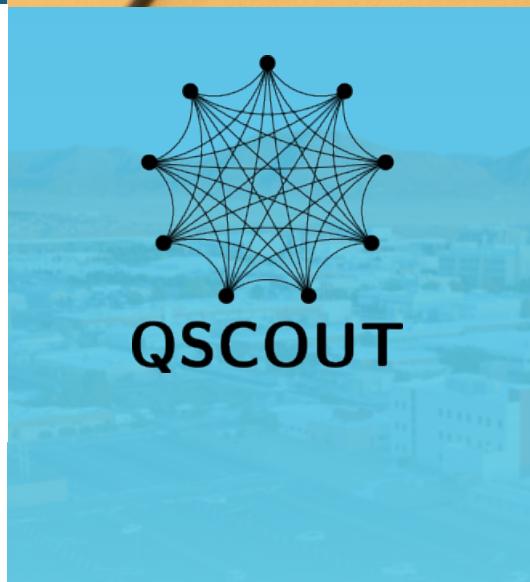
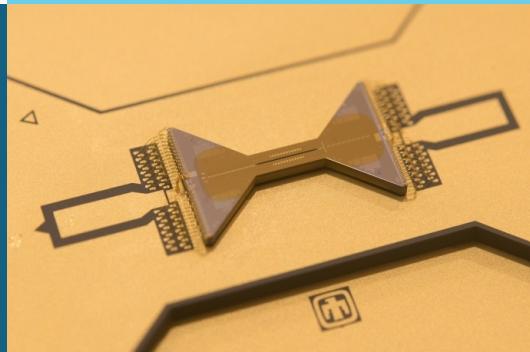
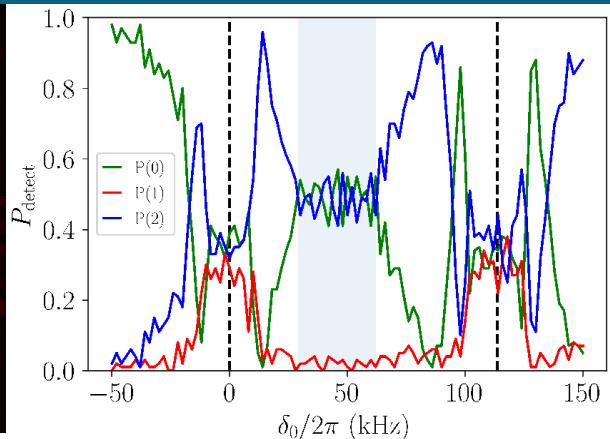
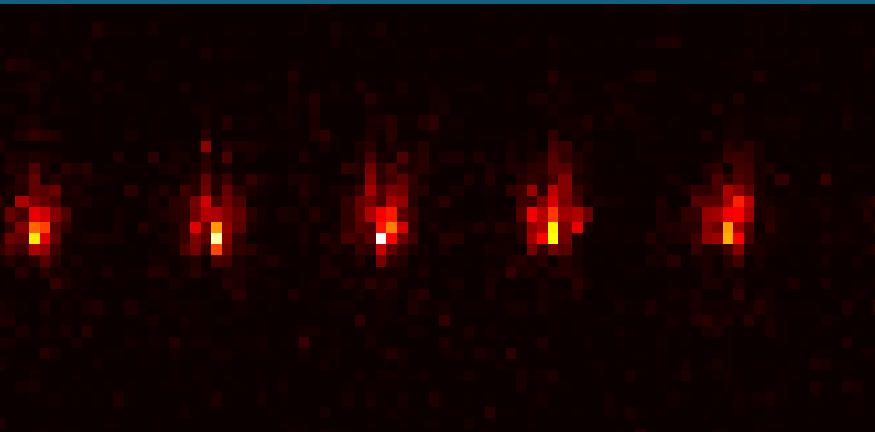




# Demonstration of Mølmer–Sørensen Gates Robust to $\pm 10$ kHz Motional Frequency Error



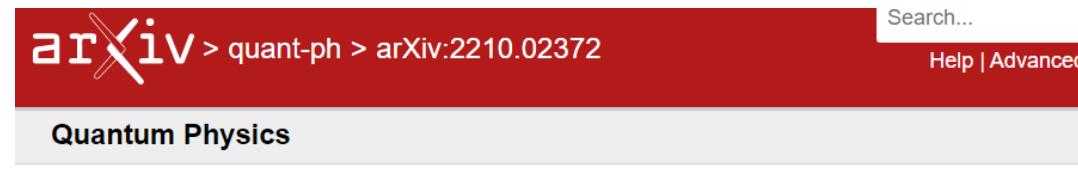
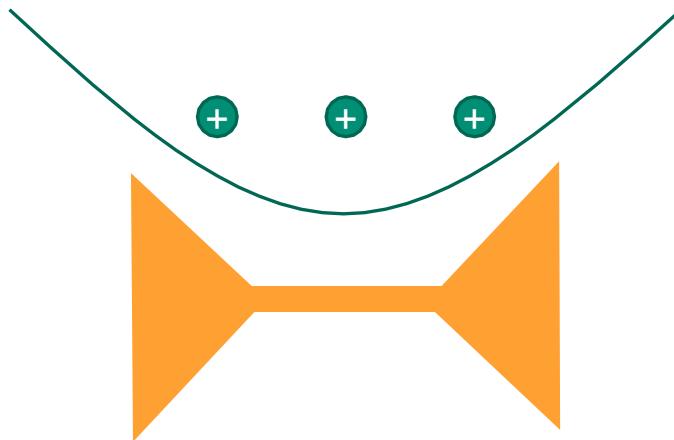
Presenting: Matthew Chow<sup>1,2,3</sup>

Team: Brandon P. Ruzic<sup>1</sup>, Ashlyn D. Burch<sup>1</sup>, Megan K. Ivory, Dan S. Lobser<sup>1</sup>, Melissa C. Revelle<sup>1</sup>, Christopher G. Yale<sup>1</sup>, Susan M. Clark<sup>1</sup>

Overview: We develop and demonstrate an entangling gate on trapped ions that is robust to a dominant noise source.



## Ion RF Paul Trap



[Submitted on 5 Oct 2022]

### Frequency-robust Mølmer-Sørensen gates via balanced contributions of multiple motional modes

Brandon P. Ruzic, Matthew N. H. Chow, Ashlyn D. Burch, Daniel Lobser, Melissa C. Revelle, Joshua M. Wilson, Christopher G. Yale, Susan M. Clark

Critical challenge: Error mitigation for entangling gates



Design pulses for robust operation

Trapped ions show great promise for quantum computing. The entangling gates suffer from technical noise.

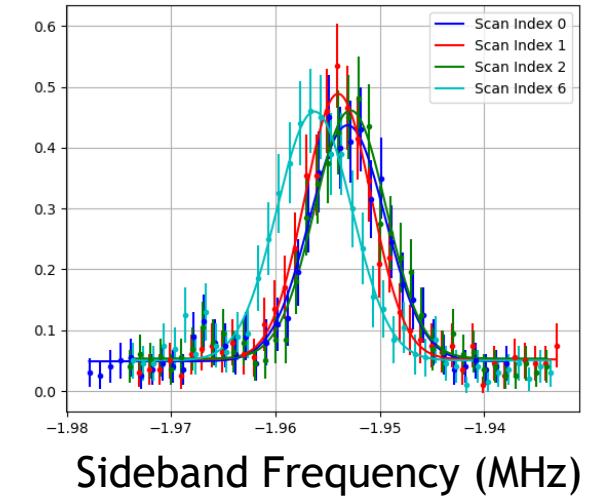


Trapped ions show great promise

- SPAM  $> 0.99999$   
[Zukas 2021]
- Single qubit rotations  $> 0.9999$   
[Ballance 2016, Gaebler 2016]
- Peak entangling gate  $\geq 0.999$   
[Ballance 2016, Gaebler 2016]

... however

- Motional frequency drifts impact the entangling gate.
- It's hard to scale to many ions.

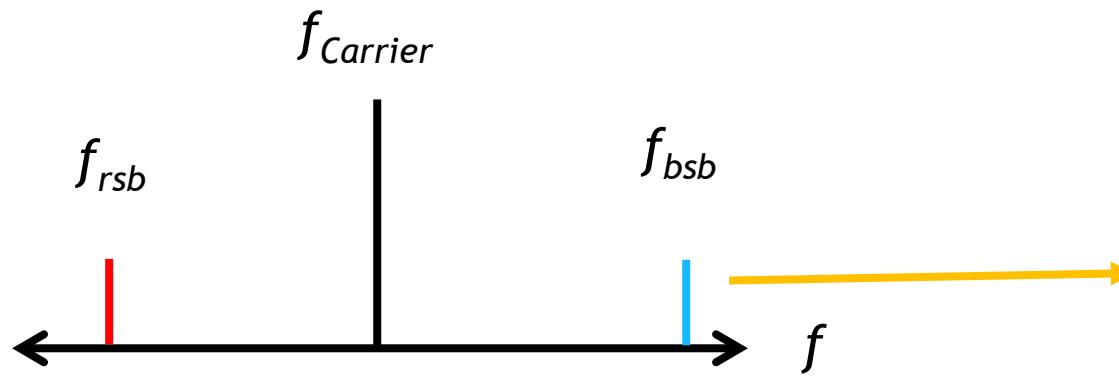
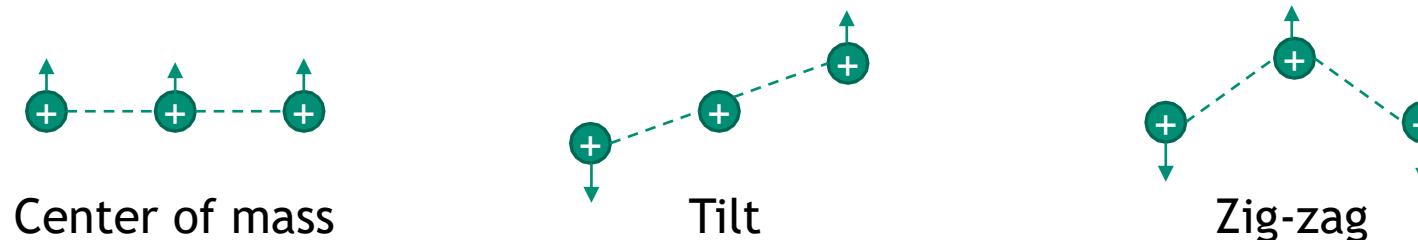


Solution? Develop gates that are robust to trap frequency drift that can be implemented on long chains.

