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Automated Model Selection for Gate Set Tomography

Stefan K. Seritan, Kenneth Rudinger, and Robin Blume-Kohout

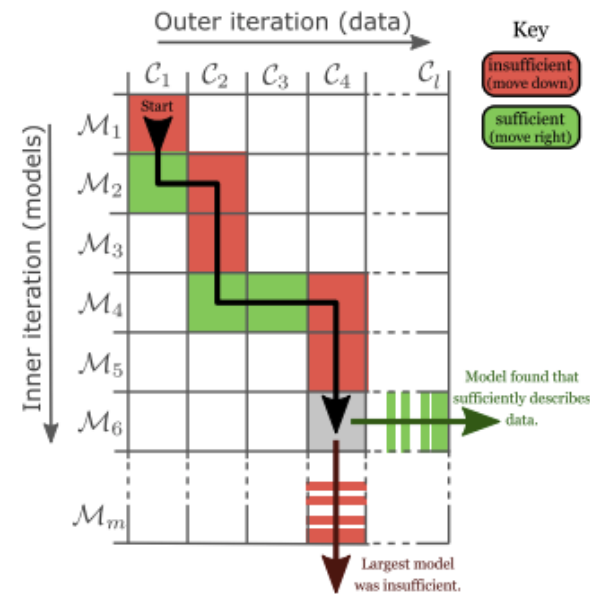
APS March Meeting 2023, Session B72

March 6th, 2022

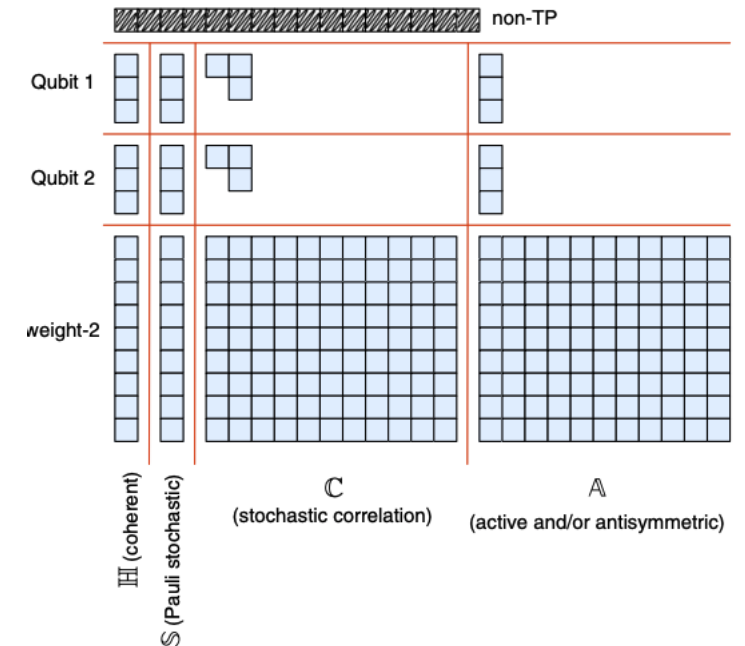


Model Reductions for Gate Set Tomography

- Gate set tomography (GST) is:
 - Accurate (Heisenberg-limited)
 - Robust to SPAM errors
 - Provides detailed process matrices for all gates
- But process matrices are not that useful always – often, we just want *simple* and *intuitive* models
- Two-part solution!
 - **Decrease model size** with a nested model reduction approach
 - Use elementary error generators for **greater interpretability**



Procedure for systematic testing of nested models¹



Elementary error generators broken up by type (coherent, Pauli stochastic, correlated stochastic, and active), weight, and support²

