SAND2023-01460C

Sandia National Laboratories

Optimal Ground State Approximations for Sparse Hamiltonians

By: Daniel Hothem, Ojas Parekh, and Kevin Thompson

Introduction

- A Hamiltonian

$$H = \sum_{\Gamma \in [2n]} H_\Gamma c^\Gamma$$

is q-local and k-sparse if each term has weight at most q and each operator appears at most k times.

- q-local, k-sparse Hamiltonians appear everywhere
  - Fermi-Hubbard model,
  - spin lattices.
- Finding ground states is hard[3], so we approximate special cases instead.

READ OUR PAPER

We developed optimal or near-optimal classical approximation algorithms for sparse fermionic and qubit Hamiltonians.

Hamiltonian		Our Result	Previous Best
fermionic	k-sparse, strictly q-local	$1/(qk+1)$	$1/\mathcal{O}(q^2k^2)$
	k-sparse, 4,2-local	$1/(4k+1)$	$1/\mathcal{O}(k^2)$
	k-sparse, q-local	$1/\mathcal{O}(qk^2)$	N/A
Qubit	k-sparse, strictly q-local	$1/(qk+1)$	$(3^{-q/2})/(4qk)$
	k-sparse, 4,2-local	$1/(2k+1)$	$1/24k$
	k-sparse, q-local	$1/\mathcal{O}(qk^2)$	$(3^{-q/2})/(4qk)$

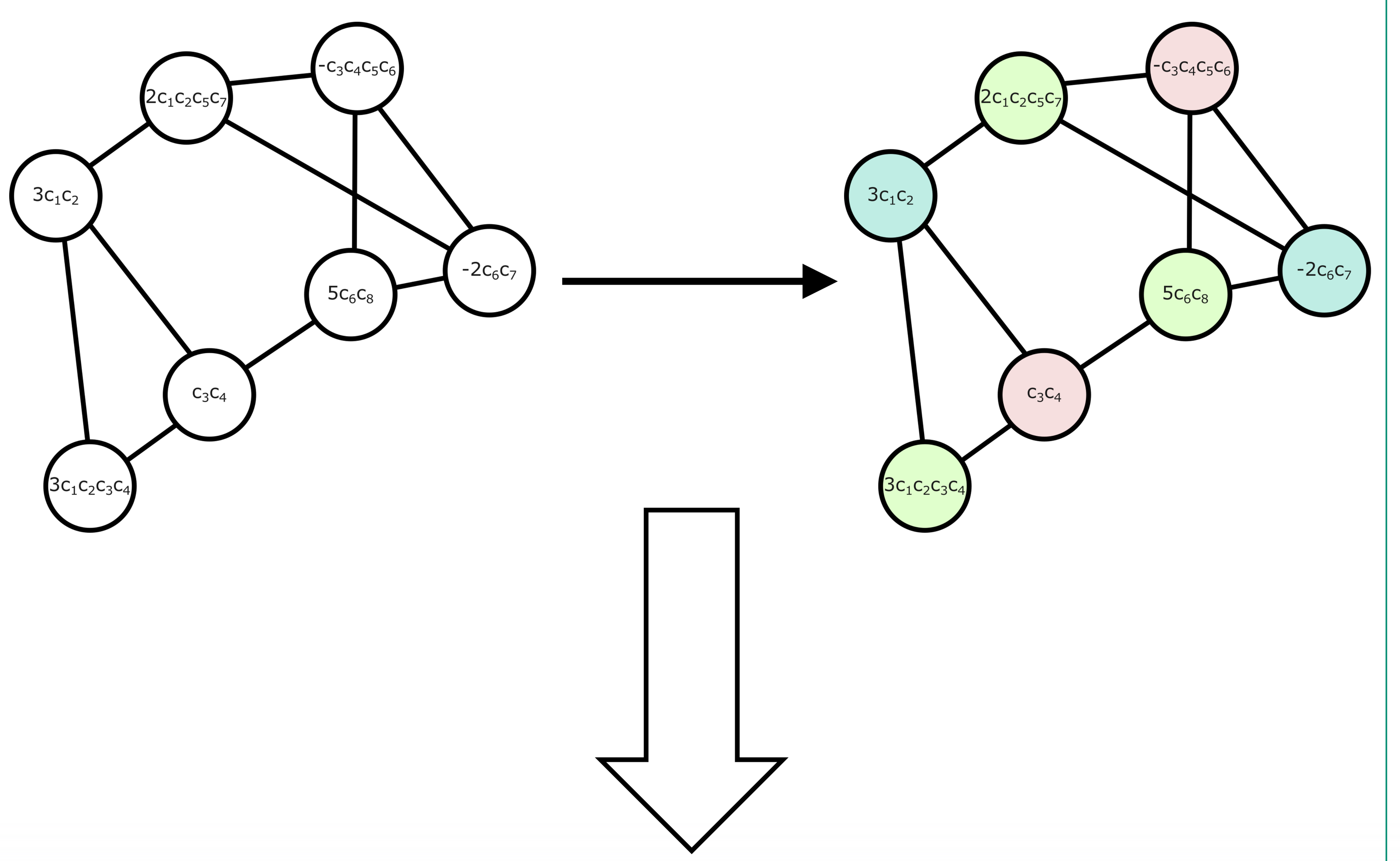
Our algorithm is asymptotically tight on

$$H_n = \sum_{i \in [n], j \in [n]} i c_i c_{j+n}$$

Our algorithm

- Build the interaction graph G
- Find a high-weight independent set
- Return a high-energy Gaussian state

$$\rho \propto \prod_{\Gamma \in S} (\mathbb{I} + sign(H_\Gamma) c^\Gamma)$$



$$\rho \propto (\mathbb{I} + c_1c_2c_3c_4)(\mathbb{I} + c_1c_2c_5c_7)(\mathbb{I} + c_6c_8)$$

Future work

- Optimality in the generic case
- Improve the non-sparse case

1. A. Harrow and A. Montanaro. Extremal eigenvalues of local Hamiltonians. *Quantum,* 1:6, April 2017. doi: 10.22331/q-2017-04-25-6.
2. Y. Harasymenko *et. al.*. Optimizing sparse fermionic Hamiltonians. *arXiv preprint arXiv:2211.16518,* 2022.
3. A. Kitaev. Quantum NP. Talk at Second Workshop on Algorithms in Quantum Information Processing . 1999.

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology &Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.
SAND No. ____________

Sandia National Laboratories

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.