# Shared motional modes mediate entangling interactions



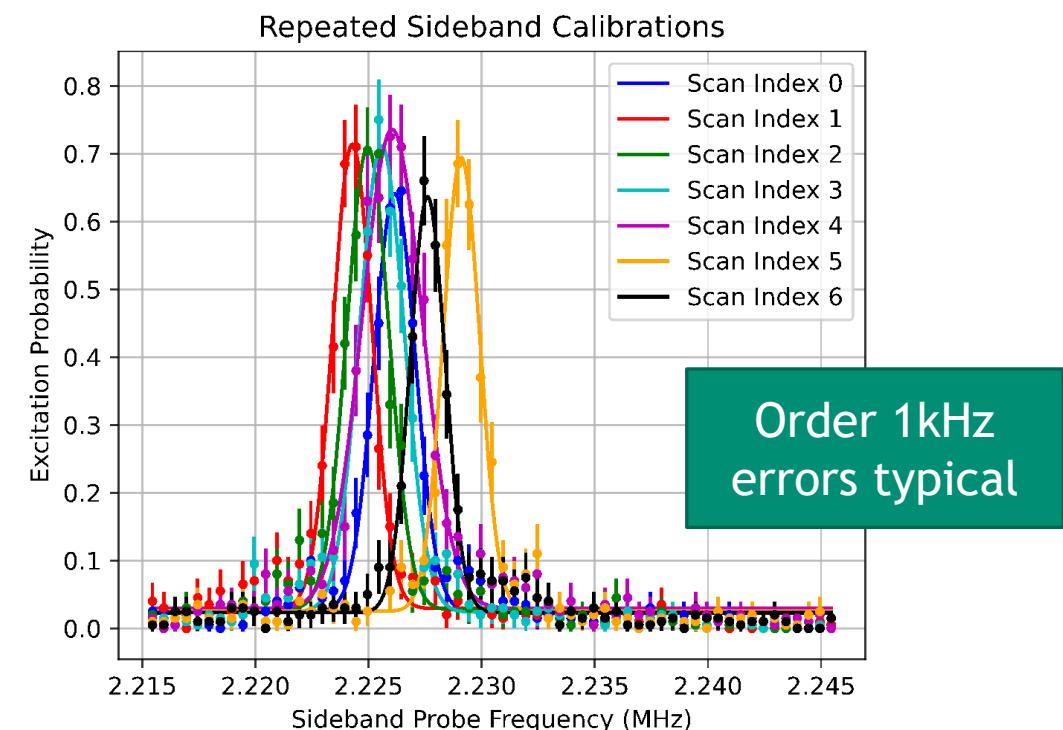
E.g: 3-ion normal modes of motion



Red sideband:  $n-1$   
Remove a phonon

Blue sideband:  $n+1$   
Add a phonon

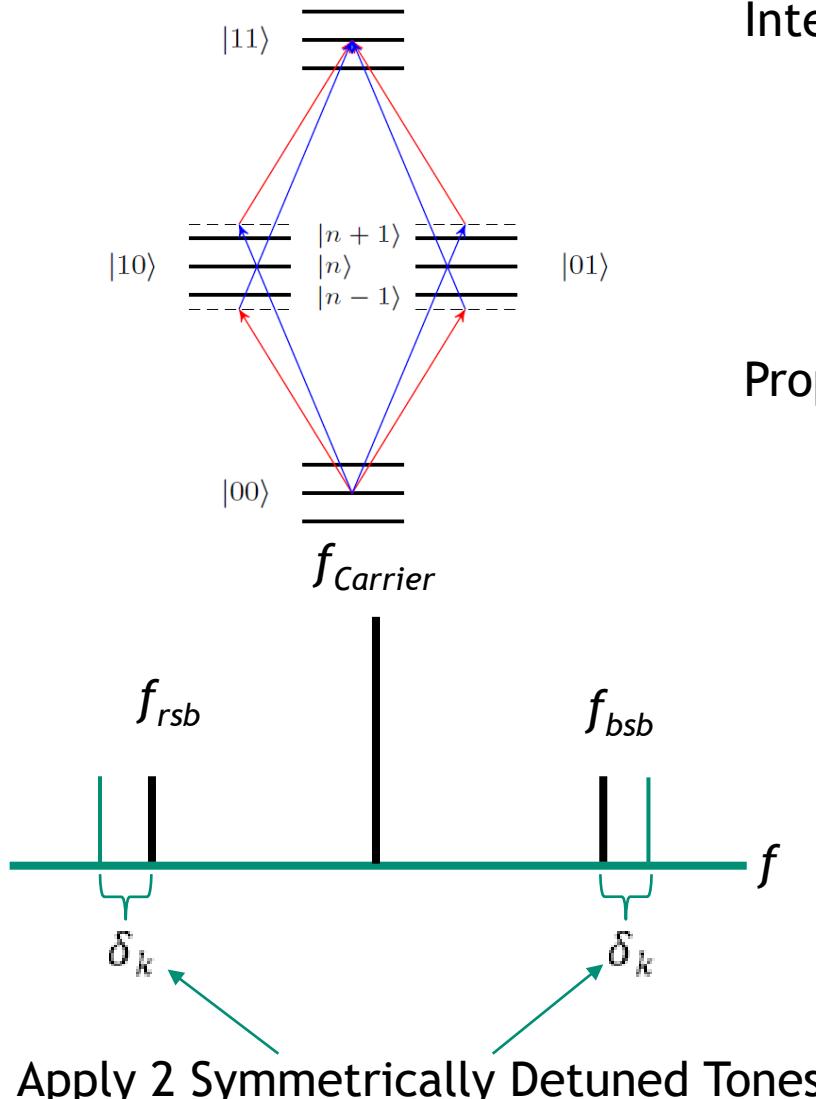
- Motional sidebands at the normal mode trap frequencies
- Coulomb interaction couples ions together
  - Vibrational levels act as “bus” connecting qubits



... however

Motional frequency drifts impact the entangling gate.

# The Mølmer-Sørensen (MS) interaction drives spin entanglement and coherent displacement.



Interaction Hamiltonian:

$$\hat{H}_I = -\Omega(t) \sum_k \hat{S}_{\phi, k} \hat{a}_k e^{i\delta_k t} + h.c.$$

Bosonic annihilation operator for mode  $k$

Spin operator,  $\hat{S}_{\phi, k} = (\eta_{1, k} \sigma_{,1} + \eta_{2, k} \sigma_{,2})/2$

Propagator:

$$\hat{U}(t) = \prod_k e^{-i\beta_k(t) \hat{S}_{\phi, k}^2} \hat{D}(\hat{S}_{\phi, k} \alpha_k(t))$$

Phase space trajectory:  $\alpha_k(t) = i \int_0^t \Omega(t') e^{-i\delta_k t'} dt'$

Governs spin entanglement:  $\beta_k(t) = \frac{i}{2} \int_0^t \frac{d\alpha_k(t')}{dt'} \alpha_k^*(t') - \alpha(t') \frac{d\alpha_k^*}{dt'} dt'$

Gate angle:  $\theta(t) = \sum_k \eta_{1, k} \eta_{2, k} \beta_k(t)$

# Experimental indicators of two error mechanisms guide analysis of robust gate implementations



If  $\alpha_k(\tau) \neq \alpha(0)$ ,

then the motional state at the end of the gate is not where it started.

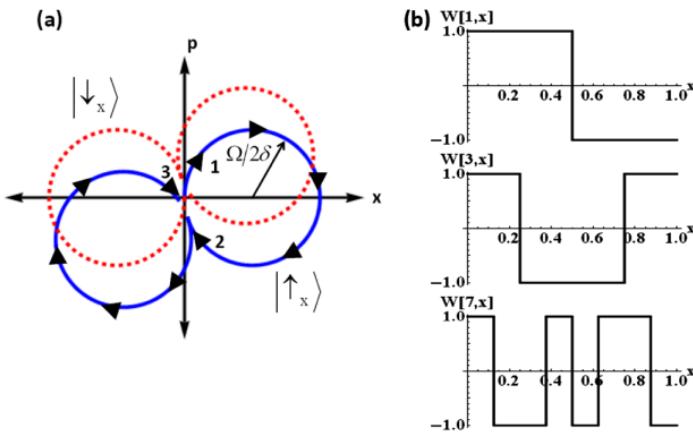
We call this **displacement error**,  $\epsilon_d$ .

Experimental Indicator :  $|01\rangle$  and  $|10\rangle$

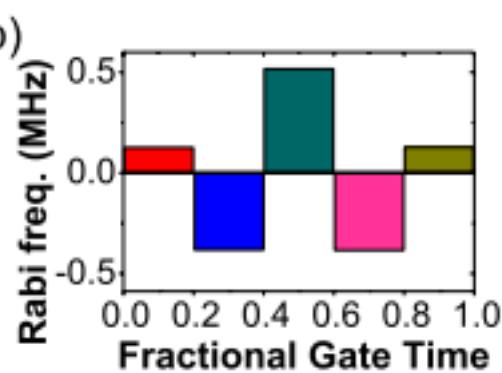
For robust gate, both  $\epsilon_d$  and  $\epsilon_r$  need to be small over a broad acceptable range of input parameters.

Pioneering work by Brown and Monroe groups have found ways to minimize  $\epsilon_d$ :

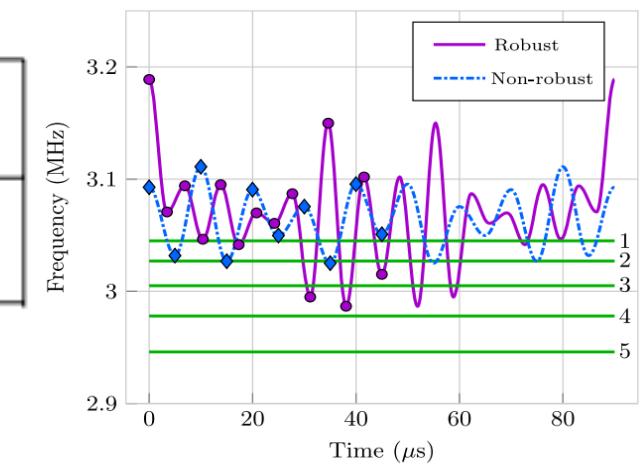
[Hayes et al 2012]



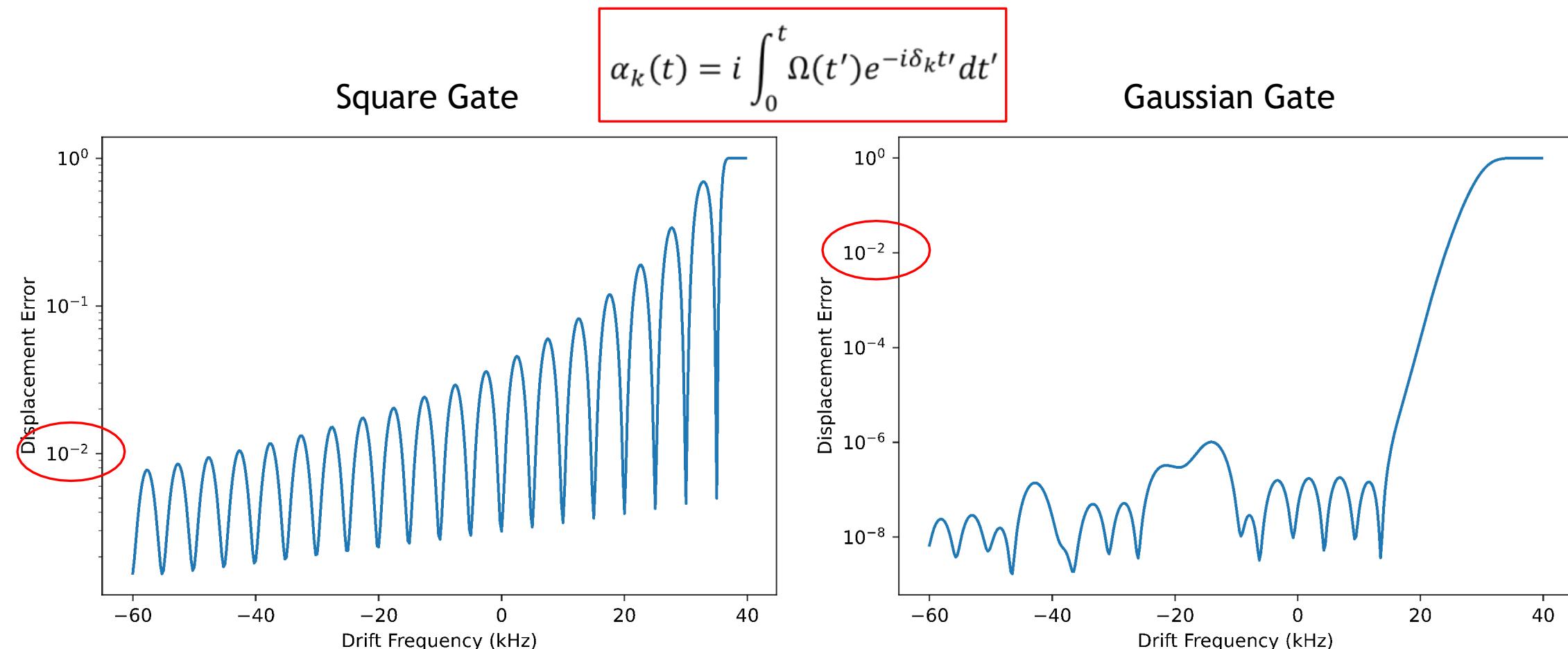
[Choi et al 2014]



[Leung et al 2018]



# Gaussian pulse shape is ‘naturally’ robust to displacement error.



In the Gaussian gate, we still need the right detuning and amplitude to get the right area enclosed, but this shows broadly robust spin-motion disentanglement at the end of a Gaussian gate.

