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Statistical Modeling of Mechanical Lifetime in Glass and Ceramic

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Statistical lifetime prediction for brittle materials

Brittle materials fail stochastically and interact with water for delayed failure or reduction in strength. How to combine these?

Probabilistic failure at short times

$$P_W = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^{\rho}\right]$$

Single crack propagation at long times

$$v\left(K_{\mathrm{I}}\right) = A\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{Ic}}}\right)^{n}$$

$$K_{\mathrm{I}} = Y(a)\sigma\sqrt{a}$$

$$t_f = \frac{2K_{\mathrm{Ic}}^2}{A\left(n-2\right)Y^2\sigma_f^2}\left[\left(\frac{\sigma_i}{\sigma_f}\right)^{n-2} - 1\right]$$

How do we predict probabilistic failure at long times? Solve the t_f equation for σ_f and assume that Weibull stress σ_0 evolves in the same way while shape ρ stays constant? Sometimes this is done, but can we do better?

Proposed method for probabilistic lifetime prediction

Solve the t_f equation for σ_i. This is the initial strength of a specimen that failed at time t_f and stress σ_f.

$$t_f = \frac{2K_{\mathrm{Ic}}^2}{A\,(n-2)\,Y^2\sigma_f^2}\left[\left(\frac{\sigma_i}{\sigma_f}\right)^{n-2} - 1\right] \longrightarrow \sigma_i = \left[\frac{A\,(n-2)\,Y^2\sigma_f^2 t_f}{2K_{\mathrm{Ic}}^2} + 1\right]^{\frac{1}{n-2}} \sigma_f$$

Since a Weibull distribution fit to strength data collected at beginning of life characterizes the un-aged flaw population we can substitute σ_i into the Weibull distribution as σ_0.

$$P_f\,(\sigma_i) = P_f\left[\sigma_i\,(\sigma_f, t_f; A, n, K_{\mathrm{Ic}}, Y)\right] = 1 - \exp\left(-\left\{\frac{\left[\frac{A(n-2)Y^2\sigma_f^2 t_f}{2K_{\mathrm{Ic}}^2} + 1\right]^{\frac{1}{n-2}}\sigma_f}{\sigma_0}\right\}^{\rho}\right)$$

If we neglect the "1" in the numerator we can get effective aged Weibull properties, note that ρ is not constant during aging

$$\tilde{\sigma_0} = \left[\frac{2K_{\mathrm{Ic}}^2}{A\,(n-2)\,Y^2 t_f}\right]^{\frac{1}{n}} \sigma_0^{1-\frac{2}{n}}$$

$$\tilde{\rho} = \left(\frac{n}{n-2}\right)\rho$$

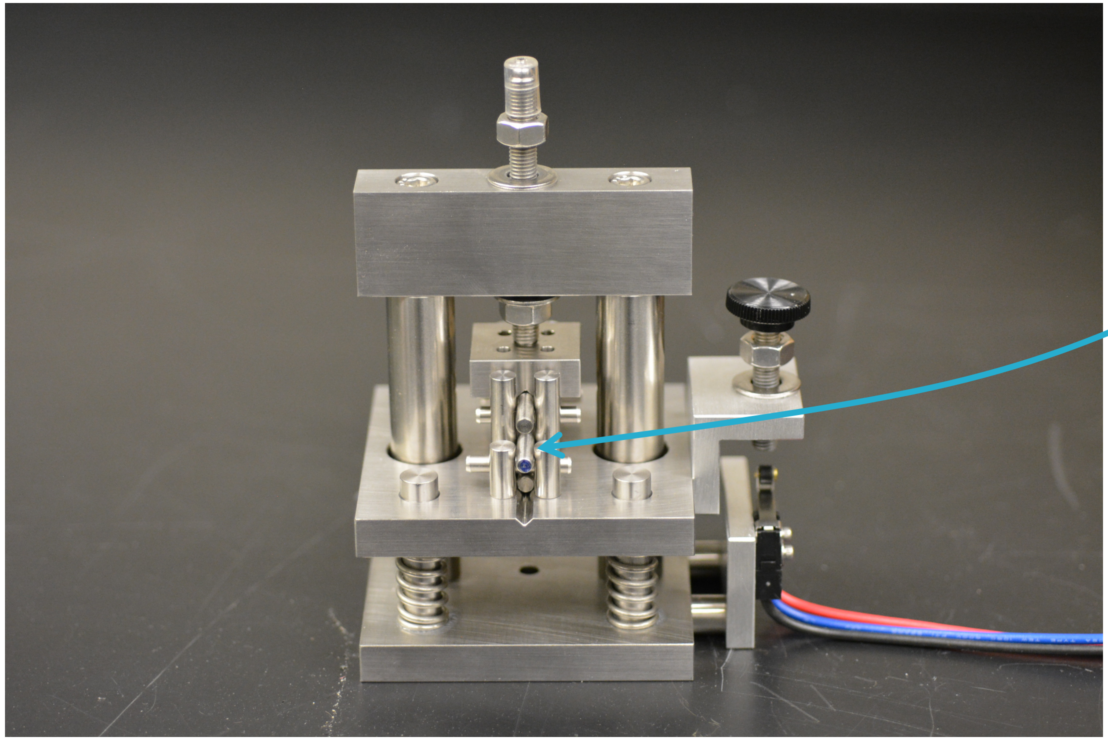
Validation experiment: static strength time to failure

The proposed method seems reasonable, but we don't know if flaws actually age in this way.