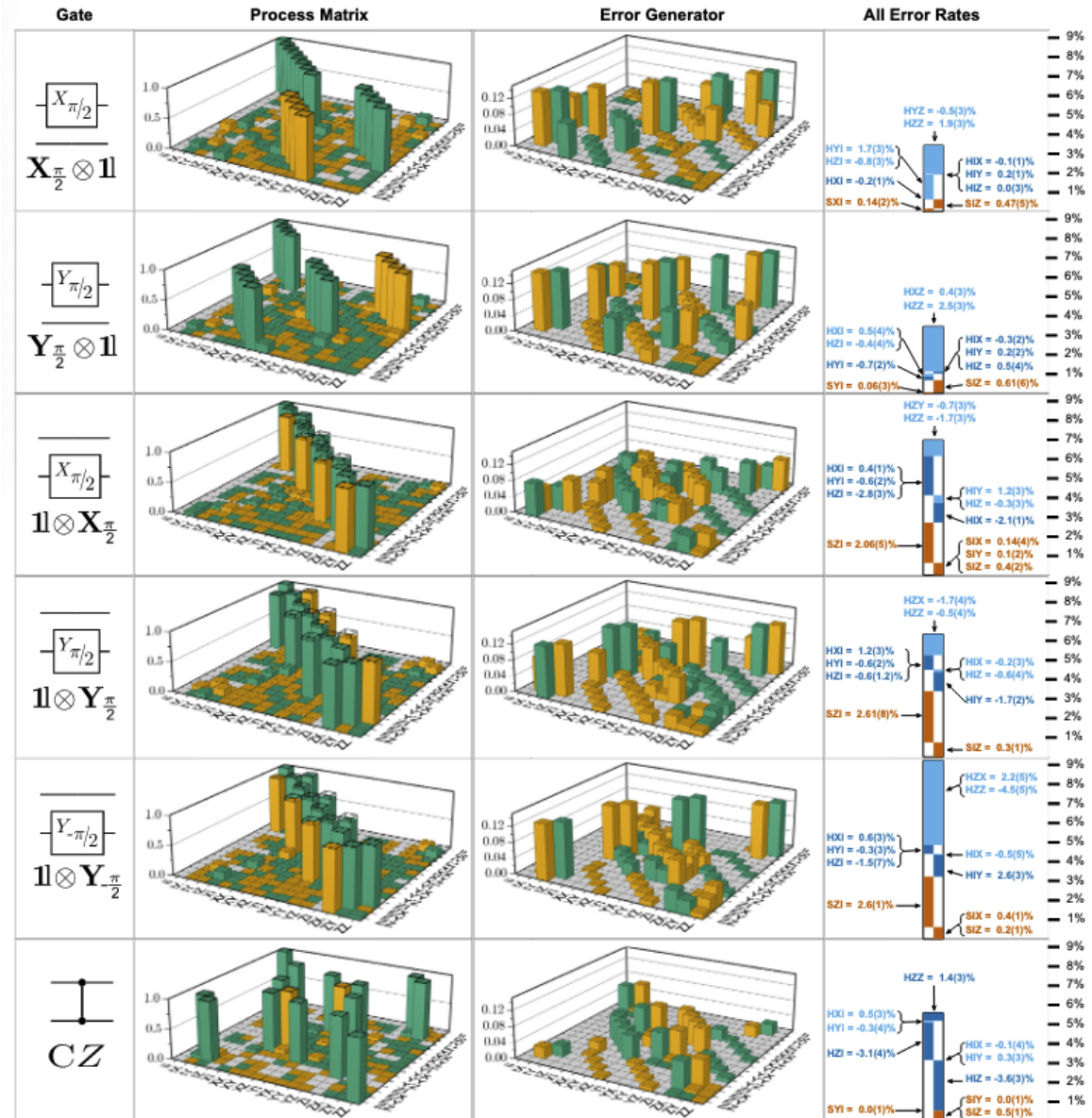
¹ Nielsen et al., *New J. Phys.*, 23, 093020 (2021)

² Blume-Kohout et al., *PRX Quantum*, 3, 020335 (2022)