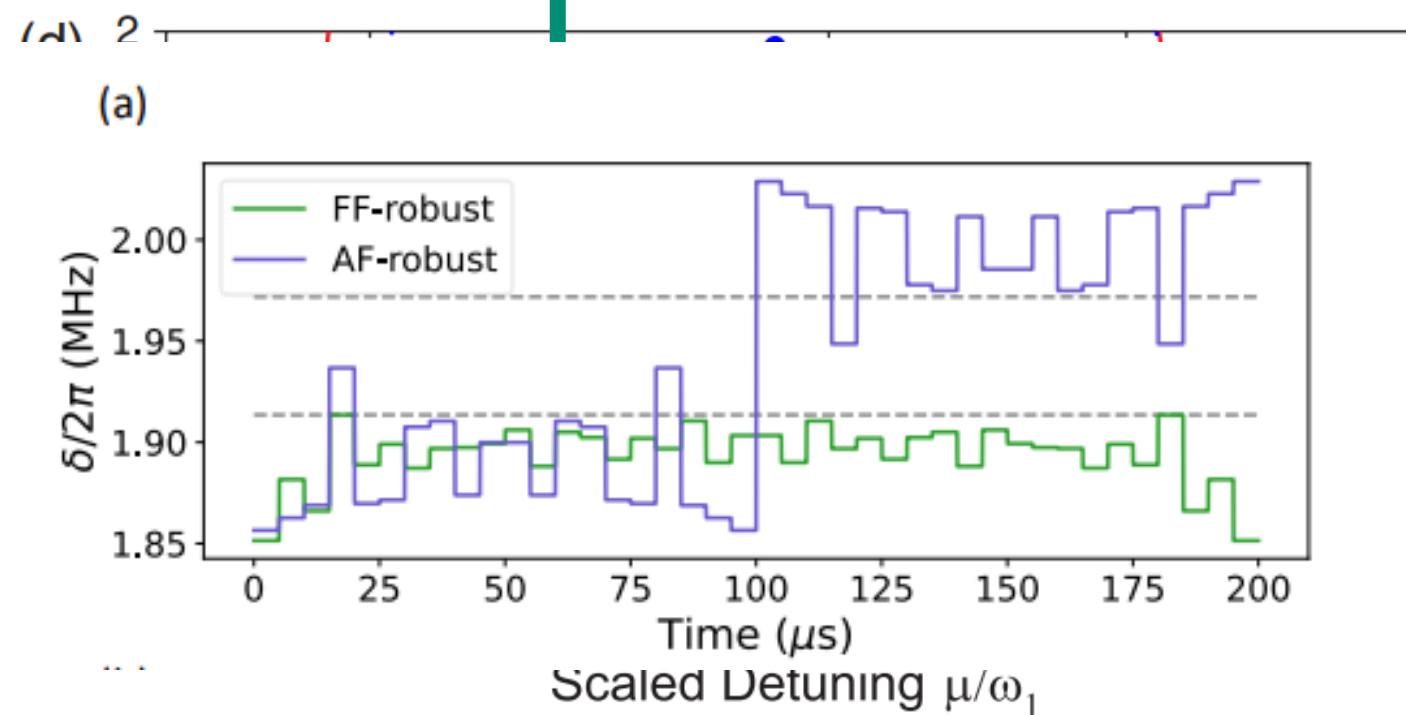
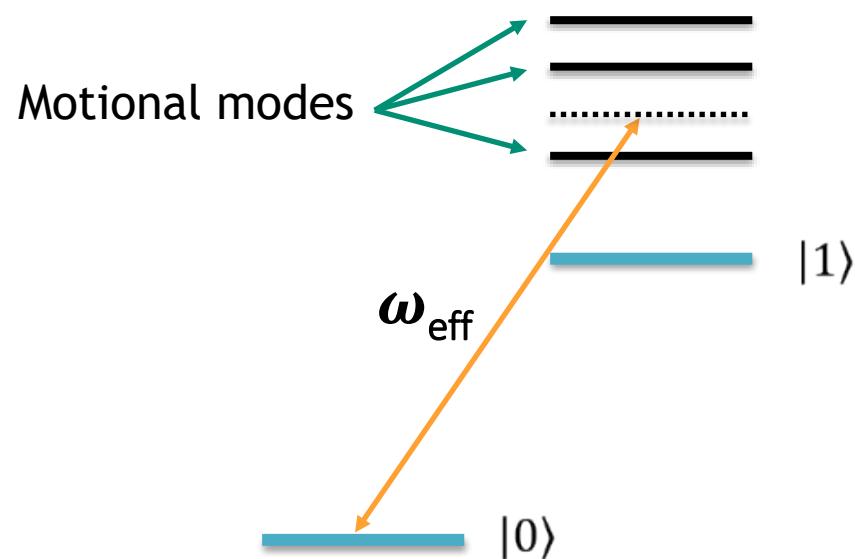
# Rotation error is suppressed by balancing the contributions of multiple motional modes.



Gaussian gives freedom to choose any sufficiently large detuning without displacement error

→ So choose  $\omega$  such that  $\frac{d\theta}{d\omega} = 0$

Jia et. Al, 2022 (Ken Brown group) recently  
showed this can be done dynamically as well.  
Operate the  
gate here

