SAND2023-12019C

Case Study on the Effect of Nonlinearity in Dynamic Environment Testing

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ND2023-12019C

Nonlinearity in Bolted Joints

Bolted joints introduce nonlinearities in structures. Accounting for these nonlinearities is important in modern design.

NASA Space Launch System (SLS)

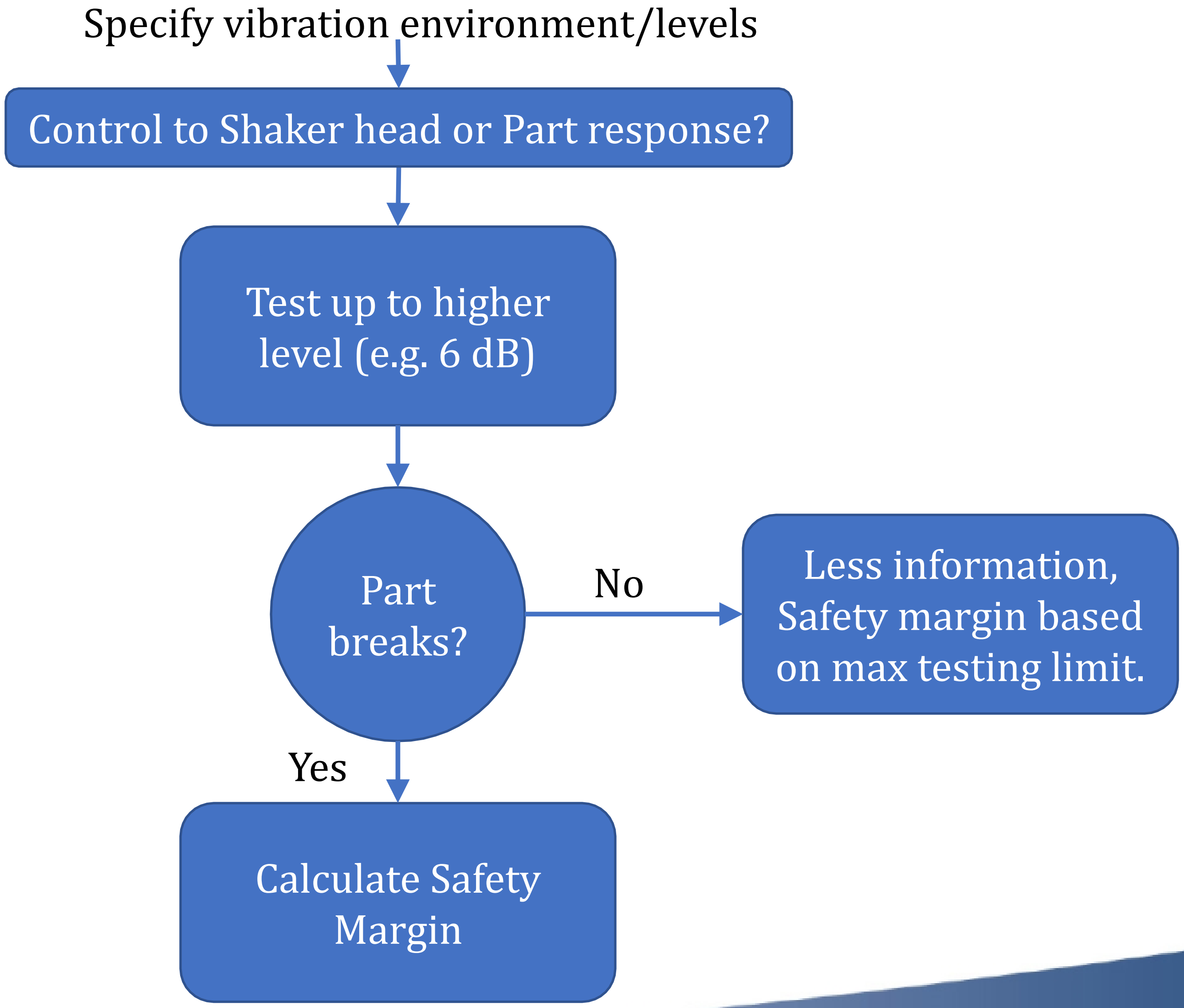
- 360 bolts are used just connecting a hydrogen fuel tank to an intertank [1]. Easily has thousands of bolts.
- Unique vibration environment the bolts must withstand.

What happens if we assume a linear model? When is this ok and when is this bad?

Image Credit: https://www.nasa.gov/sites/default/files/thumbnails/image/ksc-20220316-ph-geb01_0030large.jpg

[1] https://www.nasa.gov/exploration/systems/sls/new-milestone-reached-on-complex-large-rocket.html

Qualification Testing



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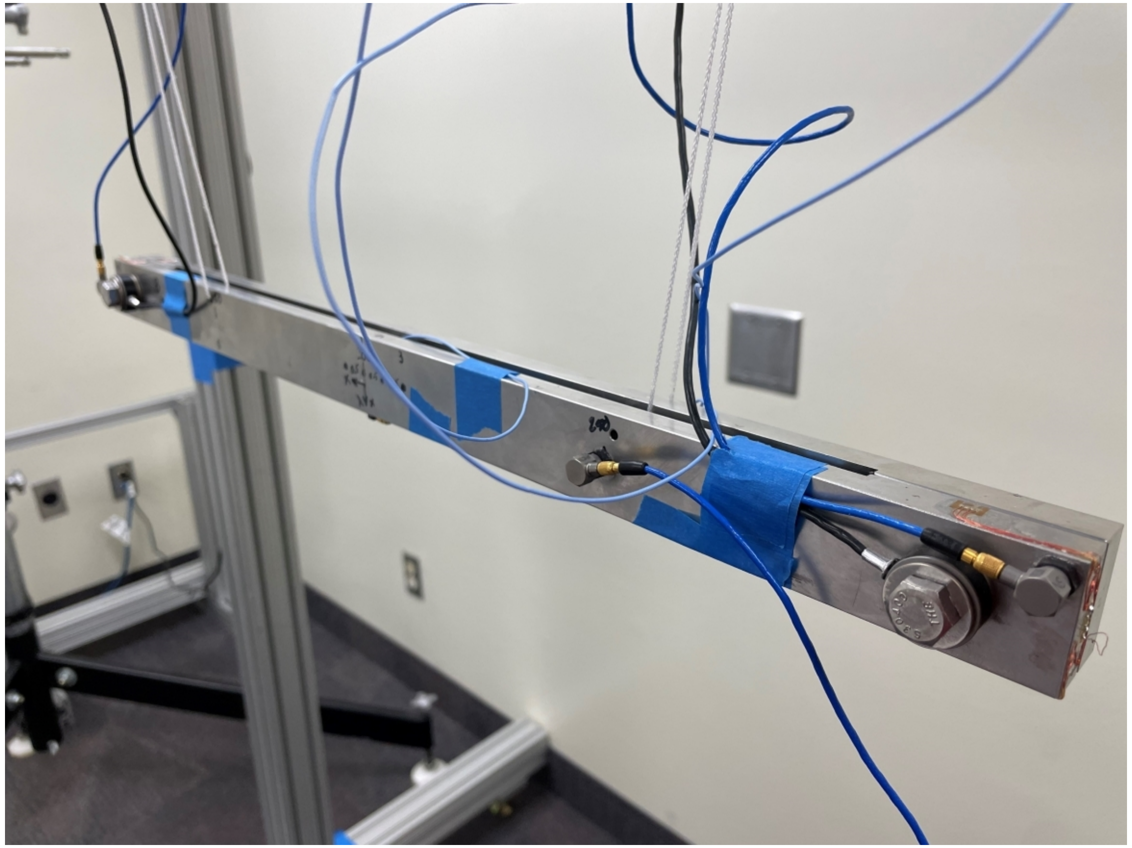
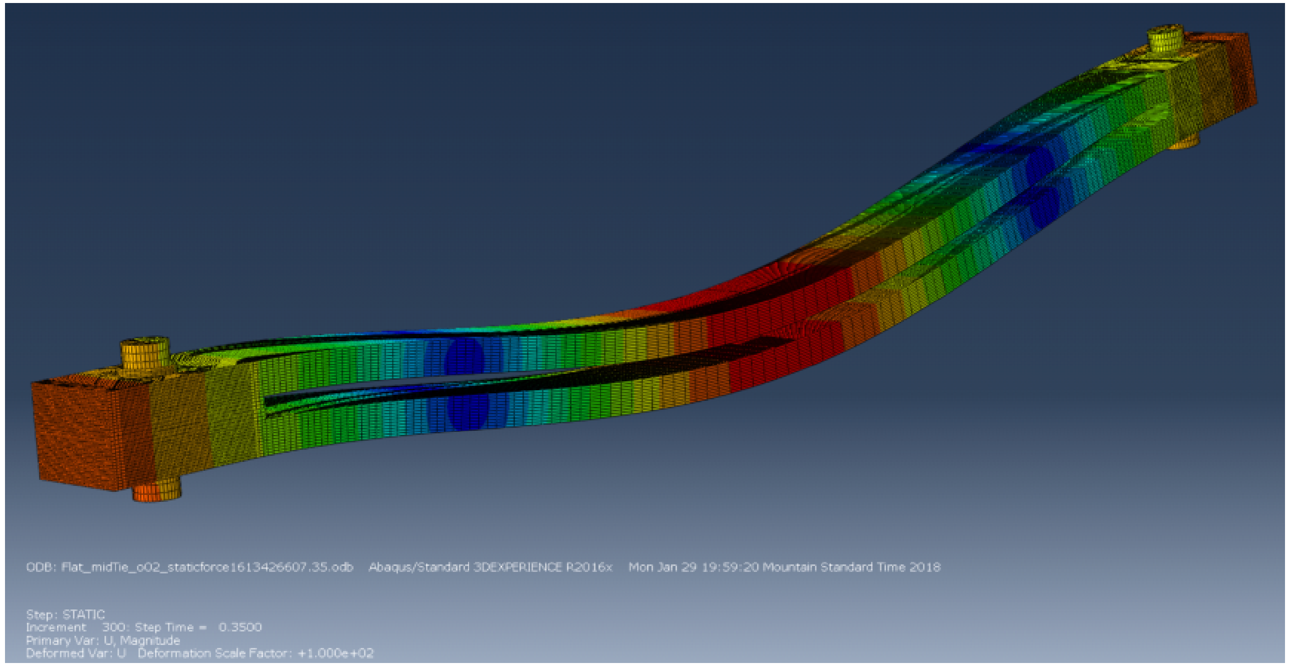
Simplified Structure Used in this Study





This structure is called the S4-beam. We focus on the 2nd vibration mode in this presentation.

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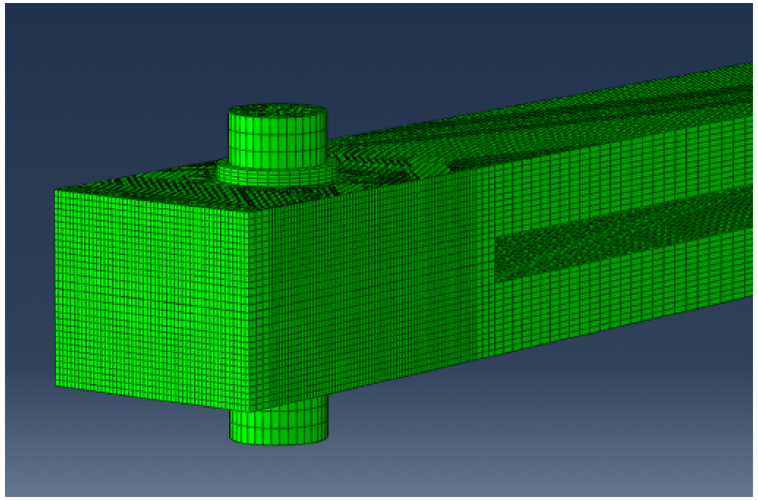
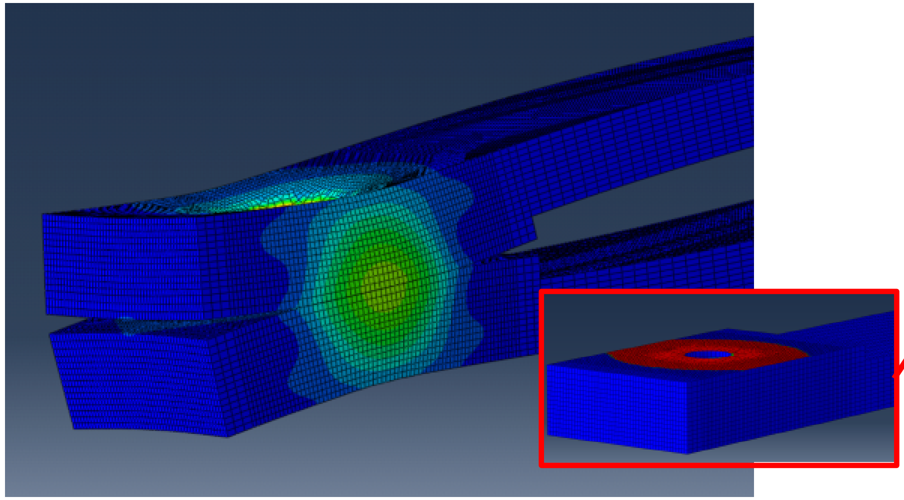
Source of Nonlinearity

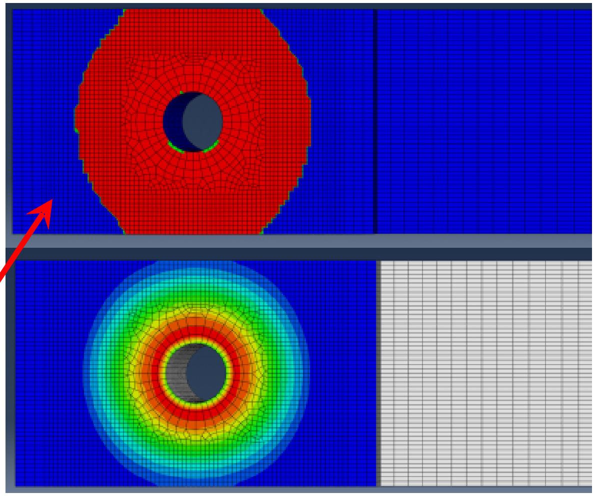
a.) Bolted Joint (undeformed)



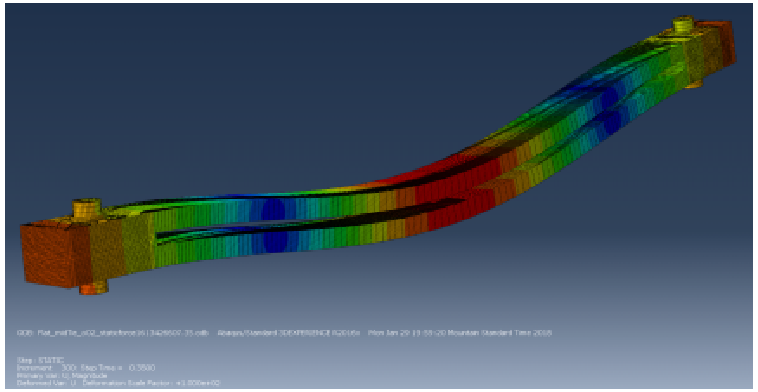
b.) Preloaded Deformation



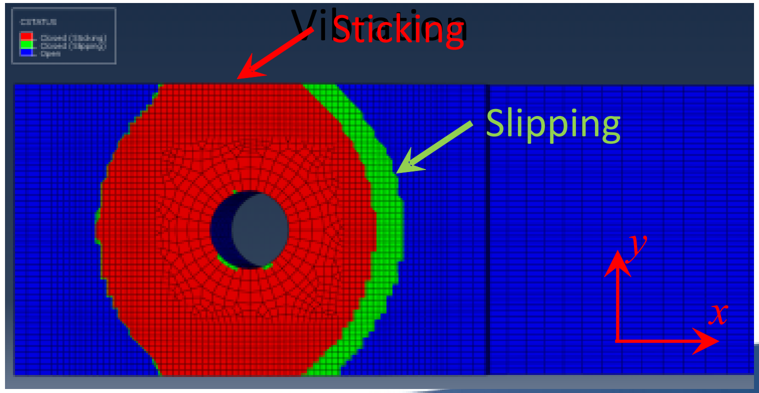
c.) Contact Area / Pressure



d.) Deformation During Vibration



e.) Slip-State of Joint During Vibration



f.) Vibration Energy is Dissipated

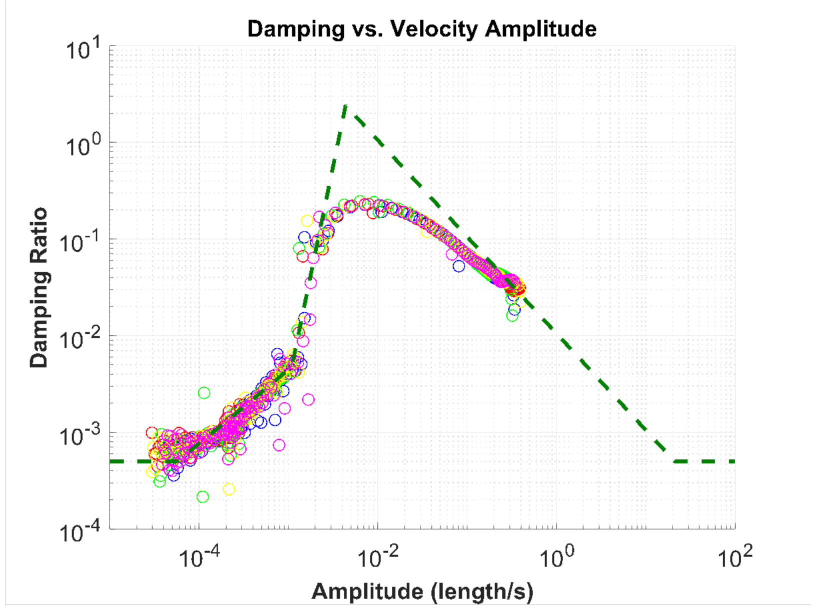
Equations of Motion

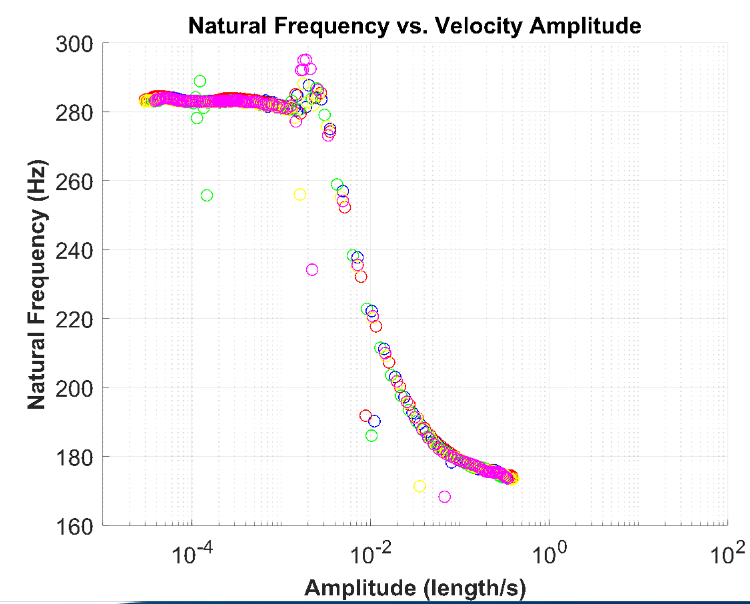
$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{f}_{\text{nl}}(\mathbf{u}, \boldsymbol{\theta}) = \mathbf{f}(t)$$ [non-modal] (MDOF)

