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Estimating Latent Fields in Stochastic Dynamical Systems - A Case Study of COVID-19 in New Mexico

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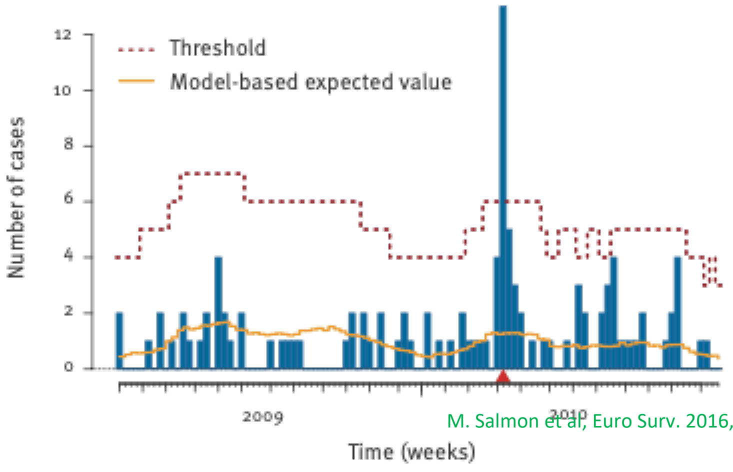
Introduction

- Aim: Devise a method to infer a spatial quantity, the spread-rate of a disease, using limited data of epidemiological dynamics (case-count data)
- Dataset: COVID-19 case-counts in the counties of New Mexico
- Why?
  - Novel outbreaks are detected by analyzing (very noisy) case-count time-series; detection often delayed
    - Reporting errors, stochastic behavior in small populations (sparsely populated areas)
    - Outbreak detections (anomalous change in epidemiological dynamics) often uncertain; wait for case-counts to increase
- Hypothesis: Detect new outbreaks using the latent spread-rate of a disease, not case-counts
- Technical challenges:
  - How to infer the spread-rate field?
  - How to impose the spatial correlations seen in data? What kind of spatial structures do we have?
  - How to compute the spread-rate fast, in a parallel manner?

The practical problem – outbreak detection

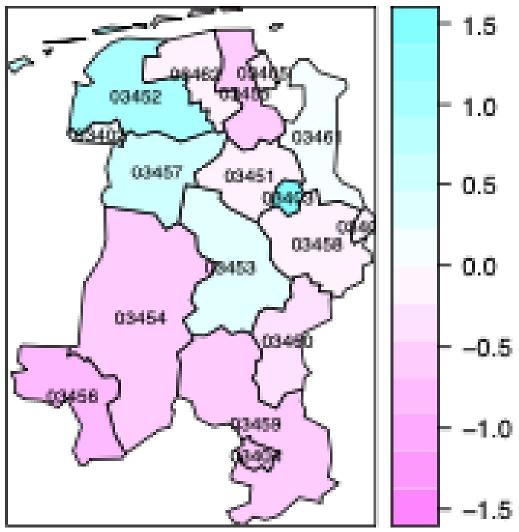
- Two ways – temporal methods (SPC) & spatiotemporal method
  - Data used: case-counts of a disease, disaggregated in time & space
- Temporal methods: Fundamentally, anomaly detection
  - Using historical data, do a 2-week forecast of case-counts & uncertainty bounds (usually 95th percentile)
  - Wait for data; if 3 consecutive days > than 95th percentile, alarm!
- Spatiotemporal methods: Use historical & neighborhood data (autocorrelation) to make forecasts
- Shortcomings
  - Need long time-series data, prefer to be high-count / low variance
  - Not really feasible for novel diseases

