SAND2023-12295C

Nonlinear dynamics, continuation, and stability analysis of a shaft-bearing assembly

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IMAC-XLI (41), submission #14375

February 13-16, 2023

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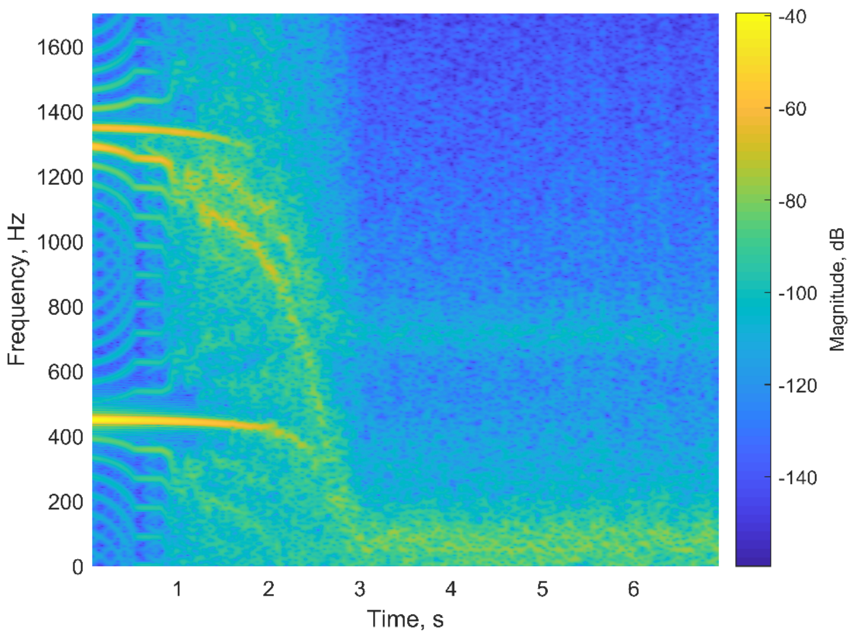
Presentation outline

- Introduction and motivation
- System formulation and numerical methods
- Linear analysis
- Nonlinear analysis
- Nonlinear system with asymmetry
- Conclusions

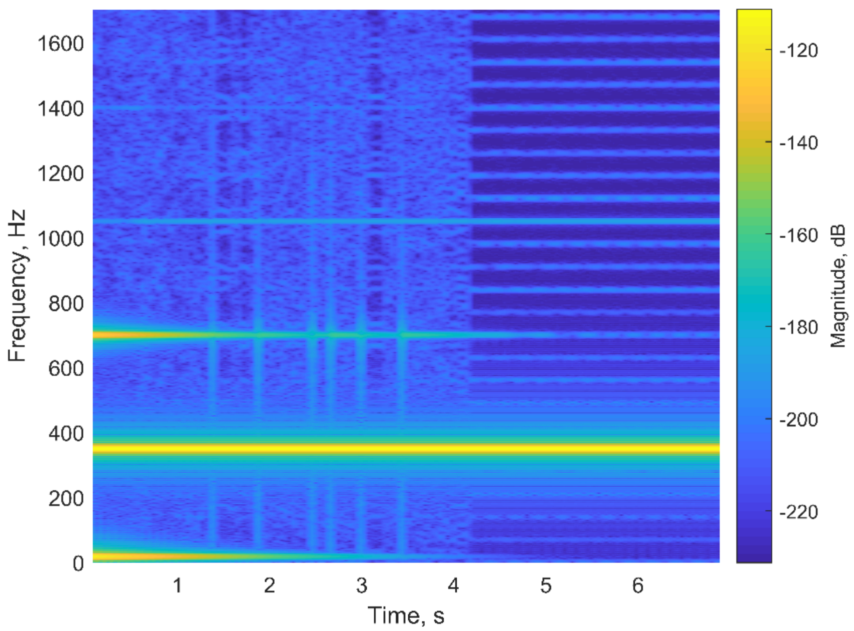
Introduction and motivation

- **Bearing-mounted shaft structures are common in mechanical engineering systems**
  - Can be subject to various nonlinear phenomena: large deformations, friction, intermittent contact, etc.
  - Micro-clearances within the bearings can lead to contact/impact nonlinearities
  - Other factors can cause shock or vibration effects: base excitations, unbalance, gear dynamics, …

- **Shock environments**
  - High amplitude, short time duration
  - Often activates broadband frequency content
  - Decays to rest after the shock event
  - Energy can transfer between modes



- **Vibrational environments**
  - Low amplitude, long time duration
  - Often activates isolated frequency content (resonances)
  - Steady, non-decaying oscillation
  - Energy can transfer between modes

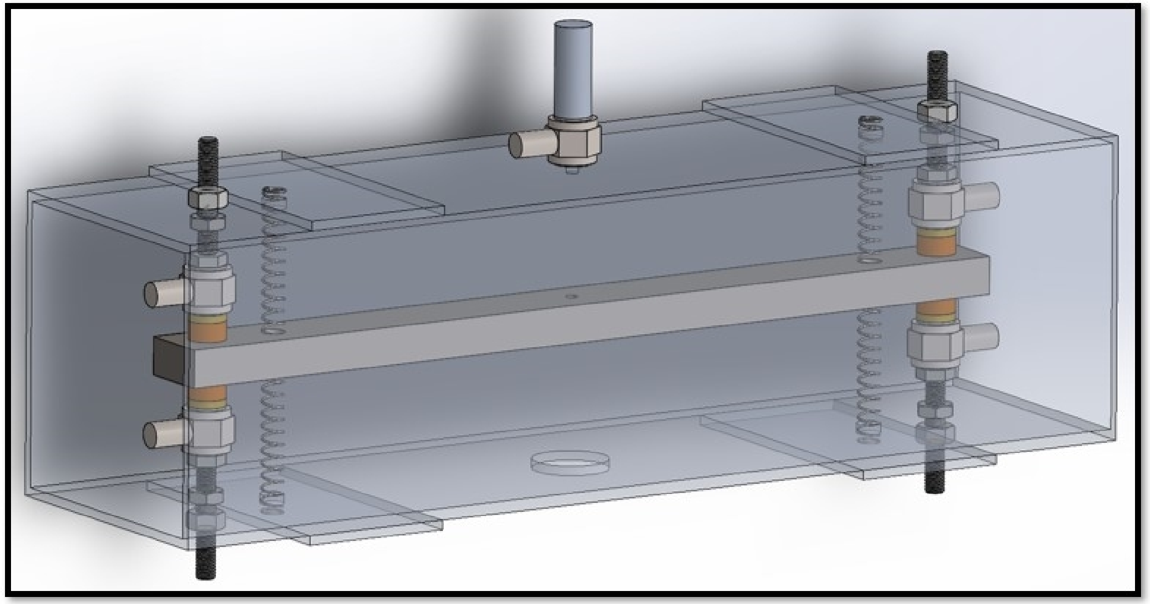


Introduction and motivation

- **System of interest:**
  - Goldberg et al. studied a non-rotating shaft-bearing assembly, both numerically and experimentally
  - Transient ringdown and nonlinear modal analysis showed interesting behavior

- **Questions:**
  - What types of behavior can we see in the system under a vibrations environment? Chaos? Nonlinear resonances? Isolas?
  - Explore the effects of asymmetries in the system, such as unequal stiffnesses or gap sizes

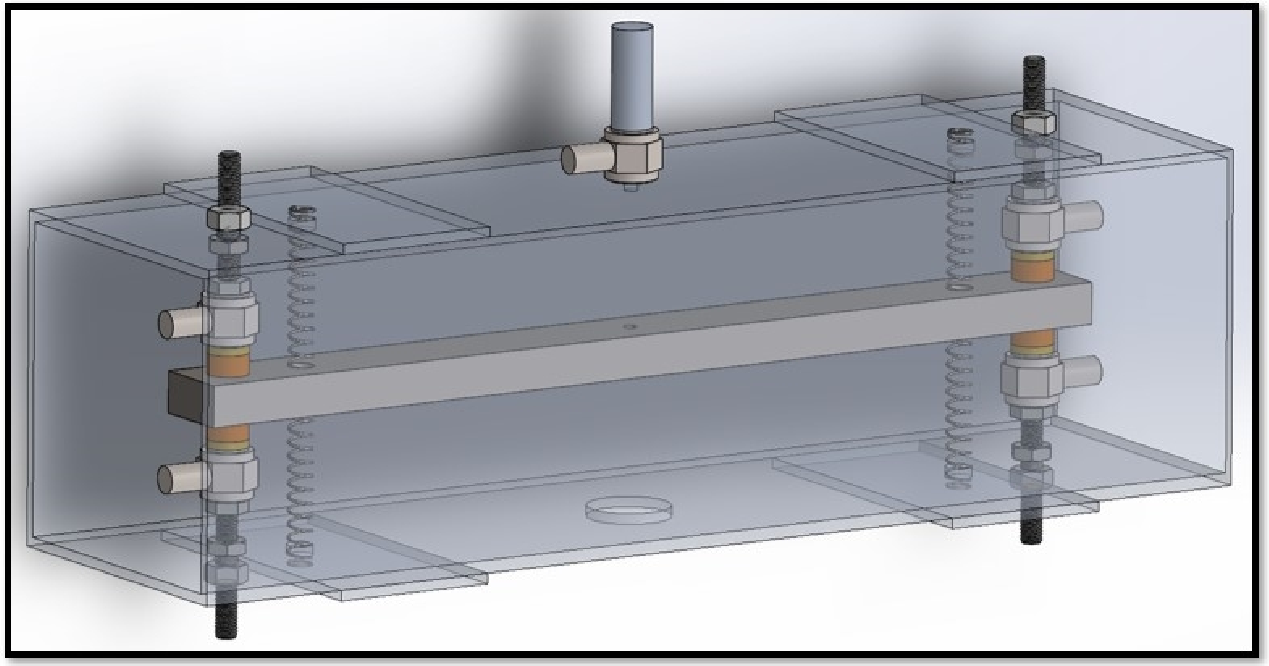
- **Goals of this work:**
  - Characterize the complex nonlinear dynamics in a non-rotating shaft-bearing assembly subject to forced vibration
  - Inform and design further experiments