Convert to quasi-linear modal model

$$\ddot{q} + 2\zeta(A)\omega_n(A)\dot{q} + \omega_n^2(A)q = f_m(t)$$ [modal] (SDOF)

$\zeta(A)$ and $\omega_n(A)$ obtained from experimental data and fitting models to

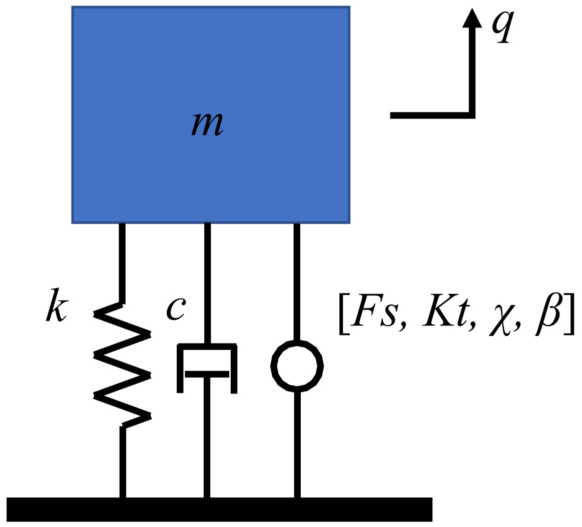


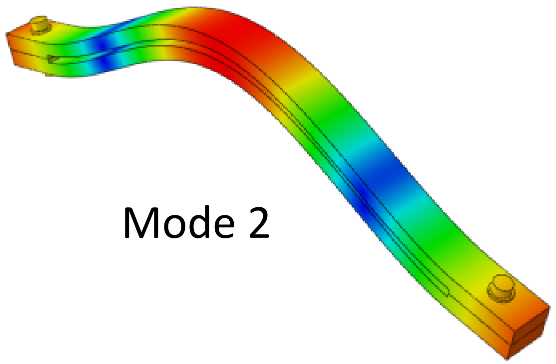


Nonlinear Model

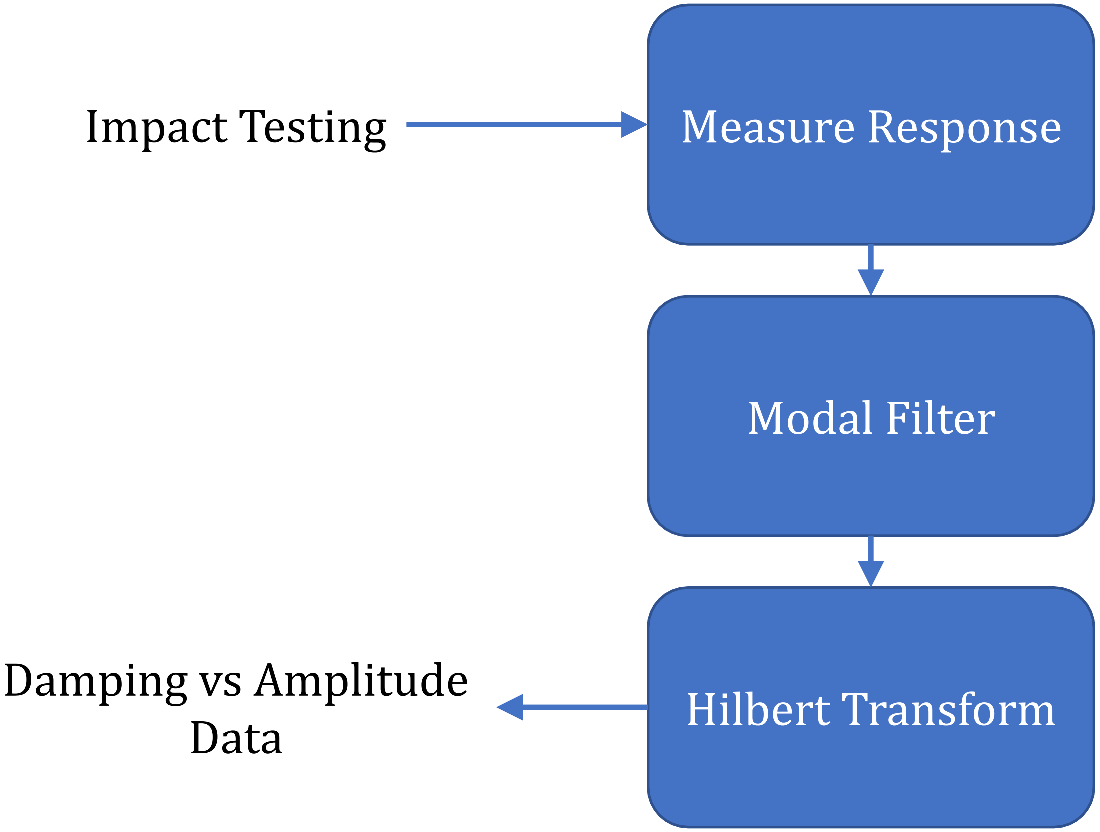
The nonlinear model approximates the actual S4-beam subject to the following assumptions:

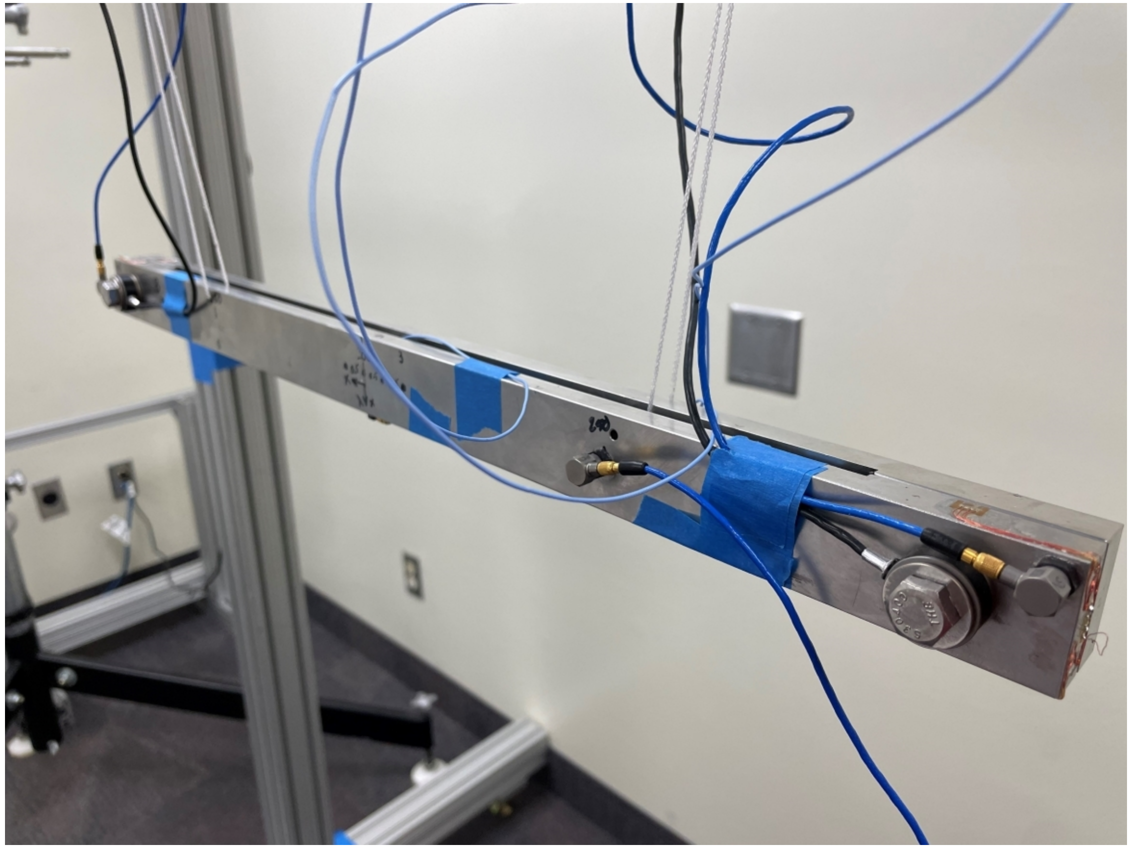
- A SDOF system captures the 2nd vibration mode.
  - Any coupling between the modes is neglected.
- Quasi-linear model (Iwan joint)
  - This is appropriate since the primary effect of the nonlinearity is amplitude dependent damping and natural frequency.
- Generalized coordinate = modal displacement.
  - Max physical displacement: $x_{\max} = \phi_{\max} q$
  - Maximum strain: $\epsilon_{\max} = C_{\epsilon} q$
- Broadband forcing





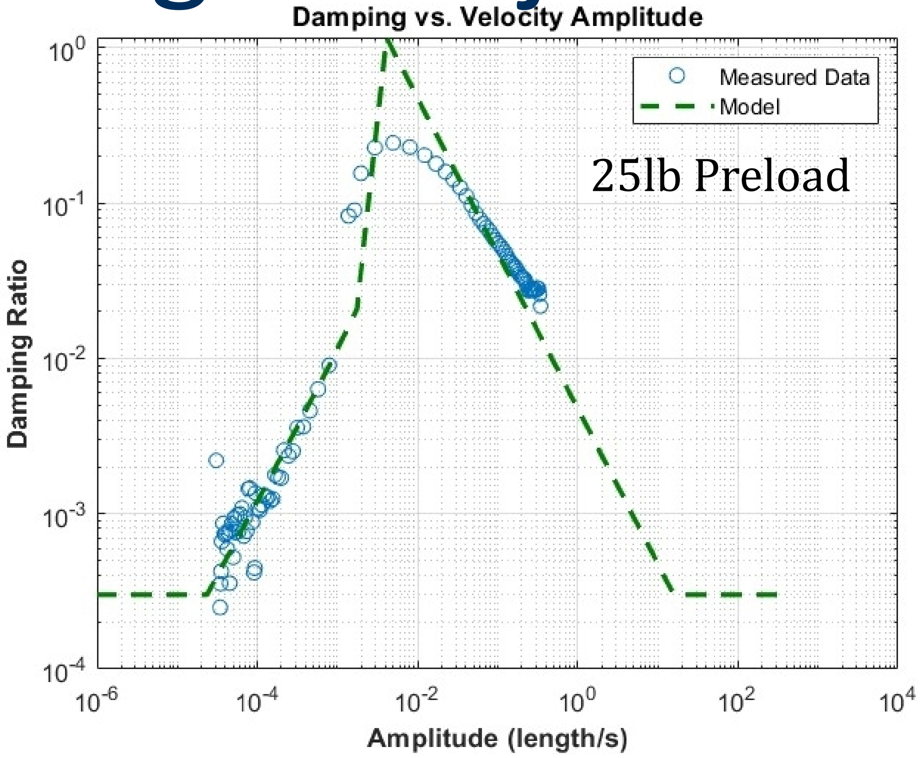
Experimental Testing

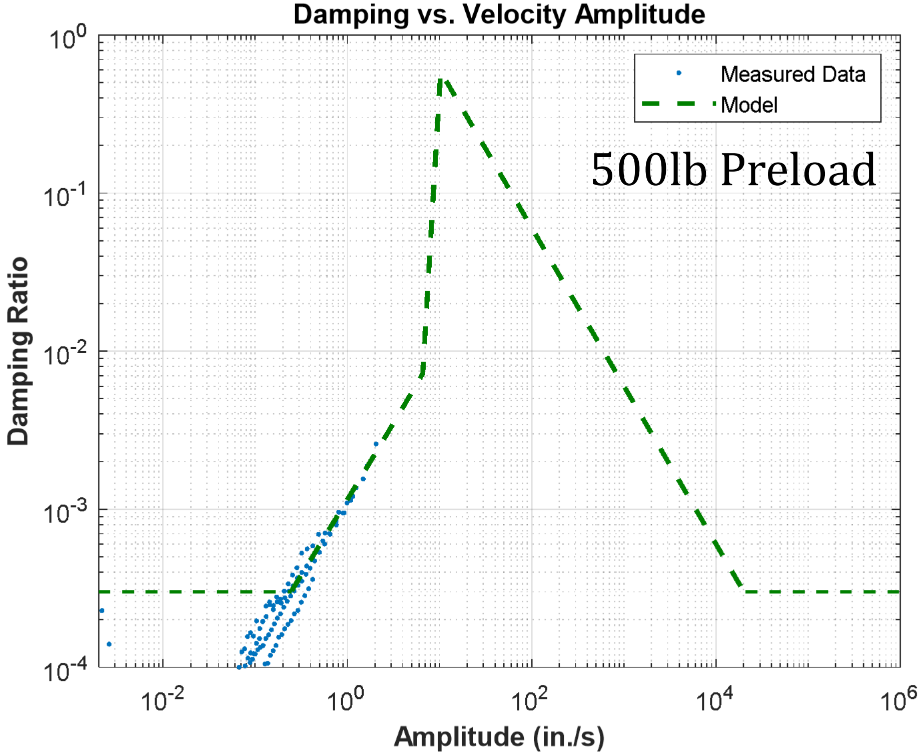




Data showing Iwan joint characteristics

This experimental data shows that an Iwan joint should be a suitable model for bolted joints in both microslip and macroslip.





At higher bolt preload, macroslip is not reached. Because of this, some model parameters are not fully constrained.

The figures show the damping ratio vs amplitude data compared to an Iwan joint model. This study assumes the case to the left with small preload follows the same trends as higher preloads. This is because macroslip data at higher preloads is harder to obtain.

Formulation of Response Magnitude

Given the SDOF modal equation of motion, the steady state velocity is

$$v = Re\left[-F_p \frac{i\omega}{(\omega_n^2 - \omega^2) + i(2\zeta\omega_n\omega)} e^{i\omega t}\right]$$

When $\omega = \omega_n$,

$$v = Re\left[-\frac{F_p}{2\zeta\omega_n} e^{i\omega t}\right]$$

So

$$V_{peak} = \frac{F_p}{2\zeta\omega_n}$$

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Formulation of Response Magnitude

The maximum velocity is

$$V_{peak} = \frac{F_p}{2\zeta\omega_n} \quad \leftrightarrow \quad F_p = 2\zeta\omega_n V_{peak}$$

Suppose that V_{peak}, F_p correspond to the TEST conditions.

Then the test results are used to predict the response when the input force is scaled by α, so $F_{p2} = \alpha F_{p1}$

$$V_{p2} = \frac{F_{p2}}{2\zeta_2\omega_{n2}} = \frac{\alpha F_{p1}}{2\zeta_2\omega_{n2}} = \alpha\left(\frac{\zeta_1}{\zeta_2}\right)\left(\frac{\omega_{n1}}{\omega_{n2}}\right)V_{p1}$$

Note: this assumes that the system is forced at the resonance in both cases, even though ω_{n1} and ω_{n2} could be different.

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Formulation of Response Magnitude

αV_{p1} is the response in a linear case, which is a result of $\left(\frac{\zeta_1}{\zeta_2}\right) = \left(\frac{\omega_{n1}}{\omega_{n2}}\right) = 1$ occurring in linear systems.

$$V_{p2} = \underbrace{\alpha V_{p1}}_{V_{LinPred}} \left(\frac{\zeta_1}{\zeta_2}\right)\left(\frac{\omega_{n1}}{\omega_{n2}}\right)$$

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Finding V_2 and ζ_2

Because ζ_2 is a function of V_2, we must solve the following system of equations:

$$V_2 = \alpha V_1 \left(\frac{\zeta_1}{\zeta_2(V_2)}\right)\left(\frac{\omega_{n1}}{\omega_{n2}(V_2)}\right) \quad \text{(Extrapolation equation)}$$

$$\zeta_2 = f_{model}(V_2) \quad (\zeta_2 \text{ vs } V_2 \text{ data or model})$$

($\omega_{n2}(V_2)$ can be incorporated into the extrapolation equation and thus is not included in the system of equations).

