Compact Parameterization of Nonrepeating FMCW Radar Waveforms

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This work was supported by Sandia National Laboratories under Contract #2237007 and the Office of Naval Research under Contract # N00014-20-C-1006. Approved for Public Release.

Background

- The choice of a particular waveform is a crucial decision that bears on the performance of a radar system.
- In contrast to using a single, non-repeating waveform for a given application, one can leverage nonrepeating waveforms to realize the benefits of high dimensionality **[1]**.
- The class of spectrally-shaped random FM (RFM) waveforms are particularly well suited for providing low range sidelobes while limiting transmitter distortion effects.

[1] S.D. Blunt, J.K. Jakabosky, C.A. Mohr, P.M. McCormick, et al, "Principles & applications of random FM radar waveform design," ***IEEE Aerospace & Electronic Systems Magazine*****, vol. 35, no. 10, pp. 20-28, Oct. 2020.**

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- The class of spectrally-shaped random FM (RFM) waveforms are particularly well suited for providing low range sidelobes while limiting transmitter distortion effects.
- Different methods of RFM have enabled physical realizations of:
  - complementary waveforms
  - intermodulation-based nonlinear radar
  - cognitive sense-and-notch, and more.

[1] S.D. Blunt, J.K. Jakabosky, C.A. Mohr, P.M. McCormick, et al, "Principles & applications of random FM radar waveform design," *IEEE Aerospace & Electronic Systems Magazine*, vol. 35, no. 10, pp. 20-28, Oct. 2020.

Problem Motivation

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- Most RFM approaches require optimization on per-waveform basis => may be prohibitive computationally for some applications
- A notable exception is Constant Envelope OFDM (CE-OFDM), which is effectively yields an optimization-free form of spectrally-shaped RFM for random symbols **[2-5]**.
- However, CE-OFDM for radar has previously only been realized in a pulsed context. Some structural changes are necessary to achieve a (nonrepeating) FMCW form.

[2] S.C. Thompson, J.P. Stralka, "Constant envelope OFDM for power-efficient radar and data communications," *Intl. Waveform Diversity & Design Conf.*, Kissimmee, FL, Feb. 2009.

[3] S. Liu, Z. Huang, W. Zhang, "A power-efficient radar waveform compatible with communication," *Intl. Conf. Communications Circuits & Systems*, Chengdu, China, Nov. 2013.

[4] Q. Zhang, et al., "Waveform design for a dual-function radar-communication system based on CE-OFDM-PM signal," *IET Radar Sonar & Navigation*, vol. 13, no. 4, pp. 566-572, Apr. 2019

[5] E.R. Biehl, C.A. Mohr, B. Ravenscroft, S.D. Blunt, "Assessment of constant envelope OFDM as a class of random FM radar waveforms", *IEEE Radar Conf.*, Florence, Italy, Sept. 2020.

OFDM Structure

- The well-known OFDM signal structure can be defined as:

$$u(t) = \sum_{n=1}^{N} \beta_n \exp(j2\pi f_n t)$$

for symbol interval T, where β_n is the communications symbol associated with subcarrier frequency f_n .

- While well-suited for communications, the significant amplitude modulation of OFDM limits utility in a radar context
  - Use of high-power amplifier on transmit to maximize "energy on target" produces severe distortion [6]

[6] J. Jakabosky, L. Ryan, S.D. Blunt, "Transmitter-in-the-loop optimization of distorted OFDM radar emissions," *IEEE Radar Conf.*, Ottawa, Canada, Apr./May 2013.

CE-OFDM Structure

- CE-OFDM provides an FM implementation by exponentiating the real part via

$$s(t) = \exp(j2\pi h\,\Re\{u(t)\}) = \exp\left(j2\pi h \sum_{n=1}^{N} |\beta_n| \cos(2\pi f_n t + \varphi_n) \right)$$

where h is the modulation index that scales FM spectral content, and $|\beta_n|$ and φ_n are the magnitude and phase of the nth symbol.

- Being constant amplitude and continuous phase, this waveform can be generated using a high-power transmitter.
  - In short, CE-OFDM can readily produce physically viable pulsed radar waveforms when symbol interval = pulse width

Implications of Periodicity

- Since subcarrier frequencies in CE-OFDM (and OFDM) are separated by $1/T$, both are defined on intervals of T.
  - Would repeat on multiples of T if the symbols are unchanged.

- The obvious approach to realize a nonrepeating signal in the CE-OFDM context is to change the symbols for each T interval.

- However, doing so introduces phase discontinuities at each symbol transition (every T interval), causing spectral spreading and distortion.

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- **Thus, our goal is to remove periodicity from the CE-OFDM structure.**

Addressing Periodicity

- Consider the subcarrier frequencies of CE-OFDM: $f_n = f_{n-1} + \frac{1}{T}$
- Clearly, they are integer multiples of $1/T$.
- It is well-known that the period of the sum of periodic functions is equal to the *least common multiple* (LCM) of the individual periods.
- In general, if $a(t)$ and $b(t)$ are periodic functions, then:
  - $a(t) = a(t + kT_a)$
  - $b(t) = b(t + lT_b)$

  where k, l are integers
- Thus, $c(t) = a(t) + b(t) = c(t + mT_{\text{LCM}})$ is likewise periodic for $T_{\text{LCM}} = kT_a = lT_b$

Addressing Periodicity

- Consequently, $f_a = 1/T_a$ and $f_b = 1/T_b$ with $f_{\text{LCM}} = 1/T_{\text{LCM}}$
- Each of these frequencies will have a ratio that is rational, such as:

$$\frac{f_a}{f_b} = \frac{k}{l}$$

(Note: this is still CE-OFDM)

- This relationship ensures the LCM exists and that the period of the sum is likewise periodic with a finite period.