Salmonella Montevideo, Germany 2009-2010



M. Salmon et al, Euro Surv. 2016, 21(13)

Measles, Weser-Ems, Germany 2001-2002



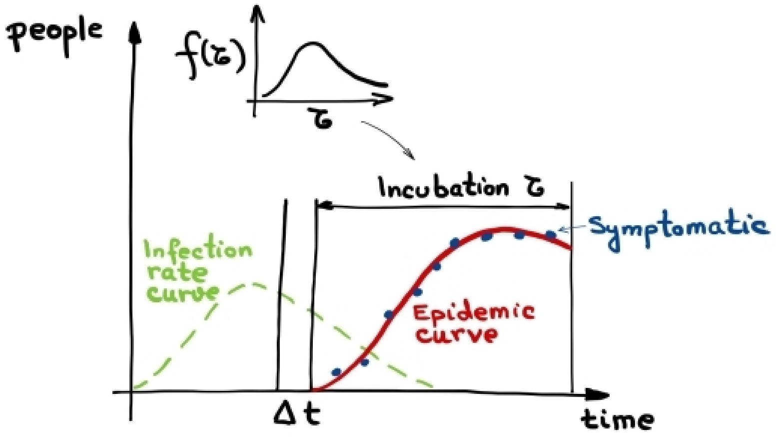
Meyer et al, JSS 2017

Approach

- Hypothesis:
  - Use (latent) spread-rate to detect outbreaks, not case-counts directly
    - Not affected by reporting errors & only depends on human mixing patterns (behavior)
- Inferring the spread-rate
  - Pose and solve an inverse problem for the spread-rate in each NM county
  - Spread-rates in counties are auto-correlated. Devise a Gaussian Markov Random Field (GMRF) model to capture spatial pattern
  - Reformulate a spatiotemporal inverse problem for spread-rates in M counties. Use GMRF to impose autocorrelation
  - Solve with MCMC (for accuracy) and Variational Inference (VI; approximate, but fast); compare estimated spread-rates
- Test: Can disease detection be done with spread-rates, even the approximate VI one?

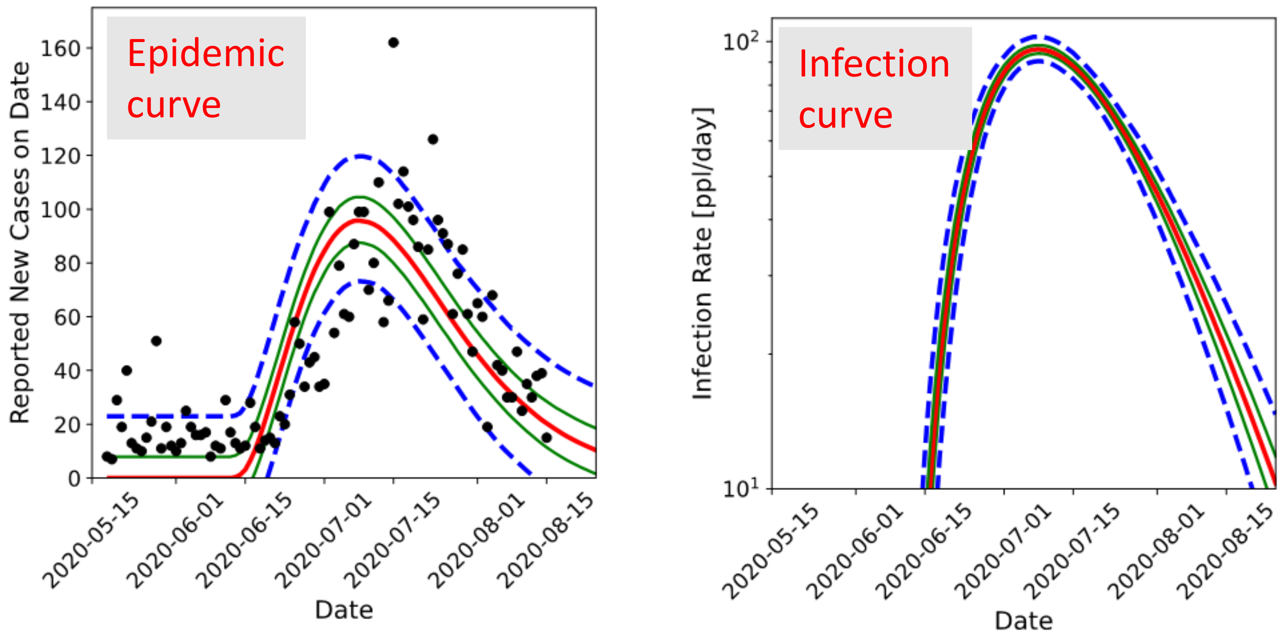
Formulating the temporal problem

- Assume $q(t;\ \theta)$, # of people infected on day t, in Area A
- $y_t^{(obs)}$: Case-counts from a location; $y_t(\theta)$: Predictions by model $M(t;\ \theta)$
- Convolve with incubation period for modeled cases
  - $y_t(\theta) = \int_{t_0}^{t} q(\tau - t_o;\ \theta) f_{inc}(t - \tau) d\tau$
- Infer $p(\theta | y_t^{(obs)})$ via Bayesian inference, using $y_t^{(obs)}$ & $y_t(\theta) = M(t;\ \theta)$
  - Provides (infers) the latent spread-rate curve



- Likelihood assumes Gaussian errors; parameter vector θ is 4-dimensional
- Inference can be done with MCMC, VI etc.
  - 4-dimension inference is easy
- Forecasting: $y_{t*}, t^* > T$ conditioned on $p(\theta | y_t^{(obs)})$

