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# Tilted stripes origin in $\text{La}_{1.88}\text{Sr}_{0.12}\text{CuO}_4$ revealed by anisotropic next-nearest neighbor hopping



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Spin- and charge- stripe order has been extensively studied in the superconducting cuprates, among which underdoped  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  (LSCO) is an archetype with static spin stripes at low temperatures. An intriguing, but not completely understood, phenomenon in LSCO is that the stripes are tilted away from the high-symmetry Cu-Cu directions. Using high-resolution neutron scattering on LSCO with  $x = 0.12$ , we find two coexisting phases at low temperatures, one with static spin stripes and the other with fluctuating ones, both sharing the same tilt angle. Our numerical calculations using the doped Hubbard model elucidate the tilting's origin, attributing it to anisotropic next-nearest neighbor hopping  $t'$ , consistent with the material's slight orthorhombicity. Our results underscore the model's success in describing specific details of the ground state of this real material and highlight the role of  $t'$  in the Hamiltonian, revealing the delicate interplay between stripes and superconductivity across theoretical and experimental contexts.

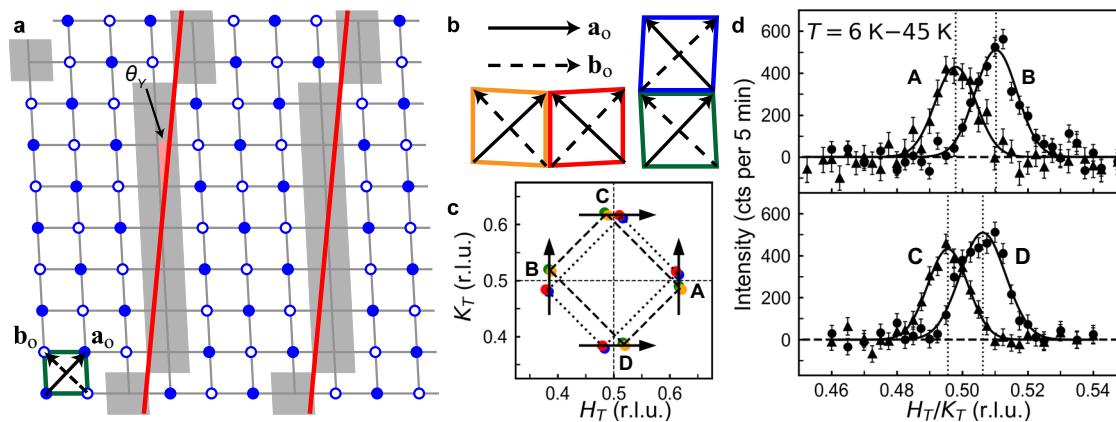
The high- $T_c$  cuprates exhibit complex physical phenomena due to the presence of various phases which may interact with the superconductivity<sup>1</sup>. In recent years, the existence of charge density wave (CDW) and spin density wave (SDW) order has been observed across many families of cuprates<sup>2,3</sup>. The La-based family is a canonical example where both the spin and charge orders form "stripes" which are especially stable near 1/8 hole doping<sup>4–6</sup>. In the stripe model, the doped holes segregate into unidirectional stripes which serve as the antiphase domain boundaries between patches of antiferromagnetically correlated spins. Since the periodicity of the spin order is twice that of the charge order, the wave vectors of these two orders satisfy the relationship  $\delta_{\text{charge}} = 2\delta_{\text{spin}}$ , which has been confirmed by extensive neutron and x-ray scattering measurements<sup>5,6</sup>.

A phenomenon that is not completely understood is that the direction of the stripes is slightly tilted from the underlying Cu-Cu direction. This follows from the observations of small in-plane shifts of both the SDW and CDW peak positions from the high-symmetry directions in orthorhombic cuprates<sup>7–16</sup>. This observation was referred to as the Y-shift in early measurements<sup>8</sup>, and we will use this term to refer to the observation that denotes tilted stripes. Specifically, in  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  (LSCO), the  $\text{CuO}_2$  square lattice is deformed upon cooling from a high-temperature tetragonal

to a low-temperature orthorhombic structure. As sketched in Fig. 1a, the average tilting of the charge domain wall boundaries has a specific orientation with the orthorhombic distortion. This phenomenon was first discovered in oxygen-doped  $\text{La}_2\text{CuO}_{4+y}$  (LCO)<sup>7</sup> and subsequently observed in other systems, including LSCO<sup>8,9,13</sup> and  $\text{La}_{1.875}\text{Ba}_{0.125-x}\text{Sr}_x\text{CuO}_4$ <sup>10</sup>. Recent x-ray scattering of the CDW order in LSCO<sup>11,15,16</sup> also confirmed the existence of the Y-shift in the charge stripes, consistent with the SDW order, further corroborating the stripe picture. Such a shift is expected in the phenomenological Landau-Ginzburg model for incommensurate stripe orders in the presence of an orthorhombic distortion<sup>17</sup>. However, the microscopic origin of the Y-shift was still not clear.

Recently, new insights regarding the stripe phases and superconductivity have come from numerical simulations using the doped Hubbard model (and  $t$ -J model) to describe the  $\text{CuO}_2$  planes. Half-filled stripes, which are consistent with the periodicities observed for doped LSCO, are found using interaction terms  $U$ ,  $t$ , and  $t'$ <sup>18–25</sup>, where  $U$  is the on-site Coulomb repulsion and  $t$  ( $t'$ ) is the (next) nearest neighbor hopping. The significance of  $t'$  in the cuprates is strongly suggested by the observation that filled stripes (with double the periodicity, inconsistent with LSCO) are favored when only  $U$  and  $t$  are used in the model<sup>22,26</sup>. Recent density-matrix

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**Fig. 1 | Tilted stripes and the crystal structure.** **a** A schematic of the  $\text{CuO}_2$  plane with orthorhombic distortion and the tilted spin- and charge- stripe order. Solid and open blue circles denote spins with opposite directions. The solid green lines show the tetragonal unit cell. The gray regions are the charge stripes served as antiphase domain walls for the magnetic stripes and the red lines show the tilted average stripe direction. **b** Tetragonal unit cells of the four structural twin domains in the orthorhombic phase. **c** Corresponding possible SDW peaks around  $(0.5, 0.5, 0)$  in reciprocal space. The two rectangles formed by the two sets of peaks (red and blue vs.

green and orange) have different centers due to orthorhombic distortion. The solid (dashed) arrows in **a**, **b** show the orthorhombic  $a_o$  ( $b_o$ ) direction. The orthorhombic distortion and  $Y$ -shift in **a**–**c** are exaggerated for clarity. **d** Elastic neutron scattering of the SDW peaks at  $6\text{ K}$  with the  $45\text{ K}$  data subtracted as a background. The solid lines are Gaussian fits to the data. The vertical dotted lines are the fitted peak centers. The trajectory of each scan is denoted in **c**. Error bars correspond to  $\pm\sigma$ , where  $\sigma$  is the standard deviation.

renormalization group (DMRG) calculations have shown that the presence of  $t'$  is also necessary to induce superconducting correlations<sup>21–23</sup>. Here, we use DMRG simulations to determine the terms in the Hubbard model that can stabilize the tilted stripes in LSCO with  $x = 0.12$ . We find that  $t'$  plays a crucial role, and that the tilted alignment of stripes is highly sensitive to small anisotropies in  $t'$ , quantitatively consistent with the experimental results.

