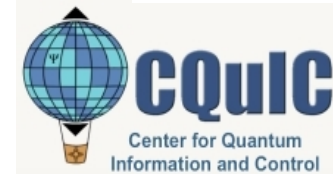
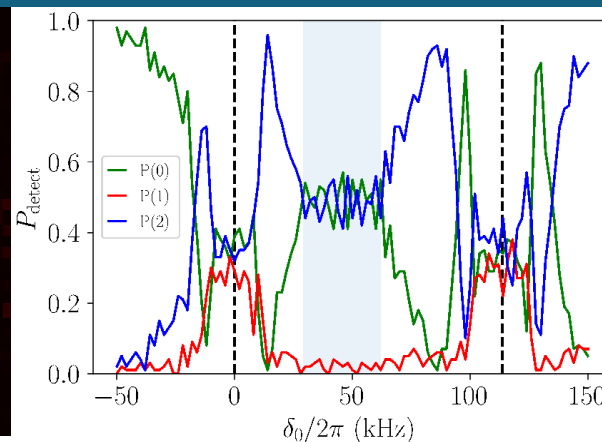
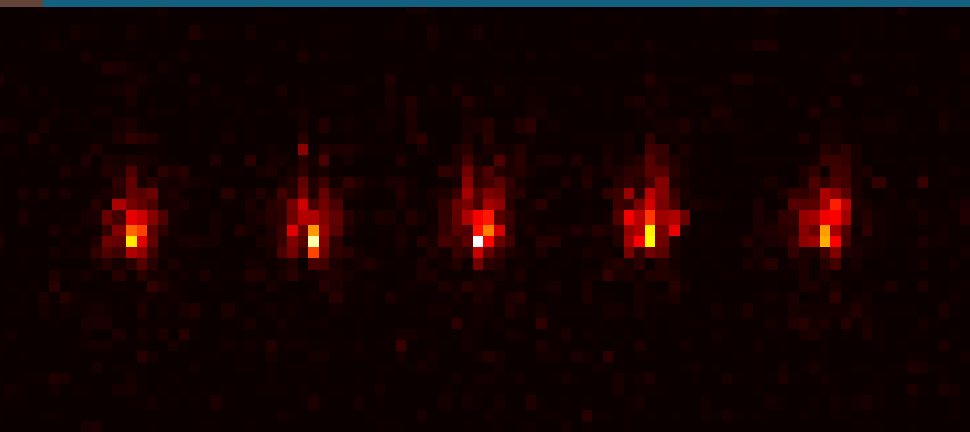
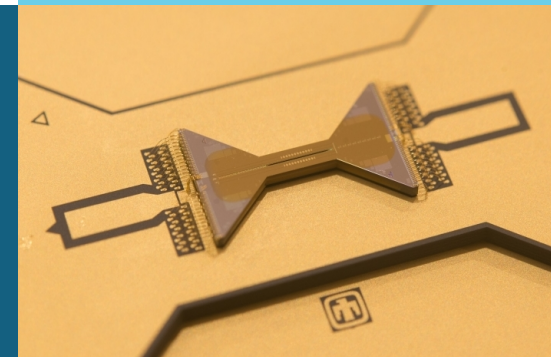




SAND2023-04555C



Demonstration of Mølmer–Sørensen Gates Robust to ± 10 kHz Motional Frequency Error



Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

SANDXXXX-

Presenting: Matthew Chow^{1,2,3}

Team: Brandon P. Ruzic¹, Ashlyn D. Burch¹, Megan K. Ivory, Dan S. Lobser¹, Melissa C. Revelle¹, Christopher G. Yale¹, Susan M. Clark¹



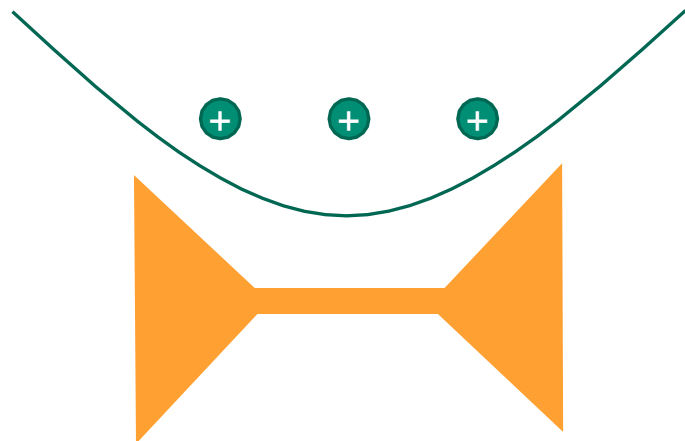
Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

3. Center for Quantum Information and Control, University of New Mexico, Albuquerque, NM

arXiv : 2210.02372

Overview: We develop and demonstrate an entangling gate on trapped ions that is robust to a dominant noise source.

Ion RF Paul Trap



Critical challenge: Error mitigation for entangling gates



Design pulses for robust operation

Trapped ions show great promise for quantum computing. The entangling gates suffer from technical noise.

Trapped ions show great promise

- SPAM > 0.99999

[Zukas 2021]

- Single qubit rotations > 0.9999

[Ballance 2016, Gaebler 2016]

- Peak entangling gate ≥ 0.999

[Ballance 2016, Gaebler 2016]

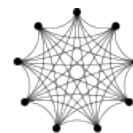
... however

- Motional frequency drifts impact the entangling gate.

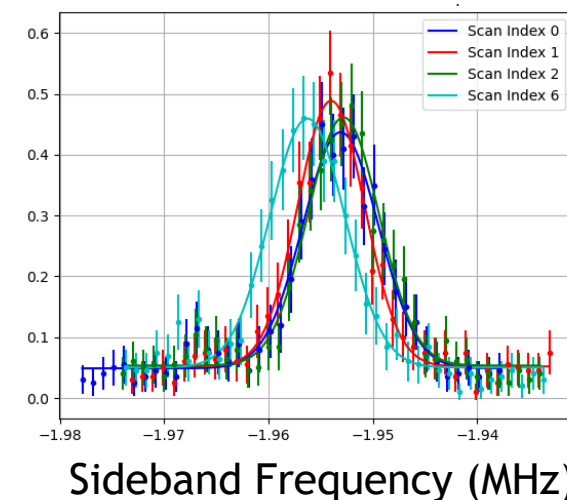
- It's hard to scale to many ions.



Solution? Develop gates that are robust to trap frequency drift that can be implemented on long chains.



QSCOUT



Shared motional modes mediate entangling interactions



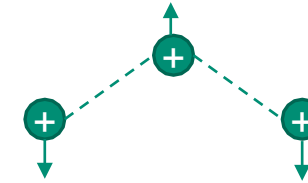
E.g: 3-ion normal modes of motion



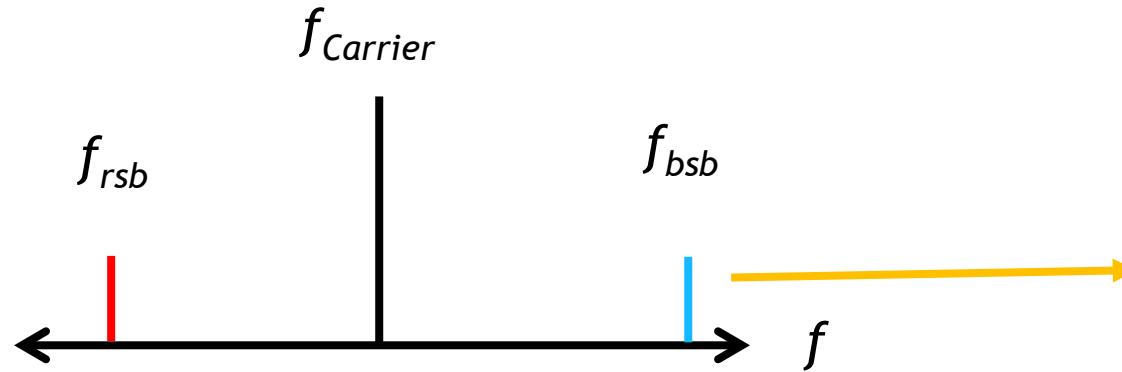
Center of mass



Tilt



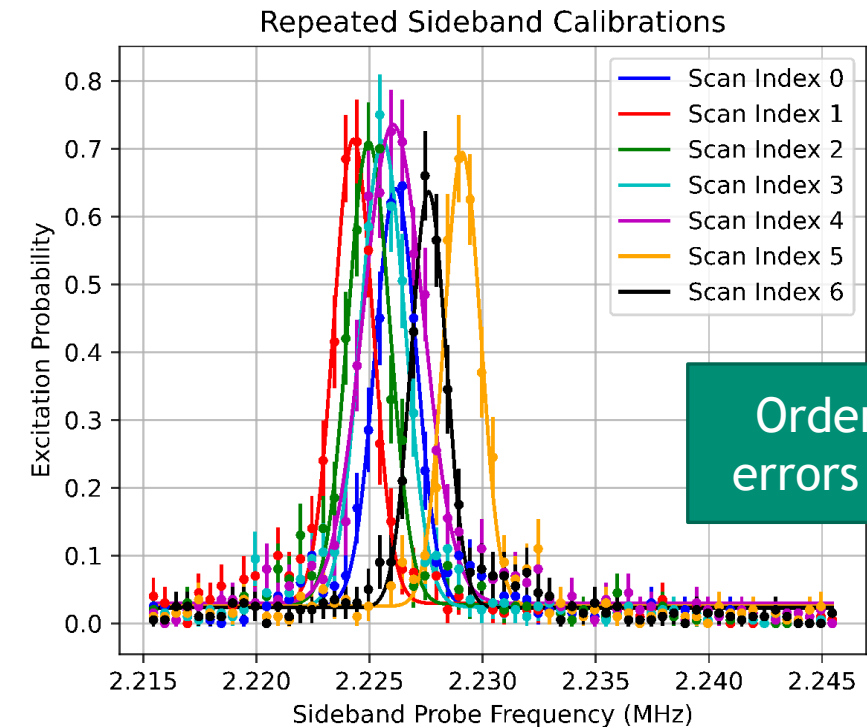
Zig-zag



Red sideband: $n-1$
Remove a phonon

Blue sideband: $n+1$
Add a phonon

- Motional sidebands at the normal mode trap frequencies
- Coulomb interaction couples ions together
 - Vibrational levels act as “bus” connecting qubits



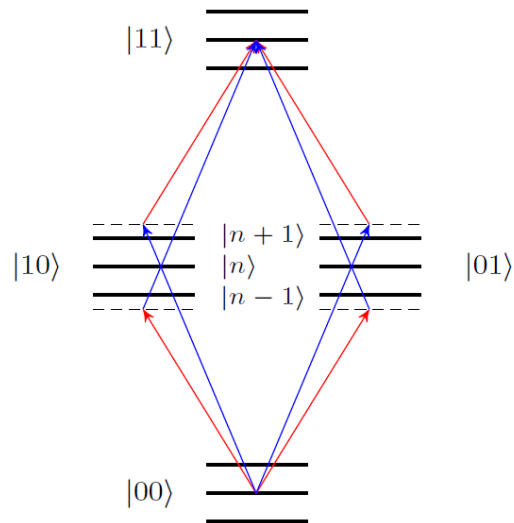
... however

Motional frequency drifts impact the entangling gate.