Reduced GST models are powerful... but not always easy to find

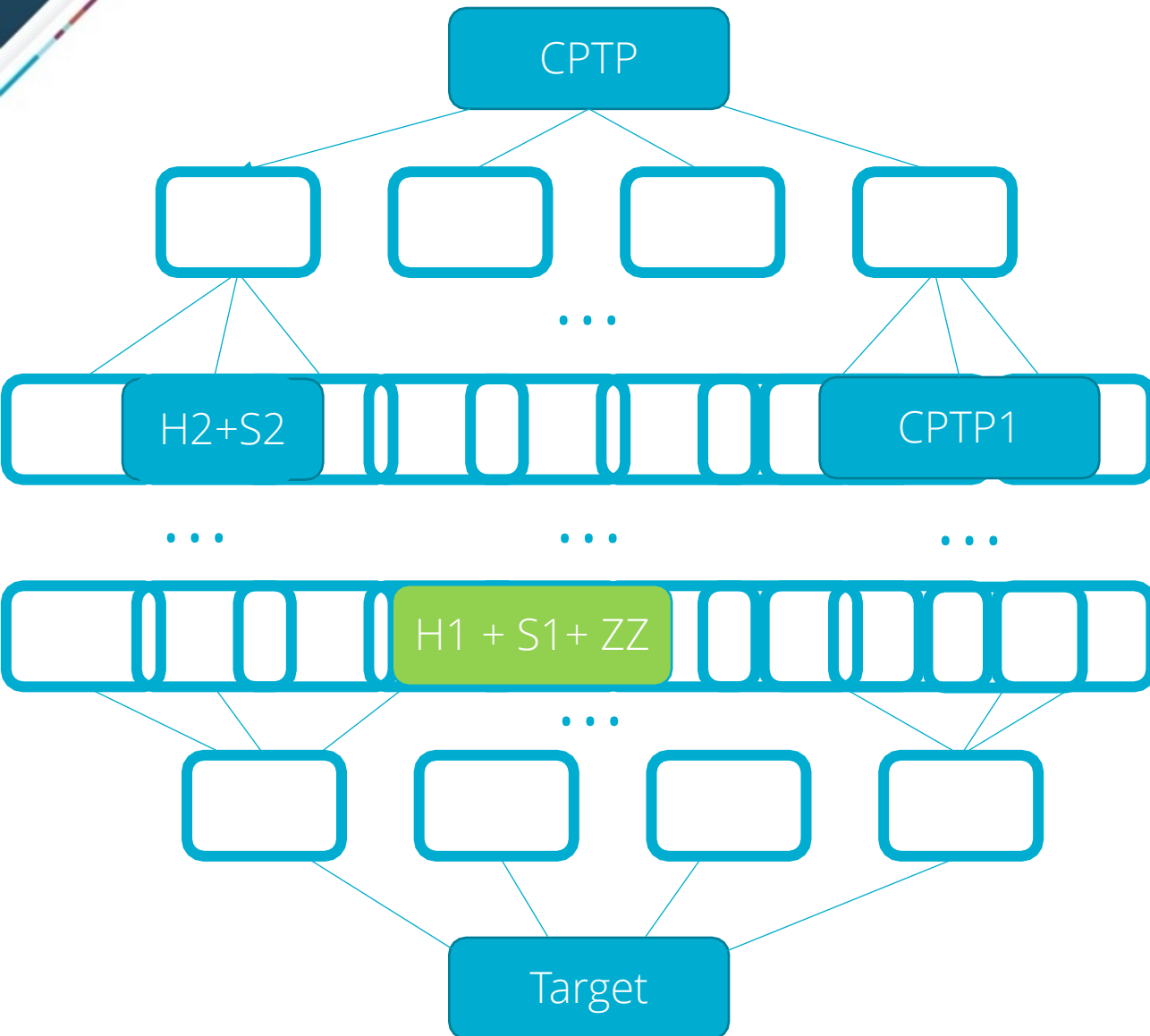
- For example: Reduced GST models provided evidence for an unexpected ZZ coherent error in a recent silicon spin qubit experiment¹
- However, this reduced model was hand-tuned with hours of data analysis and input from physics modeling
- We would like to do these types of analyses consistently – and that means doing it automatically!



¹ Madzik et al., Nature, 601, 348-353 (2022)



Searching for a Needle in a Haystack



- Unfortunately, there are a lot of valid reduced models... so brute force search is exponential
- We have some intuition that some form of sparsification should work
 - Adding the next most significant error generator seems like a reasonable thing to do?
- So we really just need to find a “good” path through this graph



How well does a greedy search do?

1. Calculate a GST fit for full model
2. Get all possible nearest-neighbor model reductions
3. Compute GST fits for reduced models
4. Calculate evidence ratio from log-likelihoods L and parameter counts N_{params}

$$\gamma = \frac{2(L^{\text{prev}} - L^{\text{curr}})}{(N_{\text{params}}^{\text{prev}} - N_{\text{params}}^{\text{curr}})}$$