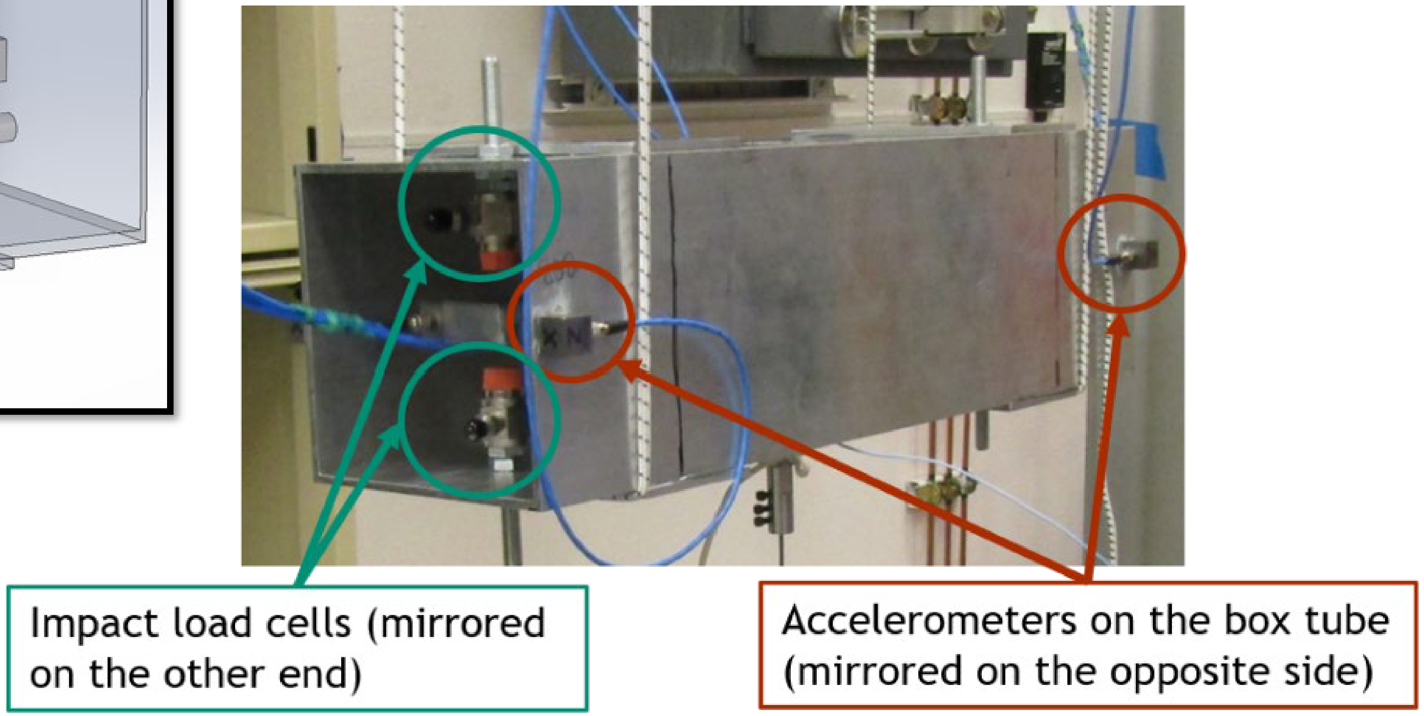
Goldberg, N.N., Demsky, S., Youssef, A.A., Carter, S.P., Fowler, D., Jackson, N., Kuether, R.J., and Steyer, A., "Experimental and computational investigation of nonlinear dynamics of a simplified bearing-and-shaft assembly," 40th International Modal Analysis Conference (IMAC XL), Orlando, FL, February 7-10, 2022.

System formulation and numerical methods

❑ Shaft-bearing assembly



- This assembly is a simplified physical model of a non-rotating, bearing-mounted shaft with micro-clearances
- The system can be excited by an impact hammer or a shaker
- Adjustable contact heads—soft, medium, hard
- Adjustable contact gaps via threaded rods
- Interchangeable springs



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System formulation and numerical methods

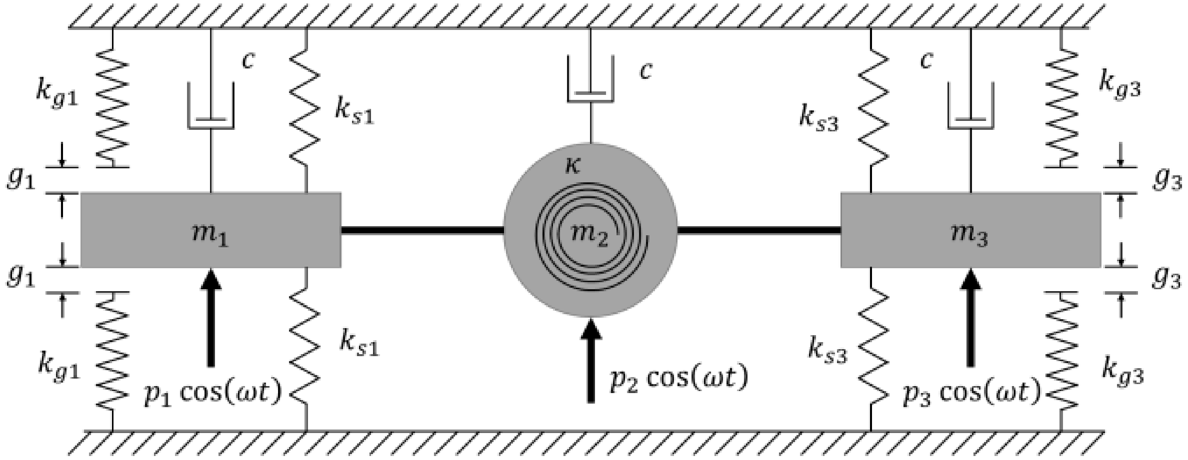
❑ 3 DOF simplified model equations of motion

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} 2k_{s1} + \frac{\kappa}{L^2} & \frac{-2\kappa}{L^2} & \frac{\kappa}{L^2} \\ \frac{-2\kappa}{L^2} & \frac{4\kappa}{L^2} & \frac{-2\kappa}{L^2} \\ \frac{\kappa}{L^2} & \frac{-2\kappa}{L^2} & 2k_{s3} + \frac{\kappa}{L^2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} f_{g1}(x_1) \\ 0 \\ f_{g3}(x_3) \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}\cos(\omega t),$$

$$f_{gi}(x_i) = \begin{cases} k_{gi}(x_i + g_i), & x_i < -g_i \\ 0, & -g_i \le x_i \le g_i, \\ k_{gi}(x_i - g_i), & x_i > g_i \end{cases} \qquad i = 1,3$$