Another issue to address is whether the tilting phenomenon is particular to static stripes, or if it is generic to the stripe correlations which may also be fluctuating. Spin fluctuations are ubiquitous in the phase diagram of cuprates<sup>2</sup>. In the stripe picture, the low-energy spin fluctuations may be thought of as spin waves associated with the ordered stripes, i.e., dynamic stripes<sup>27–30</sup>, but the universality of this description is still under debate<sup>31</sup>. Due to the broad widths and weak cross sections of the spin fluctuation peaks, it is extremely challenging to detect small peak shifts in the inelastic neutron scattering. Most of the previous studies of the  $Y$ -shift focused on static stripes, and it remains an open question whether the static and fluctuating spin correlations are characterized by the same stripe tilting.

Here, we present our comprehensive high-resolution neutron scattering study of the  $Y$ -shift phenomenon on a single crystal of LSCO with  $x = 0.12$ , which has the longest length scale for the spin correlations<sup>32</sup>. The sample was mounted in the  $(\text{HKO})$  zone, and tetragonal notation is used unless otherwise noted. More details of the experiments can be found in the Methods. First, our elastic scattering results verify the existence of the  $Y$ -shift in static SDW order. To the best of our knowledge, we determine the spin direction in LSCO for the first time, and provide a new method to determine the interlayer correlations. Then, the inelastic scattering results offer evidence for the same  $Y$ -shift in the dynamic spin stripes, where the spin fluctuation direction is found to be predominantly isotropic. Finally, our numerical calculations using the DMRG method<sup>33</sup> explain the microscopic origin of the  $Y$ -shift. The anisotropy of the next-nearest neighbor hopping term  $t'$  plays a key role here.

## Results

### Tilted static spin stripes

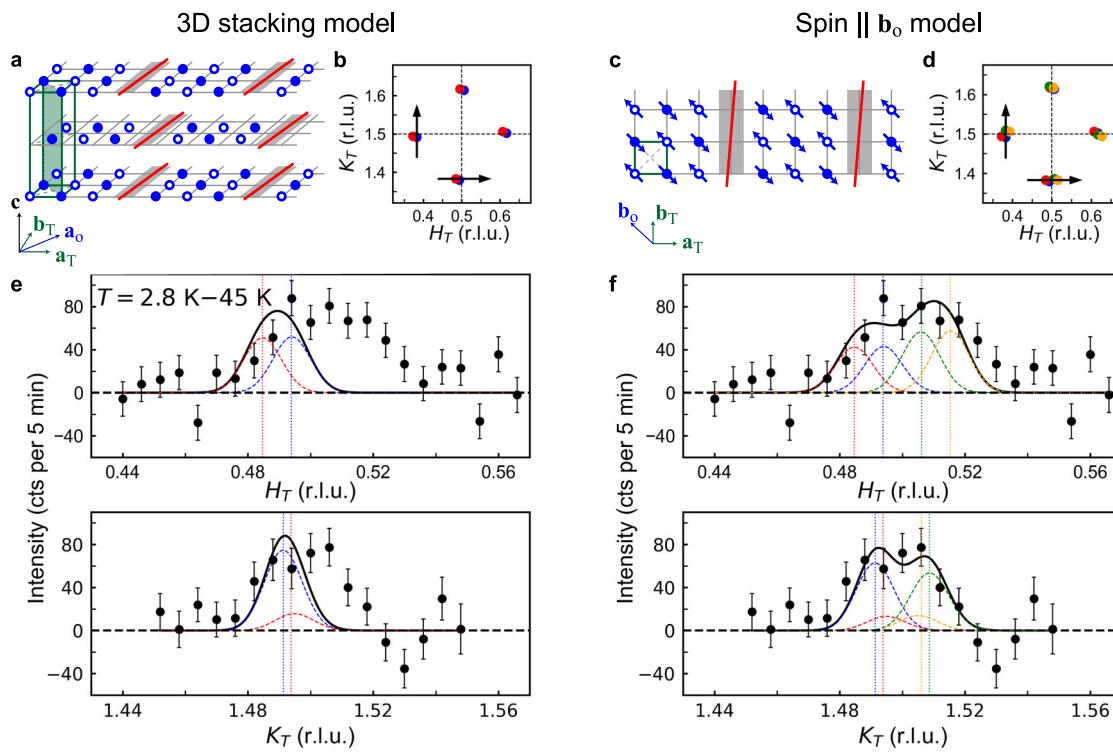
To begin, a proper interpretation of the neutron scattering data requires a careful understanding of the structural twinning in our LSCO single crystal sample. As depicted in Fig. 1b, the orthorhombic distortion leads to four possible structural twin domains<sup>34</sup>. From the precise characterization of nuclear Bragg peaks (see Supplementary Note 1, Supplementary Fig. 1, and Supplementary Tab. 1), we find that our sample has all four domains with

similar populations and the orthorhombicity is  $0.38(2)^\circ$  (calculated as  $\frac{b_o - a_o}{a_o}$ , where  $a_o$  and  $b_o$  are in-plane lattice parameters in orthorhombic notation).

We start with the elastic magnetic scattering of the SDW order near the antiferromagnetic zone center of  $(0.5, 0.5, 0)$ . A quartet of SDW peaks can be observed here due to the existence of two magnetic domains (different from the structural twin domains above) with stripe directions perpendicular to each other. Figure 1d shows representative scans along the tetragonal  $H$  or  $K$  directions. The two magnetic domains have roughly the same populations, indicated by the similar intensities between the two pairs of peaks (e.g., peak A vs. C, or peak B vs. D). Within each pair of peaks, a clear shift is observed, corresponding to a  $Y$ -shift angle of  $3.0(2)^\circ$ , consistent with previous reports<sup>8</sup>. However, a mystery observation is the presence of only a single peak in each scan, in contrast to the predicted pattern in Fig. 1c based on the existence of four equally-populated structural twin domains. Further inspection (see Supplementary Note 2) indicates that the observed peaks can only originate from the orange and green structural domains, both of which have the shorter orthorhombic  $a_o$  axis nearly aligned with  $(0.5, 0.5, 0)$ . The in-plane correlation length is determined to be  $123(7)$  Å (see Supplementary Fig. 2 and Supplementary Tab. 2).

