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Spectral Gaps via Imaginary Time

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https://arxiv.org/abs/2303.02124

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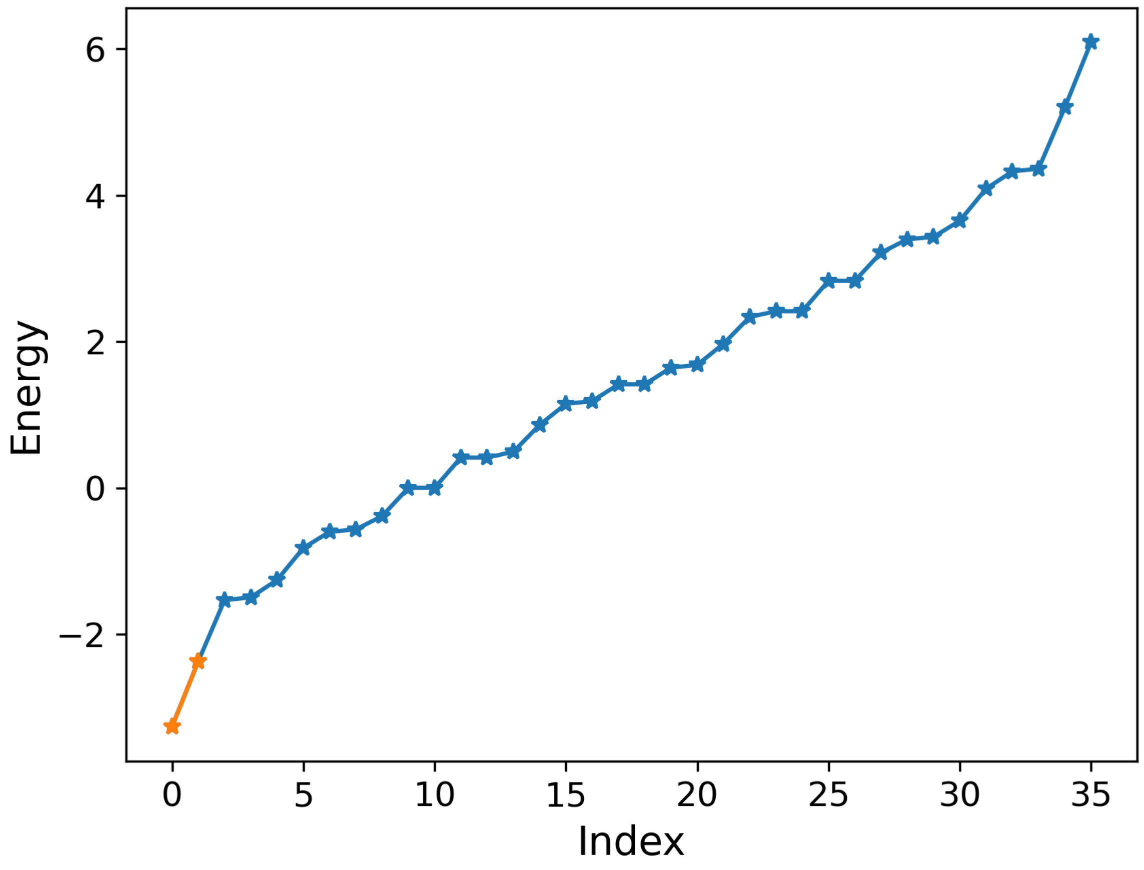
Spectral Gap

- Defined as: $\Delta E = E_1 - E_0$



Spectral Gap

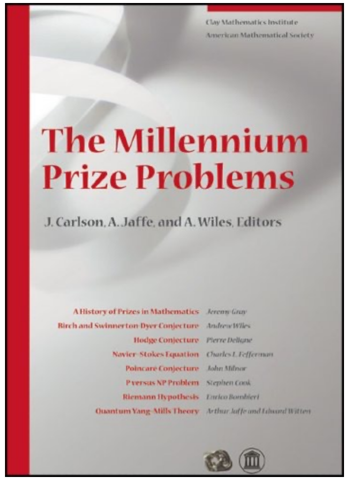
- Topological spin liquid phase
    - Artificial gauge fields
        - Superconductivity
            - Photosynthesis
                - Solar cells
                    - Fluorescence / Phosphorescence
                        - Transfer of quantum states
                            - Quark confinement
                                - and more!