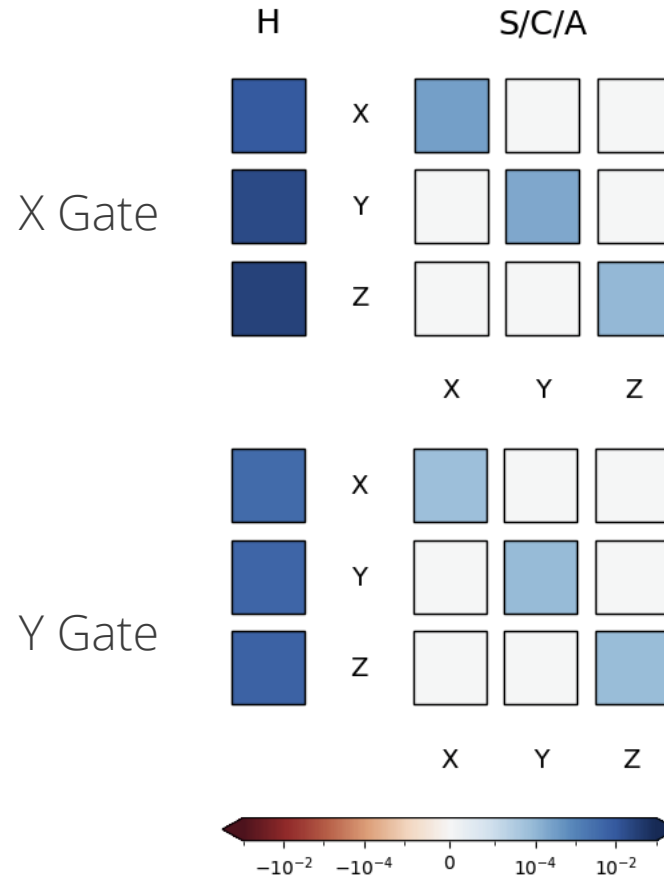
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6. If $\gamma < 2$ (the AIC), accept model
7. Repeat steps 2 – 6 until model is rejected.



