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Residual minimization formulations for model reduction of steady hypersonic flow

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Introduction

- Numerical simulations of hypersonic flow needed to fill in for a lack of easily attainable experimental data.
- Despite advancements in high-fidelity CFD, some form of surrogate model is usually needed to make *many-query* workflows tractable
- Projection-based reduced order models (ROMs) project the governing equations onto data-driven bases to generate surrogates that can offer very high accuracy and the potential for error estimates
- ROMs formulated as optimization statements have proven particularly effective for large scale flow problems, including hypersonic flow.
  - *In this work, we exploit the flexibility of this optimization statement to explore the effect of different formulations on the quality of steady hypersonic ROMs*

Residual minimization ROMs

- Focus on steady parametric problems:

$$f(x; \mu) = 0$$

- Expand the solution in a data-driven basis

$$\underset{\sim}{x}(\mu) = x^0(\mu) + \Phi \underset{\sim}{x}(\mu)$$

- Find $\tilde{x}$ by solving the minimization problem:

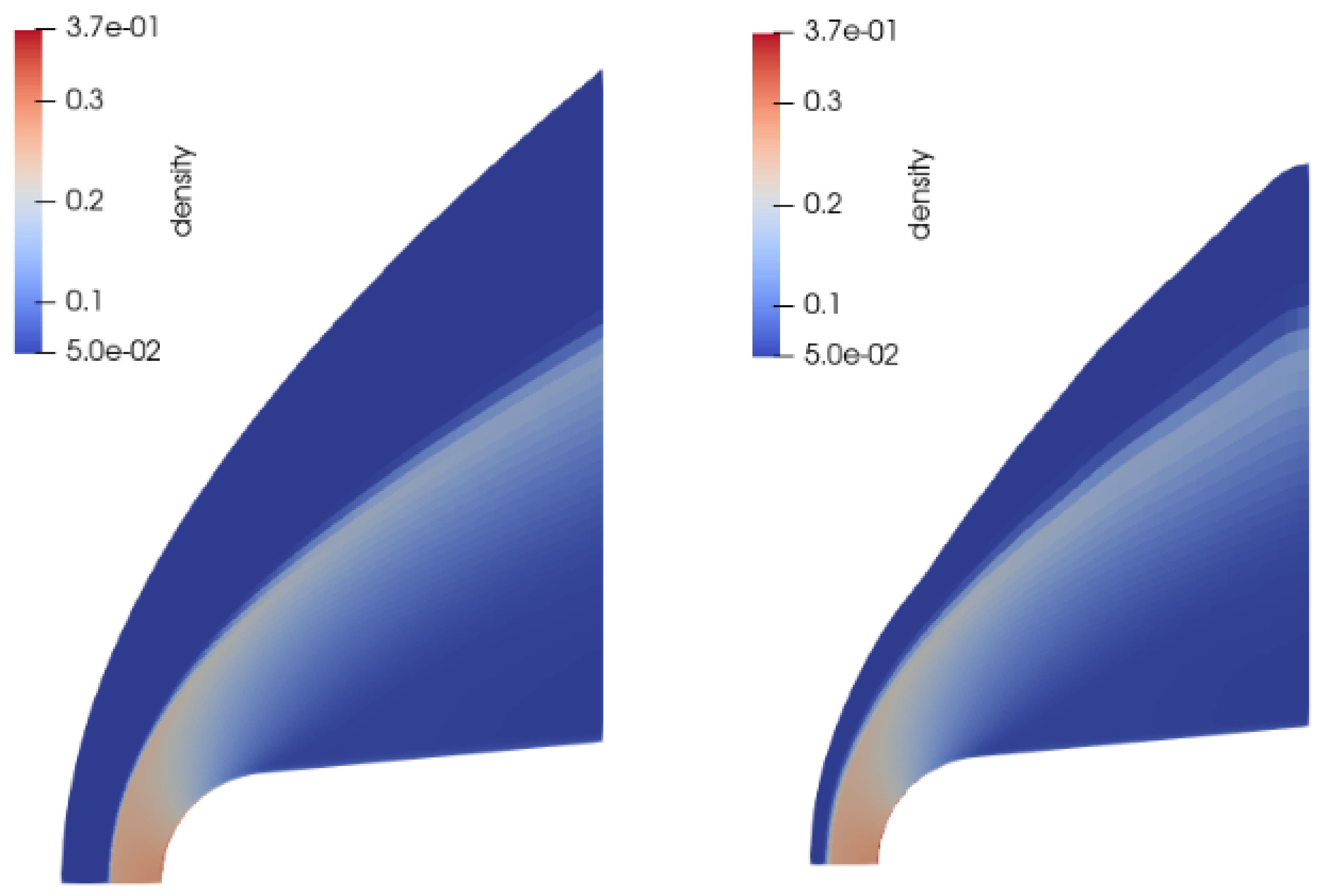
$$\underset{\sim}{x} = \min_{z} \| A f(x^0 + \Phi z) \|_2$$

For hyperreduction. Not considered here, so $A = I$

Previous work: application to hypersonic flow

- We consider flows with parametrically varying shocks
  - *Known challenge for standard methods for*
- Previous work by [1,2] addressed this and other challenges with the following:

1. A dictionary basis instead of more standard POD
2. Mesh adapted FOMs and ROMs so that shocks are aligned. Grid tailoring is applied to hypersonic FOMs so that shocks are cleanly resolved while retaining same mesh topology for each snapshot.
3. Bases built with primitive variables found to be more accurate
4. Clippers to ensure physicality of state