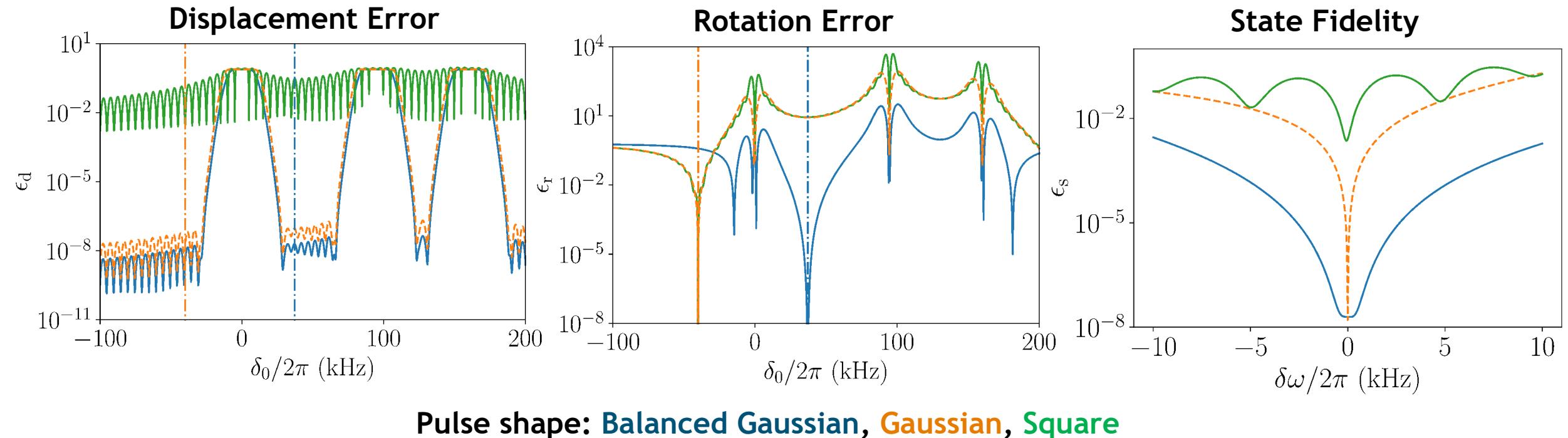


Kinoshita 2010, PRB 81, 120502 (2009) - Monroe group

# 'Balanced' gaussian gates take care of both rotation and displacement error



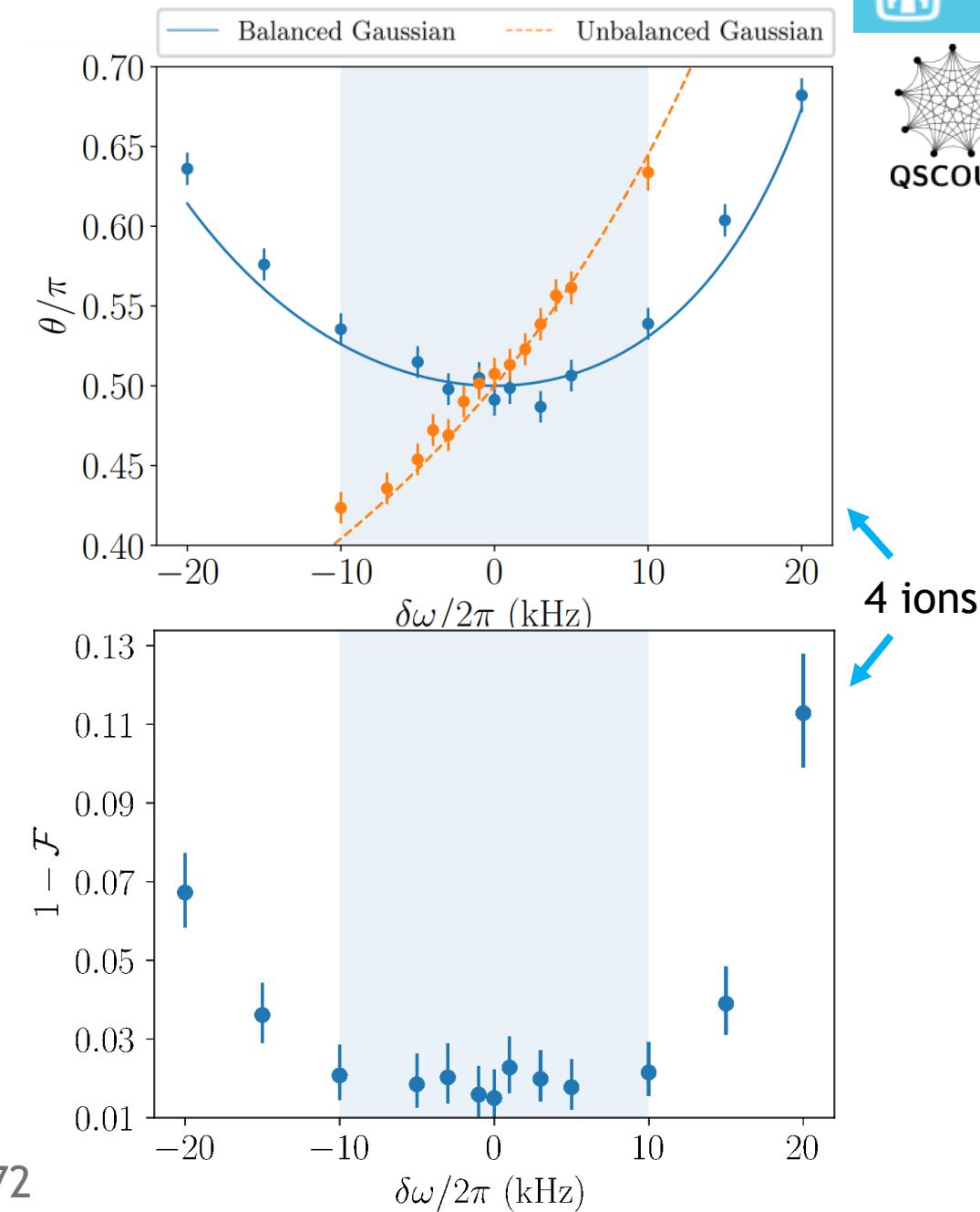
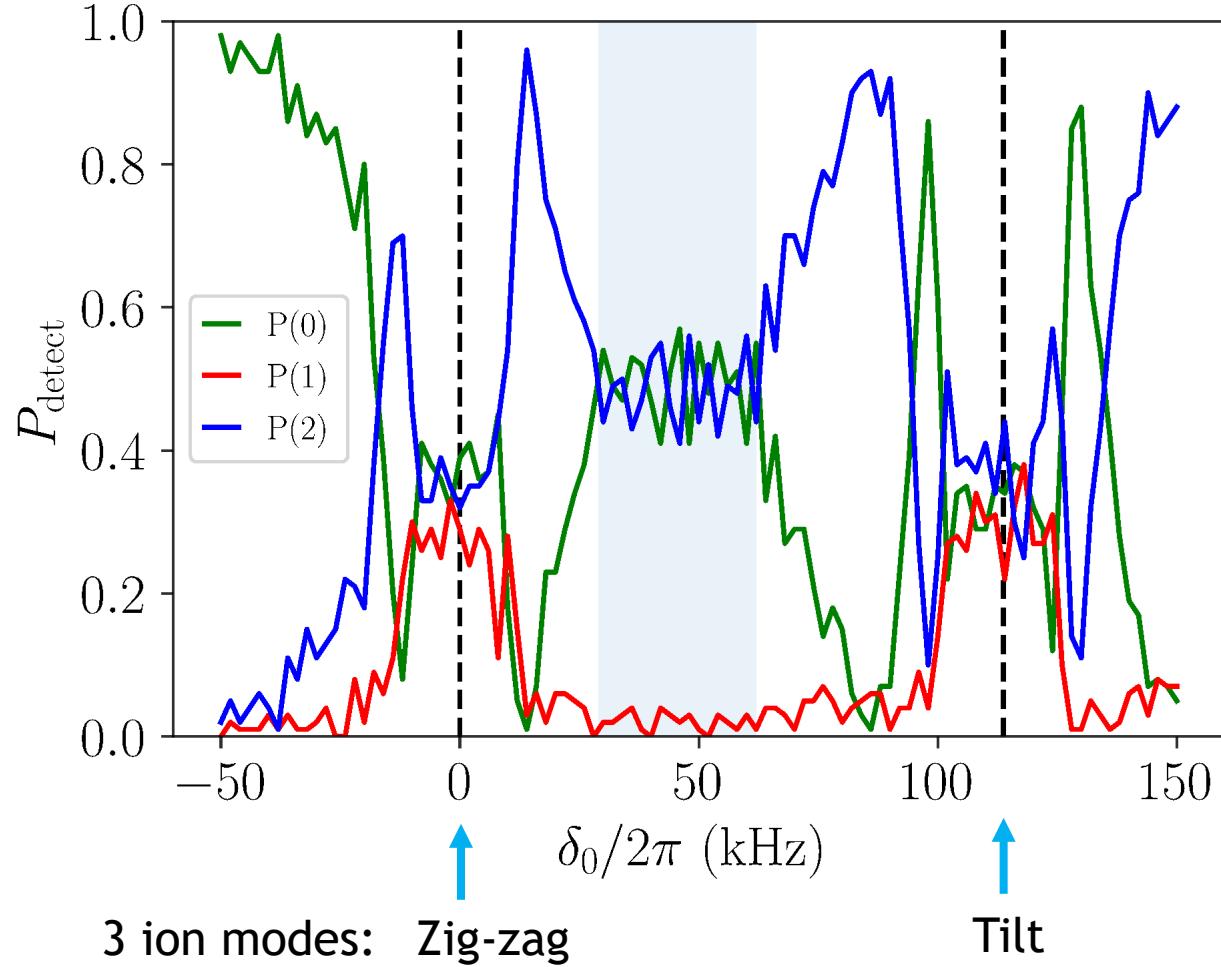
Gaussian amplitude modulation and a specific frequency give broad robustness to frequency error



Gates are simple to implement: no need to optimize tons of pulse-shape parameters

# Experiment shows balanced Gaussian is robust to $\pm 10$ kHz trap frequency drift

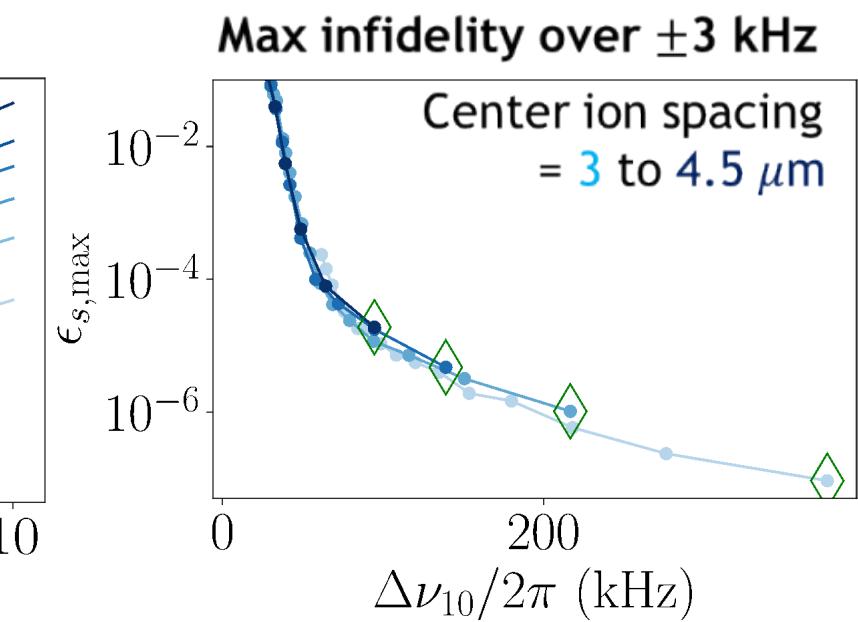
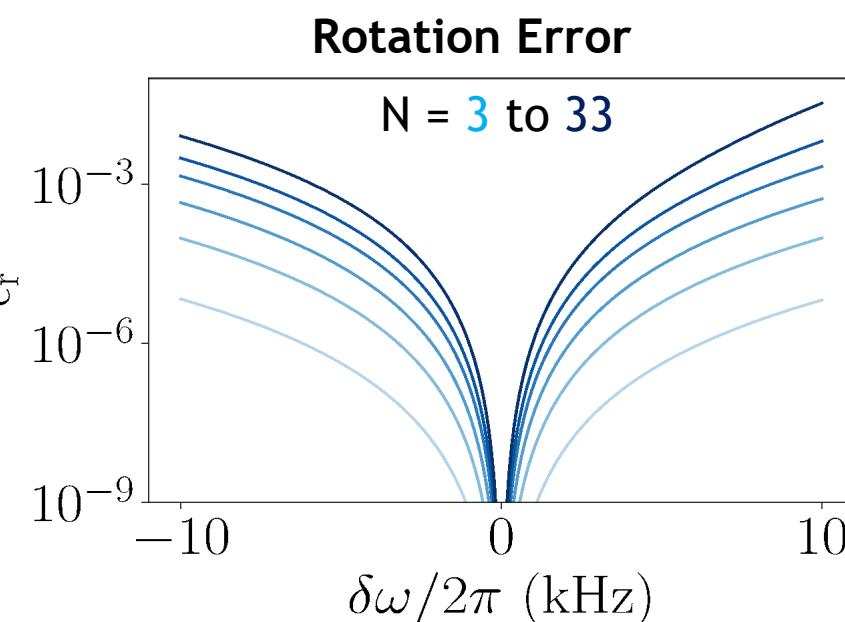
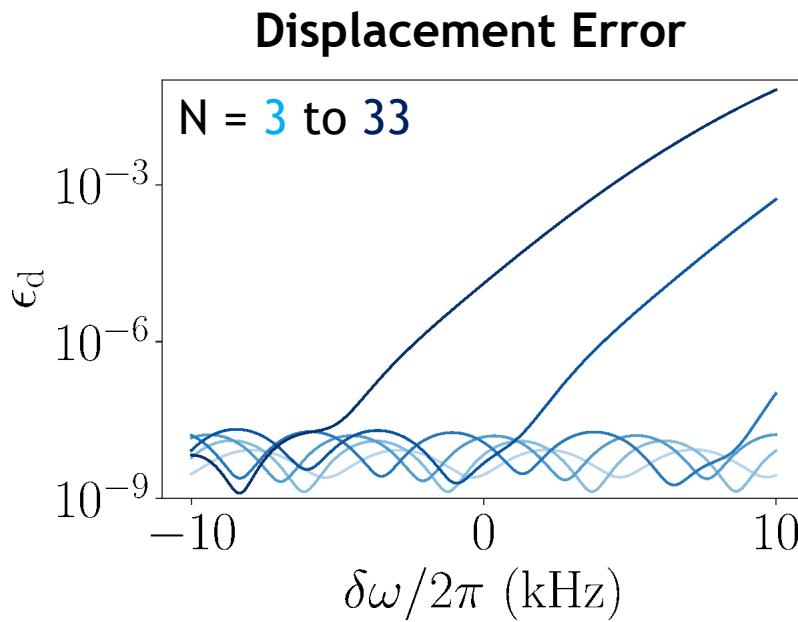
Peak fidelity = 98.5(6)%  
Drop in fidelity over  $\pm 10$  kHz < 1%



# We find good prospects for scaling to more ions.



- Numerical simulations for chains of up to 33 ions
- Fixed center ion separation: results shown are for  $3 \mu\text{m}$



- Sensitivity (right plot) depends almost entirely on the splitting of nearest two modes

# QSCOUT Team

Email: [qscout@sandia.gov](mailto:qscout@sandia.gov)

Websites: [qscout.sandia.gov](http://qscout.sandia.gov); [www.sandia.gov/quantum/trapped-ions](http://www.sandia.gov/quantum/trapped-ions)

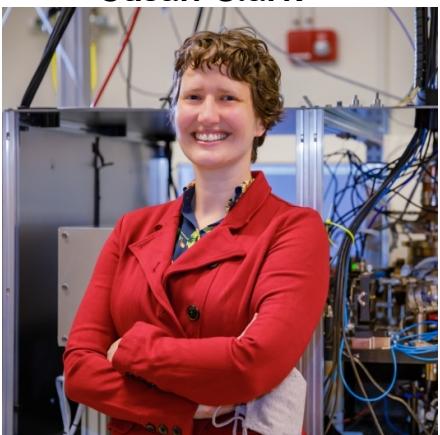
Jaql: [github/jaql](https://github/jaql)



