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Logical Majorana Fermions for Fault-Tolerant Quantum Simulation

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Quantum Simulation

- Apply unitary evolution under a Hamiltonian to a quantum system
  - Consider Hamiltonians written in terms of fermionic creation and annihilation o

$$H_{\text{HUB}} = -t \sum_{\langle p,q \rangle, \sigma} a^{\dagger}_{p,\sigma} a_{q,\sigma} + \frac{u}{2} \sum_{p,\alpha \neq \beta} n_{p,\alpha} n_{p,\beta}$$

  - Time-evolution operator cannot be implemented directly
  - Use Trotter-Suzuki approximation or block encoding instead

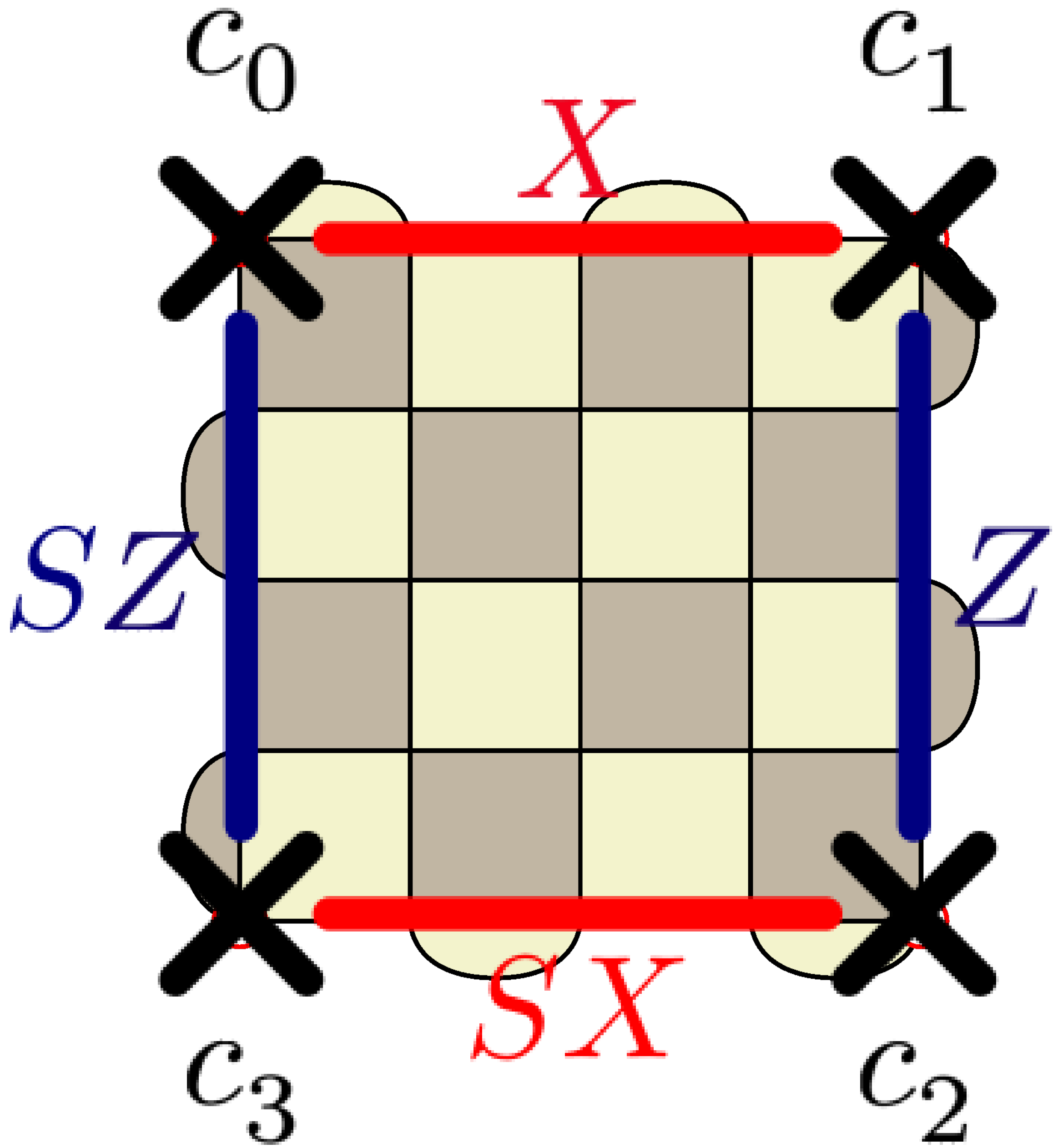
$$U = e^{-itH} = e^{-it \sum_L H_L}$$

$$\tilde{U} = \prod_L e^{-itH_L}$$

$$\text{PREPARE}\,|0\rangle = \sum \sqrt{\frac{w_l}{\lambda}}\,|l\rangle = |\mathcal{L}\rangle$$