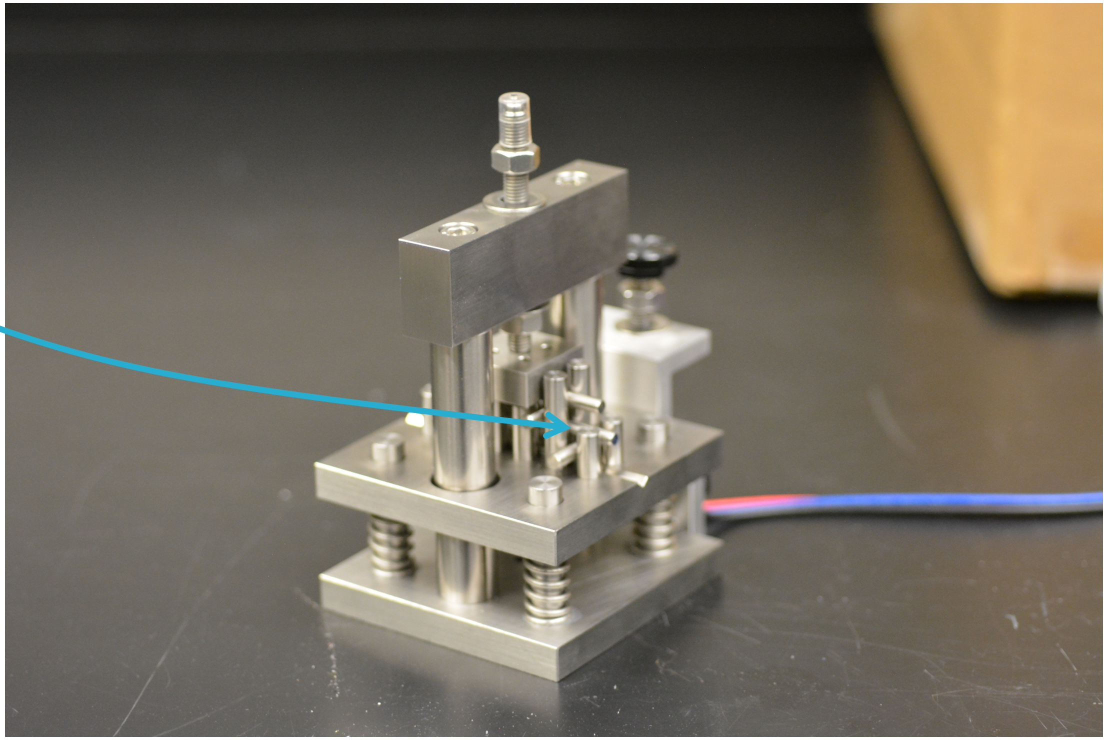
As a validation, small screw loaded four point bending load frames were designed and fabricated. The specimen is fully articulated with roller contacts.

Load is applied with dead load weights to compress the springs. A calibrated force cell is used to verify the load. Then the top nut is tightened and dead weight is removed.

An electronic switch connected to a timing circuit detects bend bar failure.



Bend bar goes here



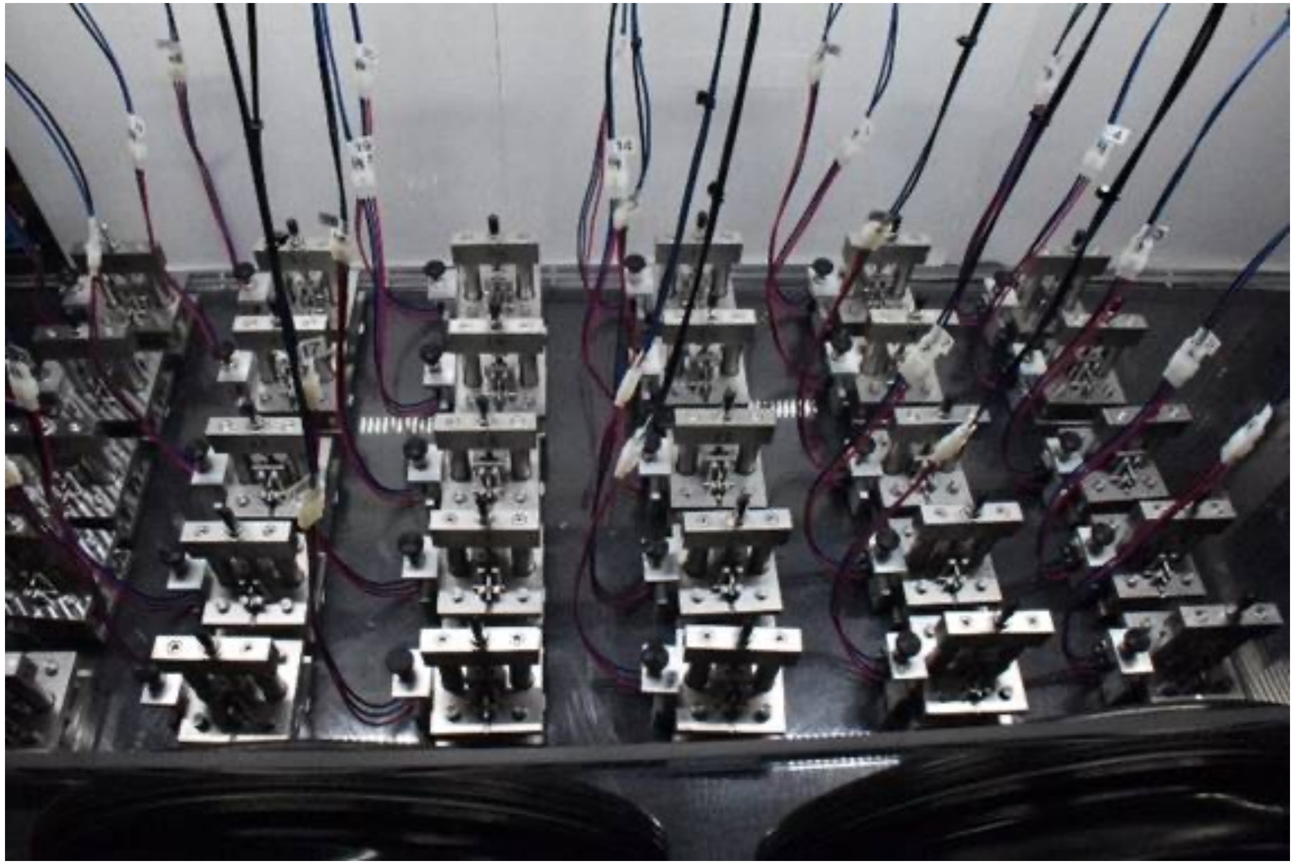
Validation experiment: static strength time to failure