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- **HOWEVER**, if the ratio between each pair of subcarriers is made to be irrational, then $T_{\text{LCM}} = \infty$ (i.e. no periodicity)

Non-Integer Constant-Envelope (NICE) OFDM

- Now let subcarriers be selected as $f_n = f_{n-1} + (1 + \alpha_n)/T$
- Here α_n is a unique, irrational, real number with $|\alpha_n| \ll 1$
- The resulting ratio between subcarriers pairs is also irrational as

$$\frac{f_n}{f_{n-1}} = \frac{f_{n-1} + (1+\alpha_n)/T}{f_{n-1}} = 1 + \frac{1}{f_{n-1}T} + \frac{\alpha_n}{f_{n-1}T}$$

- The ensuing NICE-OFDM signal never repeats.
- Moreover, the waveform can be fully characterized by the (N–1) subcarrier spacings!

NICE-OFDM Construction

- Like CE-OFDM, we can express NICE-OFDM using the Jacobi-Anger expansion [2-5]

$$s(t) = \prod_{n=1}^{N} \sum_{m=-\infty}^{\infty} d_{n,m} \exp\left(j2\pi m f_n t\right) \operatorname{rect}\left(\frac{t - T_{\mathrm{CE}}/2}{T_{\mathrm{CE}}}\right)$$

for coefficients

$$d_{n,m} = j^m J_m\left(2\pi h \left|\beta_n\right|\right) \exp\left(j m \phi_n\right)$$

- Here $J_m(\bullet)$ is the mth Bessel function of the first kind. The Fourier transform of each weighted sum becomes a weighted sum of sinc(•) functions in frequency.

- The N-fold product then becomes a repeated convolution in frequency, so the overall result tends toward a **Gaussian spectral density** via the central limit theorem for sufficient N and h.

NICE-OFDM Practical Construction

- The smallest subcarrier frequency should be enforced to be greater than the segment processing period.
  - Otherwise, a drift in apparent center frequency arises that can exacerbate clutter modulation effects.

- The factor *h* should be sufficiently modest so that the exponentiated combination of subcarriers does not produce an instantaneous frequency that is too large.
  - Could introduce distortion based on discretized implementation in hardware

- Finally, while *theoretically* nonrepeating (based on LCM), relatively high sidelobes could occur.
  - But with sufficient *N* the severity and likelihood decrease exponentially

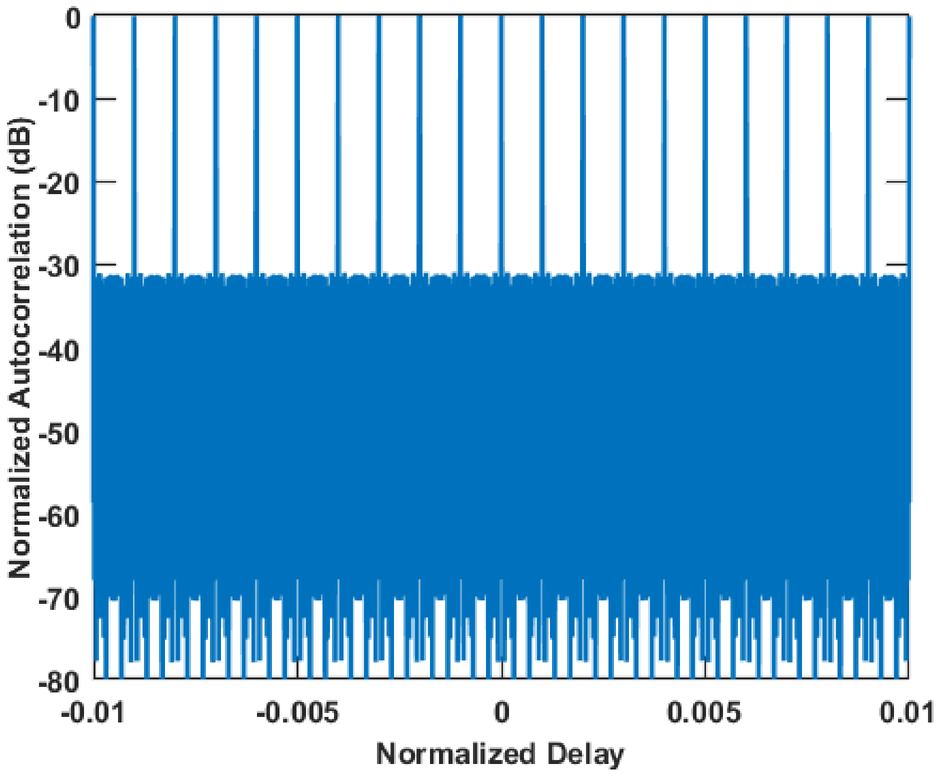
Hardware Loopback Capture

- Two 100 ms waveforms (one CE-OFDM, one NICE-OFDM) were generated for 50 MHz bandwidth, oversampled by 4 (so $f_s = 200$ MHz). Each waveform was constructed from $N = 200$ subcarriers using symbols randomly drawn from a 16-QAM constellation.
- CE-OFDM subcarrier spacing was set to 10 kHz, resulting in a repetition period of $T_{\mathrm{CE}} = 100\ \mu\mathrm{s}$, consequently repeating 1,000 times over the 100ms.
- The 199 values of α_n for NICE-OFDM were drawn from a uniform distribution on ± 1 kHz. The subcarriers were set to values between a minimum and maximum of 10 kHz and 2 MHz respectively.

