

# Modeling Capabilities of Waveform-Based Bayesian Inference for Seismic Monitoring

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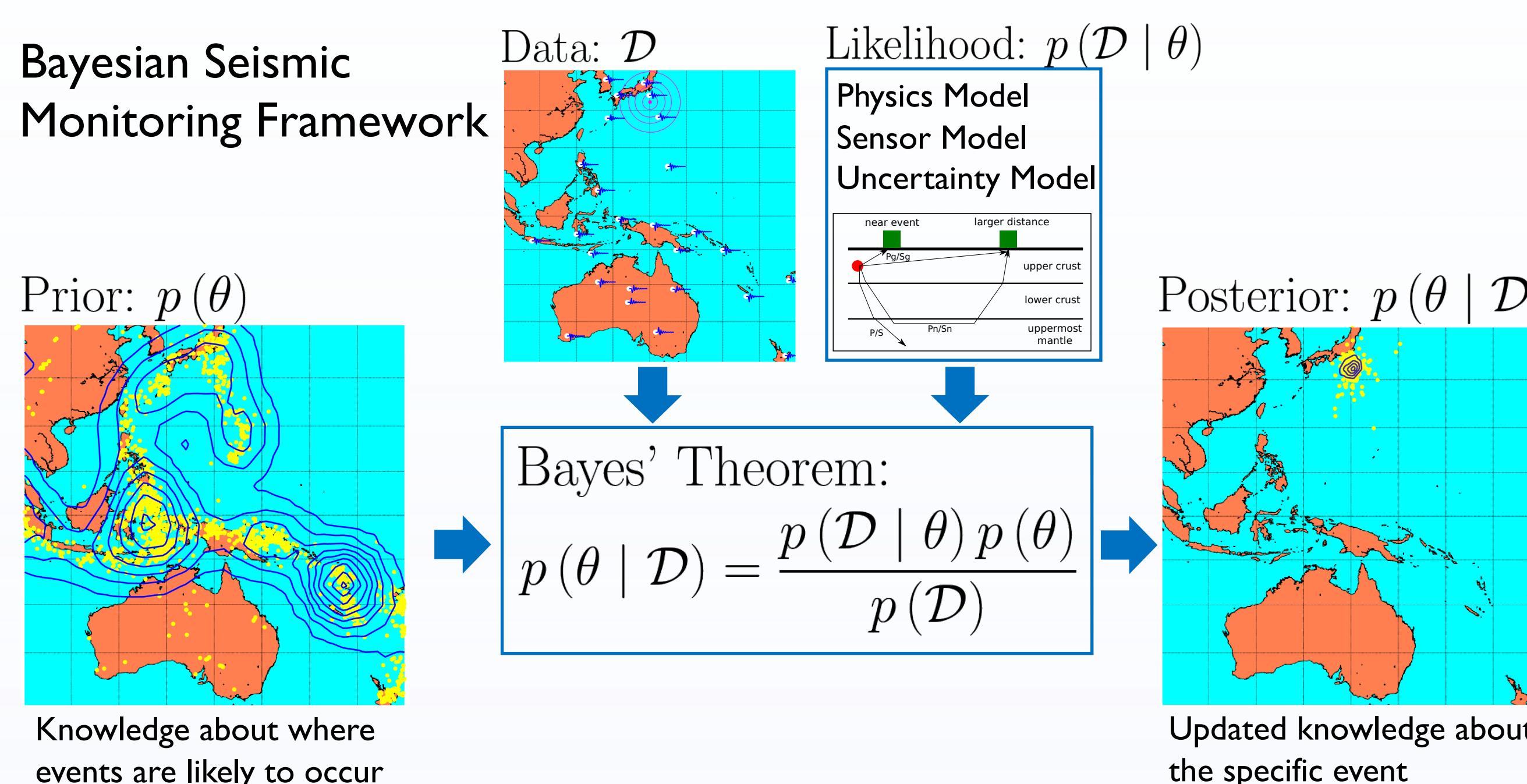
## Evaluating Bayesian Seismic Monitoring

### Goal:

- Computational tools allow high frequency ( $>1$  Hz) seismic simulation; however, modeling uncertainties limits the accuracy of these results. How would refining these simulations to higher frequencies increase Bayesian seismic monitoring capabilities? Do we benefit from higher frequencies?

