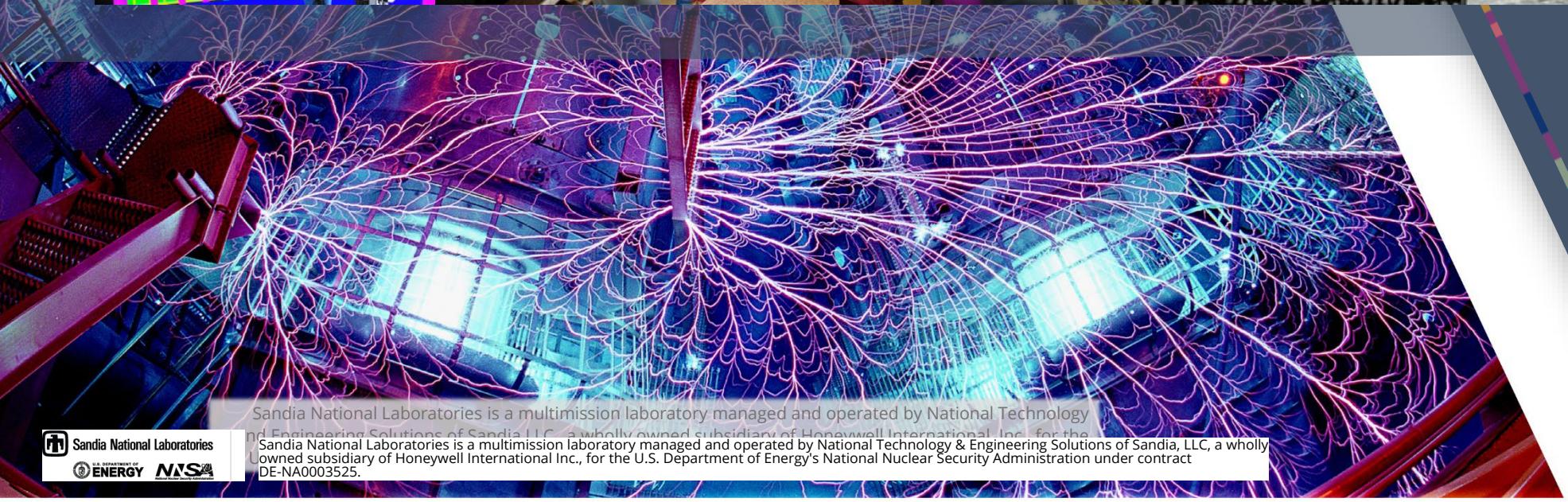
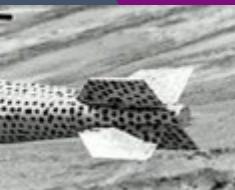
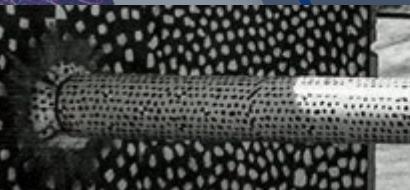
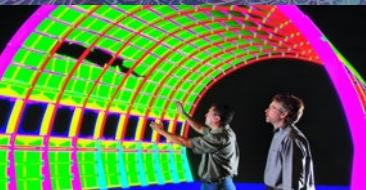




Exceptional service in the national interest



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Laboratories

## Extreme Material Dynamics Experiments on SNL's Z Machine

Dr. P. Kalita

Joint CDAC & CMEC  
Invited Seminar

02-22-23



# Acknowledgments

- **EOS design: S. D. Crockett & S. P. Rudin – LANL**
- **AIMD calculations: K. R. Cochrane – SNL**

**SNL Management: C. Seagle, T. Mattsson, D. Sinars**

**Z machine:**

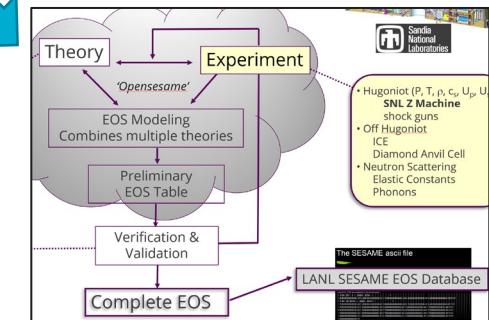
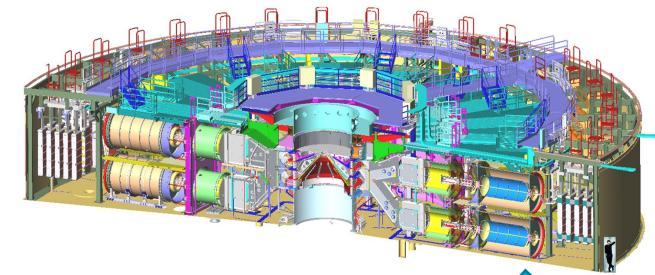
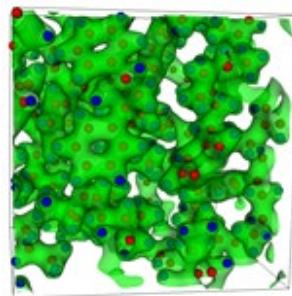
- **Z Experiments: M. Knudson, S. Root, T. Ao – SNL**
- **Z hardware design: C. Blada, J. Jackson**
- **Z pulshaping: H. Hanshaw**
- **Z Velocimetry: E. Sciglietti, J. Gluth,**
- **Z Target Fabrication Team: G. Smith, J. Taylor, A. Romero**
- **Z Machine Operations Team**

SNL is managed by NTESS, LLC under contract DE-NA0003525.

Work at Los Alamos National Laboratory is supported by the US DOE through contract number 89233218NCA000001.

# Outline

- **Intro about SNL**
- **Shock compression**
- **Z machine and shock experiments**
- **E.g. 1: Shock and re-shock of PMMA polymer to 1 TPa**
- **E.g. 2: Platinum as a shock standard to 2 TPa.**
- **E.g. 3: Ti64 alloy shocked to 1.2 TPa and a broad range EOS**



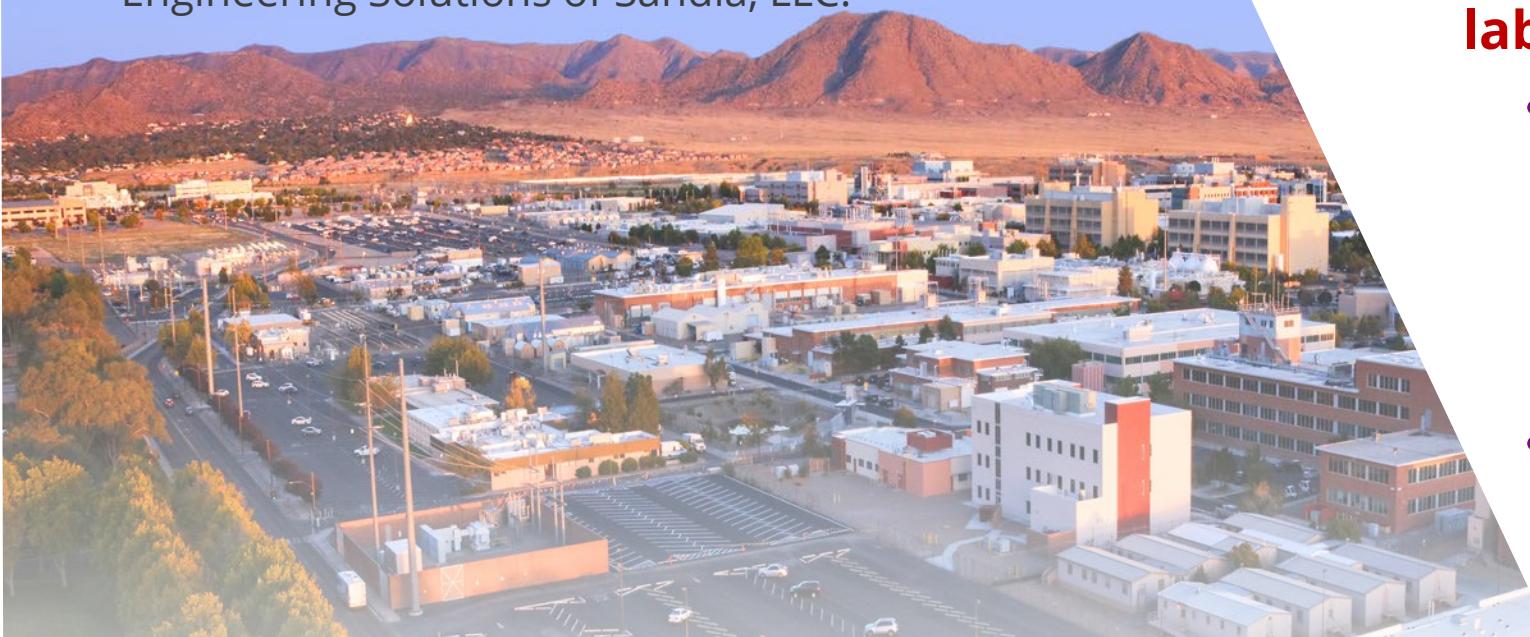
**Equation of State:** In physics and thermodynamics, an EOS is a constitutive equation describing the state of matter under a given set of physical conditions. It provides a mathematical relationship between two or more of that matter's state functions, such as its temperature, pressure, volume, or internal energy.

<https://www.lanl.gov/org/ddste/aldsc/theoretical/physics-chemistry-materials/sesame-database.php>

# SANDIA NATIONAL LABORATORIES

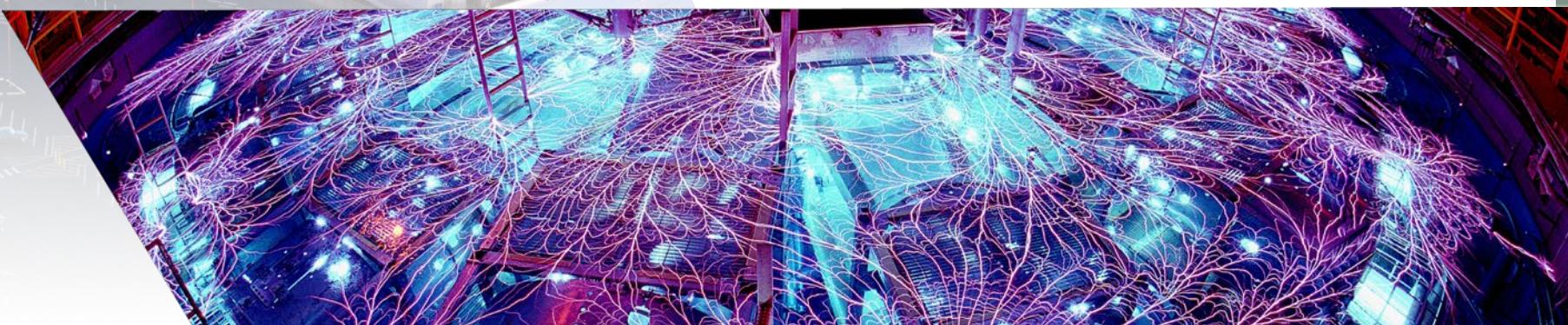


A federally funded research and development center managed and operated by National Technology & Engineering Solutions of Sandia, LLC.

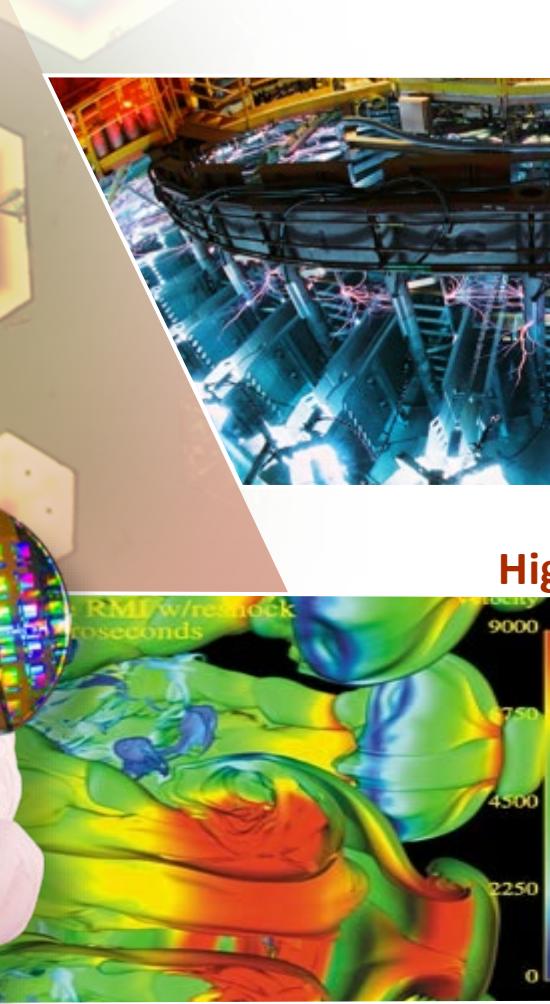


**Sandia is the largest of the 17 U.S. Department of Energy's national laboratories**

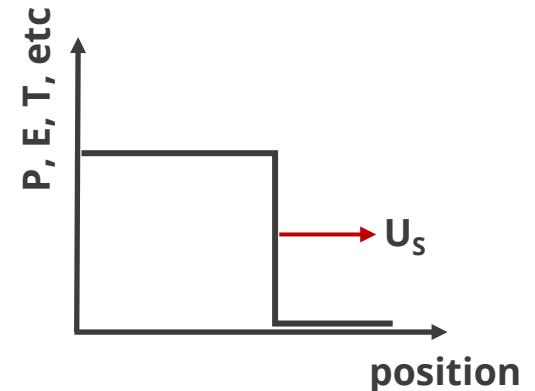
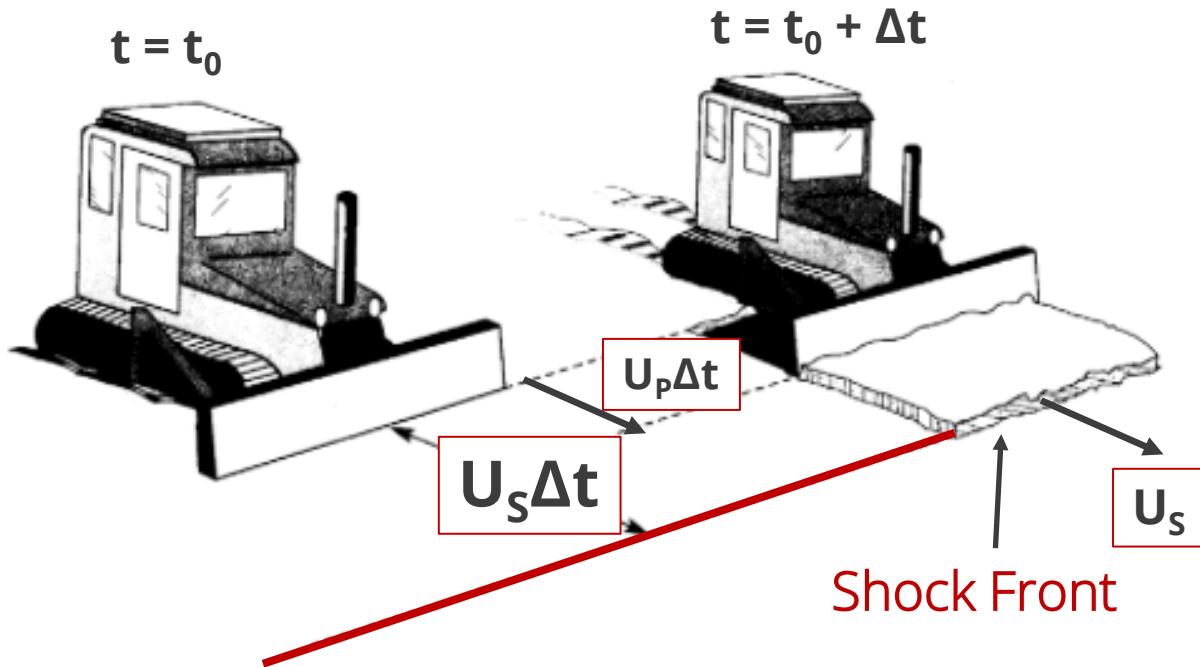
- **~14,920 employees in FY21**
- **>13,000 located in Albuquerque, New Mexico, USA**
- **~1800 located in Livermore, California, USA**
- **~\$4.5B budget in FY21**



## Sandia maintains seven Research Foundations in key areas of science



# Shock Compression: what is a Shock Wave?



- Shock wave: A propagating discontinuity in pressure, density, energy, etc.
- The pressure, density, energy are increased behind the shock front
- The material velocity behind the shock front is the *particle velocity*,  $U_p$
- **The shock wave velocity,  $U_s$ , is greater than the sound speed in the undisturbed material**
- In real life, shock waves are not perfect discontinuities

# Shock Event: the Rankine-Hugoniot Equations

**Conservation of Mass**

$$\rho_0(U_S - U_0) = \rho_1(U_S - U_1)$$

**Conservation of Momentum**

$$P_1 - P_0 = \rho_0(U_S - U_0)(U_1 - U_0)$$

$$U_0 P_0 = 0$$

**Conservation of Energy**

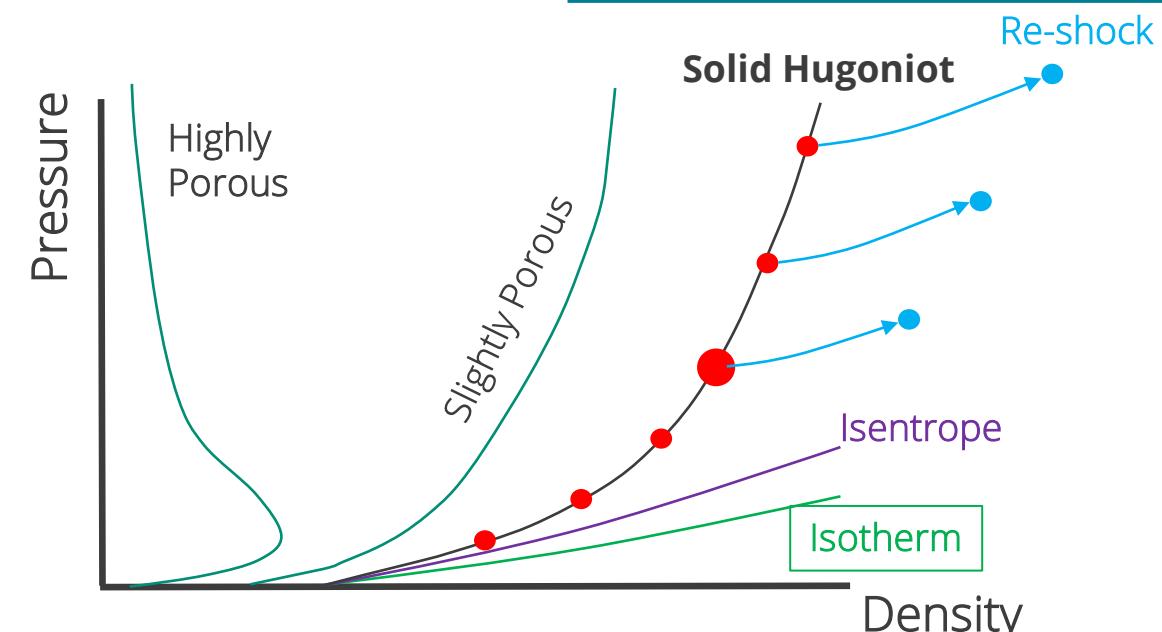
$$E_1 - E_0 = \frac{1}{2}(P_1 + P_0)(V_0 - V_1)$$

$$\frac{\rho}{\rho_0} = \frac{U_S}{U_S - U_P}$$

$$P = \rho_0 U_S U_P$$

$$E - E_0 = \frac{1}{2}P(V_0 - V)$$

- We have 3 equations, but 5 unknowns:  $U_P$  -  $U_S$  -  $\rho$  -  $P$  -  $E$
- Need to experimentally determine two unknowns
- The Hugoniot equations do not explicitly contain temperature
- Not a complete equation of state:  $E(P,V)$  – need  $T$
- *The equations are based on a specific initial state*