**Brandon Ruzic**



**Susan Clark**



**Susan Clark (PI)**

Ashlyn Burch  
**Matt Chow**  
 Craig Hogle  
 Megan Ivory  
 Dan Lobser  
 Peter Maunz  
 Melissa Revelle  
 Dan Stick  
 Josh Wilson\*  
 Chris Yale  
\*now at SDL

Brad Salzbrenner  
 Madelyn Kosednar  
 Jessica Pehr  
 Ted Winrow  
 Bill Sweatt  
 Dave Bossert

Andrew Landahl  
 Ben Morrison  
 Tim Proctor  
 Kenny Rudinger  
 Antonio Russo  
**Brandon Ruzic**  
 Jay Van Der Wall  
 Josh Goldberg  
 Kevin Young  
 Collin Epstein  
 Andrew Van Horn

Matt Blain  
 Ed Heller  
 Jason Dominguez  
 Chris Nordquist  
 Ray Haltli  
 Tipp Jennings  
 Ben Thurston  
 Corrie Sadler  
 Becky Loviza  
 John Rembetski  
 Eric Ou  
 Matt Delaney

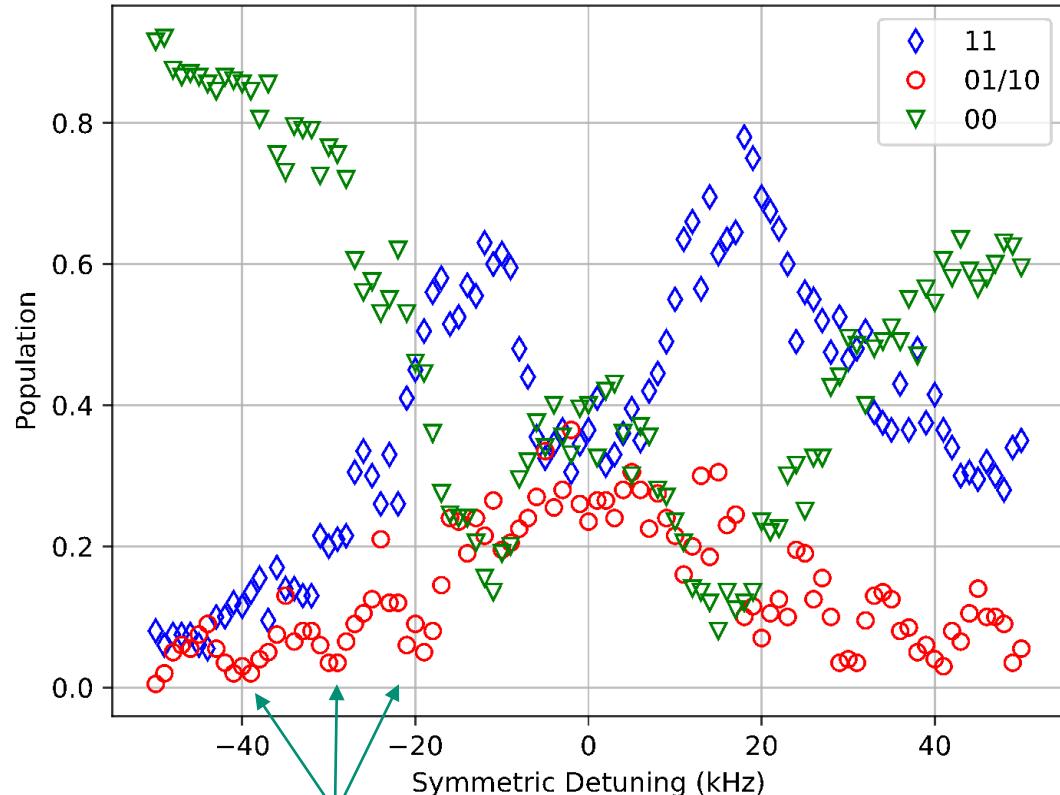
# Extra Slides Beyond



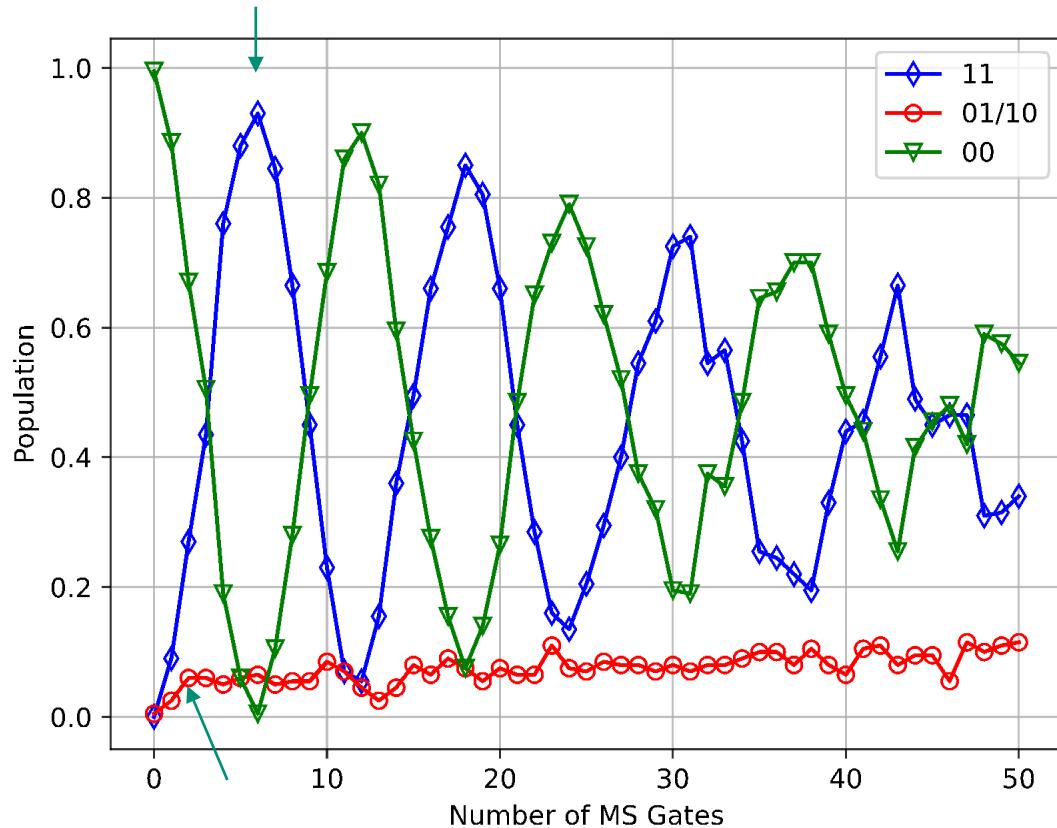
# The Standard (square, constant frequency) MS gates have issues.



Ideally, the gate generates flopping between  $|00\rangle$  and  $|11\rangle$  - never populating odd parity states.

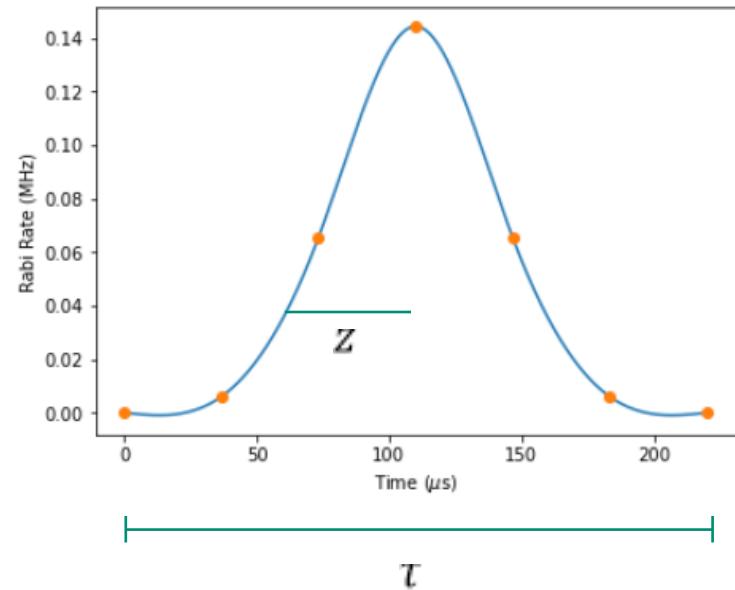


- Odd parity population does not go to zero.
- Narrow acceptable detuning range



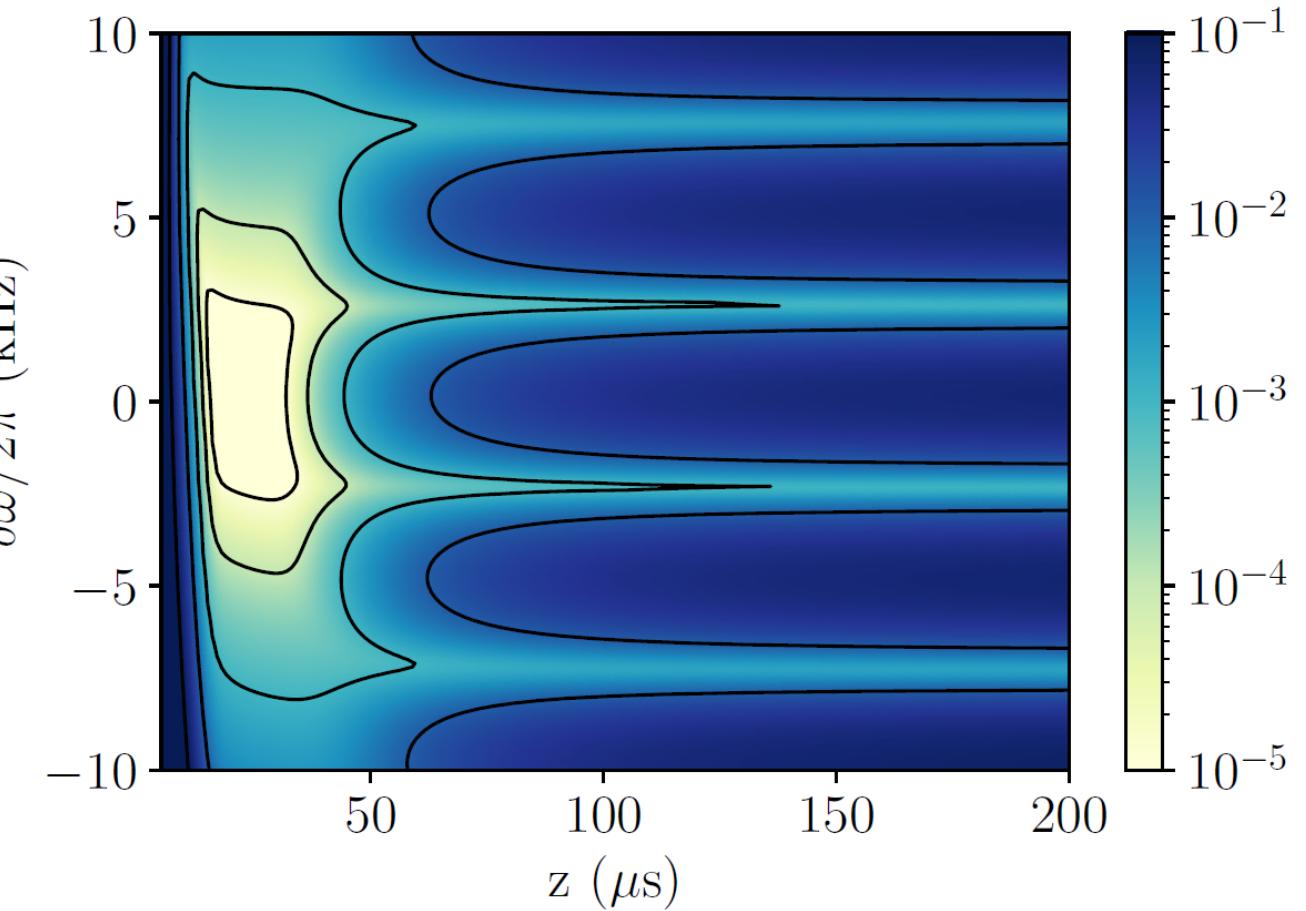
- *Best Fidelity  $\approx 96.4\%$ , estimated from the max 11 population*
- Odd parity population persistent