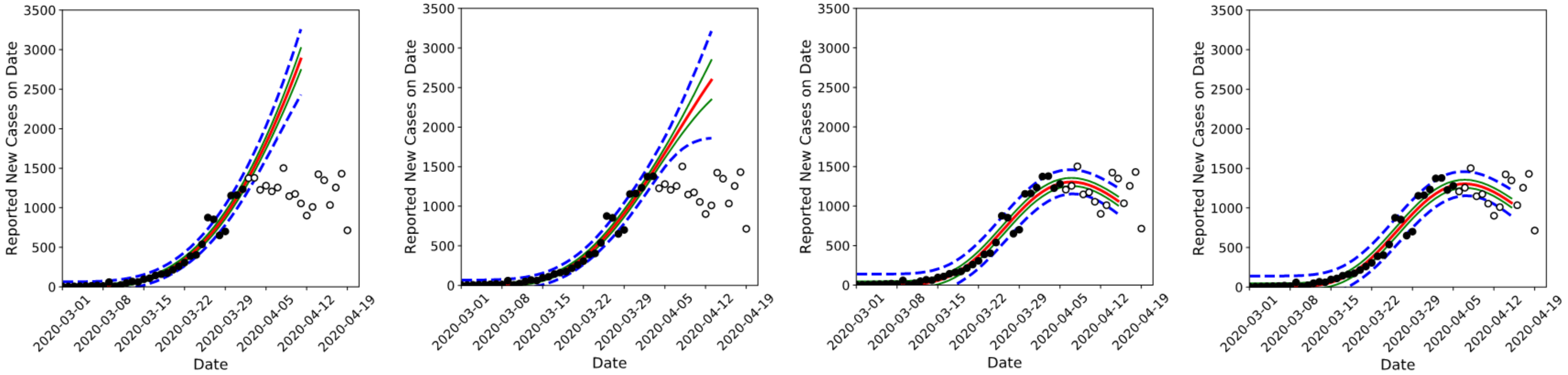
Bernalillo



Detecting change in epidemiological dynamics

- Model allows estimation of (past) infection-rate; forecasting with it assumes that it will not change drastically
- If forecasts are wrong, it implies a change in spread-rate (new variant, changes in human behavior etc.)
- Our insight: This could be formalized into a rigorous outbreak detector / change in epidemiological dynamics

Forecast on April 1 | Forecast on April 3 | Forecast on April 5 | Forecast on April 7



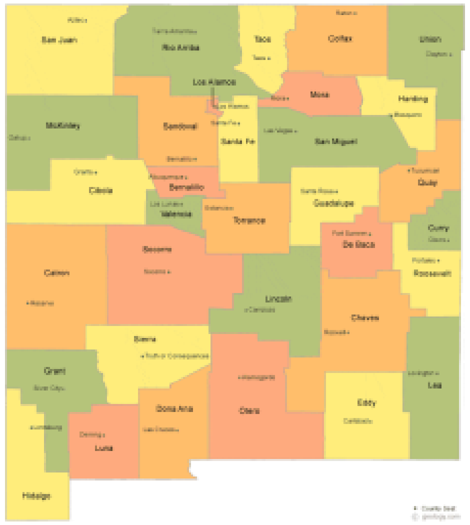
Flattening CA's curve; first lockdown in March 2020

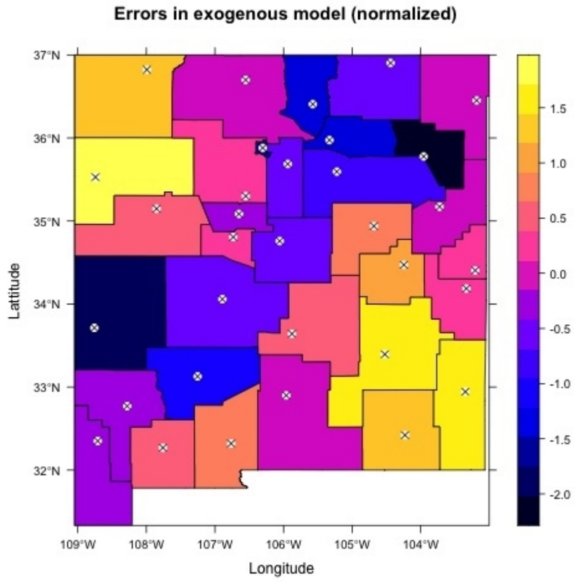
The spatiotemporal problem

- Temporal estimation problem: The posterior distribution
  - $p\left(\theta \middle| y_t^{(obs)}\right) \propto \frac{\left(y_t^{(obs)} - M(t;\theta)\right)^T \Gamma^{-\frac{1}{2}} \left(y_t^{(obs)} - M(t;\theta)\right)}{|\Gamma|^{\frac{1}{2}}} p_{prior}(\theta)\,, \Gamma = \text{diag}(\sigma_A + \sigma_M y_t^{(obs)})$
  - θ is 4-dimensional; the inversion is 6 dimensional
- The spatiotemporal estimation problem:
  - $y_t^{(obs)}$ contains case-counts for all times till t, from all areas $A_j, j = 1 \cdots J$
  - Γ spans over all time t, and all A_j and must enforce all spatial autocorrelations. What is it?
- Modeling the spatial problem:
  - Is there any spatial correlation? What form does it take?
  - What does Γ look like in a spatiotemporal inversion problem?