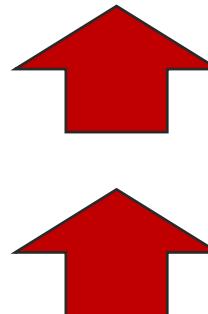


- **The Hugoniot is not a path in space**
- **Hugoniot is defined by a specific initial state**

# How to find experimentally a Hugoniot State?

- Initiate a shock wave in a sample
- $U_S$  and  $U_P$  are the easiest to measure
  - cannot always measure both
- Development of Hugoniot standards for Impedance Matching (ex: Al, Ta)
- $U_S = F(U_P) \rightarrow U_S = C_0 + S_1 U_P$
- $P$  and  $U_P$  at an interface are equal

Target/sample



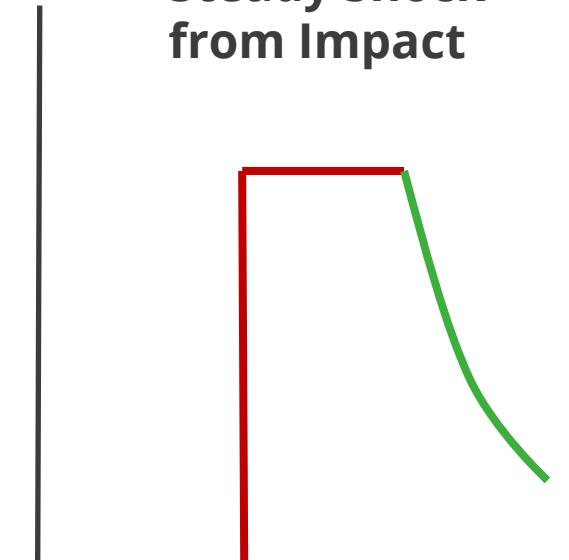
Impactor/Flyer

$$\frac{\rho}{\rho_0} = \frac{U_S}{U_S - U_P}$$

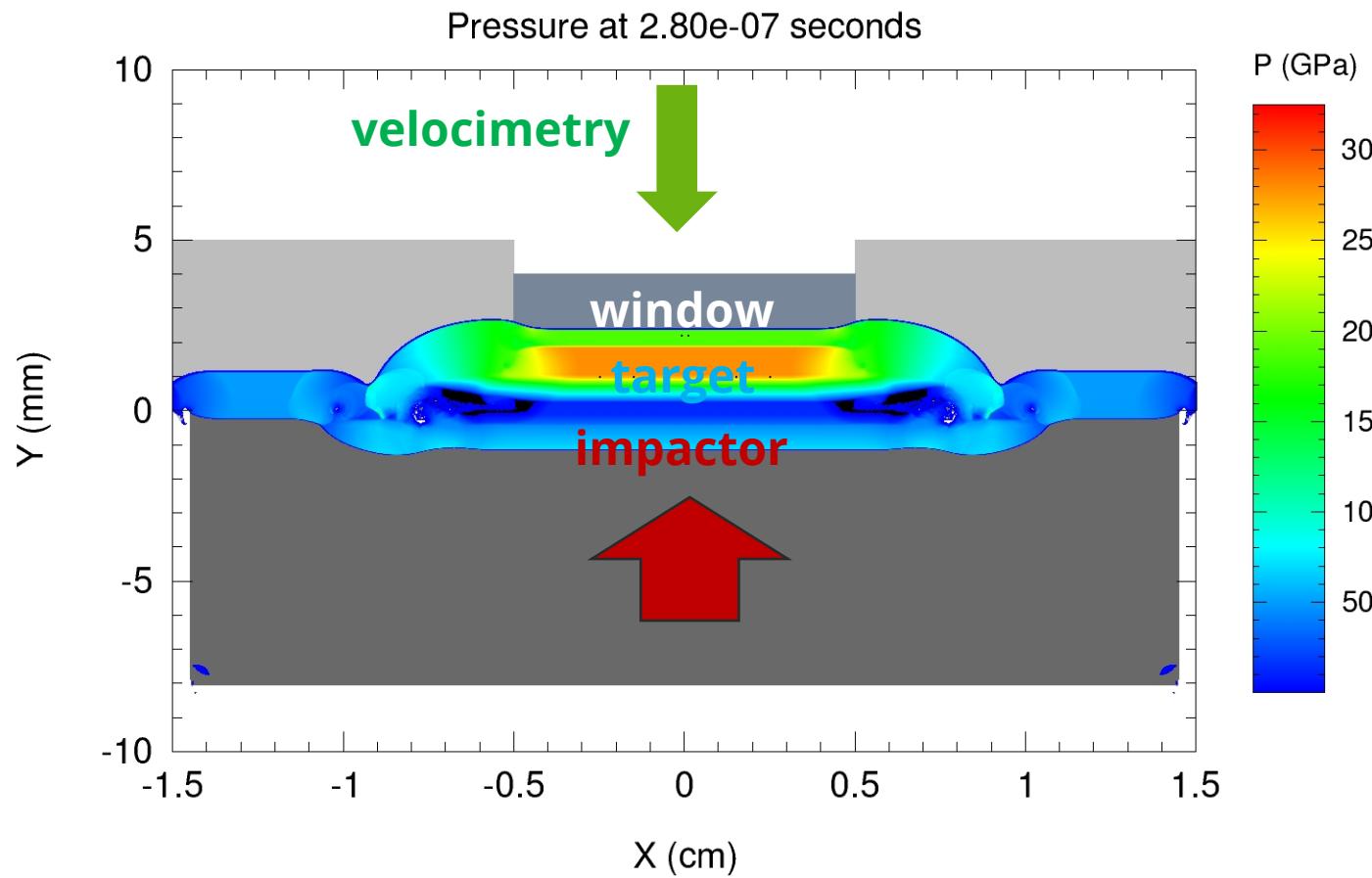
$$P = \rho_0 U_S U_P$$

$$E - E_0 = \frac{1}{2} P(V_0 - V)$$

Steady Shock from Impact



# Shock wave propagation: a simulation



CTH hydrocode simulation

Steady Shock  
from Impact

# Z is one of three flagship HEDP facilities in the U.S.



Lawrence Livermore National Laboratory

## National Ignition Facility (NIF)

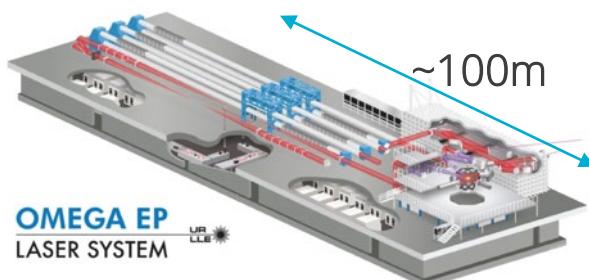
- Largest Laser on Earth
- Primary facility for Laser Indirect Drive fusion
- 400 TW / 1.8 MJ (Max Power & Energy)



University of Rochester

## OMEGA Laser Facility

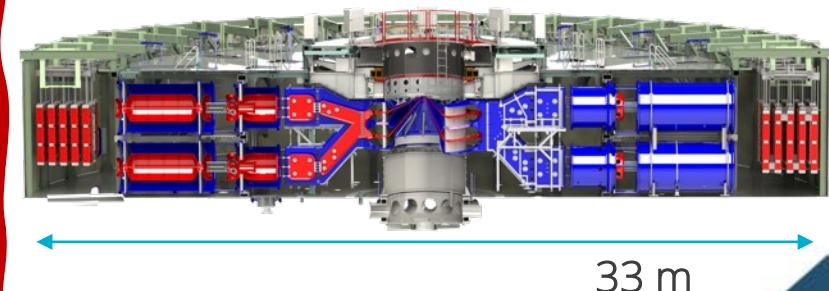
- High shot-rate academic laser facility
- Primary facility for Laser Direct Drive fusion
- 20 TW/.03 MJ (Max Power & Energy)



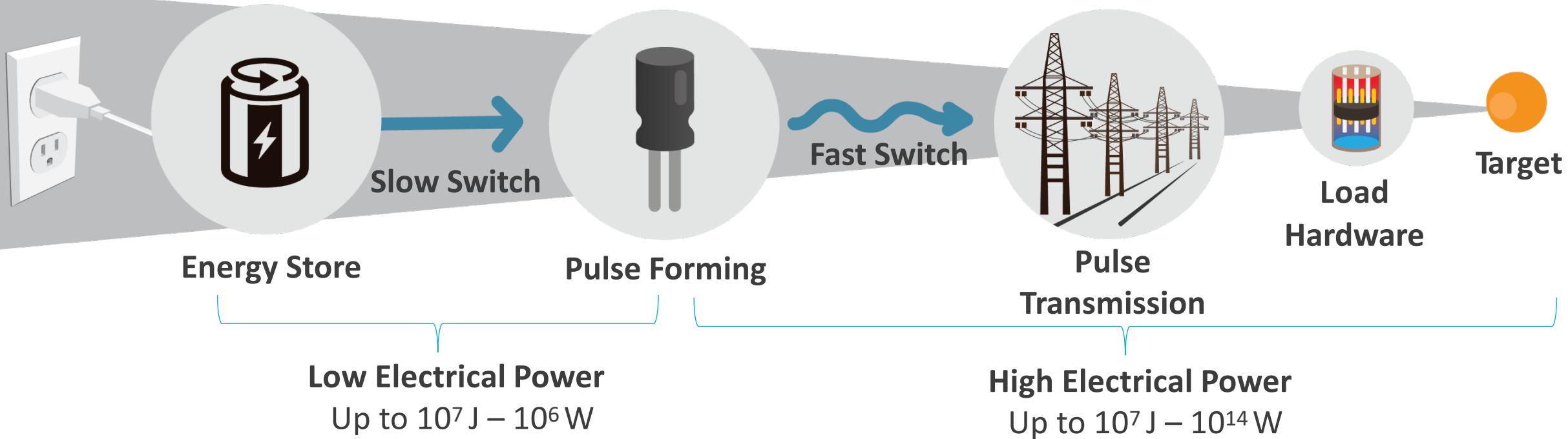
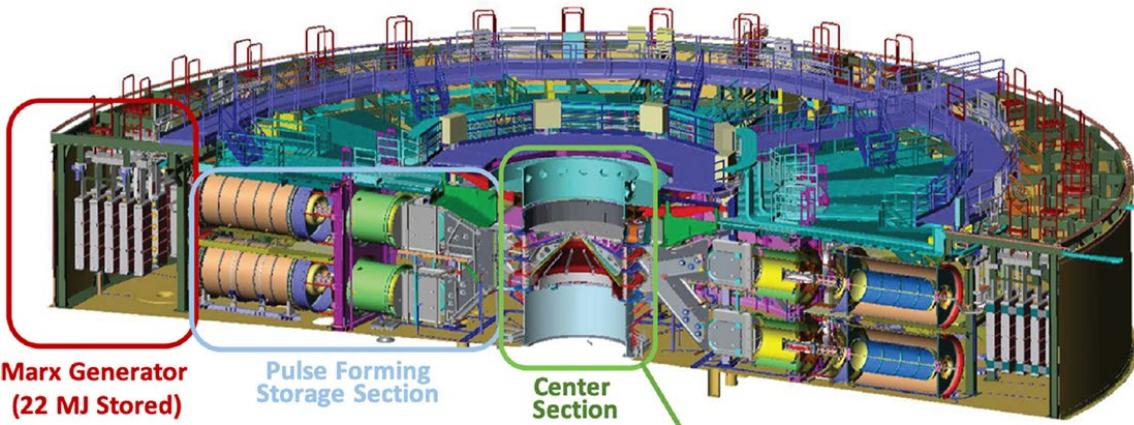
Sandia National Laboratories

## Z Facility

- Largest Pulsed Power Facility on Earth
- Primary facility for Magnetic Direct Drive fusion
- 80 TW / 3 MJ (Max Power & Energy)

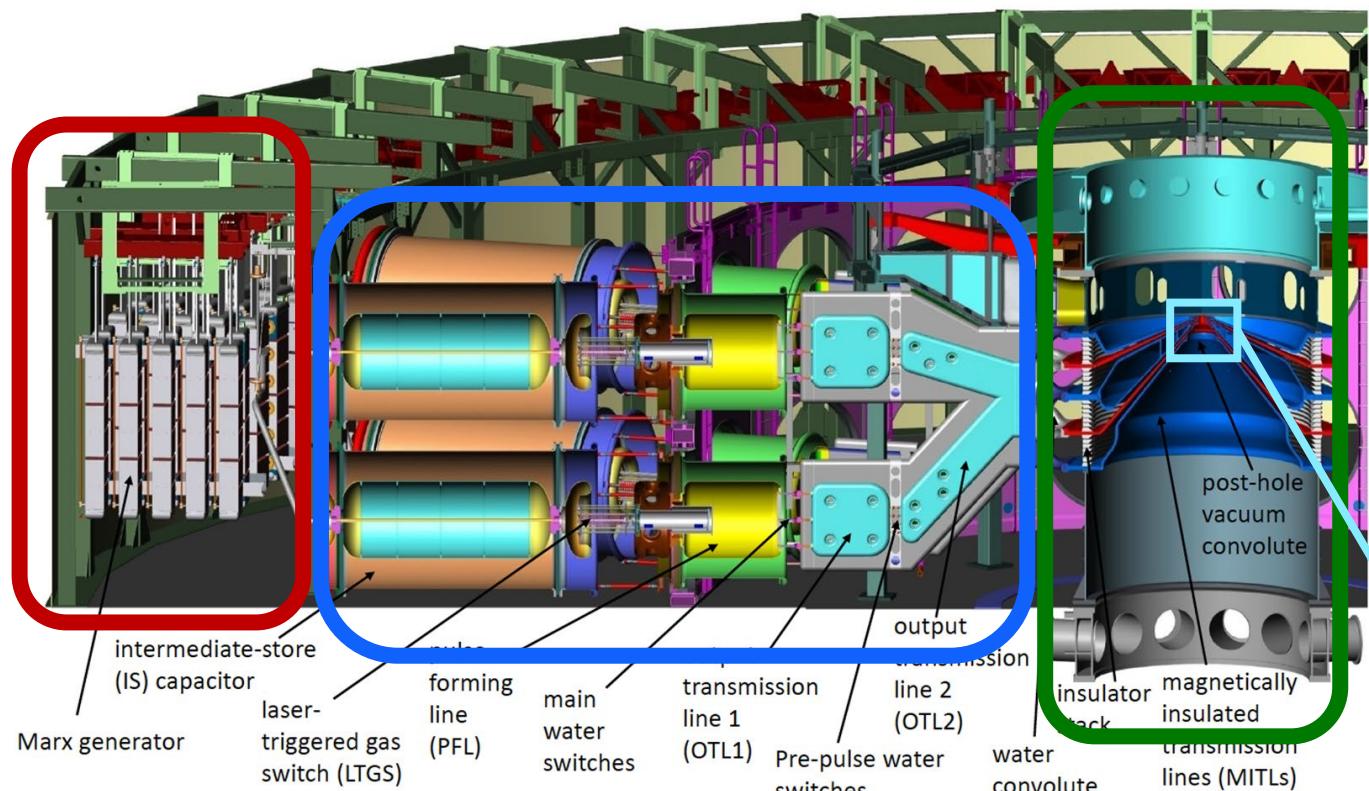


# Pulsed Power: how does it work?



**Pulsed power compresses electrical energy in both space and time to produce short bursts of high power.**

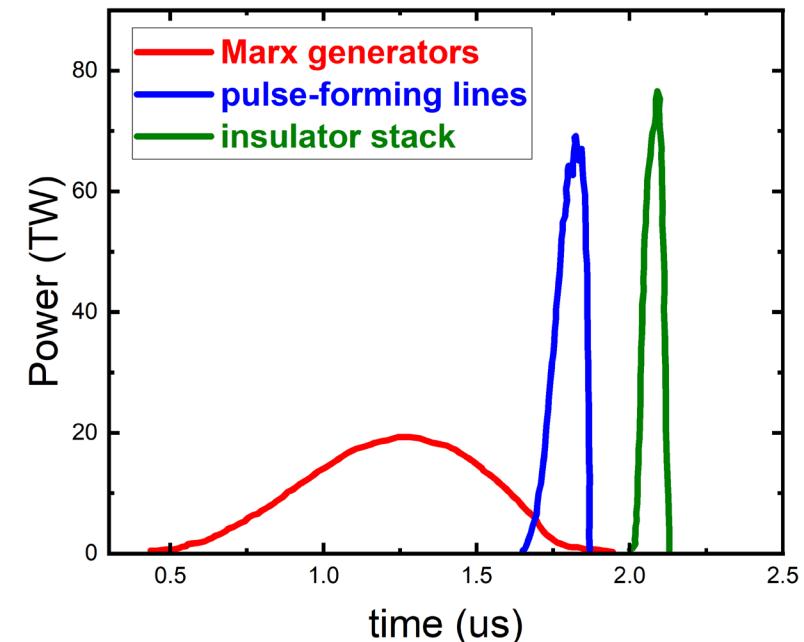
# the largest pulsed power facility in the world today: the 80-TW "Z" machine at Sandia



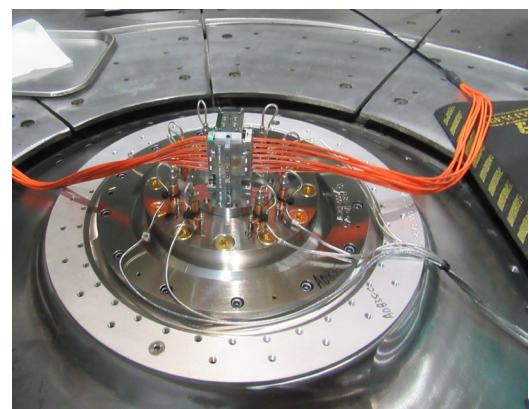
**36 Marx Banks  
(22 MJ stored)**

**Pulse forming  
storage section**  
18 independently triggerable groups  
of 2 transmission lines

**Center  
section**

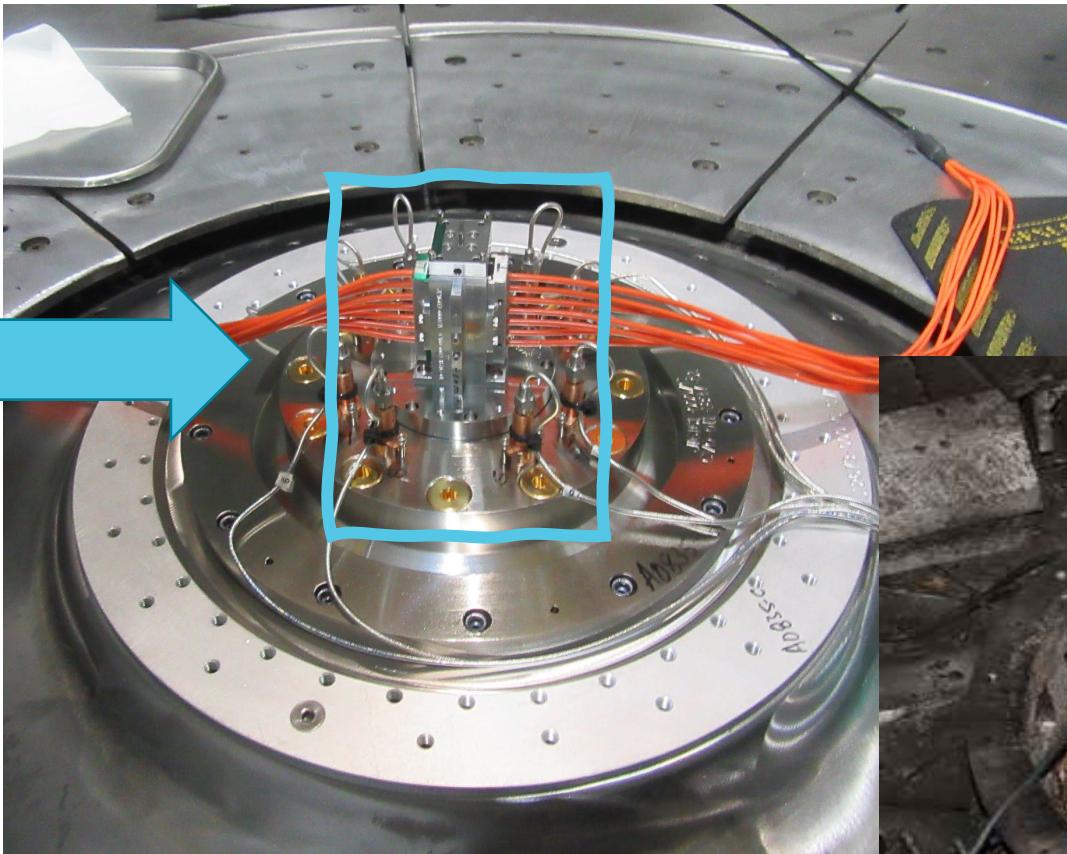
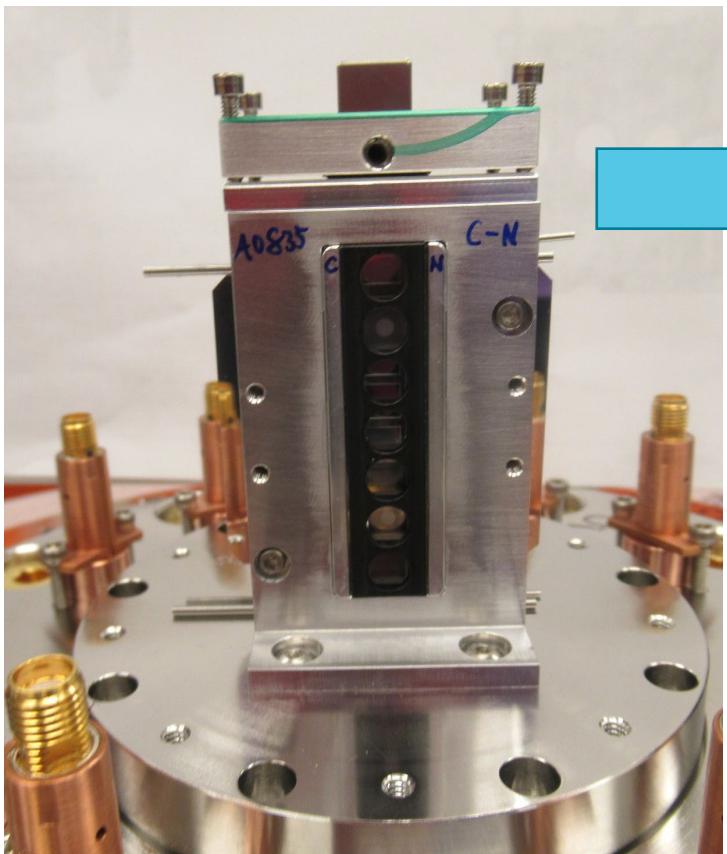


the "load" for  
a DMP shock  
compression  
experiment  
with multiple  
samples