30 static load frames were aged in controlled atmosphere at constant 95% relative humidity and room temperature

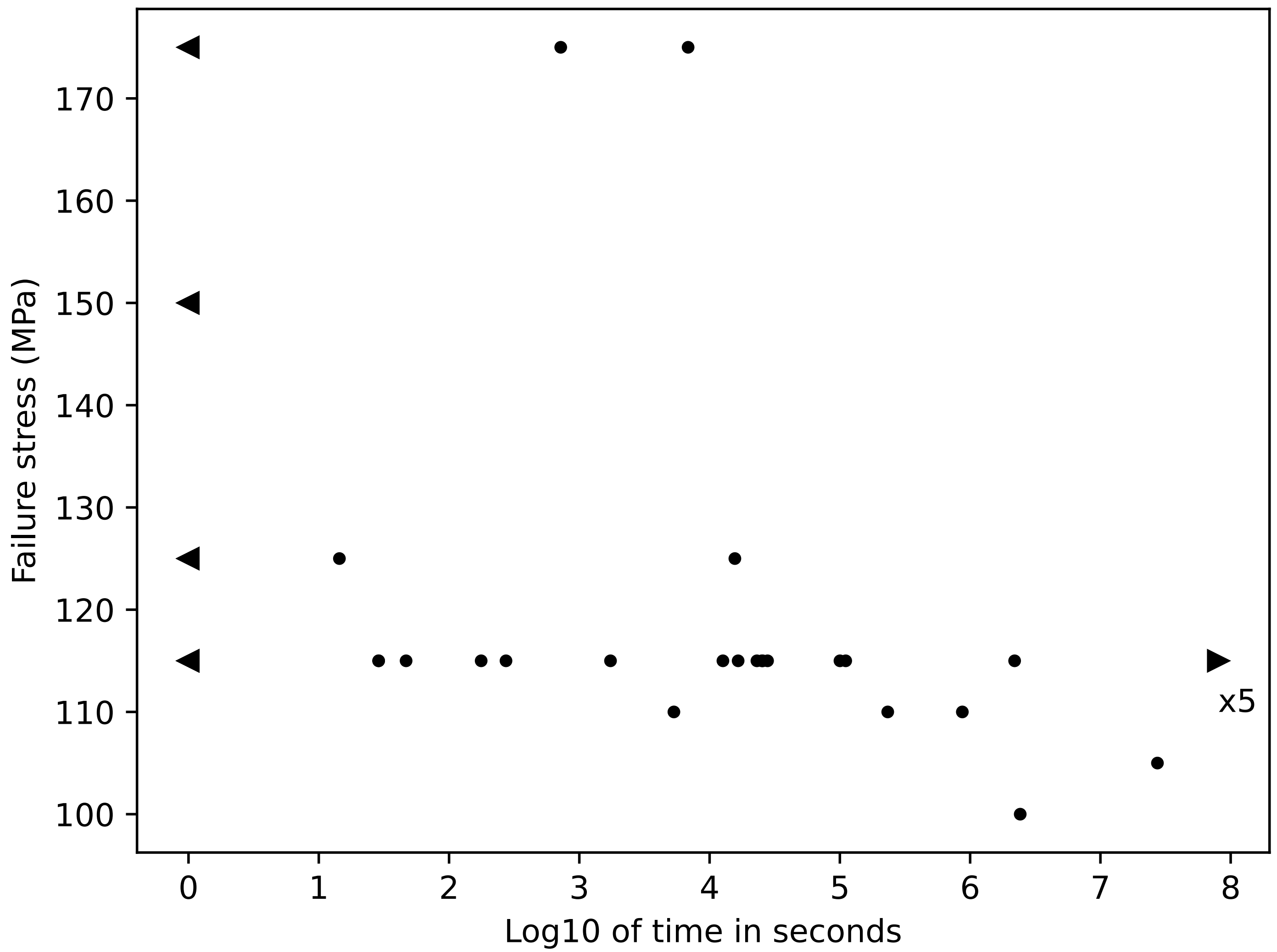
Failure times were measured at various load levels

ASTM size B bend bars were used in all cases

- 3 x 4 mm cross section
- 20 mm inner span
- 40 mm outer span
- 45 mm total length



Static strength time to failure results for barium titanate



Barium titanate preliminary characterization

Inert Weibull data collected with 4-point-bend tests at 2% RH at various stress rates, no visible tend in failure stress versus stress rate.

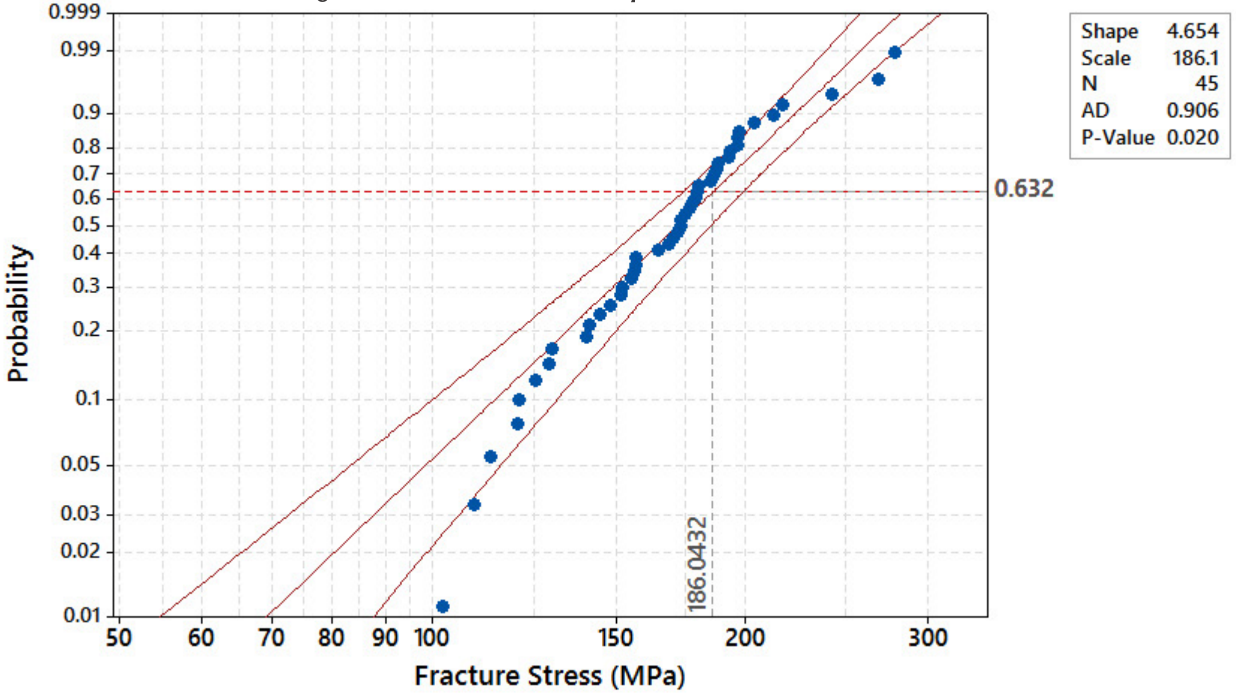
SCG parameters fit to indented stress rate strength tests at 95% RH

