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STOCHASTIC NEUROMORPHIC CIRCUITS FOR SOLVING MAXCUT

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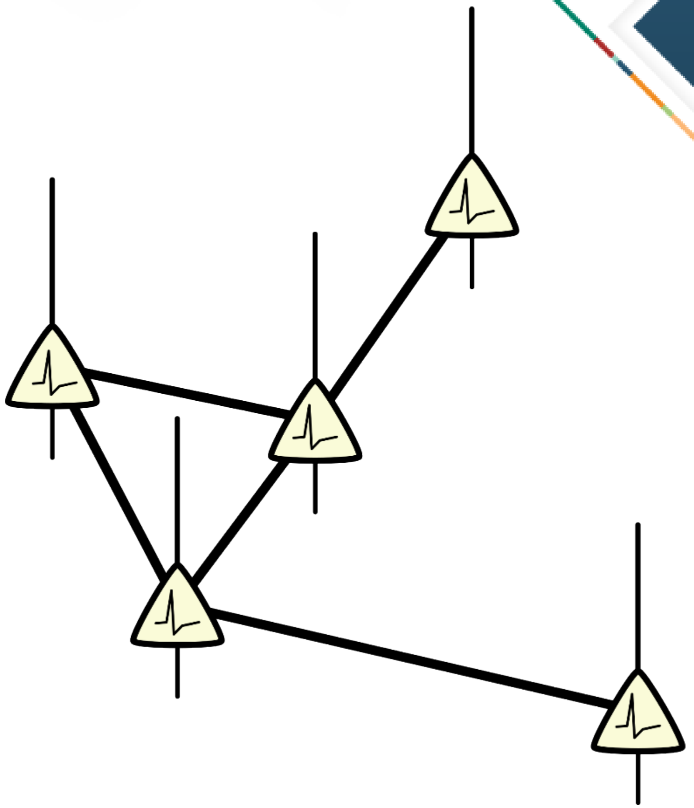
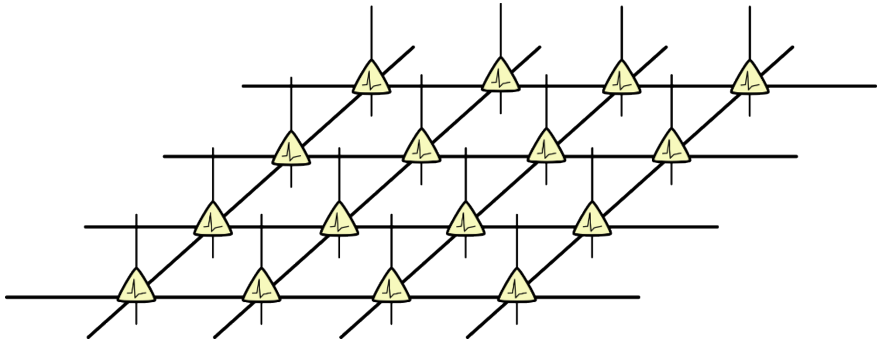
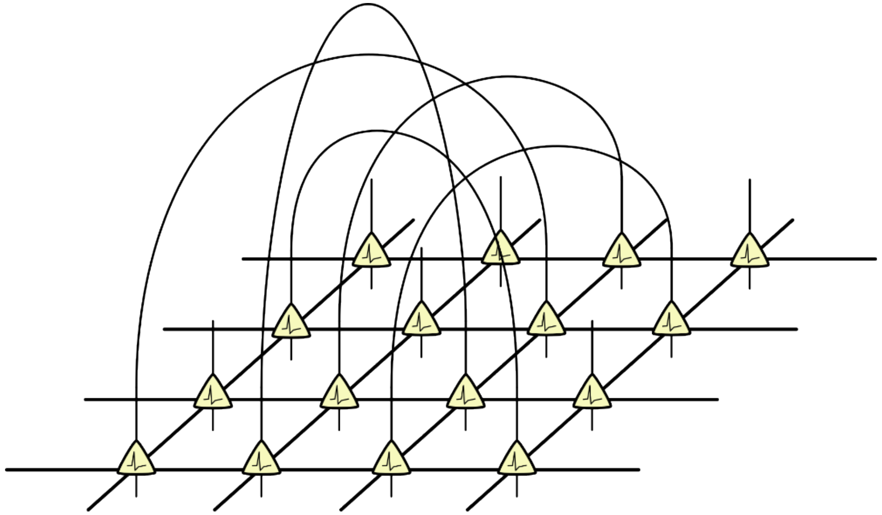
NEUROMORPHIC GRAPH ALGORITHMS