The Mølmer-Sørensen (MS) interaction drives spin entanglement and coherent displacement.



QSCOUT


 f_{Carrier}
 f_{rsb}
 f_{bsb}
 f
 δ_k
 δ_k

Apply 2 Symmetrically Detuned Tones

Interaction Hamiltonian:

$$\hat{H}_I = -\Omega(t) \sum_k \hat{S}_{\phi,k} \hat{a}_k e^{i\delta_k t} + h.c.$$

Bosonic annihilation operator for mode k

Spin operator, $\hat{S}_{\phi,k} = (\eta_{1,k}\sigma_{\phi,1} + \eta_{2,k}\sigma_{\phi,2})/2$

Propagator:

$$\hat{U}(t) = \prod_k e^{-i\beta_k(t)\hat{S}_{\phi,k}^2} \hat{D}(\hat{S}_{\phi,k}\alpha_k(t))$$

Phase space trajectory:

$$\alpha_k(t) = i \int_0^t \Omega(t') e^{-i\delta_k t'} dt'$$

$$\text{Governs spin entanglement: } \beta_k(t) = \frac{i}{2} \int_0^t \frac{d\alpha_k(t')}{dt'} \alpha_k^*(t') - \alpha(t') \frac{d\alpha_k^*}{dt'} dt'$$

Gate angle:

$$\theta(t) = \sum_k \eta_{1,k} \eta_{2,k} \beta_k(t)$$

Experimental indicators of two error mechanisms guide analysis of robust gate implementations



If $\alpha_k(\tau) \neq \alpha(0)$,

then the motional state at the end of the gate is not where it started.

We call this **displacement error**, ϵ_d .

Experimental Indicator : $|01\rangle$ and $|10\rangle$

If $\theta(\tau) \neq \pi/2$,

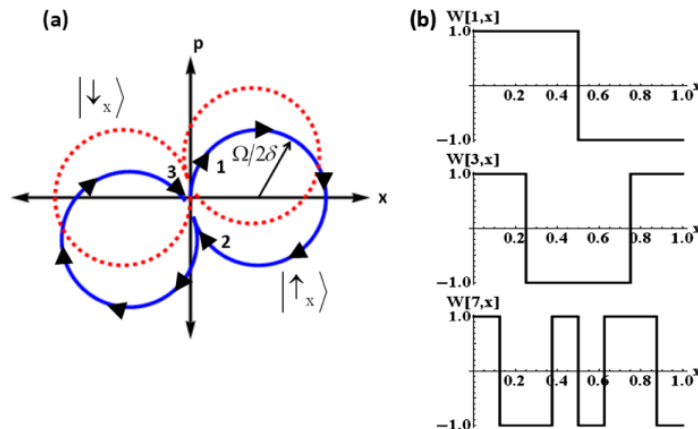
then the entangling angle accumulated is not equal to the target value.

We call this **rotation error**, ϵ_r .

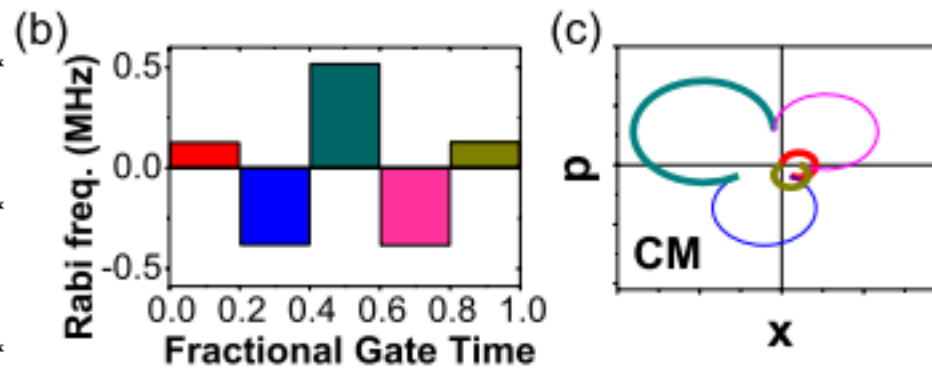
Experimental indicator: ratio of $|00\rangle$ to $|11\rangle$

For robust gate, both ϵ_d and ϵ_r need to be small over a broad acceptable range of input parameters.
Pioneering work by Brown and Monroe groups have found ways to minimize ϵ_d :

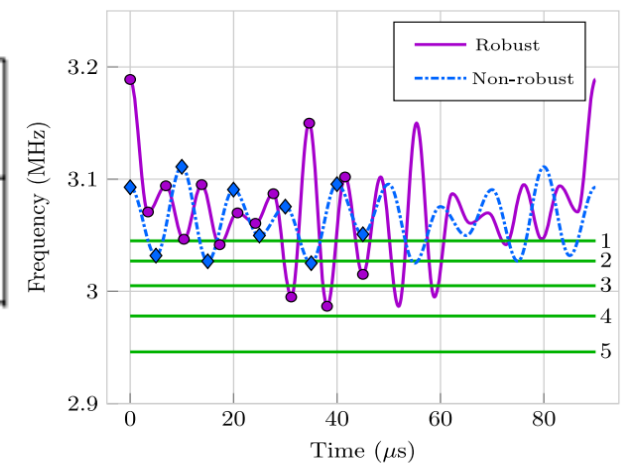
[Hayes et al 2012]



[Choi et al 2014]



[Leung et al 2018]



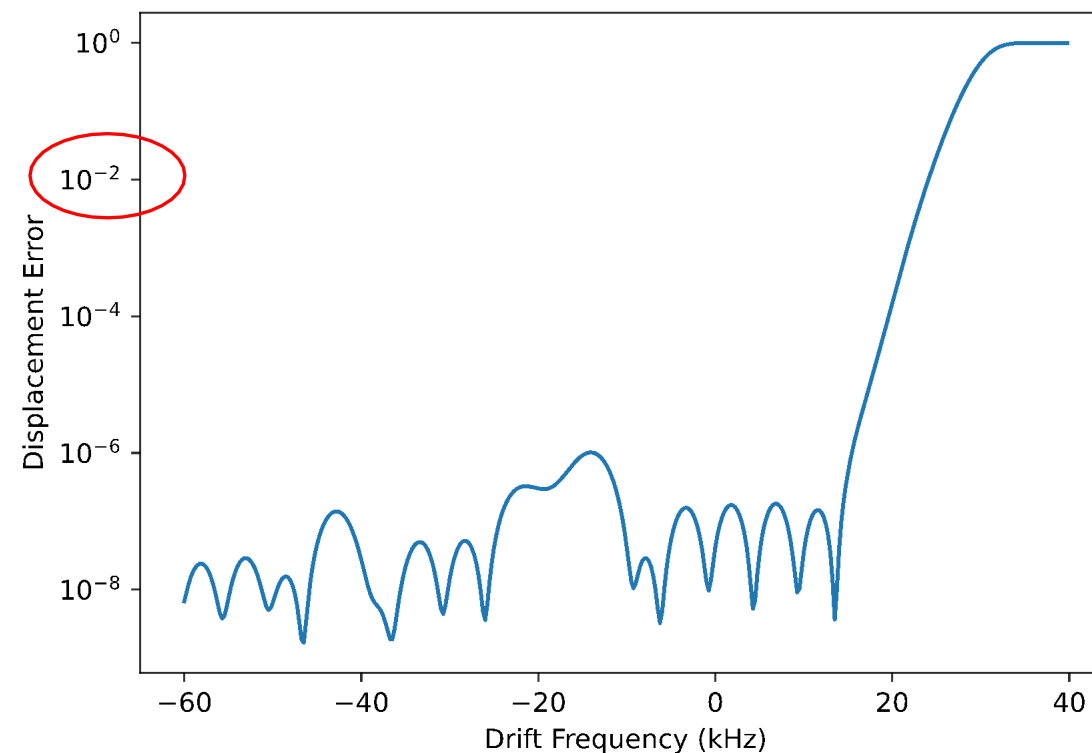
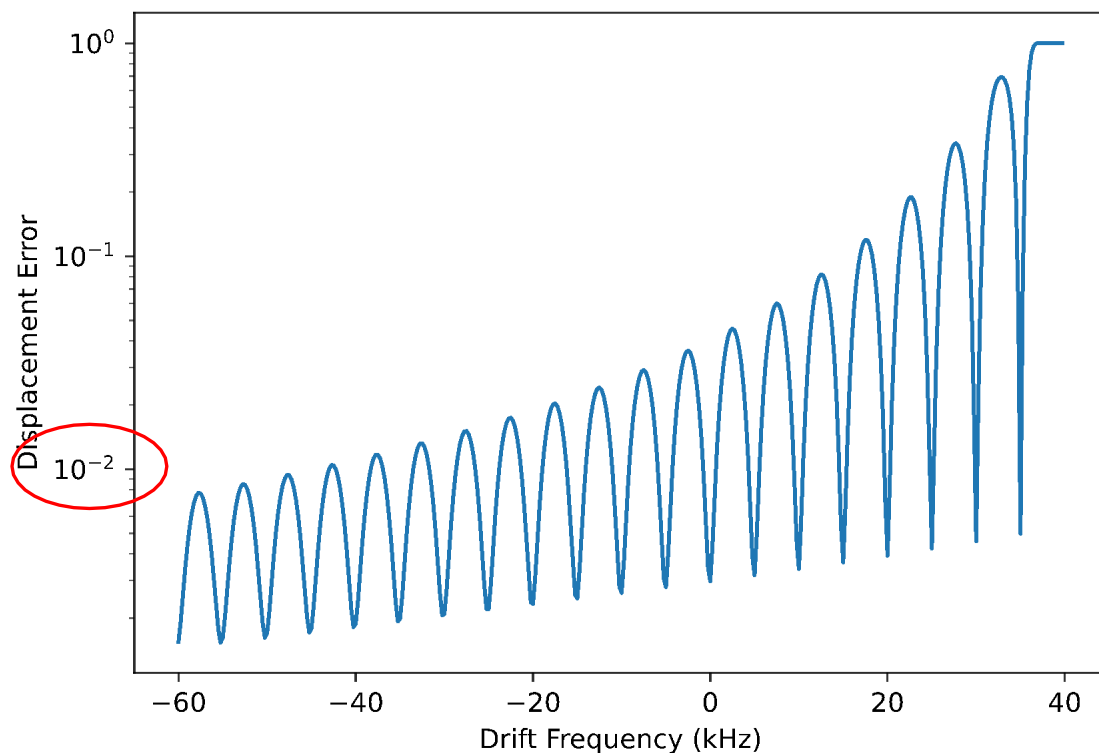
Gaussian pulse shape is 'naturally' robust to displacement error.