Spin liquids in frustrated magnets

Leon Balents[1]

Fractionalized excitations in the spin-liquid state of a kagome-lattice antiferromagnet

Tian-Heng Han[1], Joel S. Helton[2], Shaoyan Chu[3], Daniel G. Nocera[4], Jose A. Rodriguez-Rivera[2,5], Collin Broholm[2,6] & Young S. Lee[1]



Excitation and entanglement transfer versus spectral gap

M J Hartmann[1,2,3], **M E Reuter**[1,2,] **and M B Plenio**[1,2,]

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Imaginary Time Evolution

$$|\psi(t)\rangle = \sum_{n=0}^{N} e^{-itE_n} c_n |n\rangle$$

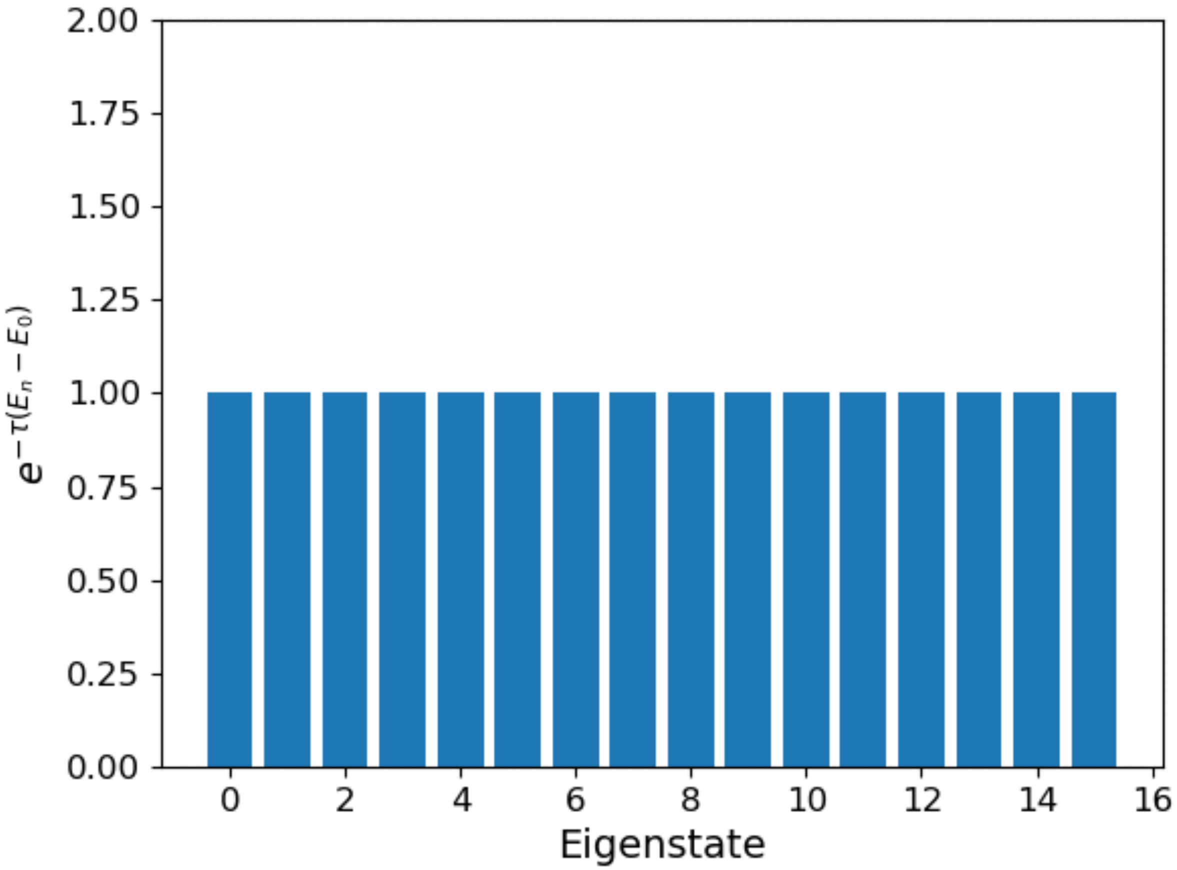
$$t = -i\tau$$

$$|\psi(\tau)\rangle = \sum_{n=0}^{N} e^{-\tau E_n} c_n |n\rangle$$

$$|\psi(\tau)\rangle = c_0 |0\rangle + \sum_{n=1}^{N} e^{-\tau(E_n - E_0)} c_n |n\rangle$$

Imaginary Time Evolution

$$|\psi(\tau)\rangle = c_0 |0\rangle + \sum_{n=1}^{N} e^{-\tau(E_n - E_0)} c_n |n\rangle$$



Our Method

Definitions:

$$\hat{H} = \sum_{n=0}^{N} E_n \hat{\Pi}_n$$

$$|\psi(\tau)\rangle = \sum_{n=0}^{N} e^{-\tau E_n} \hat{\Pi}_n |\phi_0\rangle$$

$$[\hat{H}, \hat{O}]_M = \underbrace{[\hat{H}, [\hat{H}, \ldots, [\hat{H}}_{M\text{-times}}, \hat{O}]] \ldots]$$

For 3x3 H with *non-degenerate* spectrum:

$$\hat{H} = E_0 \hat{\Pi}_0 + E_1 \hat{\Pi}_1 + E_2 \hat{\Pi}_2$$

$$\hat{\Pi}_n = |n\rangle\langle n| \qquad \delta_{nm} = \langle n|m\rangle$$

For 3x3 H with *degenerate* spectrum:

$$\hat{H} = E_0 \hat{\Pi}_0 + E_1 \hat{\Pi}_1$$

$$\hat{\Pi}_0 = |0\rangle\langle 0| + |1\rangle\langle 1|$$

$$\hat{\Pi}_1 = |2\rangle\langle 2|$$

Our Method

Definitions:

$$\hat{H} = \sum_{n=0}^{N} E_n \hat{\Pi}_n \qquad \hat{\Pi}_n = |n\rangle\langle n|$$

$$|\psi(\tau)\rangle = \sum_{n=0}^{N} e^{-\tau E_n} \hat{\Pi}_n |\phi_0\rangle$$

$$[\hat{H}, \hat{O}]_M = [\hat{H}, [\hat{H}, ..., [\hat{H}, \hat{O}]...]]$$

Conditions:

$$[\hat{H}, \hat{O}]_M \neq 0$$

$$[\hat{H}, \hat{O}]_{M+2} \neq 0$$

$$\langle\phi_0| \hat{\Pi}_0 \hat{O} \hat{\Pi}_1 |\phi_0\rangle \neq 0$$

$$\langle\psi(\tau)| [\hat{H}, \hat{O}]_M |\psi(\tau)\rangle = \sum_{l=0}^{N} \sum_{k=0}^{N} e^{-\tau(E_l + E_k)} (E_l - E_k)^M \langle\phi_0| \hat{\Pi}_l \hat{O} \hat{\Pi}_k |\phi_0\rangle$$

Our Method

- Leading order terms:

$$\langle\psi(\tau)|\,[\hat{H},\hat{O}]_M\,|\psi(\tau)\rangle = e^{-\tau(E_0+E_1)}\boxed{(E_0-E_1)^M}\langle\phi_0|\,\hat{\Pi}_0\hat{O}\hat{\Pi}_1 + \boxed{(-1)^M}h.c.\,|\phi_0\rangle + \mathcal{O}(e^{-\tau(E_0+E_2)})$$