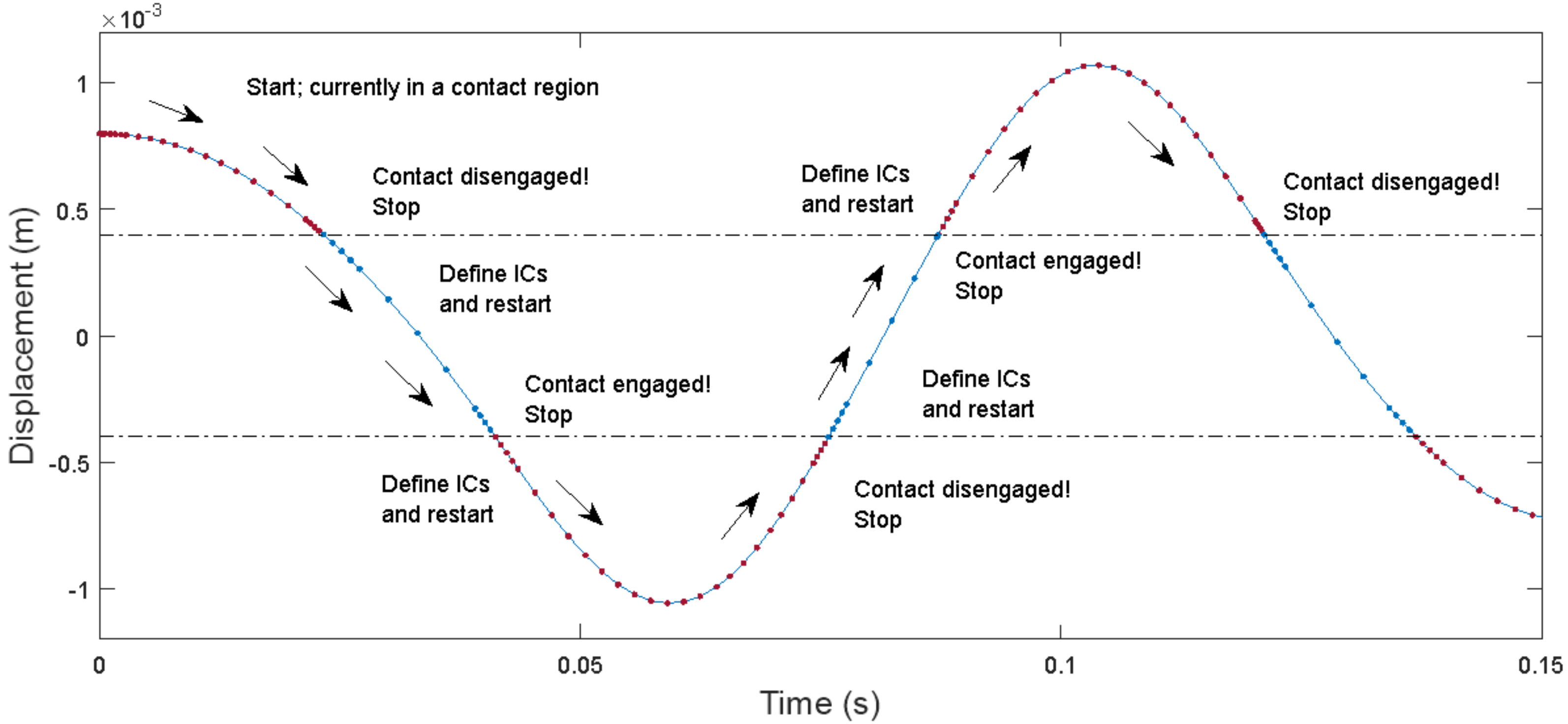
Assumptions:

- Penalty stiffness contact law
- Frictionless, undamped contacts
- Equal linear viscous damping
- Gravity ignored



System formulation and numerical methods

- **Simulations are done using Matlab® ode45 with *Event Location***
  - Piecewise time integration, which prevents accumulating roundoff error
  - A timestep is always forced at every instance of contact to ensure accuracy
  - Past validation has shown good results



Saunders, B.E., Vasconcellos, R., Kuether, R.J., and Abdelkefi, A., 2021, "Relationship between the contact force strength and numerical inaccuracies in piecewise-smooth systems," International Journal of Mechanical Sciences, 210, 106729. Doi: https://doi.org/10.1016/j.ijmecsci.2021.106729

System formulation and numerical methods

❑ Additional simulations using a multi-harmonic balance method (MHB)

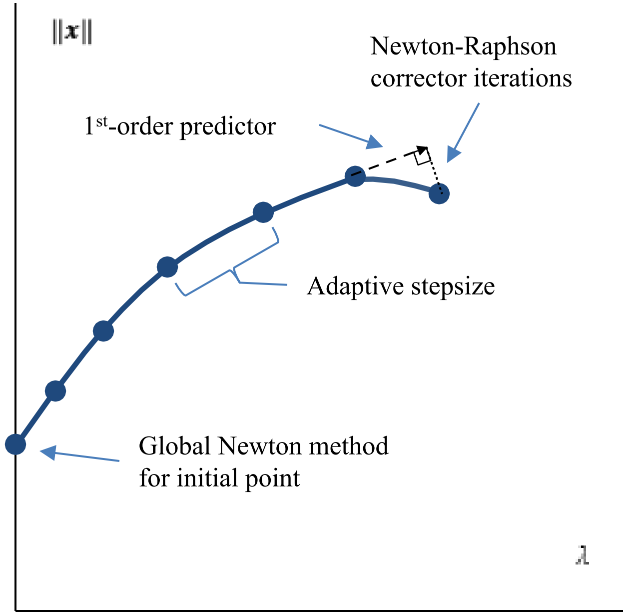
- ➢ The form of the solution is assumed to be a Fourier series:

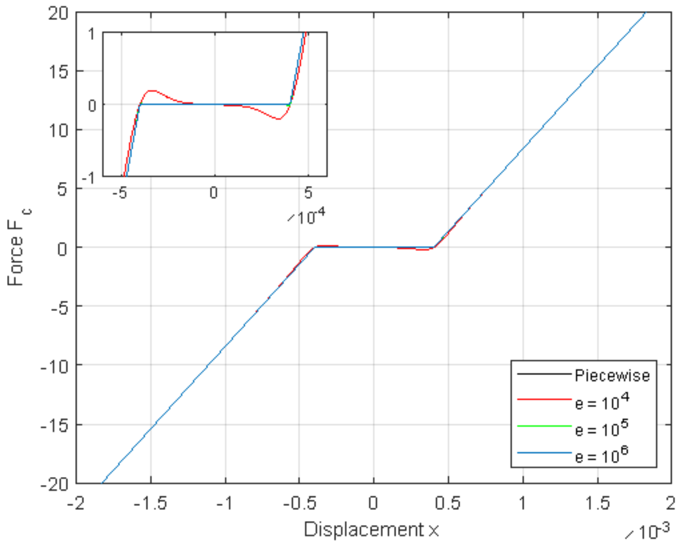
$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{f}_{nl}(\mathbf{x},\dot{\mathbf{x}}) = \mathbf{f}_{ext}(t),$$

$$\mathbf{x}(t) = \frac{\mathbf{c}_0^x}{\sqrt{2}} + \sum_{k=1}^{N_h}\left[\mathbf{s}_k^x \sin(k\omega t) + \mathbf{c}_k^x \cos(k\omega t)\right]$$

- ➢ This solution form is combined with pseudo-arclength continuation to then trace out solution branches
- ➢ The freeplay force is approximated with a fully smooth (regularized) function:

$$F_c = K_c\left(\frac{1}{2}\left[1 - tanh\left(\varepsilon(x + j_1)\right)\right](x + j_1) + \frac{1}{2}\left[1 + tanh\left(\varepsilon(x - j_2)\right)\right](x - j_2)\right)$$





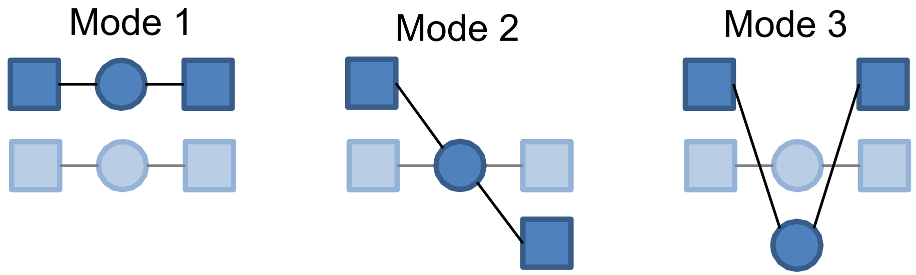
Colaïtis, Y. and Batailly, A., "The harmonic balance method with arc-length continuation in blade-tip/casing contact problems," Journal of Sound and Vibration, 502, p.116070, 2021. Doi: https://doi.org/10.1016/j.jsv.2021.116070

Saunders, B.E., Vasconcellos, R. Kuether, R.J., and Abdelkefi, A., 2022, "Insights on the continuous representation of piecewise-smooth nonlinear systems: limits of applicability and effectiveness," Nonlinear Dynamics, 107 (2), pp. 1479-1494. Doi: https://doi.org/10.1007/s11071-021-06436-w