# Z experiments release the energy of a few sticks of dynamite

before



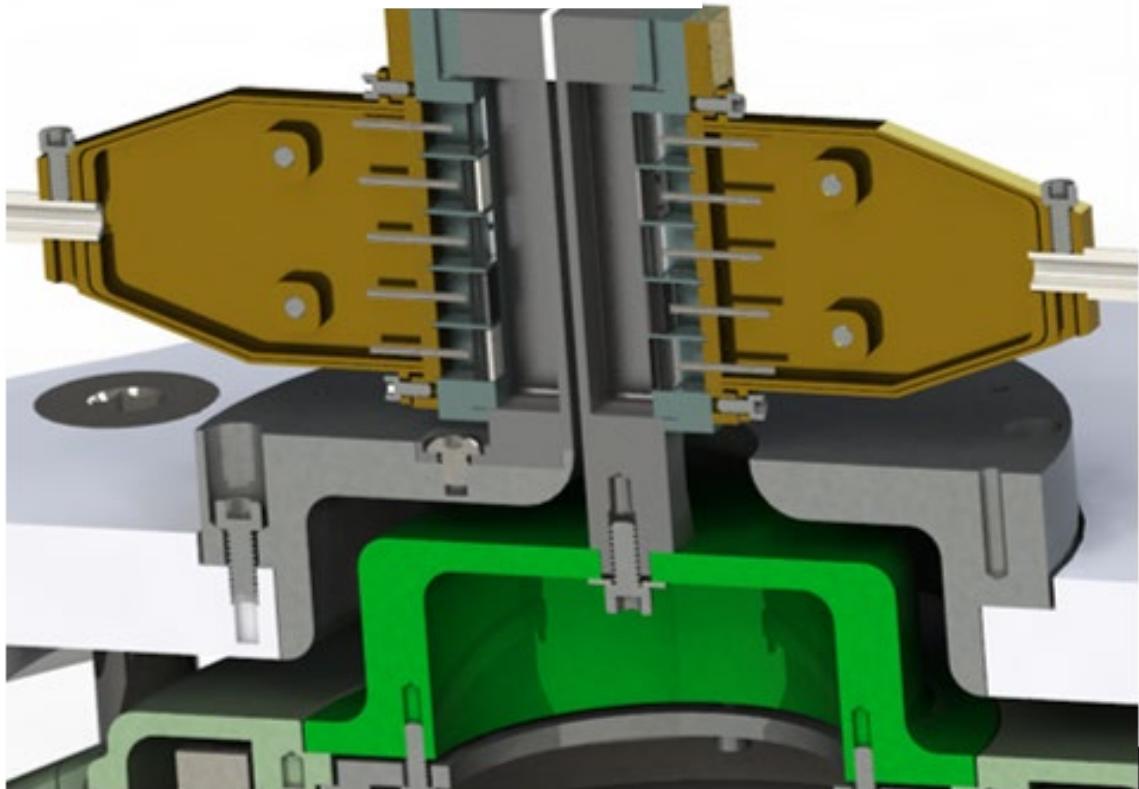
Load with column of samples

after

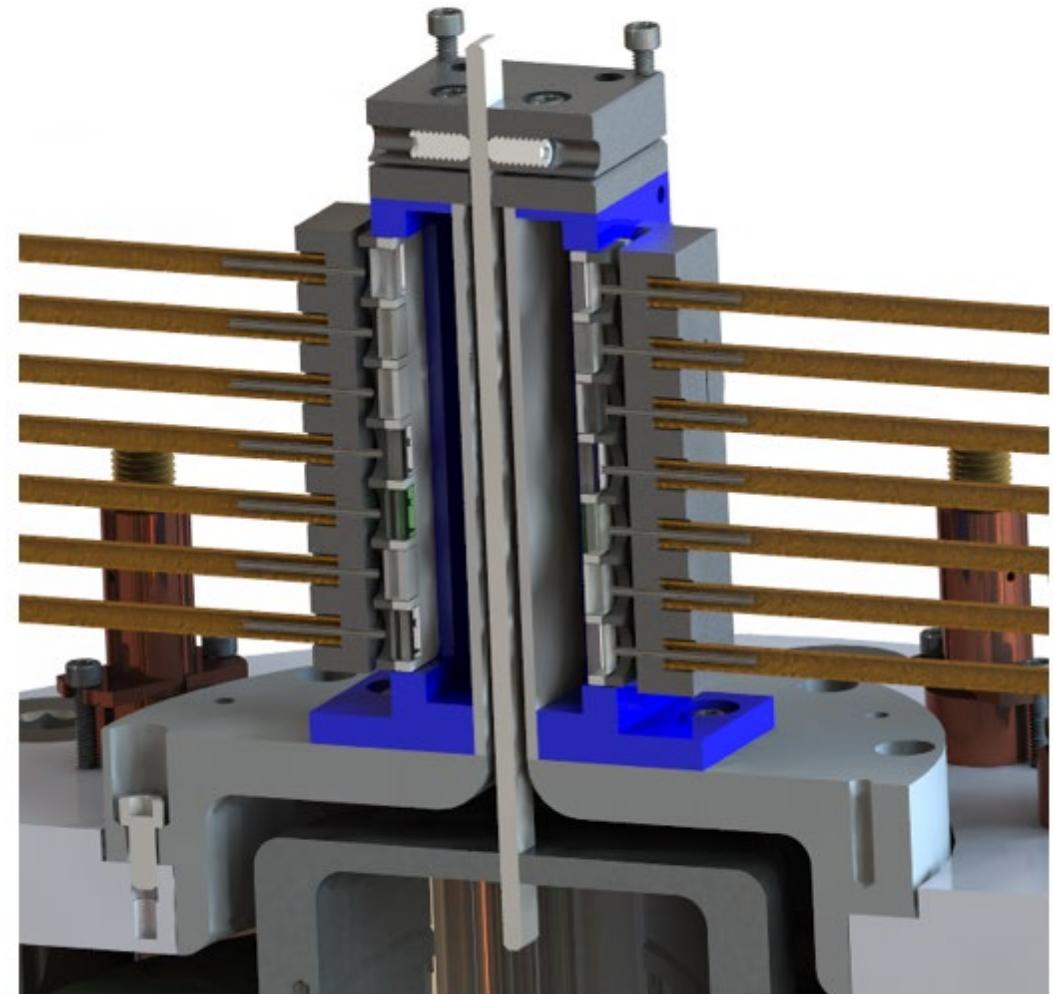


# Hugoniot shock experiments on Z Machine: 2 load geometries

2-sided **STRIPLINE** flyer velocities up to ~40 km/s



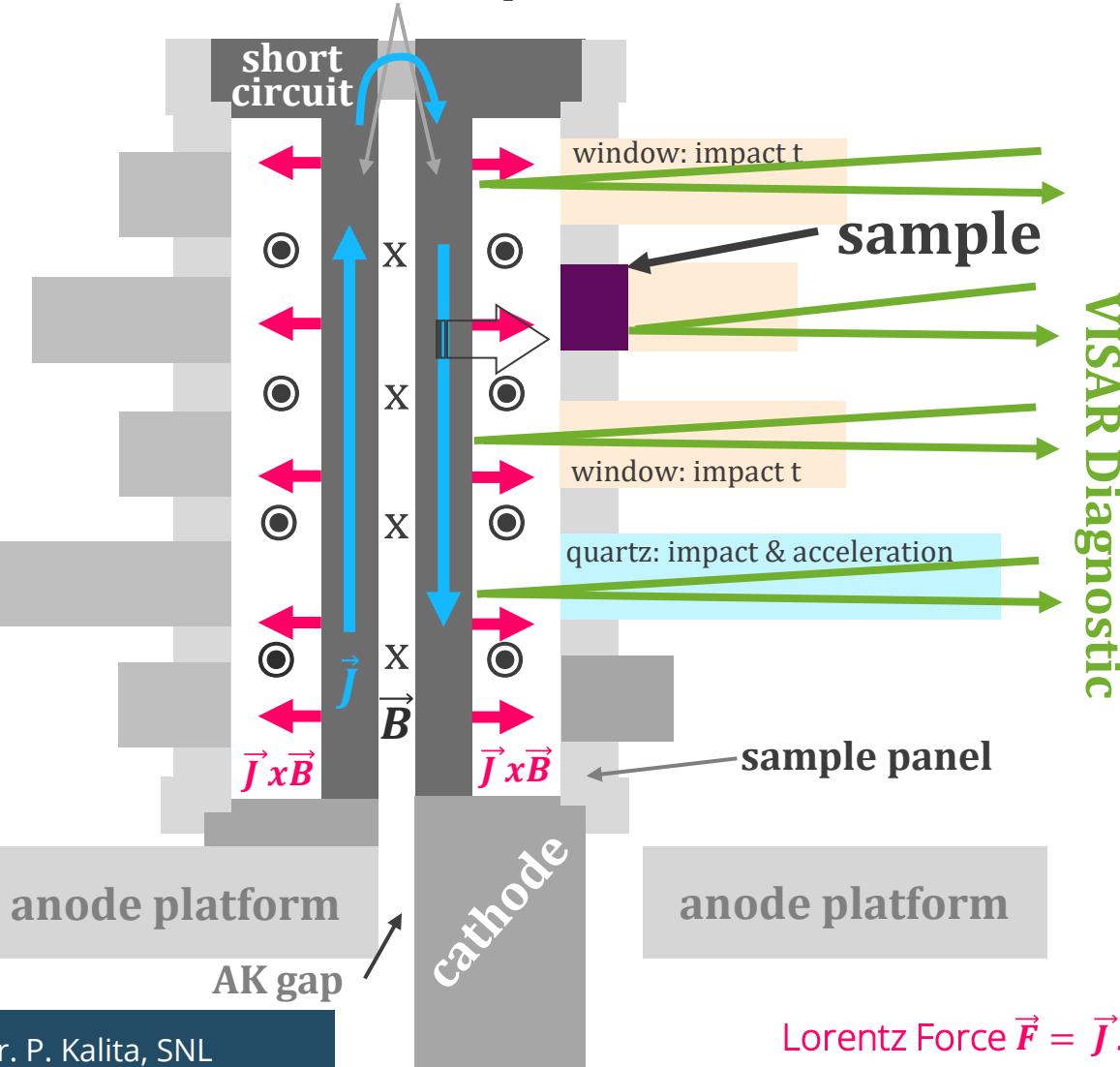
**COAXIAL** two flyer velocities 9-25 km/s



# Hugoniot shock experiments on Z Machine: 2 load geometries

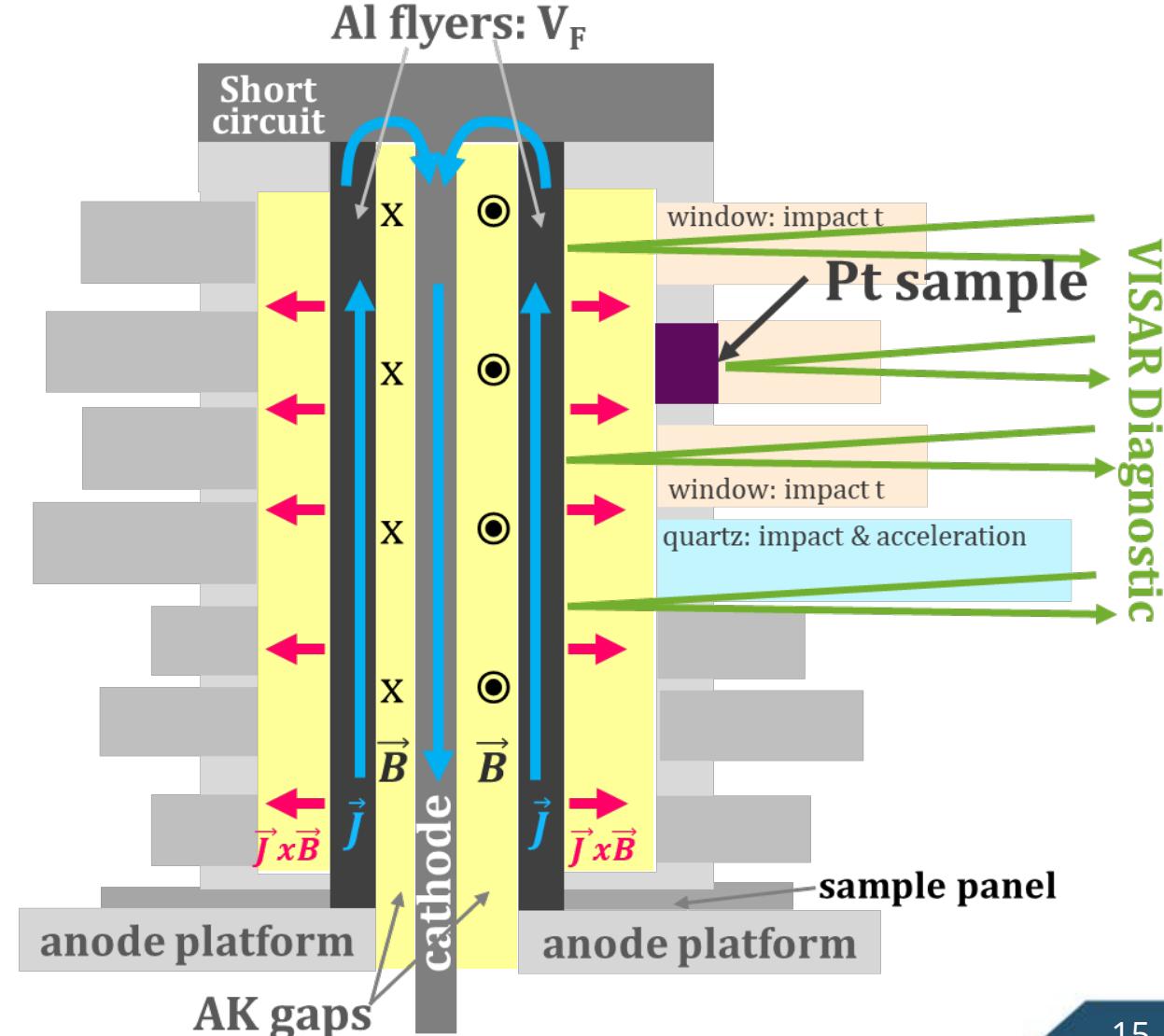
2-sided **STRIPLINE** flyer velocities up to  $\sim 40$  km/s