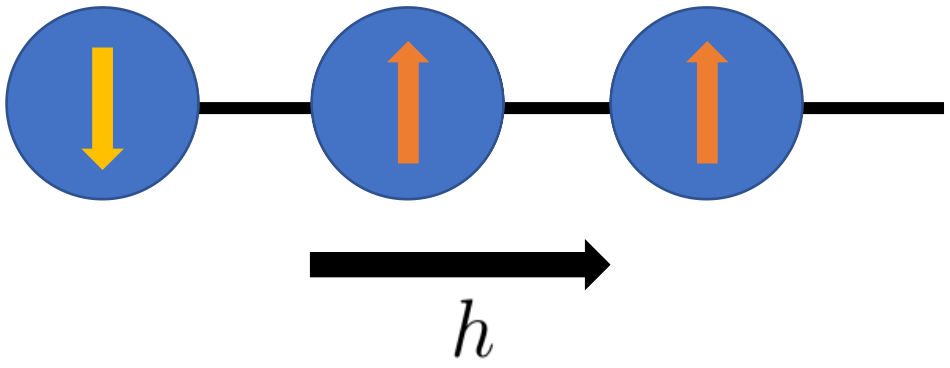
- Can obtain

$$\boxed{\frac{\langle\psi(\tau)|\,[\hat{H},[\hat{H},[\hat{H},\hat{O}]]]\,|\psi(\tau)\rangle}{\langle\psi(\tau)|\,[\hat{H},\hat{O}]\,|\psi(\tau)\rangle} = (E_0-E_1)^2 + \mathcal{O}(e^{-\tau(E_2-E_1)})}$$