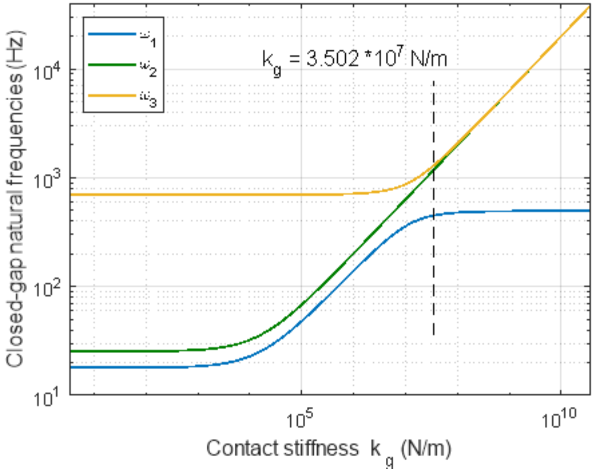
Linear analysis

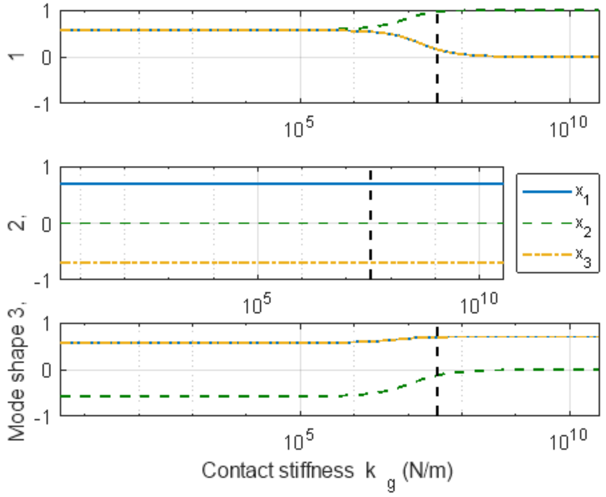
❑ Linear modal analysis

- ➢ Symmetric open-gap mode shapes:



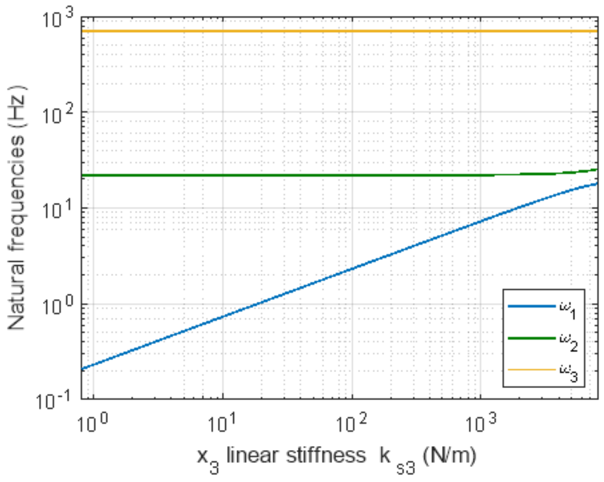
Description	Symbol	Value
Suspension spring stiffness	k_s	$8.03e3\ N/m$
Torsional spring stiffness	κ	$79161\ N*m/rad$
Gap spring stiffness	k_g	$3.502e7\ N/m$
Half-length	L	$0.1614\ m$
Gap	g	$2.54e-4\ m$
Left mass	m_1	$0.629\ kg$
Middle mass	m_2	$1.258\ kg$
Right mass	m_3	$0.629\ kg$
Damping coefficient	c	$4\ N*s/m$

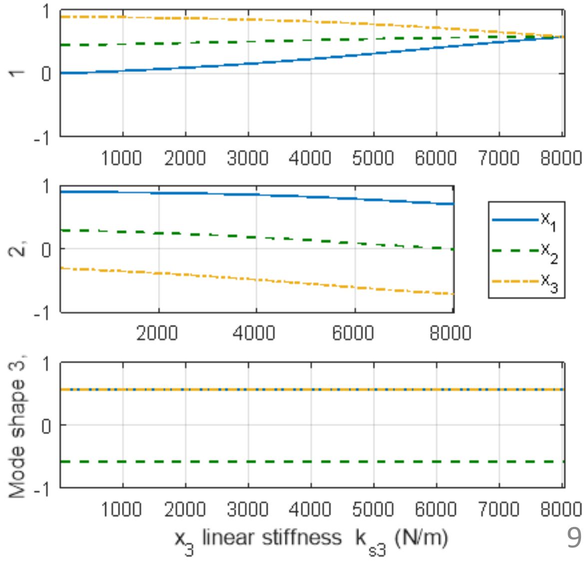




Closed-gap effects $(g = 0)$

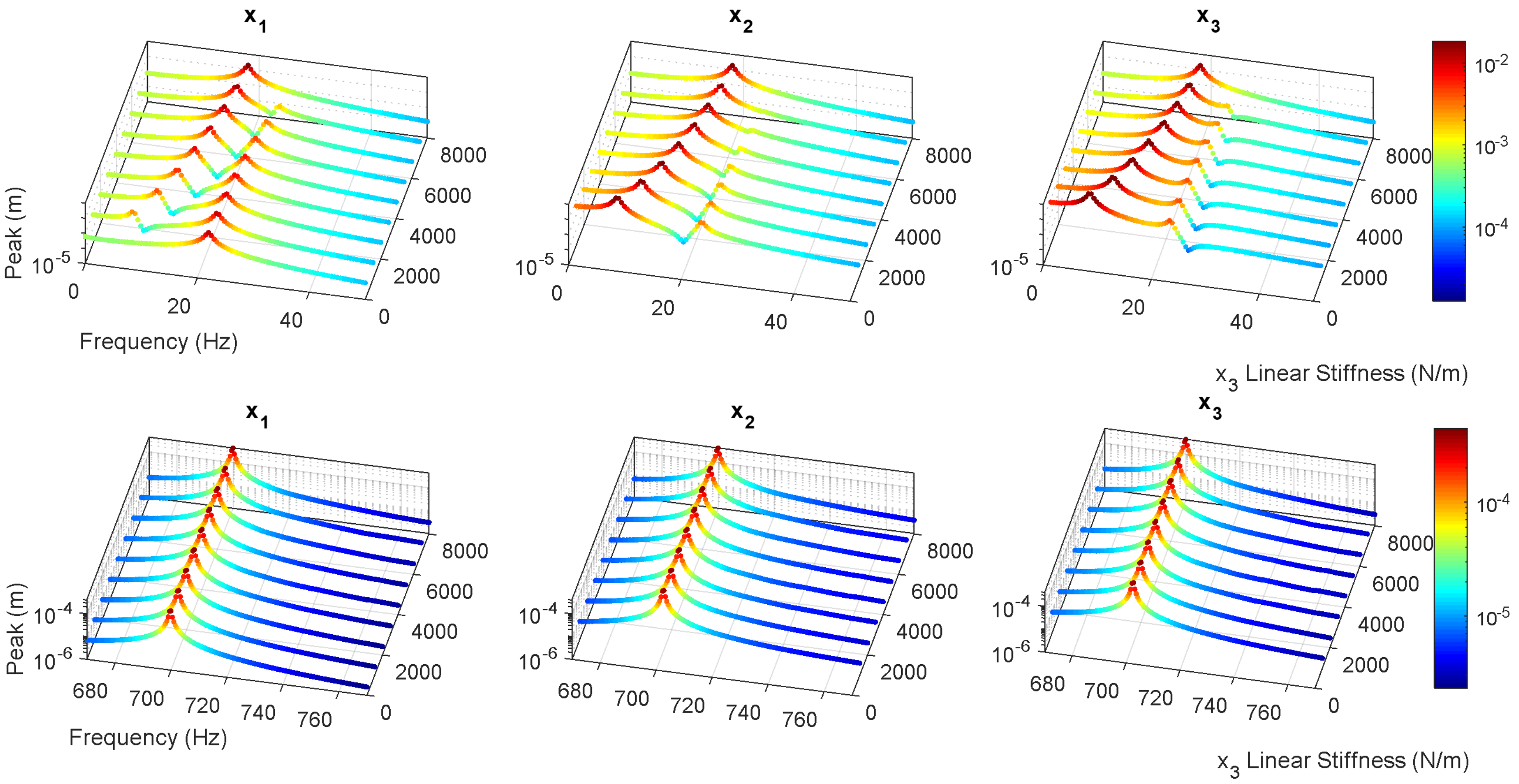
Linear asymmetry effects $(k_{s3} \neq k_{s1})$





