```

```
    BEGIN JACOBI PRECONDITIONER
```

```
    END
```

```
    MAXIMUM ITERATIONS = 10000
```

```
    RESIDUAL SCALING = RO
```

```
    CONVERGENCE TOLERANCE = 1.000000e-14
```

```
  END
```

```
END TPETRA EQUATION SOLVER
```

```
Begin Global Constants
```

```
  Stefan Boltzmann Constant = 5.67e-8 # W/m^2-K^4
```

```
  Ideal Gas Constant = 8.314 # J/mol-K
```

```
End
```

```
BEGIN ARIA MATERIAL solid
```

```
  DENSITY = constant rho = 1.
```

```
  Thermal Conductivity = constant k = 1.
```

```
  Specific Heat = Constant cp = 1.0
```

```
  Heat Conduction = basic
```

```
END ARIA MATERIAL solid
```

```
BEGIN FINITE ELEMENT MODEL VERIFY_DEATH
```

```
  DATABASE NAME = input{N}_tri3.e
```

```
  decomposition method = rcb
```

```
  COORDINATE SYSTEM = CARTESIAN
```

```
  DATABASE TYPE = EXODUSII
```

```
  USE MATERIAL solid FOR block_1
```

```
  USE MATERIAL solid FOR block_1_dead
```

```
END FINITE ELEMENT MODEL VERIFY_DEATH
```

```
Begin Universal Aria Expressions
```

```
  User Expression = Scalar_String_Function F = "ln(sqrt(x*x+y*y))*(1/ln(2-t))" User_Tag = exact_soln
```

```
  User Expression = Scalar_String_Function F = "sqrt(x*x+y*y)" User_Tag = radius
```

```
  User Expression = Scalar_String_Function F = "2-t" User_Tag = exact_interface
```

```
  User Expression = Scalar_String_Function F = "ln(sqrt(x*x+y*y))*(-1/(ln(2-t)*ln(2-t)))*(1/(2-t))*(-1)" User_Tag = exact_src
```

```
  User Expression = Vector_String_Function F_X = "(x/(x*x+y*y))*(1/ln(2-t))" F_Y = "(y/(x*x+y*y))*(1/ln(2-t))" User_Tag = exact
```

```
  User Expression = Scalar_String_Function F = "-1/((2-t)*ln(2-t))" User_Tag = exact_flux
```

```
End
```

```
Begin Procedure myProcedure
```

```
  Begin Solution Control Description
```

```
    Use System Main
```

```
    Begin System Main
```

```
      Simulation Start Time = 0.0
```

```
      Simulation Termination Time = 0.9
```

```
      Begin transient MySolveBlock
```

```
        Advance myRegion
```

```
      End
```

```
    End
```

```
    begin parameters for transient MySolveBlock
```

```
      start time = 0.0
```

```
      begin parameters for aria region myRegion
```

```
        initial time step size = {0.1*(0.5**(N-1))}
```

```

        Predictor-Corrector Tolerance = {0.05*(0.5**(N-1))}
        Maximum Time Step Size = {0.2*(0.5**(N-1))}
        Time Integration Method = bdf2
        time step variation = adaptive
    end
end
End

Begin Aria Region myRegion

Begin Heartbeat pp_out
    Legend = on
    Labels = off
    Format = csv
    At Step 1 Interval is 1
    Precision = 6
    Stream Name = errors{N}.dat
    variable is Global time
    variable is Global number_of_nodes as num_nodes
    Global l2 as l2
    Global h1 as h1
    Global linf as linf
    Global l2_interface as l2_interface
    Global linf_interface as linf_interface
End
Use Finite Element Model VERIFY_DEATH

Begin CDFEM Death death_by_temp
    add volume block_1
    Criterion is solution->Temperature > 1.0
End

Use Linear Solver solve_temperature

nonlinear solution strategy = newton
maximum nonlinear iterations = 2
nonlinear correction tolerance = 1.0e-12
nonlinear residual tolerance = 1.0e-12
nonlinear relaxation factor = 1.0

use dof averaged nonlinear residual

eq energy for temperature on all_blocks using q1 with lumped_mass diff src

IC for temperature on block_1 = Scalar_String_Function F = "exact_soln"

BC Dirichlet for Temperature on surface_2 = Scalar_String_Function F = "exact_soln"

SOURCE for Energy on block_1 = Scalar_String_Function F = "exact_src"

BC Flux for Energy on surface_1 = Scalar_String_Function F = "exact_flux"
BC Flux for Energy on surface_block_1_death_by_temp = Scalar_String_Function F = "exact_flux"

postprocess l_2_norm of scalar_string_function f = "exact_soln - (temperature)" on block_1 as l2
postprocess l_2_norm of scalar_string_function f = "sqrt((exact_soln_grad_x - (grad_temperature_x))^2+(exact_soln_grad_y - (grad_temperature_y))^2)" on block_1 as l2
postprocess l_infinity_norm of scalar_string_function f = "exact_soln - (temperature)" on block_1 as linf

Begin Postprocess l_2_norm
    Location = surface_1
    Location = surface_block_1_death_by_temp
    Output Name = l2_interface
    Source Type = Scalar_String_Function F = "exact_interface - (radius)"
End
Begin Postprocess l_infinity_norm
    Location = surface_1
    Location = surface_block_1_death_by_temp
    Output Name = linf_interface
    Source Type = Scalar_String_Function F = "exact_interface - (radius)"

```



```

End

Begin Results Output Label diffusion output
  database name = output{N}.e
  at Step 0, Interval = {2**N}
  Nodal Variables = solution->temperature as TEMP
  Global Variables = linf_interface l2_err h1_err linf_err
End Results Output Label diffusion output

End Aria Region myRegion

End Procedure myProcedure

END SIERRA ARIA

```

#### 12.4.1.2. *Tet4*

```

BEGIN SIERRA Aria

Title \$
1-d standard conduction problem, Carslaw and Jaeger P. 292\$

BEGIN TPETRA EQUATION SOLVER SOLVE_TEMPERATURE
  BEGIN CG SOLVER
    BEGIN JACOBI PRECONDITIONER
      END
    MAXIMUM ITERATIONS = 10000
    RESIDUAL SCALING = R0
    CONVERGENCE TOLERANCE = 1.000000e-14
  END
END TPETRA EQUATION SOLVER

Begin Global Constants
  Stefan Boltzmann Constant = 5.67e-8 # W/m^2-K^4
  Ideal Gas Constant = 8.314 # J/mol-K
End

BEGIN ARIA MATERIAL solid
  DENSITY = constant rho = 1.
  Thermal Conductivity = constant k = 1.
  Specific Heat = Constant cp = 1.0
  Heat Conduction = basic
END ARIA MATERIAL solid

BEGIN FINITE ELEMENT MODEL VERIFY_DEATH
  DATABASE NAME = input{N}_tet4.e
  decomposition method = rcb
  COORDINATE SYSTEM = CARTESIAN
  DATABASE TYPE = EXODUSII
  USE MATERIAL solid FOR block_1
  USE MATERIAL solid FOR block_1_dead
END FINITE ELEMENT MODEL VERIFY_DEATH

Begin Universal Aria Expressions
  User Expression = Scalar_String_Function F = "(1+t)/sqrt(x*x+y*y+z*z)" User_Tag = exact_soln
  User Expression = Vector_String_Function F_X = "(1+t)*(-1/(x*x+y*y+z*z))*(x/sqrt(x*x+y*y+z*z))" F_Y = "(1+t)*(-1/(x*x+y*y+z*z))" F_Z = "(1+t)*(-1/(x*x+y*y+z*z))"
  User Expression = Scalar_String_Function F = "sqrt(x*x+y*y+z*z)" User_Tag = radius
  User Expression = Scalar_String_Function F = "1+t" User_Tag = exact_interface
  User Expression = Scalar_String_Function F = "1/sqrt(x*x+y*y+z*z)" User_Tag = exact_src
  User Expression = Scalar_String_Function F = "-1/(1+t)" User_Tag = exact_flux
End