Square Gate

$$\alpha_k(t) = i \int_0^t \Omega(t') e^{-i\delta_k t'} dt'$$

Gaussian Gate



In the Gaussian gate, we still need the right detuning and amplitude to get the right area enclosed, but this shows broadly robust spin-motion disentanglement at the end of a Gaussian gate.

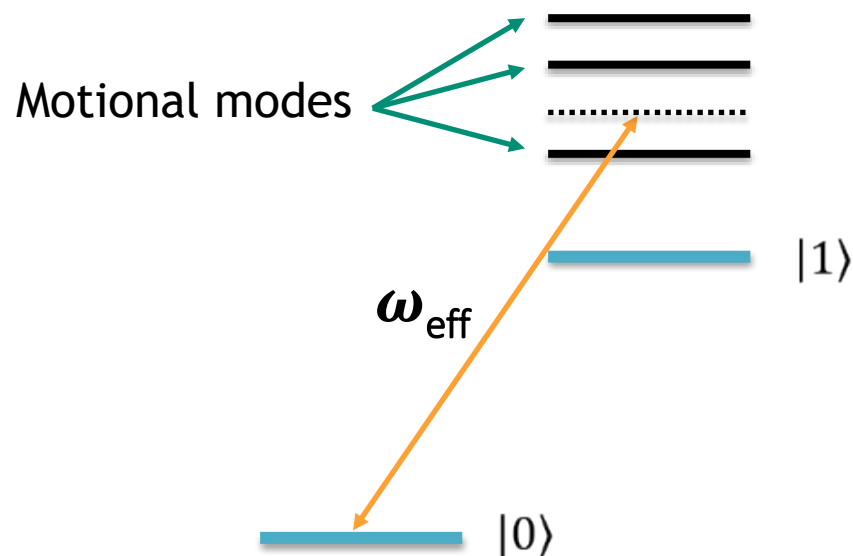
Rotation error is suppressed by balancing the contributions of multiple motional modes.



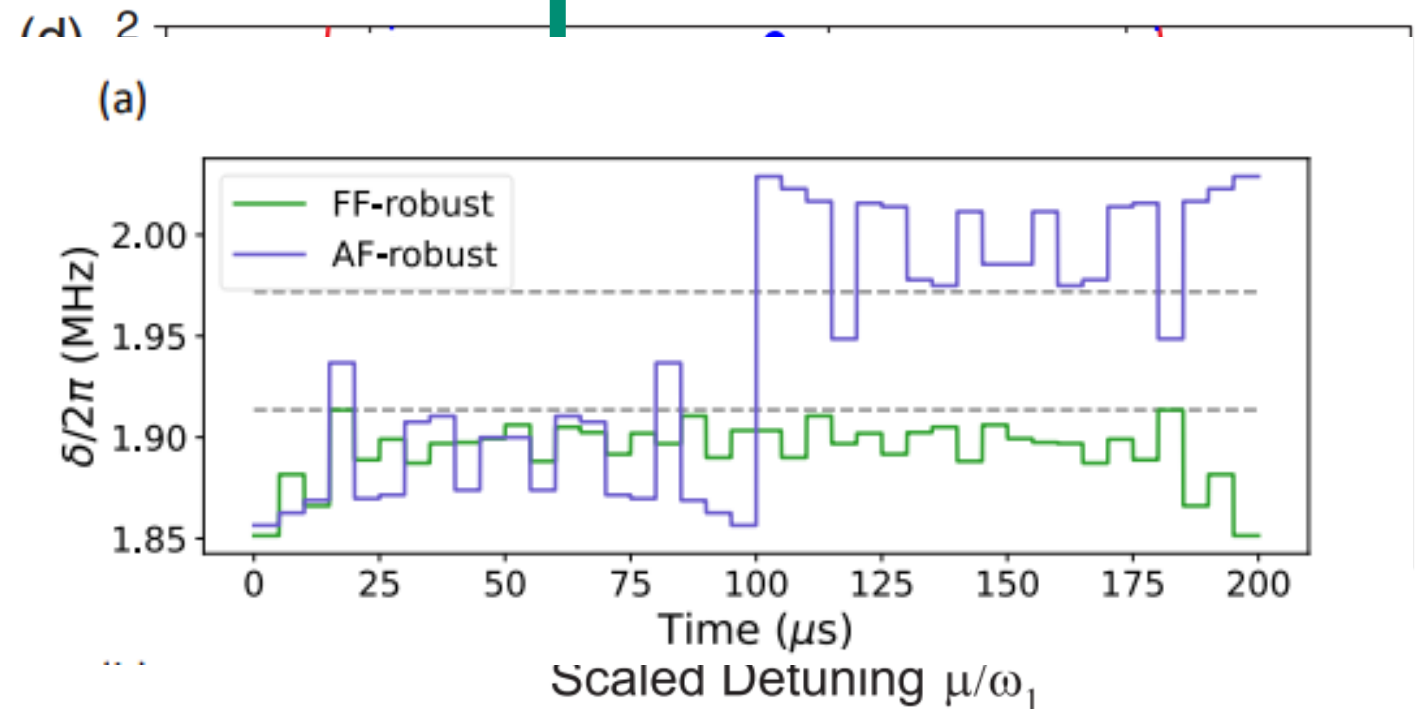
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Gaussian gives freedom to choose any sufficiently large detuning without displacement error

→ So choose ω such that $\frac{d\theta}{d\omega} = 0$



Jia et. Al, 2022 (Ken Brown group) recently showed this can be done dynamically as well.

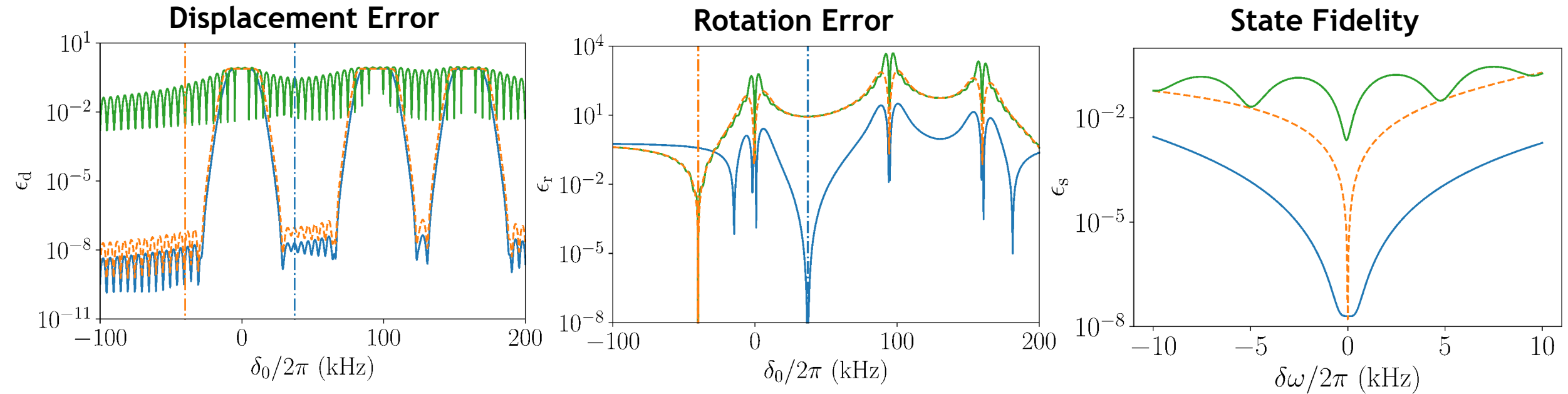


Kim et al, 2010, PRL 103, 120502 (2009) - Monroe group

9 'Balanced' gaussian gates take care of both rotation and displacement error



Gaussian amplitude modulation and a specific frequency give broad robustness to frequency error