Spatial modeling

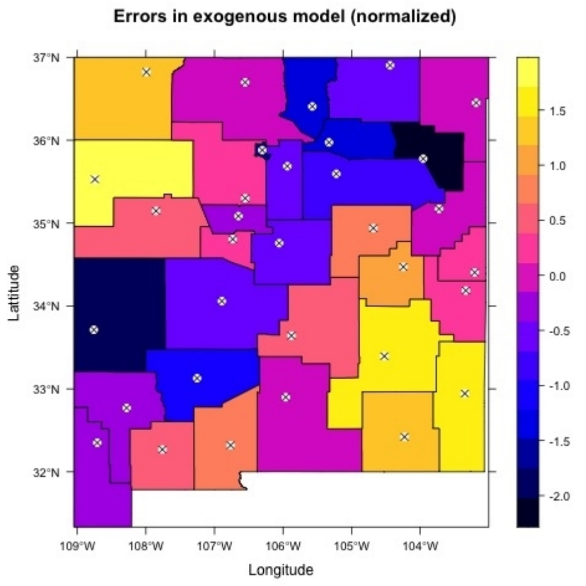
- Created a simple regression model for case-counts in NM
  - $Y = w_0 + \sum_k w_k \phi_k + \epsilon, \epsilon \sim N(0, \zeta^2)$
  - ϕ_k: exogenous covariates of epidemiology/risk factors (population, socioeconomic conditions, transport connectivity etc.)
  - ϵ shows spatial correlations in epidemiological dynamics not explained by exogenous covariates
- Clear spatial pattern
  - Rio Grande valley (inhabited; blue) shows similar ϵ
  - Further out, red counties have similar behavior
  - Northwest / Southeast counties show max ϵ
- To do:
  - Clearly, clustered, but need to get significance via a statistical test
  - Need to capture this pattern in a GMRF model





Γ for GMRF

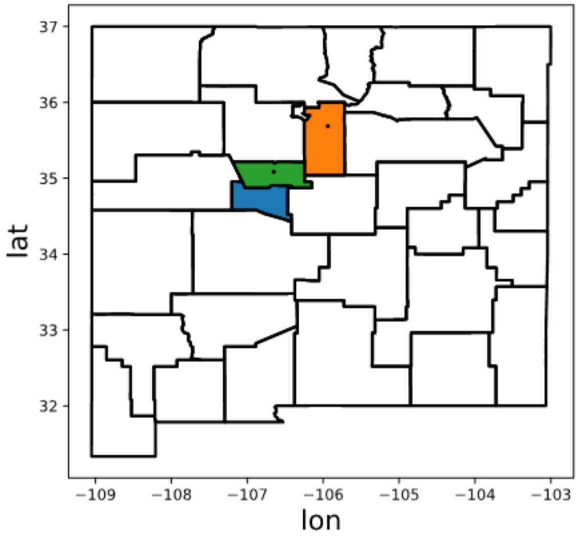
- Existence of clusters determined by Moran's I test
- How far does autocorrelation extend in the (large) counties of NM?
  - Also determined by Moran's I test, computed with 1-hop and 2-hop neighborhoods
  - **Finding:** autocorrelation is only between nearest neighbors
- **Precision matrix** $\Gamma^{-1} = \frac{1}{\tau^2}[I - \lambda W]$, W is the nearest-neighbor connectivity matrix, λ is the strength of spatial autocorrelation



