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Wave Propagation Through a Random Medium in the Short-Wavelength Limit: A Scintillation Model Without Phase Screens

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Overview

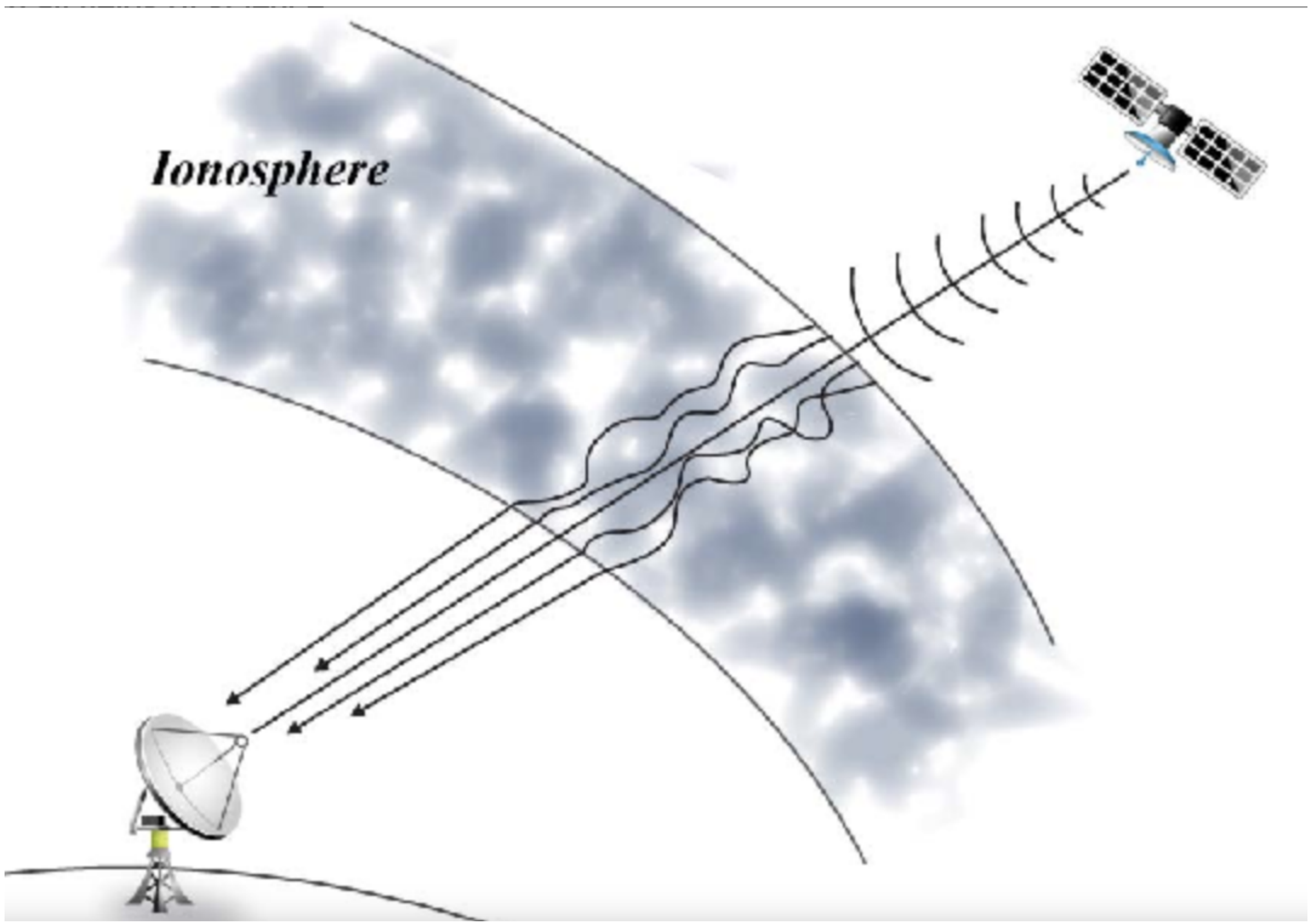
- How does the ionosphere impact radio propagation?
- Why develop a new method to predict scintillation when established ones exist?
- How does the model work?
- What do results from the model look like?
- How does it compare with established results?
- How does it fit in with the bigger picture of ionospheric science?