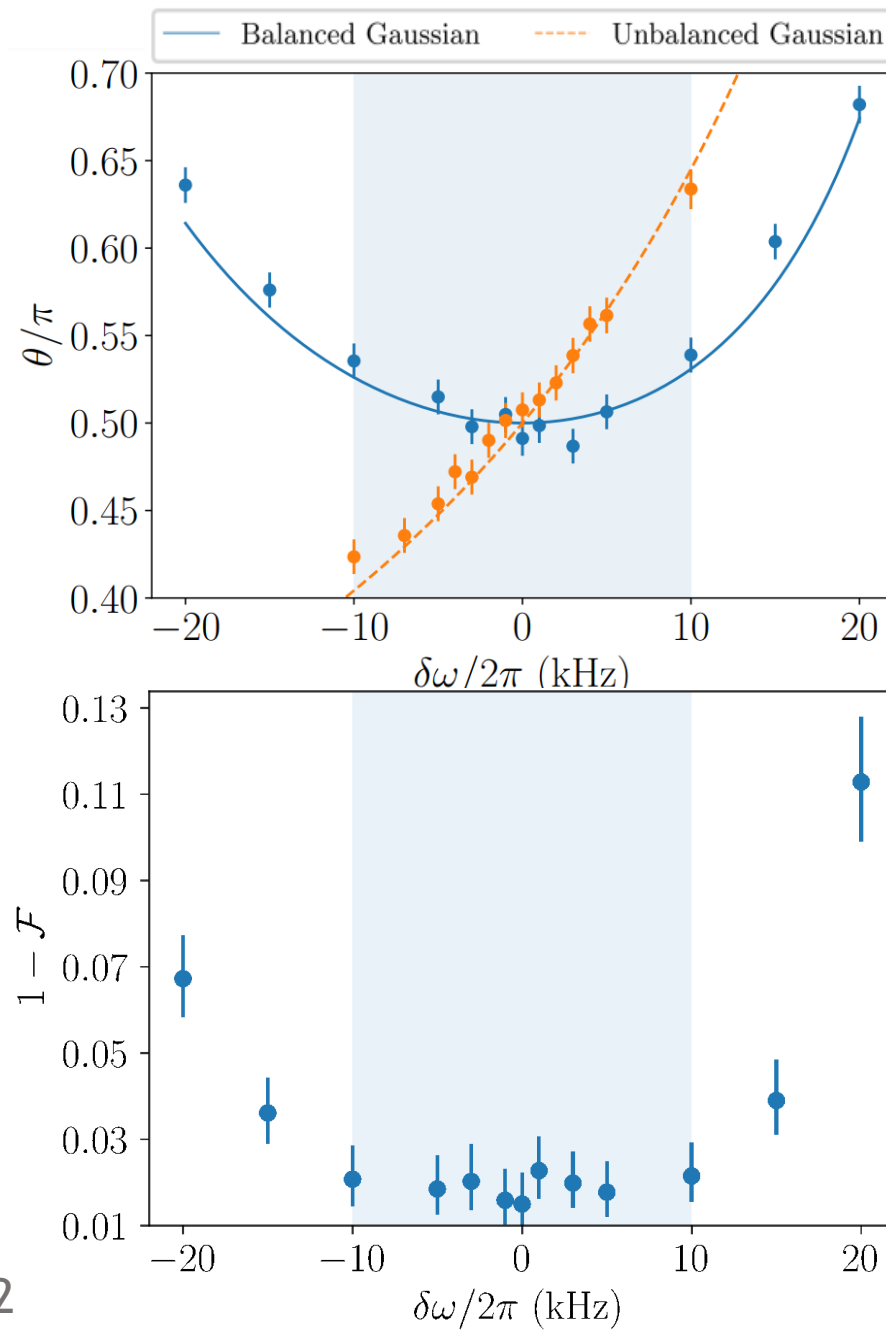
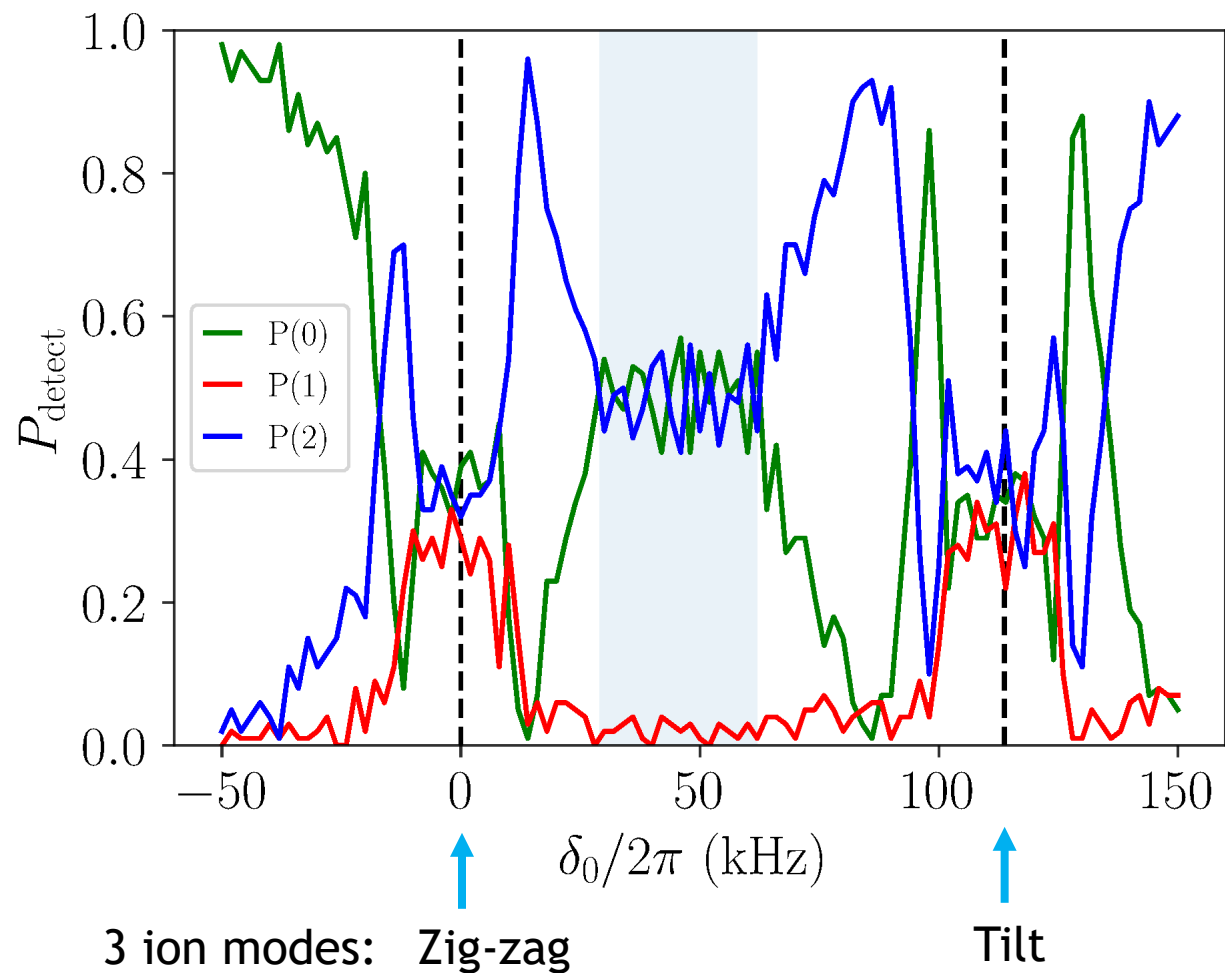
Pulse shape: **Balanced Gaussian**, **Gaussian**, **Square**

Gates are simple to implement: no need to optimize tons of pulse-shape parameters

Experiment shows balanced Gaussian is robust to ± 10 kHz trap frequency drift

Peak fidelity = 98.5(6)%

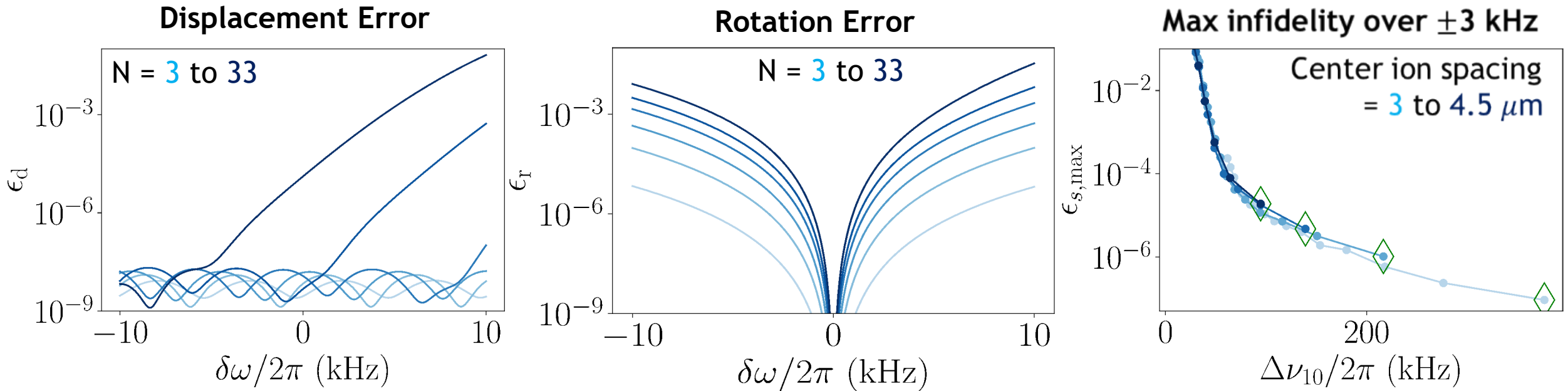
Drop in fidelity over ± 10 kHz < 1%



We find good prospects for scaling to more ions.



- Numerical simulations for chains of up to 33 ions
- Fixed center ion separation: results shown are for $3\ \mu\text{m}$



- Sensitivity (right plot) depends almost entirely on the splitting of nearest two modes

QSCOUT Team

Email: qscout@sandia.gov

Websites: qscout.sandia.gov; www.sandia.gov/quantum/trapped-ions

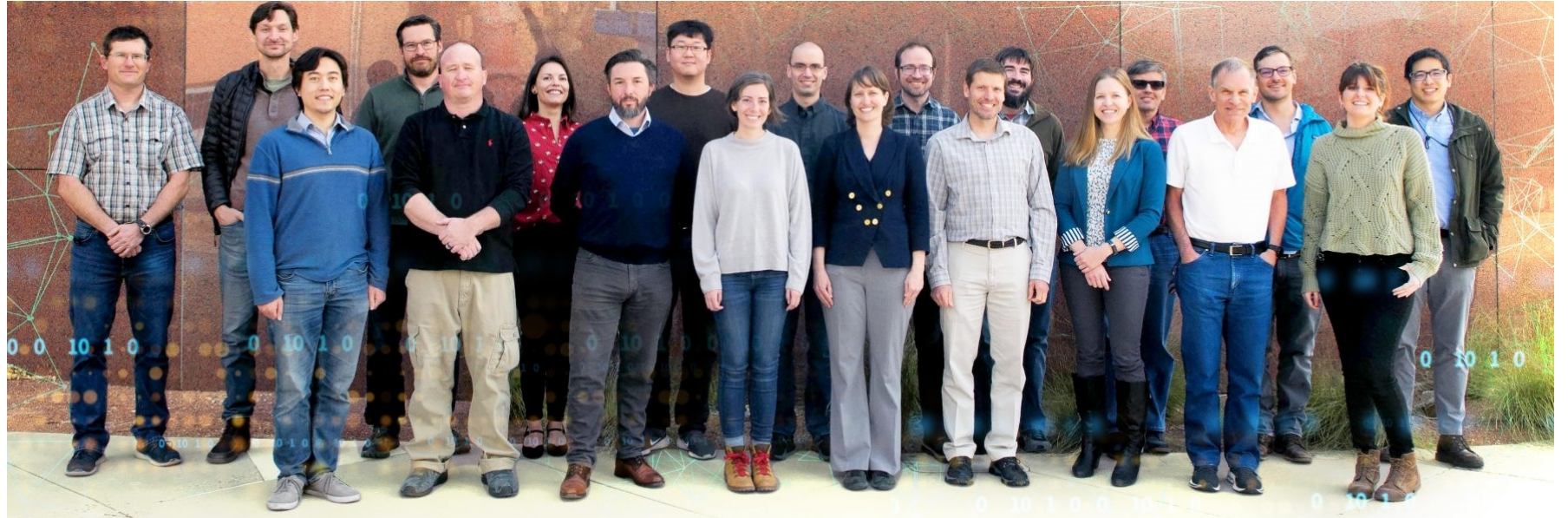
Jaqal: [github/jaqal](https://github.com/jaqal)



Brandon Ruzic



Susan Clark



Susan Clark (PI)