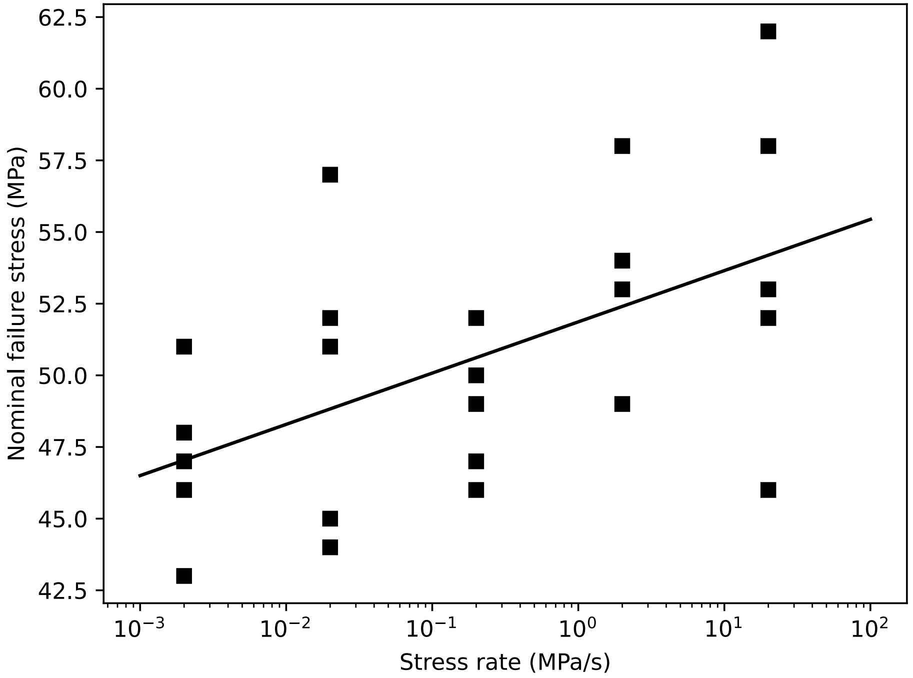
$$P_f = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^{\rho}\right]$$