Begin Procedure myProcedure

  Begin Solution Control Description
    Use System Main
  End
End

```

```

Begin System Main
  Simulation Start Time      = 0.0
  Simulation Termination Time = 0.75
  Begin transient MySolveBlock
    Advance myRegion
  End
End
begin parameters for transient MySolveBlock
  start time = 0.0
  begin parameters for aria region myRegion
    initial time step size = {0.1*(0.5**(N-1))}
    Predictor-Corrector Tolerance = {0.05*(0.5**(N-1))}
    Maximum Time Step Size = {0.2*(0.5**(N-1))}
    Time Integration Method = bdf2
    time step variation = adaptive
  end
end
End

Begin Aria Region myRegion

Begin Heartbeat pp_out
  Legend = on
  Labels = off
  Format = csv
  At Step 1 Interval is 1
  Precision = 6
  Stream Name = errors{N}.dat
  variable is Global time
  variable is Global number_of_nodes as num_nodes
  Global l2 as l2
  Global h1 as h1
  Global linf as linf
  Global l2_interface as l2_interface
  Global linf_interface as linf_interface
End
Use Finite Element Model VERIFY_DEATH

Begin CDFEM Death death_by_temp
  add volume block_1
  Criterion is solution->Temperature > 1.0
End

Use Linear Solver solve_temperature

nonlinear solution strategy = newton
maximum nonlinear iterations = 2
nonlinear correction tolerance = 1.0e-12
nonlinear residual tolerance = 1.0e-12
nonlinear relaxation factor = 1.0

use dof averaged nonlinear residual

eq energy for temperature on all_blocks using q1 with lumped_mass diff src

IC for temperature on block_1 = Scalar_String_Function F = "exact_soln"

BC Dirichlet for Temperature on surface_2 = Scalar_String_Function F = "exact_soln"

SOURCE for Energy on block_1 = Scalar_String_Function F = "exact_src"

BC Flux for Energy on surface_1 = Scalar_String_Function F = "exact_flux"
BC Flux for Energy on surface_block_1_death_by_temp = Scalar_String_Function F = "exact_flux"

postprocess l_2_norm of scalar_string_function f = "exact_soln - (temperature)" on block_1 as l2
postprocess l_2_norm of scalar_string_function f = "sqrt((exact_soln_grad_x - (grad_temperature_x))^2+(exact_soln_grad_y - (grad_temperature_y))^2)" on block_1 as l2
postprocess l_infinity_norm of scalar_string_function f = "exact_soln - (temperature)" on block_1 as linf

```

```

Begin Postprocess l_2_norm
  Location = surface_1
  Location = surface_block_1_death_by_temp
  Output Name = l2_interface
  Source Type = Scalar_String_Function F = "exact_interface - (radius)"
End
Begin Postprocess l_inf_norm
  Location = surface_1
  Location = surface_block_1_death_by_temp
  Output Name = linf_interface
  Source Type = Scalar_String_Function F = "exact_interface - (radius)"
End

Begin Results Output Label diffusion output
  database name = output{N}.e
  at Step 0, Interval = {2**N}
  Nodal Variables = solution->temperature as TEMP
  Global Variables = linf_interface l2 h1 linf
End Results Output Label diffusion output

End Aria Region myRegion

End Procedure myProcedure

END SIERRA ARIA

```

## 12.4.2. 3D Spherical Shell Enclosure

```

BEGIN SIERRA myJob

#{include(r2CoFunctions.inc)}

Title Element Death Test Problem #1

BEGIN TPETRA EQUATION SOLVER SOLVE_TEMPERATURE
  BEGIN CG SOLVER
    BEGIN JACOBI PRECONDITIONER
      END
      RESIDUAL SCALING = R0
      CONVERGENCE TOLERANCE = 1.000000e-09
    END
  END TPETRA EQUATION SOLVER

BEGIN TPETRA EQUATION SOLVER DIRECT_SOLVER
  BEGIN SUPERLU SOLVER
    END
  END TPETRA EQUATION SOLVER

BEGIN GLOBAL Constants
  Stefan Boltzmann Constant = 5.6704e-08 # W/m2-K4
End

#{is_inner = 0}
{loop(2)}
  BEGIN ARIA MATERIAL ss304_{is_inner}
    Heat Conduction      = Basic
    Density              = Constant rho = 7862.0    $ kg/m**3
    Specific Heat         = Constant Cp = 10.0      $ J/gm/K
    Thermal Conductivity = Constant K = 1.0        $ W/m/K

    {if(is_inner)}
      user expression = scalar_string_function user_tag = "T1_curr" \$
        f = "{T1} + {dT1}*(-cos(pi*t)/2.0 + 1.0/2.0)"
    }
  END
}

```

```

user expression = scalar_string_function user_tag = "T1_curr_dot" \$
f = "pi*{dT1}*sin(pi*t)/2.0"

user expression = scalar_string_function user_tag = "dTi_dT1" \$
f = "-(-1/{r1} + 1.0/radius)/(1.0/r2 - 1/{r1}) + 1"

user expression = scalar_string_function user_tag = "dTi_dr2" \$
f = "(-T1_curr + {Tc})*(-1/{r1} + 1.0/radius)/(pow(r2, 2)*pow(1.0/r2 - 1.0/{r1}, 2))"

user expression = scalar_string_function user_tag = "exact_soln" \$
f = "T1_curr + (1.0/radius - 1/{r1})*({Tc} - T1_curr)/(1.0/r2 - 1/{r1})"

user expression = scalar_string_function user_tag = "exact_soln_dot" \$
f = "dTi_dT1*T1_curr_dot + dTi_dr2*r2_dot"

user expression = scalar_string_function user_tag = "dTemp_dr" \$
f = "-(T1_curr - {Tc})/(radius*radius*(1/{r1} - 1/r2))"
{else}
user expression = scalar_string_function user_tag = "dTo_dCo" \$
f = "-(-1.0/{r3} + 1.0/radius)/(1.0/{r4} - 1/{r3}) + 1.0"

user expression = scalar_string_function user_tag = "exact_soln" \$
f = "Co + ({T4} - Co)*(1.0/radius - 1.0/{r3})/(1.0/{r4} - 1/{r3})"

user expression = scalar_string_function user_tag = "exact_soln_dot" \$
f = "dTo_dCo*Co_dot"

user expression = scalar_string_function user_tag = "dTemp_dr" \$
f = "-(Co - {T4})/(radius*radius*(1.0/{r3} - 1.0/{r4}))"
{endif}
END ARIA MATERIAL ss304_{is_inner}
#{is_inner = 1}
{endloop}

BEGIN ARIA MATERIAL fake
END ARIA MATERIAL fake

BEGIN FINITE ELEMENT MODEL myModel
Database Name = two_sphere_shells_tet4_m{N}.g
Coordinate System is cartesian
decomposition method = rcb

Use material ss304_0 for block_1 # Outer "case" block
Use material ss304_1 for block_2 # Inner block that will have death

#THIS BLOCK SHOULD BE REMOVED EVENTUALLY BUT IS CURRENTLY REQUIRED
# Commenting it out leads to a segfault because of a null field data pointer for model_coordinates
# This is very bizarre
Use material fake for block_2_dead
#BEGIN PARAMETERS FOR BLOCK block_2_dead
# Material fake
#END PARAMETERS FOR BLOCK block_2_dead

END FINITE ELEMENT MODEL myModel

begin universal aria expressions
user expression = user_function user_tag = "Co" name=Co_function x=time
user expression = user_function user_tag = "r2" name=r2_function x=time
user expression = user_function user_tag = "Co_dot" name=Co_dot_function x=time
user expression = user_function user_tag = "r2_dot" name=r2_dot_function x=time

user expression = scalar_string_function f="sqrt(x*x + y*y + z*z)" user_tag = "radius"

user expression = vector_string_function \$
f_x="x/radius" \$
f_y="y/radius" \$
f_z="z/radius" \$

```

```

user_tag = "gradRadius"

user expression = vector_string_function \$
f_x="dTemp_dr*gradRadius_x" \$
f_y="dTemp_dr*gradRadius_y" \$
f_z="dTemp_dr*gradRadius_z" \$
user_tag = "gradExact_soln"

user expression = scalar_string_function user_tag = "exact_src" \$
f = "{rho}*{cp}*exact_soln_dot"
end universal aria expressions