Ashlyn Burch

Matt Chow

Craig Hogle

Megan Ivory

Dan Lobser

Peter Maunz

Melissa Revelle

Dan Stick

Josh Wilson*

Chris Yale

*now at SDL

Brad Salzbrener

Madelyn Kosednar

Jessica Pehr

Ted Winrow

Bill Sweatt

Dave Bossert

Andrew Landahl

Ben Morrison

Tim Proctor

Kenny Rudinger

Antonio Russo

Brandon Ruzic

Jay Van Der Wall

Josh Goldberg

Kevin Young

Collin Epstein

Andrew Van Horn

Matt Blain

Ed Heller

Jason Dominguez

Chris Nordquist

Ray Haltli

Tipp Jennings

Ben Thurston

Corrie Sadler

Becky Loviza

John Rembetski

Eric Ou

Matt Delaney

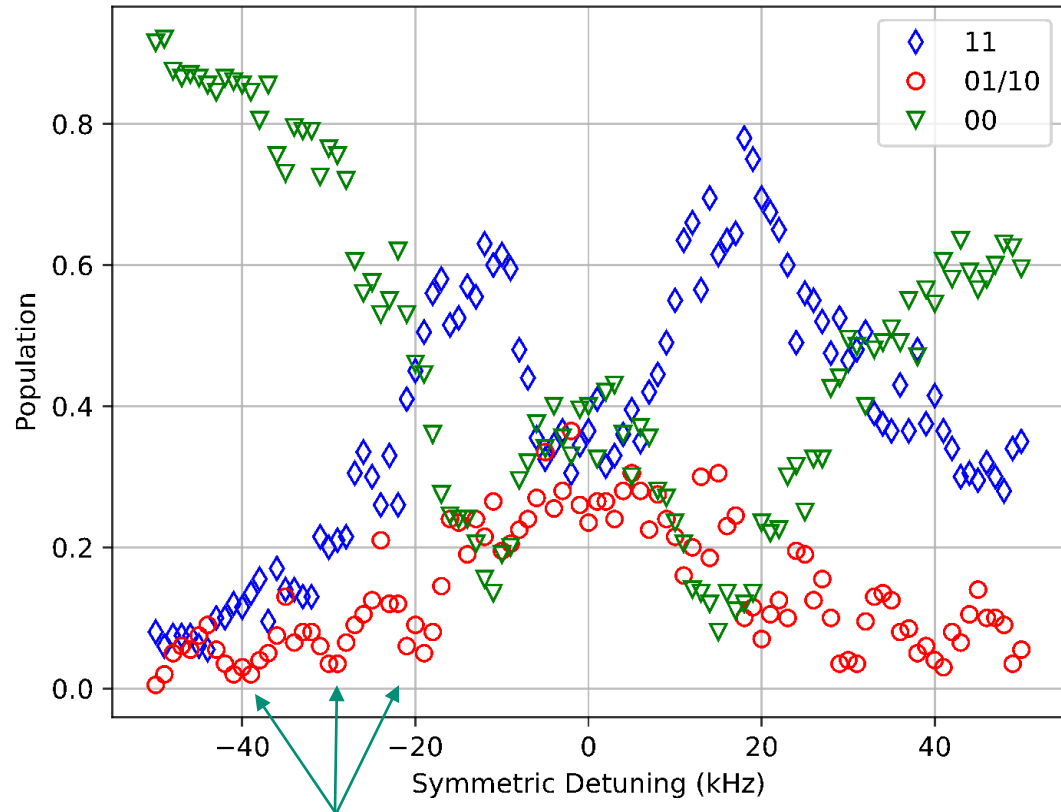
arXiv : 2210.02372

Extra Slides Beyond

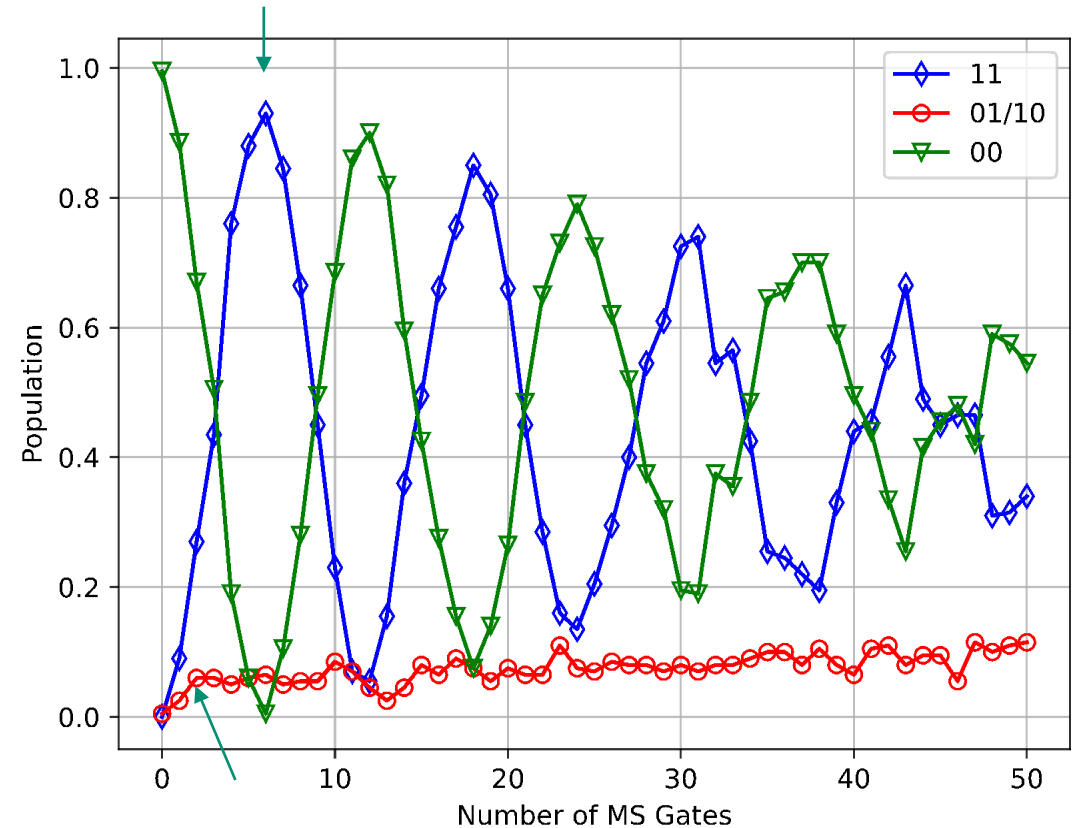


The Standard (square, constant frequency) MS gates have issues.

Ideally, the gate generates flopping between $|00\rangle$ and $|11\rangle$ - never populating odd parity states.



- Odd parity population does not go to zero.
- Narrow acceptable detuning range

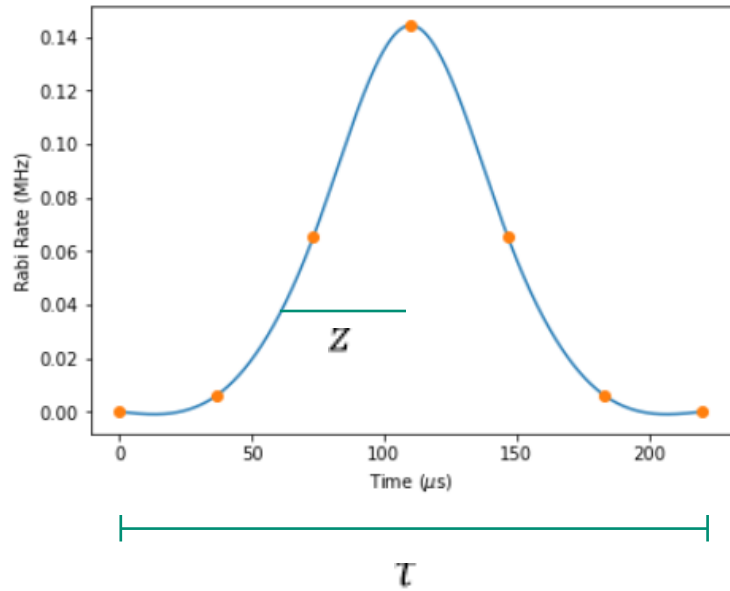


- *Best Fidelity* $\approx 96.4\%$, estimated from the max 11 population
- Odd parity population persistent