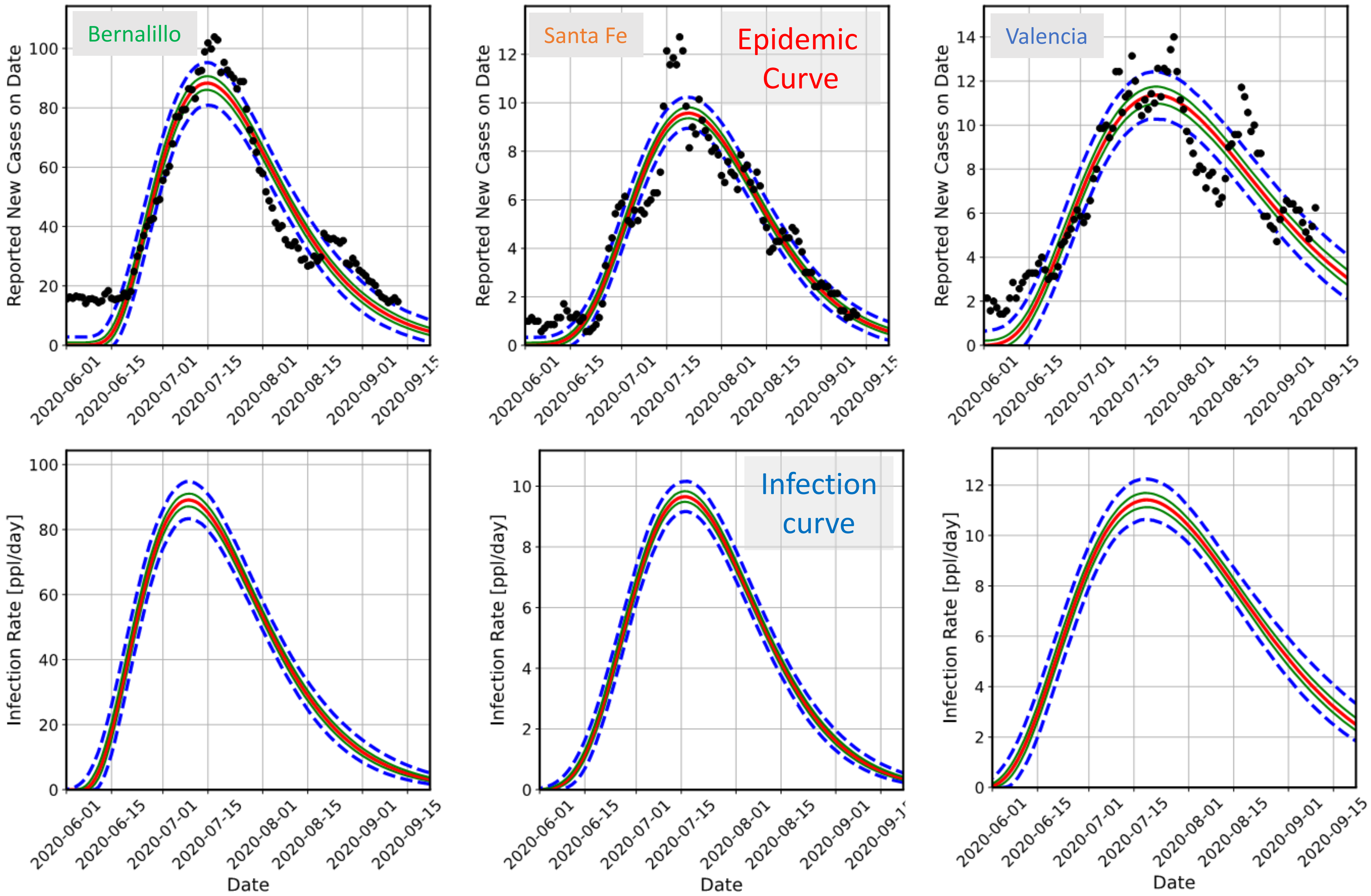
- **Posterior:**
  - $p\left(\Theta \middle| Y_t^{(obs)}\right) \propto \prod_t \frac{\left(\psi_t^{(obs)} - M(t;\Theta)\right)^T \Gamma^{-\frac{1}{2}} \left(\psi_t^{(obs)} - M(t;\Theta)\right)}{|\Gamma|^{\frac{1}{2}}} p_{prior}(\Theta), \Theta = \{\theta_j\}, j = 1 \cdots J$
  - $\psi_t = M(t;\ \Theta)$ predicts case counts on Day t
  - Θ contains $4 \times J$ parameters to infer, along with (τ, λ); high-dimension even for $J = 3$