We propose two models to explain the missing contributions from the other two twin domains. As illustrated in Fig. 2a, the first model involves the coupling between layers and therefore is named the “3D stacking model”. The depicted stacking arrangement (which is locally similar to that in  $\text{La}_2\text{CuO}_4$ <sup>35</sup>) results in nearly zero intensity around  $(0.5, 0.5, 0)$  for the red and blue domains. The second model depends on the spin direction, relying on the fact that the magnetic neutron scattering cross section is only sensitive to components of the spin  $\mathbf{S}$  that are perpendicular to the wavevector  $\mathbf{Q}$ . Hence, if the spins are pointing along the orthorhombic  $b_o$  direction, as displayed in Fig. 2c, the red and blue domains will give nearly zero intensities around  $(0.5, 0.5, 0)$  regardless of the stacking arrangement between layers. Quantitatively, considering that the intensities from the red and blue domains around  $(0.5, 0.5, 0)$  are less than  $\sim 5\%$  of those from the green and orange domains in Fig. 1d, the spins need to be either correlated over at least  $14.3$  Å in the  $L$  direction using a finite-size domain analysis (see Supplementary Note 2D)<sup>36</sup> in the first model, or fixed to the orthorhombic  $b_o$  direction within  $10^\circ$  in the second model.

Both spin models proposed above are reminiscent of the spin structure of the parent compound  $\text{La}_2\text{CuO}_4$ <sup>35</sup> and in oxygen-doped LCO<sup>7</sup>. To further distinguish the two models, one approach is to measure the  $L$ -dependence of the SDW intensity as in ref. <sup>7</sup>. However, this method involves switching



**Fig. 2 | Comparison of two models for SDW peaks around (0.5, 1.5, 0).** a, b, e are for the “3D stacking model” while c, d and f for the “Spin  $\parallel$   $\mathbf{b}_0$  model.” a, c show schematics of the two models in real space. The solid green lines denote the structural unit cell. The gray regions are the charge stripes and the red lines show the Y-shifted effective stripe direction. Solid and open blue circles denote spins with opposite directions. Arrows on top of the circles further specify spin directions in c. The spins in the neighboring layers have fixed relation in the 3D stacking model as shown in a while these long-range interlayer correlations are absent in the Spin  $\parallel$   $\mathbf{b}_0$  model.

b, d show the corresponding SDW peaks around (0.5, 1.5, 0) in reciprocal space. The colors of the peaks follow the convention in Fig. 1. e, f display elastic neutron scattering of these SDW peaks at 2.8 K with the 45 K data subtracted as a background. The solid black lines fit the data based on the two models with details in the text. The dashed curve shows the component from each peak with the center at the vertical dotted line. The trajectory of each scan is denoted in b and d. Error bars correspond to  $\pm \sigma$ , where  $\sigma$  is the standard deviation.

sample scattering geometry. As an alternate approach, we implement a strategy to distinguish these two models by going to higher Brillouin zones within the same ( $HKO$ ) geometry. As shown in Fig. 2d near the (0.5, 1.5, 0) position, since the wavevector  $\mathbf{Q}$  no longer coincides with the spin directions, peaks from four domains should all contribute to the neutron scattering. However, if the 3D stacking model is correct, then peaks from the orange and green domains would be forbidden as drawn in Fig. 2b. Figure 2e demonstrates that the measured peak profiles clearly deviate from the expectations for the 3D stacking model. Therefore, the second model with spins aligned along the orthorhombic  $\mathbf{b}_0$ -axis should be preferred. Indeed, the fits based on this “spin  $\parallel$   $\mathbf{b}_0$  model” gives significantly improved agreement, as shown in Fig. 2f. Moreover, by co-fitting with the SDW peaks near (0.5, 0.5, 0), we can further quantify the interlayer correlation length and the volume fractions of stripe ordered phases (see Supplementary Note 2). Using a finite-size domain analysis<sup>36</sup>, the interlayer correlation length is determined to be 7(1) Å, corresponding to a slight modulation of intensities by (20  $\pm$  15)% along  $L$ .

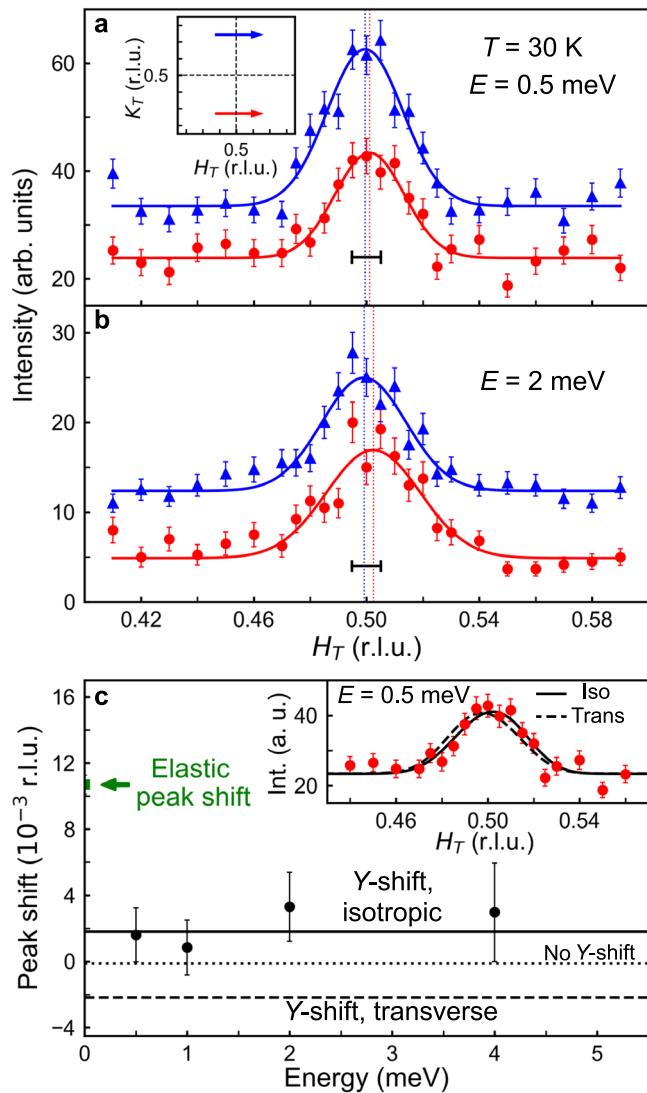
The intensities of the fits in Fig. 2f (colored lines) indicate the static SDW does not occur homogeneously throughout the sample. Rather, only a partial fraction of the sample contains static SDW, where the red and blue structural domains contain 2.9(6) times more of the static SDW phase volume fraction compared to the other two domains, as evidenced in Supplementary Fig. 3. By normalizing the SDW peak intensities with the nuclear Bragg peaks, the average ordered moment at 6 K is deduced to be at most 0.07(1)  $\mu_B$ , close to the previous reported values in LSCO<sup>37,38</sup> but only about half of that in oxygen-doped LCO<sup>7</sup>. Assuming a local moment value of 0.36  $\mu_B$  from a previous  $\mu$ SR report<sup>39</sup>, this suggests the magnetic volume fraction is less than  $\sim$ 5%. This value is lower than the ordered fraction of 18% from the  $\mu$ SR study<sup>39</sup>, possibly due to sample variation and differing

sensitivities of the probes.  $\mu$ SR is a more localized probe, whereas neutron scattering detects long-range ordered spins. The implications of the minority static SDW phase are further discussed in the Discussion Section.