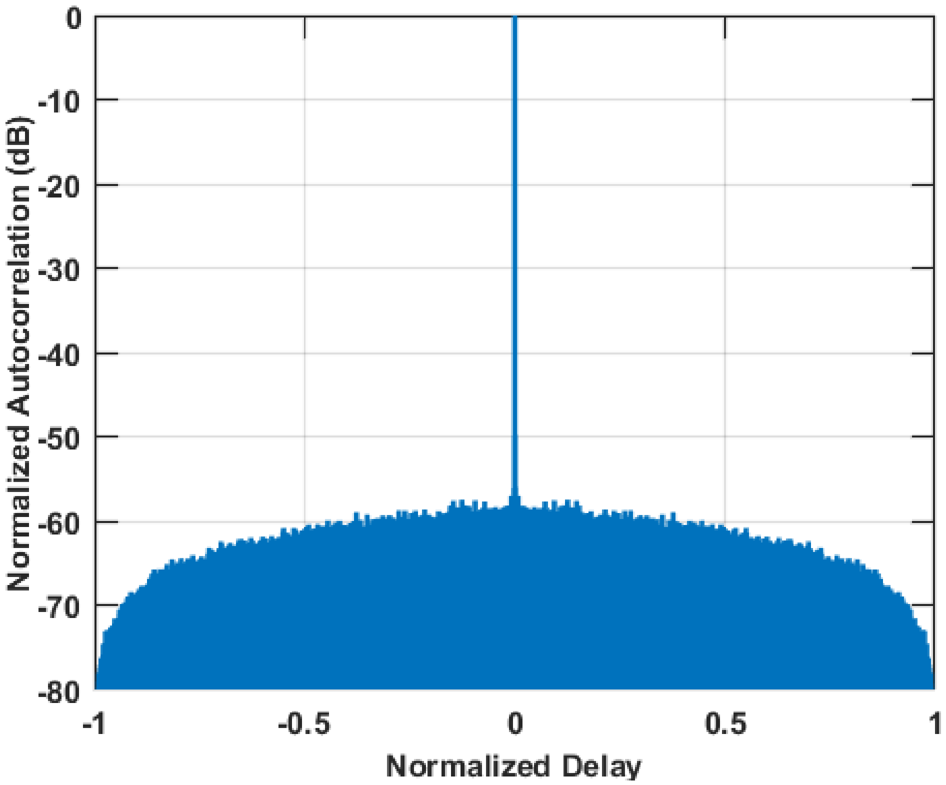
Hardware Loopback Results

Autocorrelation of each waveform

100ms "fixed symbols" CE-OFDM (**zoomed-in** view)



100ms NICE-OFDM waveform (**zoomed-out** view)



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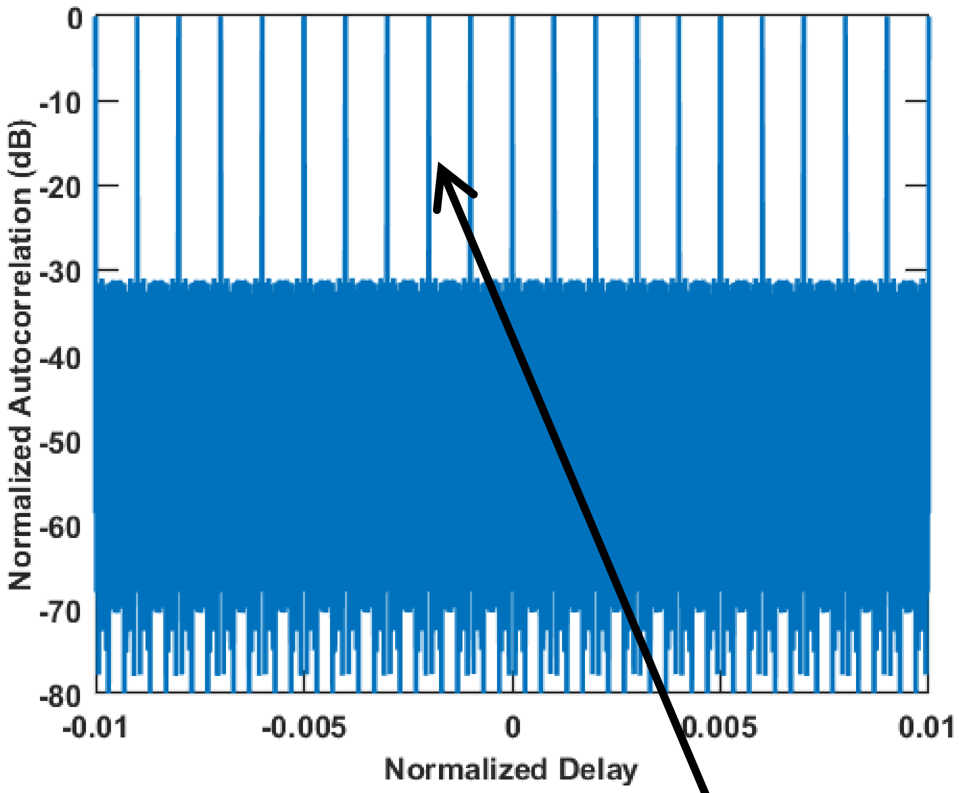