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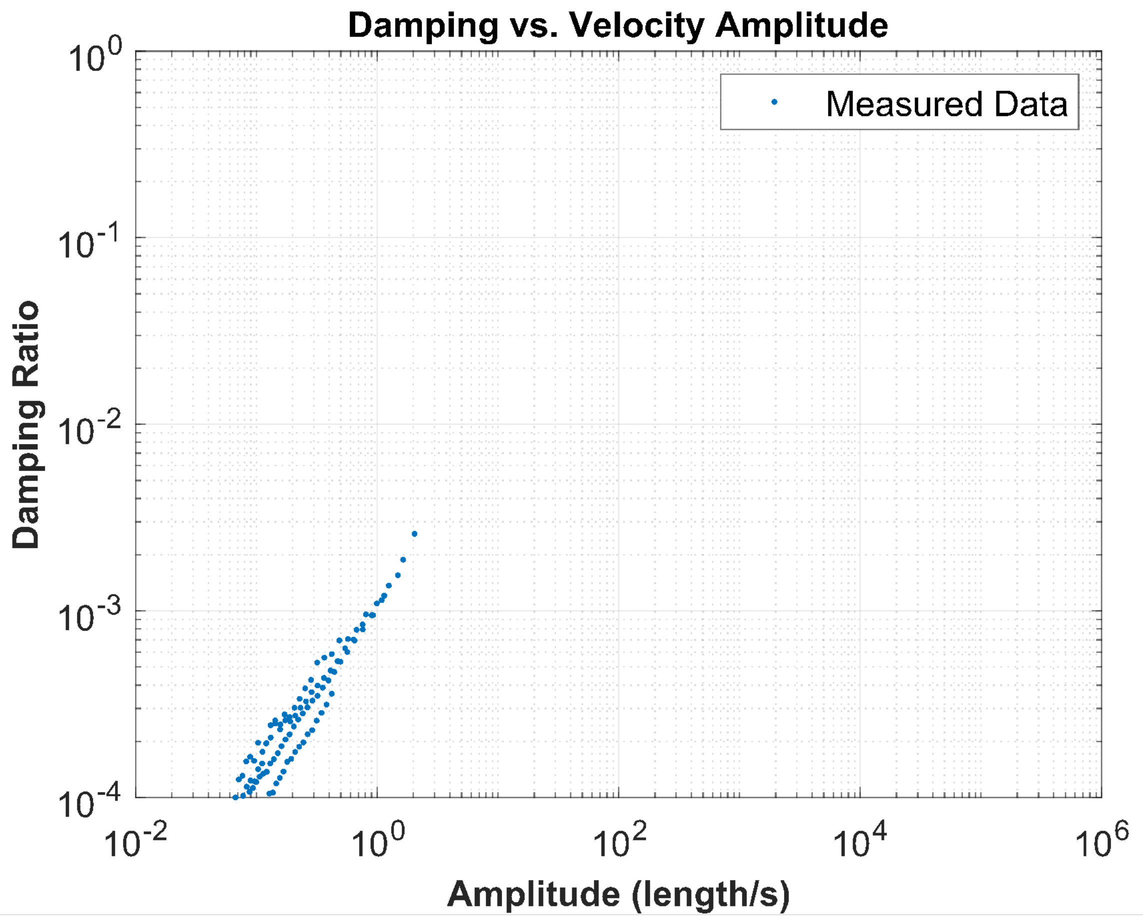
Finding V_2 and ζ_2

Finding the damping and amplitude values that fit our nonlinear model is done in the following steps:

Finding V_2 and ζ_2

Finding the damping and amplitude values that fit our nonlinear model is done in the following steps:

1) Plot measured data



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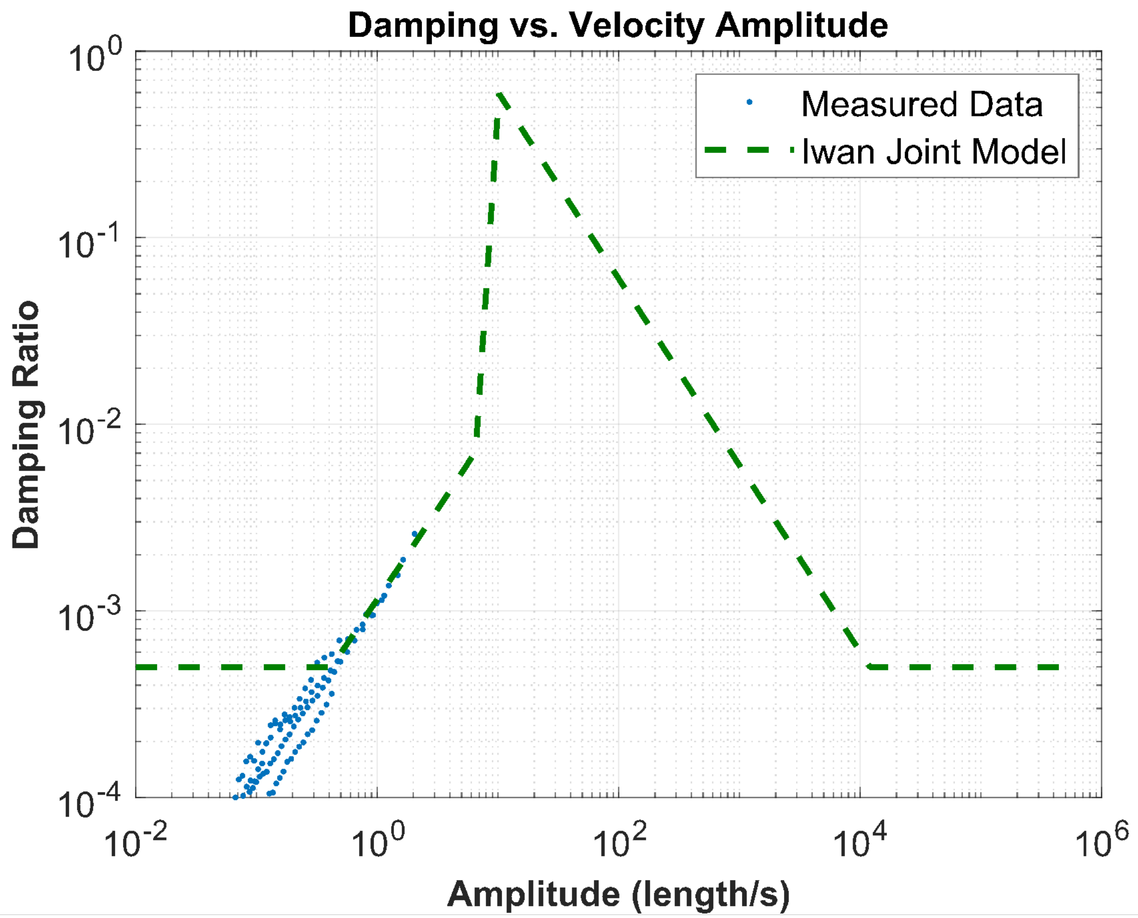
Finding V_2 and ζ_2

Finding the damping and amplitude values that fit our nonlinear model is done in the following steps:

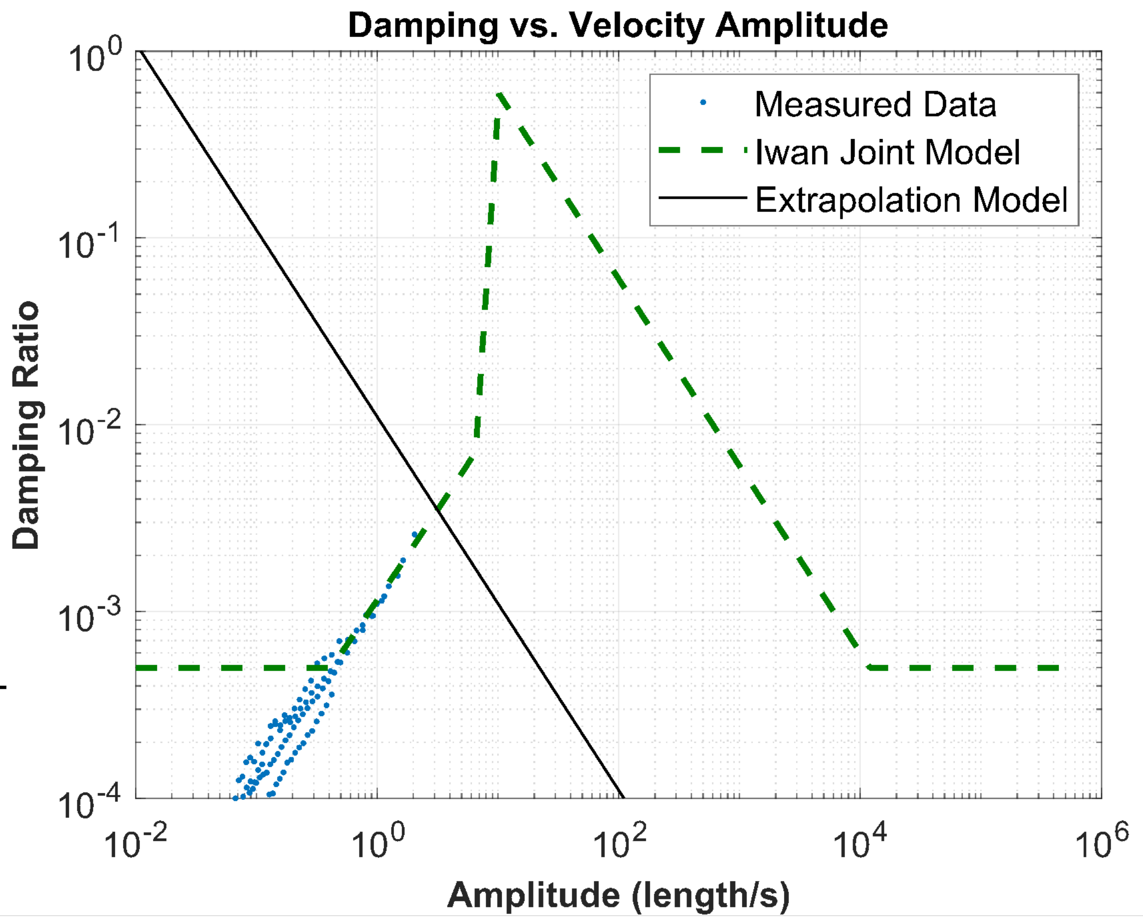
1) Plot measured data
2) Fit Iwan joint model



Finding V_2 and ζ_2

Finding the damping and amplitude values that fit our nonlinear model is done in the following steps:

1) Plot measured data
2) Fit Iwan joint model
3) Plot Extrapolation Equation



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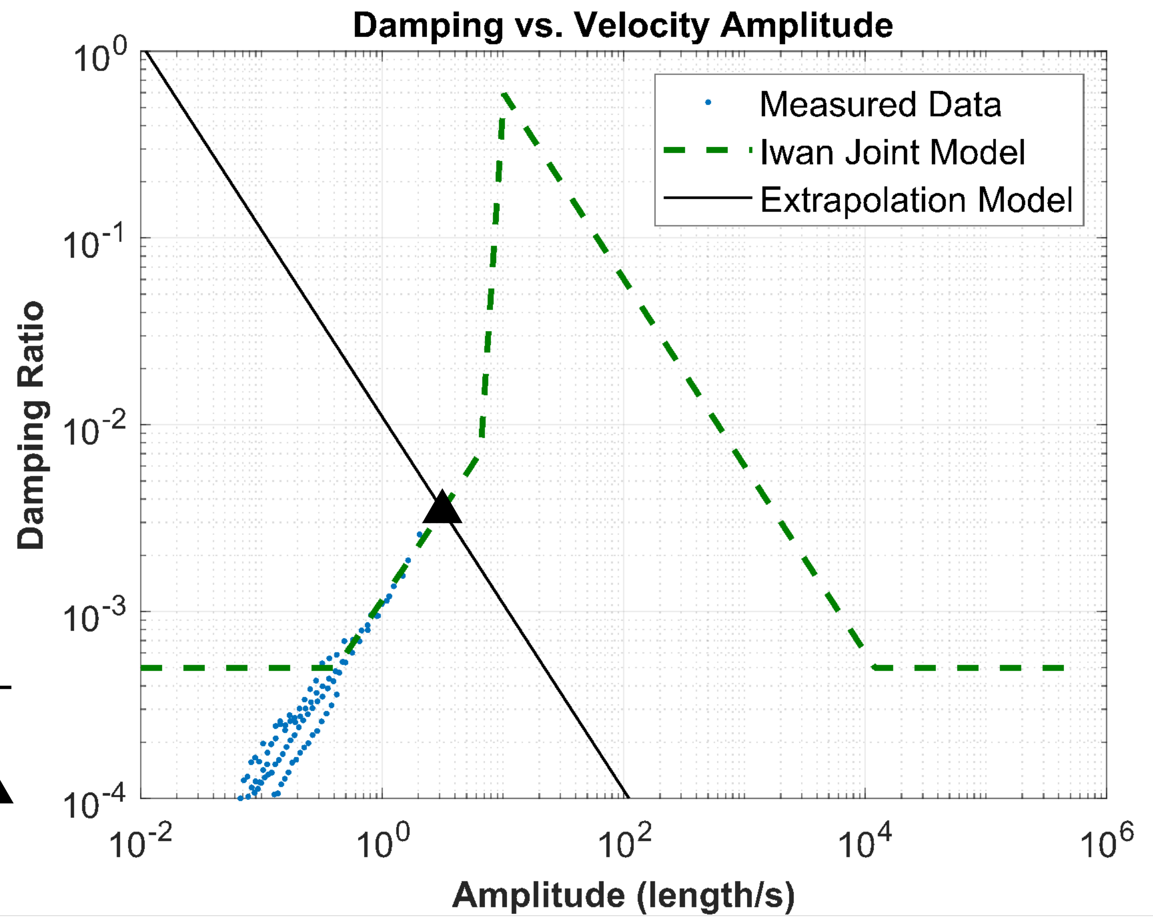
Finding V_2 and ζ_2

Finding the damping and amplitude values that fit our nonlinear model is done in the following steps:

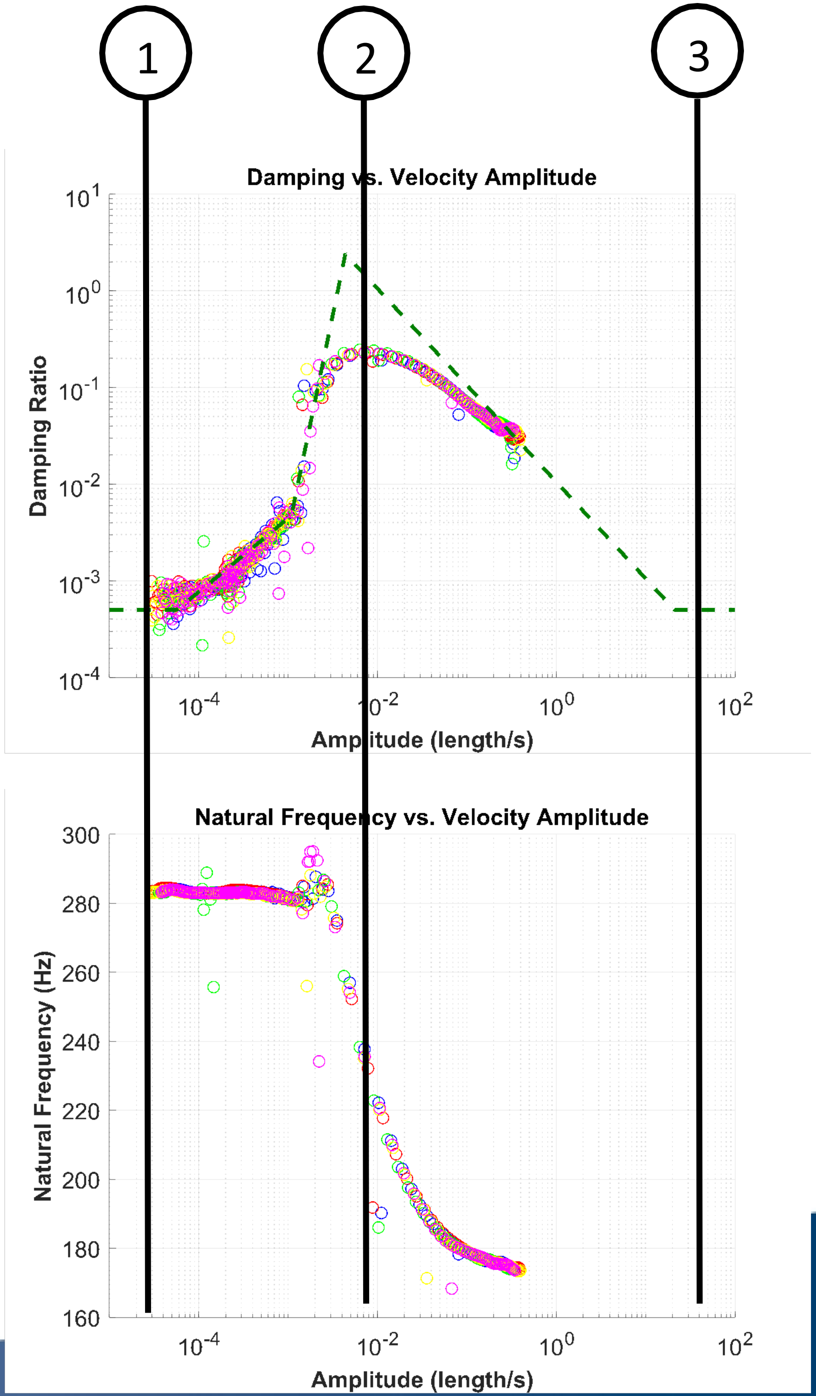
1) Plot measured data
2) Fit Iwan joint model
3) Plot Extrapolation Equation
4) Calculate intersection of models ▲



Testing Cases

There are 3 locations where testing can occur.

Different problems can occur when extrapolating data from one location to another.



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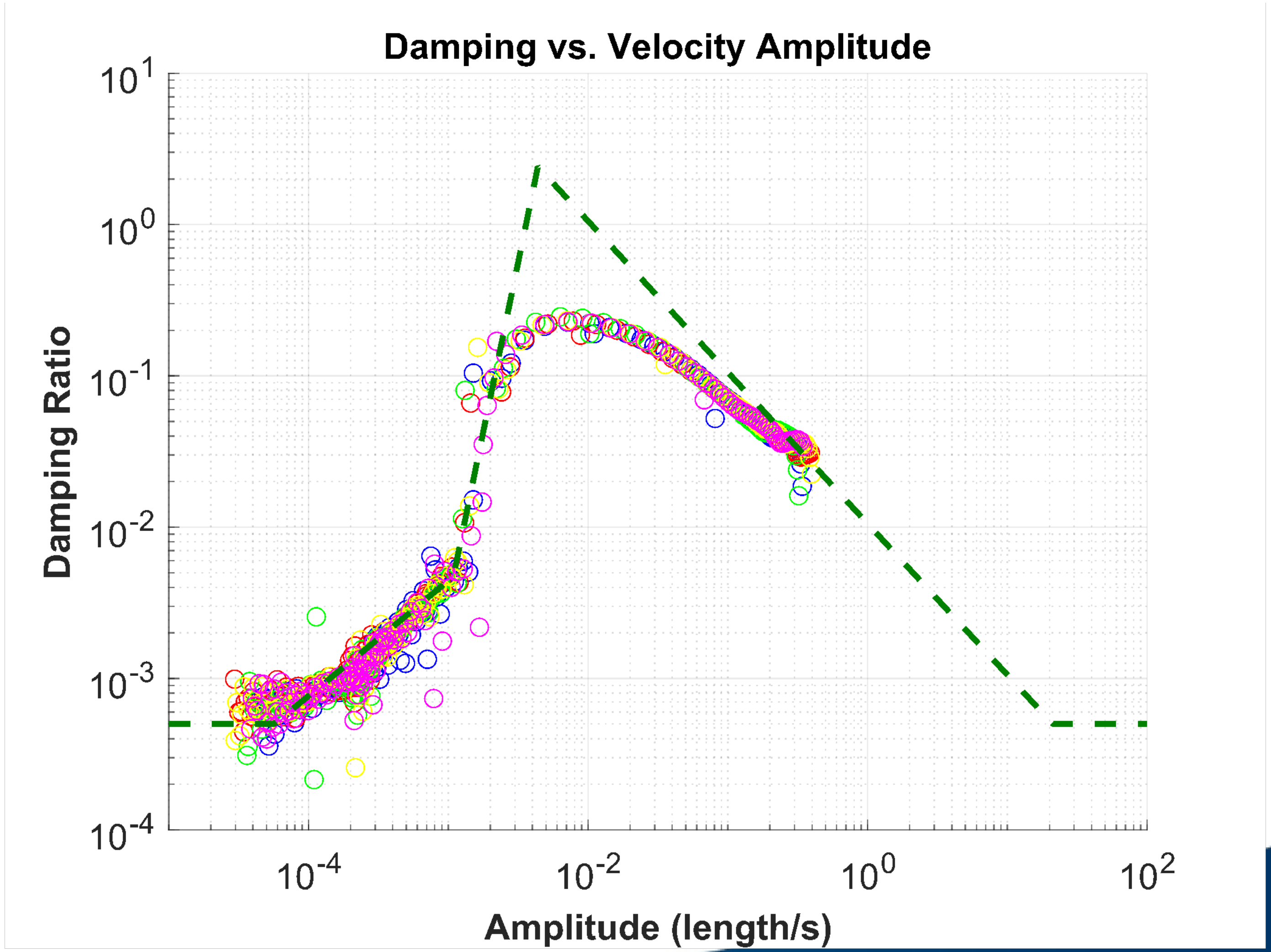
Test for Fatigue Failure (1-2)

We start with a measured amplitude and damping ratio.