#### Determination of tilting for the fluctuating spin stripes

Armed with the detailed knowledge learned from elastic neutron scattering (such as the structural domain population, magnetic structure, Y-shift angle in the static SDW phase, etc.), we next turn to high-resolution inelastic scattering to study the low-energy excitations of the spin stripes. Figure 3a–b shows scans around (0.5, 0.5, 0) at two representative energy transfers at 30 K, which is close to the onset temperature of the SDW order  $T_{\text{sdw}}$ <sup>37</sup> and has the highest intensities (evidenced in Fig. 4b). The energy dependence of the inelastic peak shift is plotted in Fig. 3c. Although the shifts here are much smaller than the elastic case, their consistently non-zero values indicate that the dynamic stripes also have a Y-shift. Note that the case assuming no Y-shift for the inelastic scattering is given by the dotted line near zero shift (this value includes a very small correction taking into account the structural domain populations). Our data show that the fluctuating stripes are tilted with the same tilt angle of  $\sim$ 3.0° as observed for the static spin stripes. That is, our analysis indicates that the inelastic peaks appearing at the same positions as their elastic counterparts in the low-energy limit, consistent with the Goldstone theorem. Recent findings that the Y-shift exists in the CDW order at  $\sim T_{\text{sdw}}$  in LSCO<sup>11,15,16</sup> with the shift angle similar to low temperatures are consistent with this notion.

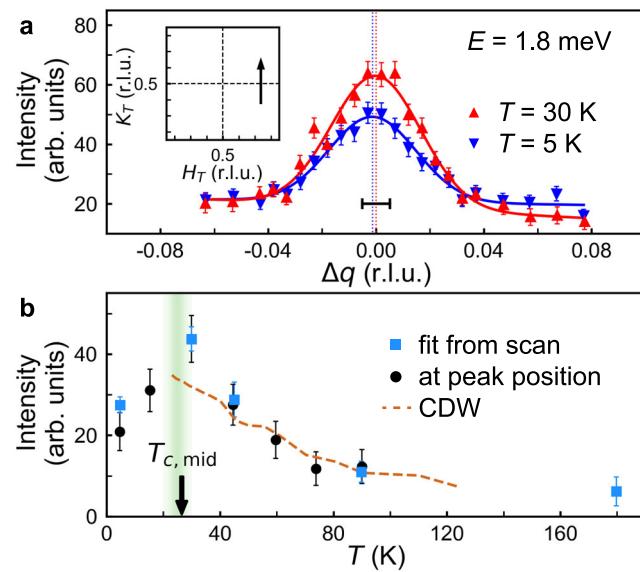
The reason that the apparent Y-shift is much smaller than that in the elastic scattering is simply an averaging effect of the structural twin domains. As seen in the analysis of the elastic magnetic cross section, the inelastic scattering intensities are also affected by the neutron polarization factor and interlayer correlations. Specifically, for the inelastic peaks around (0.5, 0.5, 0),



**Fig. 3 | Energy dependence of the dynamic spin stripes.** **a, b** Inelastic neutron scattering of the spin stripes at 30 K for an energy transfer of  $E = 0.5$  meV (**a**) and  $E = 2$  meV (**b**). The solid lines are Gaussian fits with a constant background and the vertical dotted lines are the fitted peak centers. The trajectory of each scan is denoted in the inset of **a**. The horizontal bars represent the elastic peak shift. The blue data are shifted vertically for clarity. **c** Energy dependence of the peak shift between the fitted centers of a pair of peaks as shown in **a, b** and Supplementary Fig. 5. Solid and dashed lines are the expected peak shift for isotropic and transverse spin fluctuations  $\Delta S$  with  $Y$ -shift, respectively, while the dotted line for isotropic spin fluctuations without  $Y$ -shift. Inset: the same red data in **a** with fits assuming isotropic (solid line) and transverse (dashed line) excitations with  $Y$ -shift. Error bars correspond to  $\pm\sigma$ , where  $\sigma$  is the standard deviation.

the interlayer correlations reduce the intensities of the red and blue domains by  $\sim 20\%$ . The neutron polarization factor has a bigger effect, resulting in essentially zero intensities for the longitudinal spin fluctuations  $\Delta S$  from the red and blue domains and essentially zero intensities for the in-plane transverse spin fluctuations of the orange and green domains. Here, longitudinal and transverse spin fluctuation directions are with respect to the static spin direction. With these constraints in mind, we calculate the weighted peak position for various cases.

We can first rule out the scenario in which the inelastic signal arises only from the magnetically ordered regions. In that case, the peaks have dominant contributions from the red and blue domains due to their larger stripe volume fraction and therefore shift to the opposite direction (see Supplementary Fig. 6). Hence, we can conclude that the inelastic scattering