# Gaussian pulse parameters



$z$  : time-domain standard deviation

$\tau$  : gate time, truncation window

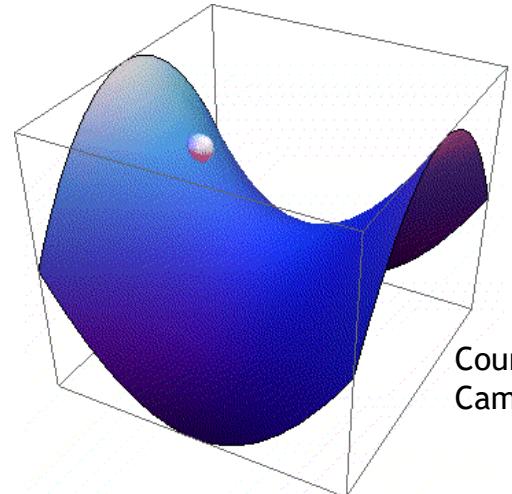


# RF Traps for Ions



Trapping requirement: A restoring force when displaced from trap center  
(in any direction)

Cannot use  
t  
Field lines  
start/en

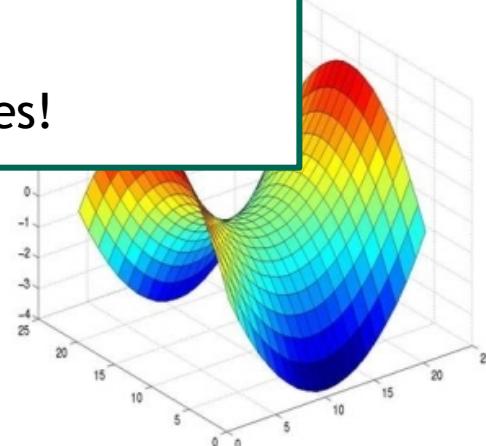
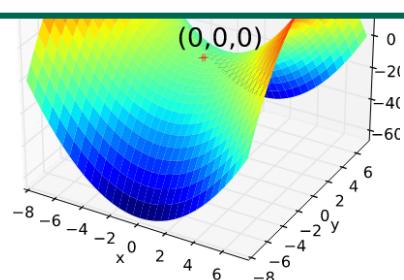


Courtesy of Wes  
Campbell

“out” and  
directions



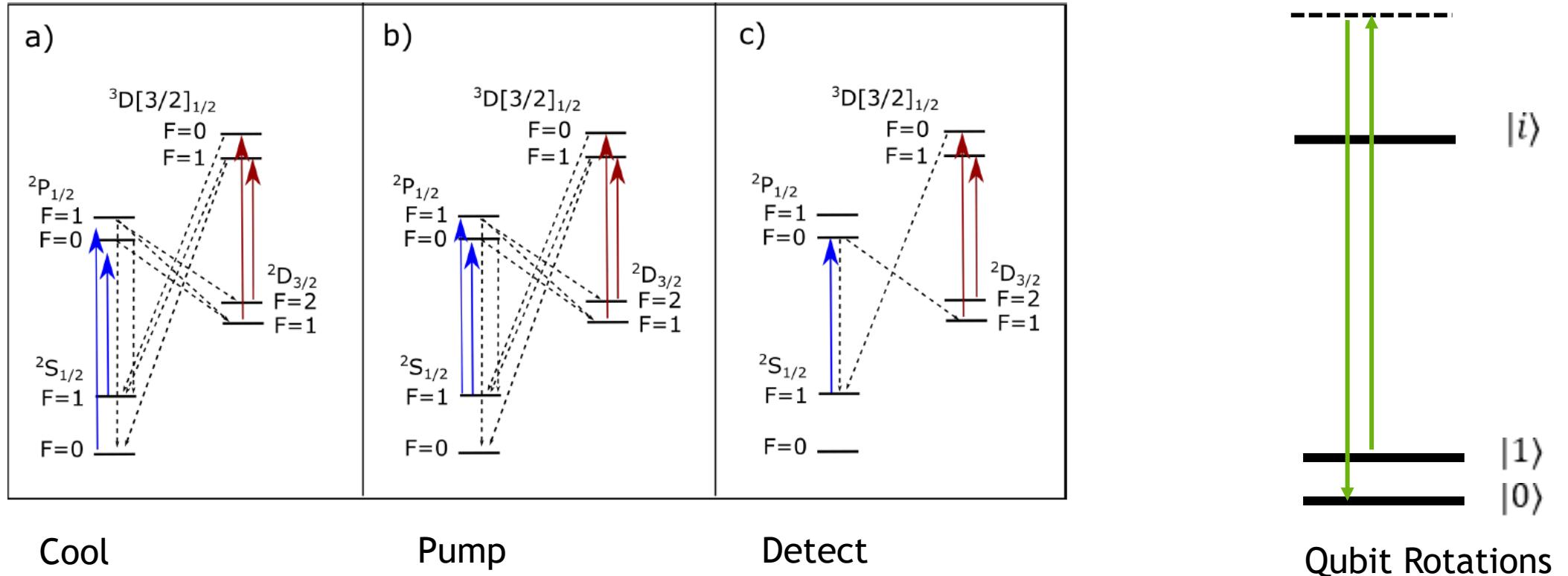
Before ion escapes, field reverses!



# Crash Course in Trapped Ion Quantum Computing

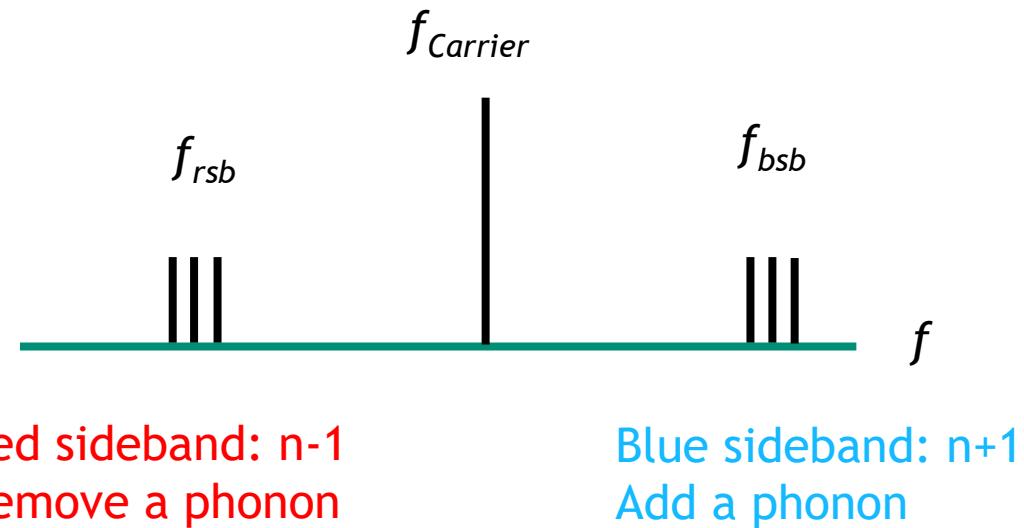
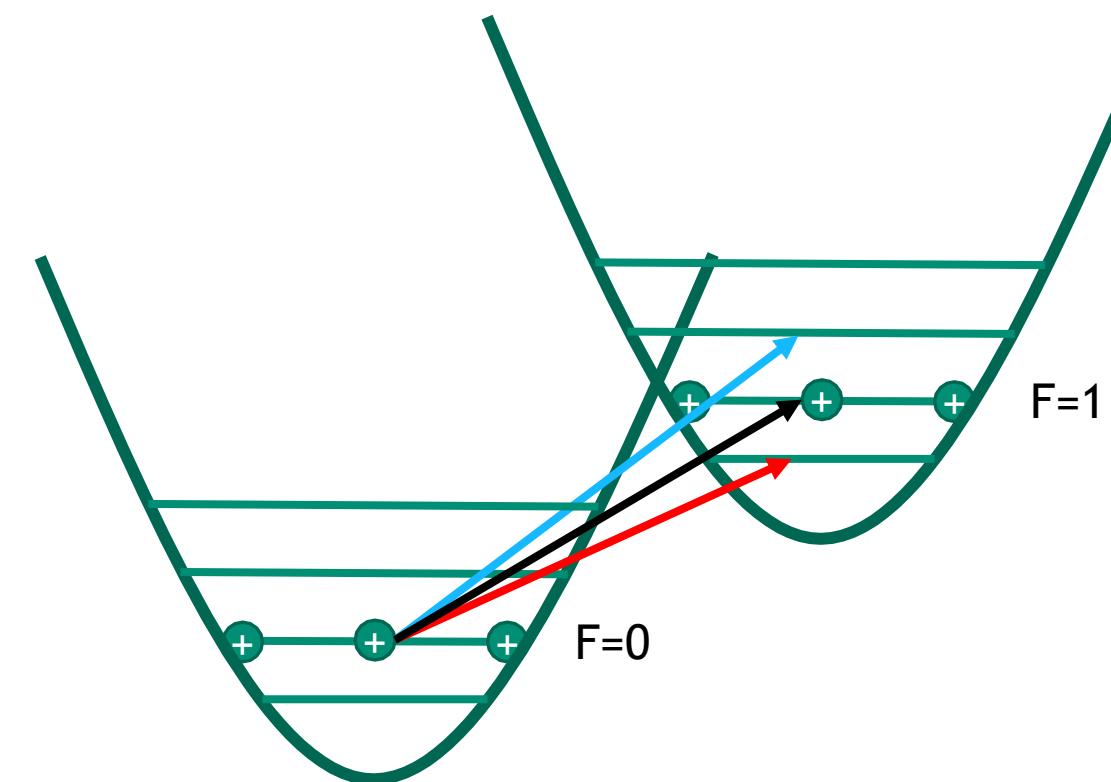


$^{171}\text{Yb}^+$



- Sidebands on cooling laser (370nm) allow incoherent control processes
- Pulsed laser (355nm) Raman transitions for qubit rotations