Results using MCMC

- Estimation with 3 counties

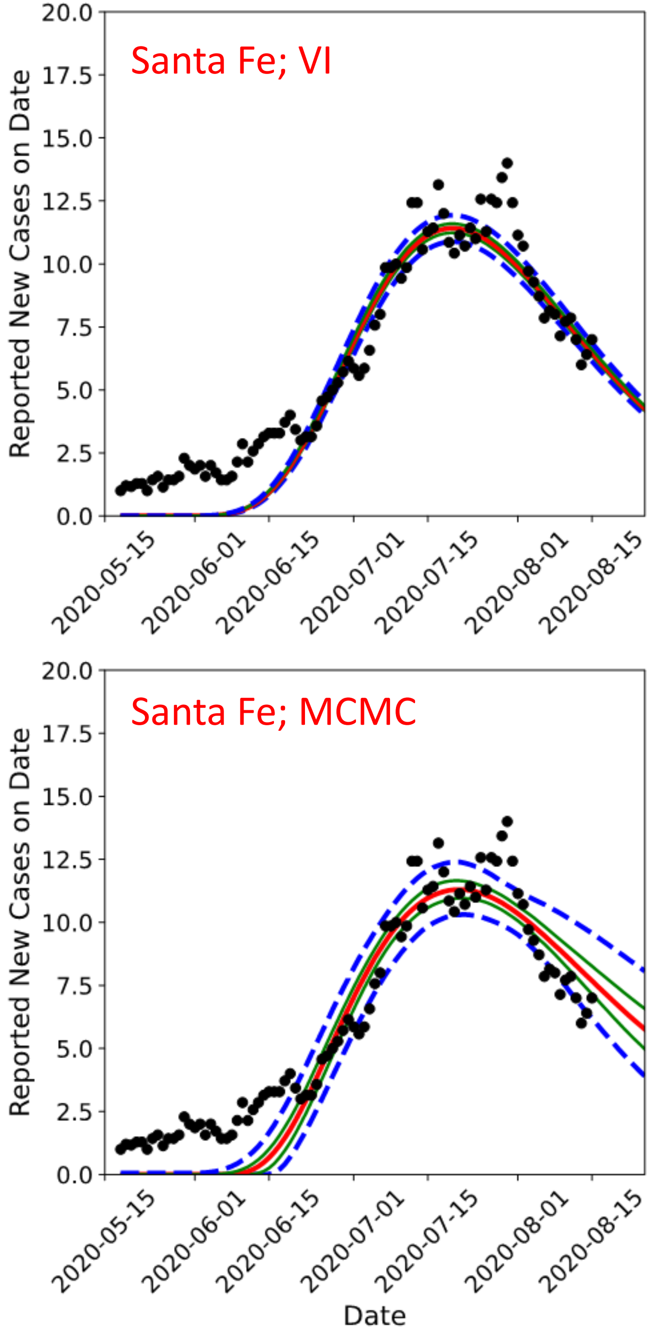


- Provides infection-rate curve too

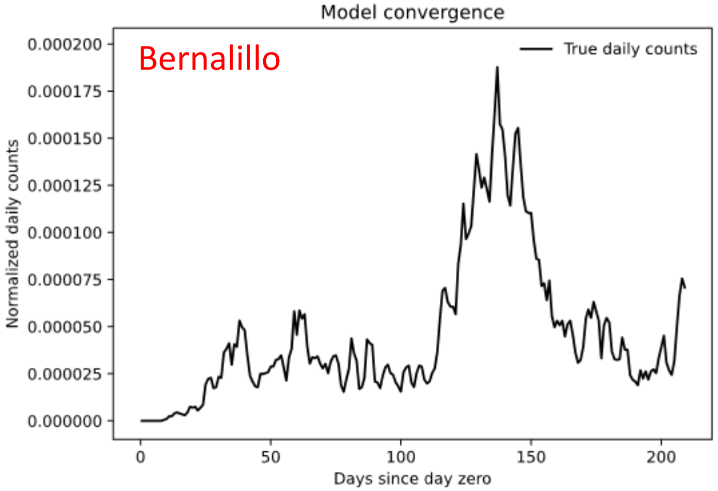


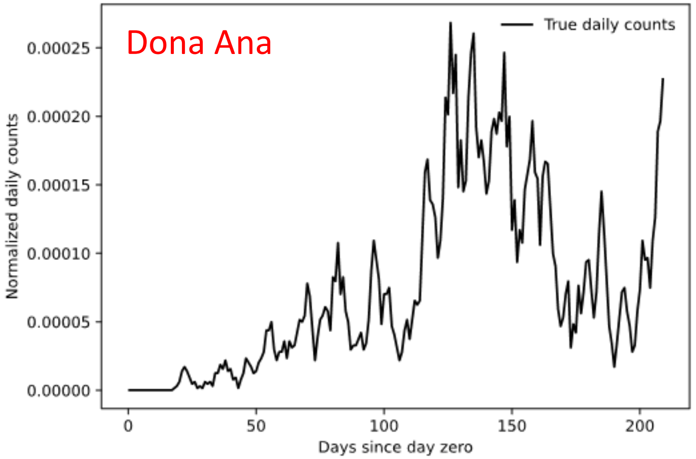
Speeding up with VI

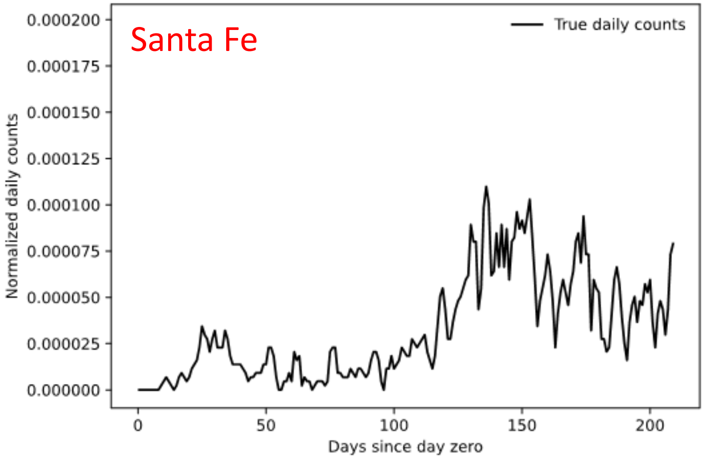
- Curse of dimensionality: Dimensionality of the inverse problem grows as ~ *4J*, J = # of areal units
  - For NM, J = 33. Too high-dimensional for MCMC
- Solution: mean-field variational inference
  - Approximate $p\left(\Theta \middle| Y_t^{(obs)}\right)$ as a multivariate Gaussian with a diagonal covariance
  - Estimation now implies estimating $(\overline{\theta_k}, \mathrm{Var}(\theta_k))$, *k = 1…K (=4J)*
  - Test on Santa Fe county
- Mathematical development
  - Objective function (likelihood) to be maximized to estimate $(\overline{\theta_k}, \mathrm{Var}(\theta_k))$
  - Parallel iterative methods to optimize (Adams)
- Effect of approximation: VI underestimates uncertainty
  - Much faster & already parallelized