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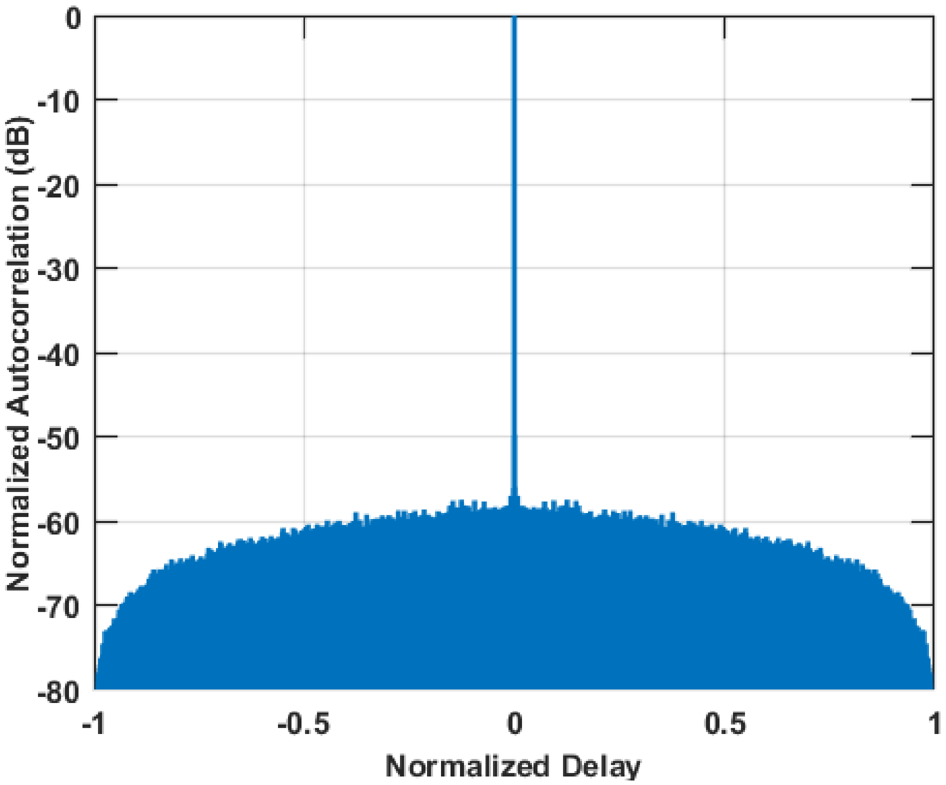

Clearly has repeated structure

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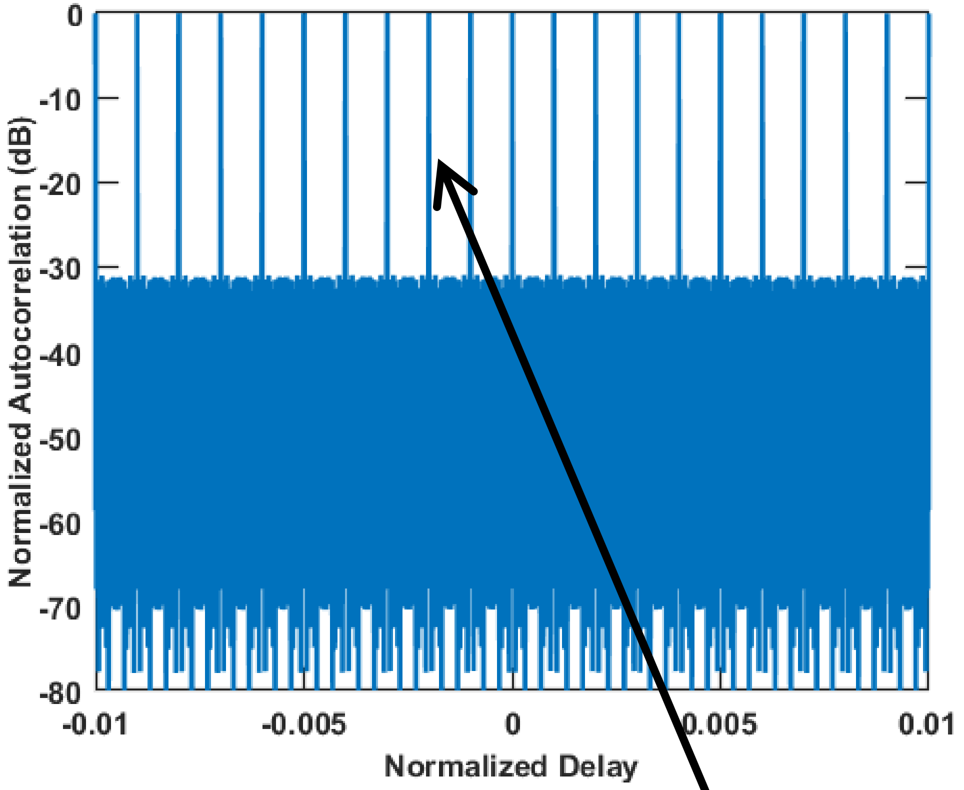

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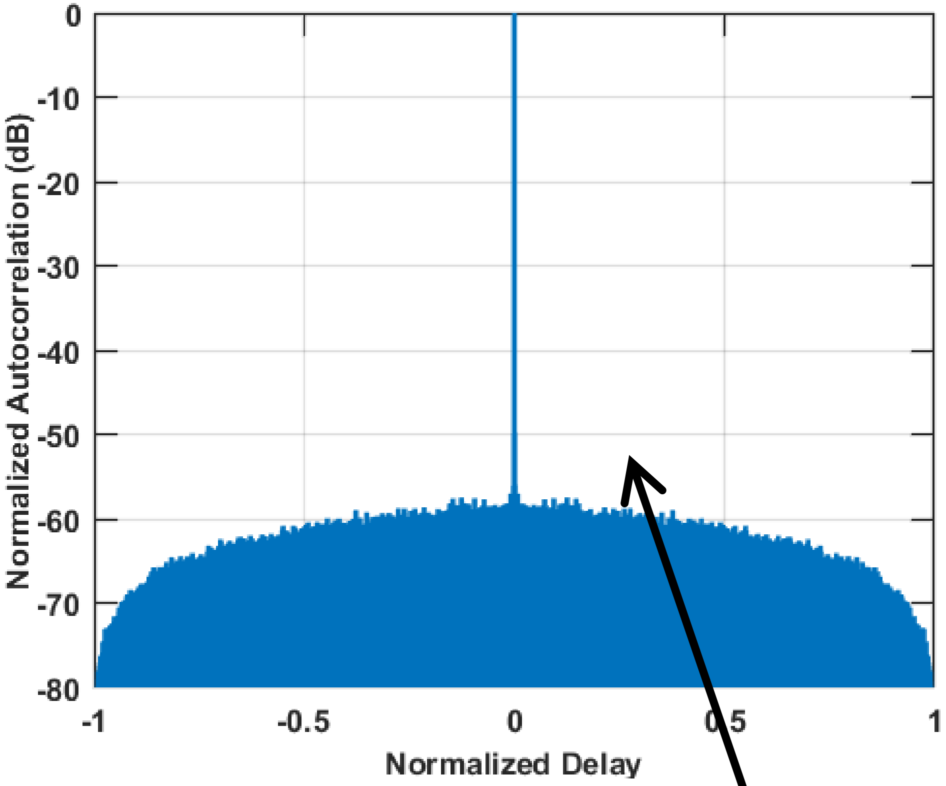
100ms NICE-OFDM waveform (**zoomed-out** view)