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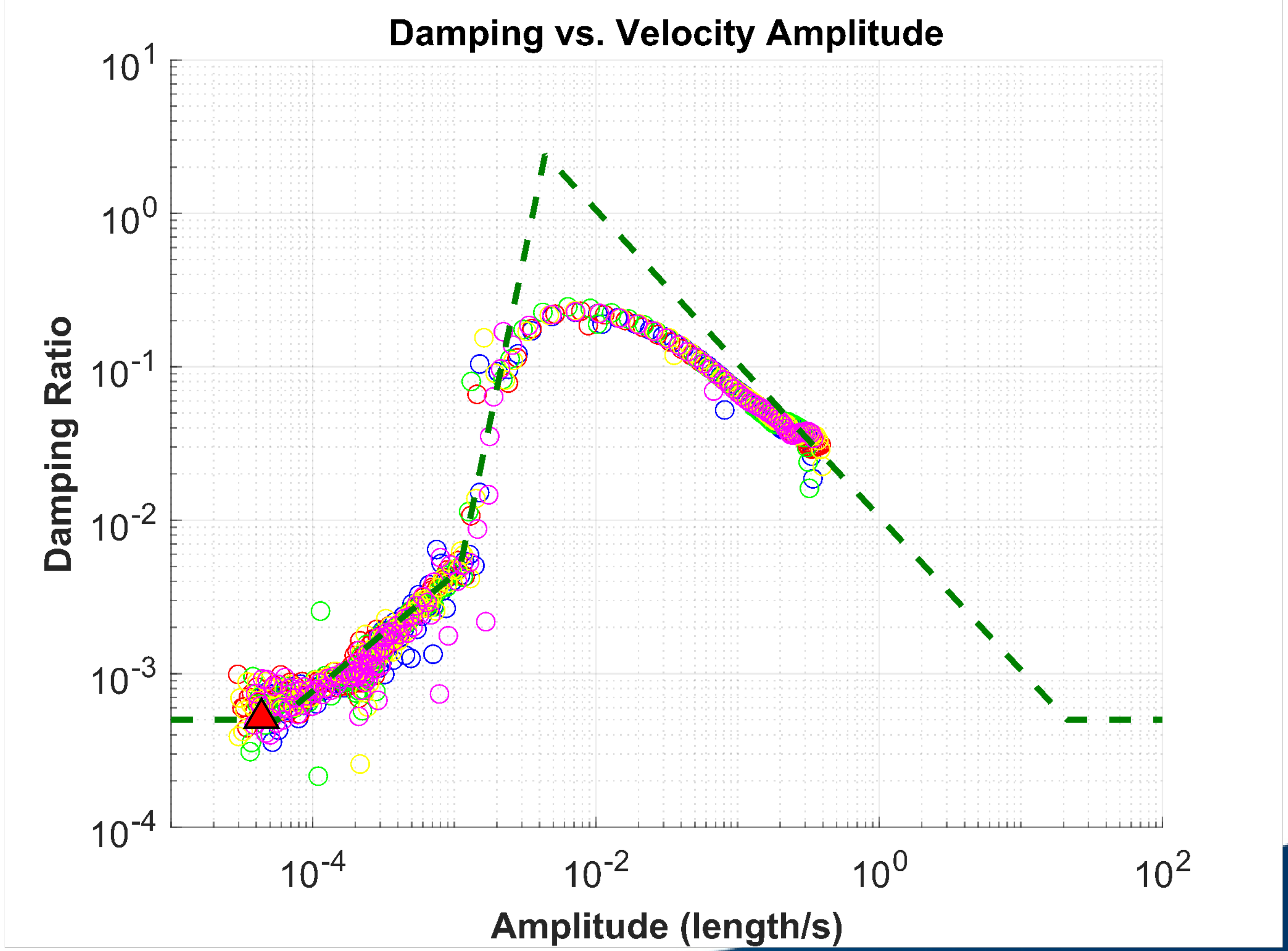
Test for Fatigue Failure (1-2)

We start with a measured amplitude and damping ratio.
$(V_1, \zeta_1) = (5 \times 10^{-5}, 5 \times 10^{-4})$



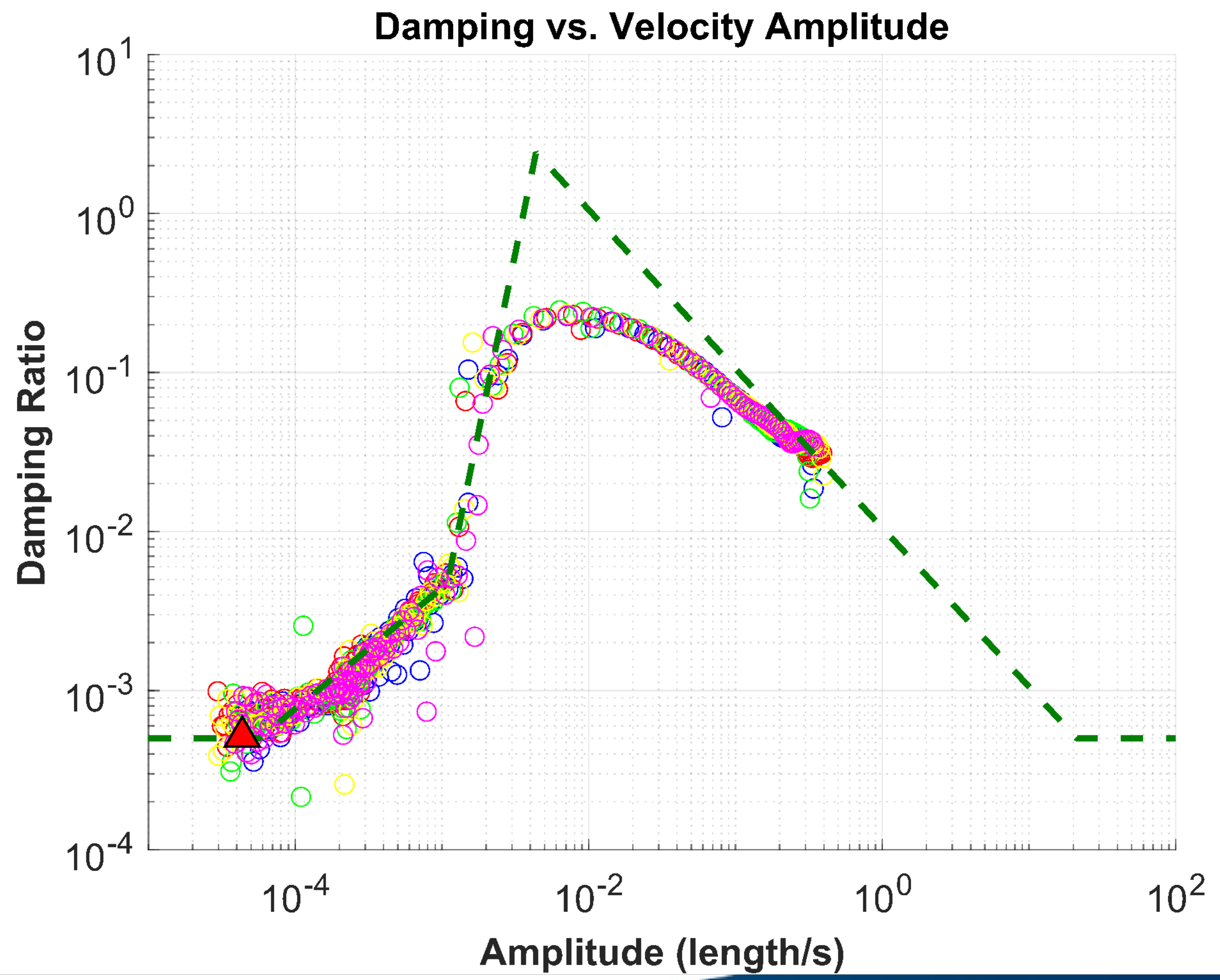
Test for Fatigue Failure (1-2)

$$(V_1, \zeta_1) = (5 \times 10^{-5}, 5 \times 10^{-4})$$

We select a forcing amplitude magnification

$$\alpha = 100$$

and plot the extrapolation equation.



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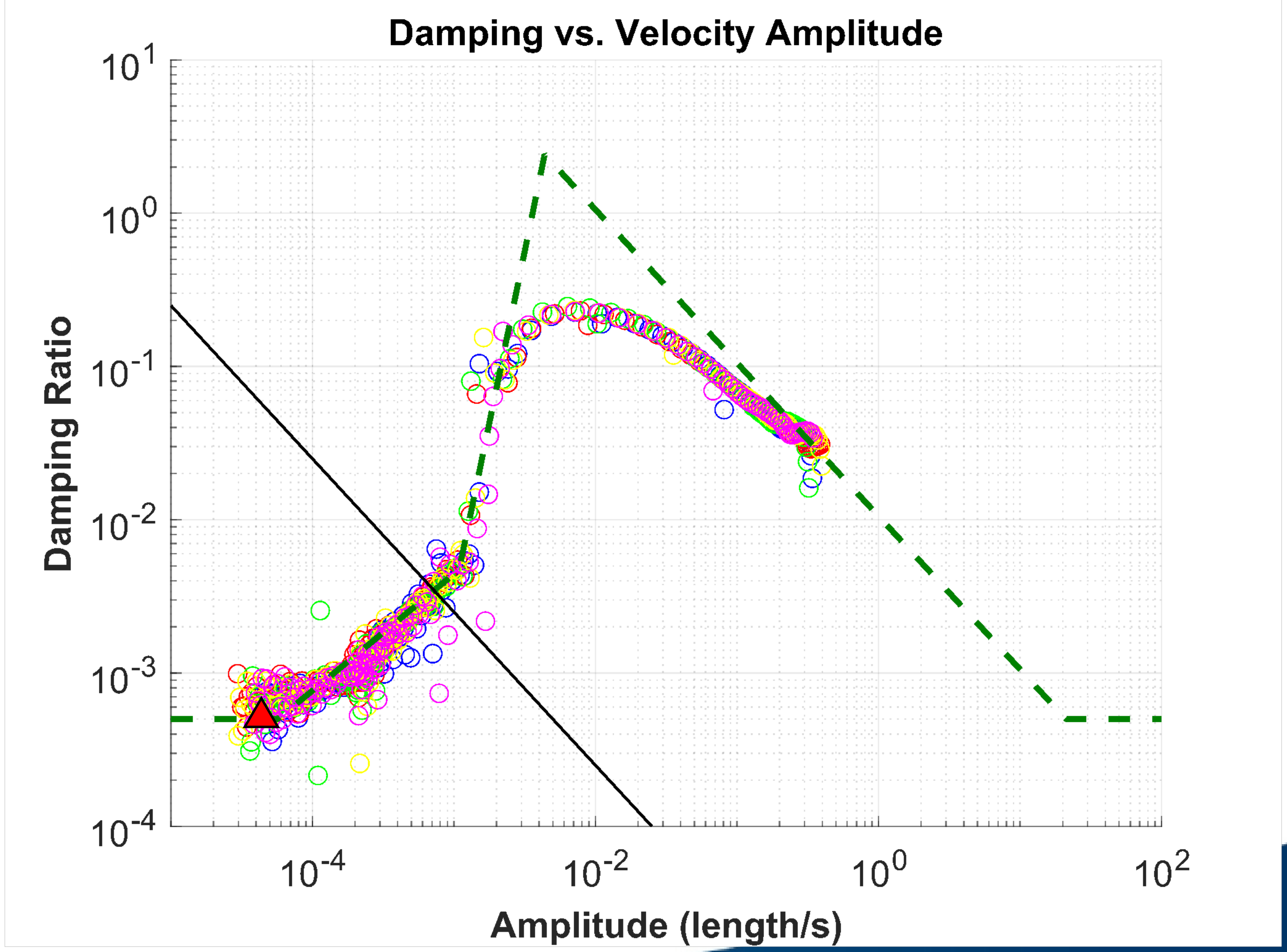

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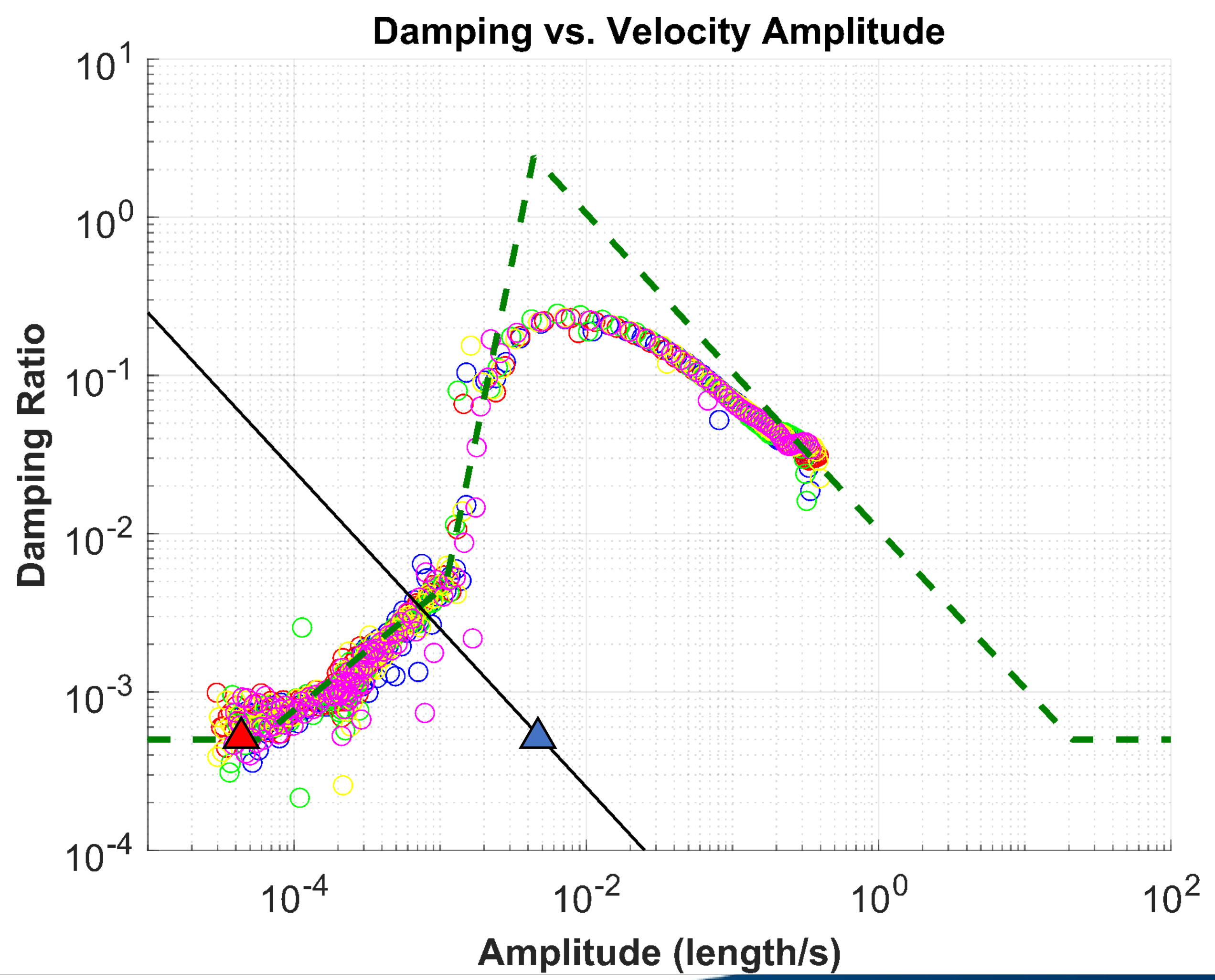
Test for Fatigue Failure (1-2)

$$(V_1, \zeta_1) = (5 \times 10^{-5}, 5 \times 10^{-4})$$

$$\alpha = 100$$

The linear prediction, with constant damping, results in:

$$(V_{2_{Lin}}, \zeta_1) = (5 \times 10^{-3}, 5 \times 10^{-4})$$



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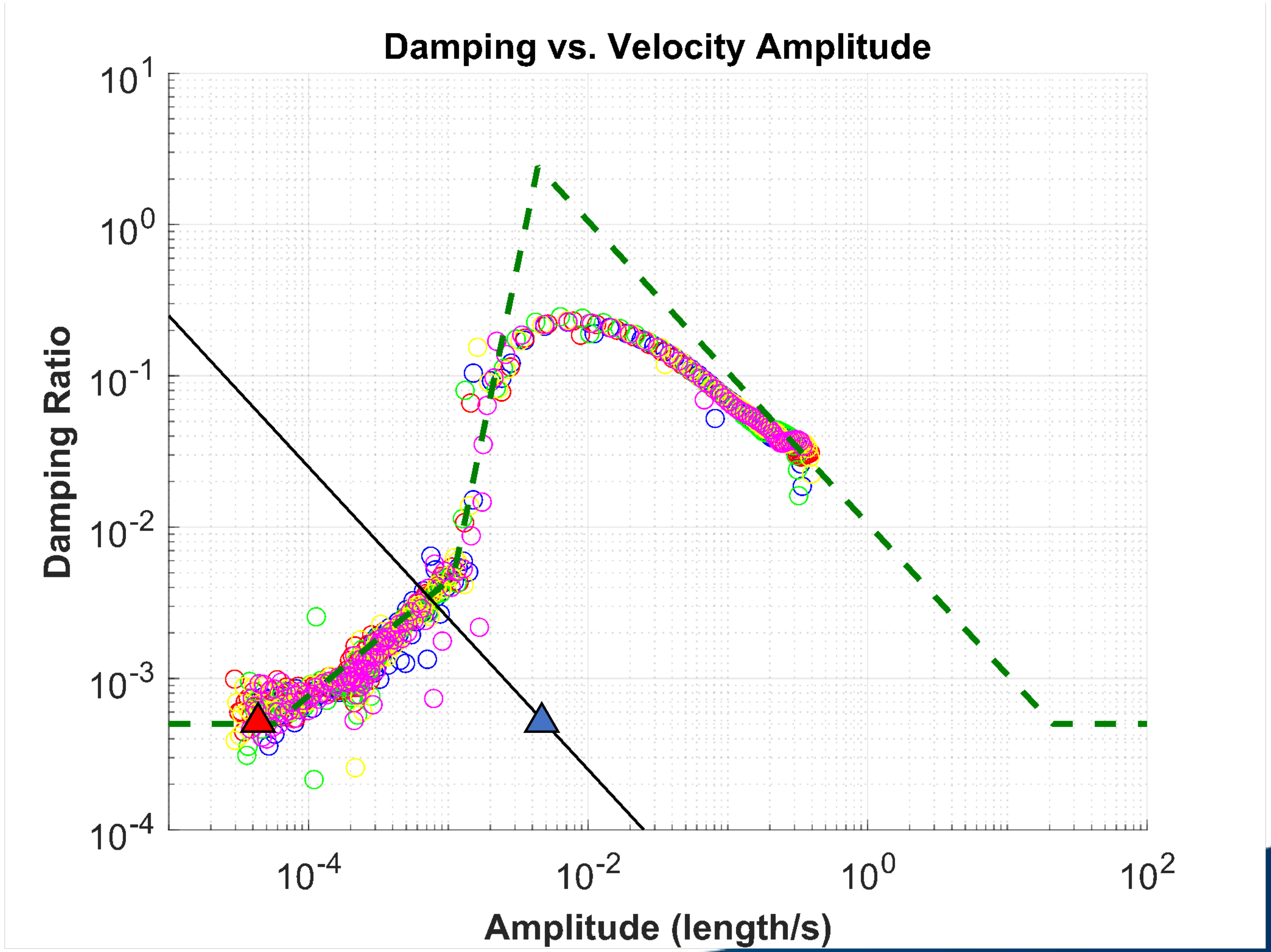
Test for Fatigue Failure (1-2)

$$(V_1, \zeta_1) = (5 \times 10^{-5}, 5 \times 10^{-4})$$

$$\alpha = 100$$

$$(V_{2_{Lin}}, \zeta_1) = (5 \times 10^{-3}, 5 \times 10^{-4})$$

The intersection of the two models is calculated.