$$\text{SELECT} = \sum_l |l\rangle\langle l| \otimes H_l$$

Fault-Tolerance

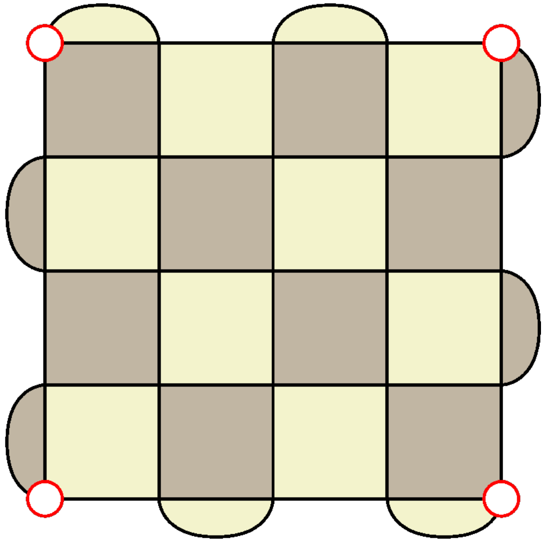


Fault-Tolerant Quantum Simulation

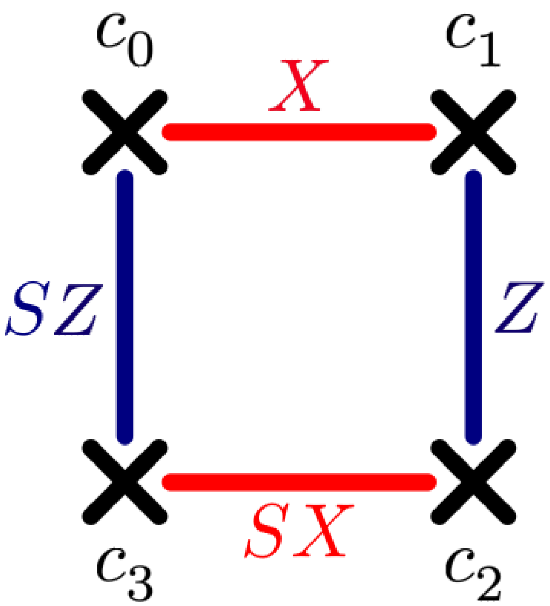
Jordan-Wigner Transformation

$$f_p = \frac{1}{2}(Z_0 \otimes \cdots \otimes Z_{p-1}) \otimes (X_p + iY_p)$$

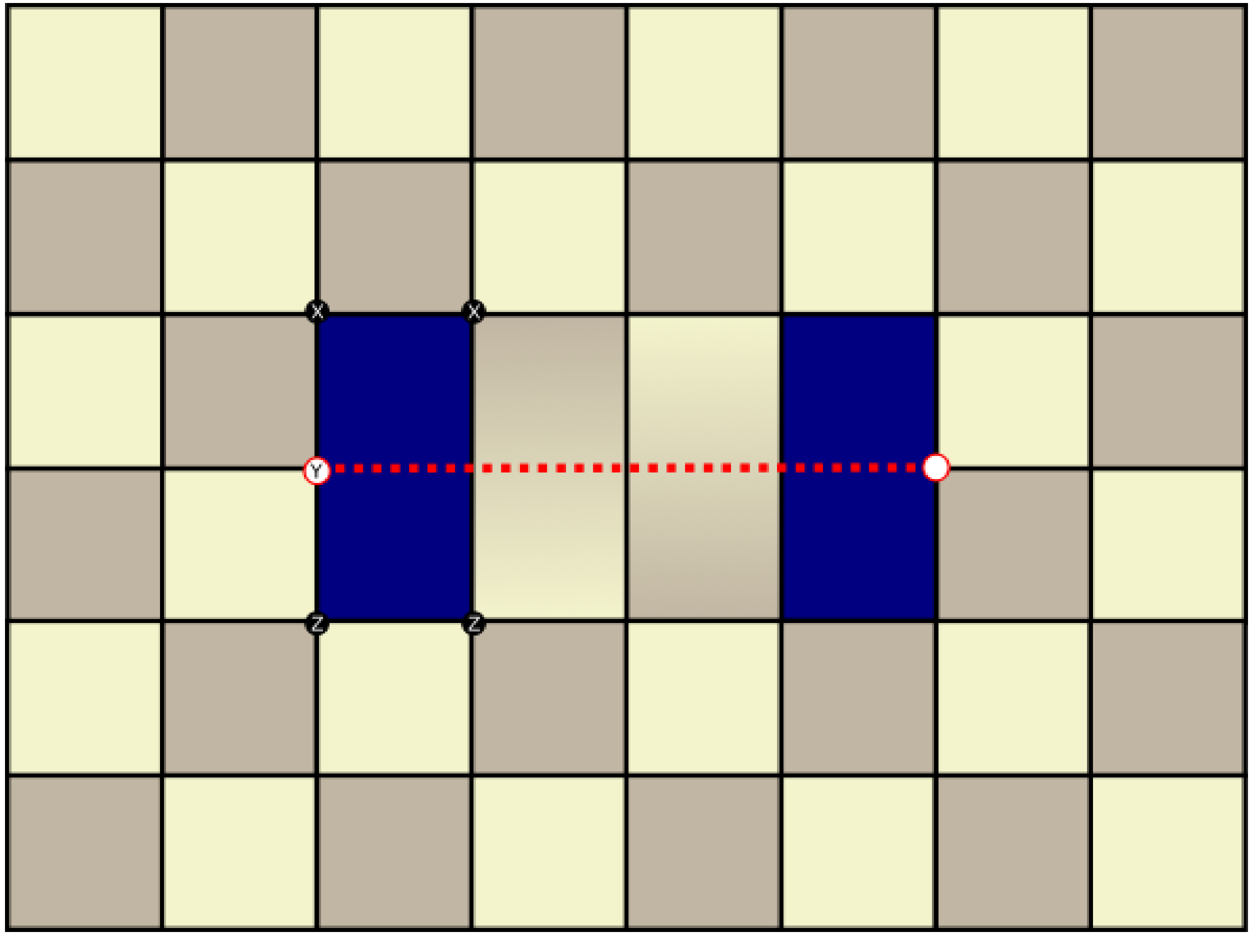
$$f_p^\dagger = \frac{1}{2}(Z_0 \otimes \cdots \otimes Z_{p-1}) \otimes (X_p - iY_p)$$



Simulation Application Dirac Fermions
Logical Qubits
Logical Majorana Fermions (Twist Defects)
Physical Qubits



Logical Majorana Fermions



Bring the twists from the boundary into the bulk