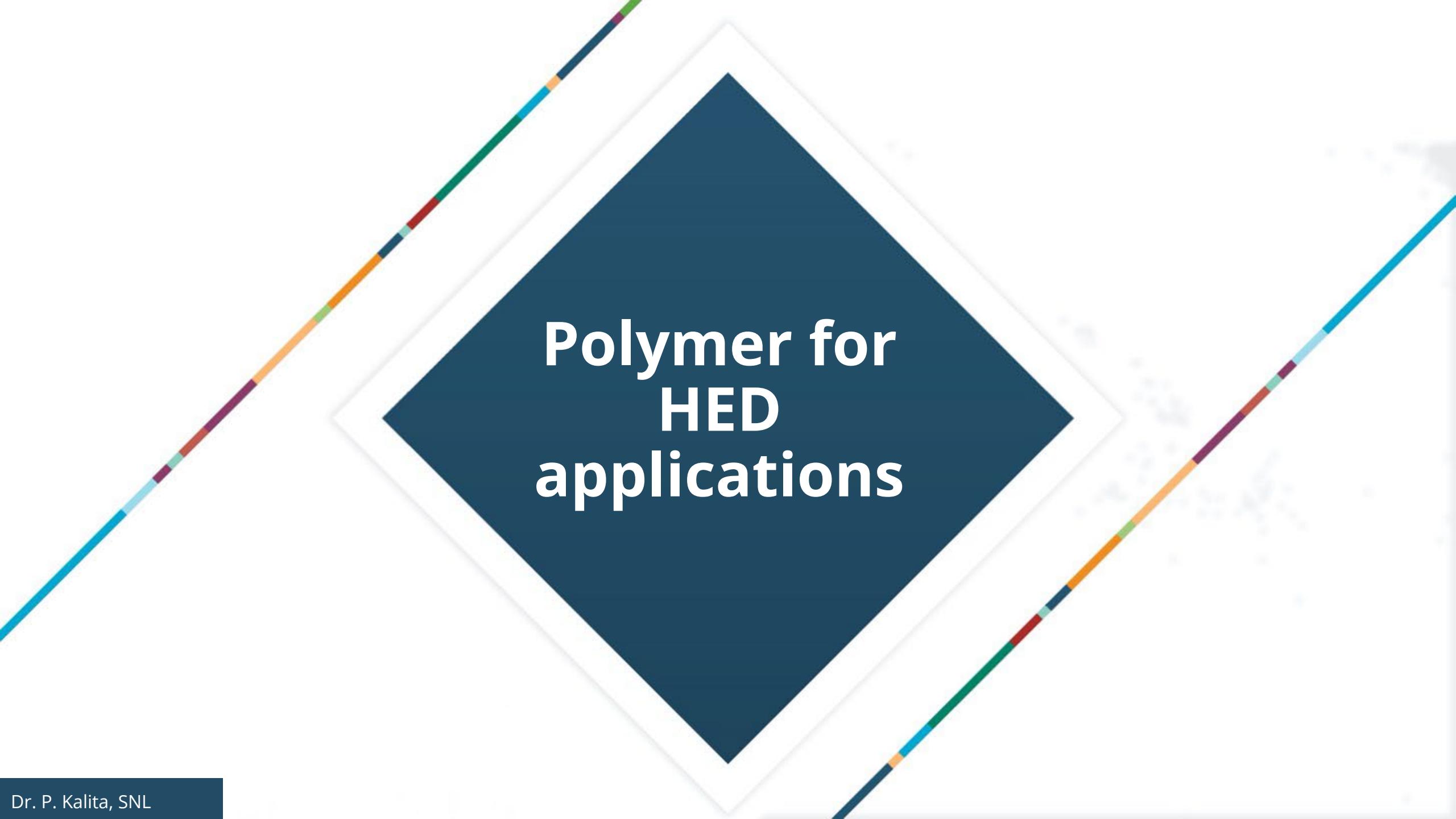
Al flyers:  $V_F$



**COAXIAL** two flyer velocities 9-25 km/s

Al flyers:  $V_F$

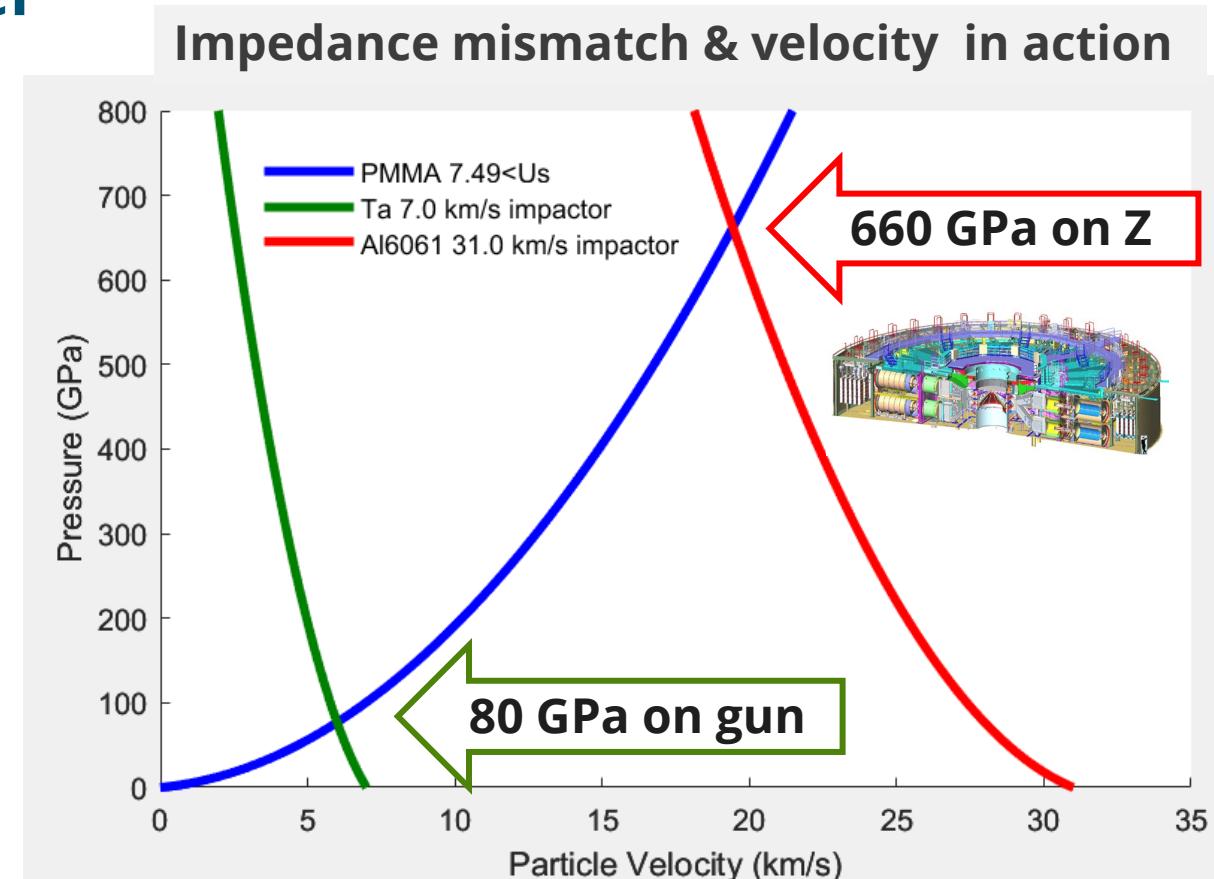




# Polymer for HED applications

## Example #1: PMMA polymer

- PMMA, Plexiglass® or Lucite®
- PMMA, belongs to the family of hydrocarbon polymers used in a wide variety of practical applications
- HED - High Energy Density facilities use these polymers in dynamic compression at extreme P
- existing Equation of State (EOS) of PMMA are benchmarked up to ~130 GPa at most!
- extrapolation is risky: the higher the pressure, the riskier the extrapolation of an EOS.
- **no data on Hugoniot above ~130 GPa**

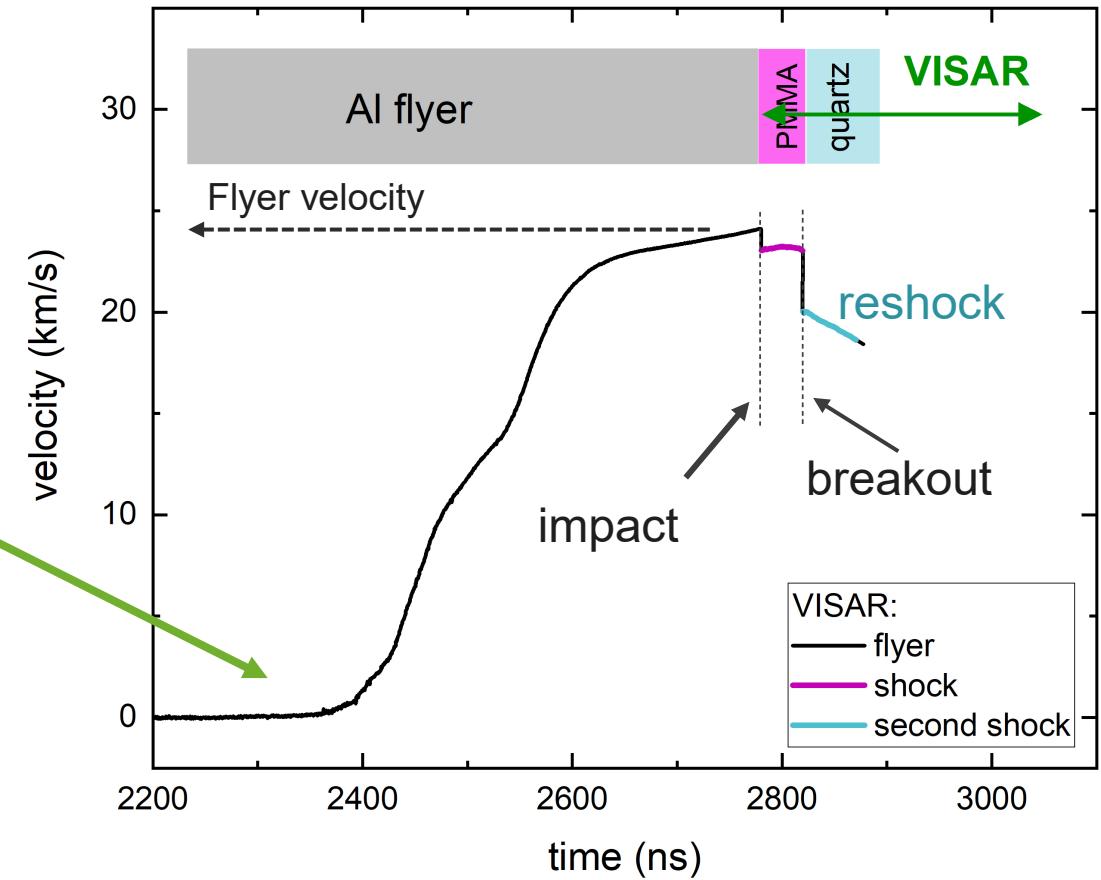


- Acoustic impedance: reference density x the speed of sound
- A polymer is a very low impedance material!
- Tantalum is a very high impedance impactor
- The **impedance difference** between flier and sample determines how much shock pressure will be generated

# PMMA - Hugoniot shock experiments on Z

## Our key diagnostic: VISAR (532nm)

## Velocity interferometer system for any reflector



# PMMA - Hugoniot shock experiments on Z

Our key diagnostic: VISAR (532nm)

Velocity interferometer system for any reflector

PMMA sample:  
experimental  $U_s \rightarrow$  reflective shock front

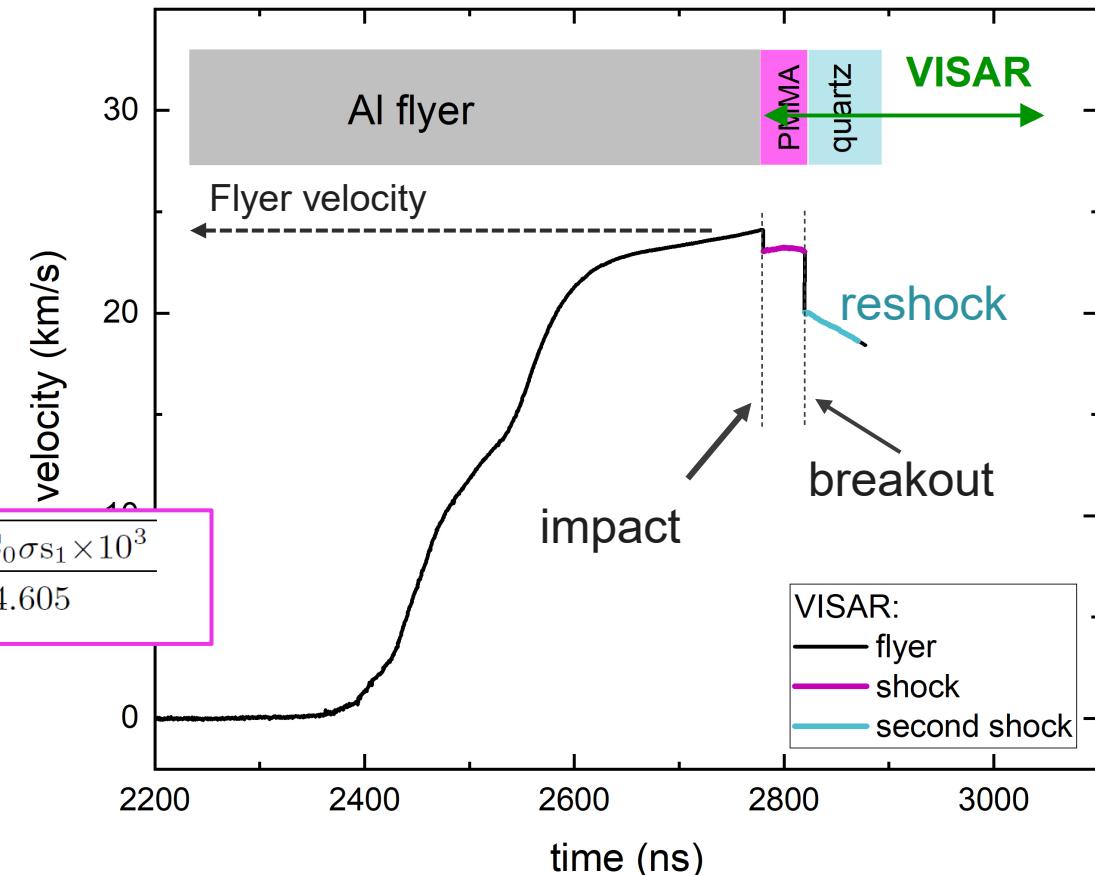
Rankine-Hugoniot eq.

Monte-Carlo impedance matching with Al flyer

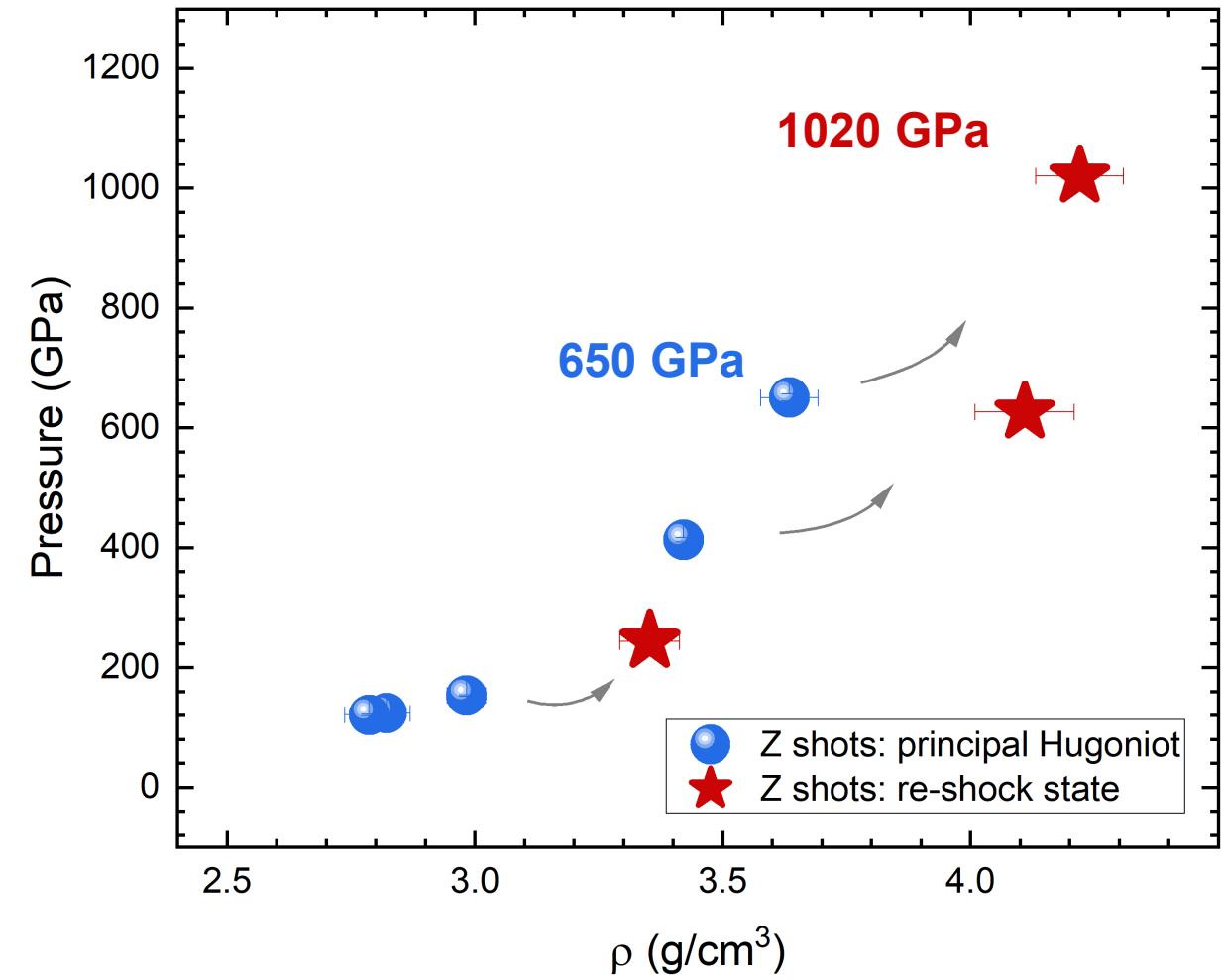
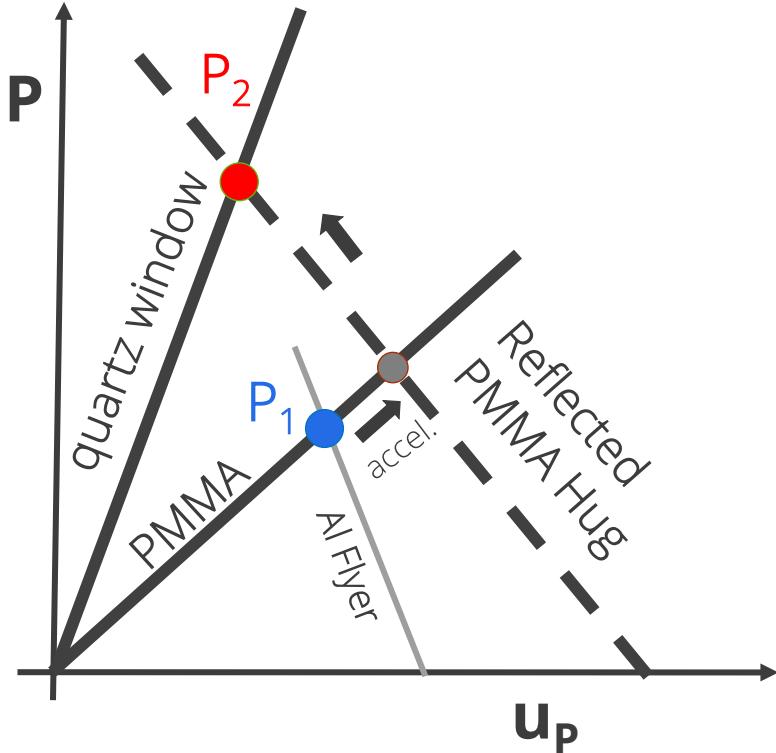


Flyer	$C_0$ (km/s)	$s_1$	$\sigma C_0 \sigma s_1 \times 10^3$
Al	$6.322 \pm 0.231$	$1.188 \pm 0.020$	-4.605

PMMA sample:  $U_p$ ,  $P$ ,  $\rho$



# PMMA Shock Hugoniot & re-shock states



# Hugoniot data form Z vs existing EOSs

Two fits to US – UP data:

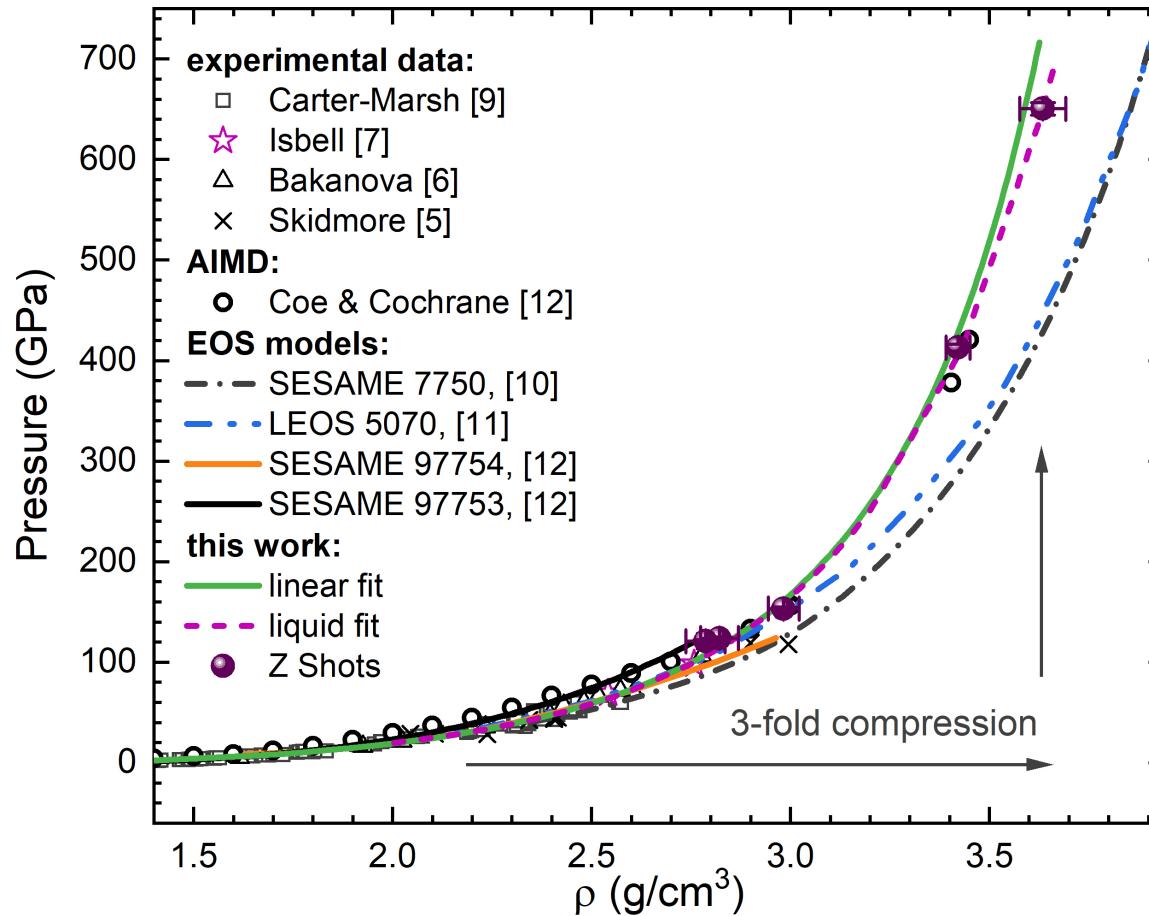
- Linear fit
- Liquid fit

TABLE VI. PMMA Hugoniot linear fit parameters and covariance matrix elements for  $U_S = C_0 + S_1 u_P$  up to 650 GPa.