# Shared motional modes mediate entangling interactions

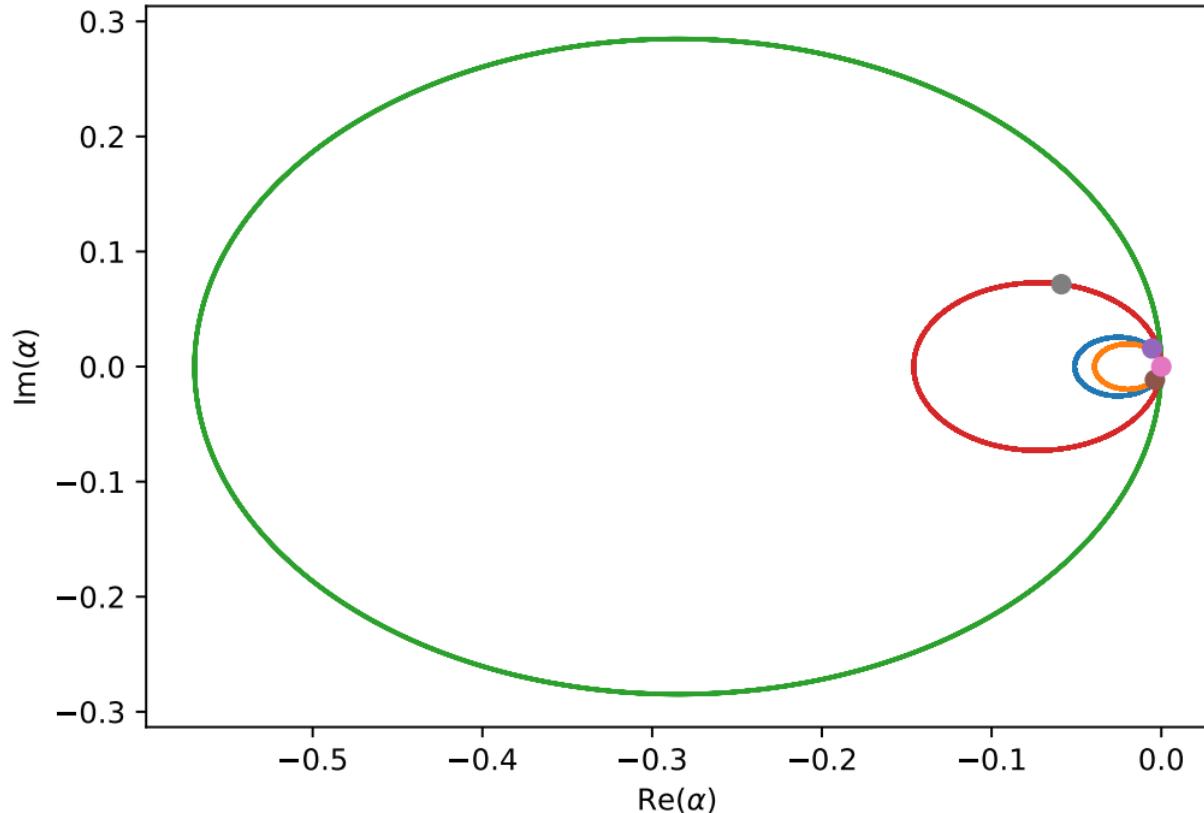


- Motional sidebands at the trap frequency
- Number of modes  $\propto$  Number of ions
- Coulomb interaction couples ions together
  - Vibrational levels act as “bus” connecting qubits

# Intuition for Phase Space Trajectories (PSTs)



PST for Square MS Gate



## How pulse parameters come into play:

- Detuning controls the angular velocity and radius in phase space.
  - Smaller detuning → Bigger, slower circles
- Rabi rate controls only radius.
- Phase can change the direction of rotation.

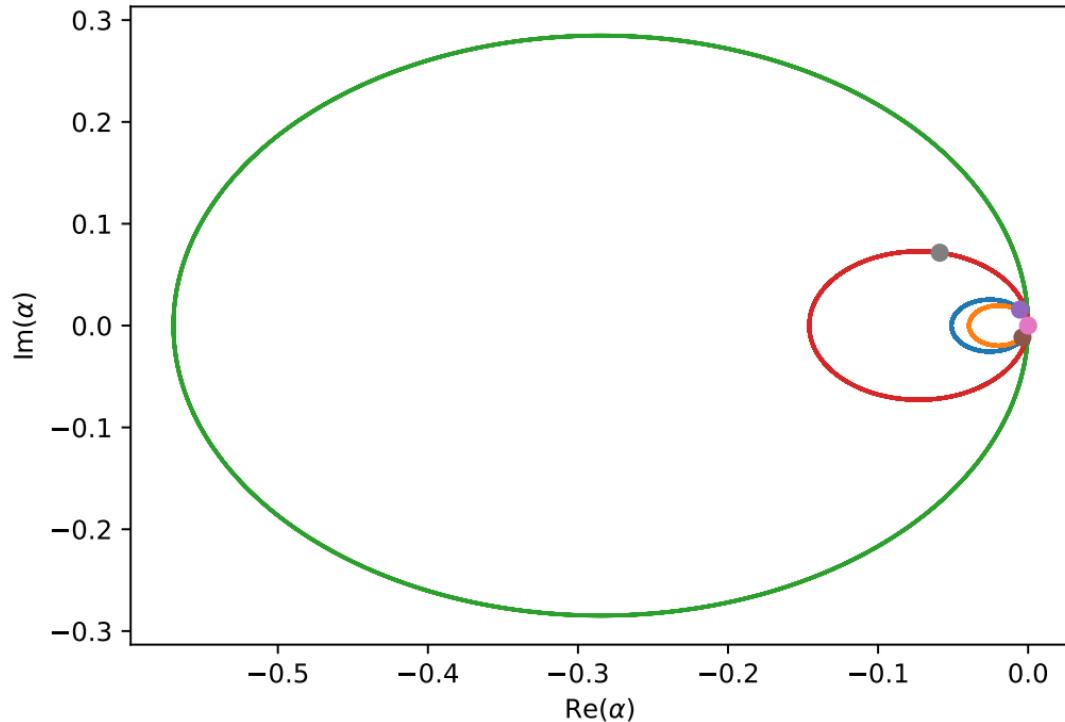
## Figures of merit:

- Loop closure of each mode:  $\alpha_k(0) = \alpha_k(t_{gate})$ ?
- Area enclosed: Is the gate angle right?

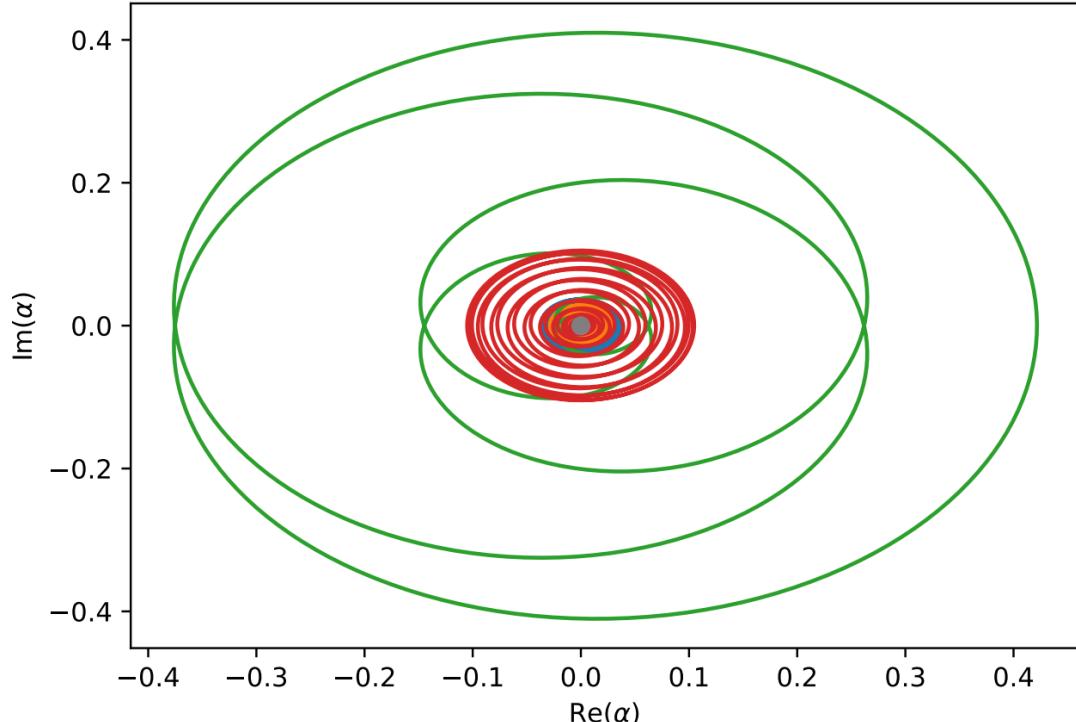
# Gaussian MS Gates Show Better Loop Closure than Square MS



Square Gate



Gaussian Gate



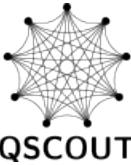
## Gate parameters (for both):

- Detuning is -40 kHz below the lowest frequency mode in a 2 ion chain.
- 200  $\mu$ s duration

## Notable differences:

- All modes end close to the starting point for Gaussian gate.
- Time averaged displacement close to zero for the Gaussian gate.

# Displacements During the MS Gate



Interaction Hamiltonian:

$$\hat{H}_I = \frac{\hbar\eta\Omega}{2} \left( \hat{a} e^{-i(\delta t + \phi^M)} + \hat{a}^\dagger e^{+i(\delta t + \phi^M)} \right) \hat{\sigma} \cdot \phi^s$$

State dependent drive force gives coherent displacement by  $\alpha_k$  on mode  $k$  [1]:

$$\hat{D}(\hat{\alpha}_k) = \exp(\hat{\alpha}_k a_k^\dagger - \hat{\alpha}_k^\dagger a_k)$$