### Bayesian Seismic Monitoring Problem:

- Infer event parameters with uncertainty: Longitude, Latitude, Depth, Origin Time, Source Time Function, and Moment Tensor
- Observations: Filtered seismic waveforms at various locations
- Uncertainty to integrate: Travel time uncertainty, earth structure heterogeneity, background noise process



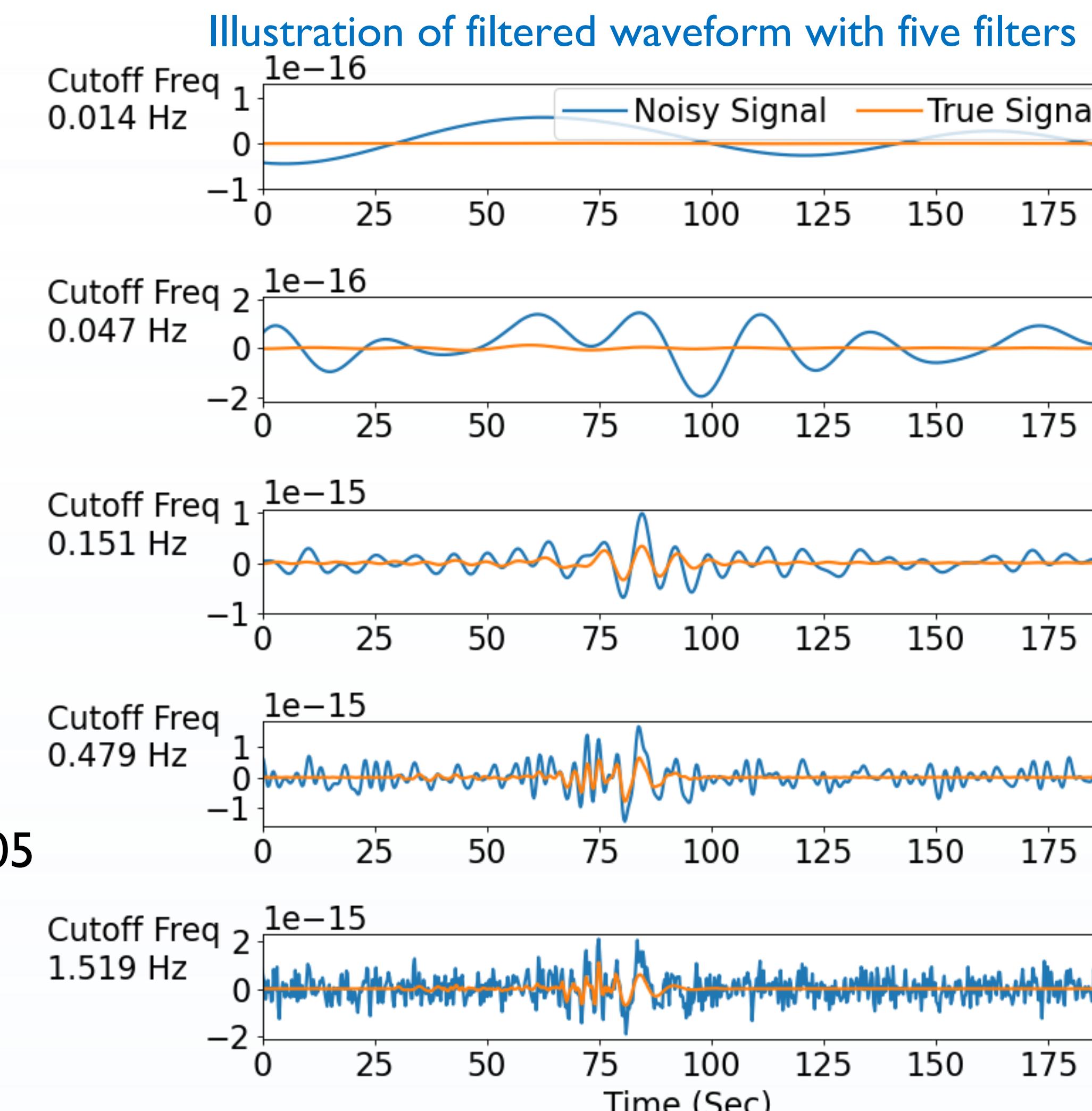
### Evaluation Approach:

- We use the approach of Bayesian experimental design where we estimate the expected information gain of the Bayesian inference problem under different low-pass filter frequency assumptions. This replicates the effect of limiting our simulations to certain frequencies.
- Expected information gain (see Mathematical Methods for definition) is an information theoretic quantity that broadly measures the change from the prior parameter distribution to the posterior distribution.
- Algorithm Outline:
  - Create a representative set of seismic sources with different locations and source properties
  - Simulate high-frequency waveforms for each of these sources and add background noise from a known noise model
  - For each waveform apply a set of low pass filters with different cutoff frequencies
  - For each representative event and filtered waveform solve the Bayesian inference problem to find the posterior distribution on the event parameters and compute the information gain
  - Average the information gain over all events for each of the filters to capture the effect of frequency content on seismic monitoring.

## Experiment and Results

### Setup:

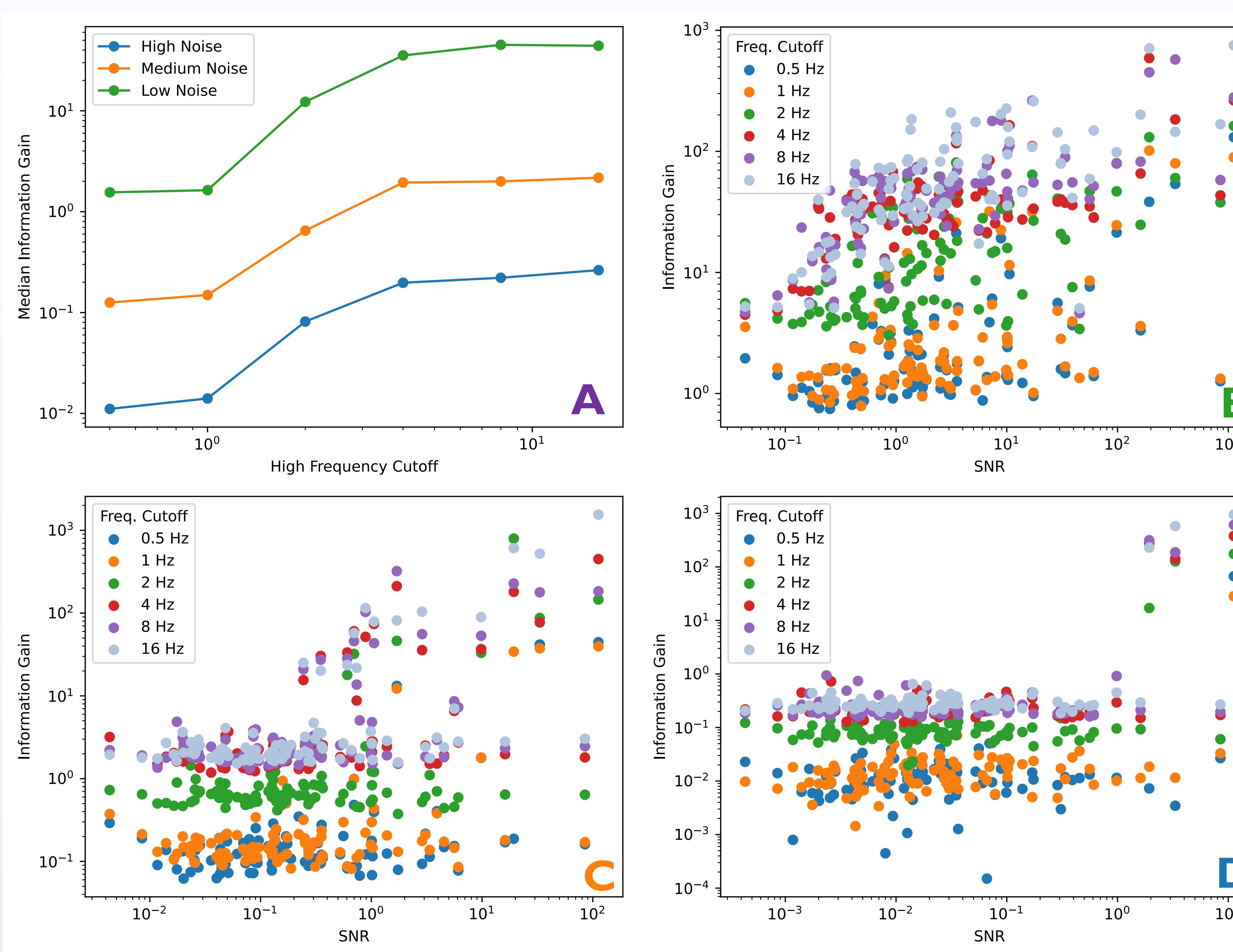
- 2D waveform simulation with an AK135 earth model + stochastic perturbations, sample rate 200 Hz and  $f_{\max} = 10$  Hz.
- Domain: 222 km wide and 40 km in depth
- Exponentially distributed  $\log_{10}$  moment magnitude factor between -2 to 2 and isotropic moment tensor
- Gaussian source time function with std 0 to 0.1 sec and origin time 0 to 60 sec
- Low additive background noise from Peterson (1993)
- Band pass filtered using sinc filter from 0.05 Hz to max frequency cutoff.
- Simulated 100 representative events