- Shortest paths (Aimone, Ho, Parekh, et al. 2020)
- Minimum spanning trees (Kay, Date, Schuman 2020)
- Subgraph enumeration (Hamilton, Mintz, Schuman 2019)
- Ising model approaches





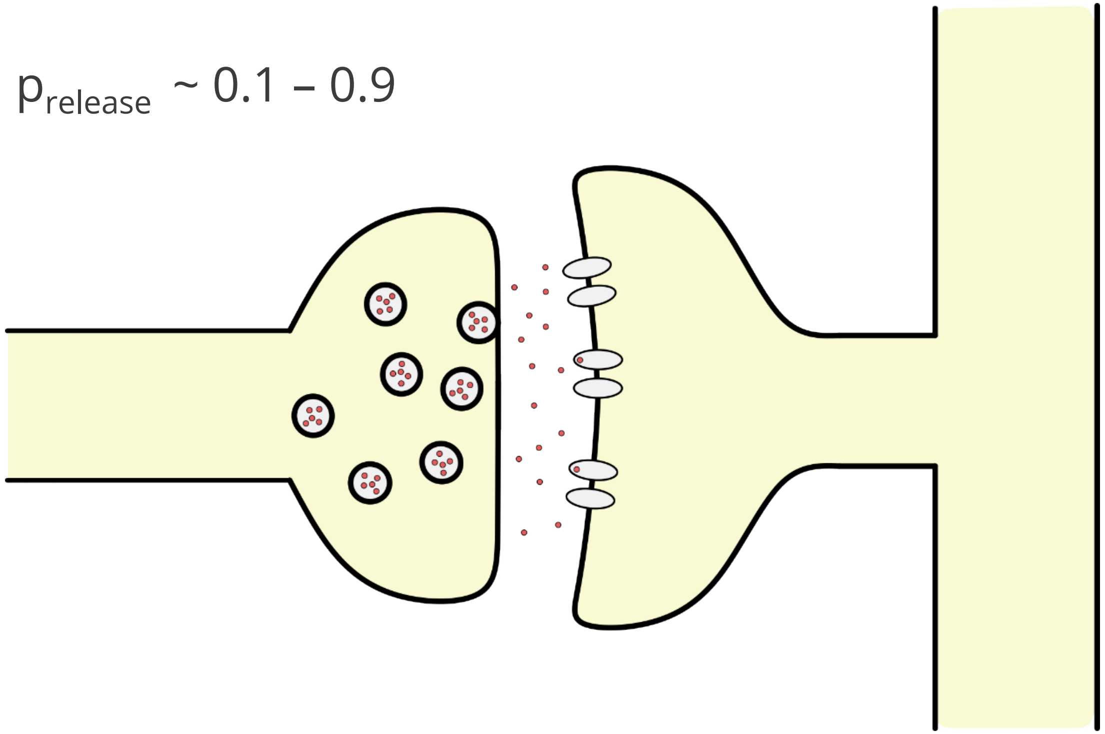
Good

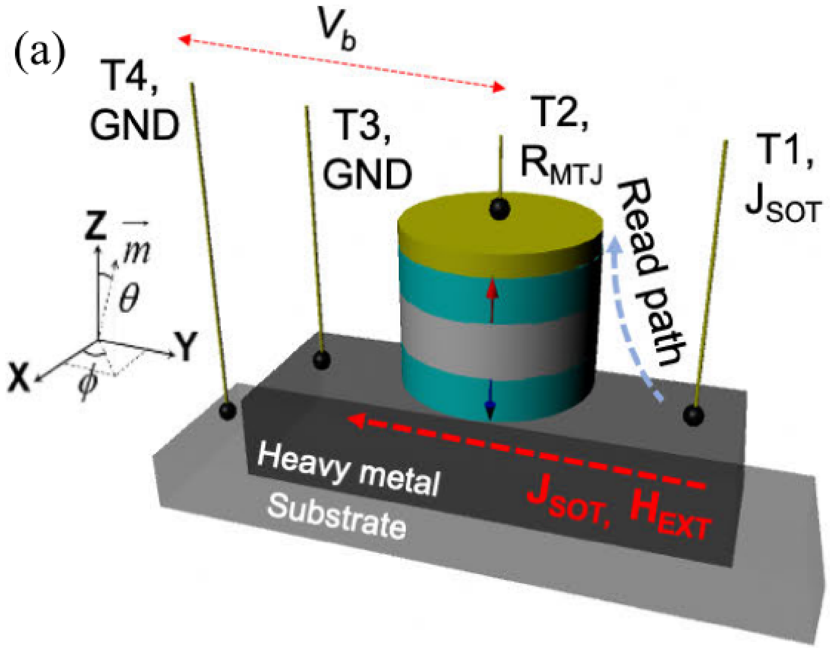


Bad

DENSE PARALLEL STOCHASTICITY

$p_{release}$ ~ 0.1 – 0.9





Liu, Kwon, Bessler, et al. 2022

~10^{15} synapses, ~1 spike/s → ~10^{15} random numbers per second

What are the computational implications of ubiquitous stochasticity in neural systems?