Braid the twists or measure strings around

Logical Majorana Fermions for Fault-Tolerant Quantum

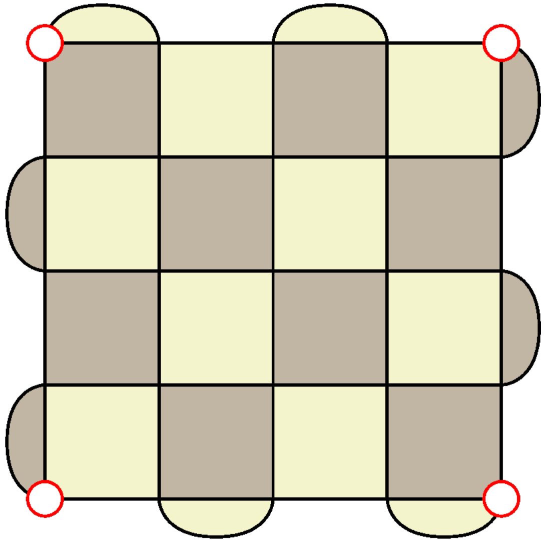
Simulation Application
Dirac Fermions

$c_{2p} := f_p^\dagger + f_p \qquad c_{2p+1} := i(f_p^\dagger - f_p)$

Logical Majorana Fermions
(Twist Defects)

Physical Qubits

- Locality allows parallelism reducing depth of Trotter steps from $\mathcal{O}(\sqrt{n})$ to $\mathcal{O}(1)$.
- Majorana-inspired speedup for block encoding SELECT oracle from $10N + \mathcal{O}(\log N)$ to $8N + \mathcal{O}(\log N)$.



- Performance improvements possible for other systems?