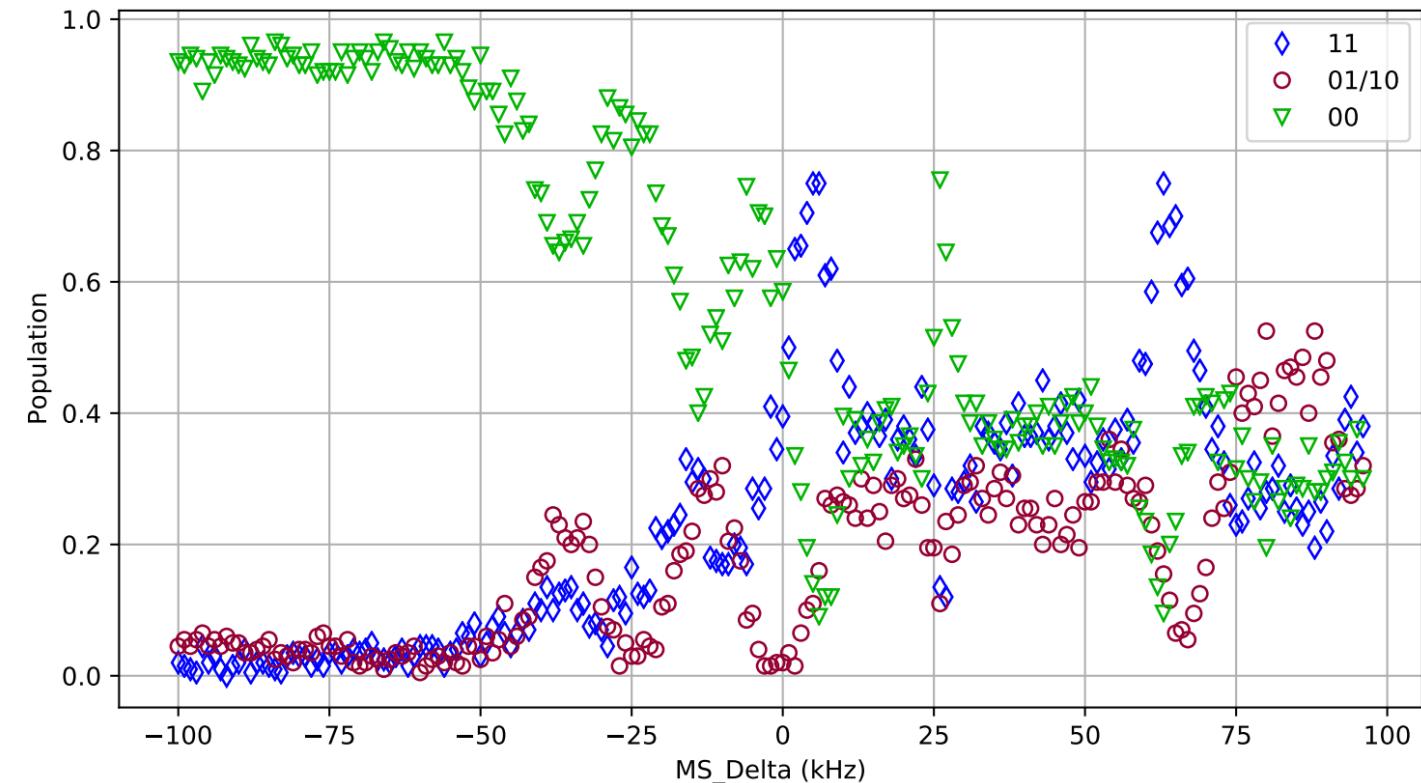
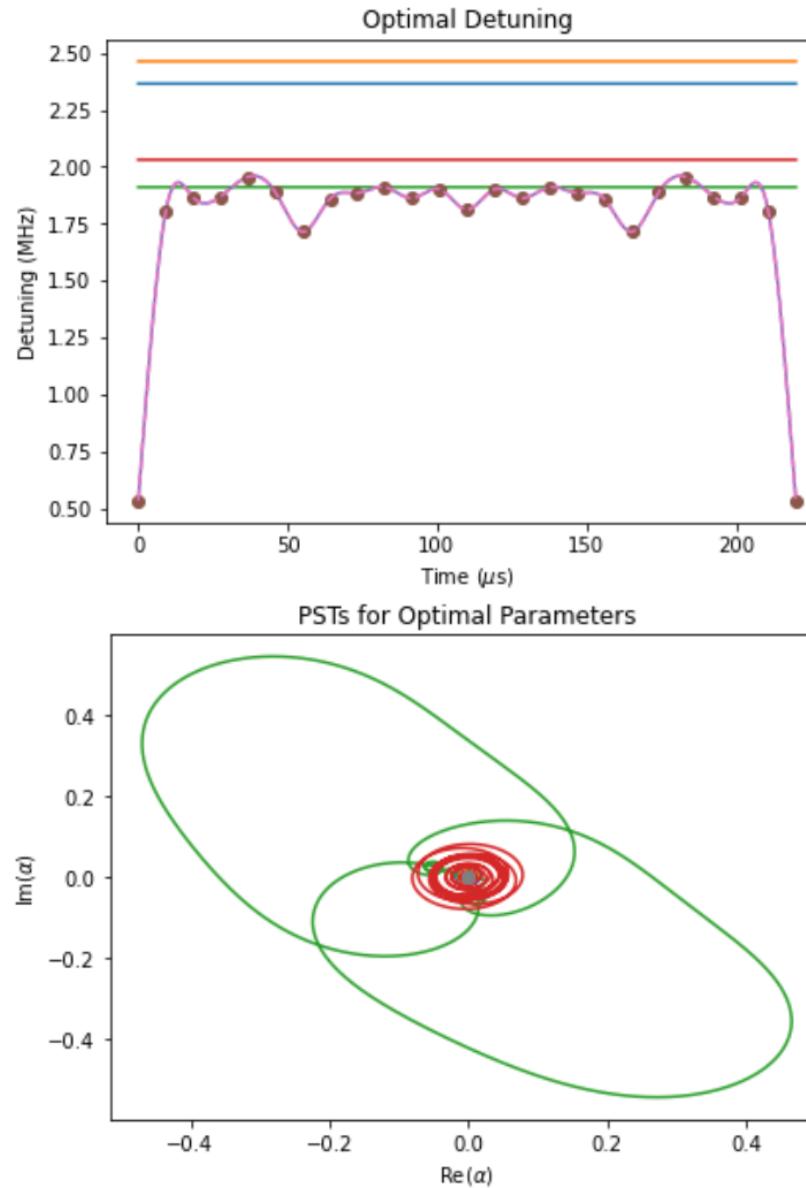
In the simplest case (+1 eigenstate of  $\hat{\sigma}_\phi$ , no laser phase difference) [1]:

$$\alpha_k = \frac{\Omega}{2} (\eta_{i,k} + \eta_{j,k}) \int_0^t e^{i\theta_k(t')} dt' \text{ where } \theta_k(t') = \int_0^{t'} \delta_k(t'') dt'' \text{ is the accumulated phase}$$

So, for a constant detuning and Rabi rate, we get circular trajectories in phase space, with closure condition:  $t_{gate} = 2\pi m / \delta_k$ .

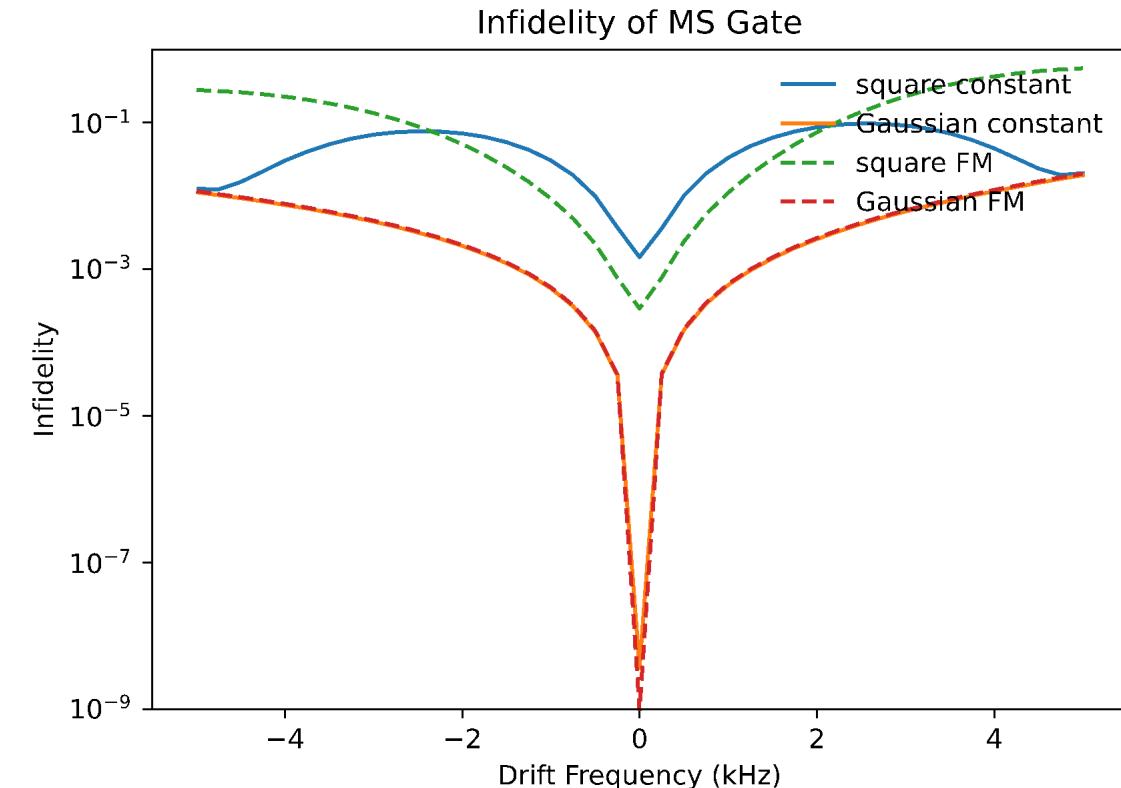
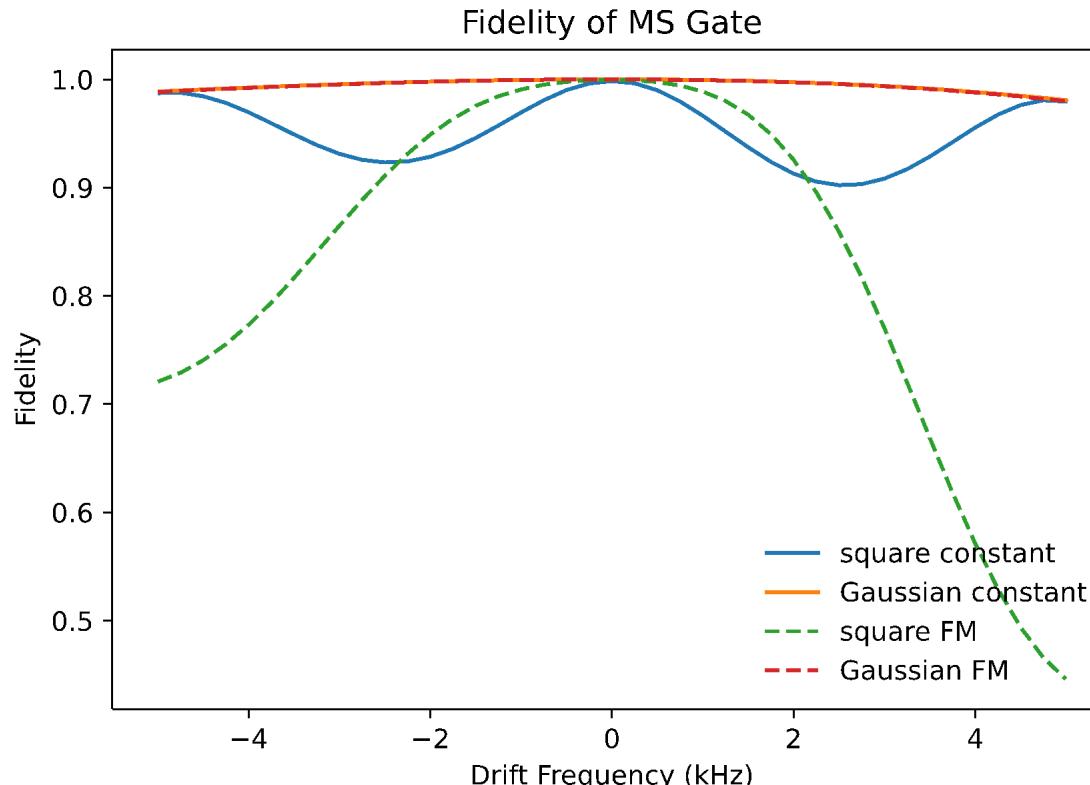
Closed loops are required for spin-motion disentanglement at the end of the gate.

# FM and Constant Frequency Gaussians Are Comparable



- See essentially the same odd parity population near optimized detuning.
- Area enclosed still has roughly the same sensitivity (slope of 11/00 crossing) as constant frequency case.

# Gaussian Pulse Shape Dominates Gate Performance (with Current Cost Functions)



## Next steps:

- Further development of gate angle cost function
- Batch optimization
- Transfer function optimization
- Larger chains

# Optimizer/Data Comparison For FM Gaussian