For the rest of this talk, the symbol  stands for an independent source of random bitstreams

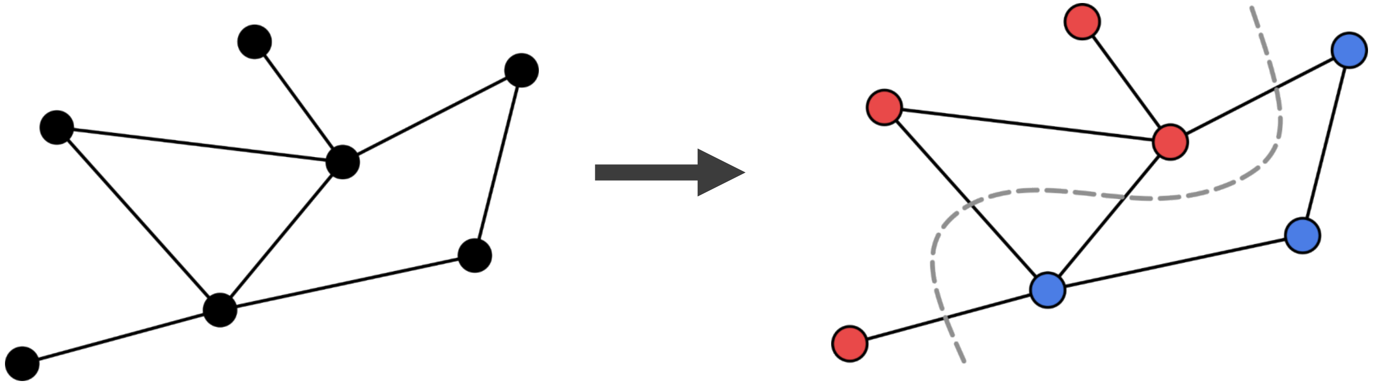
i.e. an independent, fair **coinflip**

GRAPH MAXCUT – GOEMANS WILLIAMSON ALGORITHM

Discrete optimization problem:

maximize $C = \frac{1}{4}\sum_{ij} A_{ij}(1 - y_i y_j)$

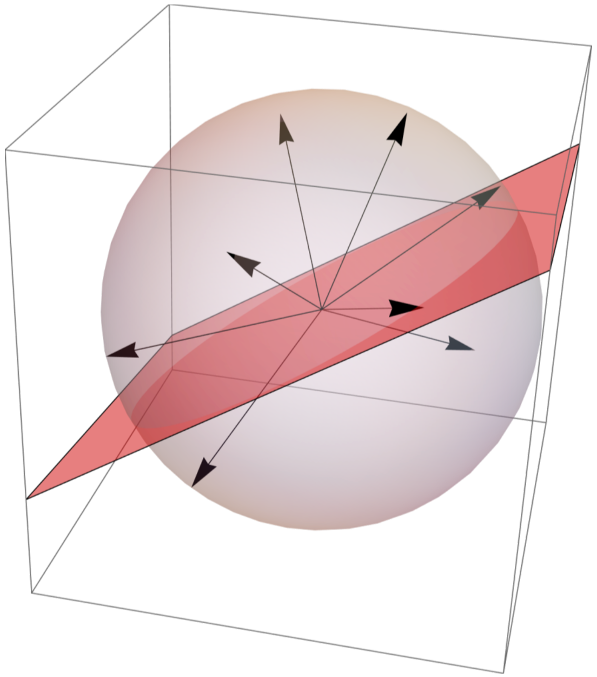
such that $y_i \in \{-1, 1\}$



Replace integer y_i with unit vector

maximize $\tilde{C} = \frac{1}{4}\sum_{ij} A_{ij}(1 - v_i \cdot v_j)$

such that $\|v_i\| = 1$



Choose random unit vector r, sample graph cut:

$$y_i = \mathrm{sgn}(r \cdot v_i)$$

STATISTICS OF LEAKY INTEGRATE-AND-FIRE NEURONS

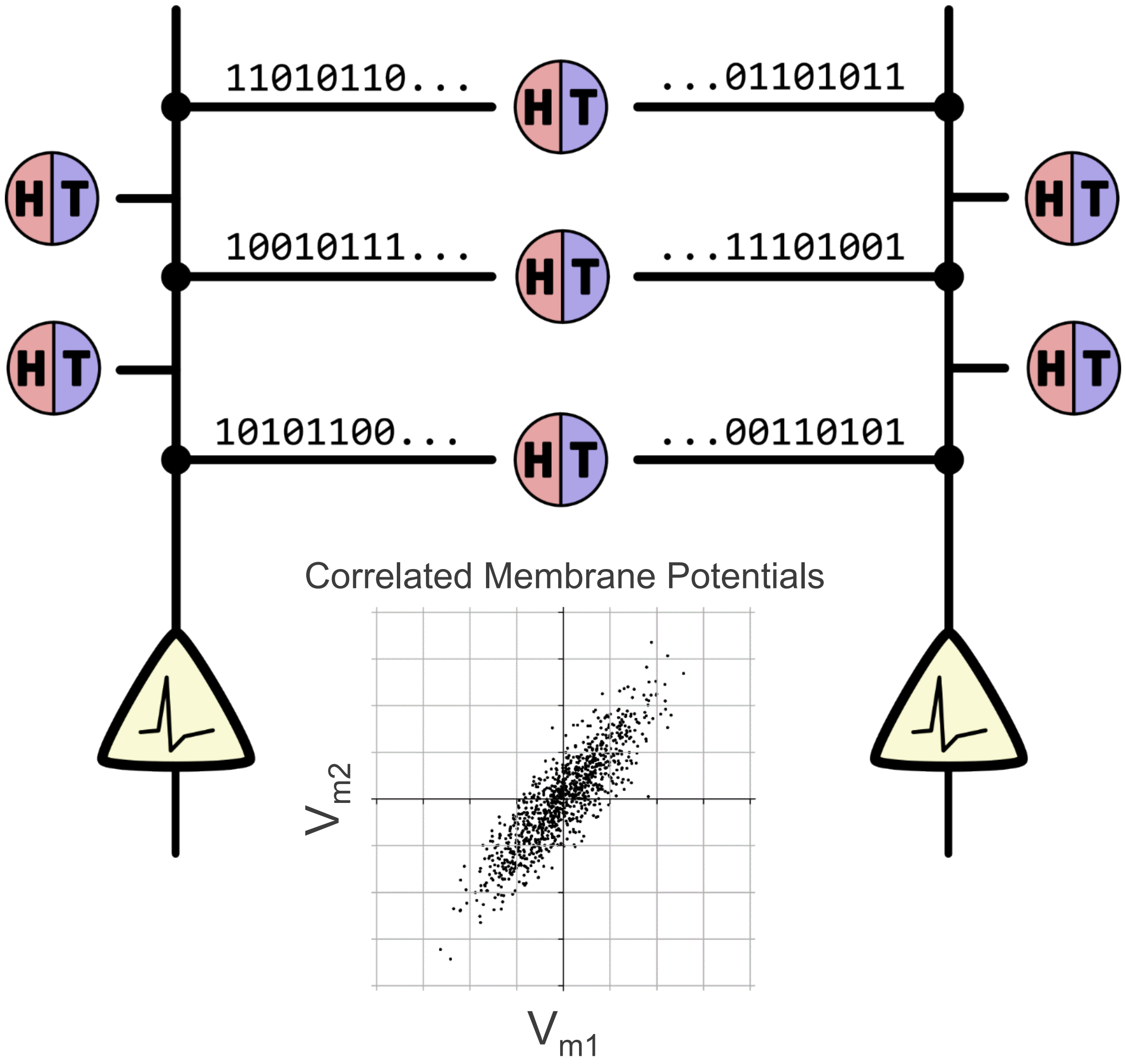
Shared synaptic input induces correlations between LIF membrane potentials