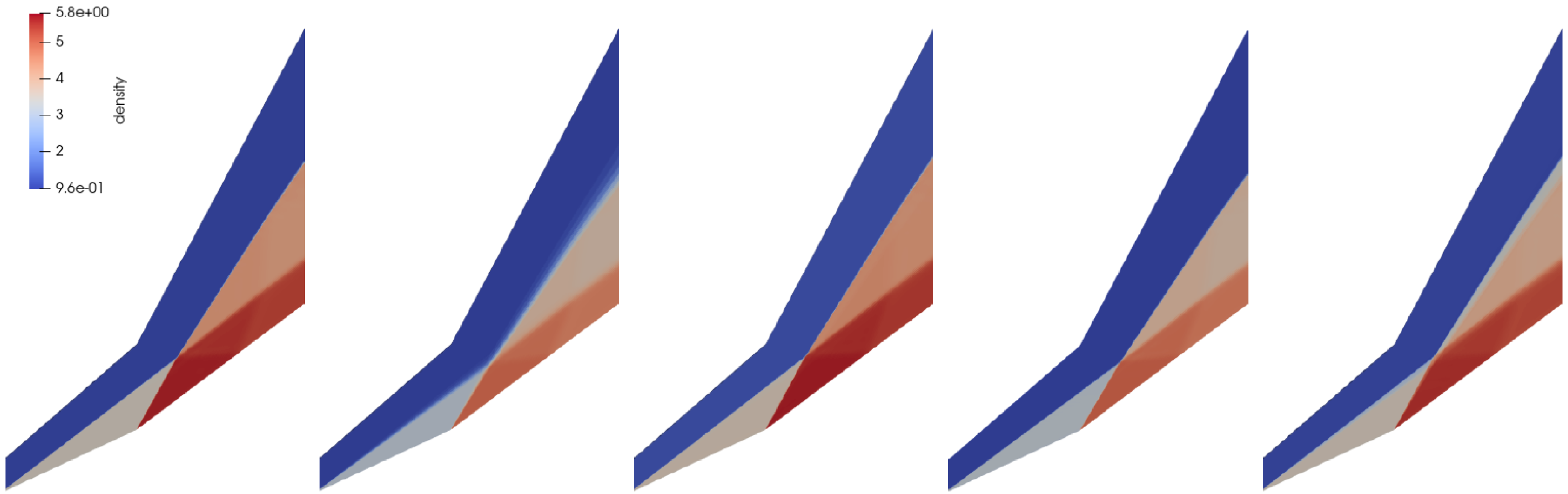
(a) Fixed mesh (b) Grid tailored

[1] Blonigan, Patrick J., et al. "Model reduction for steady hypersonic aerodynamics via conservative manifold least-squares Petrov–Galerkin projection." *AIAA Journal* 59.4 (2021): 1296-1312.

[2] Ching, David S., et al. "Model Reduction of Hypersonic Aerodynamics with residual minimization techniques." *AIAA SCITECH 2022 Forum*. 2022.

Examples

- Brief demonstration on double wedge that different formulations lead to different solutions

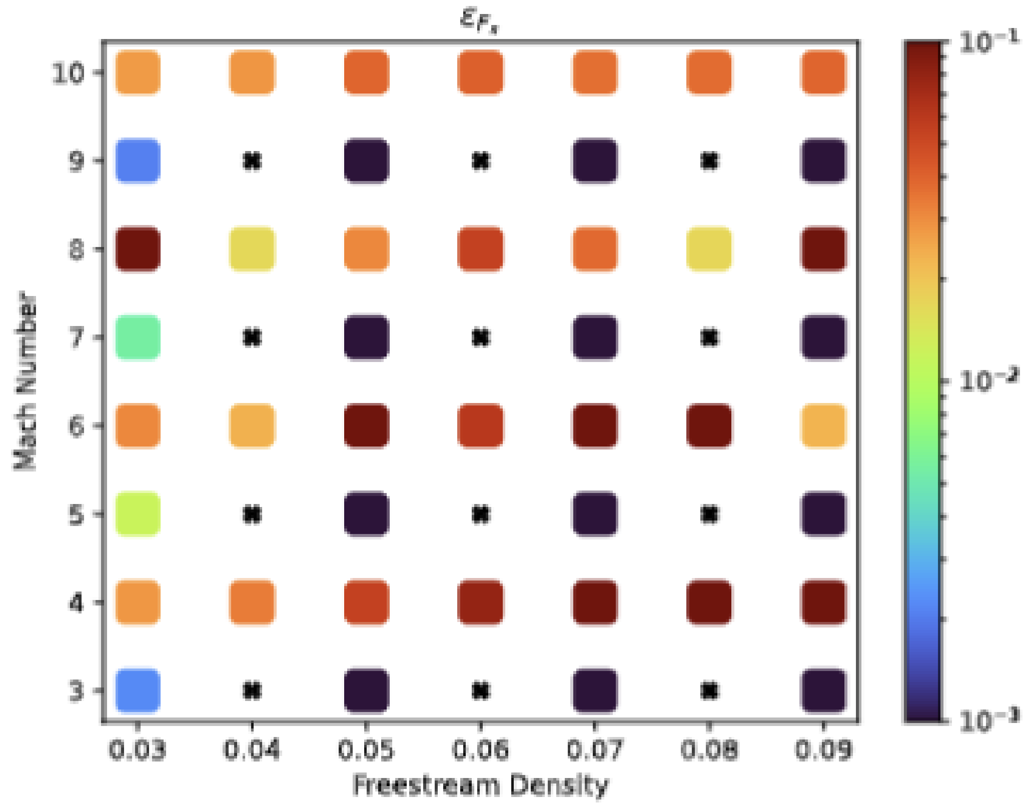


(a) FOM (b) LSPG, $\boldsymbol{W}_{diag}$ (c) LSPG, $\boldsymbol{W}_{ent}$ (d) L1 (e) C(ν)-LSPG