$C_0$ (km/s)	$S_1$	$\sigma_{C_0}^2 \times 10^4$	$\sigma_{S_1}^2 \times 10^5$	$\sigma_{C_0}\sigma_{S_1} \times 10^5$
$2.855 \pm 0.021$	$1.344 \pm 0.005$	4.59	2.36	-8.47

TABLE IV. The modified, universal liquid fit  $U_S = A + B u_P - C u_P \exp(-D u_P)$  parameters for the PMMA Hugoniot up to 650 GPa.

$A$ (km/s)	$B$	$C$	$D$ (km/s) $^{-1}$
$3.18 \pm 0.72$	$1.32 \pm 0.04$	$0.19 \pm 0.4$	$0.30 \pm 0.14$





## PMMA – takeaways

- new direct measurements of shock states for PMMA polymer
- pressure and density along the principal Hugoniot are extended from **120 to 650 GPa**.
- Re-shock states up to **1020 GPa**, which corresponds to **~3.5-fold compression** of PMMA
- the Z data are significantly stiffer than the existing EOS models
- This work shows clear evidence for the **need to revisit the wide-ranging PMMA EOS tables** for experimental and modeling applications

Journal of  
Applied Physics

### Shock compression of poly(methyl methacrylate) PMMA in the 1000 GPa regime: Z machine experiments

Cite as: J. Appl. Phys. **133**, 035902 (2023); <https://doi.org/10.1063/5.0128681>

Submitted: 29 September 2022 • Accepted: 16 December 2022 • Published Online: 19 January 2023

Pat Kalita, Marcus D. Knudson, Tom Ao, et al.





# Platinum standard

## Example #2: Platinum shock standard at multi-TPa pressures

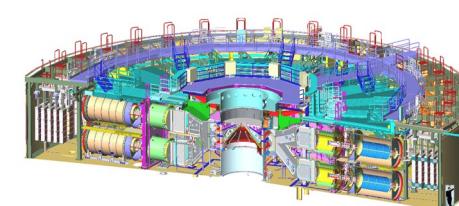
- Cutting edge shock compression research is moving towards ever increasing pressure or stress states into  $>$  TPa range
- Experiments at extreme pressures allow to test numerical methods, physics theories and uncover exotic matter behaviors.
- Existing Equation of State (EOS) standard materials are limited to  $<$  0.5 TPa,
  - no data on Pt Hugoniot above 670 GPa
  - Shockless compression to 1 TPa
- Extrapolation is risky: at shock  $P >$  1 TPa significant uncertainties.

- We conducted a series of shock compression experiments on Pt up to 2 TPa = 20 Mbar = 20 million atm on **SNL Z machine** combined with **DFT-AIMD** calculations and with **design of a broad range Hugoniot EOS**
- **Our work establishes Platinum as a shock compression standard to  $>$  2 TPa**

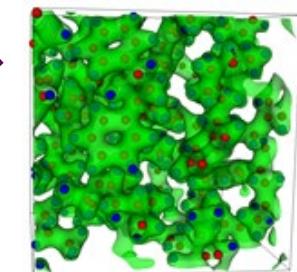
**SNL**  
shock experiments on Pt  
on Sandia's Z machine  
up to 2177 GPa = 2.17 TPa



**SNL**  
*ab initio* molecular  
dynamics (AIMD)  
simulations on Pt



**SNL & LANL**  
broad range multiphase  
EOS design for Pt  
SESAME 3732



# Pt standard: DFT calculations of the cold curve & AIMD calculations of the Hugoniot EOS

## DFT cold curve - 0K:

- Exchange correlation energy computed with **PBESol** or with **AM05** functional and a **16-electron pseudopotential**
- results are in **excellent agreement** with literature data and with the independently developed **SESAME EOS**
- DFT also validated by calculating the elastic constants and phonon dispersion (using AM05 potential)

## AIMD - ab initio molecular dynamics simulations:

- generate a tabular EOS directly useable in a hydrodynamics code
- 16-electron pseudopotential, **PBESol**, and the Mermin's generalization of the **Kohn-Sham** equations to finite T
- good agreement between **DFT** and **SESAME** Hugoniot and with principal isentrope
- **DFT melt transition is upper bound in T**
- SESAME EOS melt density is consistent with DFT

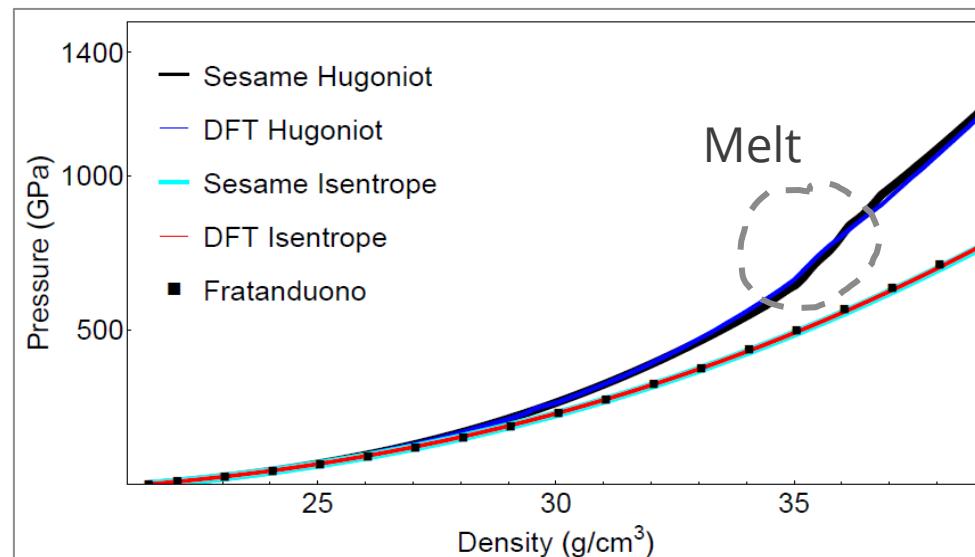


FIG. 7. Hugoniot calculated from DFT and SESAME EOSs. The isentrope is virtually identical in pressure/density space. The DFT calculated isentrope values compare well with experimental data [23].

[23] = Fratanduono et al., Science 372, 1063–1068 (2021).

# Pt standard: Broad-Range Multi-Phase EOS Design

Helmholtz free energy

$$F(T, V) = F_{cold}(V) + F_{ion}(T, V) + F_{electron}(T, V)$$

**cold curve contribution**

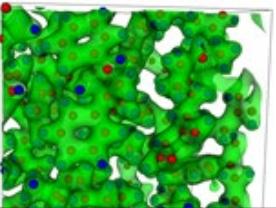
**cold + thermal ionic contribution**

**thermal electronic contribution**

## Development of SESAME EOS 3732

- standard decomposition of Helmholtz free energy
- **Cold curve**
  - determined via Mie-Grüneisen approximation by matching a quadratic to experimental data
  - intercept of the quadratic was fixed to match the adiabatic bulk modulus from ultrasonic data
- **Ion thermal component**
  - used Debye approximation for solid and corrected Debye for fluid
  - Grüneisen gamma  $\Gamma$  parameter and ref. Debye T in ion thermal model is set by matching **isobaric expansion data** and **specific heat** data
  - derivative of  $\Gamma$  with density obtained by matching to RT DAC data
- **electron thermal component** was determined using the **Thomas-Fermi-Dirac** model
- melt transition used Lindemann Melt Law.

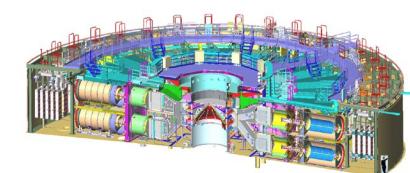
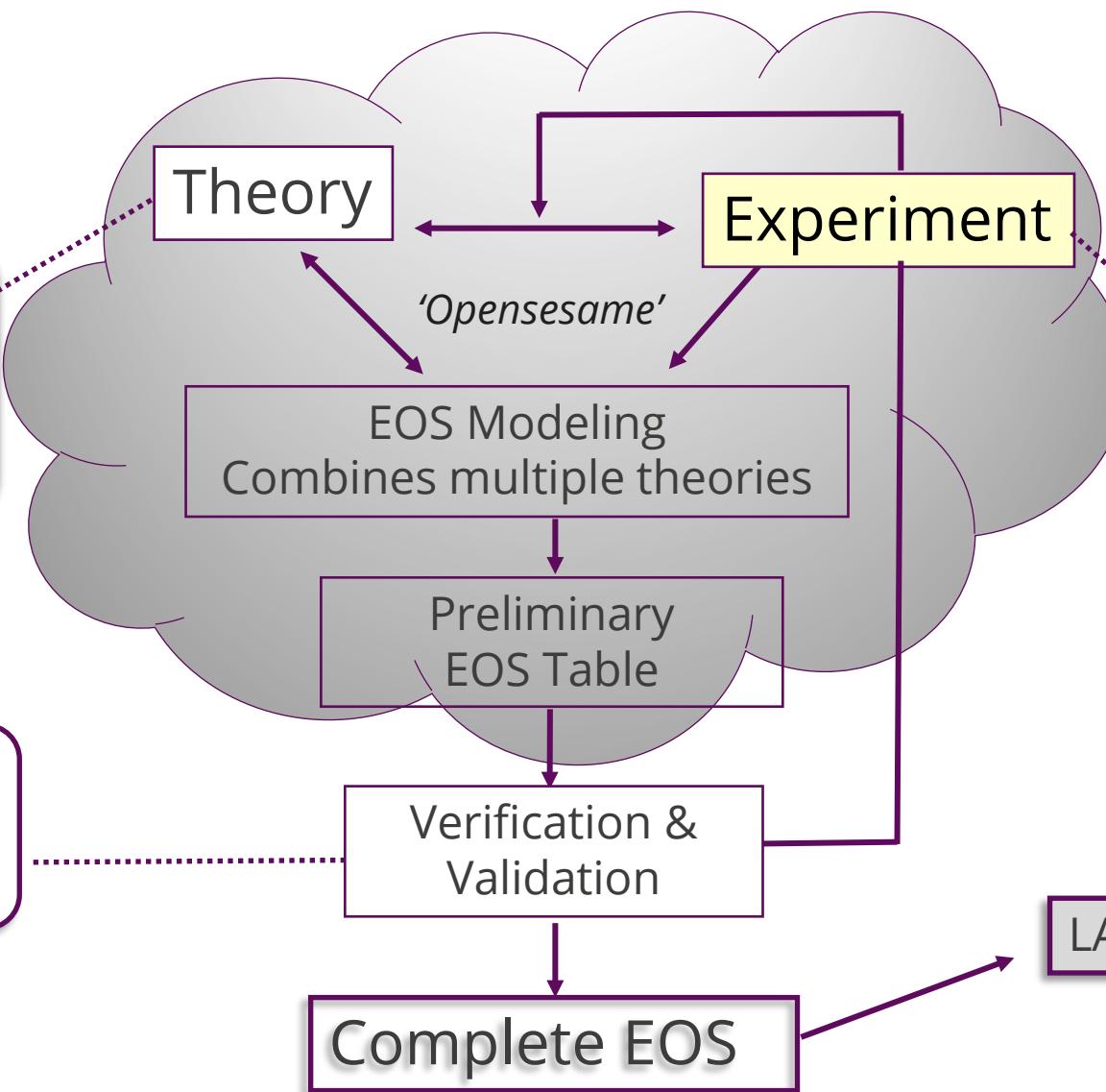
SESAME EOS development implemented in LANL code:  
**'Opensesame'**



- Density Functional Theory
- AIMD, QMD/QMC, OFMD
- Thomas-Fermi
- Lindemann melt law

VASP, RSPT, Wien2K,  
Quantum Espresso

- Hydrocode Simulation
- Additional Experiments
- Refined Theories



- Hugoniot ( $P$ ,  $T$ ,  $\rho$ ,  $c_s$ ,  $U_p$ ,  $U_s$ )  
**SNL Z Machine**  
shock guns
- Off Hugoniot  
ICE  
Diamond Anvil Cell
- Neutron Scattering  
Elastic Constants  
Phonons

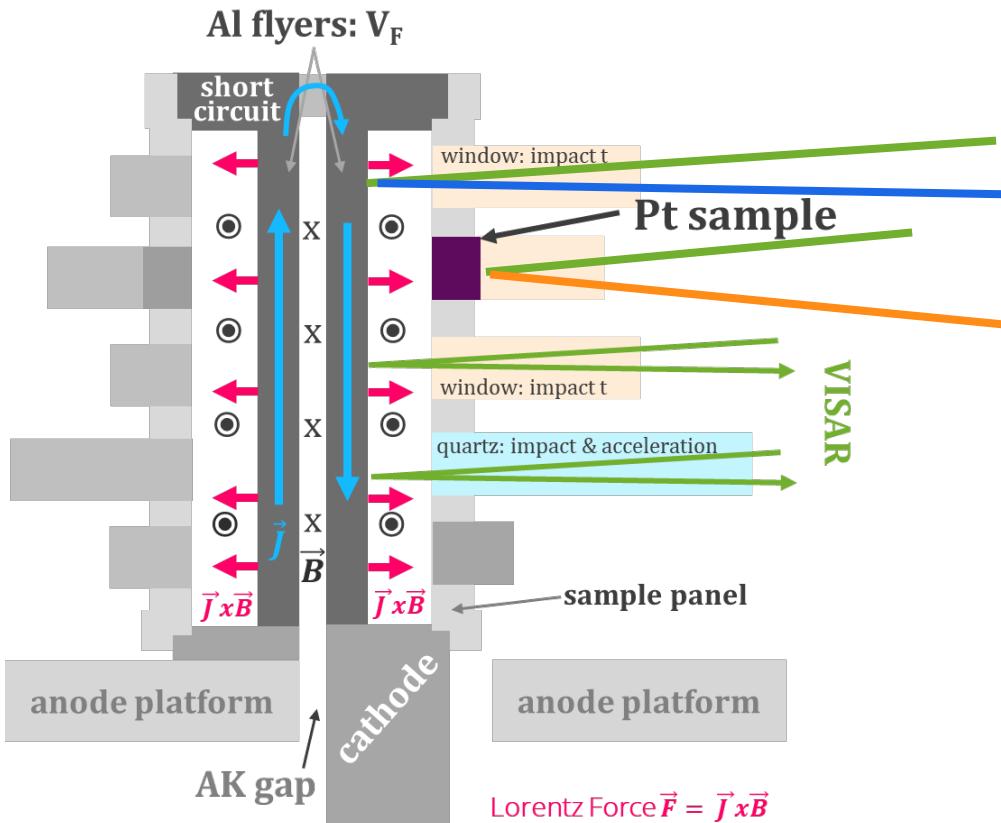
## The SESAME ascii file

LANL SESAME EOS Database

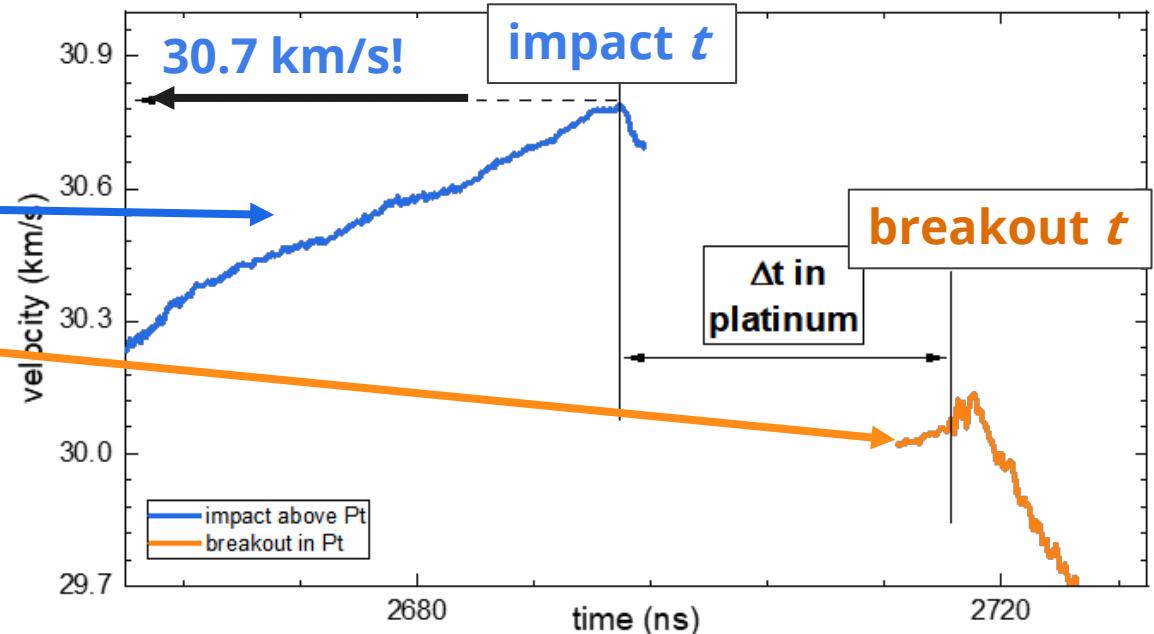
# Hugoniot shock experiments on SNL Z Machine: 2 geometries

Our key diagnostic: VISAR (532nm)

Velocity interferometer system for any reflector



Flyer	$C_0$ (km/s)	$s_1$	$\sigma C_0 \sigma s_1 \times 10^3$
Al	$6.322 \pm 0.231$	$1.188 \pm 0.020$	$-4.605$