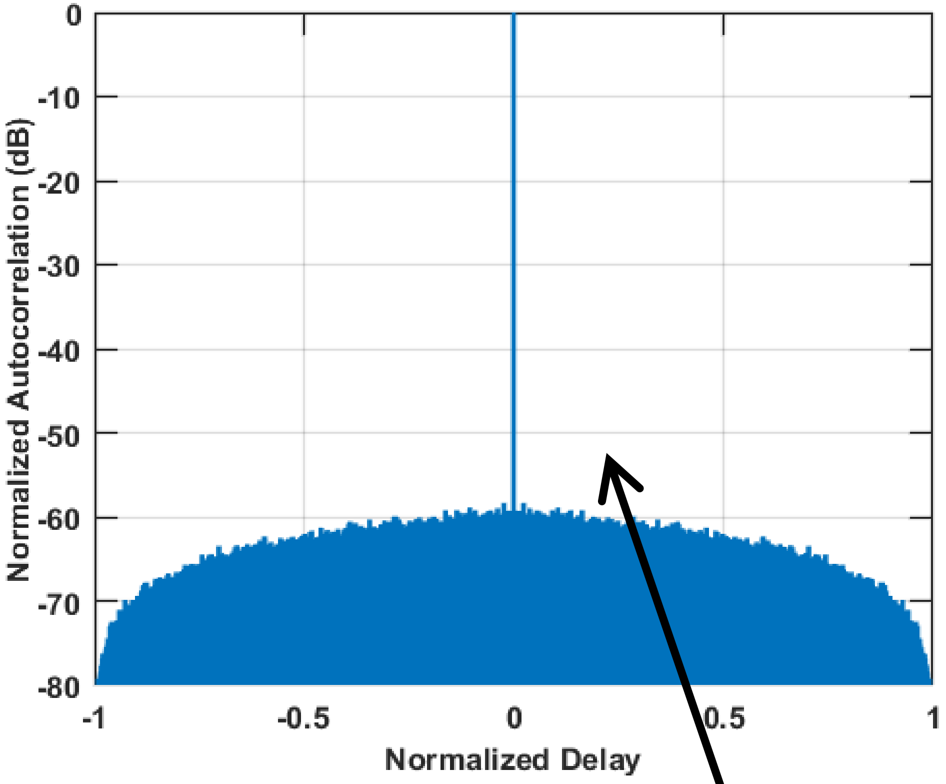


No meaningful repetition over entire 100 ms

Hardware Loopback Results

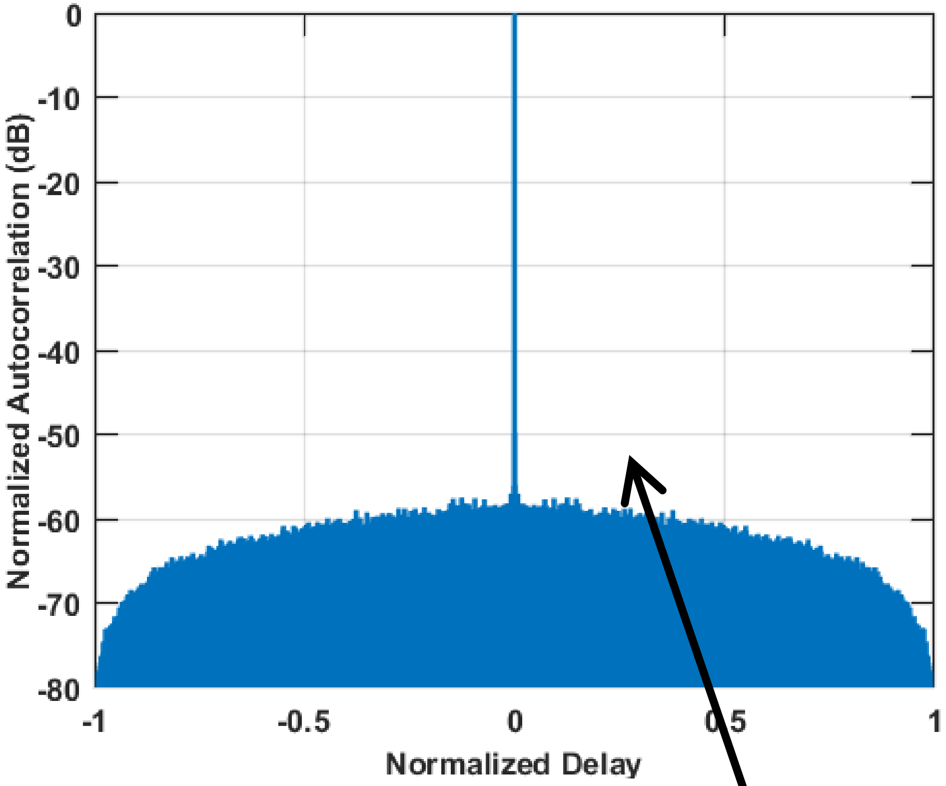
Autocorrelation of each waveform

100ms "variable symbols" CE-OFDM (**zoomed-out** view)