BEGIN PROCEDURE myAriaProcedure

Begin Solution Control Description
Use System Main
Begin System Main
Simulation Start Time      = {tStart}
Simulation Termination Time = {tEnd}
Begin Transient Stepper
Advance myRegion
End
End

Begin Parameters For Transient Stepper
Begin Parameters for Aria Region myRegion
Time Integration Method = BDF2
Time Step Variation     = fixed #Adaptive
Initial Time Step Size  = {0.05*0.5**(N)}
#Minimum Time Step Size = 1.0e-5
#Maximum Time Step Size = 1000.0
#predictor order = 0
#Predictor-Corrector Tolerance = 1.e-3
#Fail Time Step When Time Step Size Ratio Is Below 0.0
End
End
End

BEGIN ARIA REGION myRegion

Use Linear Solver solve_temperature #direct_solver
Use Finite Element Model myModel

Begin CDFEM Death death_temp1
add volume block_2
Criterion is solution->Temperature > 867.011674920813
End

Nonlinear Solution Strategy = newton
Maximum Nonlinear Iterations = 10
Nonlinear Residual Tolerance = 1.0e-06
Nonlinear Correction Tolerance = 1.0e-06
Nonlinear Relaxation Factor = 1.0
Accept Solution After Maximum Nonlinear Iterations = false

IC for Temperature for all_volumes = scalar_string_Function f="exact_soln"

BC Dirichlet for Temperature at surface_1 = scalar_string_Function f="exact_soln"
BC Dirichlet for Temperature at surface_4 = scalar_string_Function f="exact_soln"

EQ ENERGY for TEMPERATURE on block_1 using Q1 with lumped_mass Diff Src
EQ ENERGY for TEMPERATURE on block_2 using Q1 with lumped_mass Diff Src

Source For ENERGY on all_volumes = scalar_string_Function f="exact_src"

Begin Viewfactor Calculation vf_calc
Compute Rule = Hemicube
Geometric Tolerance = 1.0e-6

```

```

Hemicube Resolution      = 500
Hemicube Max Subdivides = 5
hemicube min separation  = 5.0
Output Rule              = Verbose
End

Begin Viewfactor Smoothing smooth
  Method                  = least-squares
  Convergence Tolerance   = 1.0e-06
  Maximum Iterations      = 500
  weight power            = 2.0
  Reciprocity Rule        = average
  Output Rule             = verbose
End

Begin Viewfactor Smoothing no_smooth
  Method                  = none
  Convergence Tolerance   = 1.0E-08
  Maximum Iterations      = 150
  Weight Power            = 2.0
  Reciprocity Rule        = average
  Output Rule             = Summary
End

Begin Radiosity Solver Rad_Solv
  Coupling                = mason
  Solver                  = chaparral GMRES
  Convergence Tolerance   = 1.0e-08
  Maximum Iterations      = 300
  Output Rule             = none
End

Begin Enclosure Definition enc1
  add surface surface_2
  add surface surface_3
  add surface surface_block_2_death_temp1

  meshed enclosure is block_2_dead
  disable parallel redistribution

  Emissivity = 0.6 On surface_block_2_death_temp1
  Emissivity = 0.6 On surface_2
  Emissivity = 0.7 On surface_3

  Blocking Surfaces
  Use Viewfactor Calculation vf_calc
  Use Viewfactor Smoothing no_smooth
  Use Radiosity Solver Rad_Solv
End

Postprocess Heat_Flux on all_blocks

postprocess average of expression temperature on each_surface as avg_T

postprocess l_2_norm of function "exact_soln - temperature" on all_blocks as l2_err
postprocess l_2_norm of function "exact_soln_dot - dt_temperature" on all_blocks as l2_dot_err
postprocess l_inf_norm of function "exact_soln - TEMPERATURE" on all_blocks as linf_err
postprocess integral of function "\$
  (gradExact_soln_x - grad_temperature_x)^2 + \$
  (gradExact_soln_y - grad_temperature_y)^2 + \$
  (gradExact_soln_z - grad_temperature_z)^2" on all_blocks as H1_err_sq
postprocess global_function "sqrt(H1_err_sq)" as H1_err

postprocess average of expression r2 on block_2 as r2_exact
postprocess max of expression radius on block_2 as r2_num
POSTPROCESS GLOBAL_FUNCTION "r2_exact - r2_num" as r2_err

Begin heartbeat testctrl

```

```

Stream name = errors{N}.dat
Timestamp Format      = ""
Precision            = 8
Format = csv
Labels = Off
Legend = On
At step 0, Increment is 1
variable = global time
variable = global number_of_nodes
variable = global l2_err
variable = global l2_dot_err
variable = global linf_err
variable = global h1_err
end

postprocess value of expression exact_soln on all_blocks as exact_temp
postprocess value of expression exact_src on all_blocks as exact_temp_src

BEGIN RESULTS OUTPUT myLABEL diffusion output etc
Database Name = cdfem_rad_death_m{N}.e
at step 0, increment = 1

Title CDFEM Death Test Case #1
Nodal Variables = Solution->Temperature as T
Nodal Variables = pp->heat_flux as Heat_Flux
Nodal Variables = exact_temp
# Nodal Variables = exact_temp_err
Nodal Variables = exact_temp_src
Nodal Variables = is_inside
Nodal Variables = encl_position linf_err
nodal variables = time_derivative_at_time->TEMPERATURE as TDOT
Face Variables = face_area emissivity face_coverage face_temperature irradiance rad_flux radiosity
Element Variables = current_element_volume initial_element_volume volume_change_ratio l2_err l2_dot_err h1_err
END

Begin Heartbeat thermalrace
stream Name = globals_tet4_m{N}.dat
precision = 8
Format = csv
timestamp format ''
legend = on
labels = off
Variable = Global time as time
Variable = Global l2_err
Variable = Global l2_dot
Variable = Global linf
Variable = Global h1_err
Variable = Global encl_area
Variable = Global encl_volume
Variable = Global avg_T_surface_1
Variable = Global avg_T_surface_2 # this value is mostly NaNs
Variable = Global avg_T_surface_block_2_death_temp1
Variable = Global avg_T_surface_3
Variable = Global avg_T_surface_4
Variable = Global r2_exact
Variable = Global r2_num
Variable = Global r2_err
at step 0, increment = 1
end

END ARIA REGION myRegion

END PROCEDURE myAriaProcedure

END SIERRA myJob

```

## 12.5. TIME INTEGRATION

### 12.5.1. Adaptive Time Integration

*12.5.1.1. First Order Fixed*

*12.5.1.2. First Order Adaptive*

*12.5.1.3. Second Order Fixed*

*12.5.1.4. Second Order Adaptive*

*12.5.1.5. BDF2 Fixed*

*12.5.1.6. BDF2 Adaptive*



## 12.6. ENCLOSURE RADIATION

### 12.6.1. 2D Cylindrical Shell Enclosure

```
# T1 = { T1 = 300.0 }
# T2 = { T2 = 444.7977 }
# T3 = { T3 = 956.5915 }
# T4 = { T4 = 1300.0 }
# r1 = { r1 = 0.01 }
# r2 = { r2 = 0.02 }
# r3 = { r3 = 0.03 }
# r4 = { r4 = 0.04 }
# rMidGap = { rMidGap = 0.5*(r2+r3) }
# c_in = { c_in = (T2-T1)/log(r2/r1) }
# c_out = { c_out = (T4-T3)/log(r4/r3) }

BEGIN SIERRA Aria

    Title Verification for two concentric spheres with radiation gap between them

    Begin Global Constants
        Stefan Boltzmann constant = 5.6704E-8
    End Global Constants

    Begin Aria Material inner
        Heat conduction = Basic
        Density = constant rho = 1.0
        Specific heat = constant cp = 1.0
        Thermal conductivity = constant k = 2.0
    End Aria Material inner