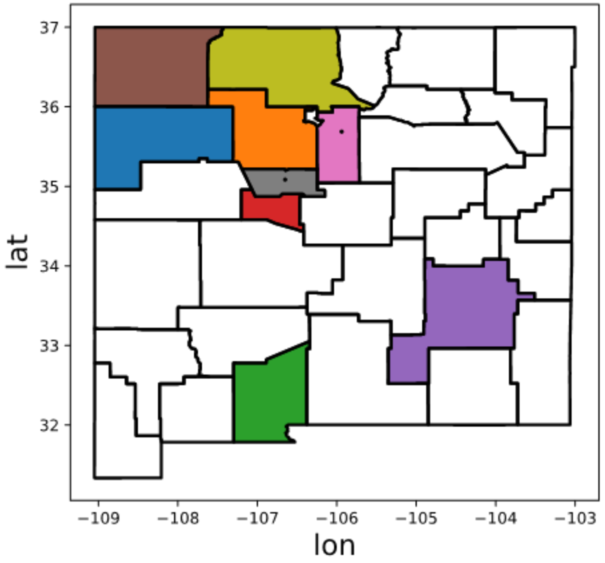
9-county inference with VI

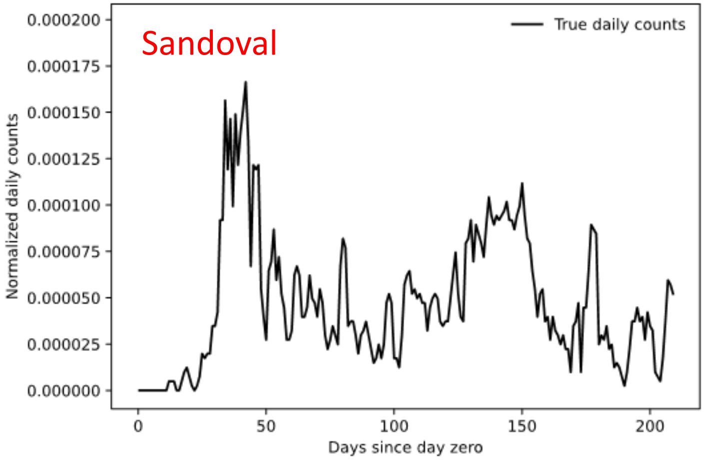


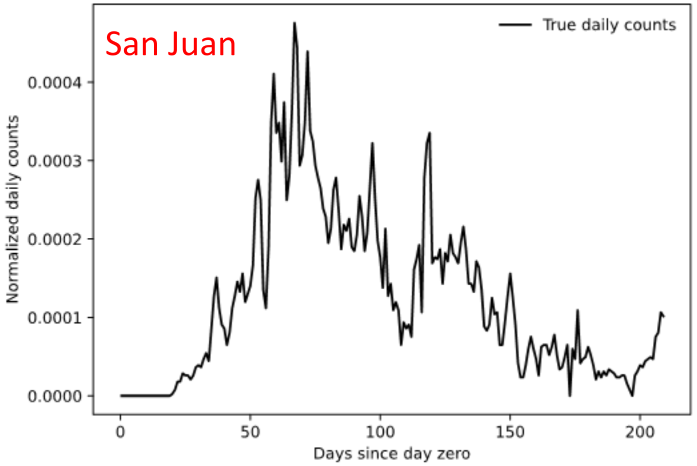




Bernalillo, Santa Fe, Valencia, Sandoval, McKinley, San Juan, Rio Arriba, Chaves, Dona Ana