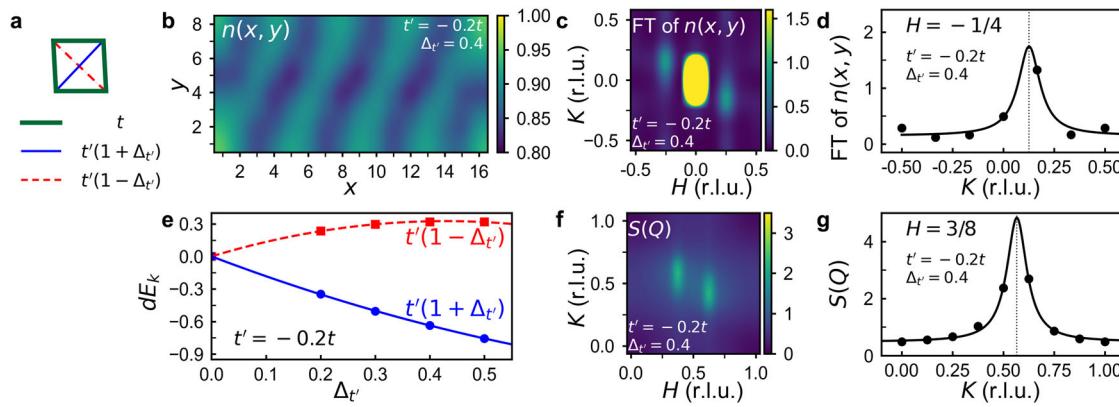


**Fig. 4 | Temperature dependence of the dynamic spin stripes.** **a** Representative scans at 5 K and 30 K for an energy transfer of  $E = 1.8$  meV. The scan trajectory is shown in the inset. The solid lines are Gaussian fits with a linear background and the vertical dotted lines are the fitted peak centers. The horizontal bar represents the elastic peak shift. **b** Integrated intensity extracted from the Gaussian fits (blue squares). The black circles are intensities from one-point measurement at the peak position and scaled to match the fitted integrated intensity at 30 K. The arrow indicates the midpoint temperature (26.5 K) of the superconducting transition  $T_{c,\text{mid}}$ <sup>8</sup>. The diffuse green vertical line indicates the SDW onset temperature varying from  $\sim 20$  K ( $\mu$ SR<sup>39</sup>) to  $\sim 30$  K (neutron<sup>37</sup>). The brown dashed line is the integrated intensity of CDW peak measured by resonant soft x-ray scattering<sup>15</sup> and scaled to match our data. Error bars correspond to  $\pm\sigma$ , where  $\sigma$  is the standard deviation.

likely originates from the whole sample rather than just the magnetically ordered regions. Then we test two extreme cases: purely isotropic excitations and purely transverse excitations. As shown in Fig. 3c and Supplementary Fig. 8, the isotropic model clearly describes the data better. If we adopt this isotropic model, the fitted intrinsic tilt angle is  $3.1(9)^\circ$ , which, within the errors, is the same as that for the static spin stripes ( $3.0(2)^\circ$ ). The observed small positive shift is primarily caused by the slight decrease of the intensities from the red and blue domains due to the finite interlayer correlations. The observed difference in the peak positions for the elastic and inelastic scattering is simply a domain-averaging effect, and phase separation explains the existence of the isotropic spin excitations (see further discussions in the Discussion Section).

We also investigate the temperature dependence of the inelastic scattering (see Fig. 4 and Supplementary Fig. 9). Even at 5 K, which is well below  $T_{\text{sdw}}$ , no obvious change of the peak position is observed relative to the 30 K data. Therefore, we can extend our previous conclusion of isotropic excitations to the low-temperature regime. This interpretation is consistent with recent results in oxygen-doped LCO<sup>40</sup>. Another type of phase separation scenario has been suggested in LSCO emphasizing a dip in the excitation spectrum  $\sim 4$  meV below  $T_c$  as a hidden-spin gap for the superconducting regions<sup>41</sup>. We do not observe a shift of the inelastic peak positions for energy transfers below 4 meV upon cooling, as shown in Fig. 4a. This could be due to a distribution of gap sizes and the energy we choose here (1.8 meV) is still above the spin gap in the majority of the sample. Energy-dependent measurements in the ordered state would be crucial to investigate this hidden-spin-gap hypothesis.

We also note that in Fig. 4b the intensities of spin fluctuations above  $T_{\text{sdw}}$  closely follow the trend of the CDW intensities recently measured using resonant soft x-ray scattering<sup>15</sup>. The extracted dynamical spin correlation length ( $62(10)$  Å at  $E = 0.5$  meV, see Supplementary Note 3 and Supplementary Fig. 7) is close to the CDW correlation length<sup>11,15,16</sup>.



**Fig. 5 | Numerical simulations of the origin of the stripe tilting.** **a** The unit cell with orthorhombic distortion to illustrate the electron hopping terms used in the calculations. **b** The electron density distribution  $n(x, y)$  calculated for  $t' = -0.2t$  and  $\Delta_{t'} = 0.4$ . **c** The Fourier transform of the electron density distribution in **b**. **d**  $K$ -cut of the charge ordering peaks at  $H = -0.25$  in **c**. **e** The kinetic energy differences as defined in the main text for  $t' = -0.2t$ . The lines are quadratic fits to the data. **f** The static spin structure factor  $S(Q)$  calculated for  $t' = -0.2t$  and  $\Delta_{t'} = 0.4$ . **g**  $K$ -cut of

the spin ordering peaks in  $S(Q)$  at  $H = 0.375$ . The maps in **b**, **c**, **f** are produced using multiquadric interpolation method<sup>69</sup>. The solid lines in **d**, **g** are Lorentzian fits with a constant background to determine the peak center (denoted by the vertical dotted lines). The Lorentzian peak widths for charge ordering and spin ordering peaks in the fitting are fixed to the best overall value based on data with all different  $\Delta_{t'}$ , respectively.

### Numerical calculations: understanding the origin of the tilting

The experimental observation that the same tilting arrangement is inherent to both static and dynamic stripes calls for a microscopic explanation. Here, we study the spatially anisotropic  $t$ - $t'$  Hubbard model on the square lattice using the DMRG method<sup>33</sup>, which is defined by the Hamiltonian

$$H = - \sum_{ij\sigma} t_{ij} \left( \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + h.c. \right) + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}. \quad (1)$$

This is one of the simplest models that respect the orthorhombic symmetry in real materials. Here  $\hat{c}_{i\sigma}^\dagger$  ( $\hat{c}_{i\sigma}$ ) is the electron creation (annihilation) operator on-site  $i = (x_i, y_i)$  with spin- $\sigma$ , and  $\hat{n}_{i\sigma}$  is the electron number operator. The electron hopping amplitude  $t_{ij} = t$  if  $i$  and  $j$  are nearest neighbors and  $t_{ij} = t'(1 \pm \Delta_{t'})$  for next-nearest neighbors where  $\Delta_{t'}$  is the spatial anisotropy related to the orthorhombic distortion as shown in Fig. 5a.  $U$  is the on-site Coulomb repulsion. We set  $t = 1$  as an energy unit with interaction  $U = 12t$  and report results for  $t' = -0.3t \sim -0.1t$ , which corresponds to the typical values reported in the literature for LSCO<sup>42–44</sup> with doping near  $\delta = 12.5\%$ , the hole concentration used in our study. The main results are shown in Figs. 5, 6 and the details are given below. Additional details can be found in Supplementary Note 4 and Supplementary Figs. 10–20.