$$C\frac{dV}{dt} = -\frac{V}{R} + \alpha \sum W_j s_j$$

$$Cov(V_i, V_j) = \frac{\alpha^2 R}{2C} W_{ia} W_{jb} Cov(s_a, s_b)$$

$$Cov(s_a, s_b) = \frac{1}{4}\delta_{ab}$$

$$Cov(V_i, V_j) = \frac{\alpha^2 R}{8C} W_i \cdot W_j$$

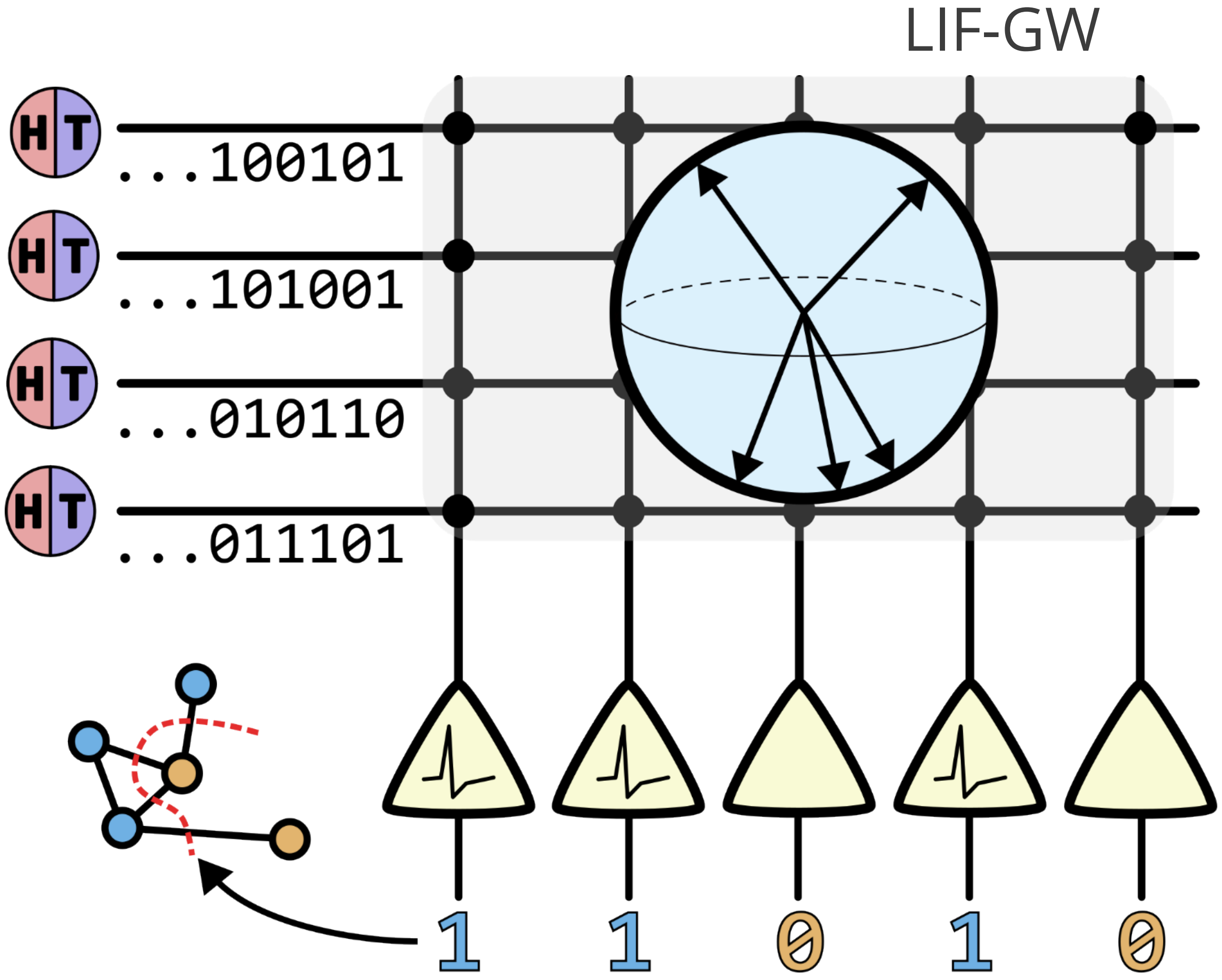


NEUROMORPHIC GOEMANS-WILLIAMSON SAMPLING

Assign one LIF neuron to each graph vertex