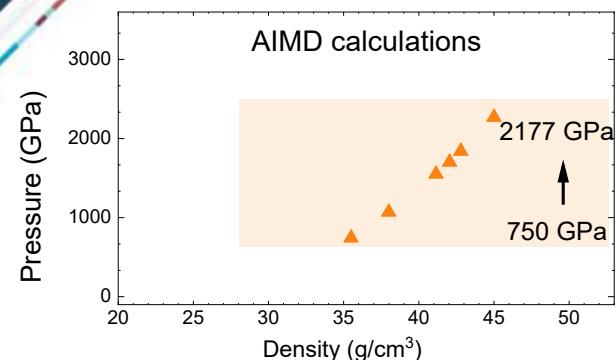


experimental  $U_s = \text{thickness} / \Delta t$

Monte-Carlo impedance matching with Al flyer

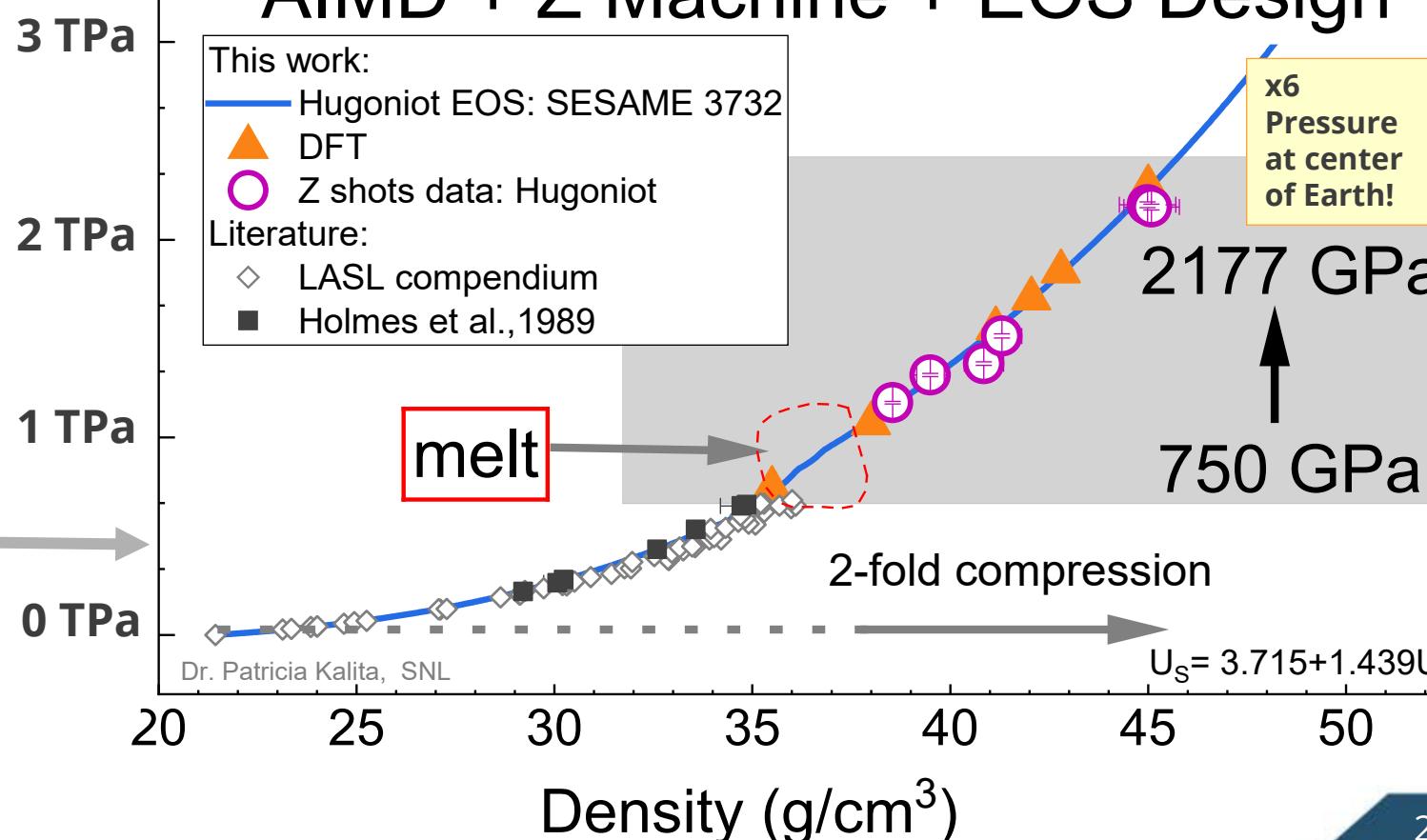
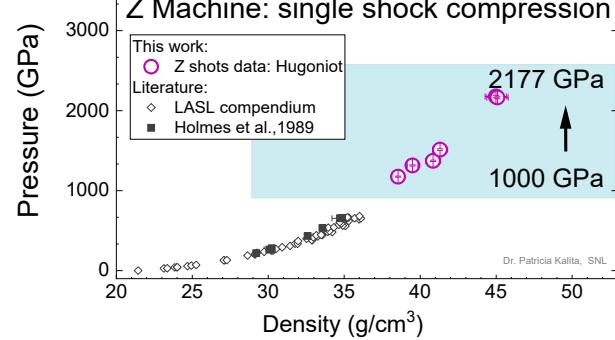
Pt sample:  $U$ ,  $P$ ,  $\rho$

# Pt: New experimental Hugoniot to 2.2 TPa + AIMD simulations + SESAME 3732 EOS



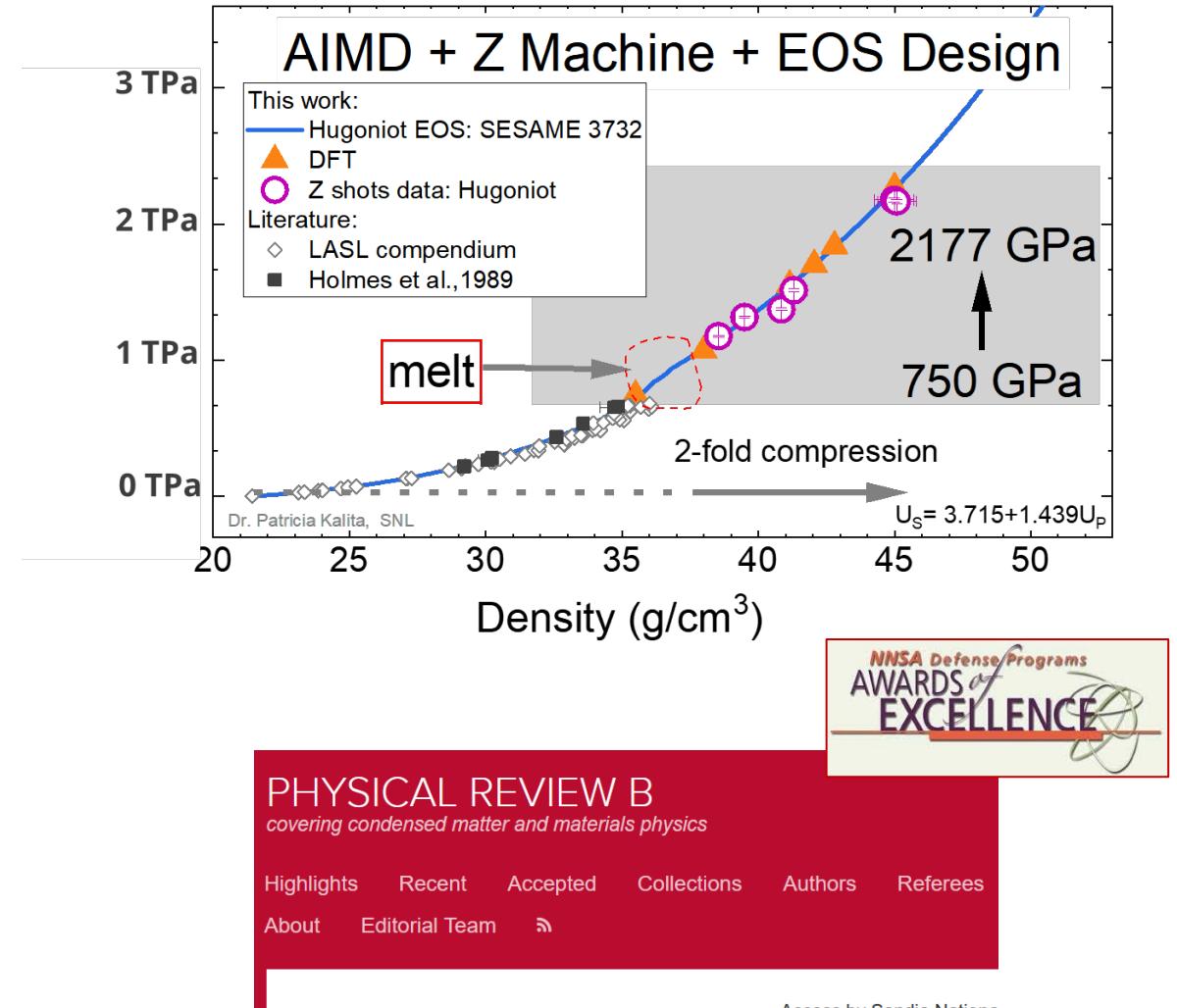
**Pt compressed to > 2 TPa =  
20 Mbar = 20 million atm.**

Establishing Platinum as a shock compression standard to more than 2 TPa.



# Pt standard - Takeaways

- Our multi-TPa EOS of platinum allows to use Pt as a dependable high impedance standard in shock experiments
- We experimentally constrained the Pt standard up to 2.2 TPa using shock compression
- we carried out AIMD simulations
- we designed a new EOS for Pt: SESAME 3732
- Achieved record shock pressure on Z machine



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Platinum equation of state to greater than two terapascals: Experimental data and analytical models

Kyle R. Cochrane, Patricia Kalita, Justin L. Brown, Chad A. McCoy, Jeffry W. Gluth, Heath L. Hanshaw, Edward Sciglietti, Marcus D. Knudson, Sven P. Rudin, and Scott D. Crockett  
Phys. Rev. B 105, 224109 – Published 17 June 2022



# Shock compression of Ti64 alloy

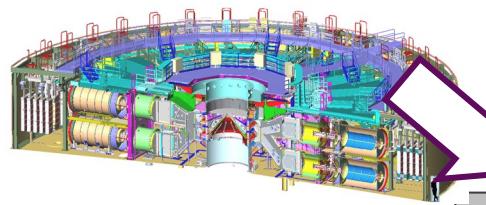
## Example #3: Ti64 alloy

- **Ti-6Al-4V** (Ti 90wt%, Al 6wt%, V 4wt%), or Ti64
- the most widely used of titanium alloys
- multitude of commercial & industrial applications
- for use in extreme conditions - defense, aerospace, or nuclear industries - it is paramount to have a good description of the mechanical response of the **Ti64 alloy** to extreme P and T
- yet the **Ti64 alloy** is much less studied, compared with pure **titanium**
- no shock data on **Ti64 alloy**  $> 250$  GPa
- extrapolation of EOS without benchmarking to experimental data is risky

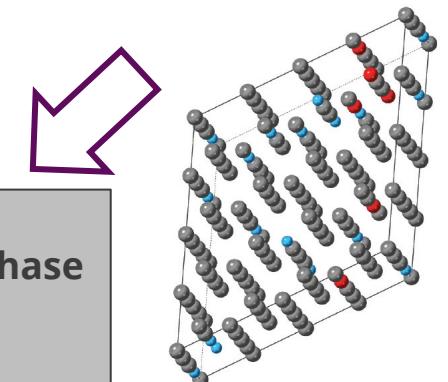
We conducted:

- a series of shock compression experiments on Ti64 up to 1.3 TPa on **SNL Z machine**
- combined with **DFT-AIMD** calculations
- and with **design of a broad range Hugoniot EOS**

**SNL**  
shock experiments on  
Ti64 on Sandia's Z  
machine up to 1.3 TPa

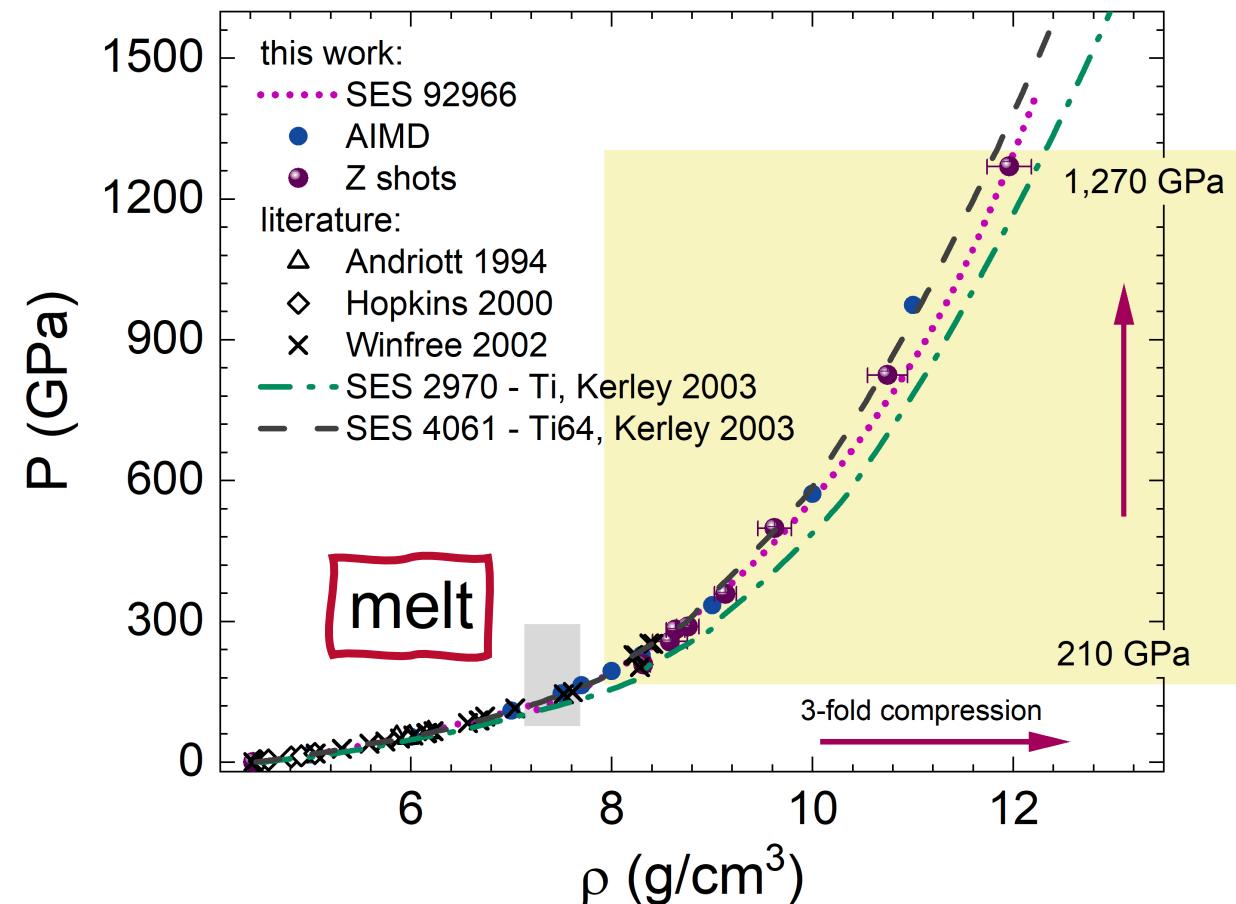


**SNL**  
*ab initio* molecular  
dynamics (AIMD)  
simulations on Ti64



**SNL & LANL**  
broad range multiphase  
EOS design for Ti64  
SESAME 92966

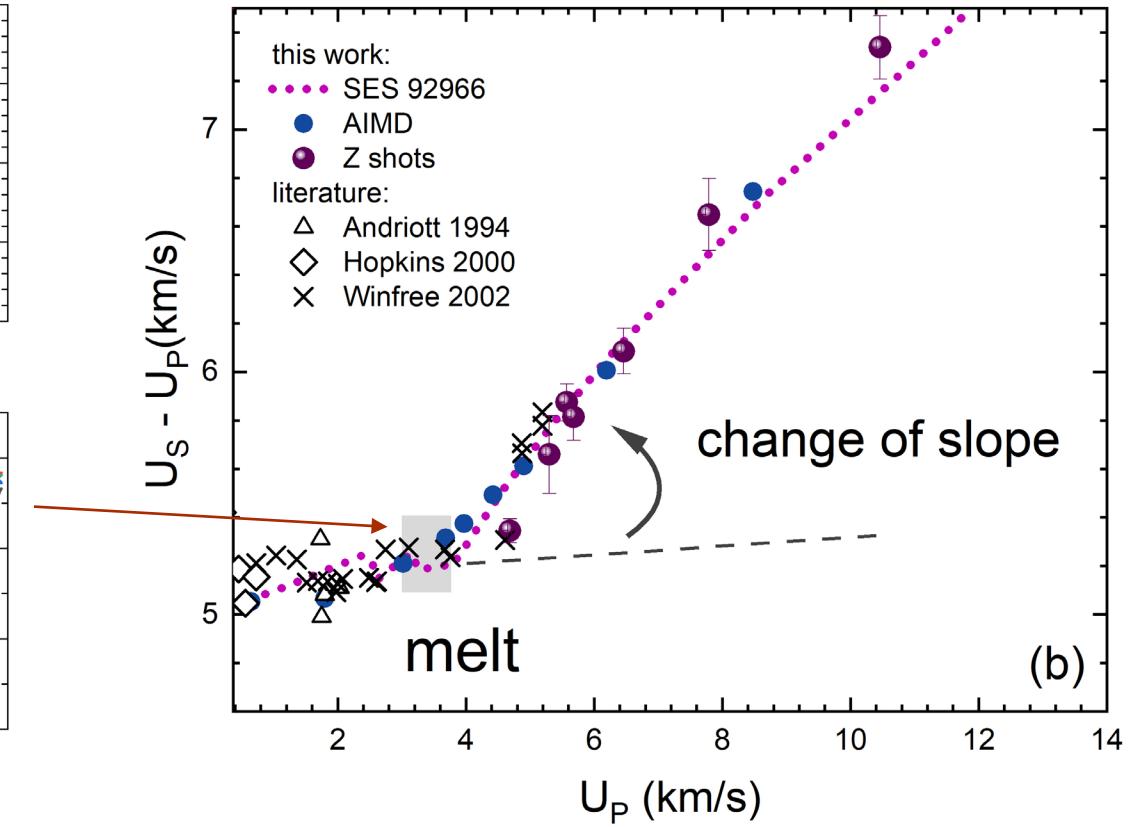
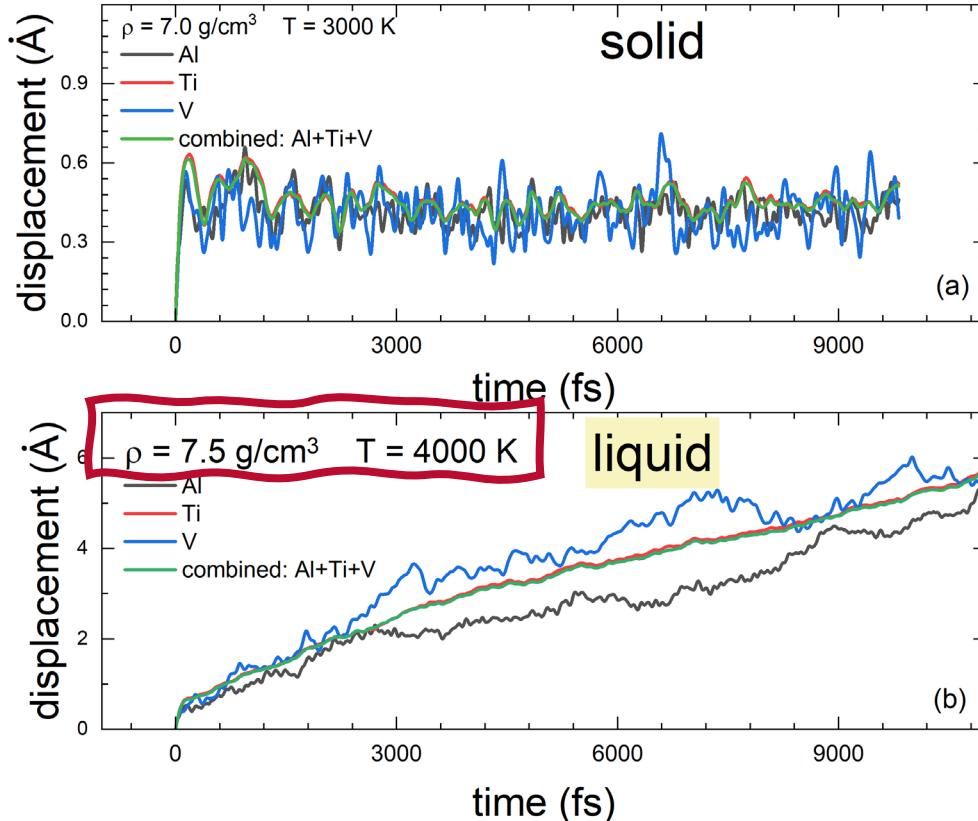
# Ti64 alloy: AIMD + Z Machine up to 1.3 TPa + SESAME 92966 EOS



**density accuracy  
better than 2%**

**Ti64 compressed to > 1.3 TPa = 13 Mbar = 13 million atm.**

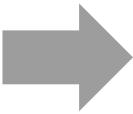
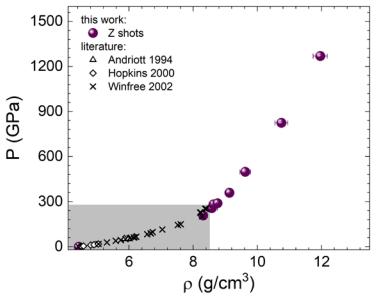
# Ti64: AIMD + Z Machine demonstrate melt on Hugoniot