Test for Fatigue Failure (1-2)

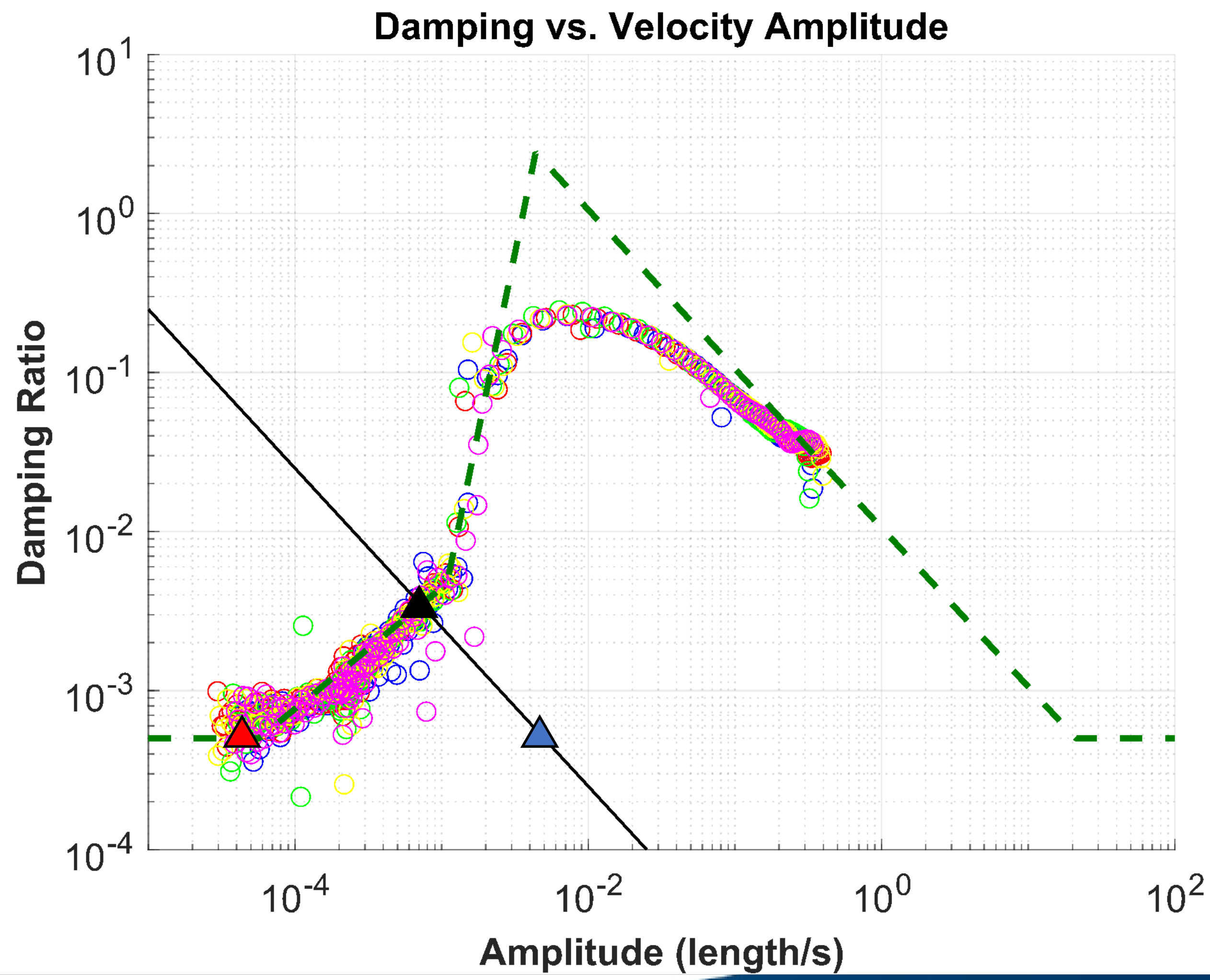
$$(V_1, \zeta_1) = (5 \times 10^{-5}, 5 \times 10^{-4})$$

$$\alpha = 100$$

$$(V_{2_{Lin}}, \zeta_1) = (5 \times 10^{-3}, 5 \times 10^{-4})$$

The intersection of the two models two models is calculated.

$$(V_2, \zeta_2) = (7.2 \times 10^{-4}, 3.5 \times 10^{-3})$$



Test for Fatigue Failure (1-2)

$$(V_1, \zeta_1) = (5 \times 10^{-5}, 5 \times 10^{-4})$$

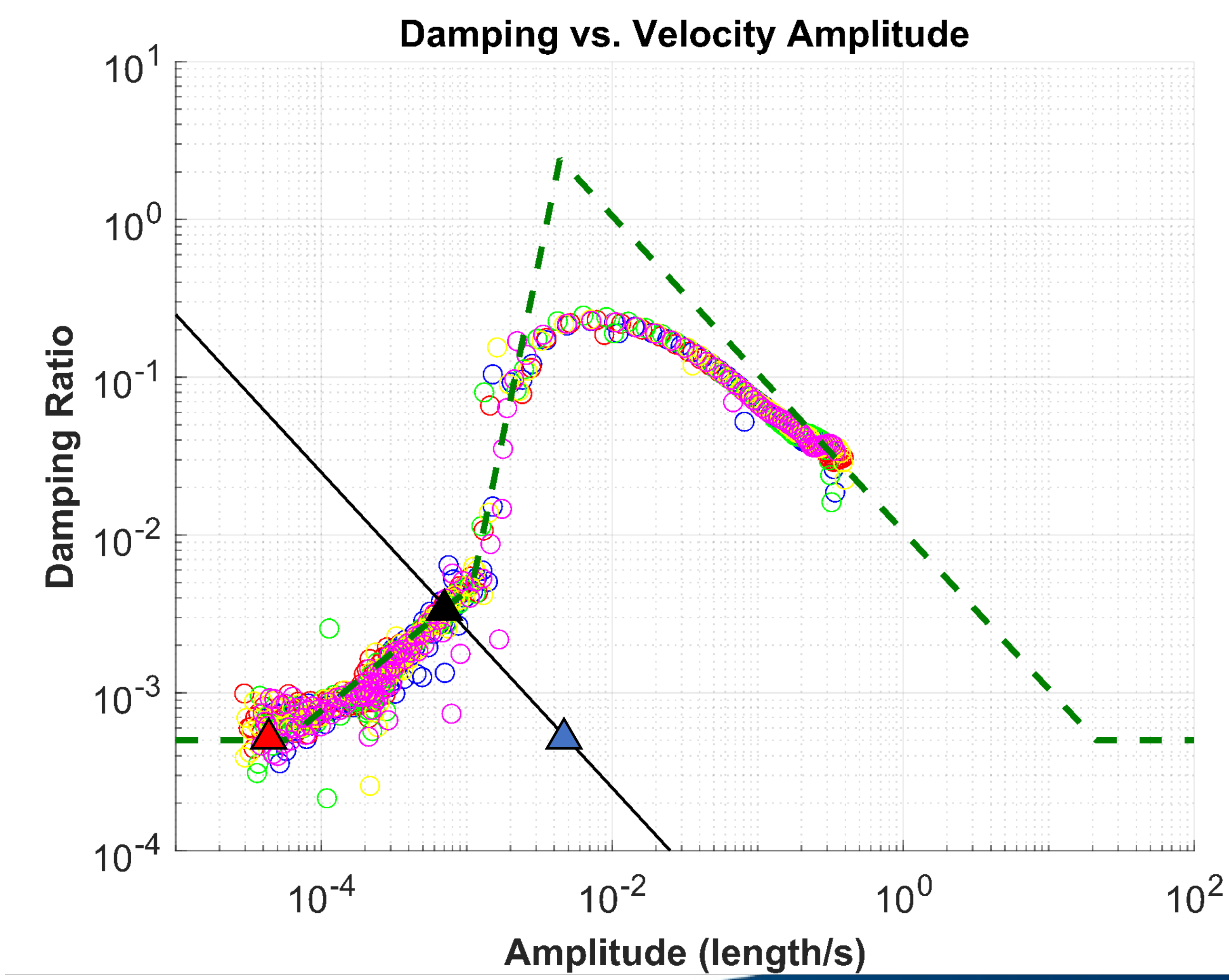
$$\alpha = 100$$

$$(V_{2_{Lin}}, \zeta_1) = (5 \times 10^{-3}, 5 \times 10^{-4})$$

$$(V_2, \zeta_2) = (7.2 \times 10^{-4}, 3.5 \times 10^{-3})$$

$$\frac{V_2}{V_{2_{Lin}}} = 0.145 = -16.8\ dB$$

Response is much smaller than expected.

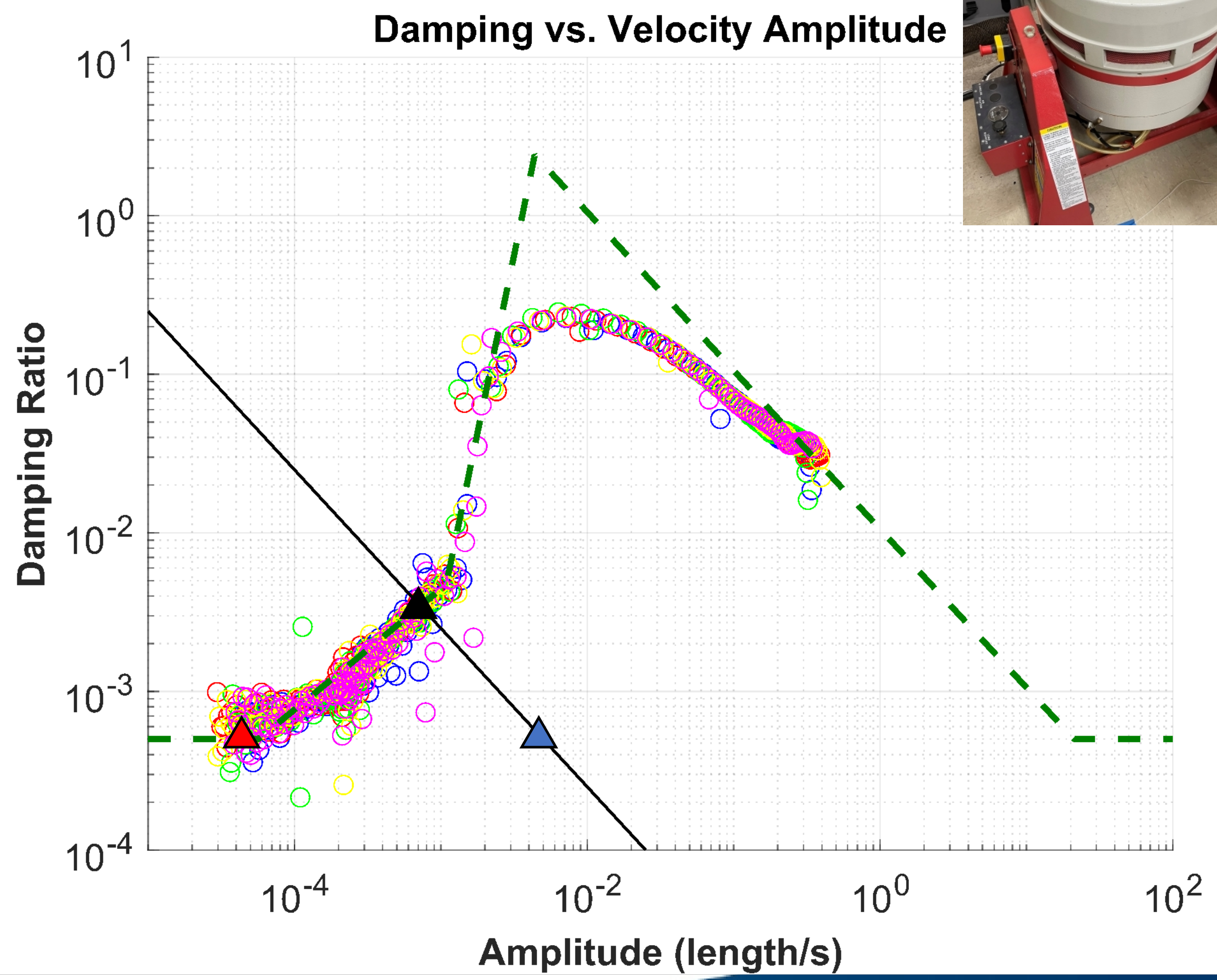


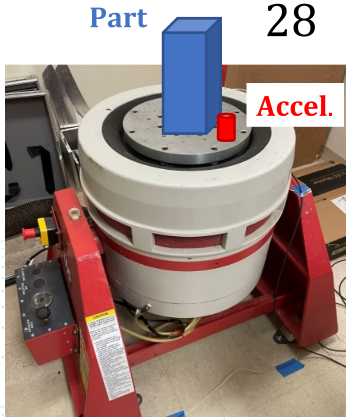
Test for Fatigue Failure (1-2)

$\alpha = 100$ – Increase in base acceleration environment

$V_2 = 14.5$ – Increase in response (or stress)

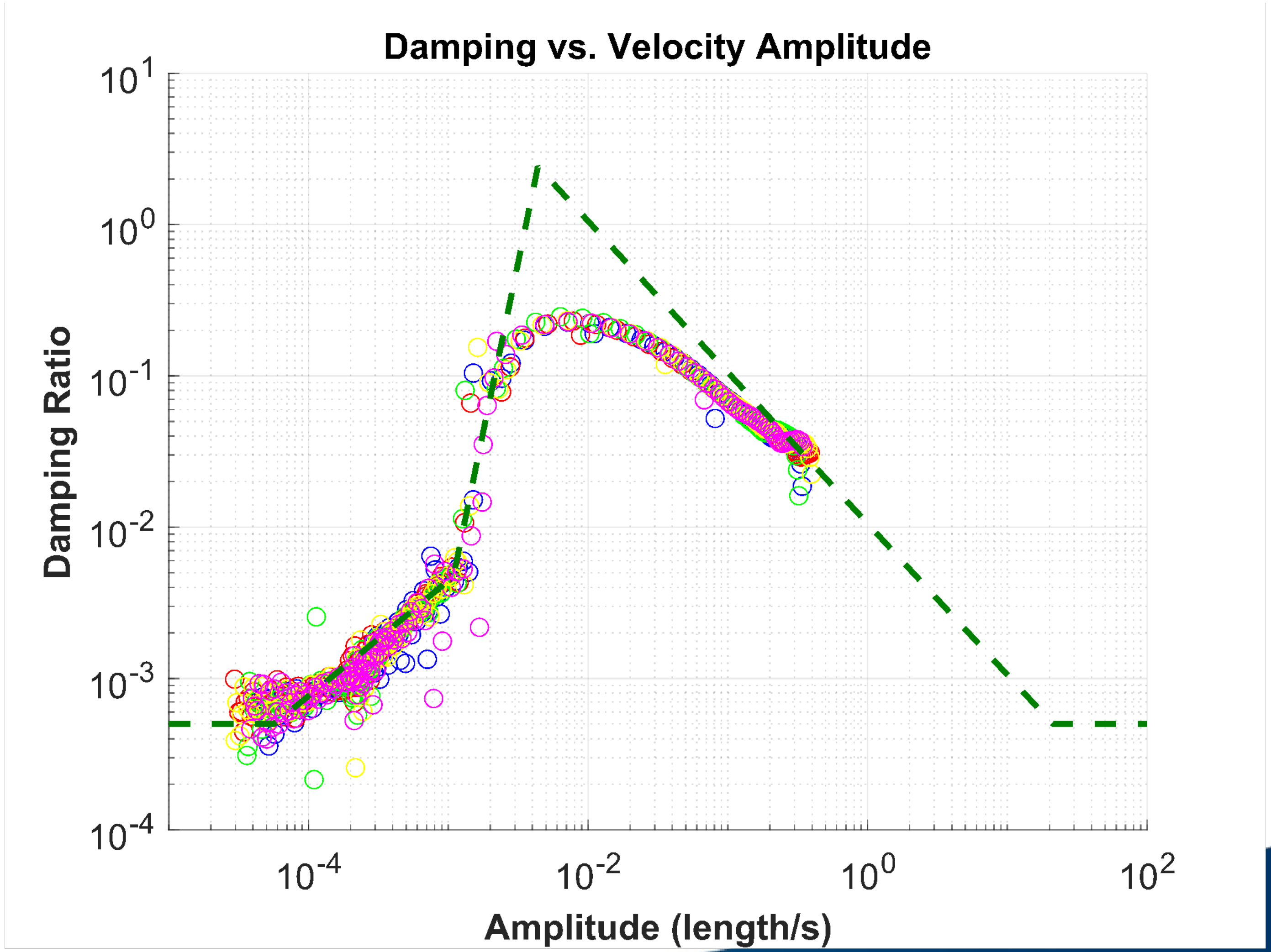
The part can probably survive a much higher base acceleration environment than expected – **overconservative design**!