Also no meaningful repetition, but 1000× higher data representation AND incurs phase discontinuities

100ms NICE-OFDM waveform (**zoomed-out** view)



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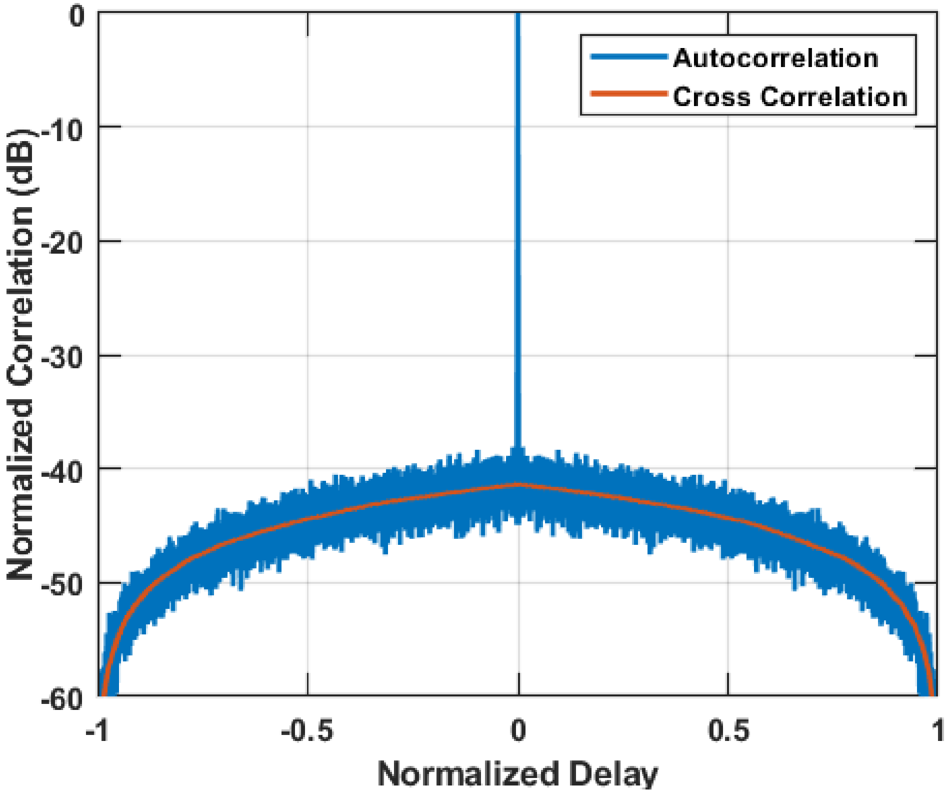
Hardware Loopback Results

- Now consider segment-wise combining akin to slow-time processing
- RMS average of autocorrelation and pairwise cross-correlation for 1000 segments of T_{seg} = 100 μs each
- Peak autocorrelation sidelobe: –39 dB
- Cross-correlation peak: –42 dB

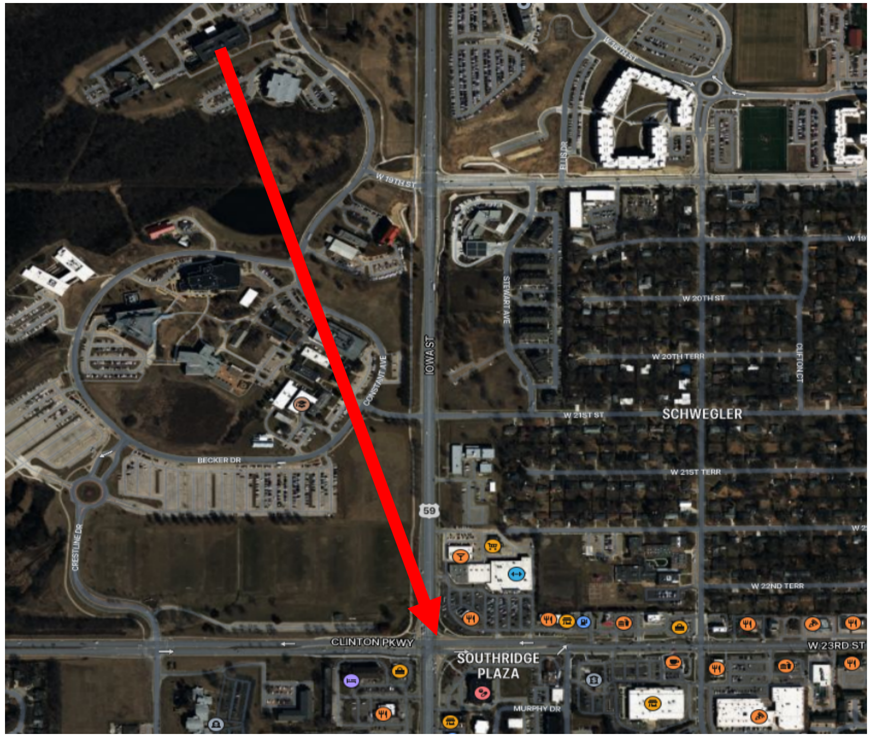
RMS per-segment performance for 100ms NICE-OFDM



Open Air Capture

Free-space measurements taken from rooftop of Nichols Hall at the University of Kansas

Moving vehicles traverse the intersection of 23rd and Iowa streets. Trees and buildings also in view.