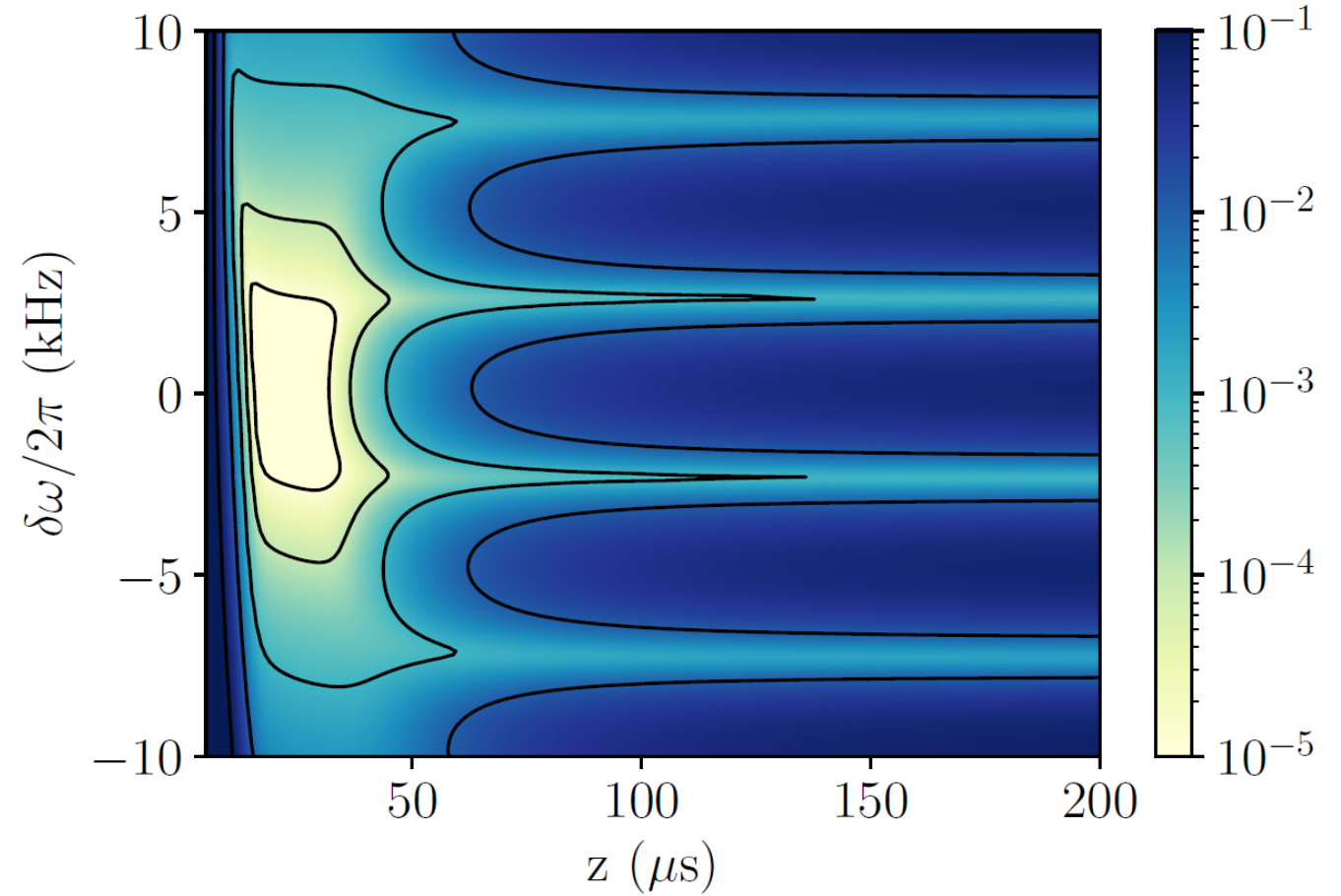
QSCOUT

Gaussian pulse parameters



z : time-domain standard deviation

τ : gate time, truncation window

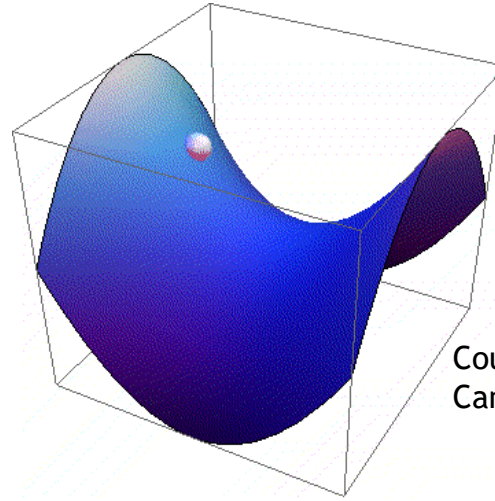


RF Traps for Ions



Trapping requirement: A restoring force when displaced from trap center
(in any direction)

Cannot use
t
Field lines
start/en

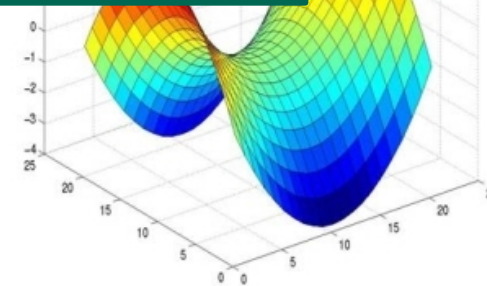
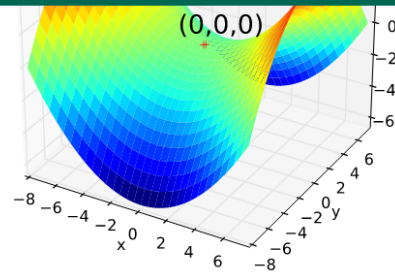


Courtesy of Wes Campbell

ut” and
rections



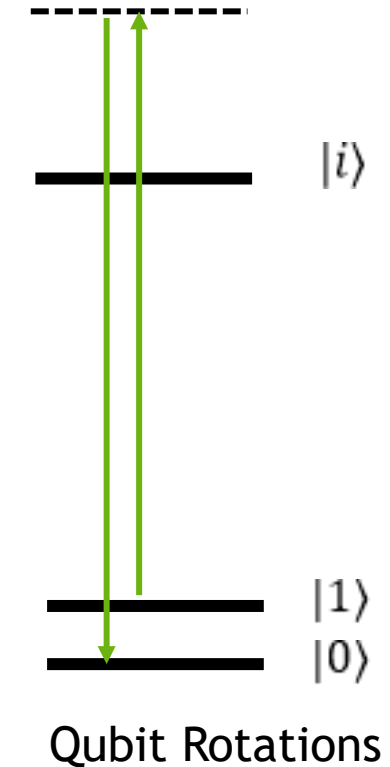
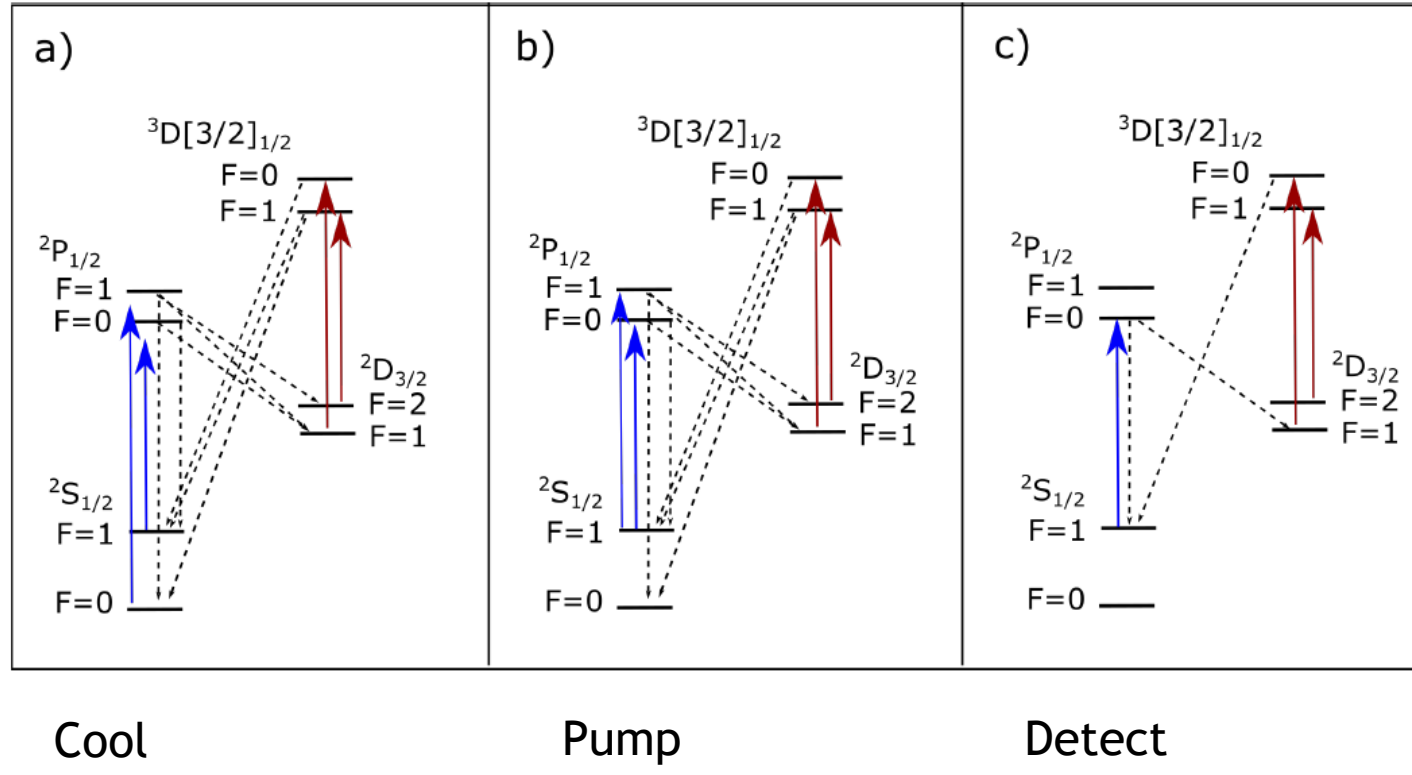
Before ion escapes, field reverses!



Crash Course in Trapped Ion Quantum Computing

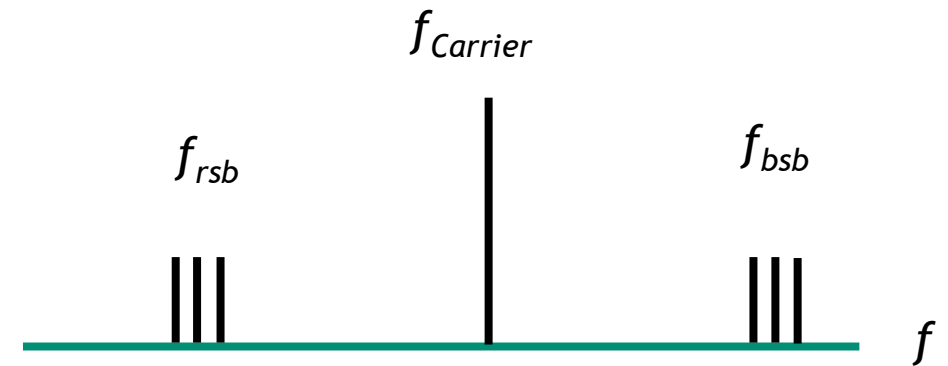
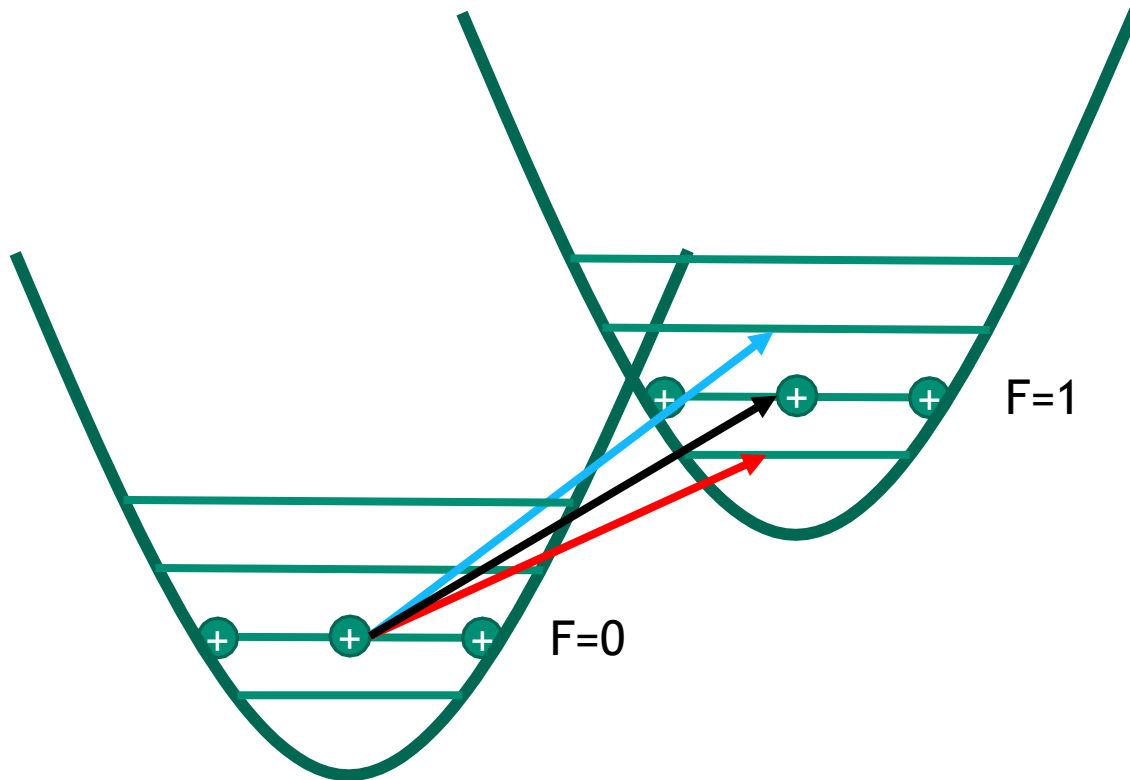