Test for Fatigue Failure (2-3)

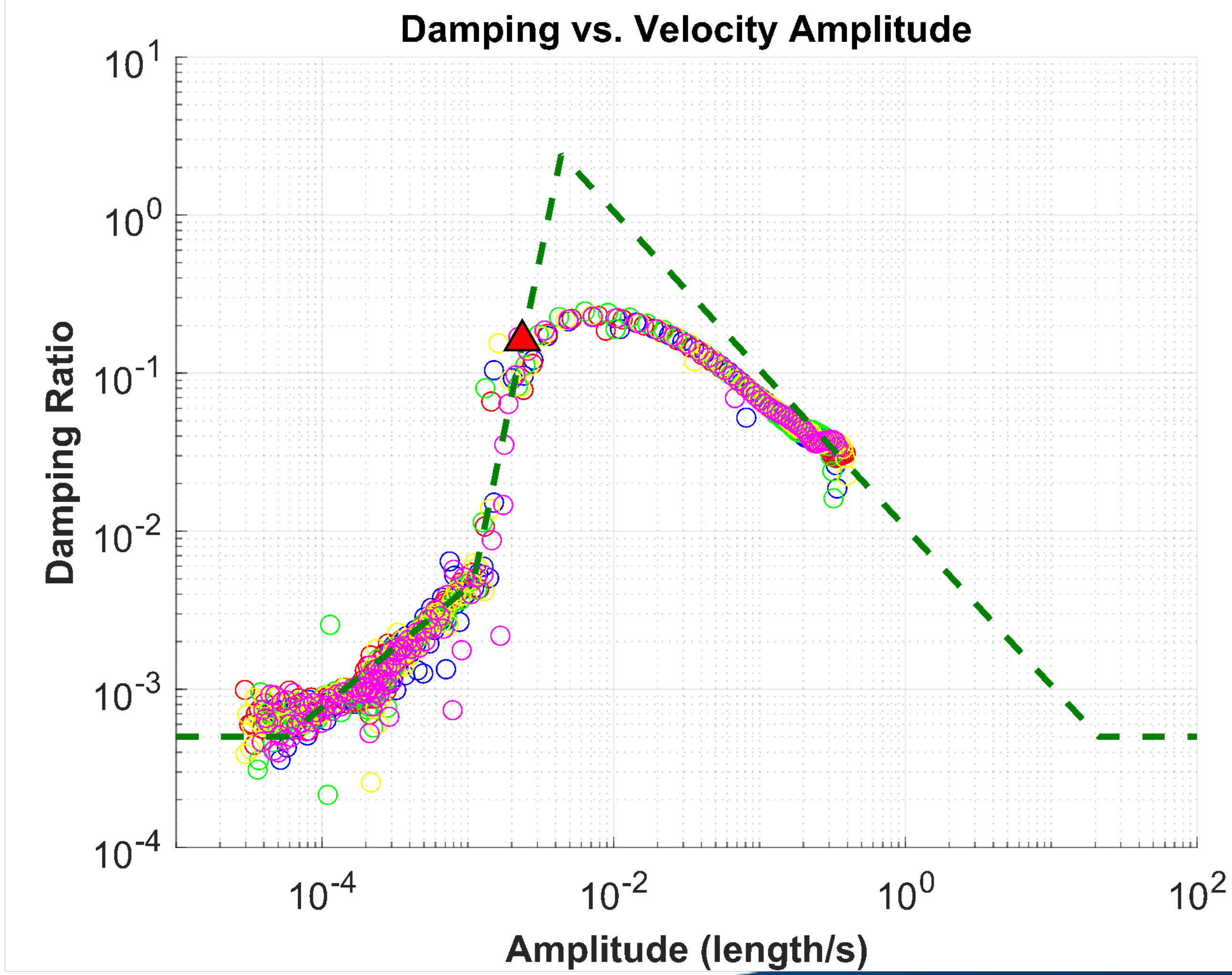
Next we assume that the first measurement occurs in region 2.



Test for Fatigue Failure (2-3)

Next we assume that the first measurement occurs in region 2.

$$(V_2, \zeta_2) = (2.2 \times 10^{-3}, 1.7 \times 10^{-1})$$



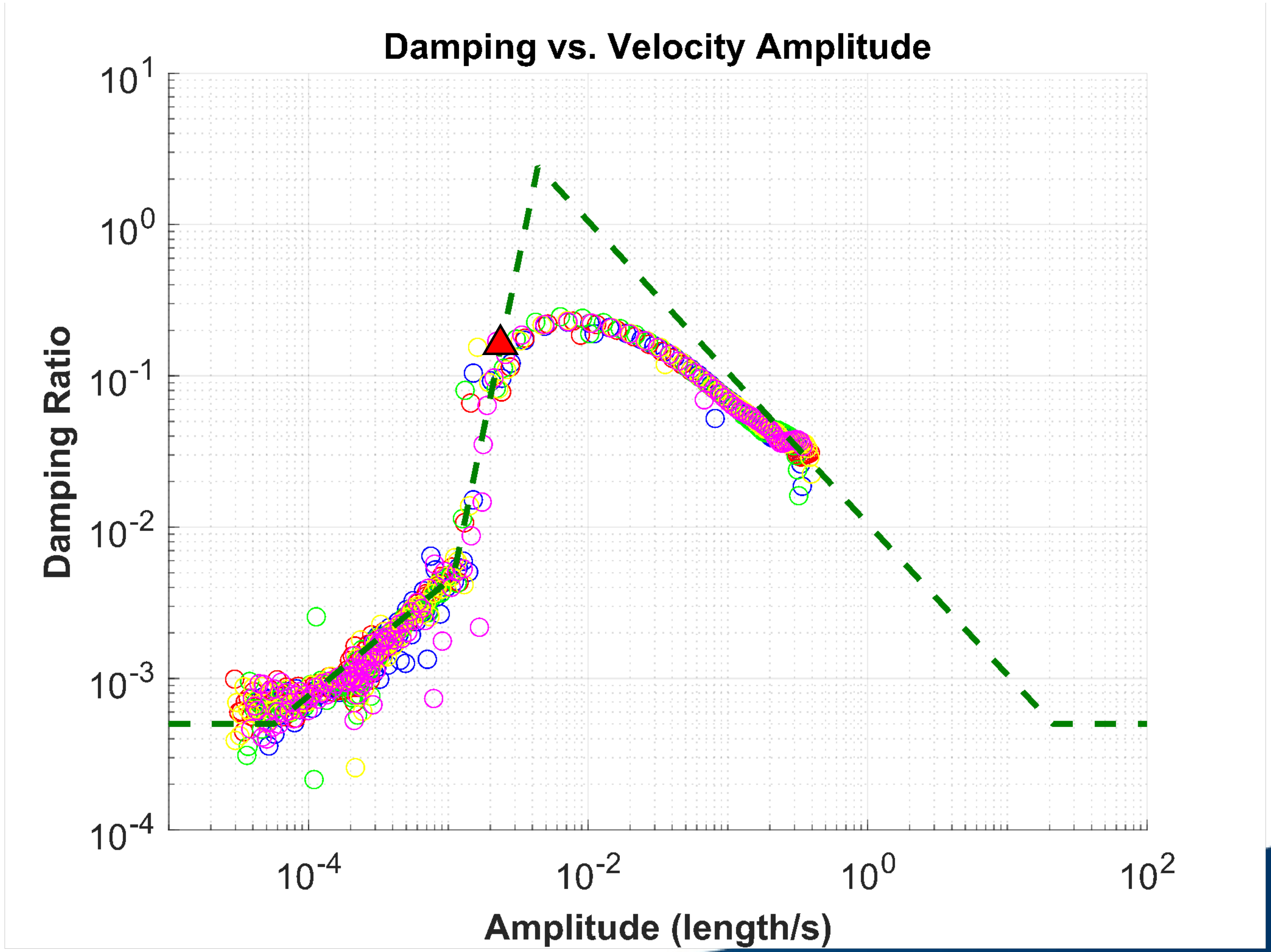
Test for Fatigue Failure (2-3)

$$(V_2, \zeta_2) = (2.2 \times 10^{-3}, 1.7 \times 10^{-1})$$

We select a forcing amplitude magnification

$$\alpha = 100$$

and plot the extrapolation equation.



Test for Fatigue Failure (2-3)

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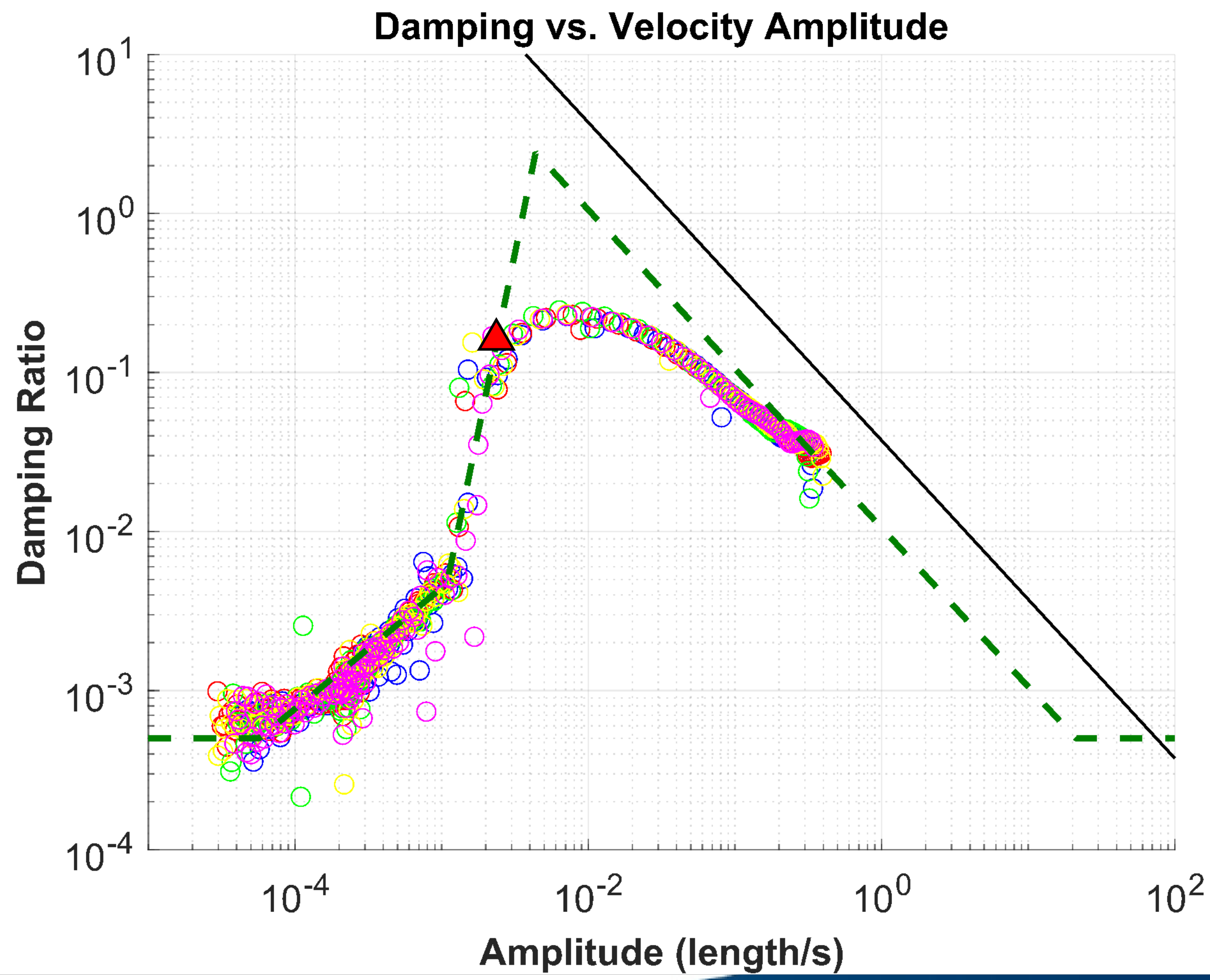


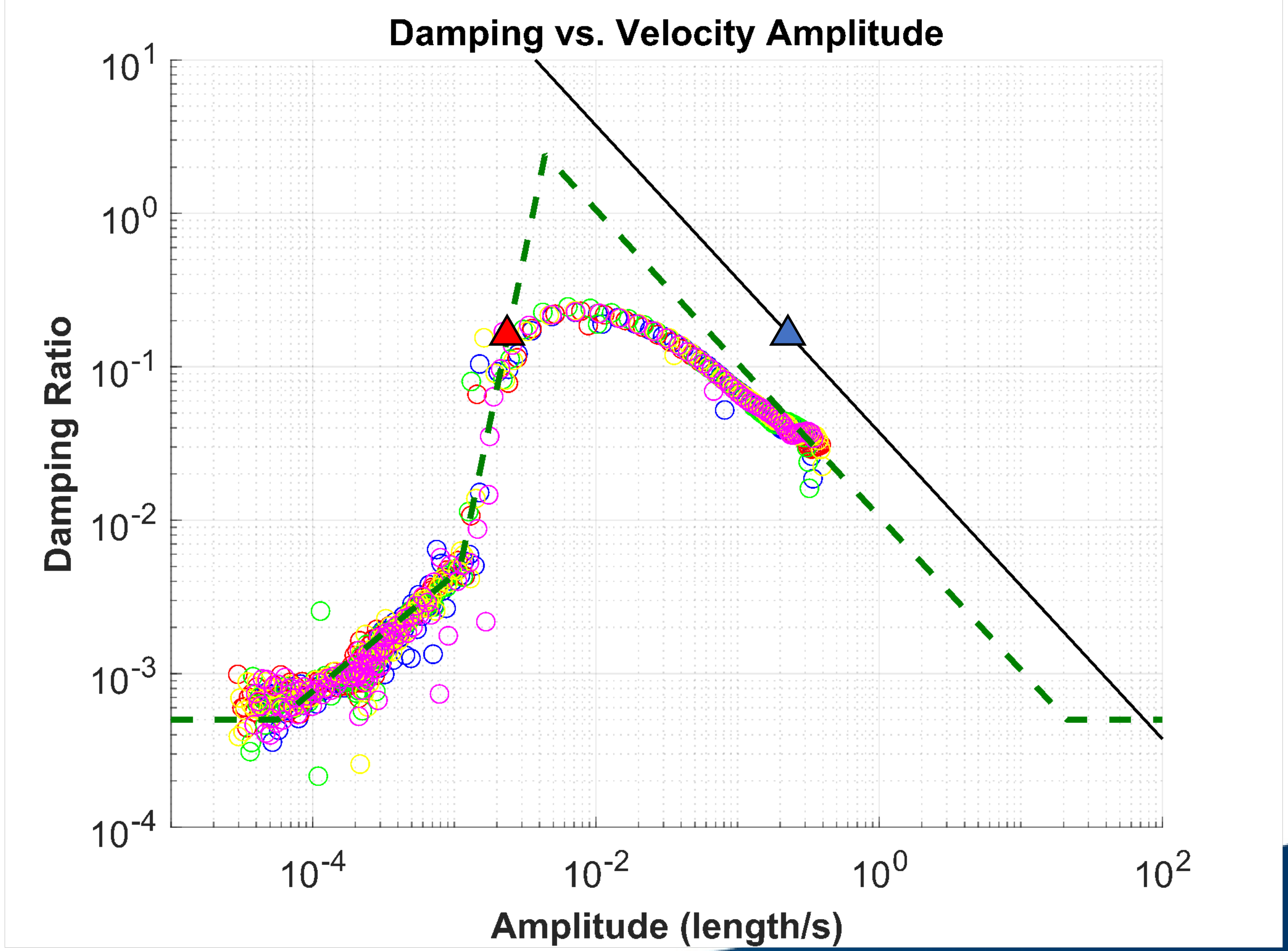
Test for Fatigue Failure (2-3)

$(V_2, \zeta_2) = (2.2 \times 10^{-3}, 1.7 \times 10^{-1})$

$\alpha = 100$

The linear prediction, with constant damping, results in:

$(V_{2_{Lin}}, \zeta_1) = (0.22\,, 1.7 \times 10^{-1})$



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Test for Fatigue Failure (2-3)

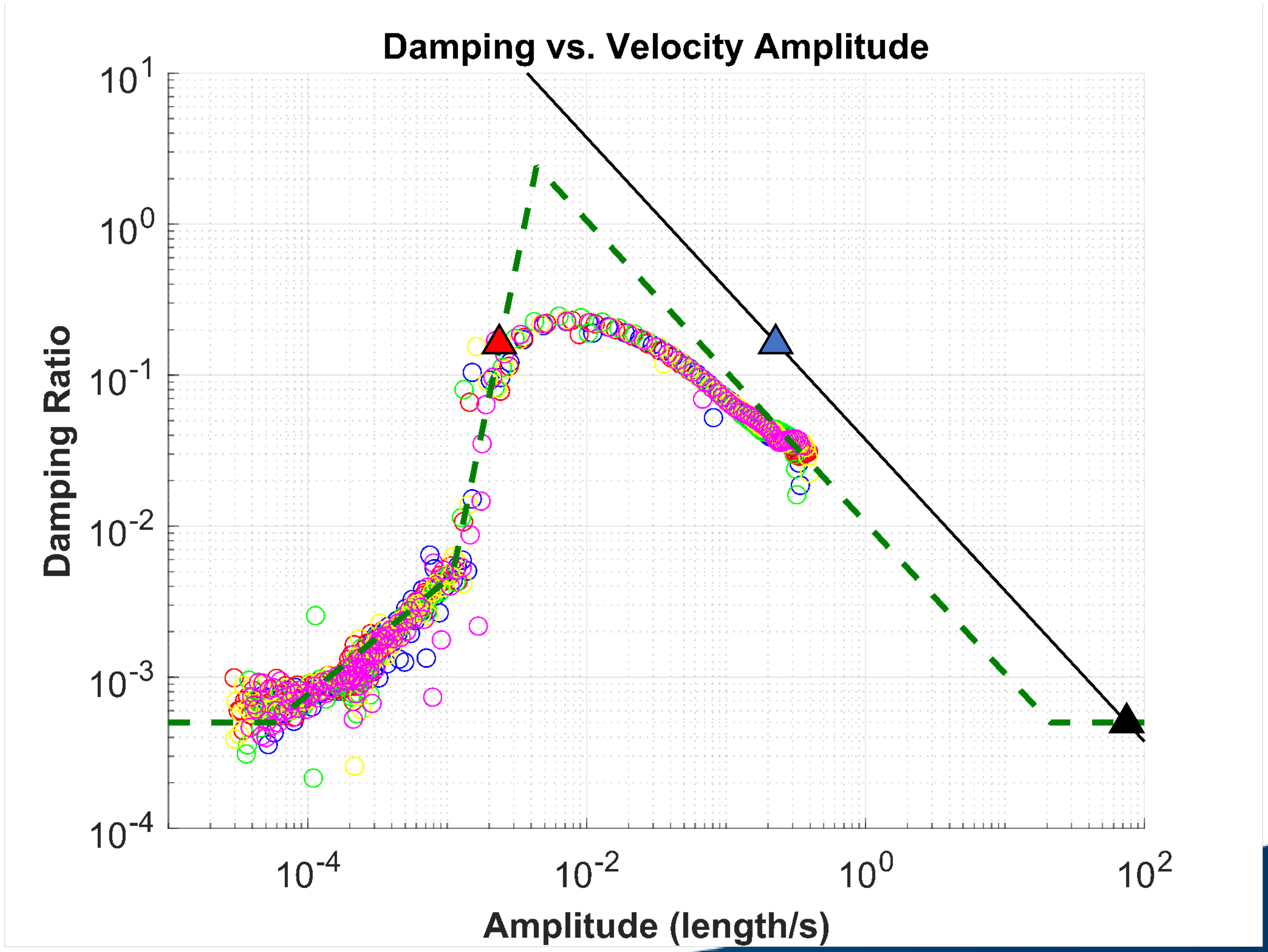
$(V_2, \zeta_2) = (2.2 \times 10^{-3}, 1.7 \times 10^{-1})$

$\alpha = 100$

$(V_{2_{Lin}}, \zeta_1) = (0.22\,, 1.7 \times 10^{-1})$

The intersection of the two models two models is calculated.