    Begin Aria Material outer
        Heat conduction = Basic
        Density = constant rho = 1.0
        Specific heat = constant cp = 1.0
        Thermal conductivity = constant k = 0.35
    End Aria Material outer

    begin universal aria expressions
        user expression = scalar_string_function f="sqrt(x*x + y*y)" user_tag = "radius"

        user expression = vector_string_function \$
            f_x="x/radius" \$
            f_y="y/radius" \$
            user_tag = "gradRadius"

        user expression = scalar_string_function f = "( (radius<={rMidGap}) ? \$
            ({T1} + {c_in}*log(radius/{r1})) : \$
            ({T4} + {c_out}*log(radius/{r4})) )" \$
            user_tag = "exact_soln"

        user expression = scalar_string_function f = "( (radius<={rMidGap}) ? \$
            ({c_in}/radius) : \$
            ({c_out}/radius) )" \$
            user_tag = "dTemp_dr"

        user expression = vector_string_function \$
            f_x="dTemp_dr*gradRadius_x" \$
            f_y="dTemp_dr*gradRadius_y" \$
            user_tag = "gradExact_soln"

    end universal aria expressions

    Begin Finite Element Model VERIFY_RAD_GAP
        Database name = input{N}.g
        Coordinate System = Cartesian
```

```

Database Type = EXODUSII
Begin Parameters for Block block_1
  Material inner
End
Begin Parameters for Block block_2
  Material outer
End
End Finite Element Model VERIFY_RAD_GAP

BEGIN TPETRA EQUATION SOLVER  SOLVE_TEMPERATURE
  BEGIN CG SOLVER
    BEGIN JACOBI PRECONDITIONER
      END
    MAXIMUM ITERATIONS = 1000
    RESIDUAL SCALING = RHS
    CONVERGENCE TOLERANCE = 1.000000e-14
  END
END TPETRA EQUATION SOLVER

Begin Procedure myProcedure

  Begin Solution Control Description
    Use System Main
    Begin System Main
      Begin Sequential Steady
        Advance myRegion
      End
    End
  End
End

Begin Aria Region myRegion

  Use Finite Element Model VERIFY_RAD_GAP
  Use Linear Solver solve_temperature

  Nonlinear Solution Strategy = Newton
  maximum nonlinear iterations = 1000
  nonlinear residual tolerance = 1.0e-10
  nonlinear relaxation factor = 1.0

  EQ energy for Temperature for all_volumes using Q1 with Diff
  IC const for all_volumes Temperature = 300.0
  BC const Dirichlet at surface_1 Temperature = 300.0
  BC const Dirichlet at surface_4 Temperature = 1300.0

  begin enclosure definition sph_shell
    add surface surface_2
    add surface surface_3
    blocking surfaces
    use viewfactor calculation vf_calc
    use viewfactor smoothing vf_smooth
    use radiosity solver rad_solver
    #      enclosure id      = 1
    emissivity = 0.50 on surface_2
    emissivity = 0.80 on surface_3
  end enclosure definition sph_shell

  begin viewfactor calculation vf_calc
    bsp tree max depth = 0 and min list length = 25
    compute rule      = hemicube
    geometric tolerance = 1.0E-6
    hemicube max subdivides = 5
    hemicube min separation = 5.0
    hemicube resolution = 500
    #      check rowsum with tolerance = .001
    output rule      = verbose
  end viewfactor calculation vf_calc

```

```

begin viewfactor smoothing vf_smooth
  convergence tolerance = 1.0E-10
  method                = least-squares
  weight power          = 2
  maximum iterations    = 150
  reciprocity rule      = average
  output rule           = verbose
end viewfactor smoothing vf_smooth

begin radiosity solver rad_solver
  coupling              = mason
  solver                = chaparral gmres
  convergence tolerance = 1.0E-8
  maximum iterations    = 800
  output rule           = verbose
end radiosity solver rad_solver

postprocess l2_norm of function "exact_soln - TEMPERATURE" on all_blocks as l2_err
postprocess l_inf_norm of function "exact_soln - TEMPERATURE" on all_blocks as lInf_err
postprocess integral of function "\$
  (gradExact_soln_x - grad_temperature_x)^2 + \$
  (gradExact_soln_y - grad_temperature_y)^2" on all_blocks as H1_err_sq
postprocess global_function "sqrt(H1_err_sq)" as H1_err

Begin heartbeat testctrl
  Stream name = errors{N}.dat
  Timestamp Format = ""
  Precision     = 8
  Format = csv
  Labels = Off
  Legend = On
  At step 0, Increment is 1
  variable = global time
  variable = global number_of_nodes

  variable = global l2_err
  variable = global lInf_err
  variable = global H1_err
end

Begin Results Output Label diffusion output
  database name = output{N}.e
  at Step 0, increment = 1
  Nodal Variables = solution->temperature as TEMP
End Results Output Label diffusion output

End Aria Region myRegion

End Procedure myProcedure

END SIERRA Aria

```

## 12.6.2. 2D Annular Enclosure

## 12.6.3. 3D Spherical Shell Enclosure

## 12.6.4. 3D Spherical Shell Partial Enclosure

```

# T1 = { T1 = 300.0 }
# T2 = { T2 = 578.225 }
# T3 = { T3 = 1035.02 }
# T4 = { T4 = 1300.0 }
# r1 = { r1 = 0.01 }

```

```

# r2 = { r2 = 0.02 }
# r3 = { r3 = 0.03 }
# r4 = { r4 = 0.04 }
# rMidGap = { rMidGap = 0.5*(r2+r3) }

BEGIN SIERRA Aria

Title Verification for two concentric spheres with radiation gap between them

Begin Global Constants
  Stefan Boltzmann constant = 5.6704E-8
End Global Constants

begin definition for function emissivity_table
  scale by 1.0
  type = piecewise linear
  begin values
    0.0 0.5
    5000 0.5
  end values
end definition for function emissivity_table

Begin Aria Material inner
  Heat conduction = Basic
  Density = constant rho = 1.0
  Specific heat = constant cp = 1.0
  Thermal conductivity = constant k = 2.0
  emissivity = user_function name=emissivity_table X=temperature
End Aria Material inner

Begin Aria Material outer
  Heat conduction = Basic
  Density = constant rho = 1.0
  Specific heat = constant cp = 1.0
  Thermal conductivity = constant k = 0.35
  emissivity = constant e = 0.80
End Aria Material outer

begin universal aria expressions
  user expression = scalar_string_function f="sqrt(x*x + y*y + z*z)" user_tag = "radius"

  user expression = vector_string_function \$
    f_x="x/radius" \$
    f_y="y/radius" \$
    f_z="z/radius" \$
    user_tag = "gradRadius"

  user expression = scalar_string_function f = "( (radius<={rMidGap}) ? \$
    ({T1} + ({T2} - {T1}) * (1/radius - 1/{r1}) / (1/{r2} - 1/{r1})) : \$
    ({T4} + ({T3} - {T4}) * (1/radius - 1/{r4}) / (1/{r3} - 1/{r4})) )" \$
    user_tag = "exact_soln"

  user expression = scalar_string_function f = "( (radius<={rMidGap}) ? \$
    (({T2} - {T1}) * (-1/(radius*radius)) / (1/{r2} - 1/{r1})) : \$
    (({T3} - {T4}) * (-1/(radius*radius)) / (1/{r3} - 1/{r4})) )" \$
    user_tag = "dTemp_dr"

  user expression = vector_string_function \$
    f_x="dTemp_dr*gradRadius_x" \$
    f_y="dTemp_dr*gradRadius_y" \$
    f_z="dTemp_dr*gradRadius_z" \$
    user_tag = "gradExact_soln"

end universal aria expressions

Begin Finite Element Model VERIFY_RAD_GAP
  Database name = sphere_cutout_h{N}.g

```

```

Coordinate System = Cartesian
Database Type = EXODUSII
Begin Parameters for Block block_1
  Material inner
End
Begin Parameters for Block block_2
  Material outer
End
End Finite Element Model VERIFY_RAD_GAP