We first calculate the electron density distribution  $n(x, y) = \langle \hat{n}(x, y) \rangle$ , where an example is shown in Fig. 5b for  $t' = -0.2t$  and  $\Delta_{t'} = 0.4$ . In the absence of spatial anisotropy, i.e.,  $\Delta_{t'} = 0$ , we find the charge stripe (parallel to the  $y$  direction) of wavelength  $\lambda_c \approx 4$  (see Supplementary Fig. 10a), consistent with the half-filled charge stripes observed in previous studies<sup>18,19,21,22</sup>. In our simulations, the charge stripe being bond-centered is due to the even number of sites along the  $x$  direction in the calculation. Our results show that the charge stripe is tilted when  $\Delta_{t'} > 0$  as shown in Fig. 5b–d, consistent with the experimental observations. This tilting reduces the kinetic energy of the charge stripe<sup>45,46</sup>. To support this, we further calculate the kinetic energy for both  $t'(1 \pm \Delta_{t'})$  bonds as  $E_k(\pm \Delta_{t'}) = -t'(1 \pm \Delta_{t'}) \sum_{ij \in (1 \pm \Delta_{t'})-\text{bonds}} \langle \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} \rangle$ . The kinetic energy difference  $dE_k(\pm \Delta_{t'}) = E_k(\pm \Delta_{t'}) - E_k(\Delta_{t'} = 0)$  is shown in Fig. 5e. While the kinetic energy along the  $(1 - \Delta_{t'})$ -bonds is increased, i.e.,  $dE_k(-\Delta_{t'}) > 0$ , the kinetic energy along the  $(1 + \Delta_{t'})$ -bonds is reduced, so that the total kinetic energy  $dE_k = dE_k(\Delta_{t'}) + dE_k(-\Delta_{t'}) < 0$  when  $\Delta_{t'} > 0$ . We note that a finite value of  $\Delta_{t'} > 0.1$  is necessary to give observable tilting of the stripes due to limited sample size along  $y$  direction in the calculations, although the eight-leg ladder size in DMRG calculations with the presented accuracy is already unprecedented as far as we are aware.

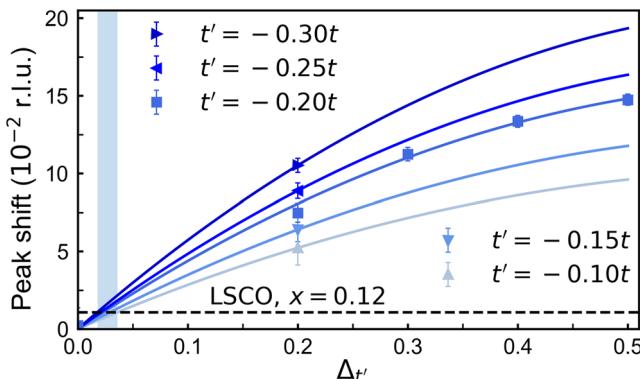
To describe the magnetic properties of the system, we have also calculated the static spin structure factor, which is defined as

$$S(Q) = \frac{1}{N} \sum_{i,j=1}^N e^{i\mathbf{Q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle. \quad (2)$$

Here the wavevector  $\mathbf{Q} = (k_x, k_y) = k_x \mathbf{b}_x + k_y \mathbf{b}_y$  in reciprocal space is defined by the reciprocal vectors  $\mathbf{b}_x = (2\pi, 0)$  and  $\mathbf{b}_y = (0, 2\pi)$ . Consistent with previous studies<sup>18,19,21,22</sup>, we find that the spin-spin correlations show a spatial oscillation of wavelength  $\lambda_s$  that is mutually commensurate with the CDW order, i.e.,  $\lambda_s \approx 2\lambda_c$ . As a result,  $S(Q)$  is peaked at the momentum  $(k_x, k_y)$  where  $k_x = 0.5 \pm 0.125$  and  $k_y = 0.5$ . Similar with the charge stripe, we find that the peak position of  $S(Q)$  is shifted with  $\Delta_{t'}$  as shown in Fig. 5f and g. The peak shift, i.e., the difference of peak positions along  $y$  direction for this pair of peaks in  $S(Q)$ , and its dependence on  $\Delta_{t'}$  is given in Fig. 6. We focus on  $t' = -0.2t$ , and find that the relationship between the peak shift and  $\Delta_{t'}$  can be well fitted by a quadratic function, which is commonly used in the analysis of DMRG data<sup>21,26</sup>. The use of the quadratic function passing origin without a threshold value in  $\Delta_{t'}$  is also consistent with the previous phenomenological study<sup>17</sup>, which showed that the tilting is proportional to the orthorhombicity to first order approximation.

Comparing our results with experimental observation allows us to estimate the spatial anisotropy of  $t'$  in the real material. If we interpolate the above empirical curve to the experimental observed value of the peak shift in our LSCO sample, the expected anisotropy is  $\Delta_{t'} = 2.3(1)\%$ . This estimation of  $\Delta_{t'}$  depends somewhat on the value of  $t'$ . For  $t' = -0.3t \sim -0.1t$ , if we assume the peak shift follows the same quadratic form, the estimated  $\Delta_{t'}$  can vary between 1.8%~3.6%, which is pretty close to the prediction ( $\Delta_{t'} \gtrsim 1.5\%$ ) from previous studies<sup>47,48</sup>. Note that  $\Delta_{t'}$  has not been determined experimentally yet, even with advanced spectroscopic techniques such as angle-resolved photoemission spectroscopy (ARPES). This could be due to its small effect on the band structure. Additionally, the presence of the structural twinning may further obscure the interpretation of such measurements.

We can estimate the energy scale of the tilted stripes. The difference between the hopping integral for the two diagonal directions is  $|2t' \Delta_{t'}|$ . Taking  $\Delta_{t'} = 0.023$ ,  $t' = -0.2t$ , and  $t = 0.43$  eV<sup>42,49</sup>, we have  $|2t' \Delta_{t'}| = 4.0$  meV  $\sim$  46 K. This is consistent with the temperature range where the  $y$ -shift can be clearly detected with neutron scattering. At higher temperatures, thermal broadening may obscure the small shift of the peak positions.



**Fig. 6 | The spin stripe tilting as a function of  $\Delta_{t'}$  for different  $t'$ .** The tilting is measured by the peak shift along K direction between the pair of peaks in  $S(Q)$  maps (shown in Fig. 5f and Supplementary Figs. 11 and 14). The solid line for  $t' = -0.2t$  is a quadratic fit to the data. Other lines are the same quadratic curve as for  $t' = -0.2t$  but scaled to match the corresponding data. The horizontal dashed line indicates the elastic peak shift measured in our LSCO sample. The vertical bar represents the range of our estimated  $\Delta_{t'}$  determined by the intersection of the fitted curves and the dashed line. Error bars correspond to  $\pm\sigma$ , where  $\sigma$  is the standard deviation.