$(V_2, \zeta_2) = (74.8, 5 \times 10^{-4})$



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Test for Fatigue Failure (2-3)

$$(V_2, \zeta_2) = (2.2 \times 10^{-3}, 1.7 \times 10^{-1})$$

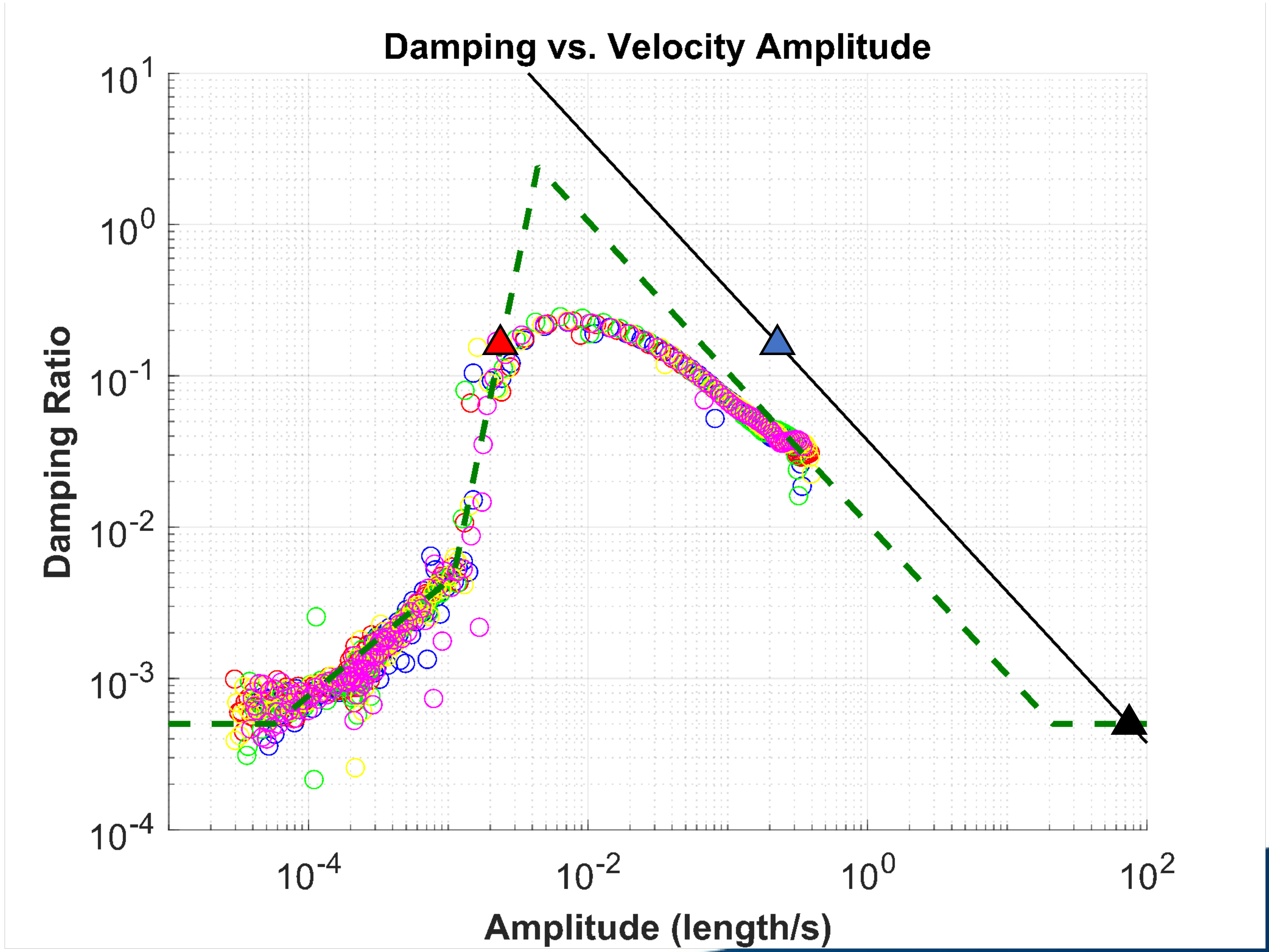
$$\alpha = 100$$

$$(V_{2_{Lin}}, \zeta_1) = (0.22\,, 1.7 \times 10^{-1})$$

$$(V_2, \zeta_2) = (74.8, 5 \times 10^{-4})$$

$$\frac{V_2}{V_{2_{Lin}}} = 340.0 = 50.6\;dB$$

Response is much larger



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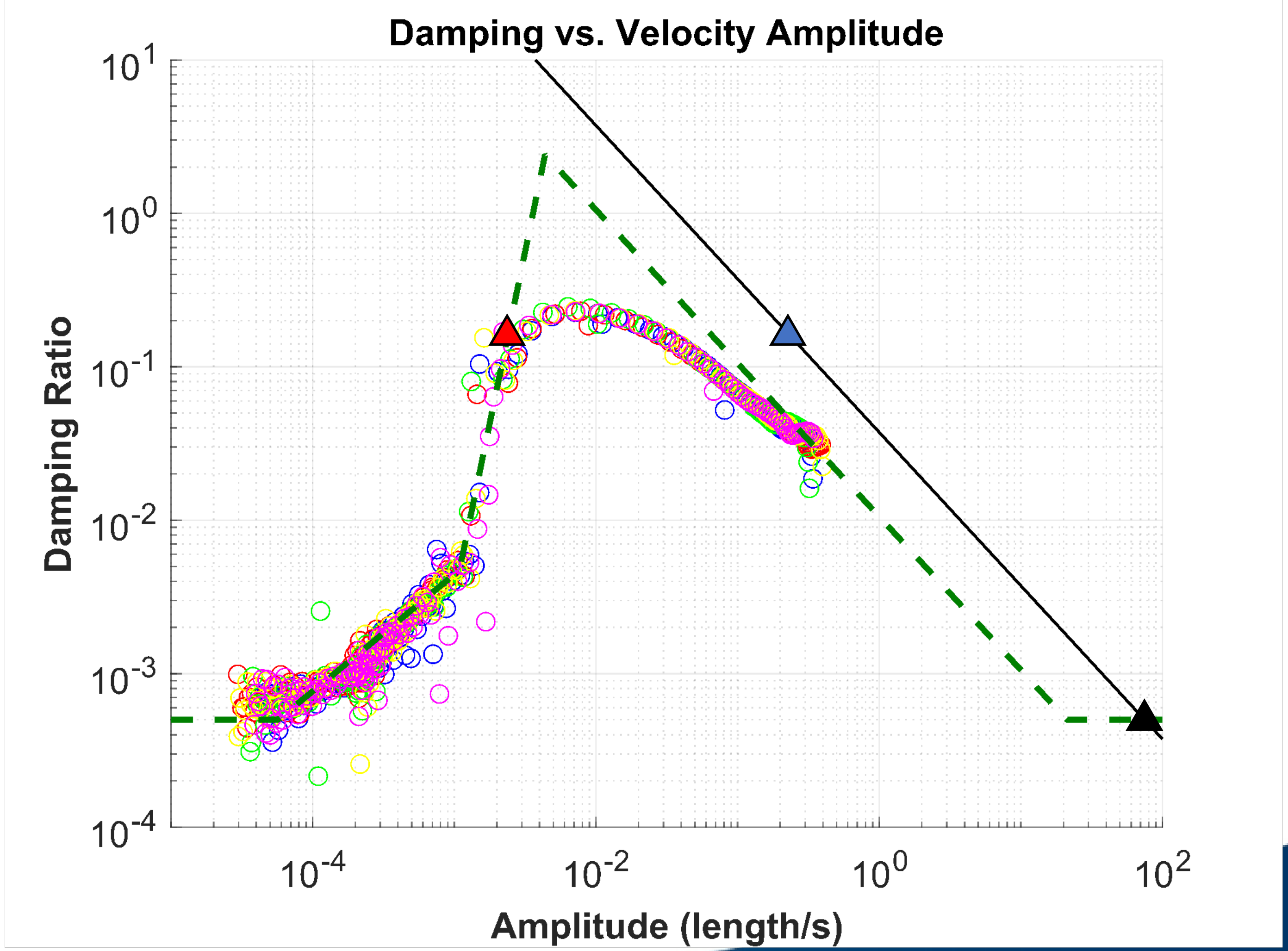
Test for Fatigue Failure (2-3)

$\alpha = 100$ – Increase in base acceleration environment

$V_2 = 34{,}000$ – Increase in response (or stress)

The part fails dramatically as the base acceleration environment is increased.



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What if we instead control to an accelerometer on the part?

Case 1-2

$\alpha = 100$ – Increase in base acceleration environment

$V_2 = 14.5$ – Increase in response (or stress)

Case 2-3

$\alpha = 100$ – Increase in base acceleration environment

$V_2 = 34{,}000$ – Increase in response (or stress)

Effect of Nonlinearity:

Case 1-2 – The shaker will exert much more power than expected to reach the desired environment.

Case 2-3 – A small increase in the shaker power will increase the response greatly – may lead to difficulties in control.

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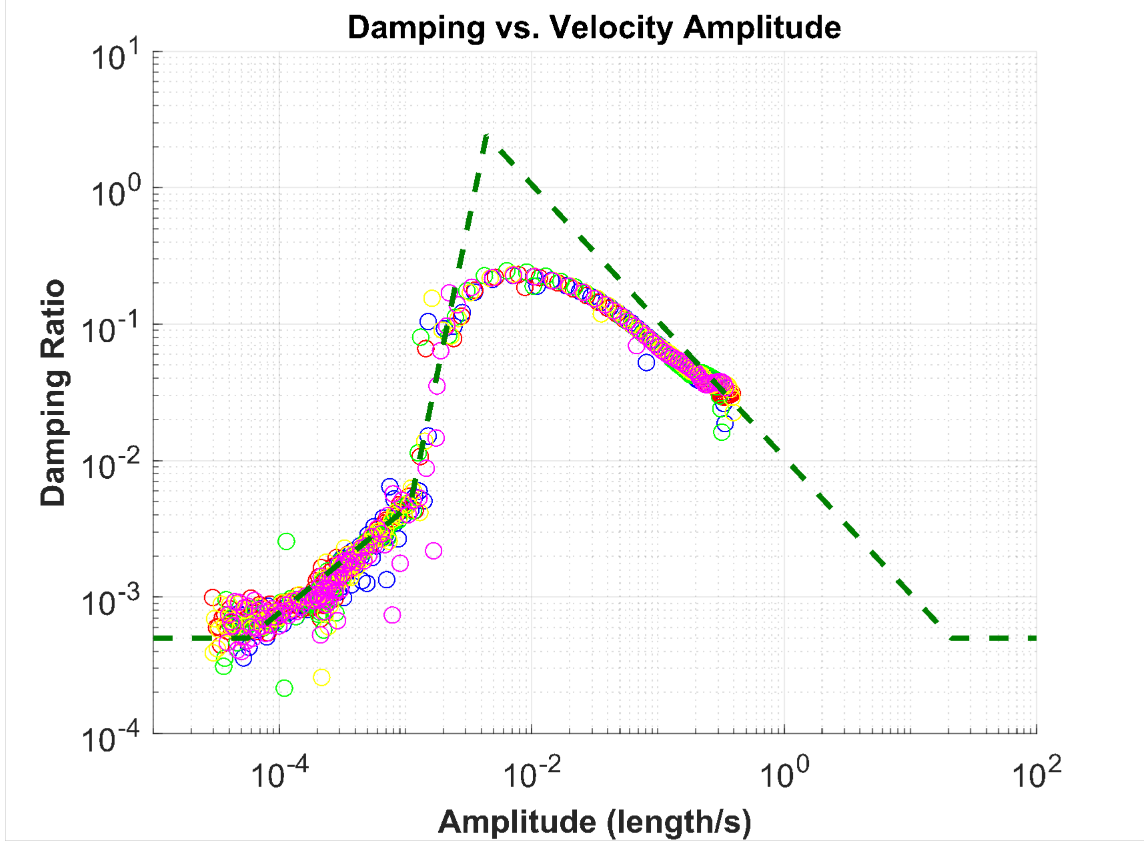
Conclusions

Assuming a linear model can result in major overprediction or underprediction, depending on the location of the initial test data on the damping vs amplitude curve.

This can result in unexpected values of response amplitude and inaccurate values of stress limits and fatigue life.

In different cases, either overprediction and underprediction may be more desirable.

typical micro-slip nonlinearity



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Future Plans

- Mode 1: Bilinear nonlinearity
- Material nonlinearity

Mode 1 Nonlinearity

Mode 1 will also be explored.

It has an open-close bilinear nonlinearity, which we plan to model using higher order polynomial spring stiffnesses.

The model will be fit to experimental data. The model will then be used to predict response amplitude, peak stress, PSD, and expected fatigue life.

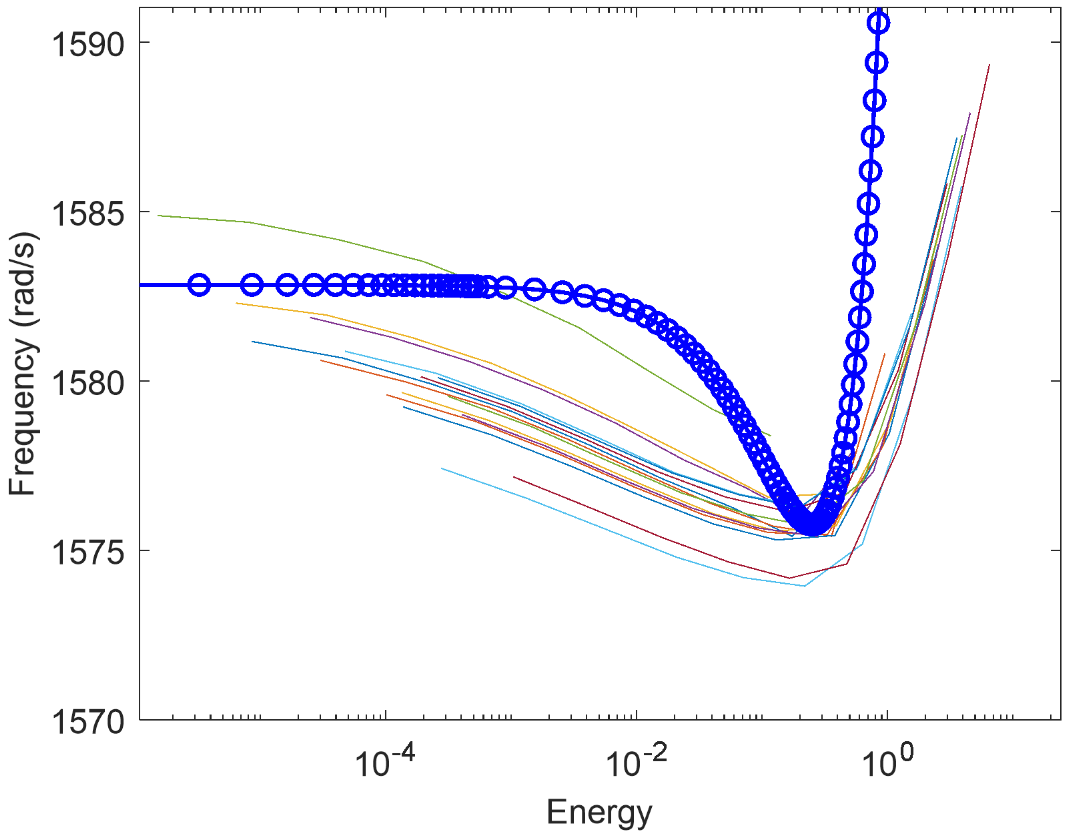

Figure showing progress in fitting an NNM branch to the experimental data.