Two cases collected back-to-back (consistent set of movers for comparison):

- 100 ms (10^4 segments) of NICE-OFDM
- 100 ms (10^4 segments) of "variable symbols" CE-OFDM

Open Air Test Results

- Open-air measurements performed at 3.45 GHz with 50 MHz bandwidth for 100ms
  - Note: simultaneous transmit/receive

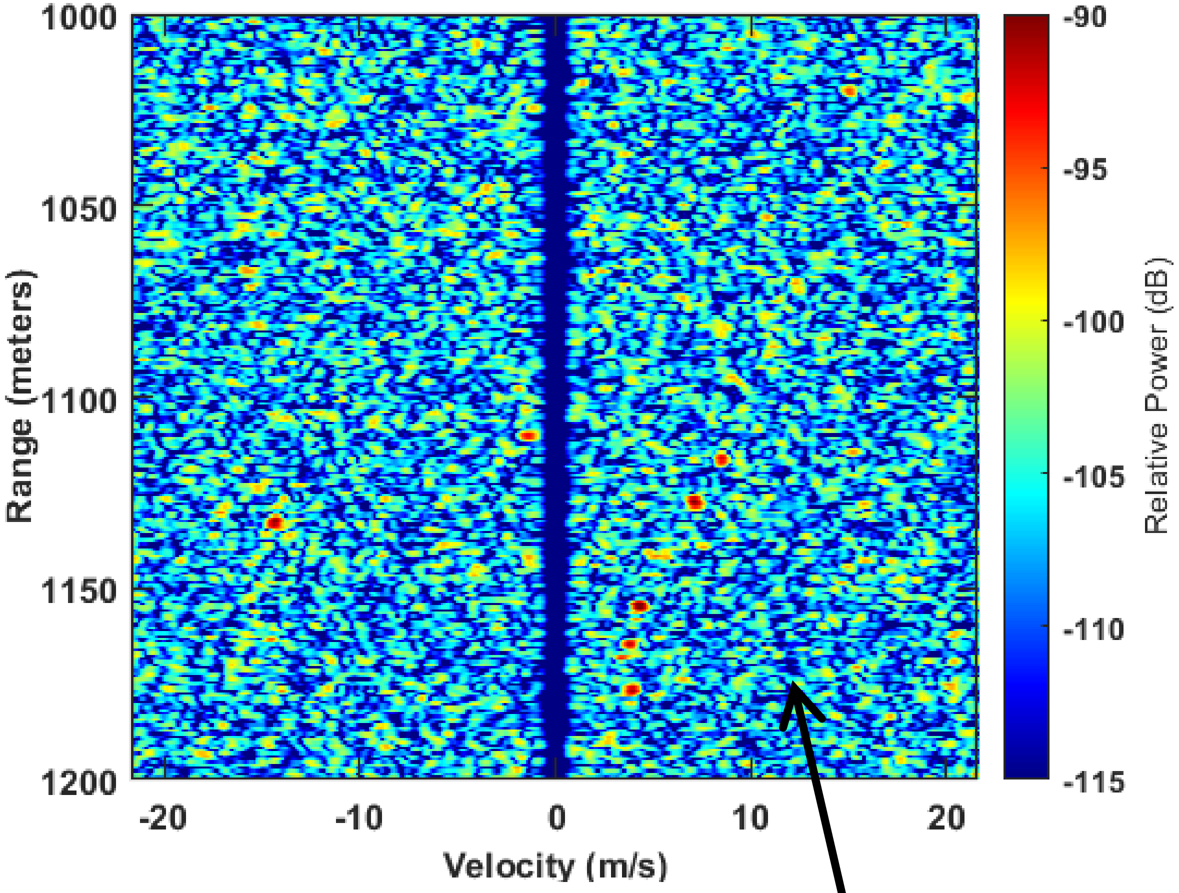
- CW signals separated into 10^4 segments of 10μs each to perform pulse compression

- Doppler processing performed using every 100^{th} segment in "quasi-pulsed" manner (emulates PRF of 1kHz and 100 pulses), with resulting 100 complex range/Doppler responses then combined via averaging.

- To address direct path leakage, a version of CLEAN accounting for range straddling was applied. Simple projection-based clutter cancellation was performed (since platform is stationary).