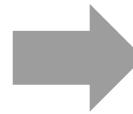
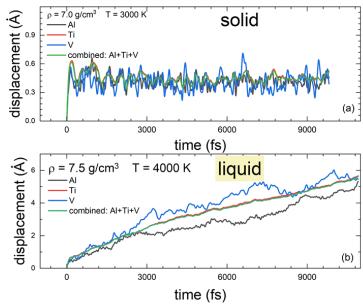
- Melt boundary on Hugoniot in disagreement with literature EOS
- previously shock melt was proposed at 6000–6800 K and 182–207 GPa
- our work shows shock melt on the Hugoniot is  $\sim 4000 \text{ K}$  and  $\sim 140 \text{ GPa}$

# Final product: broad-range phase diagram for Ti64 alloy

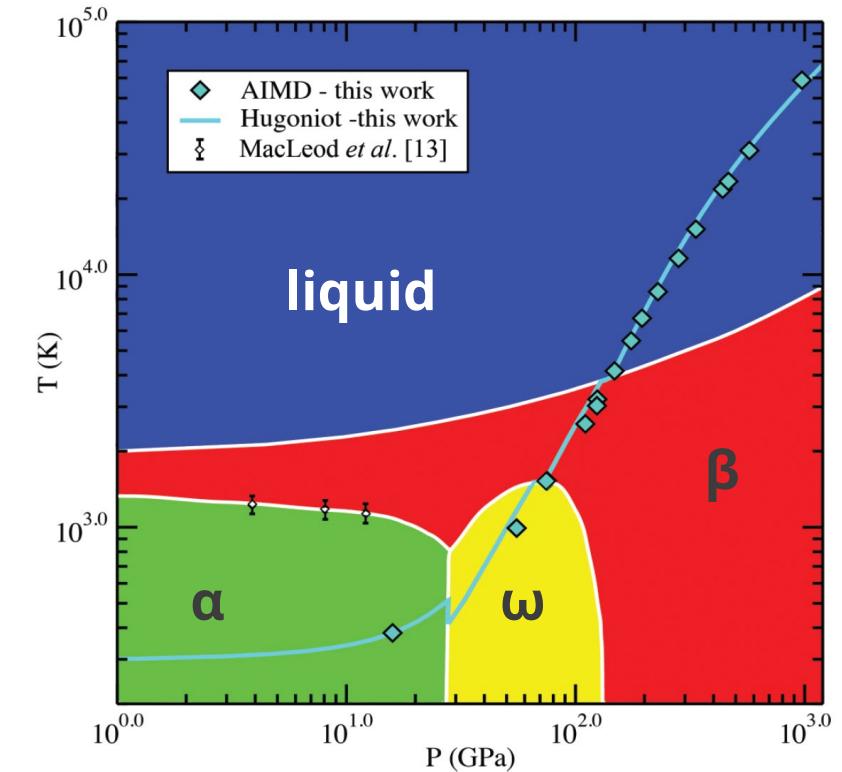
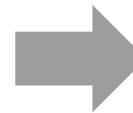
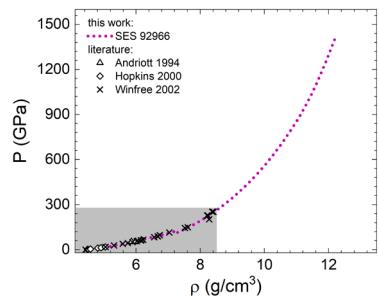
## Z machine experiments



## AIMD calculations



## New SESAME EOS



# Ti64 alloy: Takeaways

- New shock Hugoniot data are from the Sandia Z machine  
0.21 TPa - 1.27 TPa
- our AIMD calculations in excellent agreement with Z data
- Shock melt on Hugoniot is lower than previously calculated
- we developed a high-fidelity, multiphase EOS of Ti64, SESAME EOS 92966, spanning a broad range of T and P
- Hugoniot  $U_s(u_p)$  can be used in dynamic experiments

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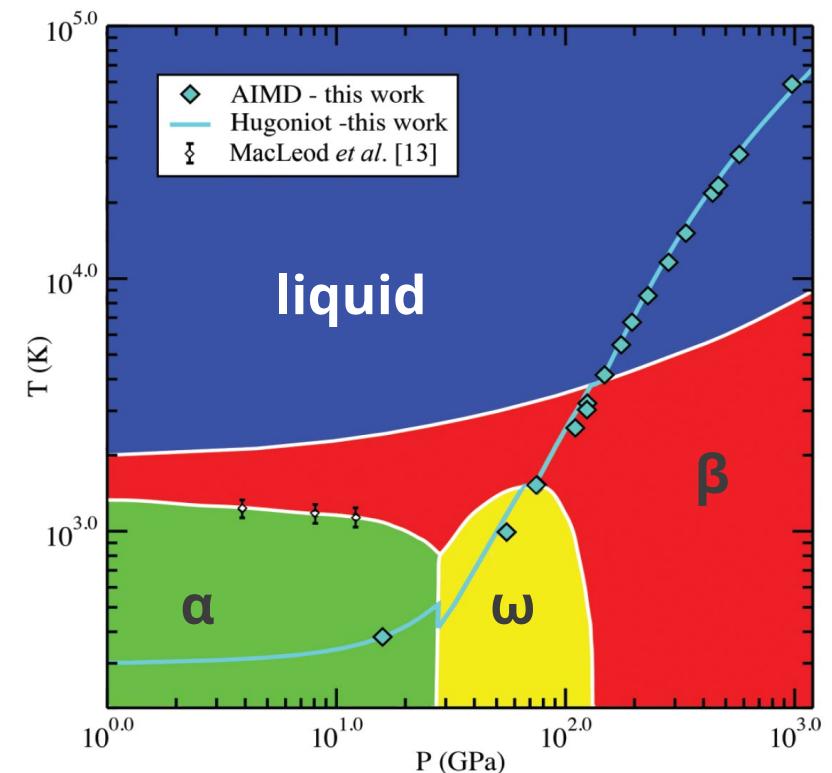
Accepted Paper

Ti-6Al-4V to over 1.2 TPa: Shock Hugoniot experiments, *ab initio* calculations, and a broad-range multiphase equation of state

Phys. Rev. B

Pat Kalita, Kyle R. Cochrane, Marcus D. Knudson, Tommy Ao, Carrie Blada, Jerry Jackson, Jeffry Gluth, Heath Hanshaw, Ed Scoglietti, and Scott D. Crockett

Accepted 30 January 2023





# Conclusions

- Z is an “engine of discovery” for a wide range of HED science
- ~1/3 of our Z shots are dynamic material properties experiments
- Cutting edge shock compression research is moving towards ever increasing pressure or stress states into > TPa range
- Shock experiments on Z allow to understand the high P-T behavior of materials and...
- ...create accurate equation of state models based on experimental data

<https://journals.aps.org/prb/accepted/3c07c090j741fd4061ee3b303fa03d1fb33f395c3>  
<https://journals.aps.org/prb/abstract/10.1103/PhysRevB.105.224109#>  
<https://aip.scitation.org/doi/10.1063/5.0128681>

**Journal of Applied Physics**

**Shock compression of poly(methyl methacrylate) PMMA in the 1000 GPa regime: Z machine experiments**

Cite as: J. Appl. Phys. 133, 035902 (2023); <https://doi.org/10.1063/5.0128681>  
Submitted: 29 September 2022 • Accepted: 16 December 2022 • Published Online: 19 January 2023

Pat Kalita, Marcus D. Knudson, Tom Ao, et al.

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# Backup slides

# Sandia's Z Pulsed Power Facility

An aerial photograph of the Sandia Z Pulsed Power Facility. The facility is a complex of industrial buildings, including a large central building labeled 'Z Building' in red text, several smaller buildings, and numerous large cylindrical storage tanks. The facility is situated in a desert-like environment with dry, sandy ground and some sparse vegetation. In the background, there are more industrial buildings and what appears to be a parking area with several vehicles.

The Earth's largest pulsed power machine: The central focus of today's talk

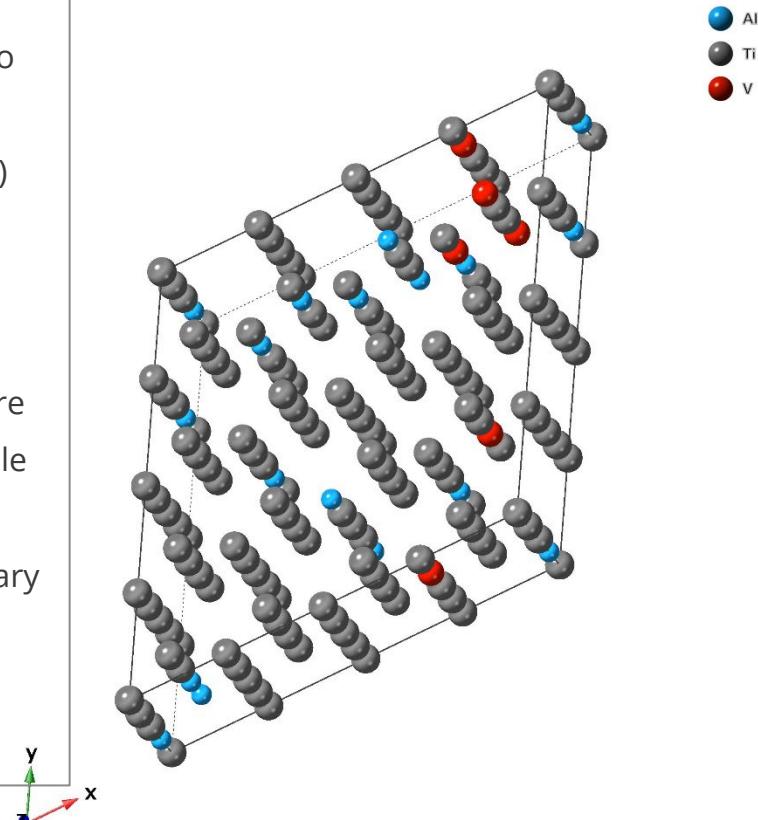
Z Building

# Ti64: DFT calculations of the cold curve & AIMD calculations of the Hugoniot EOS



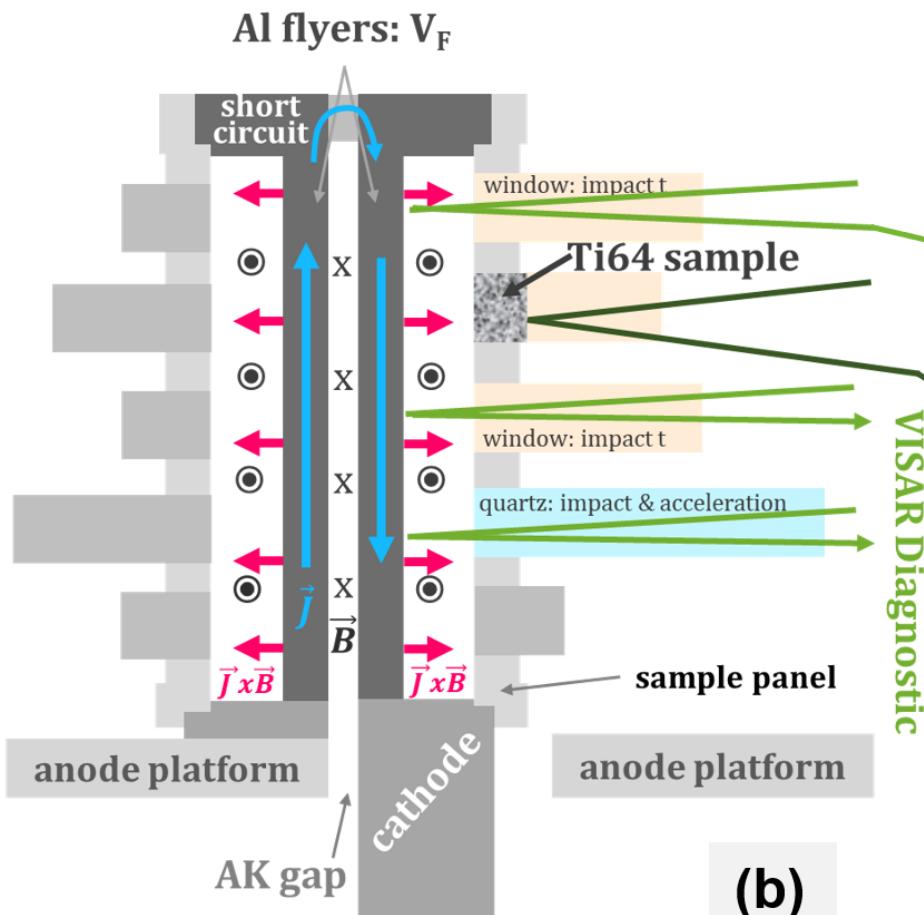
## AIMD - ab initio molecular dynamics simulations:

- performed in the canonical (NVT) ensemble with Mermin's generalization of the **Kohn-Sham** equations to finite T
- exchange-correlation energy computed with the parameterization of Perdew, Burke, and Ernzerhof (**PBE**)
- In Kohn-Sham equations, the nuclei were represented by a projector augmented wave (PAW) method. Pseudopotentials contained Al: 3 electrons; Ti: 12 electrons; V: 13 electrons
- The k-point mesh was Monkhorst-Pack [30]  $2 \times 2 \times 2$  for a 128 atom hcp cell.
- vanadium 4 wt% and aluminum 6 wt% were rounded to an integer number of 5 and 13 atoms, which were substituted randomly into the titanium lattice using the ATAT code, because DFT is not realistically capable of looking at grain boundary effects.
- only examined the  $\alpha$  Ti-64 phase assuming that the contribution to the reference state by  $\beta$  grain boundary would be small.
- good agreement between **DFT** and **SESAME** Hugoniot
- SESAME EOS melt density is consistent with DFT and experiments

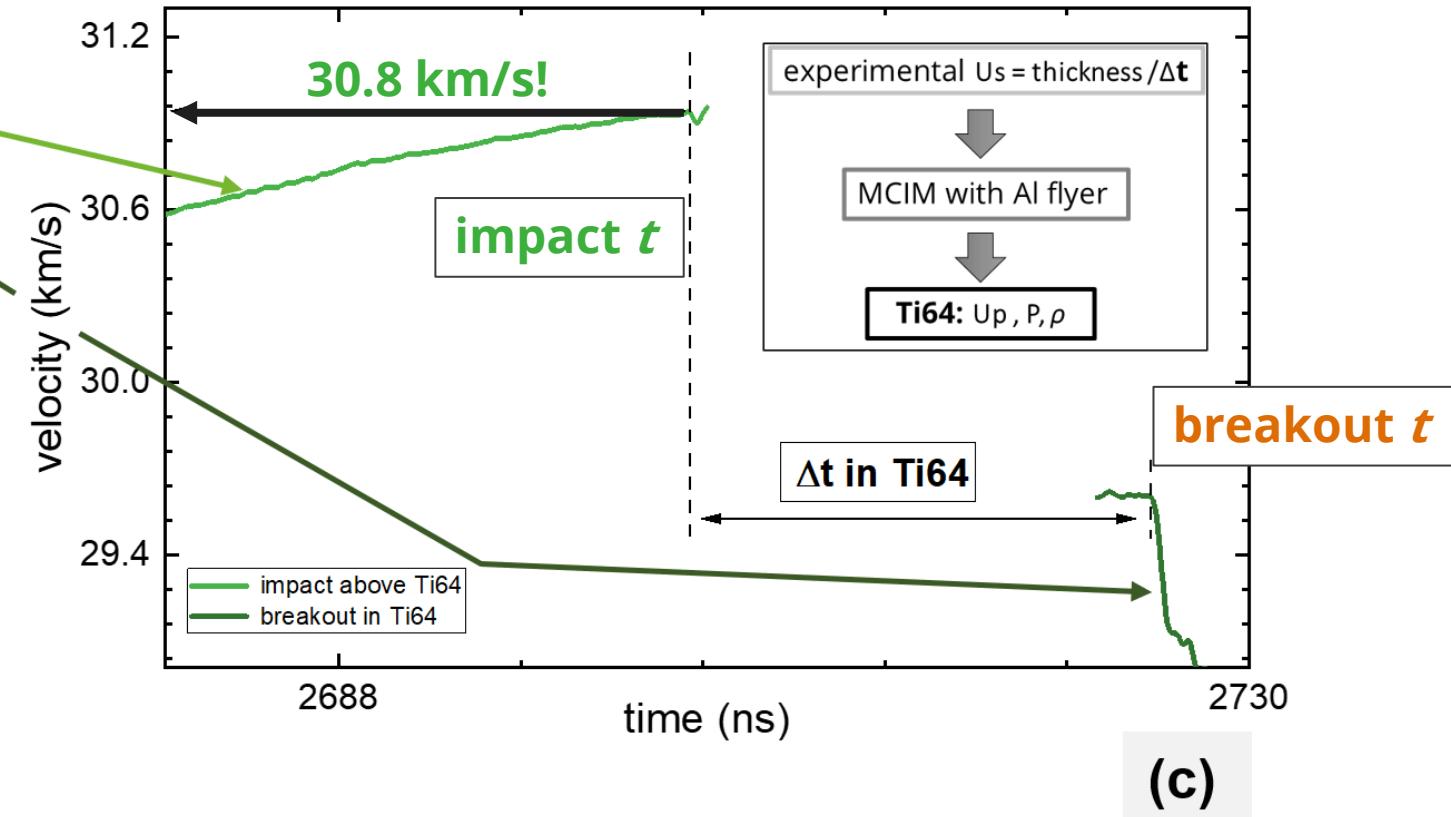


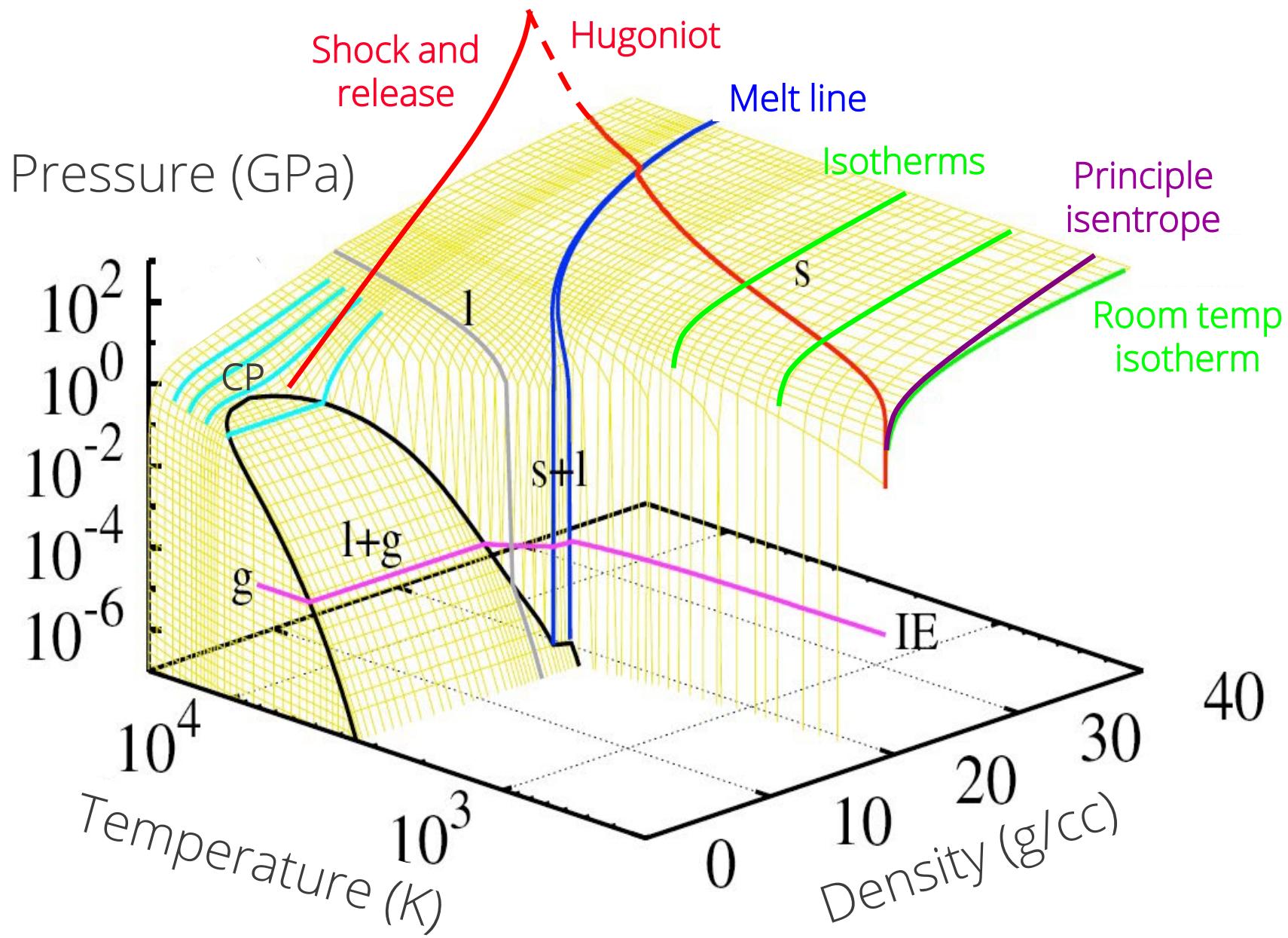
# Ti64 alloy: Hugoniot shock experiments on SNL Z Machine