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S-N Diagram for Steels

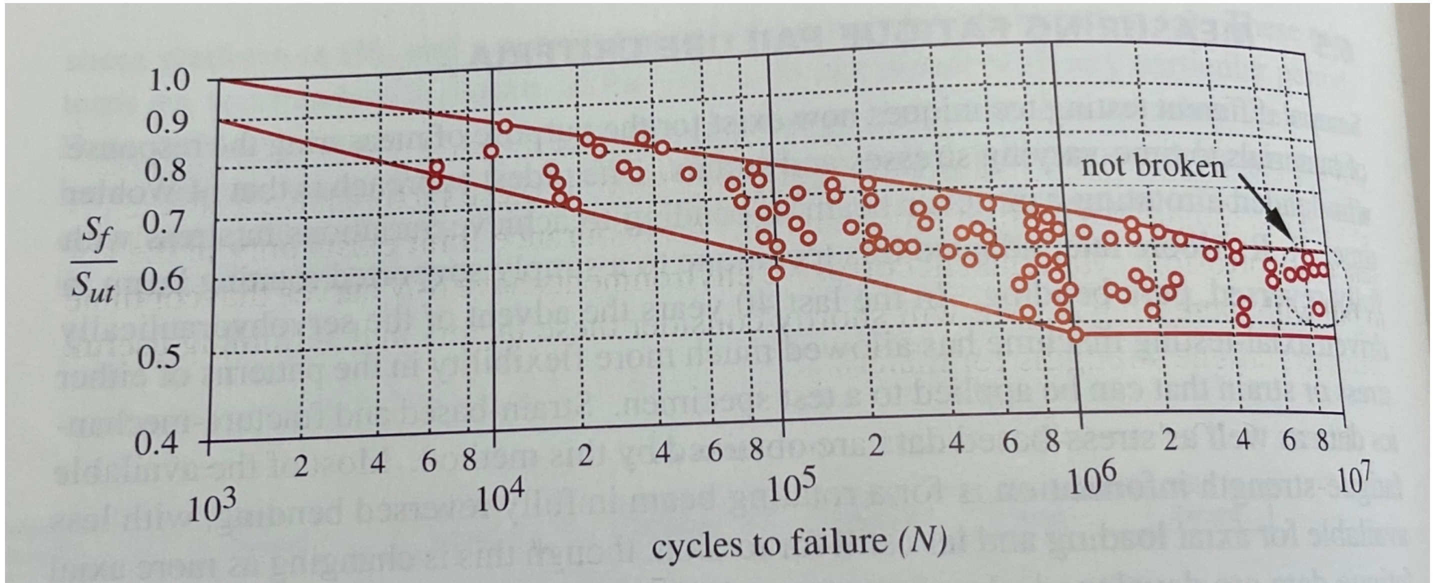
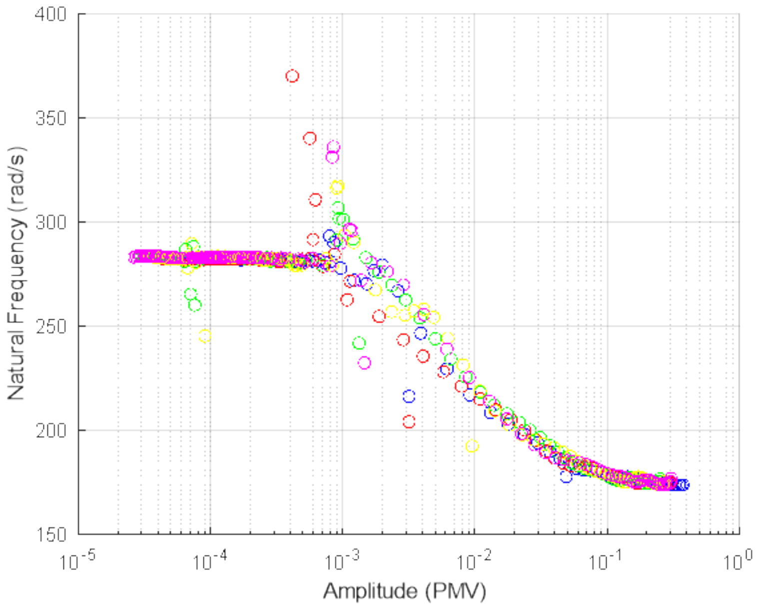


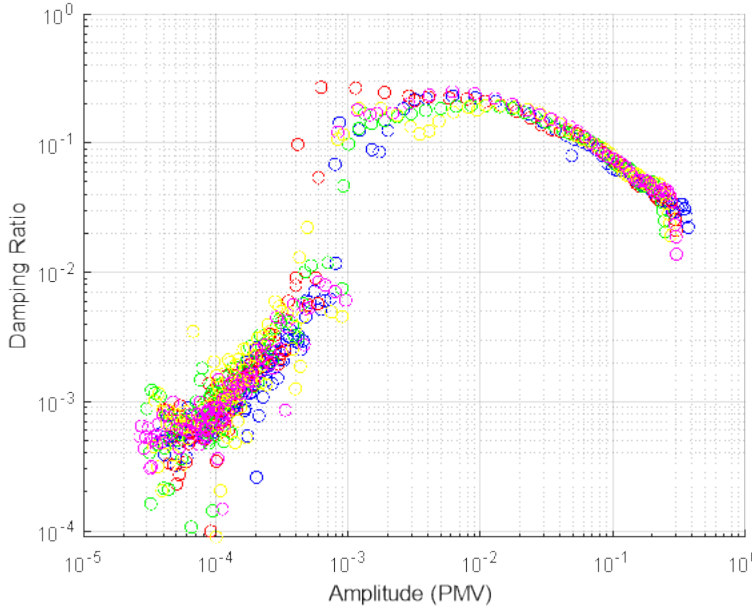
FIGURE 6-8

Log-Log Plot of Composite S-N Curves for Wrought Steels of $S_{ut} < 200$ ksi (From Fig. 11.7, p. 210, R. C. Juvinall, *Stress, Strain, and Strength*, McGraw-Hill, New York, 1967, with permission)

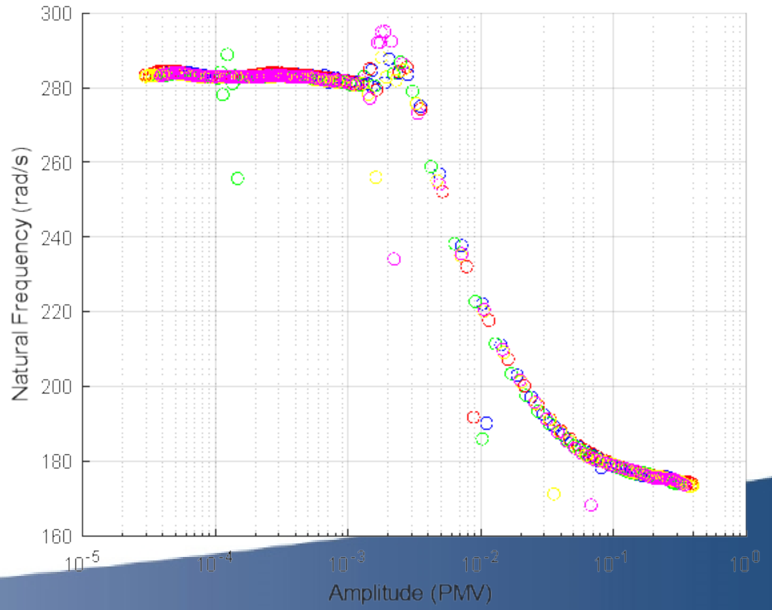
Natural Frequency and damping vs Amplitude Data

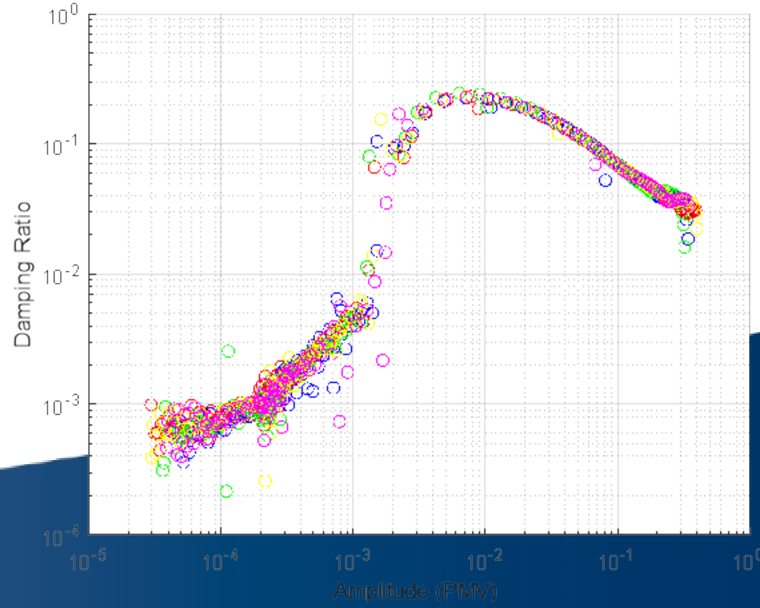
DP2





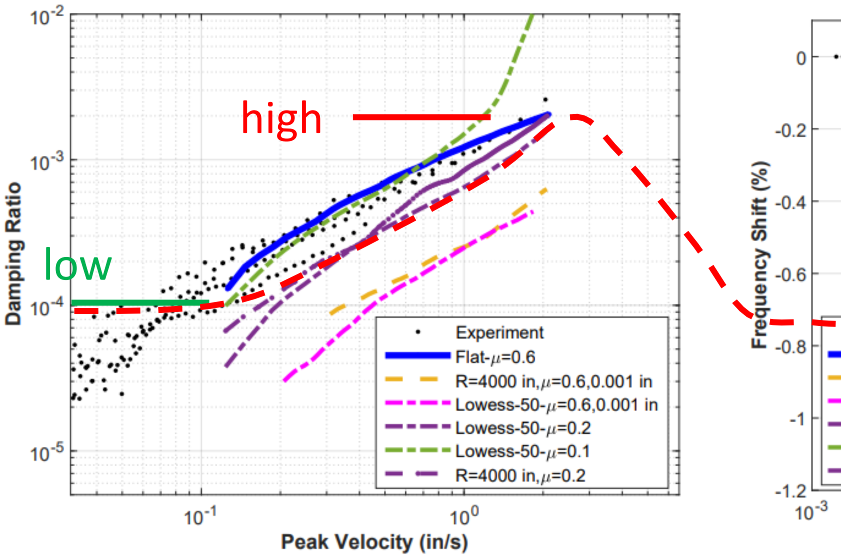
DP4

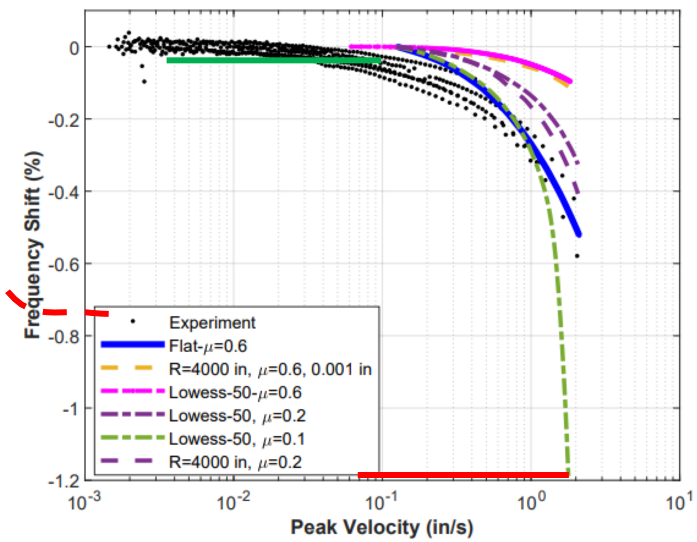




Test for Fatigue Failure (1-2)

- Tuned to low-amplitude test
  - Testing/correlation is performed at $V_{p1} = 0.1$ in/s
  - $\zeta_1 = 0.0001 \rightarrow$ linear approximation ($\zeta_2 = 0.0001$).
  - If we cannot test to higher levels: (base excitation = force)
    - Assume that the response at $10F_{p1}$ will be $V_{LinPred} = 10V_{p1} = 1.0$ in./s
    - But if $V_{p2} = 1.0$ then the damping would be 20x higher and so this response is not possible with $\alpha = 10$.





    - Solving for the intersection, we find that the equation is satisfied if the damping ratio is $\zeta_2 = 0.0003$ and $v_{p2} = 0.3$ in/s

$V_{p2} = 10\left(\frac{0.0001}{0.0003}\right)\left(\frac{\omega_{n1}}{\omega_{n2}}\right)0.1 \approx 0.3$ in/s   We thought $V_{p2} = 1.0$, Overconservative by a factor of about 3

    - $S_f(0.3 \text{ in/s}) \approx 1.85$ ksi, so $\frac{S_f}{S_{ut}} = 0.025$ and $\boldsymbol{N} = \infty$ Life still expected to be infinite

  - If we can test to higher levels:
    - Typically the base input (or force) is applied and there is no measure of response (so Vp isn't known). Hence, if the force increases 10x the internal response would only increase 3x and we would not know that we have only stressed the part 3x more rather than 10x more. The force must increase 100x to obtain a 10x increase in V_p and hence stress.
    - Because so much force is required, we're unlikely to observe failure in test.
    - Hence, all tests will succeed, and we will have as much margin as we are able to test to.

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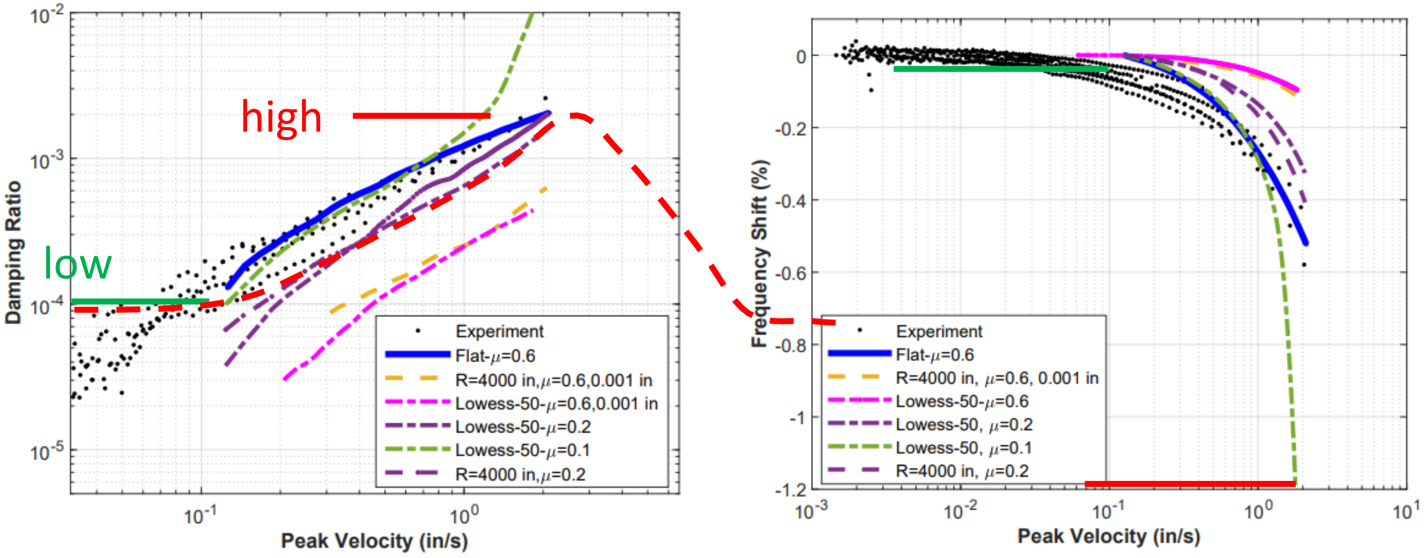
Approach for Fatigue Failure Criterion (2-3)

- Tuned to high-amplitude test
  - Testing/correlation performed at $V_{p1} = 2.0$ in/s
  - $\zeta_1 = 0.002 \rightarrow$ linear approximation ($\zeta_2 = 0.002$).
  - If we cannot test to higher levels: (base excitation = force)
    - Assume that the response at $10F_{p1}$ will be $v_{LinPred} = 10v_{p1} = 20.0$ in/s.
    - Damping ratio is actually $\zeta_2 = 0.0001$

$$V_{p2} = 10\left(\frac{0.002}{0.0001}\right)\left(\frac{\omega_{n1}}{\omega_{n2}}\right)2.0 \approx 400\ in/s$$

    - $S_f(400\ \text{in/s}) \approx 110$ ksi
    - $\frac{S_f}{S_{ut}}$=1.51
    - $\boldsymbol{N = 0}$



Our life estimate at $S_f(400)$ *is* $N = 0$ so we're underconservative.

- If we can test to higher levels:
  - We will observe that the response of the component increases quickly as the input increases, because the damping is decreasing.
  - If we can increase the forcing amplitude enough then we are likely observe failure in test and to calculate an accurate margin.

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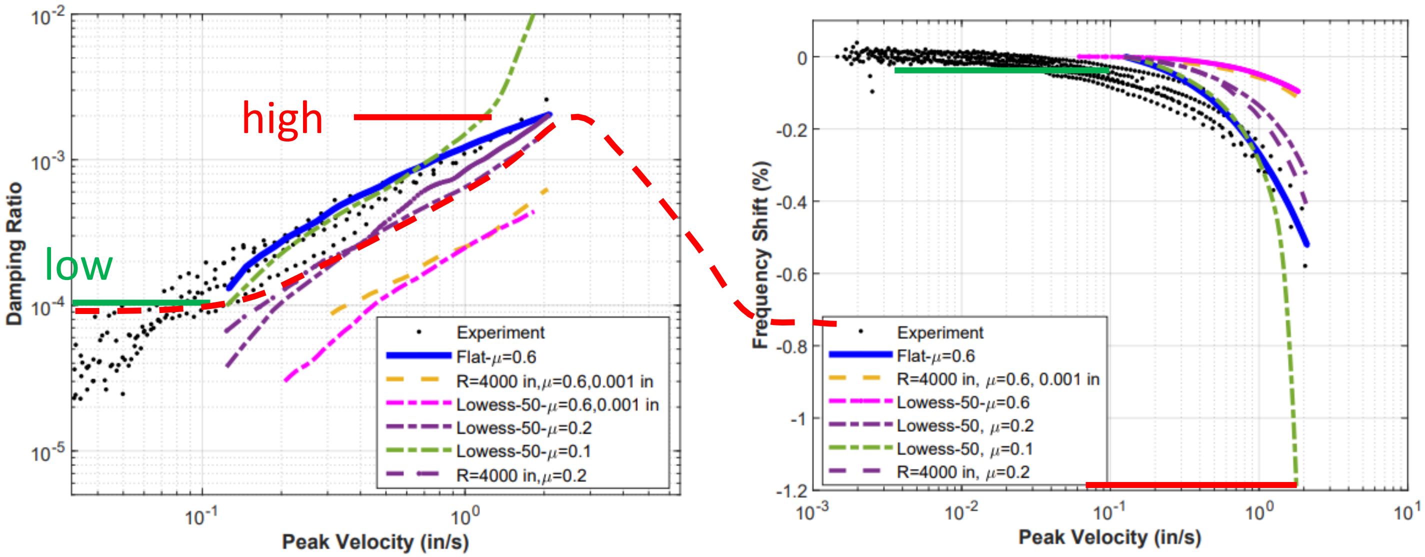
Approach for Fatigue Failure Criterion (2-2)

- Tuned to high-amplitude test
  - Testing/correlation performed at $V_{p1} = 2.0$ in/s
  - $\zeta_1 = 0.002 \rightarrow$ linear approximation ($\zeta_2 = 0.002$).
  - If we cannot test to higher levels: (base excitation = force)
    - Assume that the response at $2F_{p1}$ will be $V_{p2} = 2V_{p1} = 4.0$ in/s.
    - Damping ratio becomes $\zeta_2 = 0.001$

$$V_{p2} = 2\left(\frac{0.002}{0.001}\right)\left(\frac{\omega_{n1}}{\omega_{n2}}\right)2.0 \approx 8\ in/s$$

    - Need to iterate a bit, but our solution is near 4-8 in/s.
    - $S_f(4\text{ in/s} - 8\text{ in/s}) \approx 1.1 - 2.2$ ksi
    - $\frac{S_f}{S_{ut}}$=0.014-0.028,
    - $\boldsymbol{N} = \infty$



Our life estimate is still $N = \infty$ so we're on target.

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