Background

- The ionosphere is the region of the upper atmosphere that is partially ionized by the solar wind and solar UV radiation
- The ions and electrons in the ionosphere interact strongly with electric and magnetic fields, and thus affect propagation of EM waves
- The ionospheric plasma reflects radio waves below ~20 MHz (the plasma frequency of the quiescent ionosphere).
- EM waves above the plasma frequency are dispersed, diffracted, refracted and absorbed; higher frequencies are less strongly affected
- One phenomenon associated with refraction and diffraction from dynamic density irregularities in the ionosphere is *scintillation*
- Scintillation degrades signals, and this is often predicted through the use of *phase screens*
- We describe an alternative to the phase screen method with particular applicability for extremely high frequency waves

Scintillation overview

- A plane wave entering a region of time-dependent inhomogeneous density perturbations will follow many different ray paths to reach a receiver. This leads to a randomly fluctuating amplitude and phase at the receiver.
- The statistics of the fluctuations of the received signal are determined by the statistics of the turbulence.

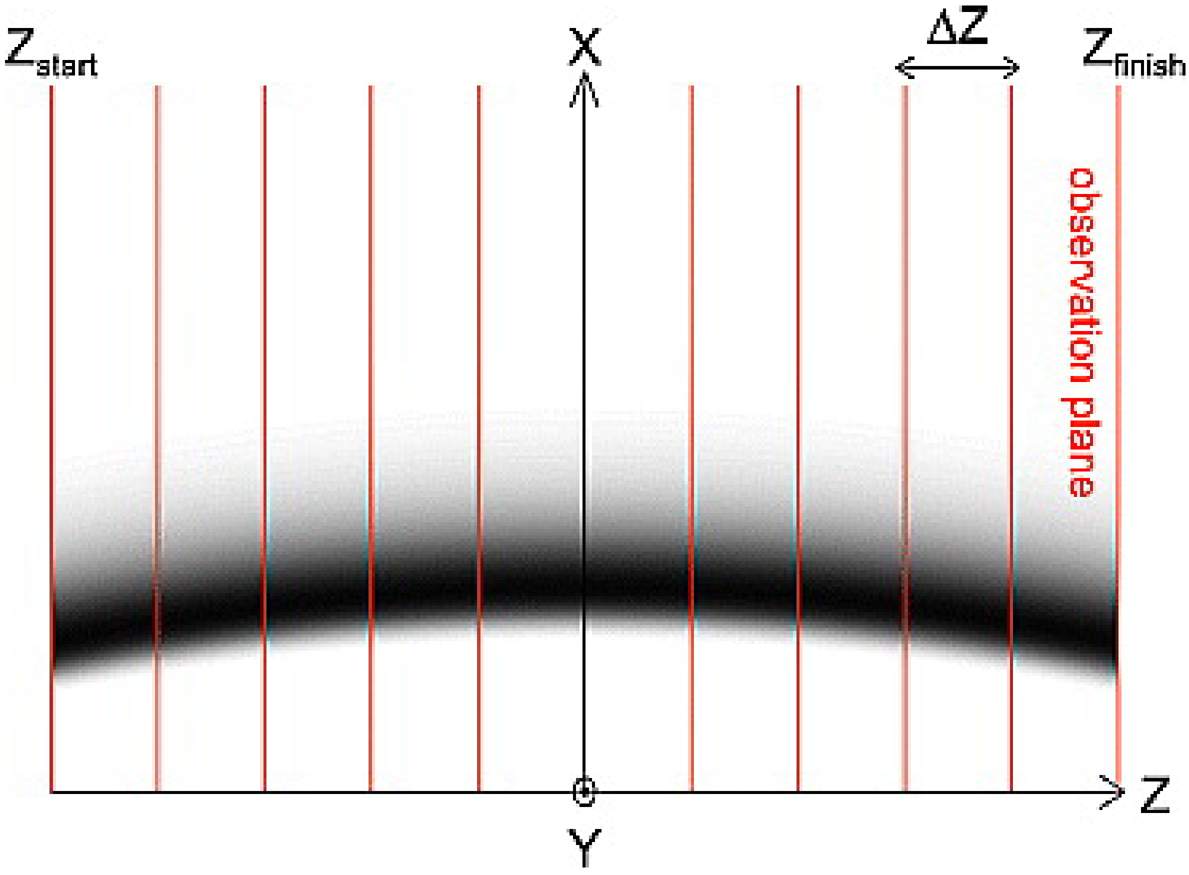


W. Sward, T. Swanson and M.D. Williams 2014 IEEE Military Communications Conference

Solving for scintillation: phase screens

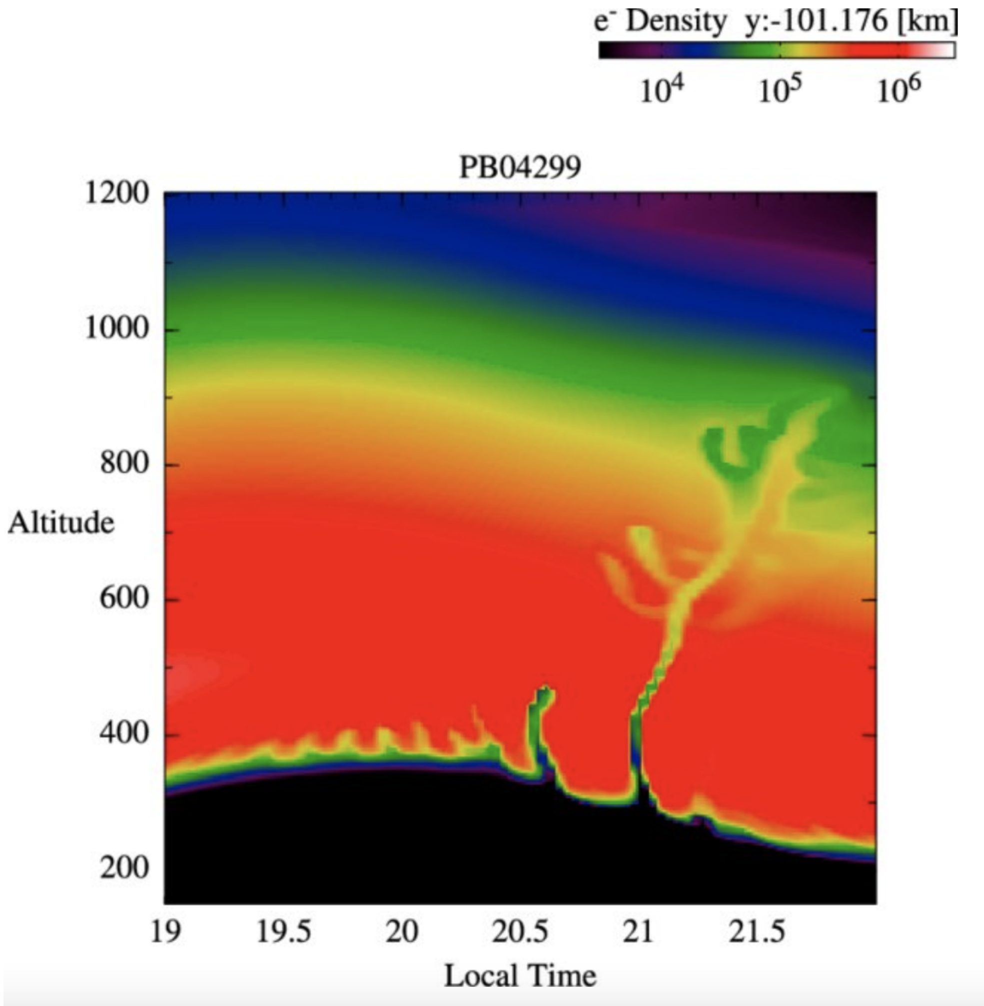
- A inhomogeneous medium with constant statistical properties can be reduced to a 2-D screen that alters the phase of an EM wave that penetrates it. The phase fluctuations are determined by the properties of the slab, including statistics of the turbulence.

- A realistic medium with varying turbulence properties can be approximated by a discrete number of such phase screens. This approximation is called the multiple phase screen approach (MPA).



When is the MPS method insufficient?

- Complex 3-D turbulence may have more structure than can be captured with a series of phase screens, or the number of phase screens can grow too big to be computationally tractable
- Natural and manmade *disturbances* can perturb the state of the ionosphere, leading to increased complexity and turbulence, as well as increased electron density.
- The applicability of the MPS in the most complex feasible scenarios should be assessed.