### Results:

**Panel A** – Expected information gain for different cutoff frequencies at different background noise levels. We see in all cases there is little change in information gain above 4 Hz

**Panel B, Panel C, Panel D** – Information gain for each representative event for the five filters vs the true signal power. Background level noise: **Panel B SNR50% = 1.5**, **Panel C SNR50% = 0.15**, and **Panel D SNR50% = 0.015**. We see similar trends between B, C, D, just shifted to higher gains.



## Conclusion

### Discussion:

- Using the framework of Bayesian experimental design we can quantify the utility of different frequency content in high fidelity seismic simulation for seismic monitoring.
- Under our assumptions of an AK135 earth model, stochastic perturbations, and background noise process, we observe that the contribution of higher frequency information is limited above 4.0 Hz for this regional setup.

### Future Directions:

- We would like to extend this analysis to cases where instead of a single earth model, we have many candidate earth models. This introduces another source of uncertainty beyond the background noise.
- We made assumptions about the background and source mechanism to facilitate computation, relaxing the assumptions and using more realistic models may influence these results.

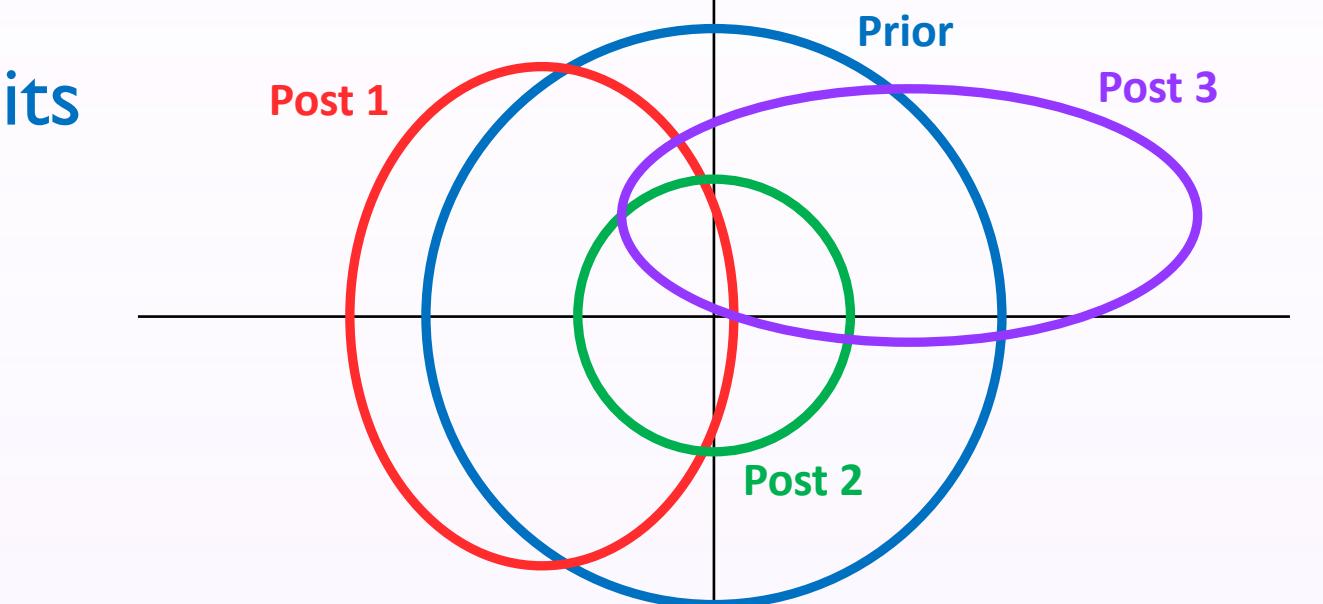
## Mathematical Methods

### Quantifying Information Gain :

- Bayesian Inference:  $p(\theta | \mathcal{D}) = \frac{p(\mathcal{D} | \theta) p(\theta)}{p(\mathcal{D})}$
- Kullback-Leibler (KL) Divergence  $KL[p(\theta | \mathcal{D}) || p(\theta)] = \int p(\theta | \mathcal{D}) \log \frac{p(\theta | \mathcal{D})}{p(\theta)} d\theta$  measures information due to inference