BEGIN TPETRA EQUATION SOLVER  SOLVE_TEMPERATURE
  BEGIN CG SOLVER
    BEGIN JACOBI PRECONDITIONER
    END
    MAXIMUM ITERATIONS = 1000
    RESIDUAL SCALING = RHS
    CONVERGENCE TOLERANCE = 1.000000e-10
  END
END TPETRA EQUATION SOLVER

Begin Procedure myProcedure

  Begin Solution Control Description
    Use System Main
    Begin System Main
      Begin Sequential Steady
        Advance myRegion
      End
    End
  End
End

Begin Aria Region myRegion

  Use Finite Element Model VERIFY_RAD_GAP
  Use Linear Solver solve_temperature

  Nonlinear Solution Strategy = Newton
  maximum nonlinear iterations = 1000
  nonlinear residual tolerance = 1.0e-8
  nonlinear correction tolerance = 1.0e-8
  nonlinear relaxation factor = 1.0

  EQ energy for Temperature for all_volumes using Q1 with Diff
  BC const Dirichlet at surface_1 Temperature = {T1}
  BC const Dirichlet at surface_4 Temperature = {T4}
  BC Dirichlet for Temperature on surface_5 = scalar_string_function f="exact_soln"
  IC for Temperature on all_blocks = scalar_string_function f="exact_soln"

  begin enclosure definition sph_shell
    add surface surface_2
    add surface surface_3
    blocking surfaces
    use viewfactor calculation vf_calc
    use viewfactor smoothing vf_smooth
    use radiosity solver rad_solver
    emissivity = 0.50 on surface_2
    emissivity = 0.80 on surface_3
    Partial Enclosure Emissivity = 0.8
    Partial Enclosure Area = {2.0*PI*0.03*(0.03-0.025)}
    Partial Enclosure Temperature = {T3}
  end enclosure definition sph_shell

  begin viewfactor calculation vf_calc
    bsp tree max depth = 0 and min list length = 25
    compute rule = hemicube
    geometric tolerance = 1.0E-10
    hemicube max subdivides = 5

```

```

    hemicube min separation      = 5.0
    hemicube resolution         = 500
    #      check rowsum with tolerance = .001
    output rule                 = verbose
end viewfactor calculation vf_calc

begin viewfactor smoothing vf_smooth
    method                     = none
end viewfactor smoothing vf_smooth

begin radiosity solver rad_solver
    coupling                   = mason
    solver                     = chaparral gmres
    convergence tolerance      = 1.0E-9
    maximum iterations         = 80
    output rule                 = summary
end radiosity solver rad_solver

postprocess l2_norm of function "exact_soln - TEMPERATURE" on all_blocks as l2_err
postprocess l_inf_norm of function "exact_soln - TEMPERATURE" on all_blocks as lInf_err
postprocess integral of function "\$
    (gradExact_soln_x - grad_temperature_x)^2 + \$
    (gradExact_soln_y - grad_temperature_y)^2 + \$
    (gradExact_soln_z - grad_temperature_z)^2" on all_blocks as H1_err_sq
postprocess global_function "sqrt(H1_err_sq)" as H1_err

postprocess value of expression exact_soln on all_blocks as analytic_temp
postprocess Value of function "exact_soln - temperature" on all_blocks as temp_error

Begin heartbeat testctrl
    Stream name = errors{N}.dat
    Timestamp Format = ""
    Precision = 8
    Format = csv
    Labels = Off
    Legend = On
    At step 0, Increment is 1
    variable = global time
    variable = global number_of_nodes

    variable = global l2_err
    variable = global lInf_err
    variable = global H1_err
end

Begin Results Output Label diffusion output
    database name = output{N}.e
    at Step 0, increment = 1
    Nodal Variables = solution->temperature as T
    nodal variables = analytic_temp
    nodal variables = temp_error
    element variables = h1_err l2_err lInf_err
End Results Output Label diffusion output

End Aria Region myRegion

End Procedure myProcedure

END SIERRA Aria

```

## 12.6.5. Fully 2D Enclosure Radiation

## 12.7. CHEMISTRY

### 12.7.1. First Order Reaction (Uniform Temperature)

### 12.7.2. First Order Reaction (Spatially Varying Temperature)

### 12.7.3. First Order Reaction

### 12.7.4. DAE and Pressure Test

BEGIN SIERRA Aria

Title Verification Problem for Coupled Chemistry Diffusion

begin definition for function A\_exact  
type = piecewise linear  
begin values