2-sided symmetric stripline config. 25-40 km/s



key diagnostic: VISAR (532nm)

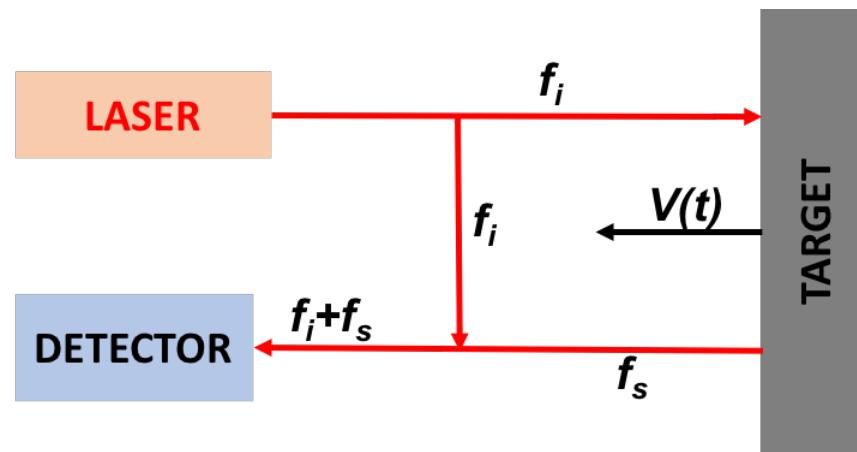






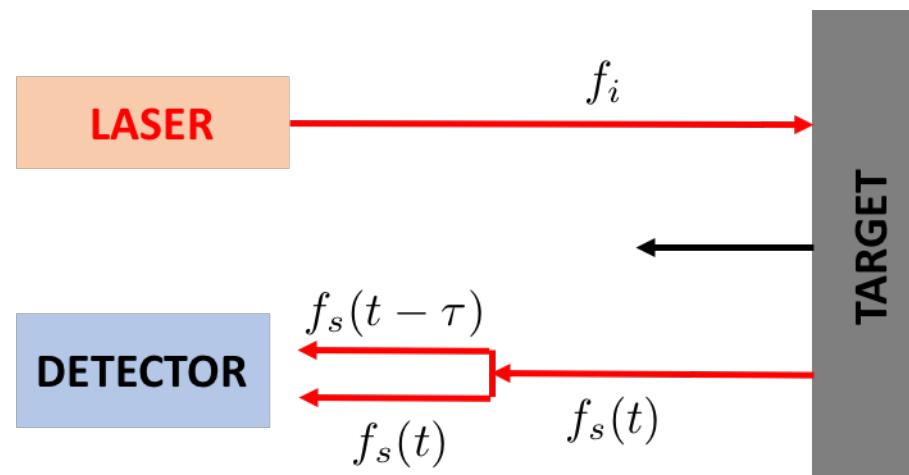
# Experimental Methods for Shock Compression

# Primary Diagnostics: Velocimetry



Displacement Interferometer (PDV)

$$B = |f_s - f_i| = \frac{2v}{\lambda_i}$$



Velocity Interferometer (VISAR)

$$v(t) \approx \frac{\lambda_i}{2\tau(1 + \delta)} g(t)$$

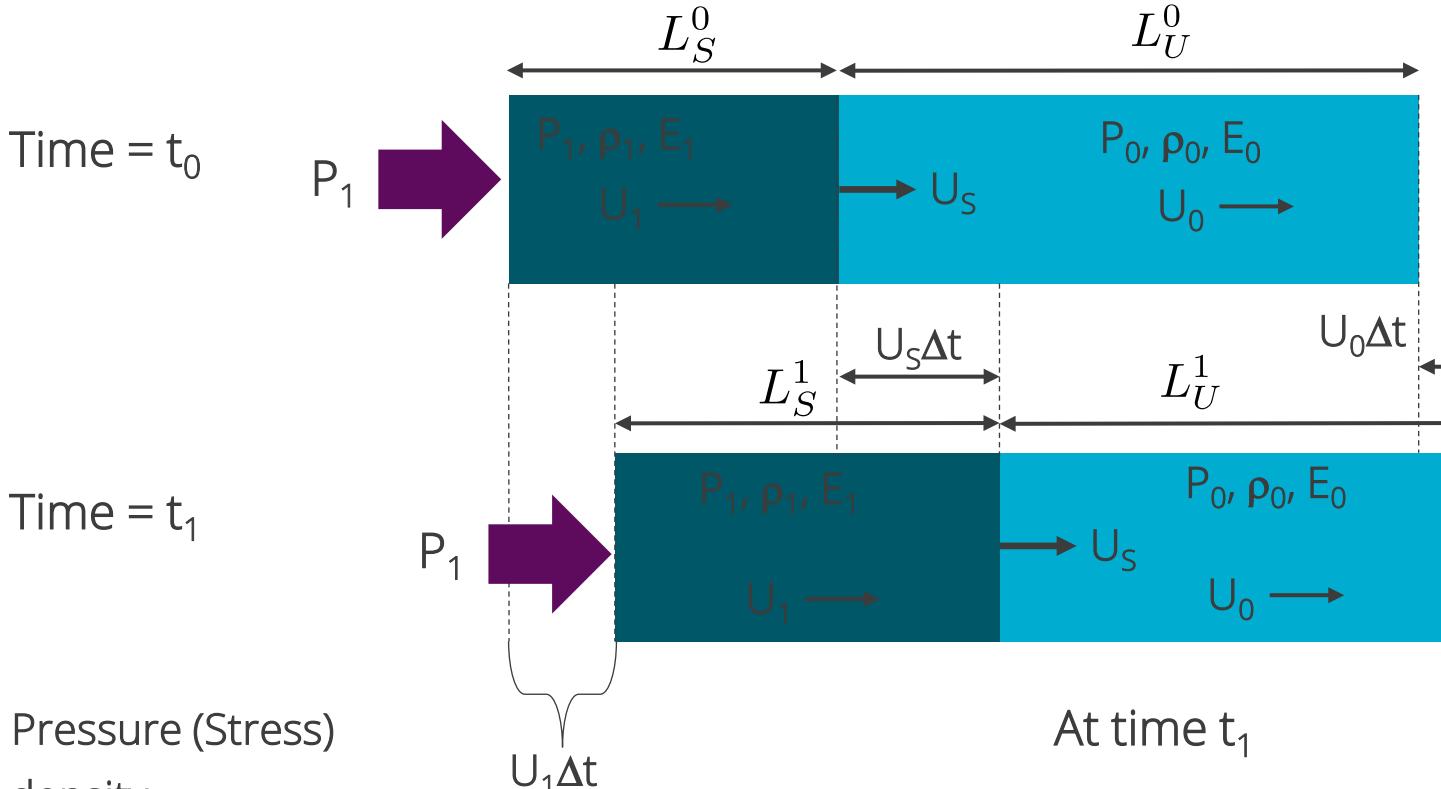
609 m/s  
1460 m/s

1060 m/s  
2290 m/s

No "fringing" at constant velocity

# Hugoniot: The Equations

*The Hugoniot equations are a results of the conservation laws*



$P$  = Pressure (Stress)

$\rho$  = density

$U_{1,0}$  = Particle velocity

$U_S$  = Shock velocity

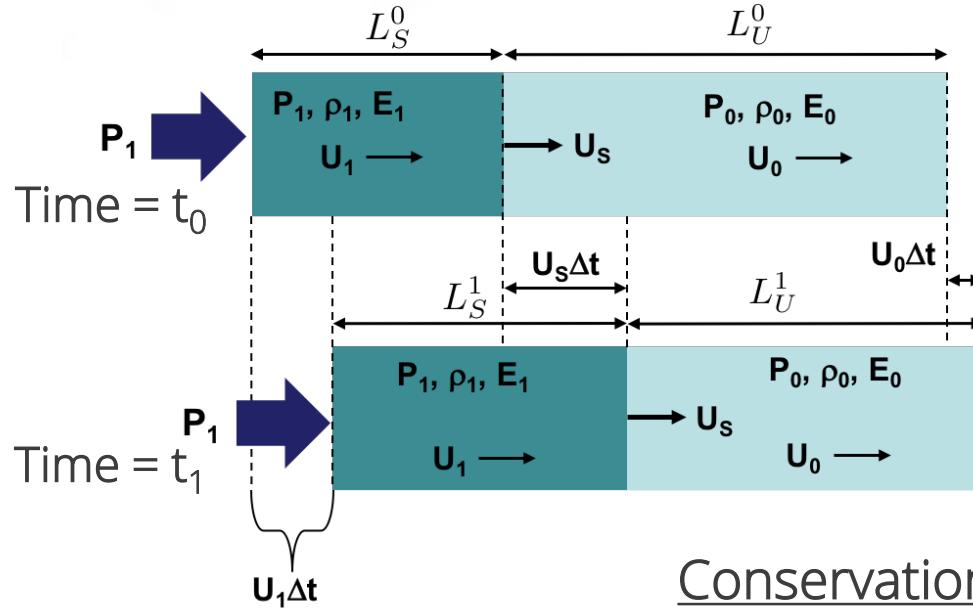
$E$  = internal energy/mass

At time  $t_1$

$$L_S^1 = L_S^0 + U_S \Delta t - U_1 \Delta t$$

$$L_U^1 = L_U^0 - U_S \Delta t + U_0 \Delta t$$

$A$  = cross-sectional area



At time  $t_1$

$$L_S^1 = L_S^0 + U_S \Delta t - U_1 \Delta t$$

$$L_U^1 = L_U^0 - U_S \Delta t + U_0 \Delta t$$

$A$  = cross-sectional area

### Conservation of Mass

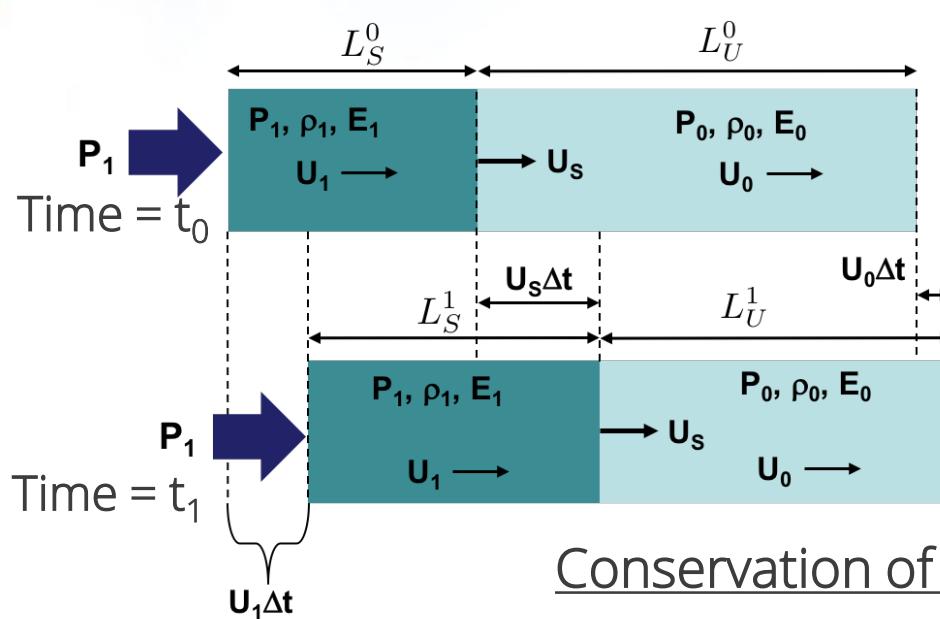
Mass at  $t_0$  = mass at  $t_1$

$$\rho_0 L_S^0 A + \rho_1 L_U^0 A = \rho_0 L_S^1 A + \rho_1 L_U^1 A$$

$$\rho_0 L_S^0 A + \rho_1 L_U^0 A = \rho_0 (L_S^0 + U_S \Delta t - U_1 \Delta t) A + \rho_1 (L_U^0 - U_S \Delta t + U_0 \Delta t) A$$

⋮

$$\rho_0 (U_S - U_0) = \rho_1 (U_S - U_1)$$



### Conservation of Momentum

A change in momentum must equal the applied impulse

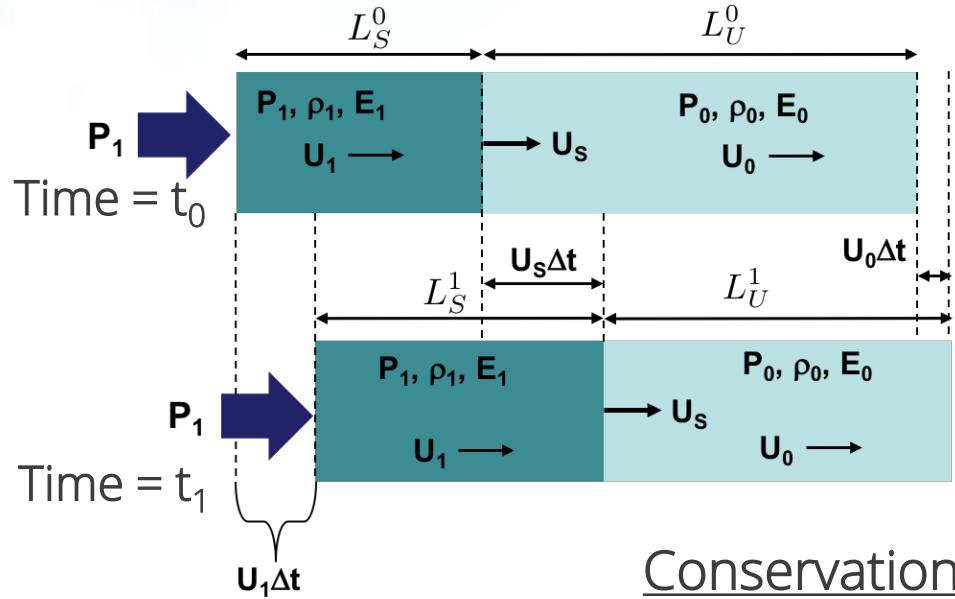
$$\text{Impulse} \quad F \Delta t = (P_1 - P_0) A \Delta t$$

$$\Delta(\text{Momentum}) \quad \rho_0 L_U^1 A U_0 + \rho_1 L_S^1 A U_1 - \rho_0 L_U^0 A U_0 - \rho_1 L_S^0 A U_1$$

$$(P_1 - P_0) A \Delta t = \rho_0 (L_U^0 - U_S \Delta t + U_0 \Delta t) A U_0 + \rho_1 (L_S^0 + U_S \Delta t - U_1 \Delta t) A U_1 - \rho_0 L_U^0 A U_0 - \rho_1 L_S^0 A U_1$$

•  
•  
•

$$P_1 - P_0 = \rho_0 (U_S - U_0) (U_1 - U_0)$$



At time  $t_1$

$$L_S^1 = L_S^0 + U_S \Delta t - U_1 \Delta t$$

$$L_U^1 = L_U^0 - U_S \Delta t + U_0 \Delta t$$

### Conservation of Energy

$$\Delta W = (P_1 A)(U_1 \Delta t) - (P_0 A)(U_0 \Delta t) = E_{Total}(t_1) - E_{Total}(t_0)$$

$$E_{Total}(1) = E_0 \rho_0 L_U^1 A + E_1 \rho_1 L_S^1 A + \frac{1}{2} \rho_1 L_S^1 A U_1^2 + \frac{1}{2} \rho_0 L_U^1 A U_0^2$$

$$E_{Total}(0) = E_0 \rho_0 L_U^0 A + E_1 \rho_1 L_S^0 A + \frac{1}{2} \rho_1 L_S^0 A U_1^2 + \frac{1}{2} \rho_0 L_U^0 A U_0^2$$

•  
•  
•

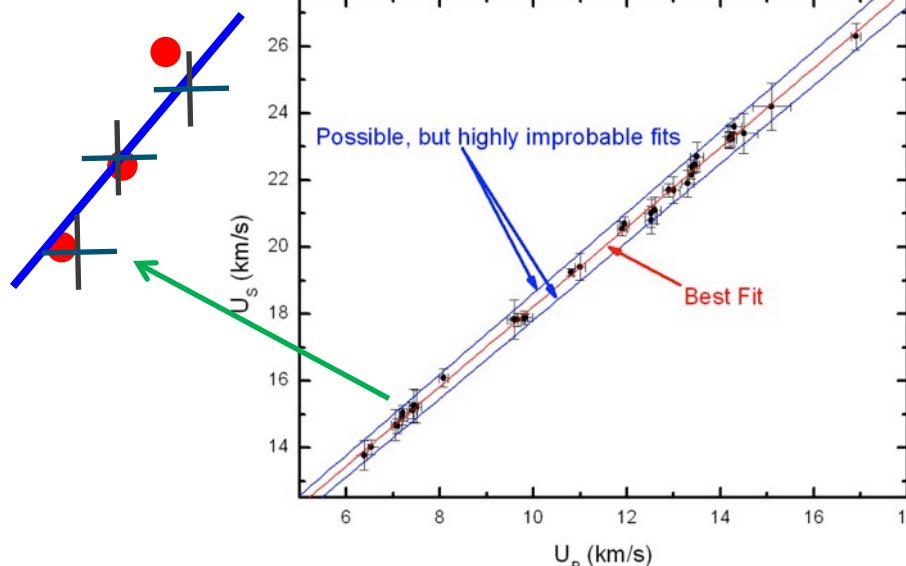
$$V = \frac{1}{\rho}$$

$$E_1 - E_0 = \frac{1}{2} (P_1 + P_0) (V_0 - V_1)$$

# Monte Carlo (MC) Impedance Matching

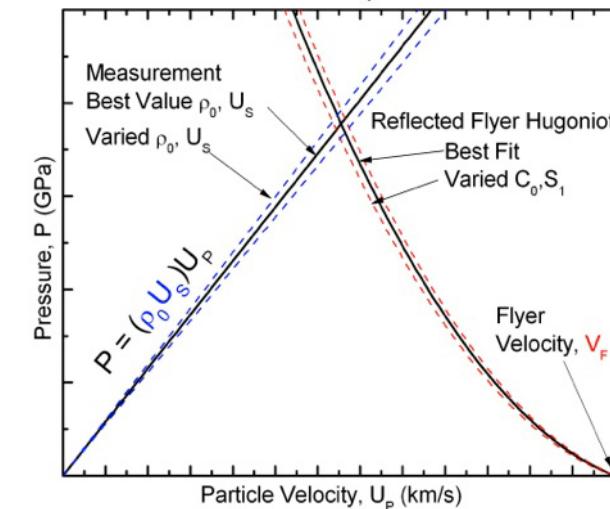
## Flyer: Aluminum

- Uncertainty in experimental data (Knudson *et al.*, JAP 2003)
- Vary each  $U_S$ - $U_P$  point by an uncorrelated random number with  $\sigma$  = expt. Uncertainty
- Solve for linear fit parameters
- Determine mean,  $\sigma$ , and correlation of fit parameters



## Sample

- Vary measured parameters ( $V_F$ ,  $U_S$ ,  $\rho_0$ ) with uncorrelated random numbers,  $\sigma$  = experimental uncertainty
- Vary Al fit parameters using correlated random numbers
- Calculate  $U_P$ ,  $P$ , and  $\rho$
- Determine mean and  $\sigma$



*Monte Carlo* technique accounts for all experimental uncertainty and propagates the Al standard's error into the sample or window data.