Linear analysis

❑ Linear modal analysis

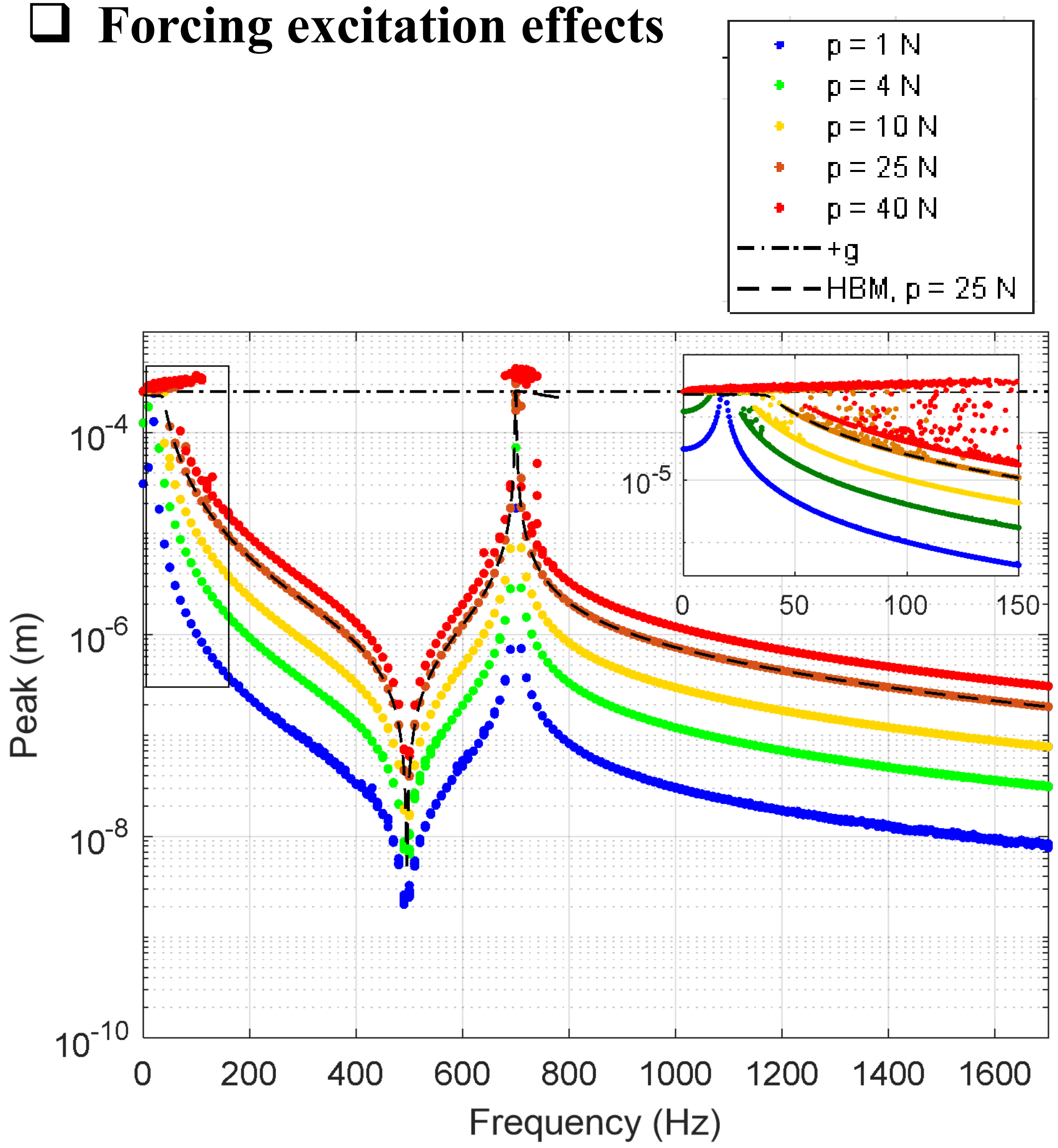


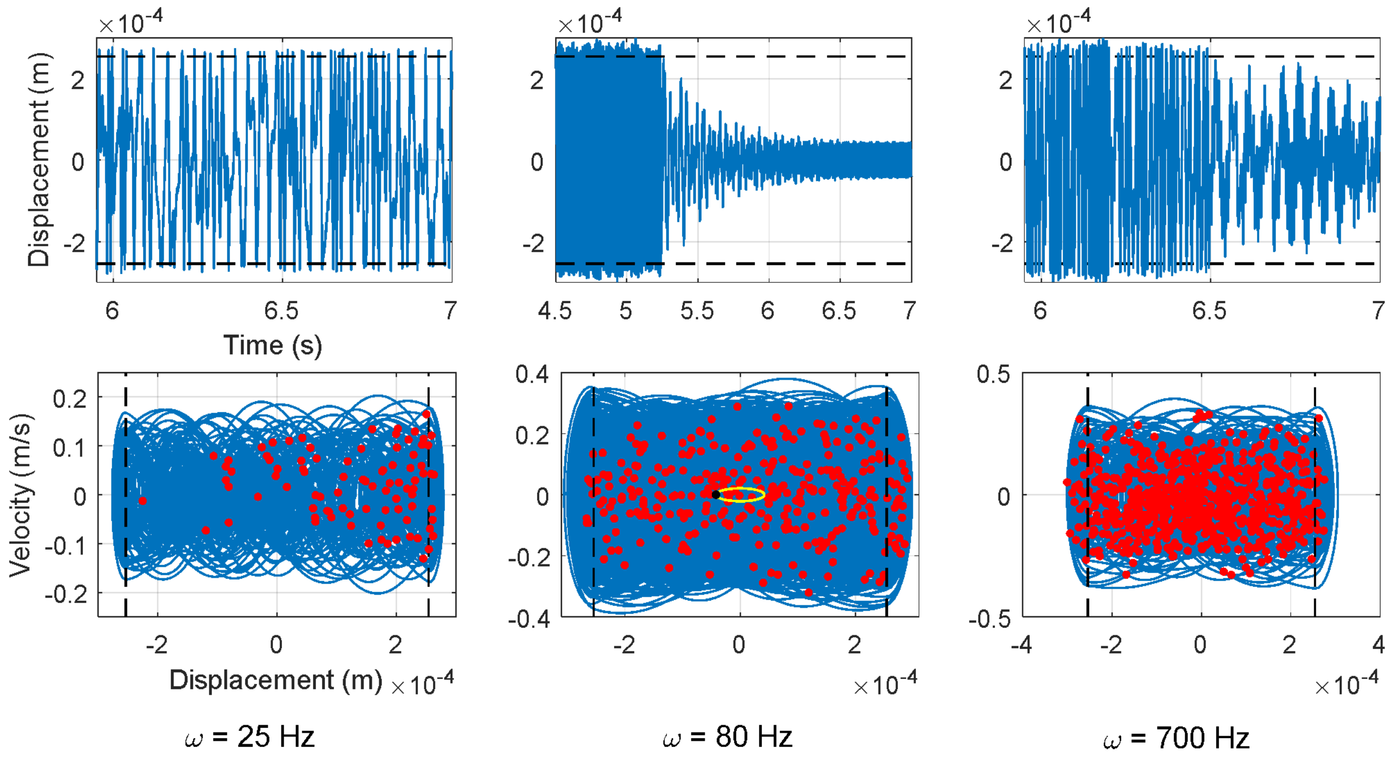
- ➢ Frequency response curves as linear stiffness becomes asymmetric
- ➢ The inherent symmetry and forcing location both contribute to the presence or absence of the second resonance peak

Nonlinear analysis

❑ Forcing excitation effects

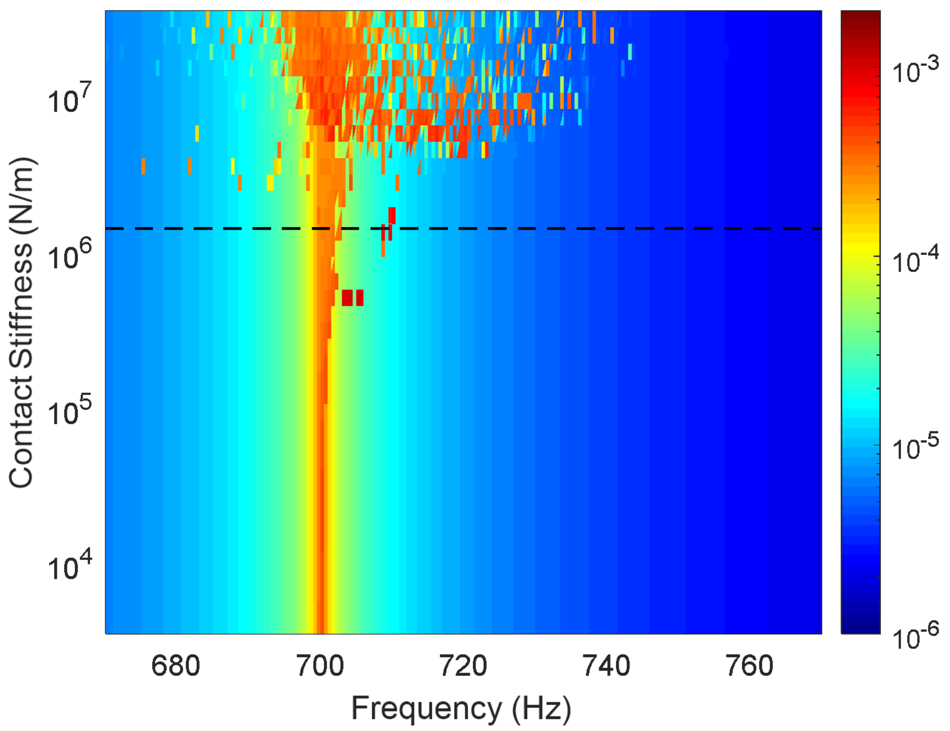
- ➢ Significant challenges due to hard contact
- ➢ Time integration shows chaos surrounding resonance peaks
- ➢ Solution jumping due to sensitivity
- ➢ MHB shows flat-top response without classical jump behavior