Model for a naturally occurring ionospheric electron density “plume”.
From: Forecasting low-latitude radio scintillation with 3-D ionospheric plume models: 2. Scintillation calculation
J. Retterer 2010

Waves in the short-wavelength limit: the WKB Approximation

- Equation for a linearly polarized wave in a 1-D varying medium

$$\frac{d^2 E_y}{dz^2} + k^2 n^2 E_y = 0.$$

- Assume solution of the form (assumes n is slowly varying compared to wavelength):

$$E_y(z) = A\, e^{i\phi(z)}$$

- Ultimately a solution is found for the phase function phi such that

$$E_y(z) \simeq A\, n^{-1/2}(z) \exp\left(\pm i\, k \int^z n(z')\, dz'\right)$$

Integral solution for the 3D E-field in the short-wavelength limit

Two strong approximations available:
$0<n-1<<1$ & $\lambda<<L$ (n = index of refraction)

$$\nabla\times\boldsymbol{B} = \mu_0\,\mathbf{j}+\frac{1}{c^2}\frac{\partial\boldsymbol{E}}{\partial t}$$

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}$$

$$\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}-\nabla^2\mathbf{E} = -\left(\frac{1}{\varepsilon_0}\nabla\rho+\mu_0\frac{\partial\mathbf{J}}{\partial t}\right)$$

$$\frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}-\nabla^2\mathbf{B} = \mu_0\nabla\times\mathbf{J}\,.$$

Assume small gradients in **J** and ρ

3D integral solution for **E** relying on *dispersion relation*

- Dispersion relation determined by dielectric function. Zeroth order approximation: assume the form of dielectric function from homogenous plasma but with varying plasma parameters.
- A given electric field entering a medium will be modulated according to the dielectric function.

The index of refraction of a magnetized cold plasma

The Appleton-Hartree Equation

$$n^2 = \left(\frac{ck}{\omega}\right)^2$$

$$n^2 = 1 - \frac{X}{1 - iZ - \frac{\frac{1}{2}Y^2 \sin^2\theta}{1-X-iZ} \pm \frac{1}{1-X-iZ}\left(\frac{1}{4}Y^4 \sin^4\theta + Y^2 \cos^2\theta(1-X-iZ)^2\right)^{1/2}}$$

$X = \frac{\omega_0^2}{\omega^2}$ Plasma frequency (small!)

$Y = \frac{\omega_H}{\omega}$ Cyclotron frequency (very small!)

$Z = \frac{\nu}{\omega}$ Scattering frequency (very small!)

For very high frequencies we can safely neglect the magnetic field and collisional effects.
For simplicity, we take them to be zero for this work.

The scintillation index

- The scintillation index is a measure of the strength of scintillation in terms of fluctuations in the intensity of a wave at a point.

$$\sigma_I^2(\mathbf{r},L)=\frac{\langle I^2\rangle-\langle I\rangle^2}{\langle I\rangle^2}$$

- For the von Karman spectrum toy model the scintillation index can be written out

$$\sigma_I^2(\mathbf{r},L)=2.6056C_n^2k^2L\int_0^1\int_0^\infty \kappa\frac{\exp\left[-\kappa^2/\kappa_m^2\right]}{\left(\kappa^2+\kappa_0^2\right)^{11/6}}\left\{1-\cos\left[\frac{L\kappa^2}{k}\xi\right]\right\}d\kappa d\xi$$