```
0 1
0.005000000000000000 0.994785408602410
0.010000000000000000 0.990262801895543
0.015000000000000000 0.985996477466782
0.020000000000000000 0.981896043746815
0.025000000000000000 0.977918836400346
0.030000000000000000 0.974039821314594
0.035000000000000000 0.970242500122094
0.040000000000000000 0.966515169579351
0.045000000000000000 0.962849092341749
0.050000000000000000 0.959237494595795
0.055000000000000000 0.955674970187395
0.060000000000000000 0.952157103941654
0.065000000000000000 0.948680221790434
0.070000000000000000 0.945241218451135
0.075000000000000000 0.941837434719033
0.080000000000000000 0.938466567707596
0.085000000000000000 0.935126603669111
0.090000000000000000 0.931815766712535
0.095000000000000000 0.928532478977252
0.100000000000000000 0.925275329232036
0.105000000000000000 0.922043047782560
0.110000000000000000 0.918834486178550
0.115000000000000000 0.915648600625087
0.120000000000000000 0.912484438289564
0.125000000000000000 0.909341125898756
0.130000000000000000 0.906217860166351
0.135000000000000000 0.903113899697718
0.140000000000000000 0.900028558097496
0.145000000000000000 0.896961198064531
0.150000000000000000 0.893911226303453
0.155000000000000000 0.890878089116401
0.160000000000000000 0.887861268564900
0.165000000000000000 0.884860279112581
0.170000000000000000 0.881874664675716
0.175000000000000000 0.878903996021445
0.180000000000000000 0.875947868463928
0.185000000000000000 0.873005899816941
0.190000000000000000 0.870077728568197
0.195000000000000000 0.867163012246165
0.200000000000000000 0.864261425954650
0.205000000000000000 0.861372661054132
0.210000000000000000 0.858496423971940
0.215000000000000000 0.855632435125884
0.220000000000000000 0.852780427948145
```

|                    |                   |
|--------------------|-------------------|
| 0.2250000000000000 | 0.849940147998007 |
| 0.2300000000000000 | 0.847111352153539 |
| 0.2350000000000000 | 0.844293807873648 |
| 0.2400000000000000 | 0.841487292522989 |
| 0.2450000000000000 | 0.838691592753186 |
| 0.2500000000000000 | 0.835906503934613 |
| 0.2550000000000000 | 0.833131829633665 |
| 0.2600000000000000 | 0.830367381131057 |
| 0.2650000000000000 | 0.827612976977205 |
| 0.2700000000000000 | 0.824868442581183 |
| 0.2750000000000000 | 0.822133609830147 |
| 0.2800000000000000 | 0.819408316736447 |
| 0.2850000000000000 | 0.816692407109965 |
| 0.2900000000000000 | 0.813985730253449 |
| 0.2950000000000000 | 0.811288140678874 |
| 0.3000000000000000 | 0.808599497843030 |
| 0.3050000000000000 | 0.805919665900755 |
| 0.3100000000000000 | 0.803248513474344 |
| 0.3150000000000000 | 0.800585913437838 |
| 0.3200000000000000 | 0.797931742715021 |
| 0.3250000000000000 | 0.795285882090032 |
| 0.3300000000000000 | 0.792648216029640 |
| 0.3350000000000000 | 0.790018632516291 |
| 0.3400000000000000 | 0.787397022891117 |
| 0.3450000000000000 | 0.784783281706188 |
| 0.3500000000000000 | 0.782177306585320 |
| 0.3550000000000000 | 0.779578998092846 |
| 0.3600000000000000 | 0.776988259609771 |
| 0.3650000000000000 | 0.774404997216808 |
| 0.3700000000000000 | 0.771829119583831 |
| 0.3750000000000000 | 0.769260537865287 |
| 0.3800000000000000 | 0.766699165601200 |
| 0.3850000000000000 | 0.764144918623373 |
| 0.3900000000000000 | 0.761597714966469 |
| 0.3950000000000000 | 0.759057474783649 |
| 0.4000000000000000 | 0.756524120266475 |
| 0.4050000000000000 | 0.753997575568824 |
| 0.4100000000000000 | 0.751477766734556 |
| 0.4150000000000000 | 0.748964621628705 |
| 0.4200000000000000 | 0.746458069871993 |
| 0.4250000000000000 | 0.743958042778454 |
| 0.4300000000000000 | 0.741464473295995 |
| 0.4350000000000000 | 0.738977295949722 |
| 0.4400000000000000 | 0.736496446787873 |
| 0.4450000000000000 | 0.734021863330195 |
| 0.4500000000000000 | 0.731553484518662 |
| 0.4550000000000000 | 0.729091250670362 |
| 0.4600000000000000 | 0.726635103432462 |
| 0.4650000000000000 | 0.724184985739139 |
| 0.4700000000000000 | 0.721740841770347 |
| 0.4750000000000000 | 0.719302616912347 |
| 0.4800000000000000 | 0.716870257719902 |
| 0.4850000000000000 | 0.714443711880037 |
| 0.4900000000000000 | 0.712022928177299 |
| 0.4950000000000000 | 0.709607856460437 |
| 0.5000000000000000 | 0.707198447610420 |
| 0.5050000000000000 | 0.704794653509741 |
| 0.5100000000000000 | 0.702396427012934 |
| 0.5150000000000000 | 0.700003721918249 |
| 0.5200000000000000 | 0.697616492940422 |
| 0.5250000000000000 | 0.695234695684496 |
| 0.5300000000000000 | 0.692858286620640 |
| 0.5350000000000000 | 0.690487223059917 |
| 0.5400000000000000 | 0.688121463130957 |
| 0.5450000000000000 | 0.685760965757493 |
| 0.5500000000000000 | 0.683405690636729 |
| 0.5550000000000000 | 0.681055598218483 |
| 0.5600000000000000 | 0.678710649685093 |



|                    |                   |
|--------------------|-------------------|
| 0.5650000000000000 | 0.676370806932034 |
| 0.5700000000000000 | 0.674036032549222 |
| 0.5750000000000000 | 0.671706289802978 |
| 0.5800000000000000 | 0.669381542618620 |
| 0.5850000000000000 | 0.667061755563646 |
| 0.5900000000000000 | 0.664746893831506 |
| 0.5950000000000000 | 0.662436923225919 |
| 0.6000000000000000 | 0.660131810145710 |
| 0.6050000000000000 | 0.657831521570168 |
| 0.6100000000000000 | 0.655536025044874 |
| 0.6150000000000000 | 0.653245288668007 |
| 0.6200000000000000 | 0.650959281077084 |
| 0.6250000000000000 | 0.648677971436139 |
| 0.6300000000000000 | 0.646401329423306 |
| 0.6350000000000000 | 0.644129325218799 |
| 0.6400000000000000 | 0.641861929493273 |
| 0.6450000000000000 | 0.639599113396547 |
| 0.6500000000000000 | 0.637340848546679 |
| 0.6550000000000000 | 0.635087107019378 |
| 0.6600000000000000 | 0.632837861337737 |
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| 0.6700000000000000 | 0.628352749781338 |
| 0.6750000000000000 | 0.626116831101620 |
| 0.6800000000000000 | 0.623885302639199 |
| 0.6850000000000000 | 0.621658139010656 |
| 0.6900000000000000 | 0.619435315224536 |
| 0.6950000000000000 | 0.617216806673033 |
| 0.7000000000000000 | 0.615002589123926 |
| 0.7050000000000000 | 0.612792638712738 |
| 0.7100000000000000 | 0.610586931935126 |
| 0.7150000000000000 | 0.608385445639480 |
| 0.7200000000000000 | 0.606188157019737 |
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| 0.7300000000000000 | 0.601806083269682 |
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| 0.7650000000000000 | 0.586597854228808 |
| 0.7700000000000000 | 0.584441359296086 |
| 0.7750000000000000 | 0.582288832110322 |
| 0.7800000000000000 | 0.580140253517936 |
| 0.7850000000000000 | 0.577995604633457 |
| 0.7900000000000000 | 0.575854866834605 |
| 0.7950000000000000 | 0.573718021757508 |
| 0.8000000000000000 | 0.571585051292031 |
| 0.8050000000000000 | 0.569455937577236 |
| 0.8100000000000000 | 0.567330662996938 |
| 0.8150000000000000 | 0.565209210175392 |
| 0.8200000000000000 | 0.563091561973075 |
| 0.8250000000000000 | 0.560977701482574 |
| 0.8300000000000000 | 0.558867612024580 |
| 0.8350000000000000 | 0.556761277143975 |
| 0.8400000000000000 | 0.554658680606020 |
| 0.8450000000000000 | 0.552559806392627 |
| 0.8500000000000000 | 0.550464638698732 |
| 0.8550000000000000 | 0.548373161928744 |
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| 0.8650000000000000 | 0.544201219804817 |
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| 0.9000000000000000 | 0.529713488675192 |

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| 1.0550000000000000  | 0.467584207031921 |
| 1.0600000000000000  | 0.465632821016487 |
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| 1.1250000000000000  | 0.440551765487674 |
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| 1.1850000000000000  | 0.417862515162895 |
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| 1.2400000000000000  | 0.397443146884633 |

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| 1.3750000000000000 | 0.348810370484198 |
| 1.3800000000000000 | 0.347048868040494 |
| 1.3850000000000000 | 0.345290162229364 |
| 1.3900000000000000 | 0.343534249440206 |
| 1.3950000000000000 | 0.341781126129396 |
| 1.4000000000000000 | 0.340030788820022 |
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| 1.4150000000000000 | 0.334796459126890 |
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| 1.4250000000000000 | 0.331320775241834 |
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| 1.4400000000000000 | 0.326127990992461 |
| 1.4450000000000000 | 0.324402582125912 |
| 1.4500000000000000 | 0.322679928110465 |
| 1.4550000000000000 | 0.320960026188650 |
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| 1.4800000000000000 | 0.312401705970768 |
| 1.4850000000000000 | 0.310698262299481 |
| 1.4900000000000000 | 0.308997553215040 |
| 1.4950000000000000 | 0.307299576471783 |
| 1.5000000000000000 | 0.305604329887650 |
| 1.5050000000000000 | 0.303911811344139 |
| 1.5100000000000000 | 0.302222018786264 |
| 1.5150000000000000 | 0.300534950222526 |
| 1.5200000000000000 | 0.298850603724898 |
| 1.5250000000000000 | 0.297168977428815 |
| 1.5300000000000000 | 0.295490069533177 |
| 1.5350000000000000 | 0.293813878300363 |
| 1.5400000000000000 | 0.292140402056254 |
| 1.5450000000000000 | 0.290469639190267 |
| 1.5500000000000000 | 0.288801588155400 |
| 1.5550000000000000 | 0.287136247468288 |
| 1.5600000000000000 | 0.285473615709267 |
| 1.5650000000000000 | 0.283813691522453 |
| 1.5700000000000000 | 0.282156473615829 |
| 1.5750000000000000 | 0.280501960761346 |
| 1.5800000000000000 | 0.278850151795028 |

|                    |                   |
|--------------------|-------------------|
| 1.5850000000000000 | 0.277201045617101 |
| 1.5900000000000000 | 0.275554641192117 |
| 1.5950000000000000 | 0.273910937549103 |
| 1.6000000000000000 | 0.272269933781716 |
| 1.6050000000000000 | 0.270631629048411 |
| 1.6100000000000000 | 0.268996022572616 |
| 1.6150000000000000 | 0.267363113642927 |
| 1.6200000000000000 | 0.265732901613310 |
| 1.6250000000000000 | 0.264105385903313 |
| 1.6300000000000000 | 0.262480565998301 |
| 1.6350000000000000 | 0.260858441449687 |
| 1.6400000000000000 | 0.259239011875192 |
| 1.6450000000000000 | 0.257622276959107 |
| 1.6500000000000000 | 0.256008236452574 |
| 1.6550000000000000 | 0.254396890173877 |
| 1.6600000000000000 | 0.252788238008748 |
| 1.6650000000000000 | 0.251182279910685 |
| 1.6700000000000000 | 0.249579015901287 |
| 1.6750000000000000 | 0.247978446070603 |
| 1.6800000000000000 | 0.246380570577489 |
| 1.6850000000000000 | 0.244785389649987 |
| 1.6900000000000000 | 0.243192903585719 |
| 1.6950000000000000 | 0.241603112752287 |
| 1.7000000000000000 | 0.240016017587701 |
| 1.7050000000000000 | 0.238431618600813 |
| 1.7100000000000000 | 0.236849916371772 |
| 1.7150000000000000 | 0.235270911552493 |
| 1.7200000000000000 | 0.233694604867143 |
| 1.7250000000000000 | 0.232120997112647 |
| 1.7300000000000000 | 0.230550089159203 |
| 1.7350000000000000 | 0.228981881950829 |
| 1.7400000000000000 | 0.227416376505910 |
| 1.7450000000000000 | 0.225853573917778 |
| 1.7500000000000000 | 0.224293475355308 |
| 1.7550000000000000 | 0.222736082063521 |
| 1.7600000000000000 | 0.221181395364229 |
| 1.7650000000000000 | 0.219629416656675 |
| 1.7700000000000000 | 0.218080147418217 |
| 1.7750000000000000 | 0.216533589205014 |
| 1.7800000000000000 | 0.214989743652743 |
| 1.7850000000000000 | 0.213448612477341 |
| 1.7900000000000000 | 0.211910197475758 |
| 1.7950000000000000 | 0.210374500526747 |
| 1.8000000000000000 | 0.208841523591664 |
| 1.8050000000000000 | 0.207311268715303 |
| 1.8100000000000000 | 0.205783738026752 |
| 1.8150000000000000 | 0.204258933740271 |
| 1.8200000000000000 | 0.202736858156203 |
| 1.8250000000000000 | 0.201217513661905 |
| 1.8300000000000000 | 0.199700902732713 |
| 1.8350000000000000 | 0.198187027932927 |
| 1.8400000000000000 | 0.196675891916838 |
| 1.8450000000000000 | 0.195167497429766 |
| 1.8500000000000000 | 0.193661847309149 |
| 1.8550000000000000 | 0.192158944485651 |
| 1.8600000000000000 | 0.190658791984302 |
| 1.8650000000000000 | 0.189161392925677 |
| 1.8700000000000000 | 0.187666750527110 |
| 1.8750000000000000 | 0.186174868103935 |
| 1.8800000000000000 | 0.184685749070769 |
| 1.8850000000000000 | 0.183199396942837 |
| 1.8900000000000000 | 0.181715815337323 |
| 1.8950000000000000 | 0.180235007974774 |
| 1.9000000000000000 | 0.178756978680533 |
| 1.9050000000000000 | 0.177281731386227 |
| 1.9100000000000000 | 0.175809270131281 |
| 1.9150000000000000 | 0.174339599064499 |
| 1.9200000000000000 | 0.172872722445669 |