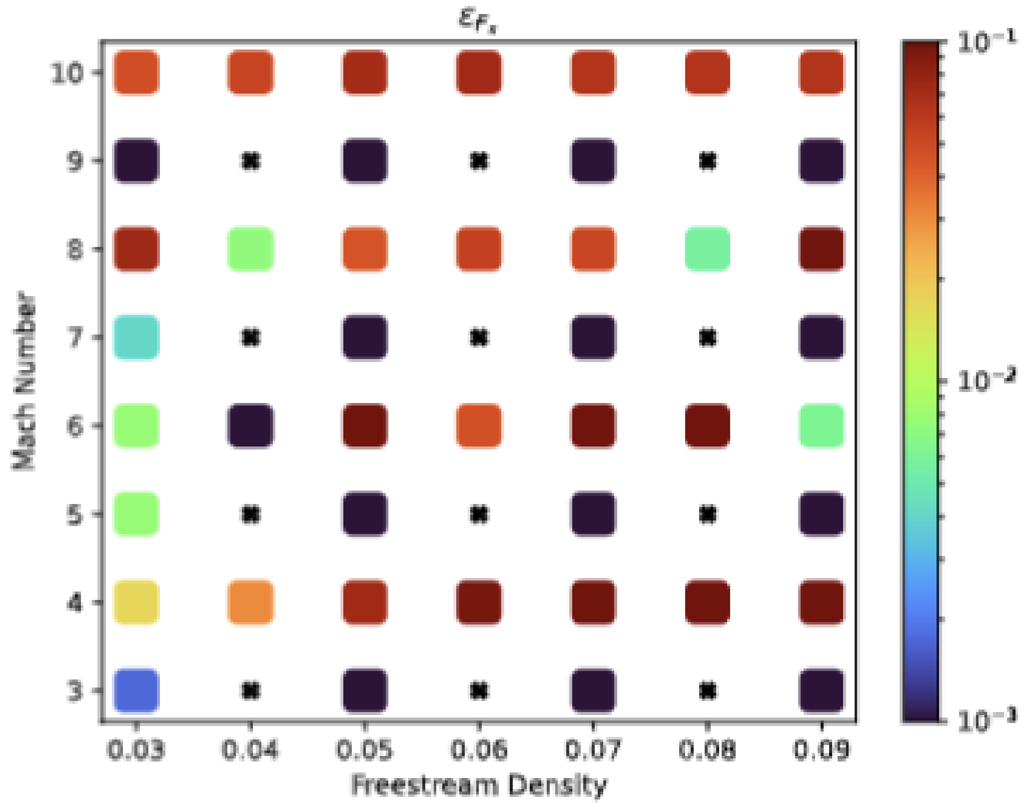
- **2D blunt wedge with**
- 3D HiFIRE with

Results: blunt wedge, fixed mesh

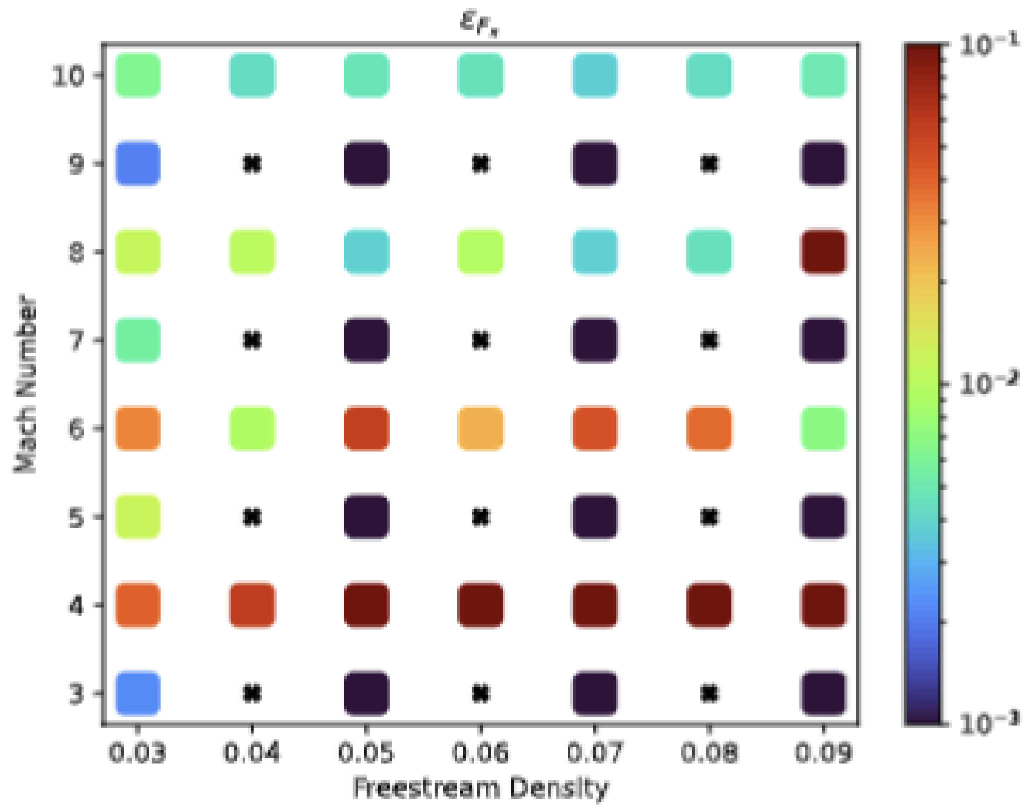
- Axial force errors on a fixed mesh
- L1 and C()-LSPG show improvements