Set COINFLIPS -> LIF weights proportional to Goemans-Williamson vectors

$$Cov(V_i, V_j) = \frac{\alpha^2 R}{8C} W_i \cdot W_j$$



“Spiking” threshold turns fluctuations into graph cuts

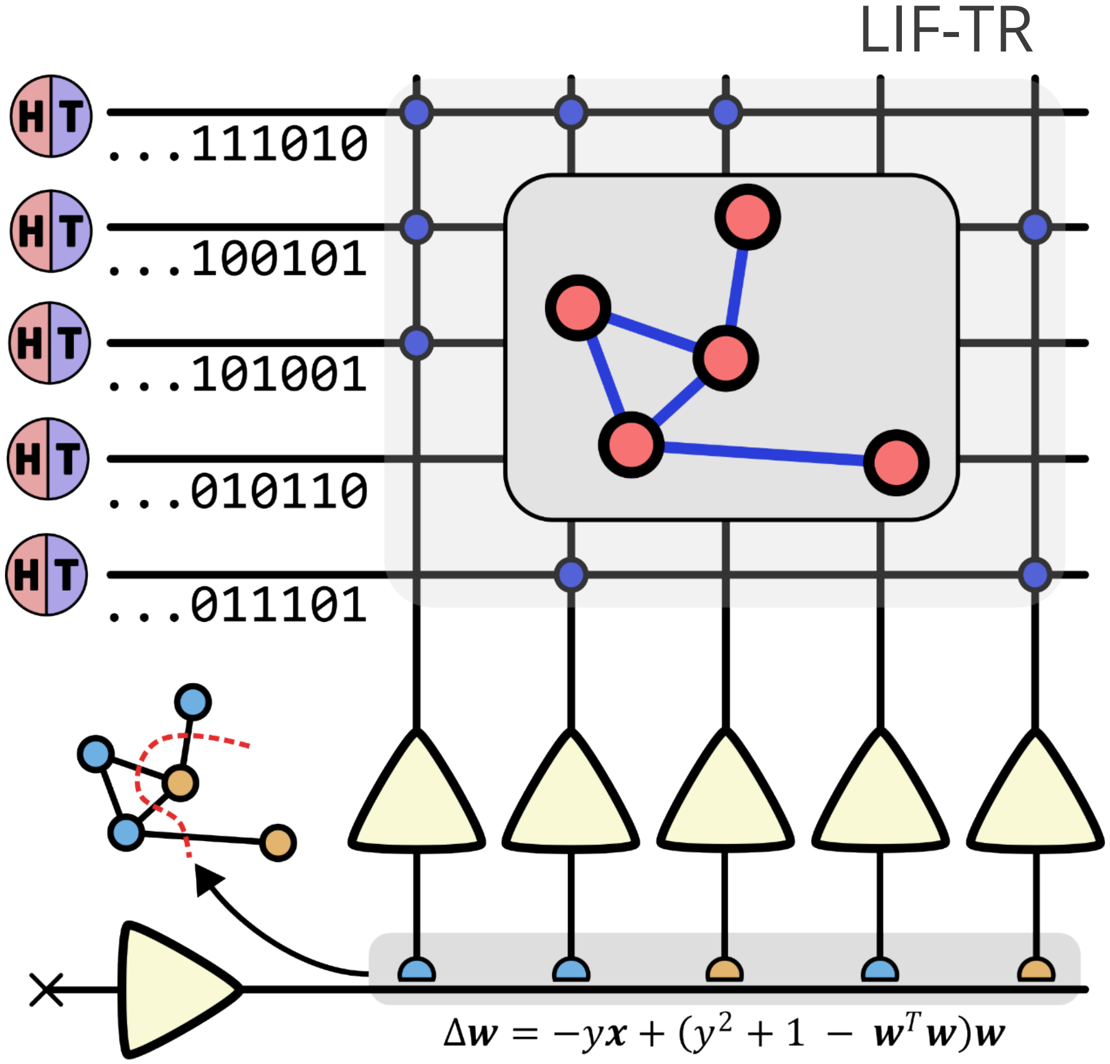
LIF-TREVISAN CIRCUIT

- Oja's *antihebbian* plasticity rule approximates minimum eigenvector:

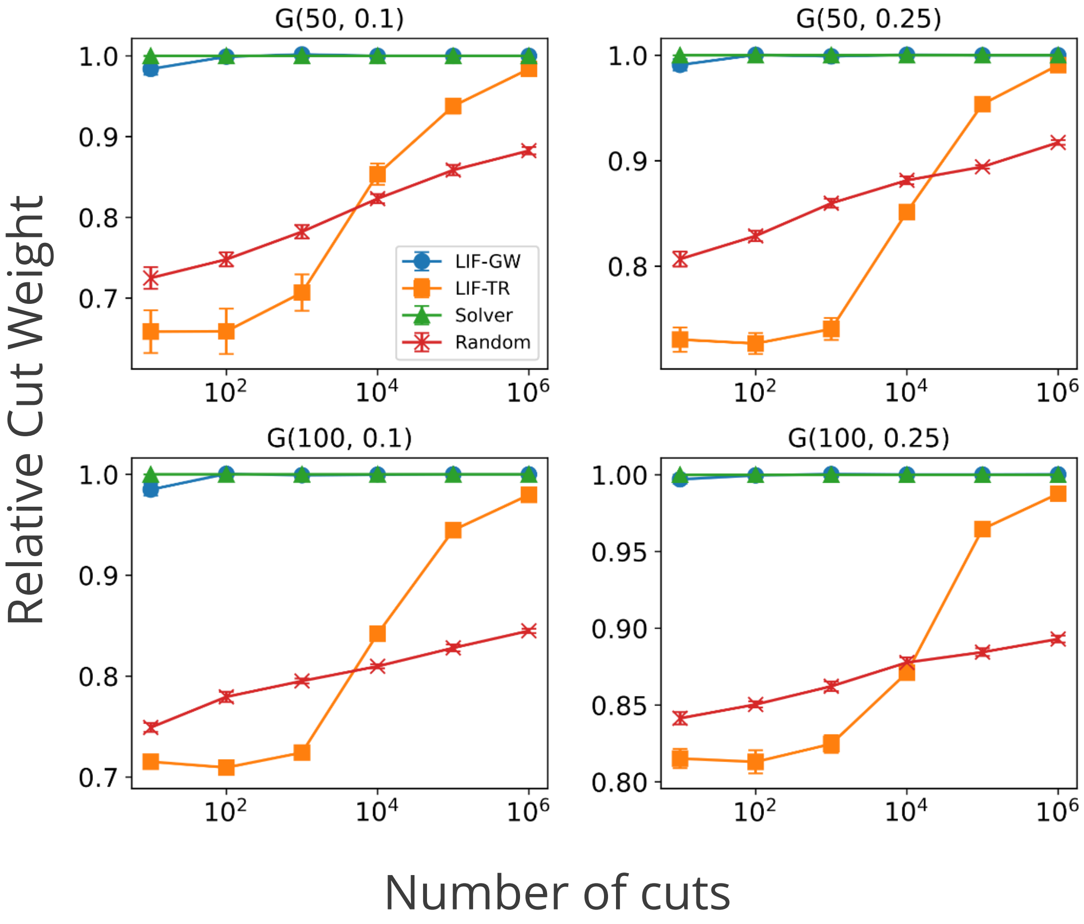
$$\Delta w = -yx + (y^2 + 1 - w^T w)w$$

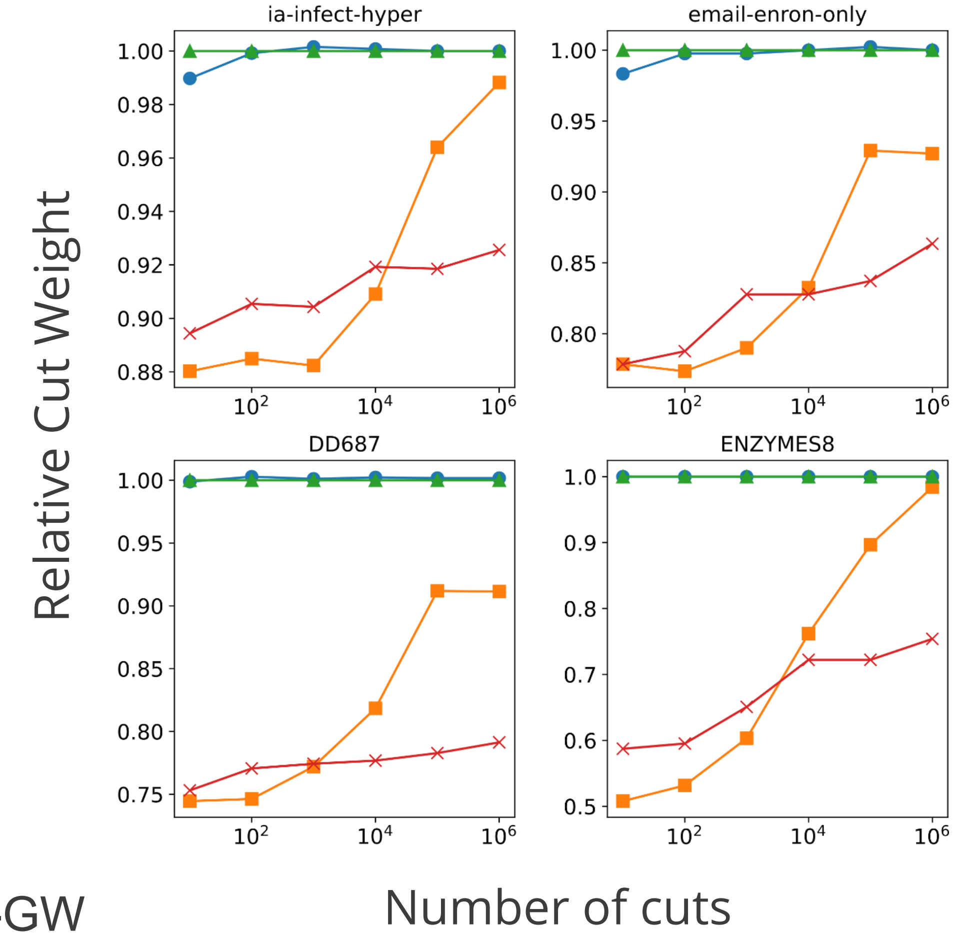
- Correlation element generates correlated activity from random devices