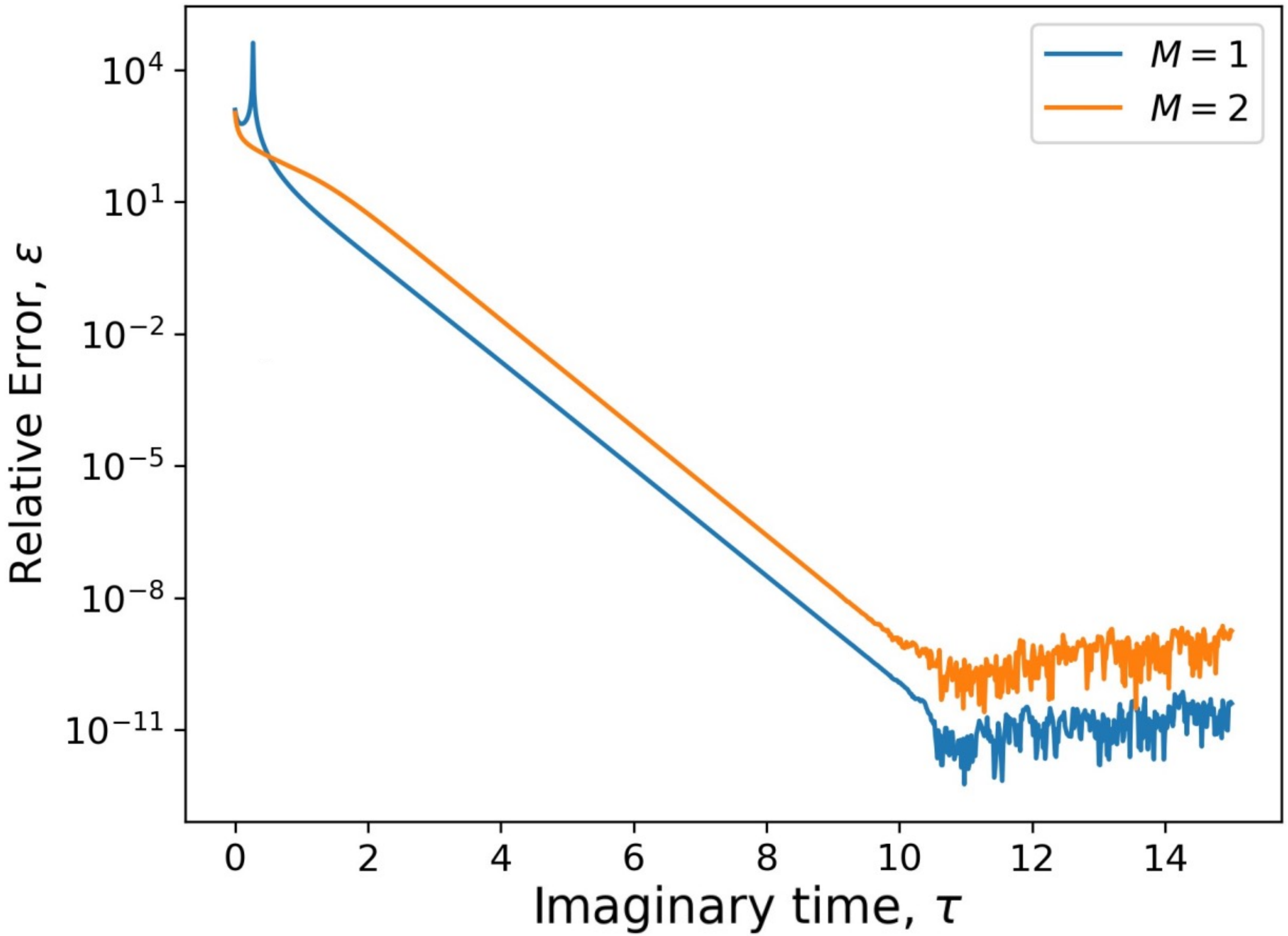
Transverse Field Ising Model

$$\hat{H} = \sum_{k=0}^{K-1} -J\sigma_{k+1}^z \sigma_k^z - h\sigma_k^x$$

- Very simple model
- Nearest neighbor interactions governed by spins



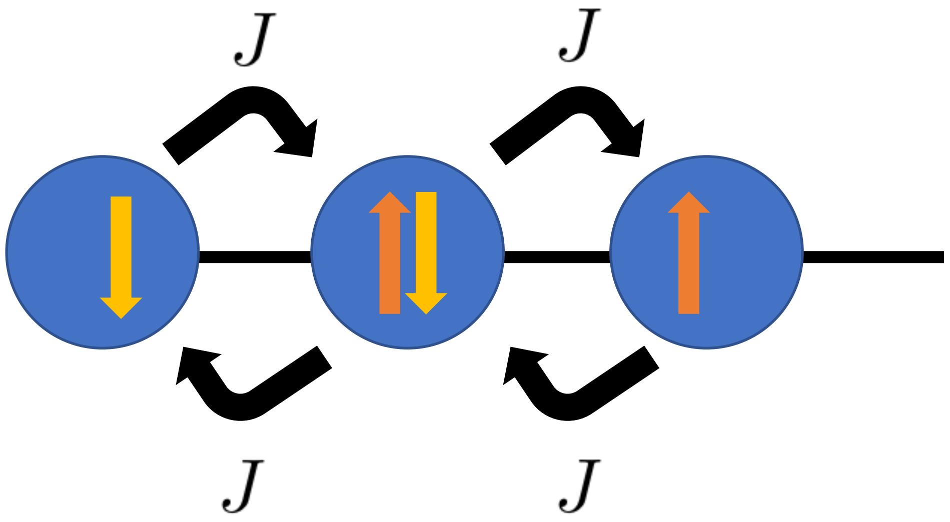
Transverse Field Ising Model Results



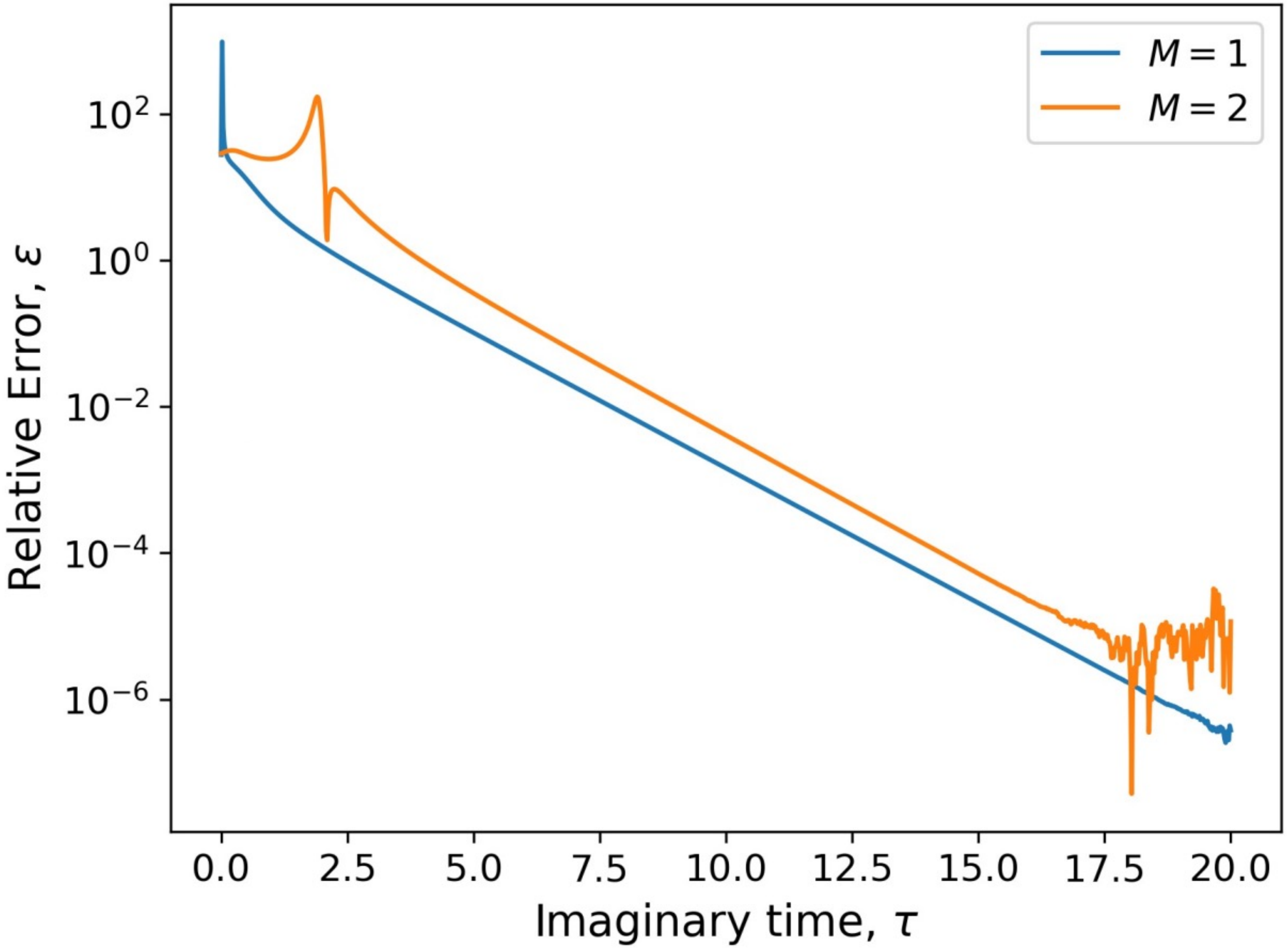
Fermi-Hubbard Model

$$\hat{H} = -J \sum_{k=0,\sigma}^{K-1} (c_{k,\sigma}^{\dagger} c_{k+1,\sigma} - c_{k,\sigma} c_{k+1,\sigma}^{\dagger}) - \mu \sum_{k=0,\sigma}^{K-1} n_{k\sigma} + U \sum_{k=0}^{K-1} n_{k\uparrow} n_{k\downarrow}$$

- Simple model for correlated systems



Fermi-Hubbard Model Results



Conclusion

- Presented a method for calculating spectral gaps using imaginary time propagation to quench out higher energy states
- Demonstrated effectiveness using two paradigmatic models
- More details -> https://arxiv.org/abs/2303.02124

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