The above calculations are performed on a system size of  $N = 16 \times 8$ , where  $L_x = 16$  and  $L_y = 8$  are the number of sites in the  $\hat{x}$  and  $\hat{y}$  directions, respectively. The reason that we use eight-leg ladder system size is because this appears to be the minimal size required to produce half-filled stripes with ordinary  $d$ -wave symmetry compatible with experimental observations. For comparison, the ground state superconducting pairing symmetry is the plaquette  $d$ -wave on the hole doped side ( $t' < 0$ ) for the four-leg ladder system<sup>23</sup>, while half-filled charge stripes are forbidden for the six-leg ladder system if superconducting Cooper pairs are present<sup>50</sup>. To show that the boundary effects and finite-size effects have negligible impacts on our fitted peak shifts, we performed more numerical calculations with different system sizes ( $N = 12 \times 8$  and  $N = 8 \times 8$ ). The results indeed show good convergence between  $N = 12 \times 8$  and  $N = 16 \times 8$  system sizes, supporting the accuracy of our calculations. More details can be found in Supplementary Figs. 17–20. We note that our results are based on local charge and spin modulations instead of the long-range decaying behavior of the correlation functions and therefore should not be affected by the exact nature of the ground state being stripes or  $d$ -wave superconductivity in the model.

## Discussion

The primary conclusions from our combined experimental and numerical work can be summarized in three main points: (1) two types of phases coexist in our sample—a minority phase with static SDW and a majority phase with fluctuating spin stripes, (2) both phases have tilted stripes (Y-shift) with the same degree of tilting, and (3) the origin of the tilting can be explained by a small anisotropy in the hopping interaction  $t'$ . We elaborate on the implications of these results below.

Our elastic and inelastic neutron scattering results are naturally explained by the presence of two distinct phases (one with static spin stripes and one with fluctuating spin stripes). As discussed in the elastic scattering results, we estimate that only a small fraction (~5%) of the sample is magnetically ordered. The observation of a partially magnetically ordered phase in LSCO has been previously reported by  $\mu$ SR studies<sup>39,51</sup>. This is further supported by recent x-ray scattering studies that two types of CDW orders coexist in LSCO, where the longer-range ordered CDW takes place in a minority fraction of the sample<sup>15,16</sup>. This suggests that the phase with static SDW has a type of CDW order which is distinguished by having longer-range correlations.

The low symmetry of the Y-shift positions combined with high-resolution measurements enabled us to determine the spin direction and interlayer correlations. So far as we know, this is the first time that the spin direction is explicitly determined to be along the orthorhombic  $\mathbf{b}_0$  direction

in LSCO. The rather short interlayer correlation length of  $\sim 7$  Å is consistent with the previous reports in both SDW and CDW studies in LSCO<sup>12,52–54</sup>, affirming the 2D nature of the stripe order. Such short interlayer correlations could be explained by stacking faults between layers (see Supplementary Fig. 4). The spin direction and local interlayer correlations are similar to those in the parent compound  $\text{La}_2\text{CuO}_4$ . This reiterates that doped anti-ferromagnets are essential to the description of high- $T_c$  cuprates<sup>55</sup>.

The inelastic scattering measurements of the isotropic nature of the spin fluctuations requires the presence of an additional phase. Since spin-wave excitations can only contain transverse fluctuations below  $T_{\text{sdw}}$ , the measured isotropic excitations indicates that there is a majority phase that is not magnetically ordered, characterized with isotropic spin excitations. In contrast, the minority phase with static SDW order should have only transverse spin excitations. Therefore, the peak shift in Fig. 3c represents the average of the isotropic and transverse contributions, weighted by their respective volume fractions, which is dominated by the non-magnetically ordered phase. This majority phase should also be responsible for the superconductivity in the sample, which is a bulk superconductor. The minority phase may not be superconducting due to competition with the static SDW.

As discussed in the inelastic scattering results, the intensities and widths of the fluctuating spin stripe peaks has a close correlation to the CDW peaks observed with x-rays. These intimate relationships suggest that the two signals detected by different tools probably originate from the same majority phase. This is consistent with the recent NMR finding that increasing charge ordered domains trigger the glassy freezing of Cu spins below  $T_{\text{cdw}}$ <sup>56</sup>.

We find that both phases contain tilted stripe correlations with the same tilt angle of  $\sim 3.0^\circ$ . Our data and analysis indicate that the same Y-shift is an intrinsic property of both static and fluctuation phases. This contrasts with a previous study of oxygen-doped LCO where it was concluded that the spin fluctuations have a significantly different stripe tilt angle compared to the static SDW order<sup>14</sup>. Our numerical results which explain the Y-shift indicate that the static and fluctuating stripes should have the same tilting, as it depends only on the underlying orthorhombicity.

DMRG calculations provide a direct and unbiased way to understand the origin of the Y-shift phenomenon. The parameters in our state-of-the-art calculations on eight-leg ladders are consistent with the structure of real materials. The anisotropy of  $t'$  may arise from inequivalent  $\text{CuO}_6$  octahedron tilts and unequal lattice parameters between the orthorhombic  $a_0$  and  $b_0$  axes. Here, it is estimated to be  $\gtrsim 1.5\%$  in LSCO with  $x = 0.12$ <sup>17,48</sup>. Our DMRG prediction for the anisotropy  $\Delta_{t'} = 1.8\% \sim 3.6\%$  matches this expected value for LSCO very well, and much better than the mean field approach based on Fermi surface nesting<sup>47,48</sup>. Moreover, our results reveal the importance of kinetic energy considerations in stripe formation<sup>57</sup>. Microscopically, the anisotropy of  $t'$  causes the stripes to tilt towards the short  $a_0$  axis, the direction with larger hopping magnitude and gain in kinetic energy. This is an obvious manifestation of the role of kinetic energy term in stabilizing charge stripes by having delocalized holes along the stripes<sup>45,58</sup>. Our DMRG results also qualitatively agree with an early exact diagonalization study<sup>59</sup>, which found that an anisotropic  $t'$  in the extended  $t$ -J model results in a preferred orientation of stripes formed by doped holes.

The explanatory power of our results can further be tested by their applicability to similar orthorhombic cuprate materials. Indeed, the Y-shift has been reported in both CDW and SDW orders with various dopants in La-based cuprates<sup>12,15,16</sup>. Tilted SDW order was also found in LSCO at lower doping level of  $x = 0.07$ <sup>13</sup>, with a surprisingly large tilt angle. In the above cases, the larger tilt angle goes hand-in-hand with a larger structural orthorhombicity, as expected in our explanation based on anisotropic  $t'$ . A different class of stripes (i.e., diagonal stripes) have been observed in lightly-doped LSCO<sup>60–62</sup> and  $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$  (LBCO)<sup>63</sup> below  $x \sim 0.055$ , coincident with a superconductor-to-insulator transition. Since these diagonal stripes are aligned along the shorter  $a_0$  direction, this could be regarded as the extreme case of our tilted stripe explanation. Such unidirectional diagonal stripes are not limited to cuprates—they were also found in the insulating

nickelates<sup>64</sup>, implying the possible generality of this mechanism for the selection of the stripe direction in systems with similar orthorhombic symmetry.