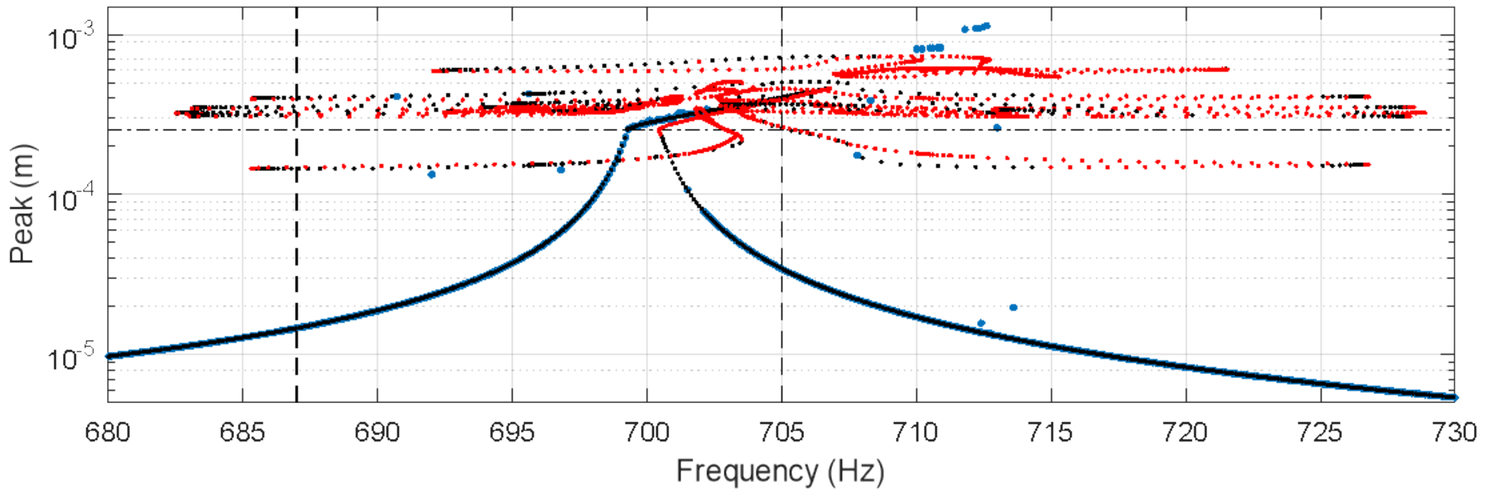




Nonlinear analysis

❑ Nonlinear behavior due to contact stiffness—high frequency

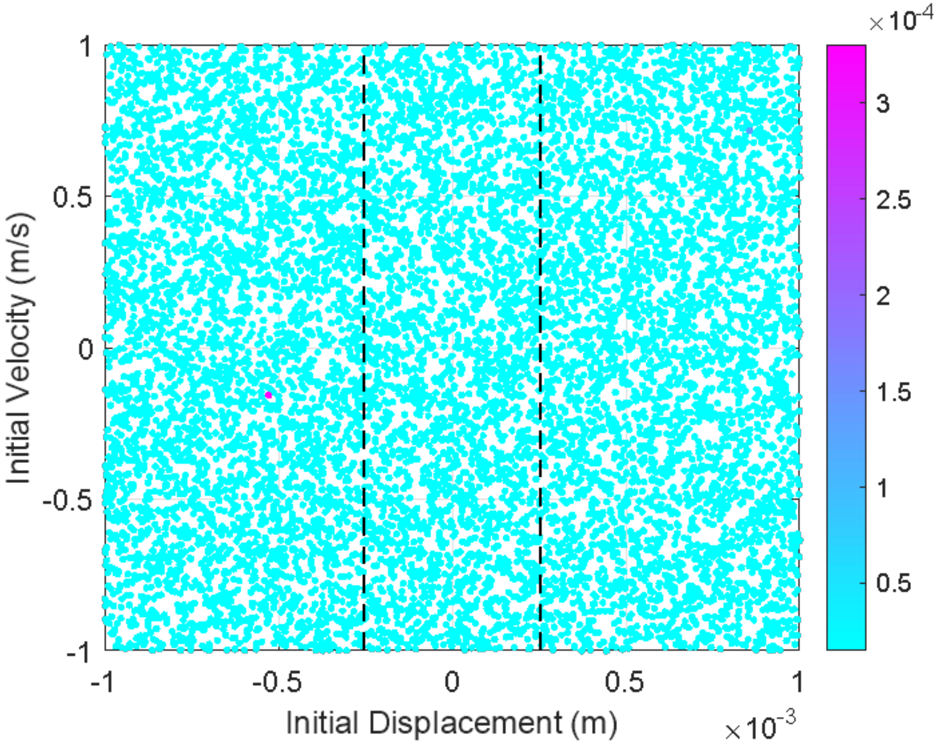


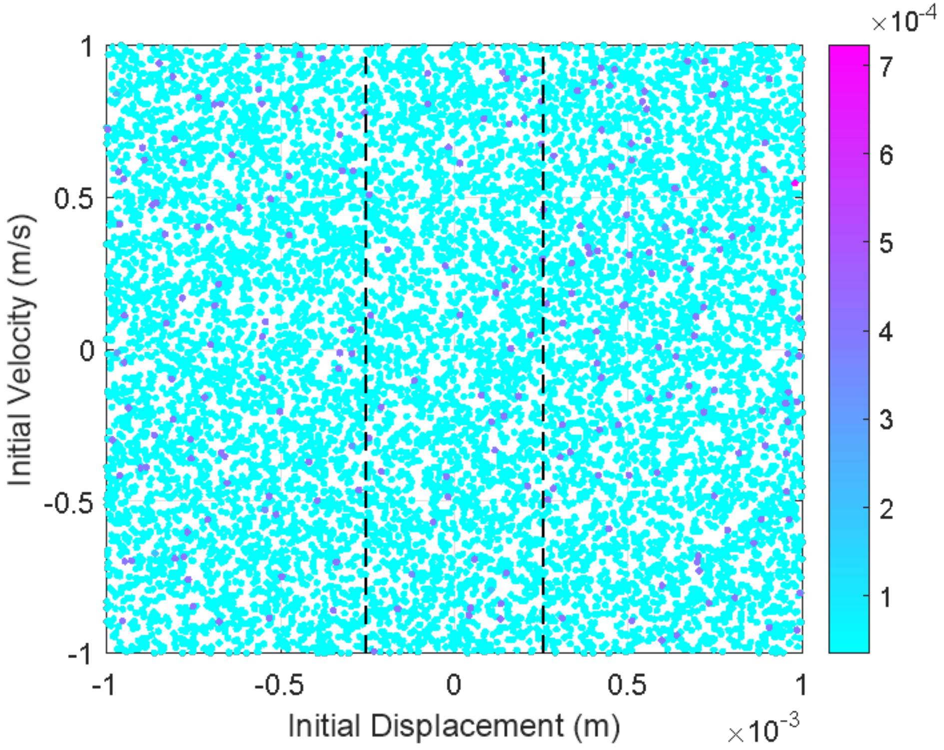


- Time integration shows a few isolas around the third resonance peak
- MHB shows *many* isolas are present
- Period-5, period-7, up to period-13
- The majority of each isola is unstable
- Quasiperiodic isolas cannot be traced out by the MHB