- And simplified to

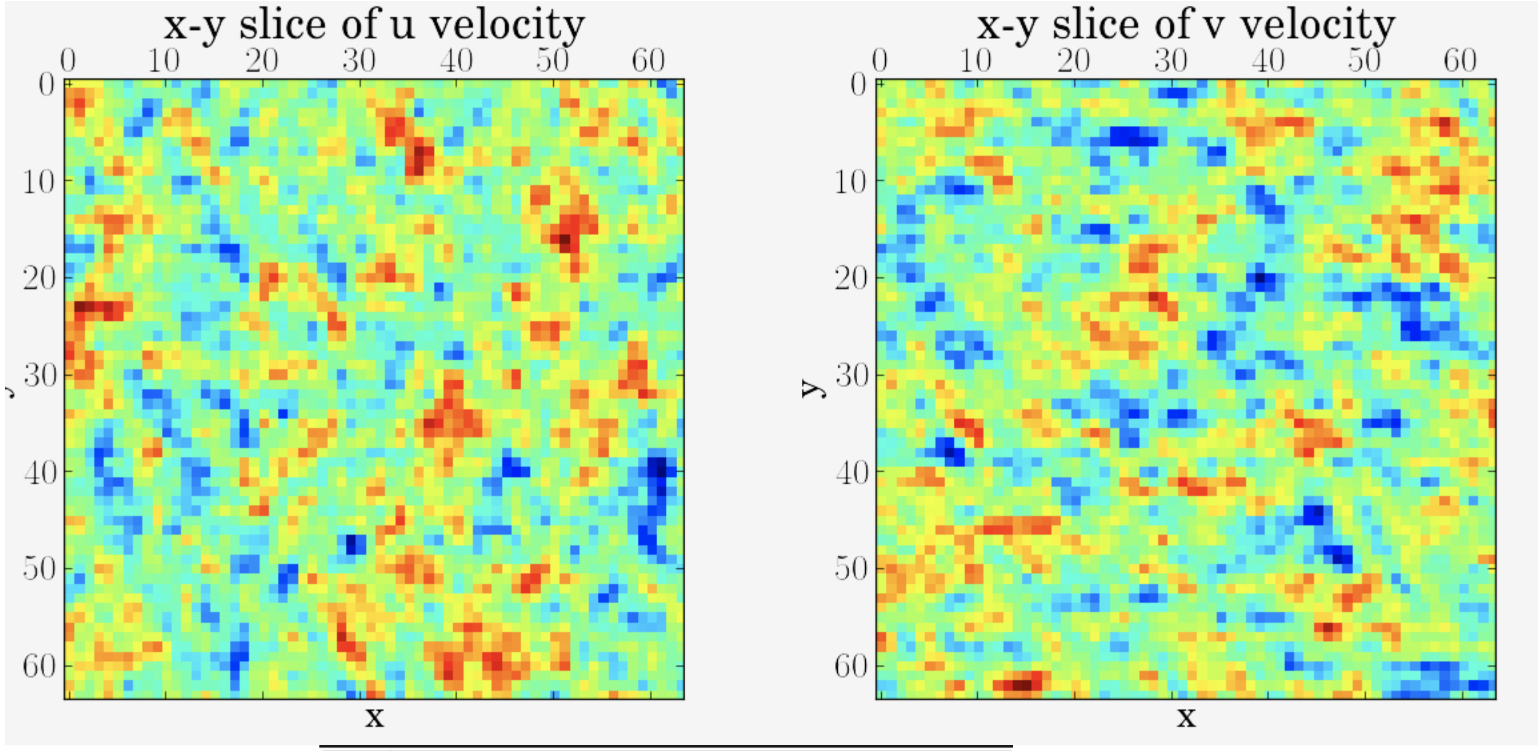
$$\sigma_I^2(\mathbf{r},L)=2.6056C_n^2k^2L\times.4715\left(\frac{L}{k}\right)^{5/6}$$

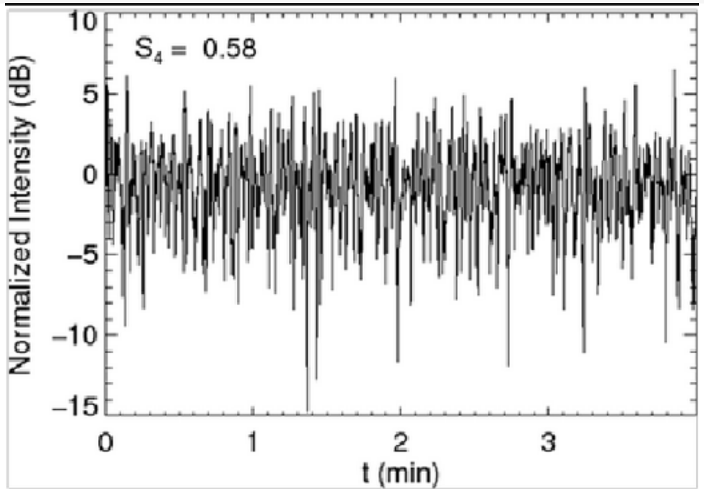
Toy problem for the new model: a turbulent slab