```

1.925000000000000 0.171408644647231
1.930000000000000 0.169947370155987
1.935000000000000 0.168488903574865
1.940000000000000 0.167033249624732
1.945000000000000 0.165580413146267
1.950000000000000 0.164130399101882
1.955000000000000 0.162683212577707
1.960000000000000 0.161238858785634
1.965000000000000 0.159797343065421
1.970000000000000 0.158358670886860
1.975000000000000 0.156922847852013
1.980000000000000 0.155489879697517
1.985000000000000 0.154059772296954
1.990000000000000 0.152632531663307
1.995000000000000 0.151208163951476
2 0.149786675460890
end
end

BEGIN Aria MATERIAL hmx
  density = constant rho = 1
  specific heat = constant cp = 1
  heat conduction = basic
  thermal conductivity = constant k = 1
  pressure = constant value=3

  species names are AA
  species of AA = from_chemeq

begin parameters for chemeq model hmx
  number of reactions = 1

  species names are AA
  species phases are Condensed
  Concentration units = moles

  Condensed Fraction = 0.0
  Steric Coefficients are 0.0
  Log Preexponential Factors are {log(5)}
  Activation Energies are 10.0
  Energy Releases are 0.0

  Concentration Exponents for AA are 0.0

  Stoichiometric coefficients for AA are -1.0

  # Pressure dependence
  Reference pressure = 2.
  Pressure exponents are 2.
  Pressure = From_Material_Definition

  #Distributed activation energy
  Activation energy st devs are 1.
  extent of reaction based on AA

end parameters for chemeq model hmx

end Aria MATERIAL hmx

BEGIN GLOBAL CONSTANTS
  Ideal Gas Constant = 1.
END GLOBAL CONSTANTS

BEGIN FINITE ELEMENT MODEL block
  Database Name = 1block.g
  decomposition method = rib

  Use material hmx for block_1

```

```

END FINITE ELEMENT MODEL block

BEGIN TPETRA EQUATION SOLVER SOLVE_TEMPERATURE
  BEGIN CG SOLVER
    BEGIN JACOBI PRECONDITIONER
      END
    MAXIMUM ITERATIONS = 1000
    RESIDUAL SCALING = NONE
    CONVERGENCE TOLERANCE = 1.000000e-12
  END
END TPETRA EQUATION SOLVER

Begin Universal Aria Expressions
  user expression = user_function name = A_exact user_tag = "exactAA" x = "time"
End

BEGIN procedure myProcedure

begin solution control description
  Use System Main
  Begin System Main
    Simulation Start Time      = 0.0
    Simulation Termination Time = 2.0
    Begin Transient time_block
      advance myregion
    End Transient time_block
  End System Main

  BEGIN parameters for transient time_block
    start time = 0.0
    termination time = 2.0

    BEGIN PARAMETERS FOR Aria REGION myRegion
      time step variation = fixed
      time integration method = second_order
      initial time step size = {0.01*0.5}
    END PARAMETERS FOR Aria REGION myRegion

  END parameters for transient time_block

end solution control description

  begin aria region myRegion

  postprocess L_2_norm of function "species_AA - exactAA" on all_blocks as l_2_A

  EQ energy for temperature on all_blocks using Q1 with diff mass src
  Source for Energy on all_blocks = chemeq_heating MODEL = hmx
  IC for temperature on all_blocks = constant value=3

  maximum nonlinear iterations = 10
  nonlinear residual tolerance = 1.0E-10
  nonlinear correction tolerance = 1.0E-10
  Nonlinear Relaxation Factor = 1.

  BEGIN CHEMEQ SOLVER FOR hmx
    ODE SOLVER = CVODE ADAMS 12 FUNCTIONAL
    Absolute Tolerance = 1e-12
    Relative Tolerance = 1e-10
    Activation Temperature = 0.0
    species AA = 1.0
  END CHEMEQ SOLVER FOR hmx

  BEGIN RESULTS OUTPUT LABEL diffusion output
    database Name = output.e
    At Step 0, Increment = 1
    Timestep Adjustment Interval = 1
    Title Aria Chem/Diffusion Verification

```

```

        Nodal Variables = solution->temperature as T
        Nodal Variables = exact_A
        Element Variables = AA
END RESULTS OUTPUT LABEL diffusion output

USE FINITE ELEMENT MODEL block
use LINEAR SOLVER solve_temperature

Begin Heartbeat pp_out
    Legend = off
    Labels = off
    At Step 1 Interval is 1
    Precision = 6
    Stream Name = error.txt
    variable is Global time
    Global L_2_A
End

END Aria REGION myRegion

END procedure myProcedure

END SIERRA Aria

```

## 12.8. MISCELLANEOUS

### 12.8.1. Thermal Postprocessing

```
BEGIN SIERRA myJob
```

```
BEGIN ARIA MATERIAL Kryptonite
  Density          = CONSTANT rho = 1
  Thermal Conductivity = Constant k  = 1
  Specific Heat     = Constant cp  = 1
  heat conduction   = basic
END
```