$^{171}\text{Yb}^+$



- Sidebands on cooling laser (370nm) allow incoherent control processes
- Pulsed laser (355nm) Raman transitions for qubit rotations

Shared motional modes mediate entangling interactions



Red sideband: $n-1$
Remove a phonon

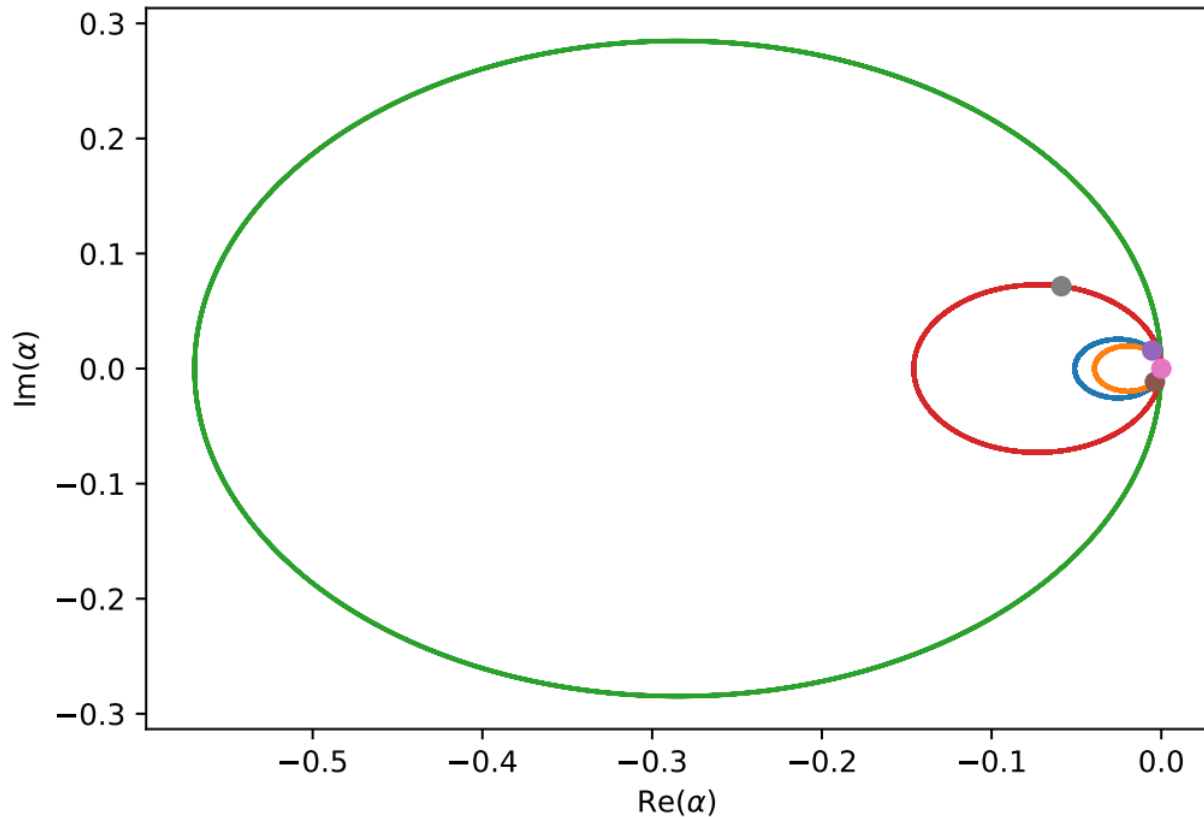
Blue sideband: $n+1$
Add a phonon

- Motional sidebands at the trap frequency
- Number of modes \propto Number of ions
- Coulomb interaction couples ions together
 - Vibrational levels act as “bus” connecting qubits

Intuition for Phase Space Trajectories (PSTs)



PST for Square MS Gate



How pulse parameters come into play:

- Detuning controls the angular velocity and radius in phase space.
 - Smaller detuning \rightarrow Bigger, slower circles
- Rabi rate controls only radius.
- Phase can change the direction of rotation.

Figures of merit:

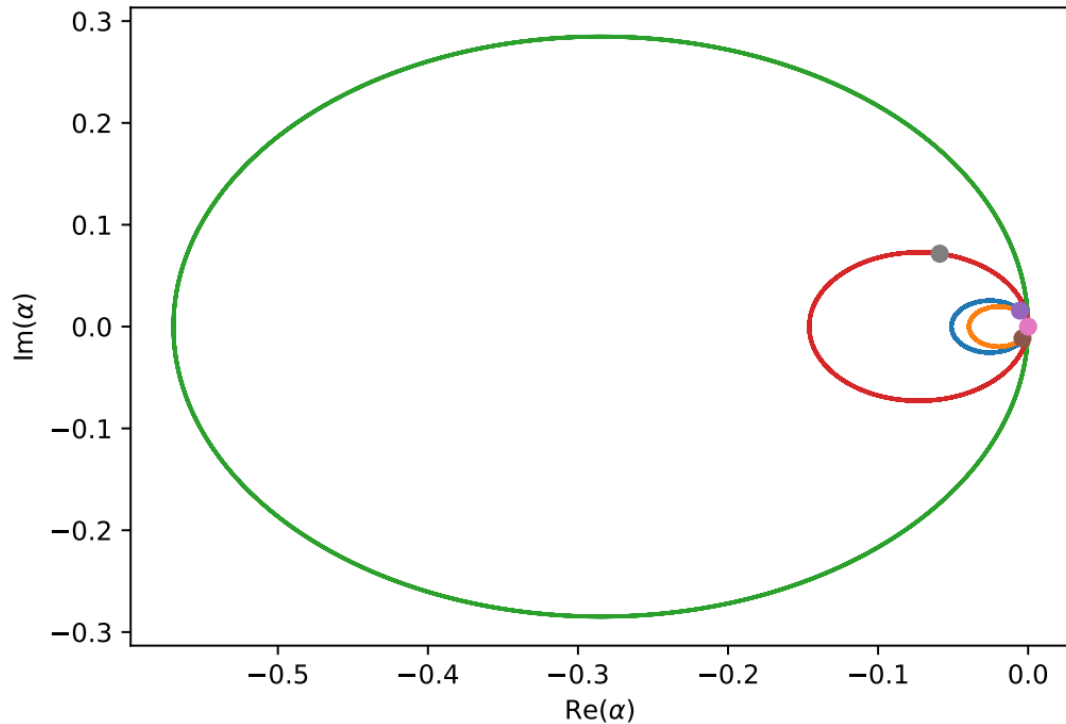
- Loop closure of each mode: $\alpha_k(0) = \alpha_k(t_{gate})$?
- Area enclosed: Is the gate angle right?

Gaussian MS Gates Show Better Loop Closure than Square MS

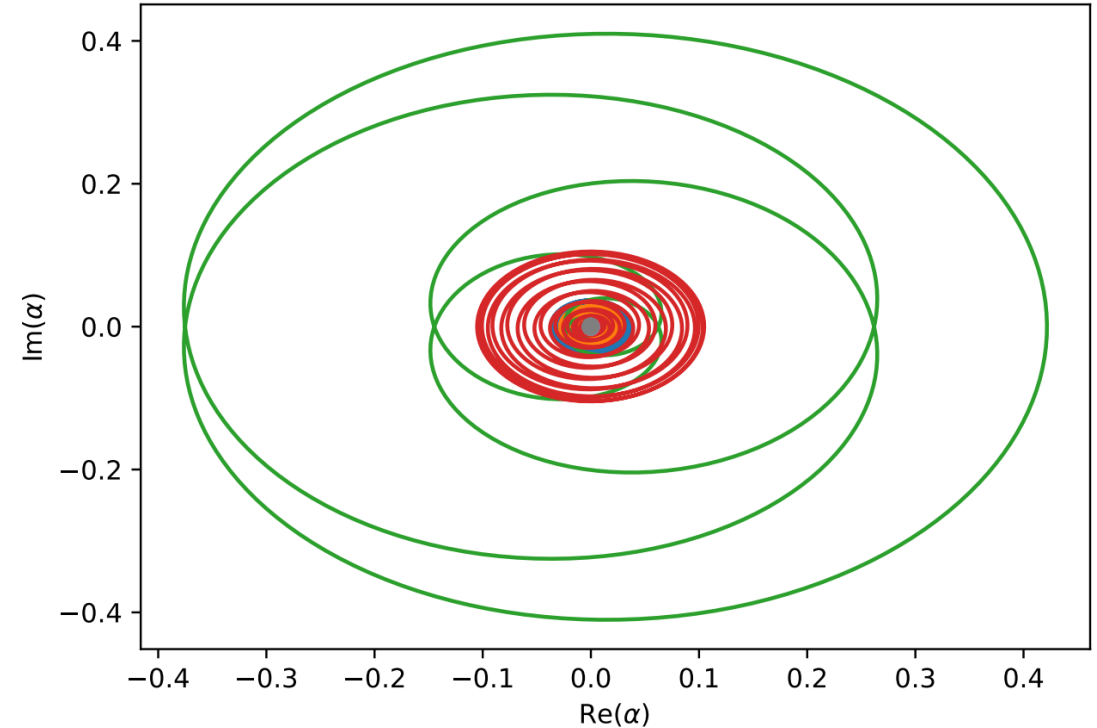


QSCOUT

Square Gate



Gaussian Gate



Gate parameters (for both):

- Detuning is -40 kHz below the lowest frequency mode in a 2 ion chain.
- 200 μs duration

Notable differences:

- All modes end close to the starting point for Gaussian gate.
- Time averaged displacement close to zero for the Gaussian gate.