$$v = \dot{a} = A\left(\frac{K_I}{K_{Ic}}\right)^{n}$$

$\sigma_0 = 186.1$ MPa, $\rho = 4.654$

$A = 1.83$ mm/s, $n = 90.7$



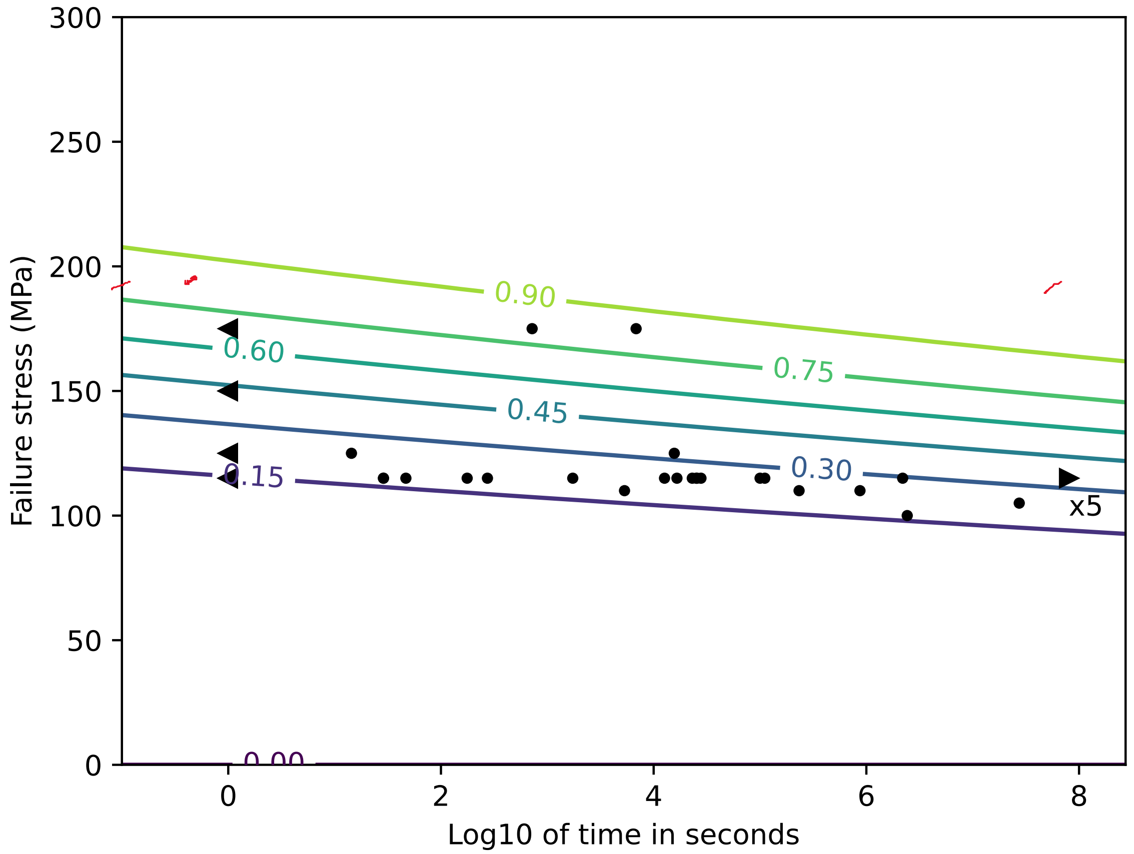


Probabilistic prediction of measured data

Measured $BaTiO_3$ parameters

- σ_0 = 186 MPa
- ρ = 4.654
- A = 1.83 mm/s
- n = 90.7

The resulting predicted probability distribution over time is shown at right



$$P_W = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^{\rho}\right]$$

$$v\left(K_{\mathrm{I}}\right) = A\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{Ic}}}\right)^{n}$$

$$P_f = 1 - \exp\left(-\left\{\frac{\left[\frac{A(n-2)Y^2\sigma_f^2 t_f}{2K_{\mathrm{Ic}}^2}+1\right]^{\frac{1}{n-2}}\sigma_f}{\sigma_0}\right\}^{\rho}\right)$$

Fitting parameters to measured time-to-failure

Can we use our proposed P_f to calibrate parameters from static failure data?

Note $\lim_{t\to\infty} P_f = \lim_{\sigma_f\to\infty} P_f = 1$, so P_f is a cumulative failure function in both t and σ_f

Calculate marginal probability over log$t$ as $\frac{\partial P_f}{\partial(\log t)}$

Time-to-failure should be conditional on surviving initial load application

Run-out specimens and those failing on load-up should also be included

$$P_f = 1 - \exp\left(-\left\{\frac{\left[\frac{A(n-2)Y^2\sigma_f^2 t_f}{2K_{\mathrm{Ic}}^2}+1\right]^{\frac{1}{n-2}}\sigma_f}{\sigma_0}\right\}^{\rho}\right) \qquad P_W = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^{\rho}\right]$$

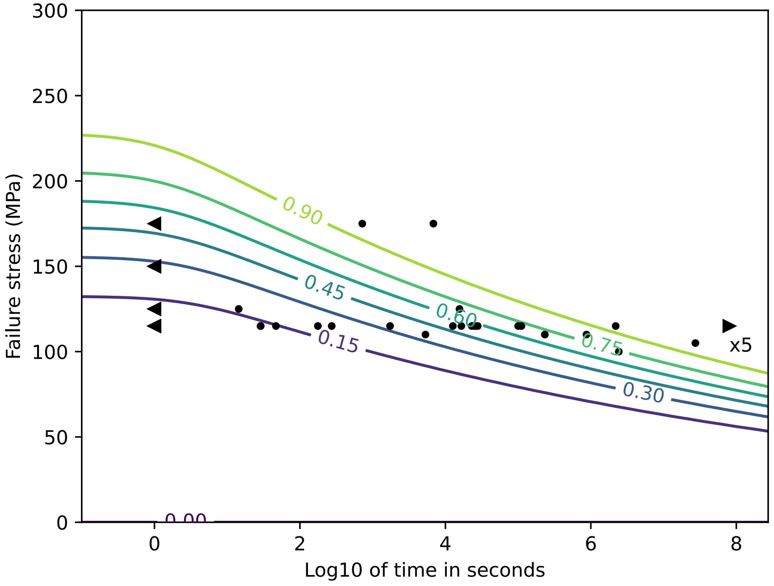
$$\begin{aligned} p_{MLE} &= \max_{\{\sigma_0,\rho,A,n\}} \mathrm{loglike}\left(\sigma_0,\rho,A,n|t_f,\sigma_f\right) \\ &= \max_{\{\sigma_0,\rho,A,n\}} \sum_{j\in\mathrm{loadup}} \log P_W\left(\sigma_j|\sigma_0,\rho\right) + \sum_{j\in\mathrm{delayed}} \log\left\{\left[\frac{\partial P_f\left(\sigma_j,t_j|\sigma_0,\rho,A,n\right)}{\partial\left(\log t\right)}\right]\left[1-P_W\left(\sigma_j|\sigma_0,\rho\right)\right]\right\} + \sum_{j\in\text{run-out}} \log\left[1-P_f\left(\sigma_j,t_{cut}|\sigma_0,\rho,A,n\right)\right] \end{aligned}$$

Fitting parameters to measured time-to-failure

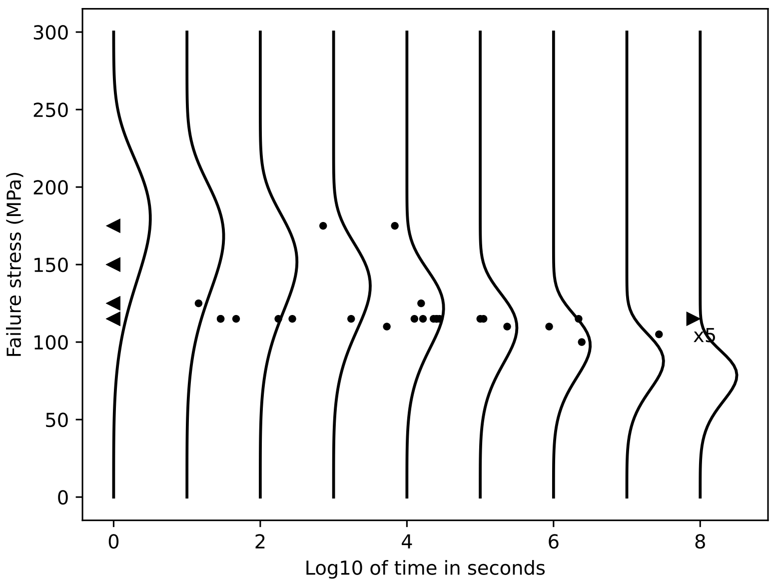
Prior parameters were $\sigma_0 = 186$ MPa, $\rho = 4.654$, $A = 0.00183$ m/s, $n = 90.7$