```
BEGIN ARIA MATERIAL Mathite
  Density          = CONSTANT rho = 1
  Thermal Conductivity = Constant k  = 1
  Specific Heat     = Constant cp  = 1
  heat conduction   = basic
END
```

```
Begin Global Constants
  Stefan Boltzmann Constant = 5.67e-8
End
```

```
# T0 = {T0 = 400}
# h = {h = 10}
# C0 = {C0 = 2}
# C1 = {C1 = 3}
# C2 = {C2 = 4}
# C3 = {C3 = 0.4}
# eps = { eps = 0.6 }
# sigma = { sigma = 5.67e-8}
```

```
Begin universal aria expressions
```

```
User Expression = Scalar_String_Function \$
  f = "{C0}*(x^2-1) + {C1}*(y^2-0.25) + {C2}*(z^2-0.25) + {C3}*t" \$
  user_tag = arg
```

```
User Expression = Scalar_String_Function \$
  f = "{T0} + exp(arg)" \$
  user_tag = T_mms
```

```
User Expression = Vector_String_Function \$
  f_x = "{C0}*2*x*exp(arg)" \$
  f_y = "{C1}*2*y*exp(arg)" \$
  f_z = "{C2}*2*z*exp(arg)" \$
  user_tag = gradT
```

```
User Expression = Scalar_String_Function \$
  f = "2*exp(arg)*({C0}*(2*{C0}*x^2+1) + {C1}*(2*{C1}*y^2+1) + {C2}*(2*{C2}*z^2+1))" \$
  user_tag = laplacianT
```

```
User Expression = Scalar_String_Function \$
  f = "-laplacianT" \$
  user_tag = mms_src
```

```
User Expression = Scalar_String_Function \$
  f = "T_mms + gradT_z/{h}" \$
  user_tag = conv_Tref
```

```
User Expression = Scalar_String_Function \$
  f = "(T_mms^4 + gradT_y/({eps*sigma}))^0.25" \$
  user_tag = rad_Tref
```

```
User Expression = Scalar_String_Function \$
```



```

        f = "{h}*(T_mms - conv_Tref)" \$
        user_tag = conv_flux_mms

User Expression = Scalar_String_Function \$
f = "{sigma*eps}*(T_mms^4 - rad_Tref^4)" \$
user_tag = rad_flux_mms

End

Begin Aria Material surf_2_models
BC Reference Temperature = Scalar_String_Function f = "conv_Tref"
Heat Transfer Coefficient = constant h={h}
End

Begin Aria Material surf_3_models
BC Rad Reference Temperature = Scalar_String_Function f = "rad_Tref"
Emissivity = Constant E={eps}
Radiation form factor = Constant F=1.0
End

BEGIN TPETRA EQUATION SOLVER SOLVE_TEMPERATURE
BEGIN CG SOLVER
BEGIN JACOBI PRECONDITIONER
END
MAXIMUM ITERATIONS = 1000
RESIDUAL SCALING = R0
CONVERGENCE TOLERANCE = AUTOMATIC
END
END TPETRA EQUATION SOLVER

BEGIN FINITE ELEMENT MODEL cube
database name = cube_two_blocks_hex8_h{N}.g
coordinate system is cartesian

# [0,1] x [-0.5,0.5] x [-0.5,0.5]
BEGIN PARAMETERS FOR BLOCK block_1
material Kryptonite
END

# [-1,0] x [-0.5,0.5] x [-0.5,0.5]
BEGIN PARAMETERS FOR BLOCK block_2
material Mathite
END

Use Material surf_2_models for surface_2
Use Material surf_3_models for surface_3

END FINITE ELEMENT MODEL cube

BEGIN PROCEDURE myAriaProcedure

Begin Solution Control Description
Use System Main
Begin System Main
Begin Transient MySolveBlock
Advance myRegion
End
Simulation Max Global Iterations = 1
Simulation Start Time = 0
Simulation Termination Time = 1
End
Begin Parameters for Transient MySolveBlock
End
End

BEGIN ARIA REGION myRegion

```

```

use finite element model cube
use linear solver solve_temperature

nonlinear solution strategy = newton

maximum nonlinear iterations = 10
nonlinear residual tolerance = 1.0e-16
nonlinear correction tolerance = 1.0e-12
nonlinear relaxation factor = 1.0
use dof averaged nonlinear residual

EQ ENERGY for TEMPERATURE on all_blocks using Q1 with DIFF SRC

MESH GROUP Dirichlet_Surface = surface_4 surface_5 surface_6 surface_7
mesh group block_1_2 = block_1 block_2
BC Dirichlet for Temperature on Dirichlet_Surface = scalar_string_function f = "T_mms"

BC Flux for Energy on surface_2 = Generalized_Nat_Conv Power_Output=cf_bc_ipo Flux_Output=cf_bc_ipo
BC Flux for Energy on surface_3 = Generalized_Rad Power_Output=rf_bc_ipo Flux_Output=rf_bc_ipo

SOURCE for ENERGY on all_blocks = scalar_string_function f = "mms_src" Power_Output=src_ipo

postprocess integral of expression conv_flux_mms on surface_2 as cf_bc_ipo_ex
postprocess integral of expression rad_flux_mms on surface_3 as rf_bc_ipo_ex

postprocess integral of expression mms_src on block_1_2 as src_ipo_ex

postprocess average of expression conv_flux_mms on surface_2 as cf_bc_ipo_ex
postprocess average of expression rad_flux_mms on surface_3 as rf_bc_ipo_ex

postprocess value of expression T_mms on block_1_2 as T_exact
postprocess l2_norm of function "T_mms - TEMPERATURE" on block_1_2 as l2_error

postprocess value of field nonlinear_solution->temperature at point -0.151720462393008 0.146935733548329 -0.393641401879319 as eval_b1_ex
postprocess value of field T_exact at point -0.151720462393008 0.146935733548329 -0.393641401879319 as eval_b1_ex
postprocess value of field nonlinear_solution->temperature at point 0 0.162595269728099 -0.377464159584852 as eval_b1b2_ex
postprocess value of field T_exact at point 0 0.162595269728099 -0.377464159584852 as eval_b1b2_ex
postprocess value of field nonlinear_solution->temperature at point -0.855758209426849 0.159603369582751 0.5 as eval_s2_ex
postprocess value of field T_exact at point -0.855758209426849 0.159603369582751 0.5 as eval_s2_ex

postprocess global_function "cf_bc_ipo_ex - cf_bc_ipo" as cf_bc_ipo_err
postprocess global_function "rf_bc_ipo_ex - rf_bc_ipo" as rf_bc_ipo_err
postprocess global_function "cf_bc_ipo_ex - cf_bc_ipo" as cf_bc_ipo_err
postprocess global_function "rf_bc_ipo_ex - rf_bc_ipo" as rf_bc_ipo_err
postprocess global_function "src_ipo_ex - src_ipo" as src_ipo_err
postprocess global_function "eval_b1_ex - eval_b1" as eval_b1_err
postprocess global_function "eval_b1b2_ex - eval_b1b2" as eval_b1b2_err
postprocess global_function "eval_s2_ex - eval_s2" as eval_s2_err

Begin Heartbeat The_Heartbeat
Stream Name = globals{N}.txt
Timestamp Format = "" # Omit time stamps for diffing purposes.
Precision = 8
At Step 0 increment = 1
Legend = on
Labels = off
Format = csv
variable is Global time
variable is Global number_of_nodes as num_nodes
variable is Global l2_error
variable is Global cf_bc_ipo_err

```

```

        variable is Global rf_bc_ipo_err
        variable is Global cf_bc_ifo_err
        variable is Global rf_bc_ifo_err
        variable is Global eval_b1_err
        variable is Global eval_b1b2_err
        variable is Global eval_s2_err
    End

    END ARIA REGION myRegion

    END PROCEDURE myAriaProcedure

END SIERRA myJob

```

## **12.8.2. Postprocess Min/Max**

## **12.8.3. Local Coordinates: Cartesian**

## **12.8.4. Local Coordinates: Cylindrical**

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# Appendices

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