- Generate random fluctuations in a density field using von Karman spectrum

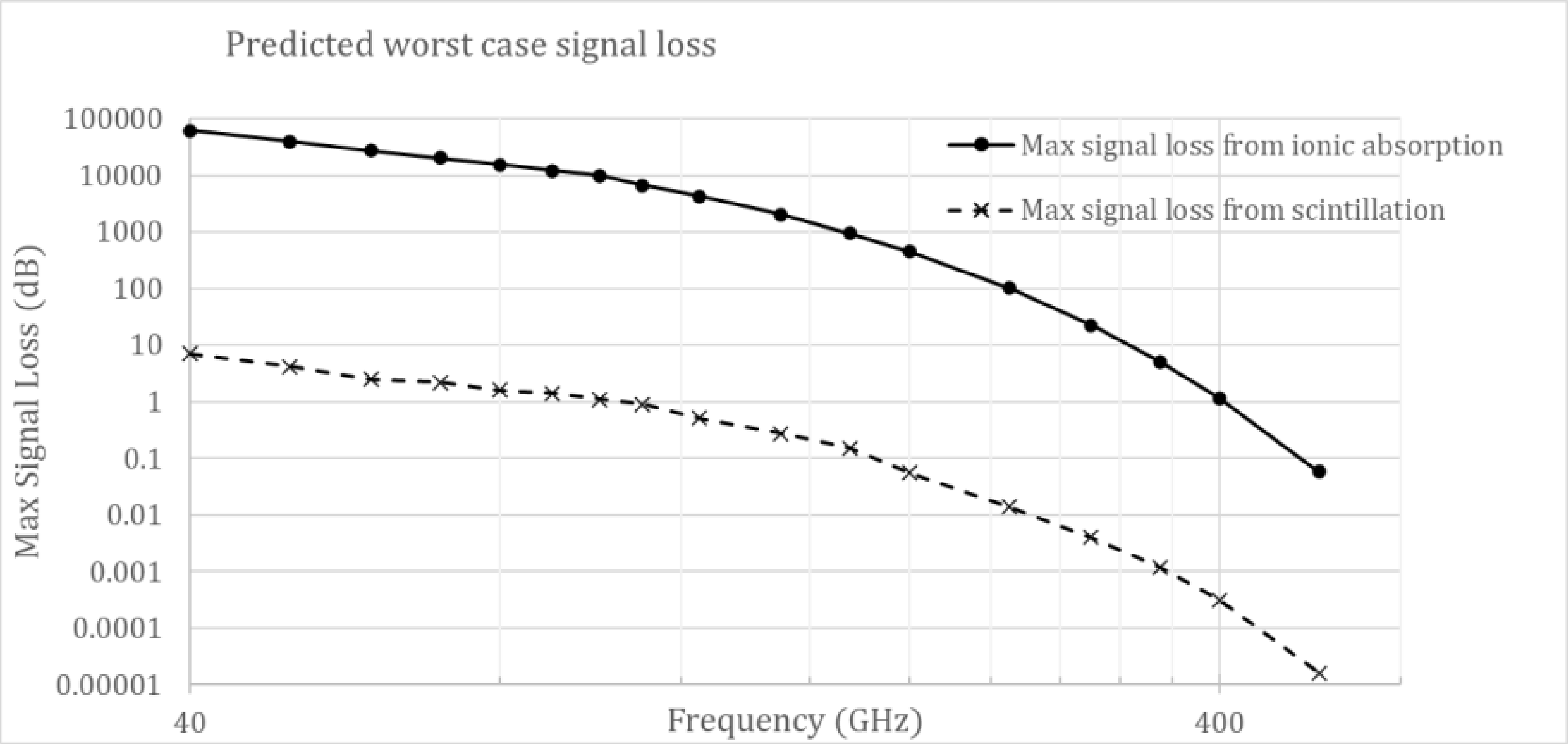
$$E(\kappa) = \alpha \frac{u'^2}{\kappa_e} \frac{(\kappa/\kappa_e)^4}{[1 + (\kappa/\kappa_e)]^{17/6}} \exp\left[-2\left(\frac{\kappa}{\kappa_\eta}\right)^2\right]$$

- Convert density field into index of refraction, assume incident plane wave on left side of slab, numerically integrate for E-field on right side of slab





Comparison to phase screens



Conclusions

- The ionosphere can have complex turbulent structure. The question of whether the method of phase screens can adequately describe all scenarios is open
- In order to assess the validity of phase screens in more complex cases, an alternative model that does not make approximations regarding the geometry of the ionospheric structure is needed
- Full finite difference time-domain techniques are too computationally expensive for most cases
- We presented a full-wave solution for the electric field that is valid in the limit of short wavelengths that can account for otherwise arbitrary turbulence statistics and plasma geometry
- The model needs further validation but seems to agree with phase screen methods in regimes where they would be expected to be valid