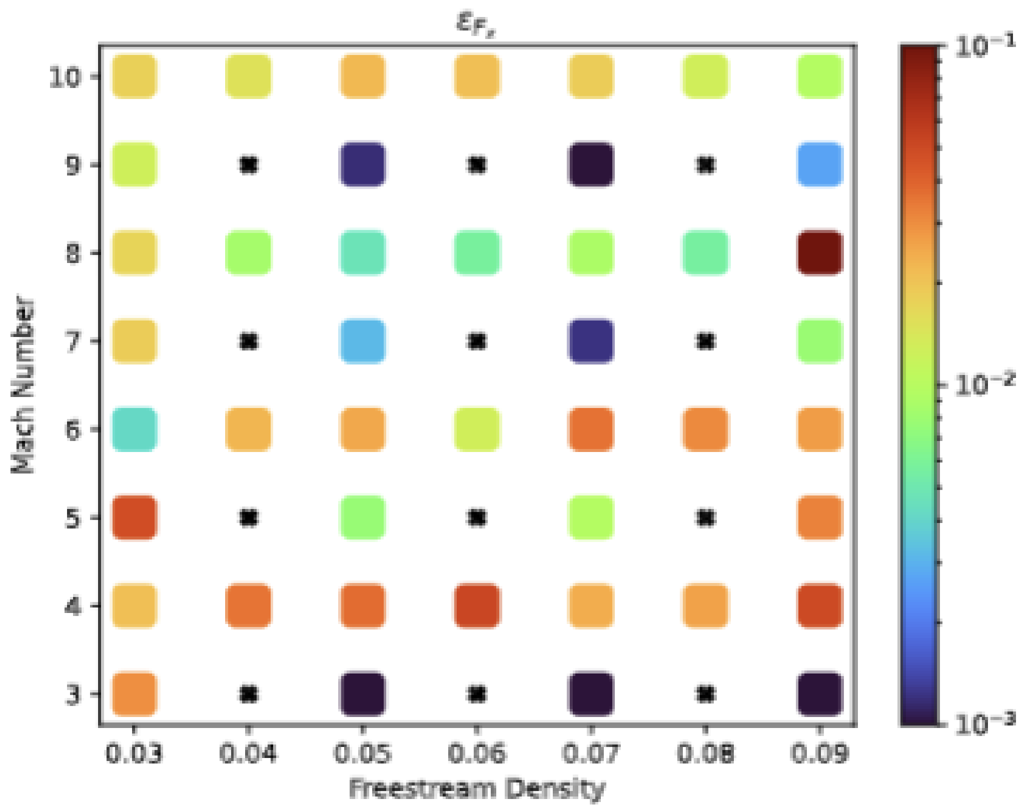
(a) LSPG, $\boldsymbol{W}_{diag}$



(b) LSPG, $\boldsymbol{W}_{ent}$



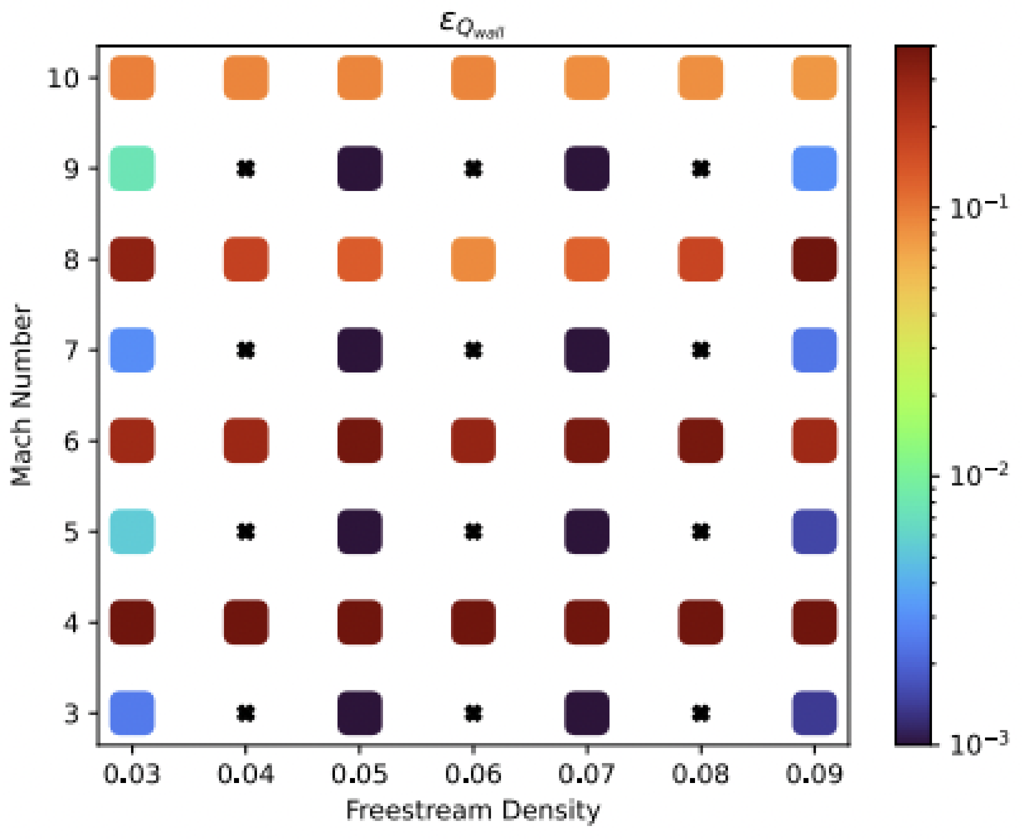
(c) ℓ^1, $\boldsymbol{W}_{diag}$



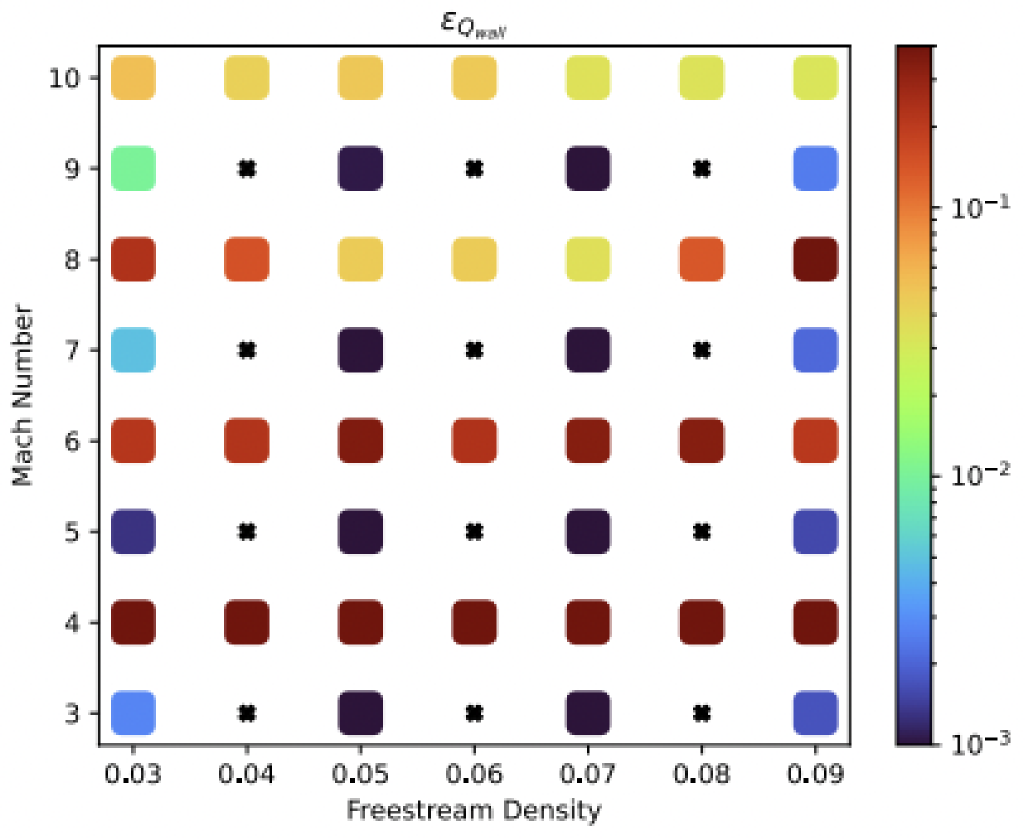