Nonlinear analysis

❑ Nonlinear behavior due to contact stiffness – high frequency

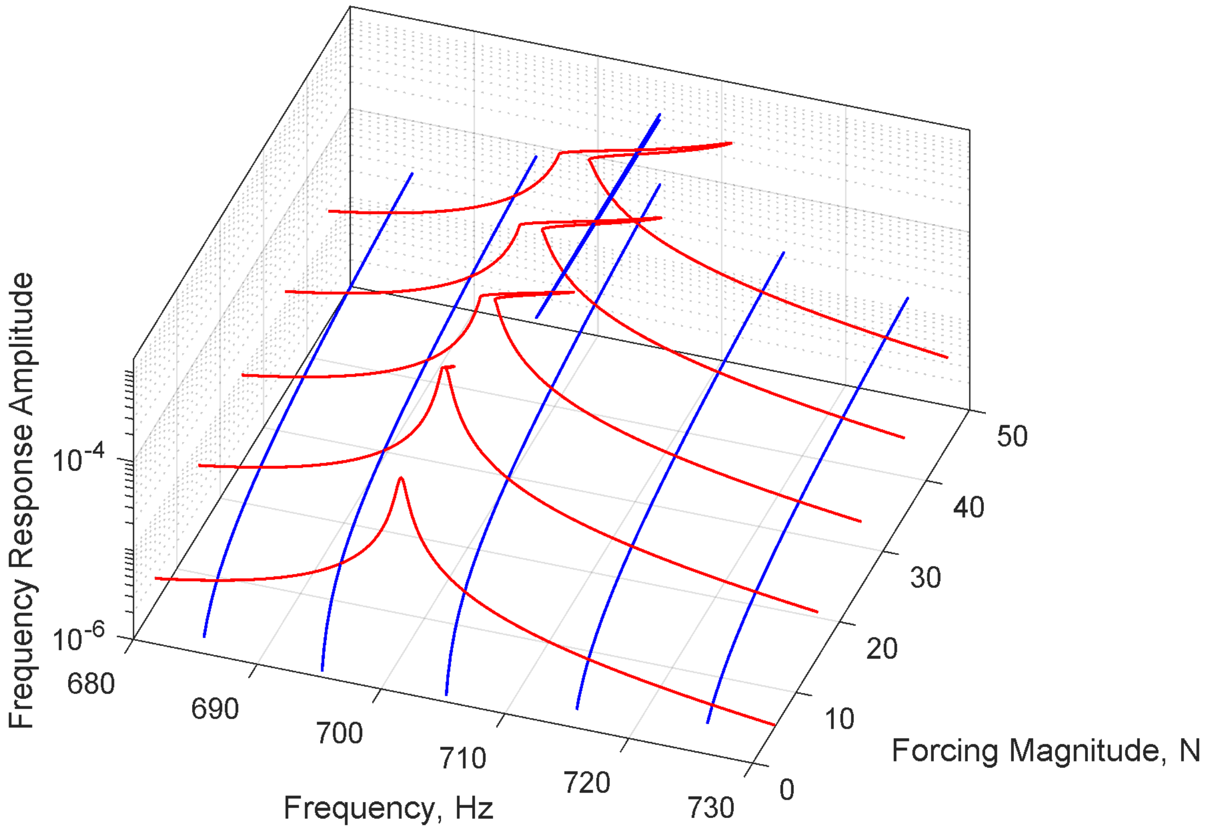


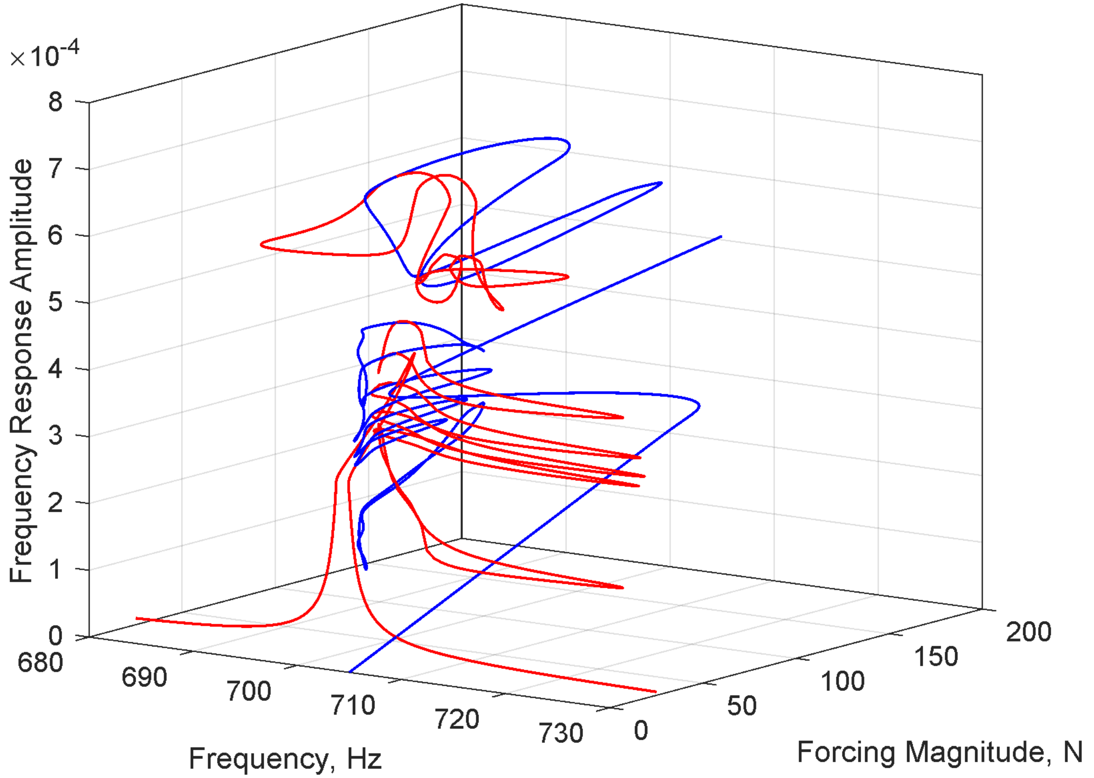


- Basins of attraction at two frequencies
- Left: 687 Hz. Right: 705 Hz. Both cut through several isolas
- The low-amplitude attractor on the main branch is dominant
- Low likelihood of the system settling onto an isola here

Nonlinear analysis

- **Perform continuation with respect to the forcing magnitude**
  - These "S-curves" show the response at a fixed frequency
  - Reveal the minimum forcing level required for various nonlinear phenomena
  - Below: **NLFRs** and **S-curves** for main solution branches and some isolas near the high-frequency resonance