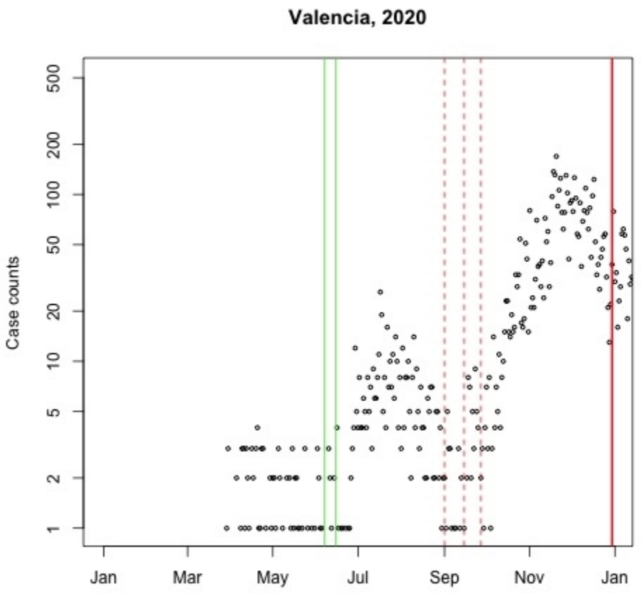


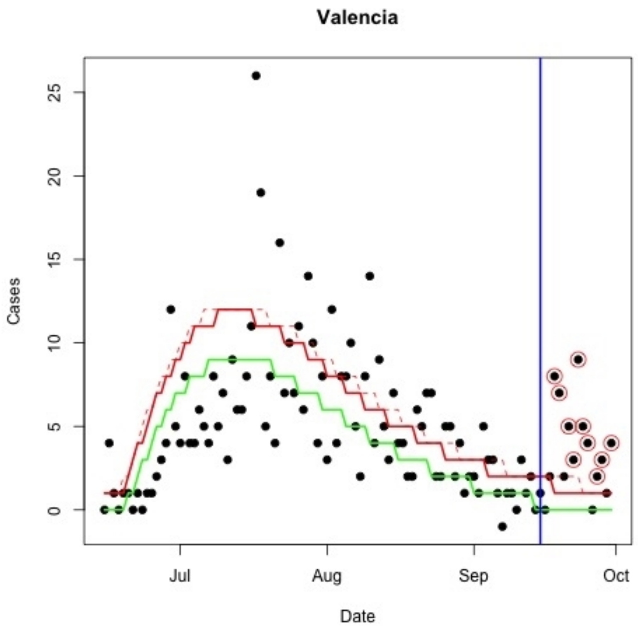


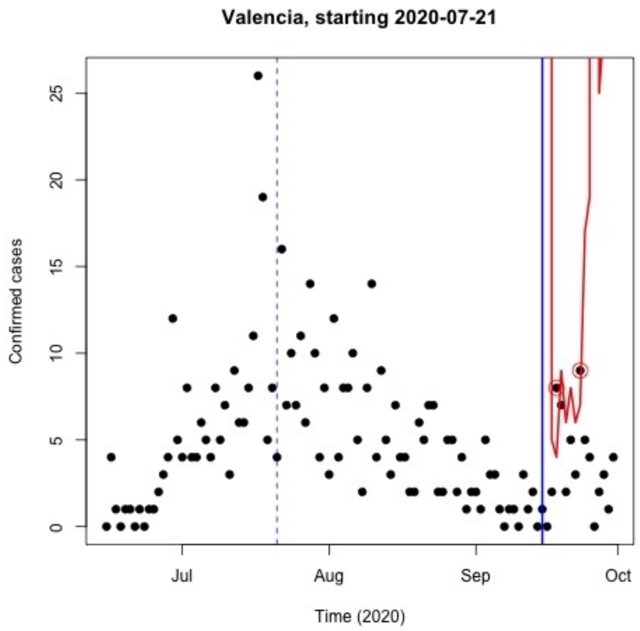


Detecting the fall wave, 2020

- Detect the arrival of the Fall wave of 2020 in Valencia county
- Process:
  - Infer spread-rate using data till Sept 15th ; forecast ahead w/ 95th percentile; detect outliers
  - Redo with negative binomial fit (RKI; Hohle & Paul, 2008)
- Result:
  - Our method detects the start of the fall wave; RKI method fails
  - RKI's time-series method needs long training data (>2 months)
  - We exploit knowledge of incubation period & parameterized infection-rate profile







Conclusions

- We have developed a VI method to infer a latent field, give indirect observations
  - Our case: latent infection-rate or spread-rate field, from case-count data
  - Requires a forward problem (epidemiological problem); spread-rate is smooth in space-time
- Algorithmic innovations: Estimation is high-dimensional; MCMC not up-to-the-task
  - Requires a Gaussian Markov Random Field model to spatially regularize (enforce spatial auto-correlation)
  - Estimation performed using Variational Inference
  - Tested on the counties of New Mexico, COVID-19 data
- Final use: Detect arrival of Fall wave in NM, posing it as an anomalous epidemiological behavior
  - Detect better than conventional detectors that employ case-counts natively
  - Better detection artefact of exploiting a smooth infection-rate, unaffected by reporting errors etc.

Acknowledgements

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