(e) C(ν)-LSPG, $\nu = 1.0$

Results: blunt wedge, fixed mesh

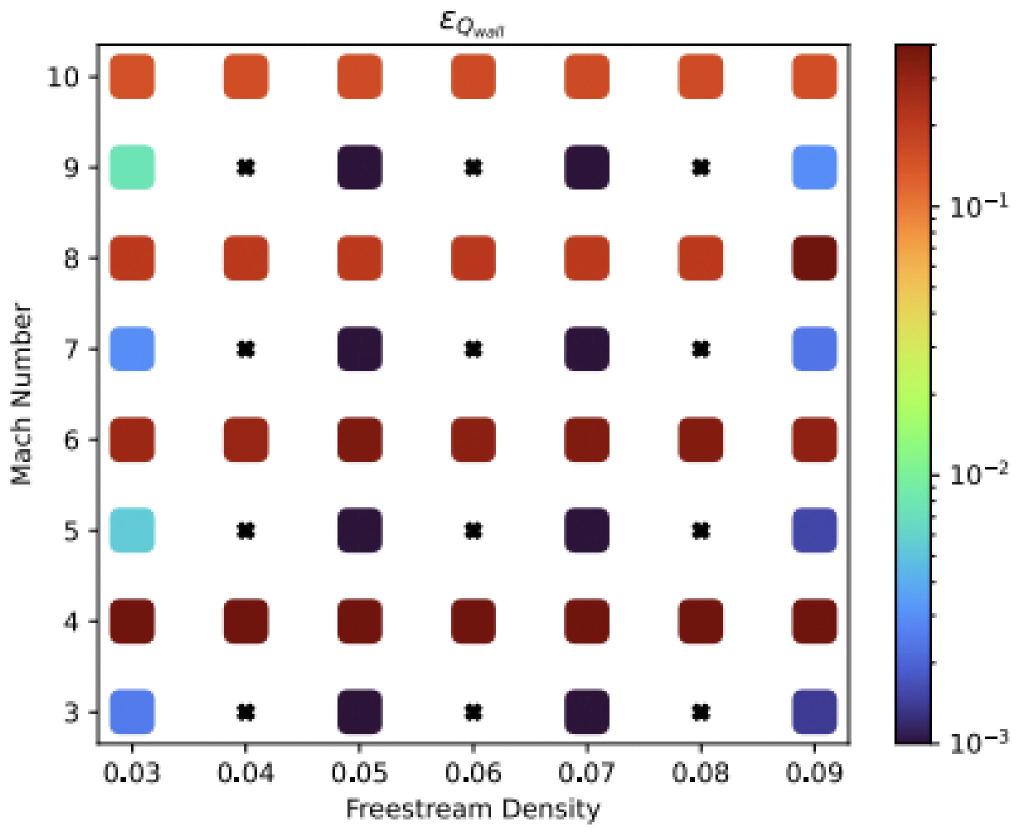
- Heat flux errors with a fixed mesh
- For this case, conservative penalty works well