MLE fit parameters are $\sigma_0 = 192$ MPa, $\rho = 4.89$, $A = 10^{-5.92}$ m/s, $n = 20.9$

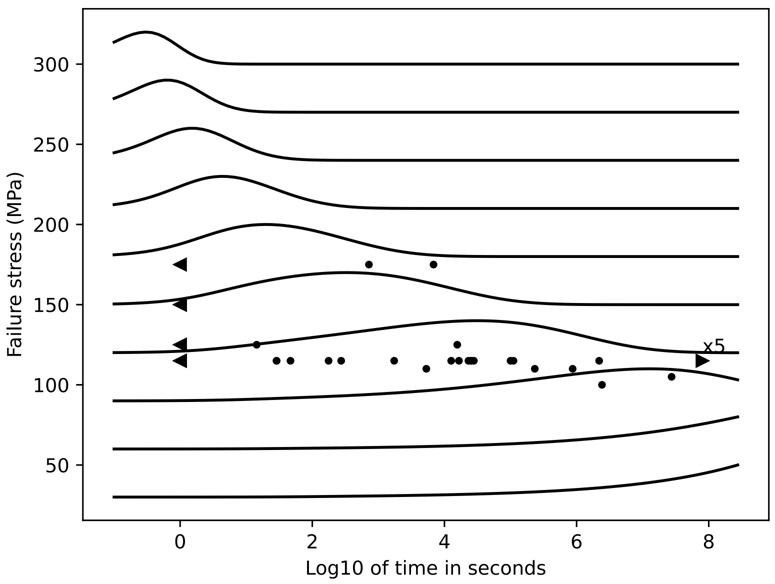
Cumulative failure probability, P_f



Marginal probability,= $p_{\sigma_f} = \frac{\partial P_f}{\partial \sigma_f}$



Marginal probability,= $p_{t_f} = \frac{\partial P_f}{\partial (\log t)}$

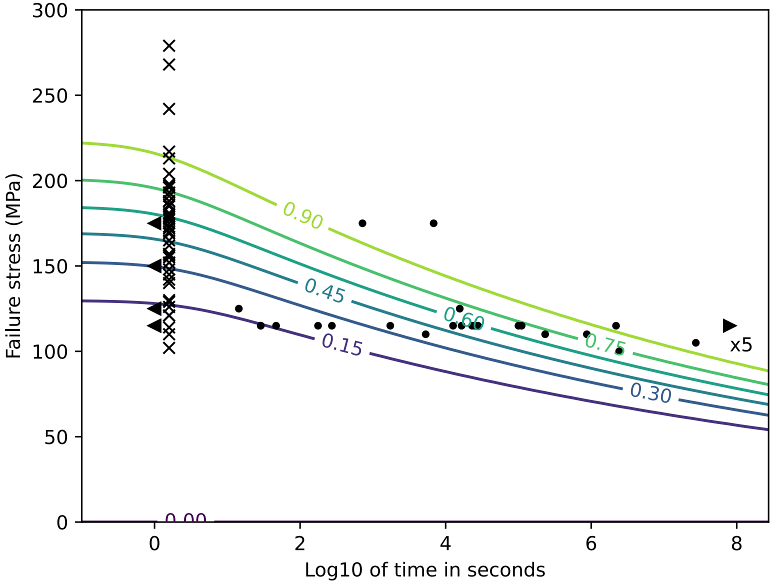


Fitting parameters including inert strength data

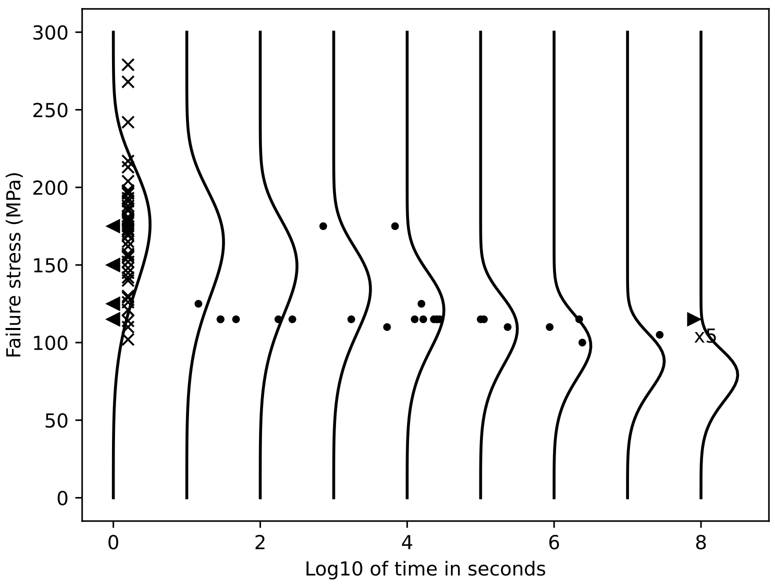
Prior parameters were $\sigma_0 = 186$ MPa, $\rho = 4.654$, $A = 0.00183$ m/s, $n = 90.7$

MLE fit parameters are $\sigma_0 = 188$ MPa, $\rho = 4.90$, $A = 10^{-5.88}$ m/s, $n = 21.8$