Illustration of information gain in bits for three posteriors

Prior  $\rightarrow$  Post 1: 0.5 Bits  
 Prior  $\rightarrow$  Post 2: 1 Bit  
 Prior  $\rightarrow$  Post 3: 1 Bit



- Expected Information Gain (EIG) from a model with filter frequency (S):

$$\begin{aligned} \mathcal{I}(S) &= E [KL[p(\theta | \mathcal{D}) || p(\theta)] | \mathcal{D} \sim p(\mathcal{D} | S)] \\ &= \int p(\mathcal{D} | S) \int p(\theta | \mathcal{D}, S) \log \frac{p(\theta | \mathcal{D}, S)}{p(\theta)} d\theta d\mathcal{D} \end{aligned}$$

Distribution of hypothetical data  
 $p(\mathcal{D} | S) = \int p(\mathcal{D} | \theta', S) p(\theta') d\theta'$

KL Divergence to measure information gain

### Likelihood model in the frequency domain:

- Let  $\omega_j$  be the Discrete Fourier Transform (F) of the predicted waveform at frequency  $j$  for an event characterized by (Lat-Lon  $\mathcal{L}$ , Depth  $z$ , Magnitude  $m$ , Origin Time  $t_o$ , and STF width  $\lambda$ )  
 $\omega_j(\mathcal{L}, z, m, t_o, \lambda) = F_j \bar{w}(\mathcal{L}, z, m, t_o, \lambda) \mathbb{1}[s_j < f]$
- Then the likelihood of the observed Discrete Fourier Transform ( $\xi$ ) up to frequency  $f$  given the predicted waveform and white noise is  
 $p(\xi_1 \dots \xi_{n_f} | \mathcal{L}, z, m, t_o, \lambda) = p(\gamma_{01} = \text{Real}[\xi_1 - \omega_1(\mathcal{L}, z, m, t_o, \lambda)]) \times \prod_{j=2}^{n_f} p(\gamma_{0j} = \text{Real}[\xi_j - \omega_j(\mathcal{L}, z, m, t_o, \lambda)]) p(\gamma_{1j} = \text{Imag}[\xi_j - \omega_j(\mathcal{L}, z, m, t_o, \lambda)])$

- Where  
 $\gamma_{01} \sim \mathcal{N}(0, \sigma^2), j = 1 \quad \gamma_{0j} \sim \mathcal{N}(0, \frac{n}{2} \sigma^2), j = 2 \dots n$   
 $\gamma_{11} = 0, j = 1 \quad \gamma_{1j} \sim \mathcal{N}(0, \frac{n}{2} \sigma^2), j = 2 \dots n$