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True Model





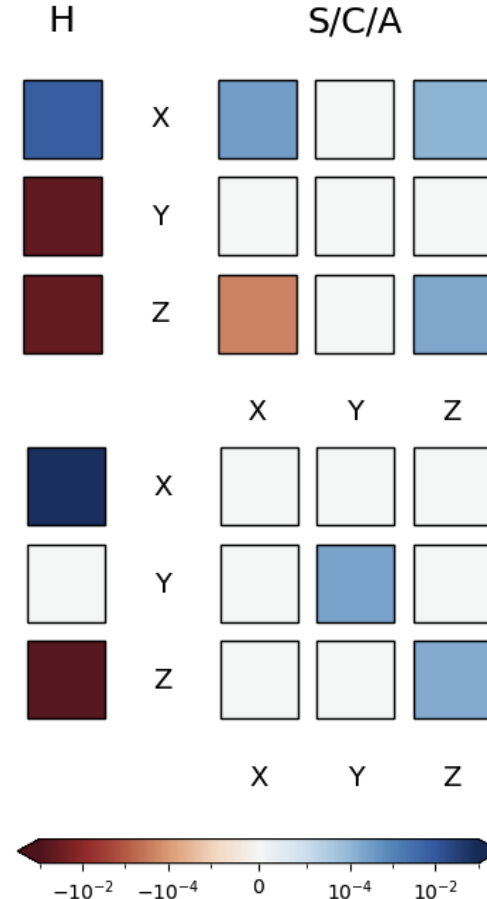
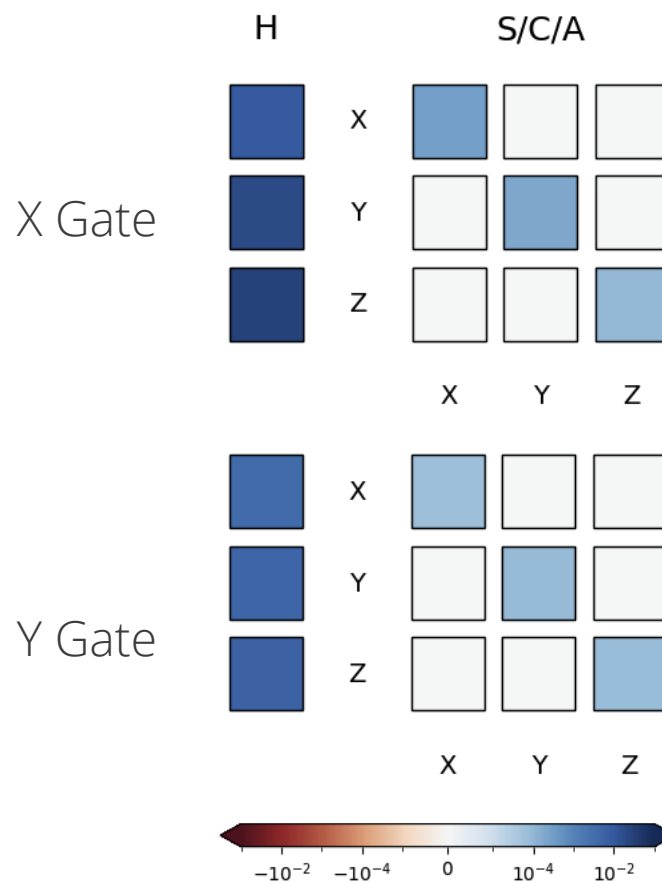
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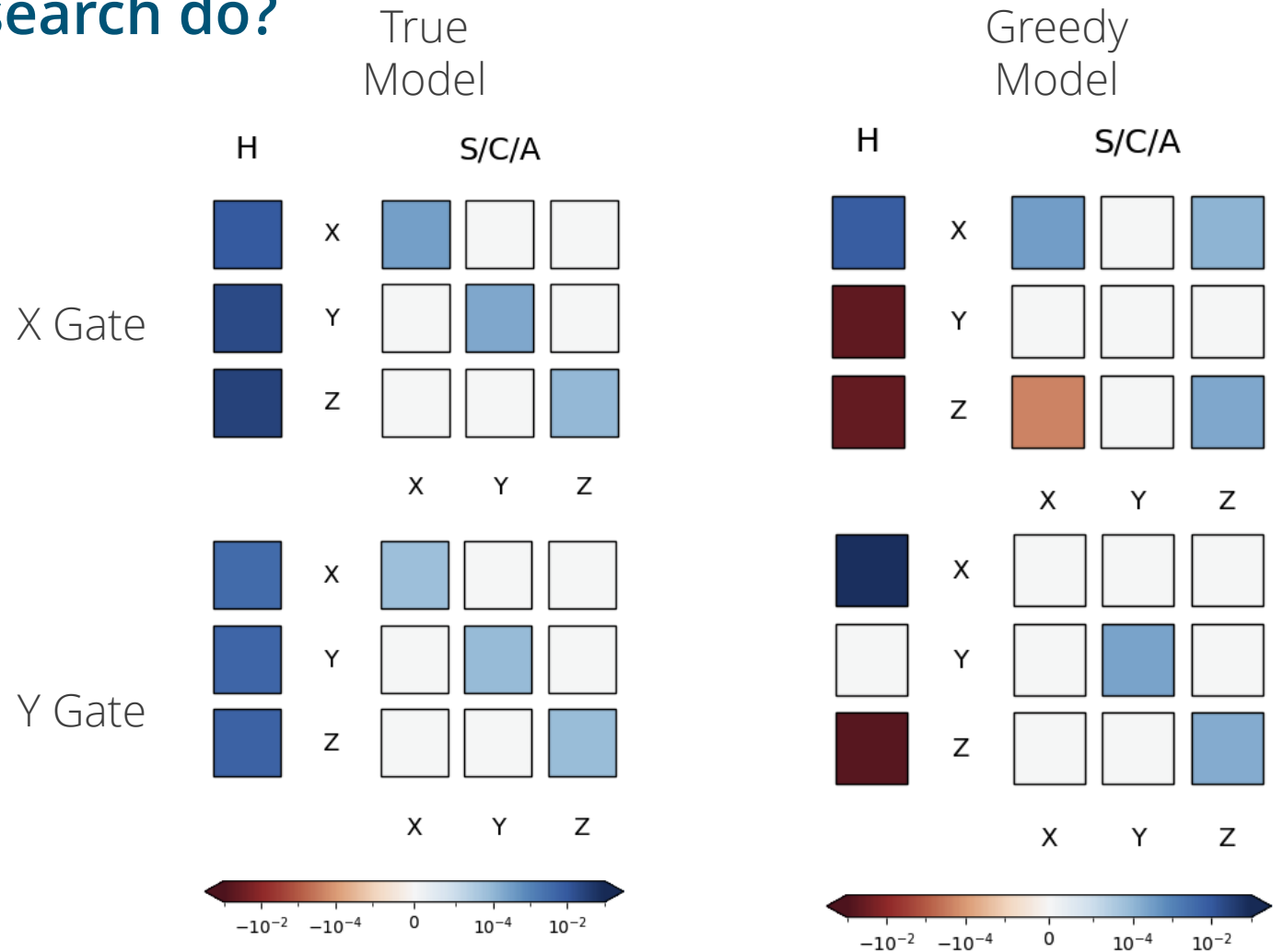




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That's a fail...