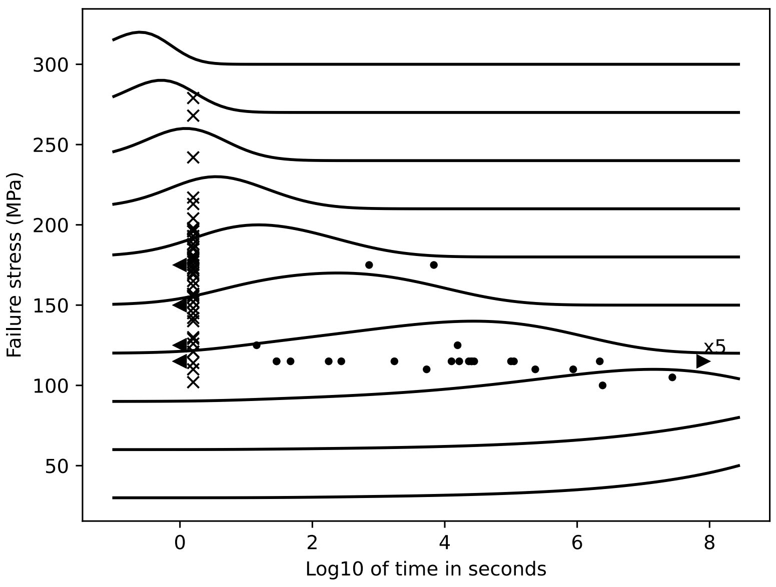
Cumulative failure probability, P_f

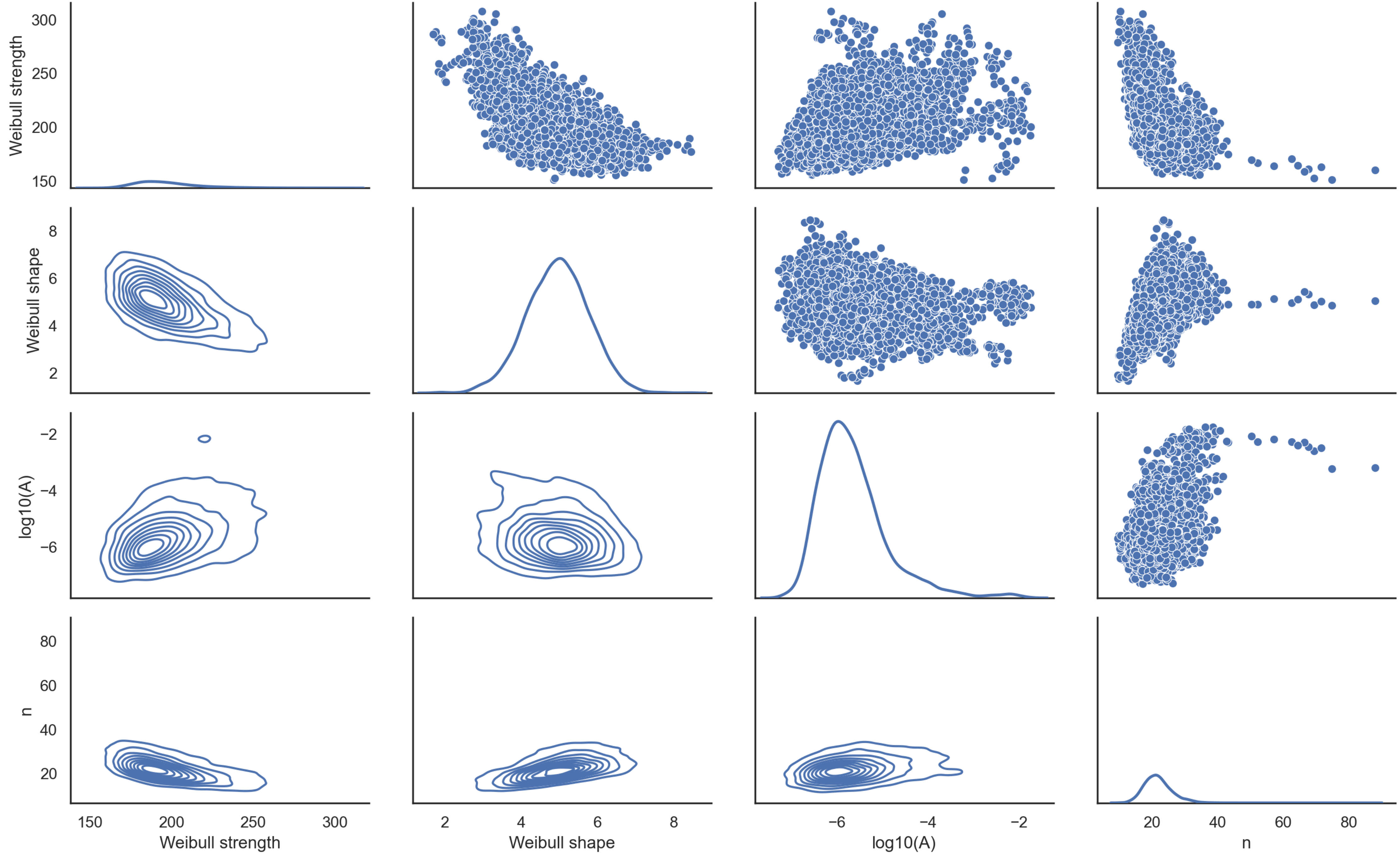


Marginal probability,= $p_{\sigma_f} = \frac{\partial P_f}{\partial \sigma_f}$



Marginal probability,= $p_{t_f} = \frac{\partial P_f}{\partial (\log t)}$




Weibull strength
Weibull shape
log10(A)
n
300
250
200
150
8
6
4
2
−2
−4
−6
80
60
40
20
150
200
250
300
2
4
6
8
−6
−4
−2
20
40
60
80
Weibull strength
Weibull shape
log10(A)
n

Time-to-failure with glass specimens

Try tests instead with a material which shows more SCG behavior: Schott S8061 glass

We have thus far been unable to measure any SCG in un-notched specimens held at approximately 50% of inert Weibull strength

What could be happening here?

Maybe threshold effects are important?