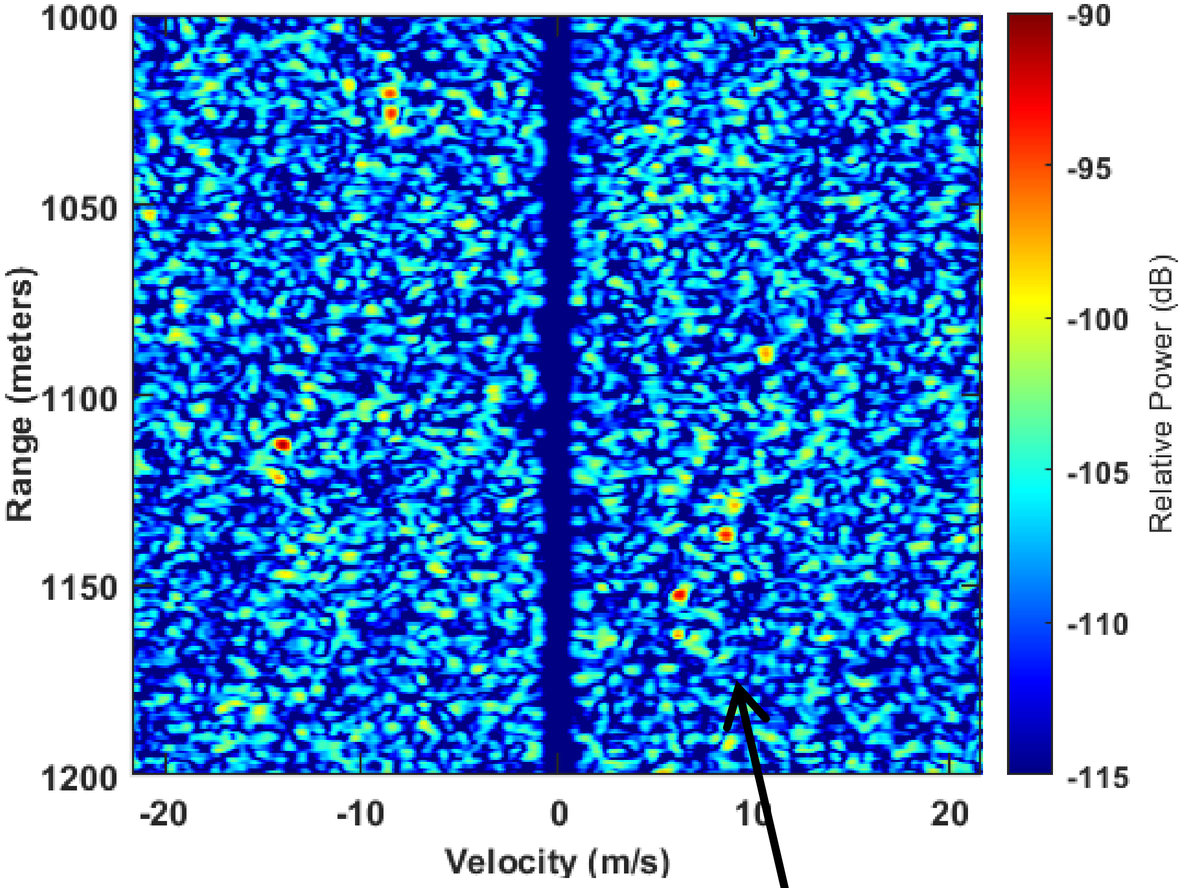
Open-Air Range-Doppler Responses

Range-Doppler response for "variable symbols" CE-OFDM



Moderately higher background

Range-Doppler response NICE-OFDM



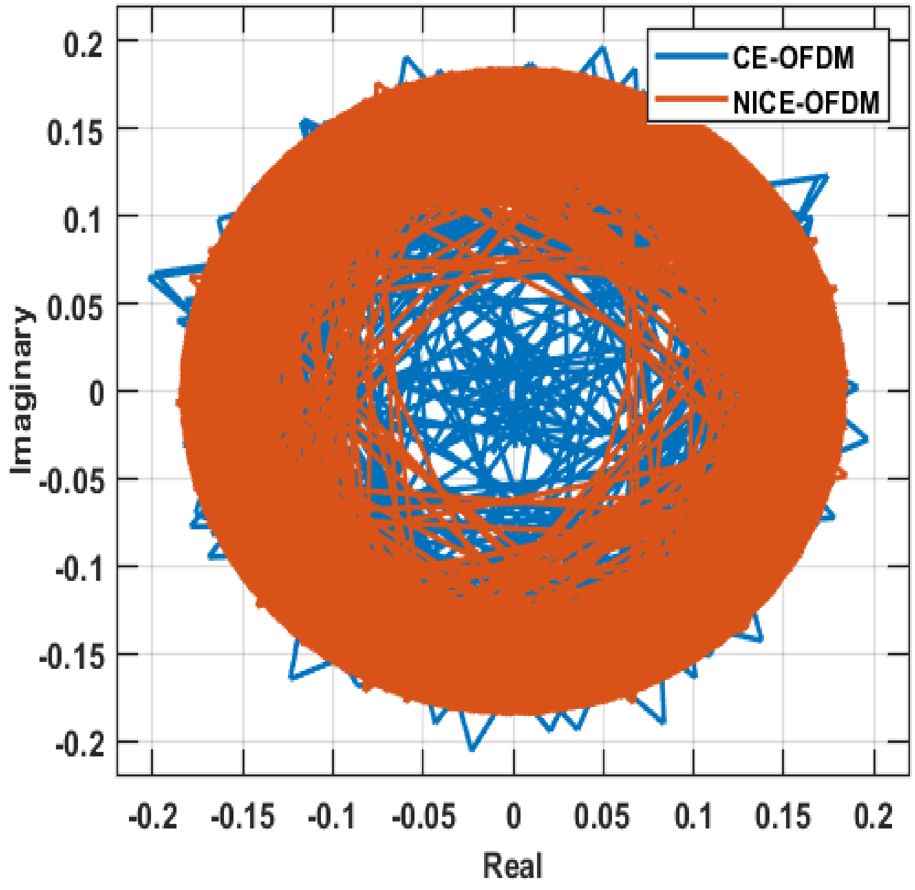
Demonstrates viable radar capability

Assessing Transmitter Distortion

- Consider the I/Q samples of each loopback-captured waveform
  - Would ideally conform to the unit phase circle

- “variable symbols” CE-OFDM exhibits abrupt phase changes that traverse thru the middle (i.e. more severe distortion)

- NICE-OFDM deviation from ideal is more modest, and could be reduced further via higher AWG “over-sampling” for spectral roll-off
  - Here only 4× the 3-dB bandwidth due to hardware limit



Conclusions

- Nonrepeating nature of NICE-OFDM can provide a computationally-light, compact representation of a random FMCW signal.

- Still maintains desirable spectral characteristics and low range sidelobes.

- (Given sufficient transmit/receive isolation) Compared to pulsed operation this nonrepeating CW structure provides:
  - Lower peak transmit power (for same "energy on target") to lessen induced interference when spectrum sharing
  - Higher dimensionality for better **separability** from received interference