Improving our search strategy

Faster Evaluations

- In the case of model reduction, we can reuse information to speed up some evaluations
- Do a quadratic expansion around GST (maximum likelihood) estimate \vec{x}_0 with Hessian H

$$L(\vec{x}) = \frac{1}{2}(\vec{x} - \vec{x}_0)^T H (\vec{x} - \vec{x}_0)$$

- Use Lagrange multipliers to do constrained optimization, where Π is the projector that sets the desired parameters of the new estimate \vec{x}' to 0

$$L(\vec{x}') = \frac{1}{2}\vec{x}_0^T \Pi (\Pi H^{-1} \Pi)^{-1} \Pi \vec{x}_0$$

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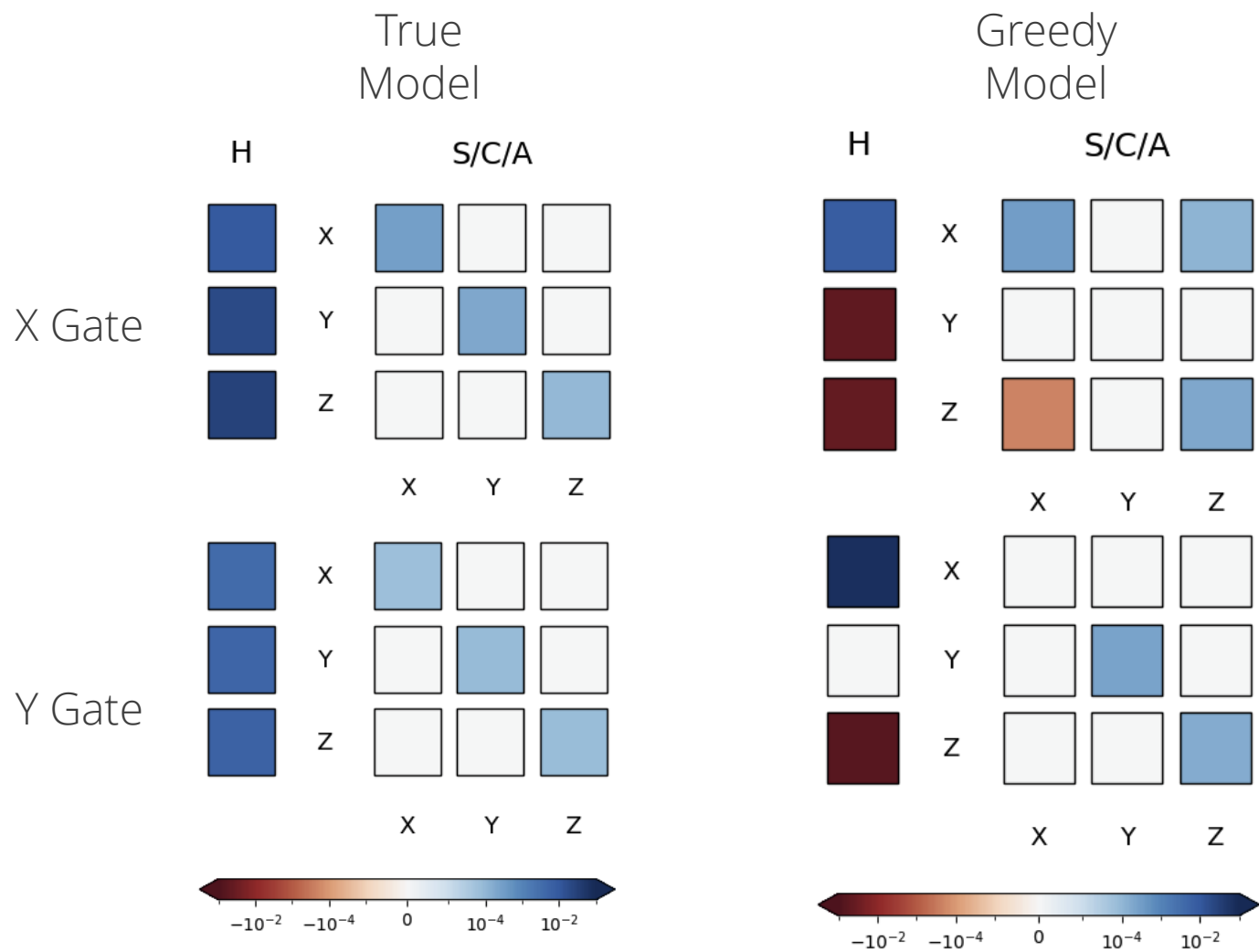
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Beam Search