Uniaxial strain engineering offers a powerful tuning knob to artificially modify the hopping strengths along specific directions. We note that a recent x-ray scattering study<sup>65</sup> observed a decrease in stripe tilt angle in LSCO under uniaxial pressure applied along one of the tetragonal directions. This agrees with our kinetic energy argument that the stripe direction should be sensitive to the anisotropy of the hopping terms. Such uniaxial pressure can result in an anisotropy in  $t$ . In another recent elastic neutron scattering experiment<sup>66</sup>, uniaxial pressure was used to select a single magnetic domain in LSCO with the stripe direction aligning along the larger  $t$  direction. LBCO has intrinsic anisotropy in  $t$  due to octahedron rotation along the Cu-Cu bond directions. A recent uniaxial strain experiment in LBCO reports surprisingly different behaviors in elastic and inelastic channels in terms of incommensurability<sup>67</sup>. In the future, it would be appealing to investigate numerically how the tilt angle would change with both anisotropic  $t$  and  $t'$  terms in the Hamiltonian.

These results show an excellent match between DMRG calculations for the doped Hubbard model and the specific ground state details observed in  $\text{La}_{1.88}\text{Sr}_{0.12}\text{CuO}_4$ —attesting to the appropriateness of the model for the cuprates. Our results further stress the importance of  $t'$  in the Hamiltonian. The presence of  $t'$  is known to stabilize half-filled stripes and superconductivity, which are both present in our LSCO sample<sup>19,21,22</sup>. The specific alignment direction of the stripes is so sensitive to  $t'$  where even a small anisotropy in  $t'$  can result in the subtle observed tilting. This highlights how the phases of stripes and superconductivity are sensitively intertwined at the level of model calculations and accounts for the appearance of these phases in a real material. Of course, the phenomenology of density wave ordering is different in LSCO compared to YBCO. This calls for a better understanding of how small changes in the Hamiltonian parameters, perhaps arising from subtle structural differences, can account for variations in the competing states across the families of cuprate materials.

## Methods

### Sample details

The single crystal of LSCO with a nominal doping of  $x = 0.12$  was grown by the traveling-solvent-floating-zone method. As-grown crystal was annealed in oxygen gas flow at 900 °C for 72 h. At room temperature, the lattice constants are  $a = b = 3.780 \text{ \AA}$  and  $c = 13.218 \text{ \AA}$  determined by powder x-ray diffraction on a crushed piece of the single crystal. The structural transition temperature from the high-temperature tetragonal phase to the low-temperature orthorhombic phase is  $T_s = 242 \text{ K}$  determined by extinction of the nuclear Bragg peak (2, 0, 0) in neutron diffraction measurements upon warming. Both the lattice constants and the structural transition temperature  $T_s$  agree pretty well with the previous reported results from powder and single crystal samples<sup>8,32,68</sup>, reassuring the Sr content. To avoid the large mosaic that could be introduced from the coalignment of multiple samples, only one large single crystal with a mass of 12.8 g was used in our neutron scattering experiments. The size of the crystal is 8 mm  $\phi \times 40 \text{ mm}$ .

### Neutron measurements

The neutron experiments were carried out at the thermal-neutron beamline HB-1A at the high flux isotope reactor (HFIR) at Oak Ridge National Laboratory and the cold-neutron beamline at the Spin Polarized Inelastic Neutron Spectrometer (SPINS) at the Center for Neutron Research (NCNR) at the National Institute of Standards and Technology (NIST). For HB-1A, the incident energy was fixed at 14.65 meV with horizontal collimations of 40°-40°-sample-40°-80°. For SPINS, the final energy was fixed at 5 meV, and most of measurements were done with horizontal collimations of guide-open-sample-80°-open, except for the mesh-scan near nuclear Bragg peak (1, 1, 0) (see Supplementary Fig. 1), which was with tighter horizontal collimations of guide-20°-sample-20°-open. The sample was mounted on an aluminum holder and aligned in the (HK0) plane. Base temperatures of  $T = 5 \text{ K}$  (HB-1A) and  $T = 2.8 \text{ K}$  (SPINS) were achieved

using closed-cycle refrigerators. The sample was cooled slowly with a rate of  $\sim 2 \text{ K}$  per minute from room temperature.

Both the elastic scattering (Fig. 1) and temperature dependence of the inelastic scattering near (0.5, 0.5, 0) (Fig. 4) were measured at HB-1A. The elastic scattering near (1, 1, 0) (Supplementary Fig. 1) and (0.5, 1.5, 0) (Fig. 2) and energy dependence of the inelastic scattering near (0.5, 0.5, 0) (Fig. 3) were measured at SPINS. We note that the sample was rotated by 90° in-plane between the HB-1A and SPINS experiments, so the same structural domains between the two experiments were related by this 90° rotation. For example, the green domain measured at SPINS corresponds to the red one at HB-1A. This has been taken into account in the analysis of HB-1A data when domain population information is needed (including Supplementary Fig. 2 and the moment size calculation).

### Numerical calculations

We employed the DMRG<sup>33</sup> method to study the ground state properties of the  $t$ - $t'$  Hubbard model as defined in Eq. (1). We considered the square lattice with open boundary conditions in both directions specified by the basis vectors  $\hat{x} = (1, 0)$  and  $\hat{y} = (0, 1)$ . The total number of sites is  $N = 16 \times 8$ , where  $L_x = 16$  and  $L_y = 8$  are the number of sites in  $\hat{x}$  and  $\hat{y}$  directions, respectively. The doped hole concentration is defined as  $\delta = (N - N_e)/N$  with  $N_e$  the number of electrons. Additional calculations were performed with smaller sample sizes ( $N = 12 \times 8$  and  $N = 8 \times 8$ ) at the same hole concentration. We performed around 60 sweeps in the current DMRG simulation and kept up to  $m = 25,000$  number of states with a typical truncation error  $\epsilon \approx 2 \times 10^{-5}$ .

### Data availability

All data needed to evaluate the findings in this paper are present in the paper and/or the Supplementary Information, and are available from the corresponding authors upon reasonable request.

### Code availability

The codes implementing the calculations of this study are available from the corresponding authors upon reasonable request.

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## Author contributions

The project was conceived by Y.S.L. W.H., J.W., G.X., and W.T. performed the neutron scattering measurements, and W.H. analyzed the data. T.T., Y.I., and M.F. synthesized the sample. H.C.J. performed the numerical calculations. The paper was written by W.H., H.C.J., and Y.S.L. with input from all co-authors.

## Competing interests

The authors declare no competing interests.

## Additional information

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