- "Output" neuron computes minimum eigenvector via Oja's antihebbian rule



RESULTS





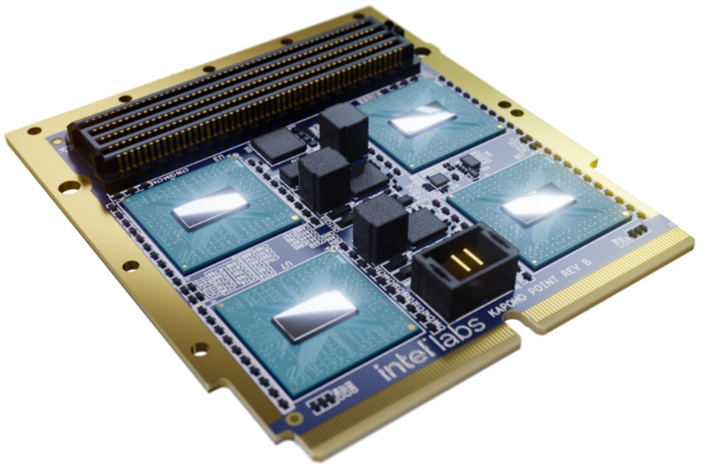
More comparisons in the paper

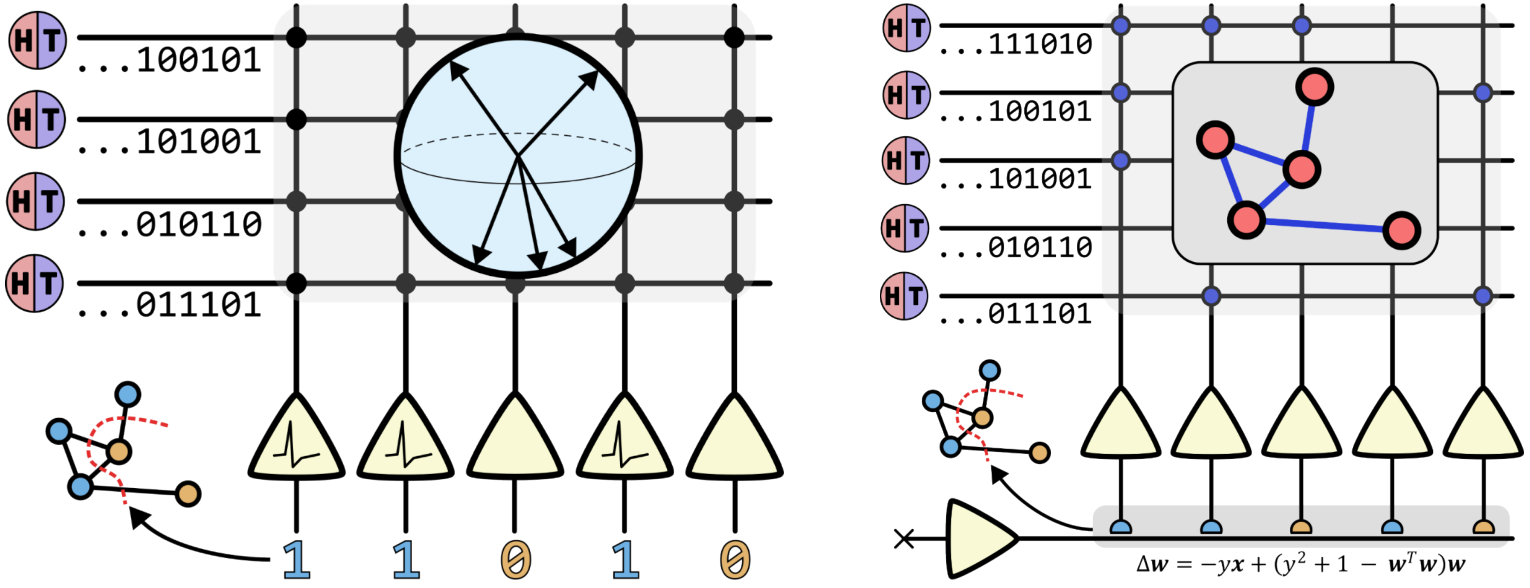
CONCLUSIONS & FUTURE WORK

- Two neuromorphic circuits using random devices to produce computationally-useful correlations

- Statistical approximation algorithms offer a new route to neuromorphic applications

- This approach has architectural advantages compared to other kinds of neuromorphic solvers (e.g. Ising models)





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