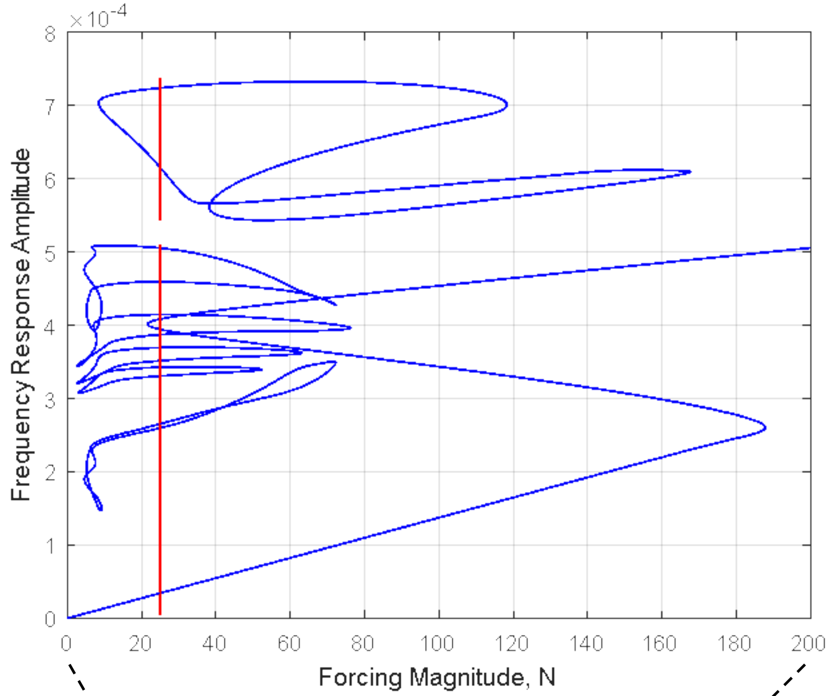




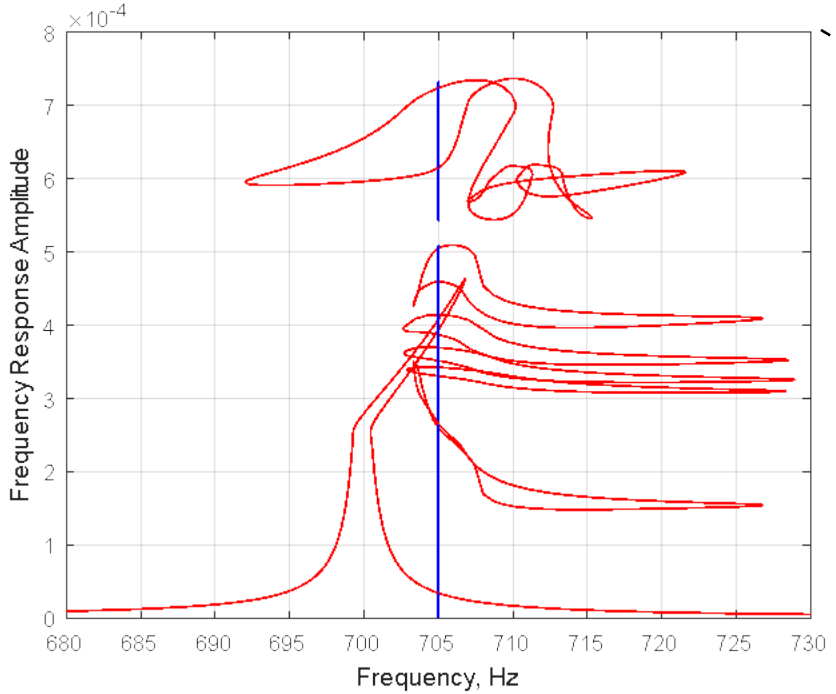
Nonlinear analysis

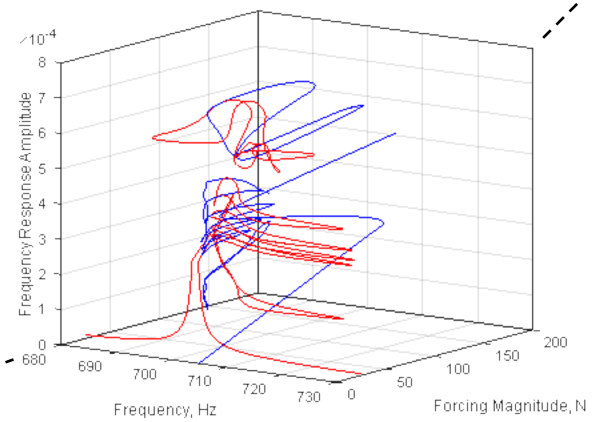
❑ NLFRs and S-curves of isolas

- ➢ NLFRs at $p = 25\ N$, S-curves at $\omega = 705\ Hz$
- ➢ NLFRs: some isolas intersect the main solution branch
- ➢ S-curves: some isolas also intersect the main branch
- ➢ Main S-curve does not intersect NLFR isolas
- ➢ Isolas can form with as little as $\approx 2.25\ N$ forcing



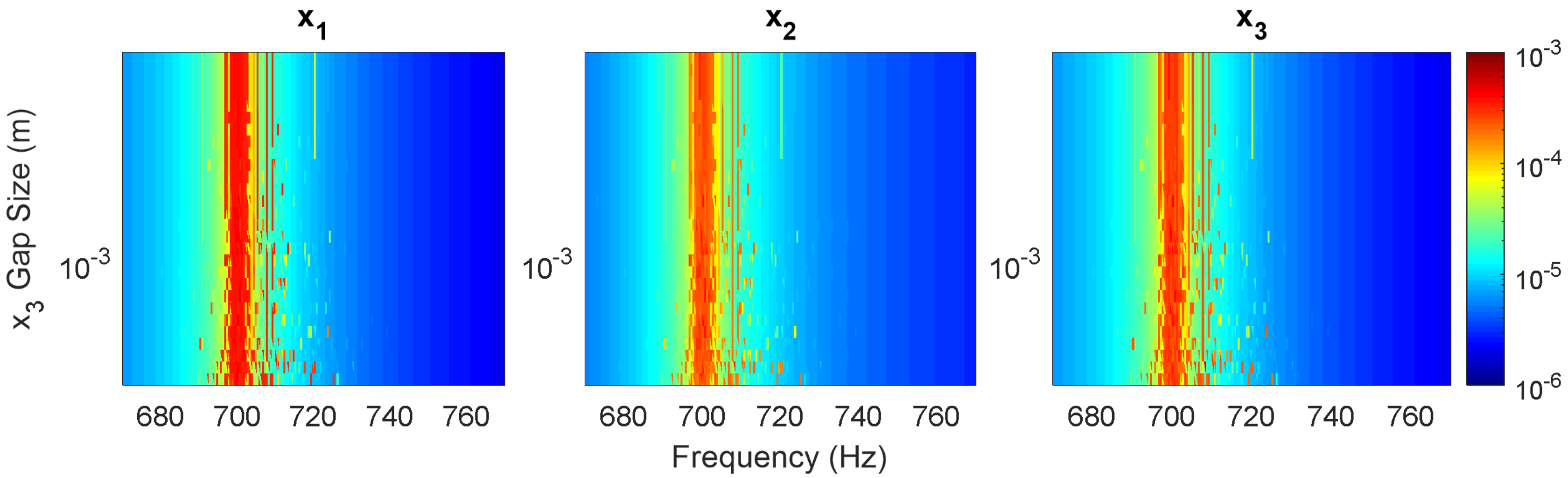
NLFRs
S-curves





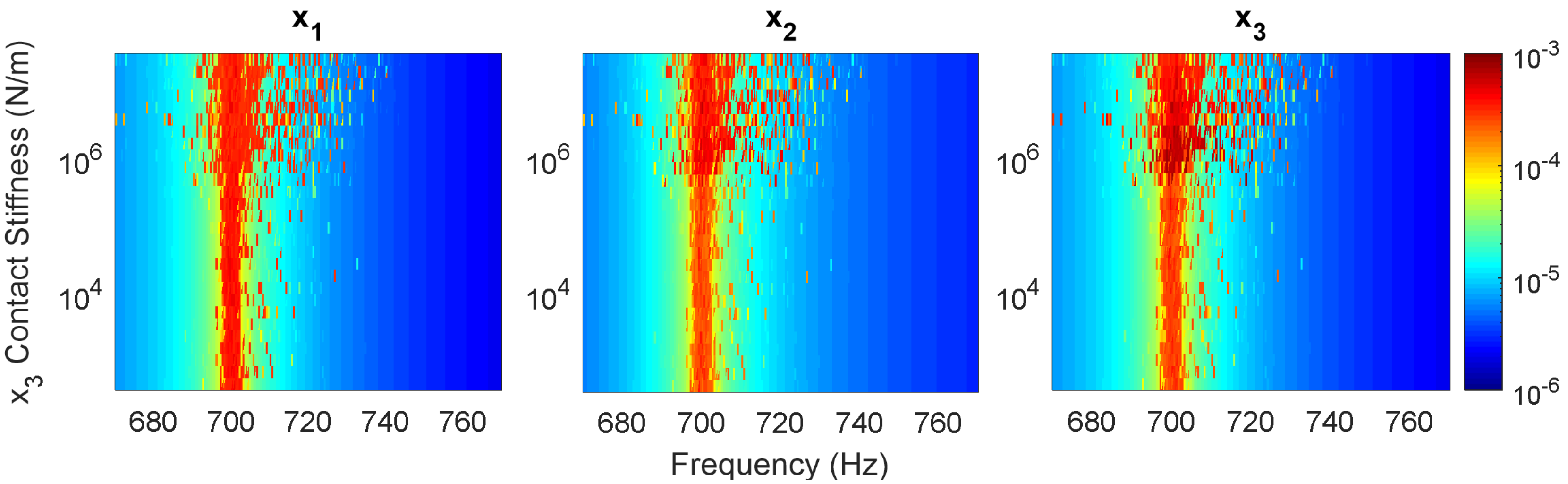
Nonlinear system with asymmetry

❑ Existence of asymmetry—gap size and contact stiffness



$$k_{g1} = k_{g3} = 3.502 * 10^7 \frac{N}{m},$$
$$g_1 = 2.54 * 10^{-4} m$$

- ➢ All three resonance peaks present for largest gap size
- ➢ Isolas may occur at high frequency



$$k_{g1} = 3.502 * 10^7 \frac{N}{m},$$
$$g_1 = g_3 = 2.54 * 10^{-4} m$$

- ➢ All three resonance peaks present for softest contact stiffness
- ➢ Isolas do occur at high frequency

Conclusions

- The nonlinear frequency responses and characteristics of an idealized shaft-bearing assembly have been investigated
- Several complex nonlinear dynamical behaviors were observed and characterized using both time integration and harmonic balance methods
- Chaos near primary resonances grows with larger forcing and disappears with softer contact stiffnesses
- A cloud of many isolas forms at high frequency for softer contact
- Isolas can form for forces as low as $2.25\ N$
- Asymmetric system configurations showed similar presence of resonances and isolas
- The experimental setup is likely robust to small discrepancies in spring stiffnesses, gap sizes, etc.

Future work

- Vibrational experiments on the simplified physical system to complement past shock experiments
- More certain representation of the damping in the system, along with the influence of any contact damping
- Improved research on combining harmonic balance and continuation principles for quasiperiodic motions, especially for isolas
- How to capture chatter behavior efficiently

Acknowledgements

New Mexico State University

Sandia National Laboratories—Laboratory-Directed Research and Development (LDRD)

São Paulo State University (UNESP)

Thank you for your attention!

Please ask any questions

This study describes objective technical results and analysis. Any subjective views or opinions that might be expressed in the paper do not necessarily represent the views of the U.S. Department of Energy or the United States Government. This article has been authored by an employee of National Technology & Engineering Solutions of Sandia, LLC under Contract No. DE-NA0003525 with the U.S. Department of Energy (DOE). The employee owns all right, title and interest in and to the article and is solely responsible for its contents. The United States Government retains and the publisher, by accepting the article for publication, acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this article or allow others to do so, for United States Government purposes. The DOE will provide public access to these results of federally sponsored research in accordance with the DOE Public Access Plan https://www.energy.gov/downloads/doe-public-access-plan. SAND2023-12295C. R. Vasconcellos acknowledges the financial support of the Brazilian agency CAPES (grant 88881.302889/2018-01).