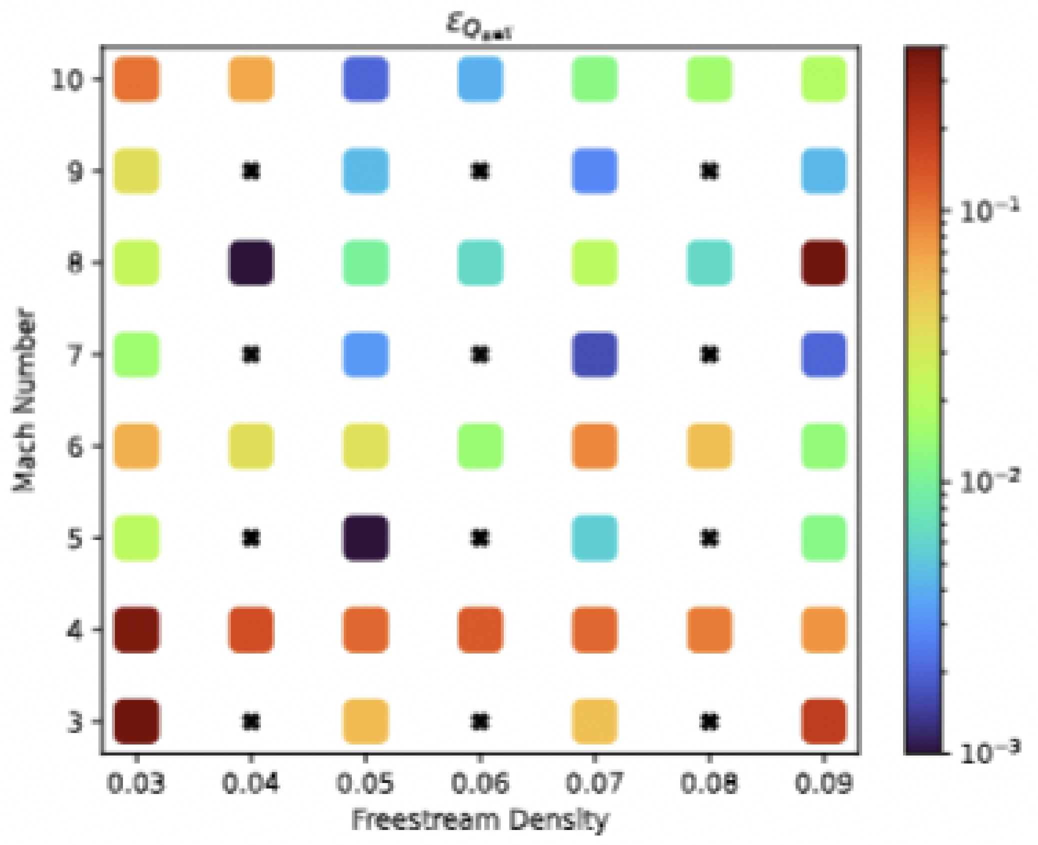
(a) LSPG, $\boldsymbol{W}_{diag}$



(b) LSPG, $\boldsymbol{W}_{ent}$



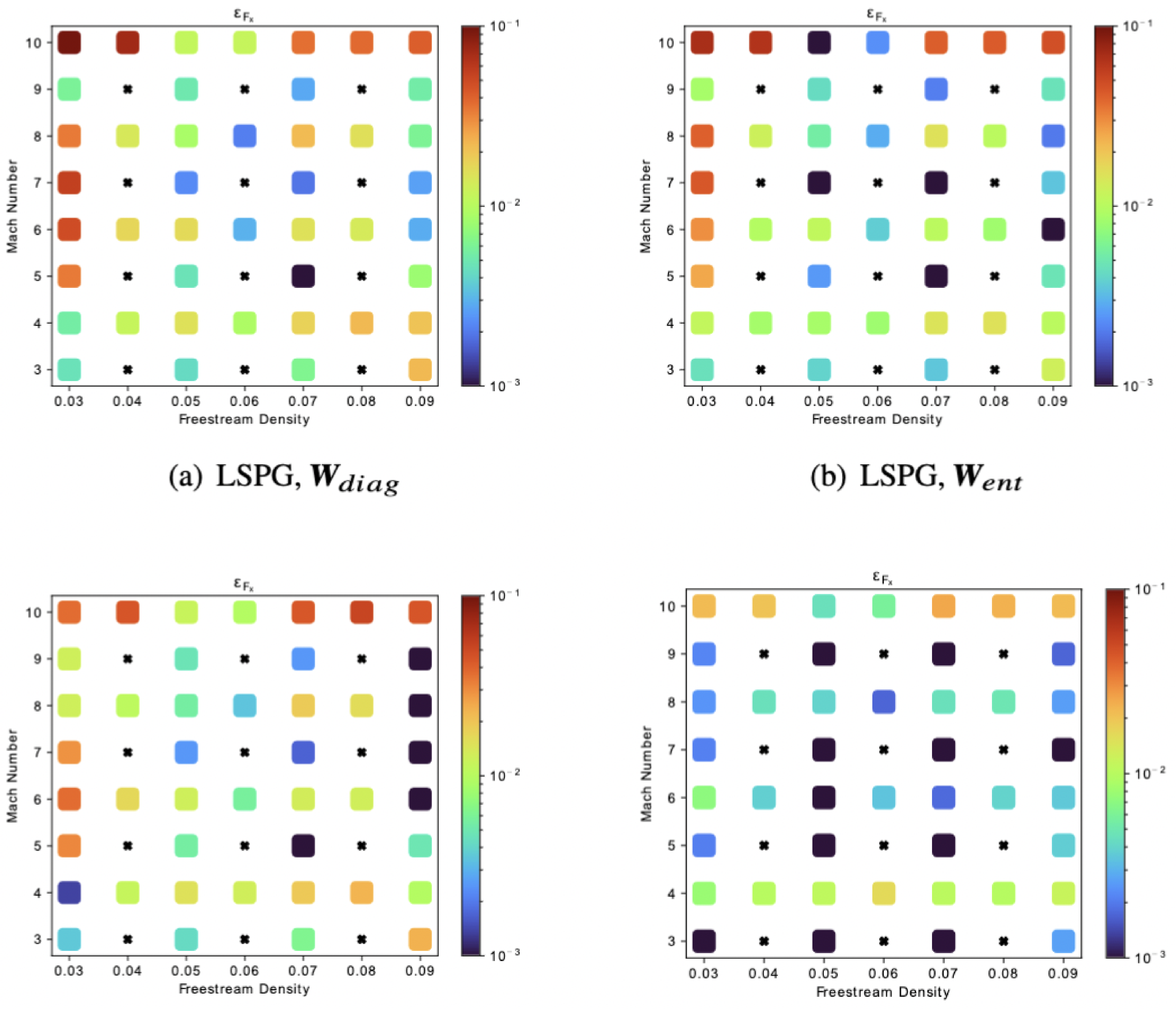
(c) ℓ^1, $\boldsymbol{W}_{diag}$



(e) C(ν)-LSPG, $\nu = 1.0$

Results: blunt wedge, grid tailored