We will consider threshold through viscoelastic crack tip stress relaxation

Recent results (doi.org/10.1016/j.nocx.2022.100134) show threshold can be calculated through viscoelastic relaxation, for long lifetimes it should be considered here too

Threshold effects in SCG time-to-failure

Take time derivative of $K_I = Y\sigma\sqrt{a}$ and combine with, $v = \dot{a} = A\left(\frac{K_I}{K_{Ic}}\right)^n$, add viscoelastic relaxation of K_I with time constant β

$$\dot{K}_I = \frac{AK_I}{2a}\left(\frac{K_I}{K_{Ic}}\right)^n - \beta K_I, \quad \dot{a} = A\left(\frac{K_I}{K_{Ic}}\right)^n$$

Non-dimensionalize to get

$$\kappa = \frac{K_I}{K_{Ic}}, \quad \alpha = \frac{a\beta}{A}, \quad \tau = \beta t$$

$$\frac{\partial\kappa}{\partial\tau} = \frac{\kappa^{n+1}}{2\alpha} - \kappa, \quad \frac{\partial\alpha}{\partial\tau} = \kappa^n$$

We can interpret $\frac{\partial\kappa}{\partial\tau} = 0$ as SCG threshold

$$\kappa_{th} = (2\alpha)^{1/n} \quad \rightarrow \quad K_{I,th} = \left(\frac{2a\beta}{A}\right)^{1/n} K_{Ic}$$

Probabilistic SCG with threshold effects

Same procedure as before (but somewhat messier):

Assume σ_f is measured at t_f

We use the initial conditions $\alpha(0) = \left(\frac{K_{Ic}}{Y\sigma_i}\right)^2 \frac{\beta}{A}, \quad \kappa(0) = \frac{\sigma_f}{\sigma_i}$

Integrate from $\{\alpha(0), \kappa(0)\}$ to $\{\alpha(\tau_f), 1\}$

Express as function $\tau_f = f(\sigma_i, \sigma_f, A, n, \beta)$

Invert to get function $\sigma_i = g(\tau_f, \sigma_f, A, n, \beta)$

Substitute into beginning-of-life Weibull distribution, $P_f = 1 - \exp\left[-\left(\frac{g(\beta t_f, \sigma_f, A, n, \beta)}{\sigma_0}\right)^\rho\right]$

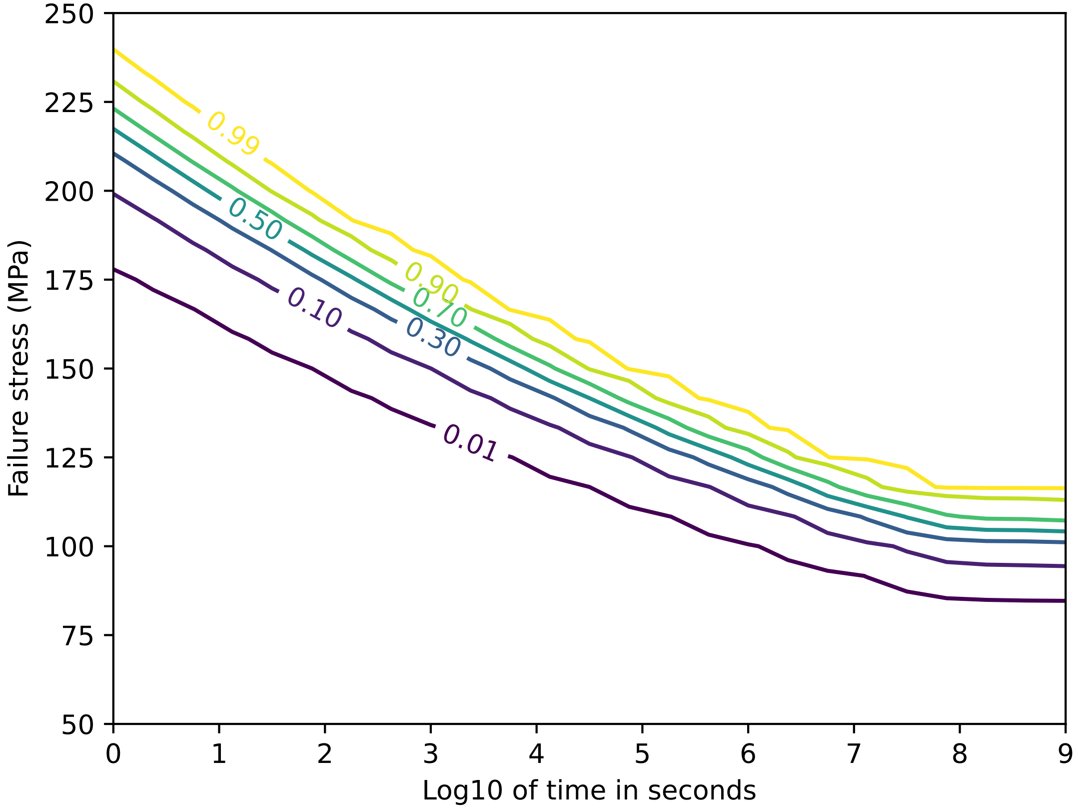
If $\kappa < \kappa_{th}$ then $\tau_f = \infty$ and $\sigma_i = \sigma_f$

Time-to-failure predictions for S8061 glass

Parameters for S8061 glass, $\sigma_0 = 252$ MPa, $\rho = 10.3$, $A = 0.325$ m/s, $n = 24$

Assume $\kappa_{th} \approx 0.5$ for 1 mm crack, estimate $\beta \approx 10^{-9}$ 1/s

Cumulative failure probability, P_f



For simplicity, here we assume $Y(a) = \text{CONSTANT}$, this can be relaxed and in some cases can significantly affect threshold prediction

We can also allow for time-evolving stress state

Given assumptions stated here, minimal flaw population aging occurs after $t \approx 1/(10\beta)$

Even with a threshold stress intensity, there is no threshold for applied load

Questions?

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Threshold calculation

Threshold is dependent on crack size, larger threshold for larger cracks

$$\kappa_{th} = (2\alpha)^{1/n} \quad \rightarrow \quad K_{I,th} = \left(\frac{2a\beta}{A}\right)^{1/n} K_{Ic}$$

Can an initially super-threshold crack become sub-threshold as it grows?

Yes, but this regime is small and we can neglect it