- Part of greedy failure was immediately due to a borderline decision to drop a Hamiltonian generator due to gauge...
- Can we be more robust to these close calls, especially at the beginning?
- Yes! We can do a beam search
 - Instead of keeping single best model at each step, keep some small number of instances instead
 - Still not guaranteed to be optimal, but a good heuristic in practice

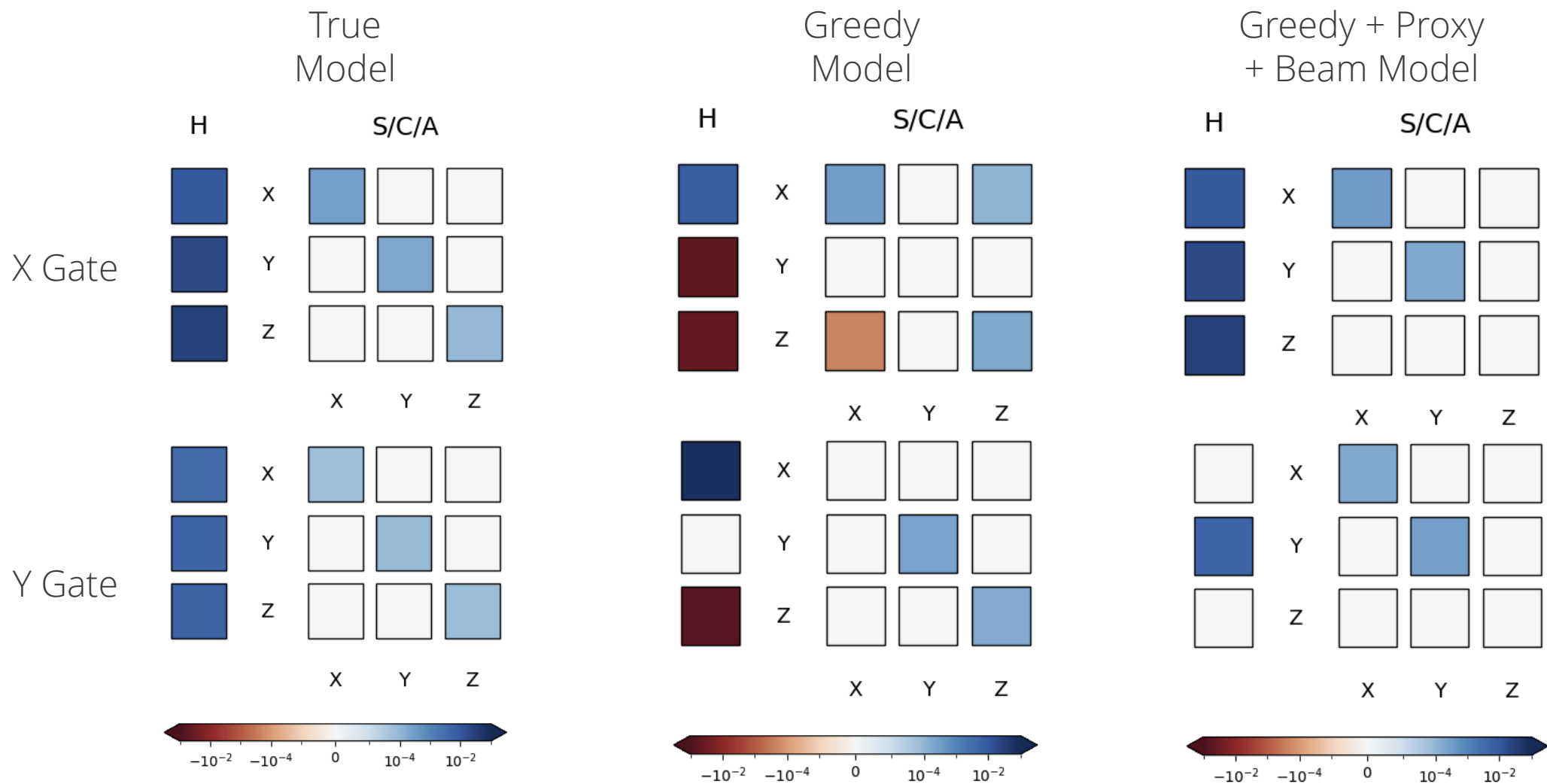


Semi-reasonable Automated Model Selection



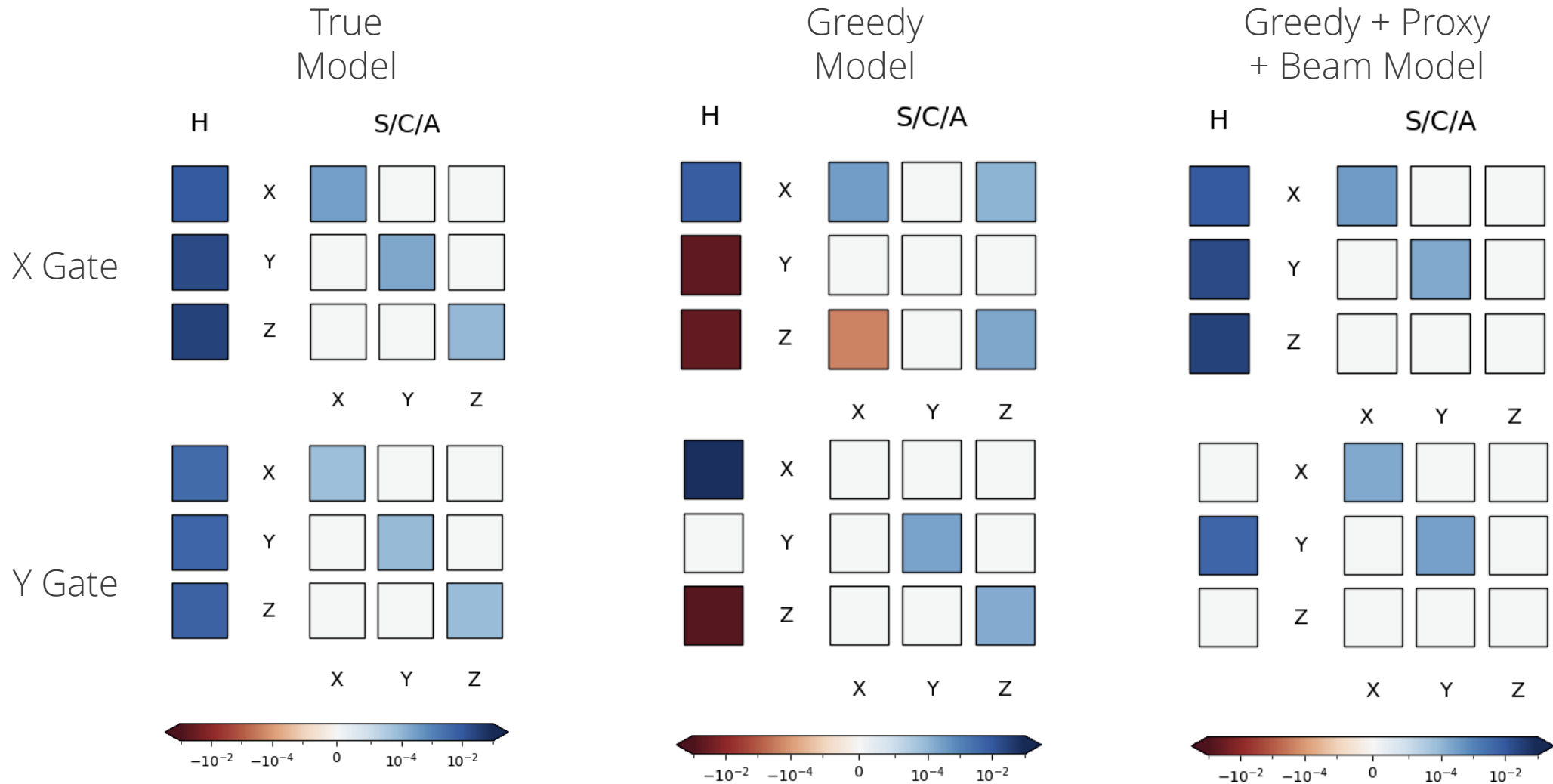


Semi-reasonable Automated Model Selection





Semi-reasonable Automated Model Selection



Still some room for improvement, but on the right track!



Future directions

- Other search strategies
 - Preliminary results for L1 regularization/least-angle regression show comparable performance to greedy or beam, but this is still under investigation
- Use of first-order gauge invariant (FOGI) parameters instead of bare error generators
- Application to existing experimental data – could we have found the ZZ error?
- Currently this is a top-down approach
 - Good for model reduction from a full traditional GST experiment
- Ideally, we also want a bottom-up approach
 - Maybe given some initial data (e.g., RB circuits), guess a good initial model
 - Construct a heavily reduced GST experiment based on that generated model
 - Iterate upwards if needed



Q & A



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