- Axial force errors with a grid tailored mesh
- C()-LSPG leads to improvements

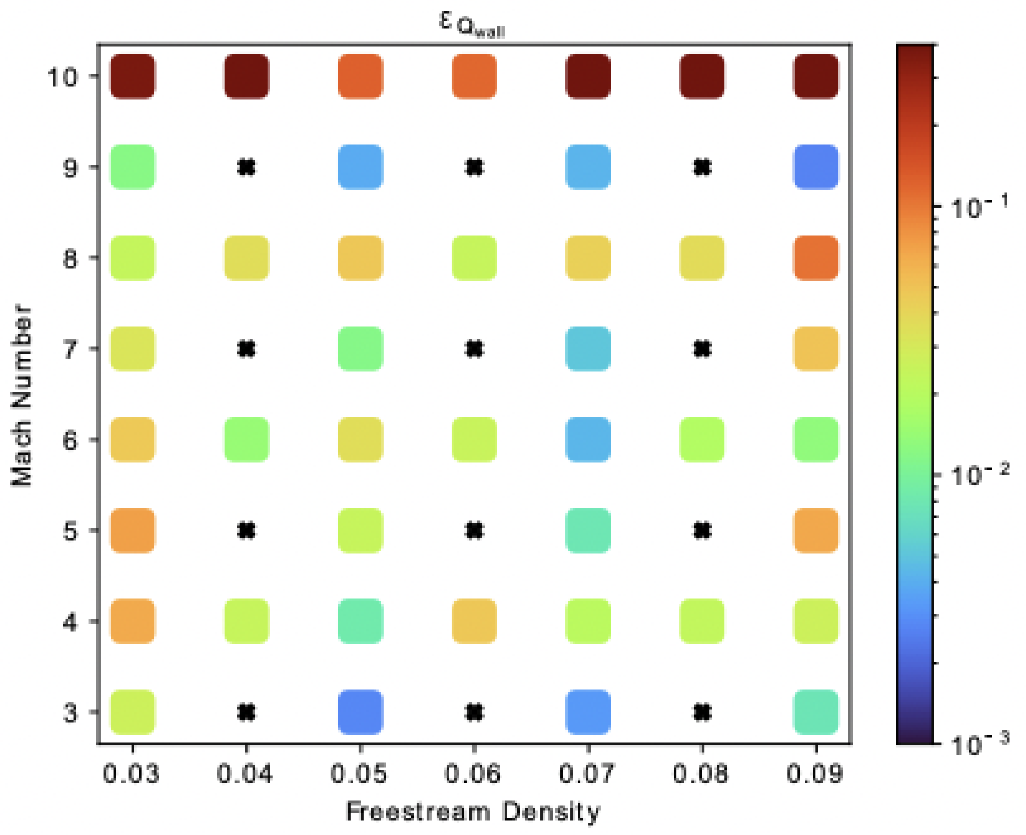


(a) LSPG, $\boldsymbol{W}_{diag}$ (b) LSPG, $\boldsymbol{W}_{ent}$

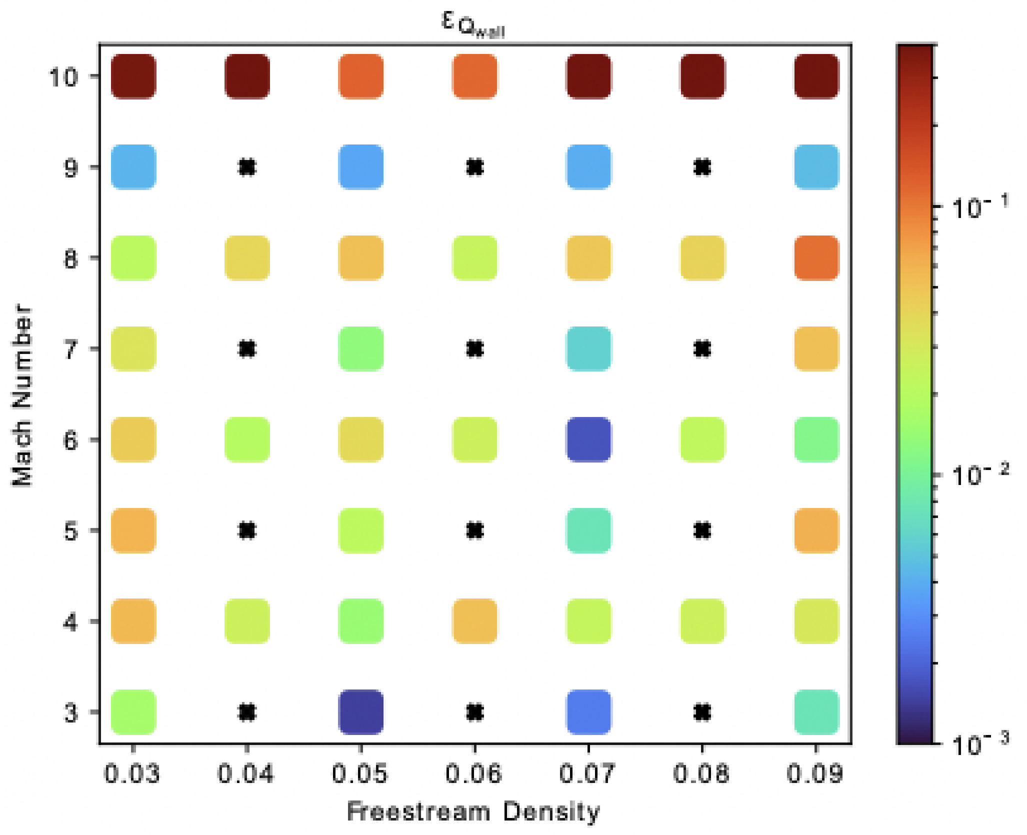
(c) ℓ^1, $\boldsymbol{W}_{diag}$ (e) C(ν)-LSPG, $\nu = 0.001$

Results: blunt wedge, grid tailored

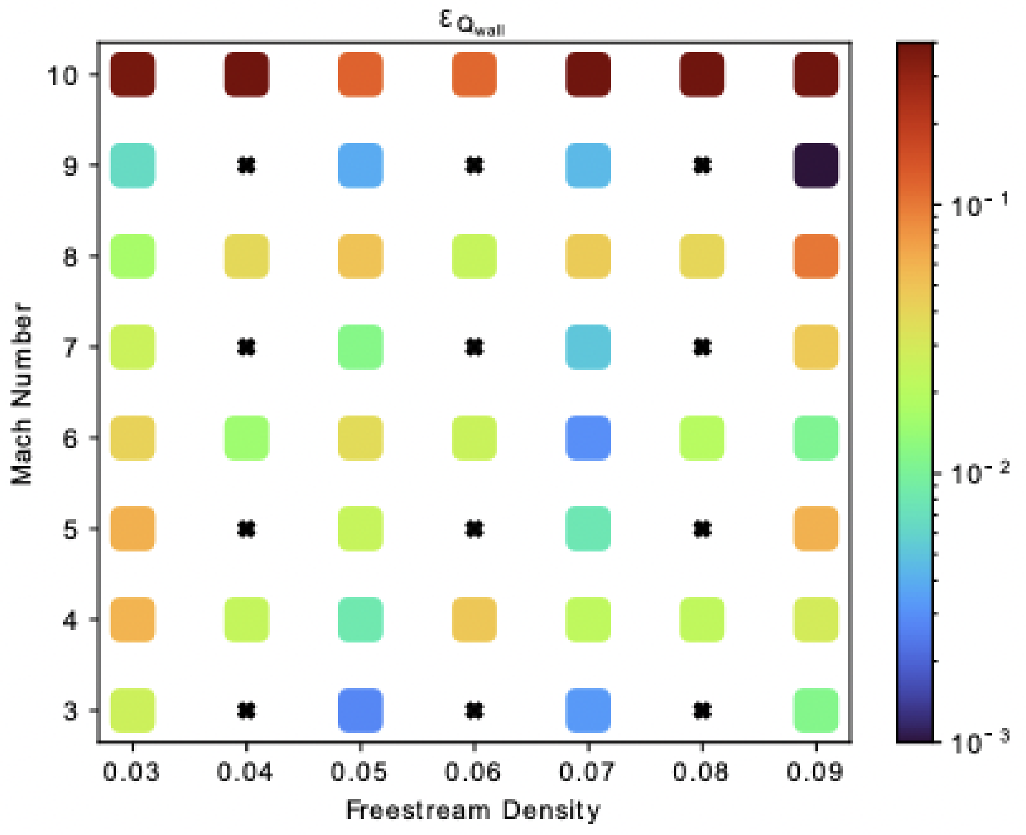
- Heat flux errors with a grid tailored mesh
- Less of a difference between the methods



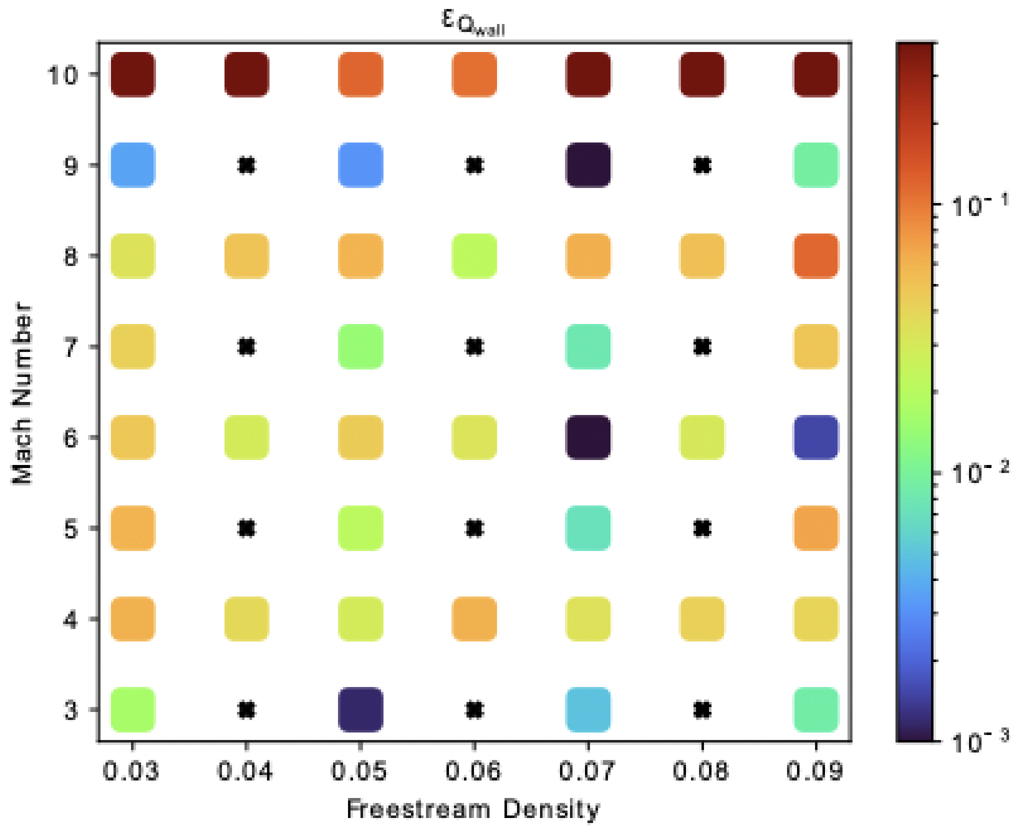
(a) LSPG, $\boldsymbol{W}_{diag}$



(b) LSPG, $\boldsymbol{W}_{ent}$



(c) ℓ^1, $\boldsymbol{W}_{diag}$



(e) C(ν)-LSPG, $\nu = 0.001$

Extensions

- Applied to 3D RANS equations
- Conclusions explored in the paper:
  - Choice of norm does not have a substantial influence
  - C()-LSPG on a fixed mesh helps for blunt wedge but not for HiFIRE, shows some sensitivity to choice of parameter
  - With grid tailoring, C()-LSPG tends to help for axial force while not improving heat flux

- Future work
  - evaluate with hyperreduction
  - process for choosing
  - integration with goal-oriented ROMs

Method	time (s)	speedup
FOM	7606	-
LSPG	160.5	47.4
C(1.0)-LSPG	49.94	152

Computational time for HiFIRE-1 FOM and ROMs, run on 48 cores.