Displacements During the MS Gate



Interaction Hamiltonian:

$$\hat{H}_I = \frac{\hbar\eta\Omega}{2} \left(\hat{a} e^{-i(\delta t + \phi^M)} + \hat{a}^\dagger e^{+i(\delta t + \phi^M)} \right) \hat{\sigma} \cdot \phi^S$$

State dependent drive force gives coherent displacement by α_k on mode k [1]:

$$\hat{D}(\hat{a}_k) = \exp(\hat{a}_k a_k^\dagger - \hat{a}_k^\dagger a_k)$$

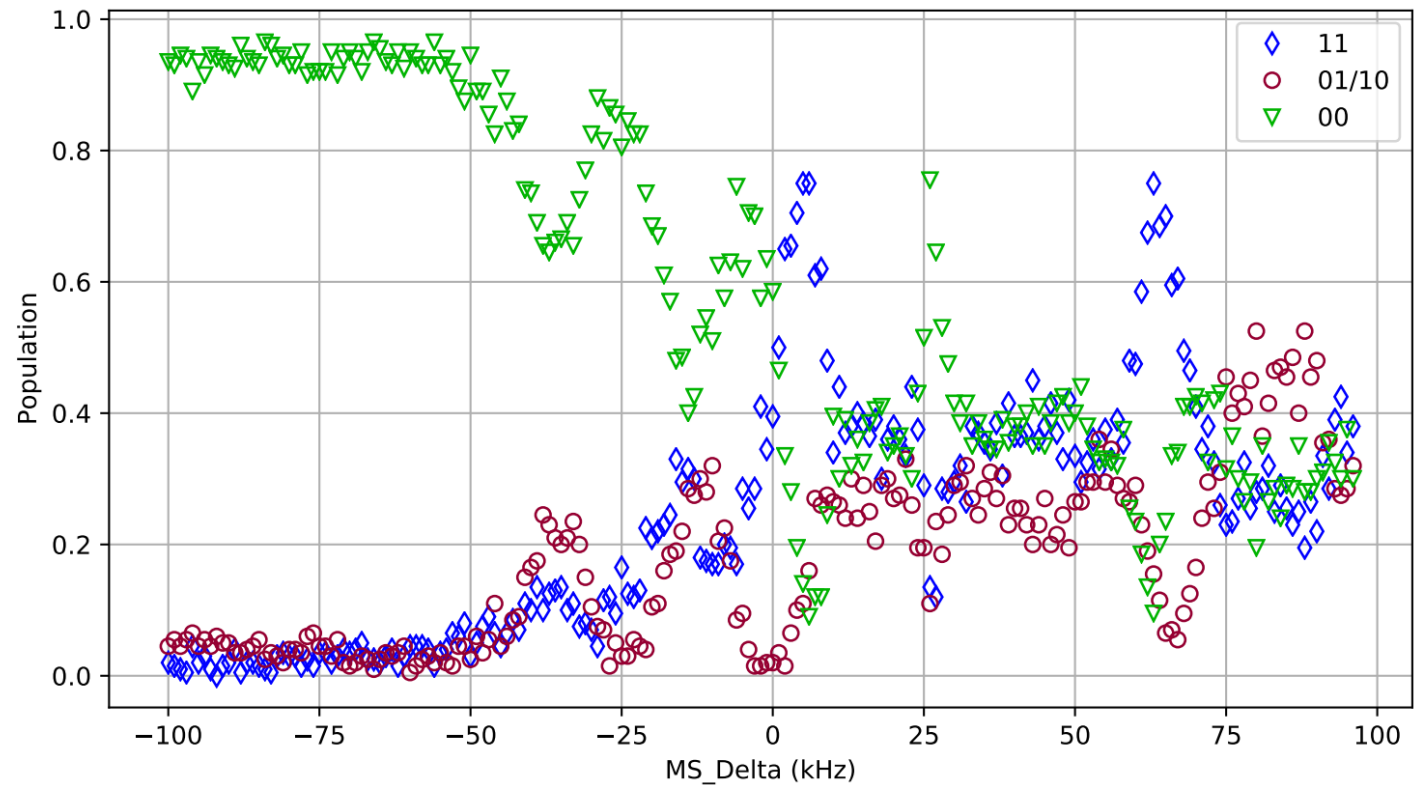
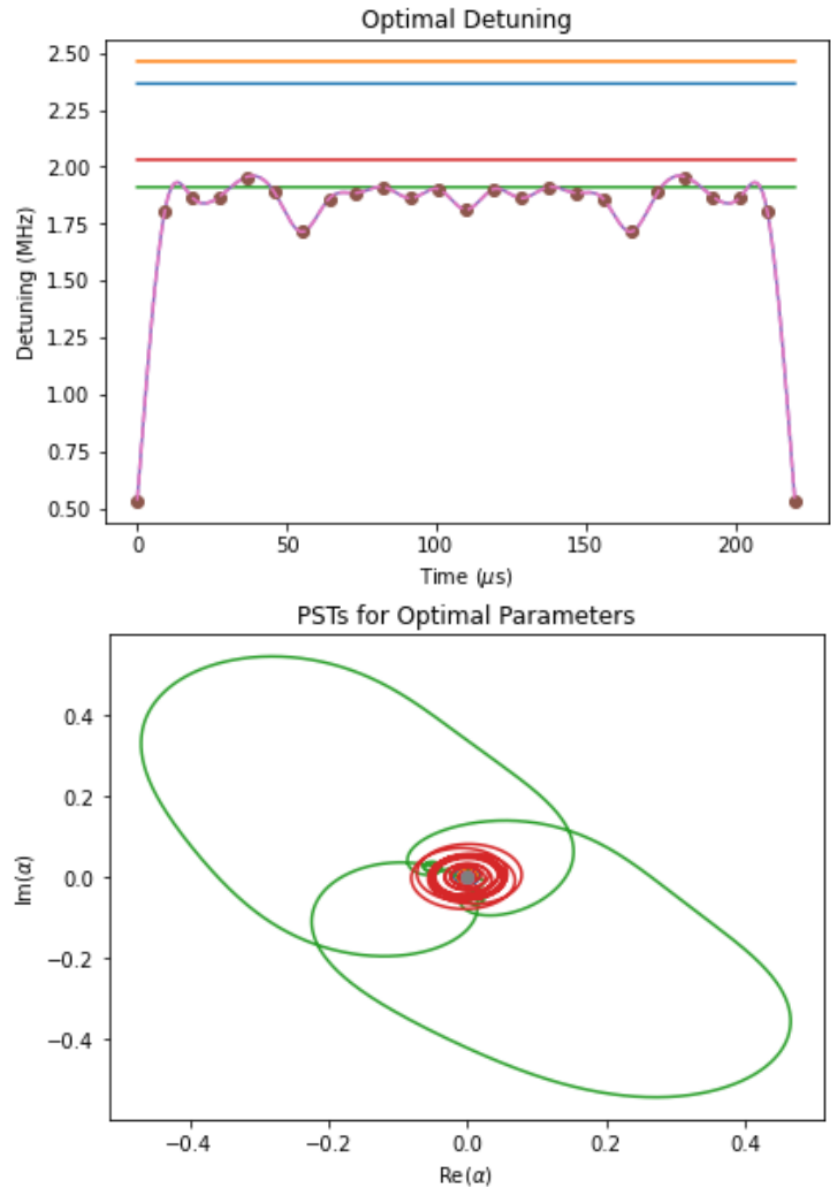
In the simplest case (+1 eigenstate of $\hat{\sigma}_\phi$, no laser phase difference) [1]:

$$\alpha_k = \frac{\Omega}{2} (\eta_{i,k} + \eta_{j,k}) \int_0^t e^{i\theta_k(t')} dt' \text{ where } \theta_k(t') = \int_0^{t'} \delta_k(t'') dt'' \text{ is the accumulated phase}$$

So, for a constant detuning and Rabi rate, we get circular trajectories in phase space, with closure condition: $t_{gate} = 2\pi m / \delta_k$

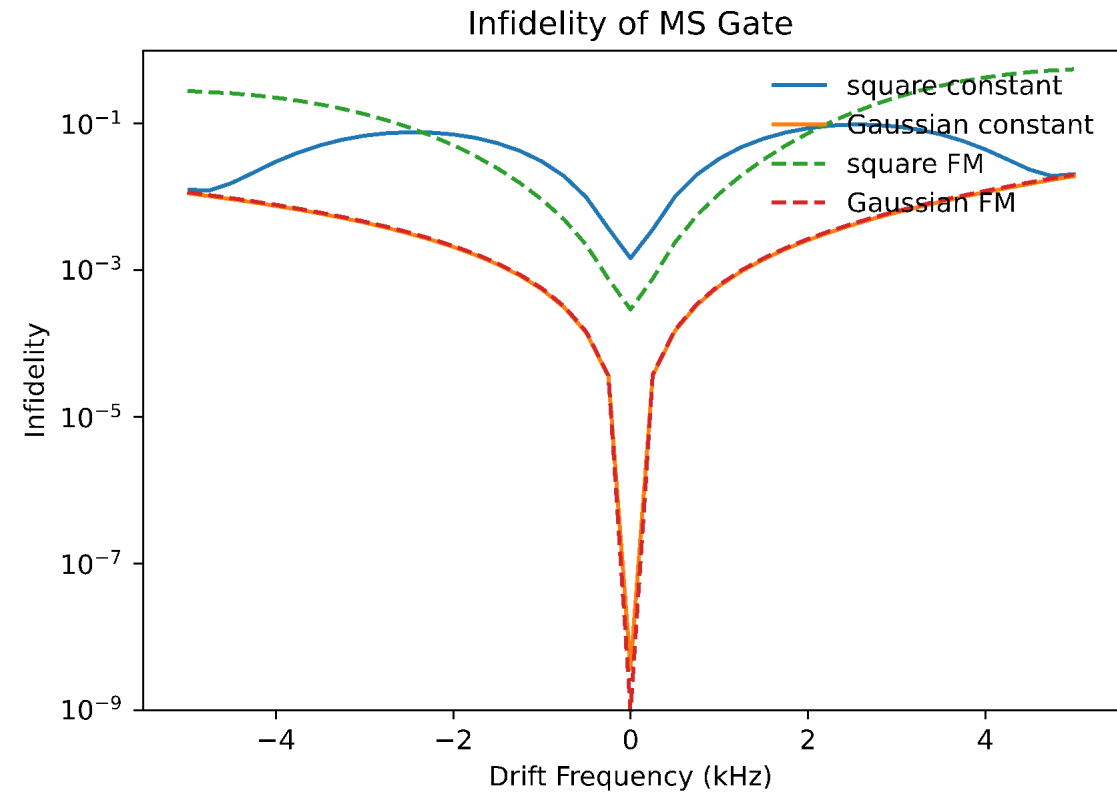
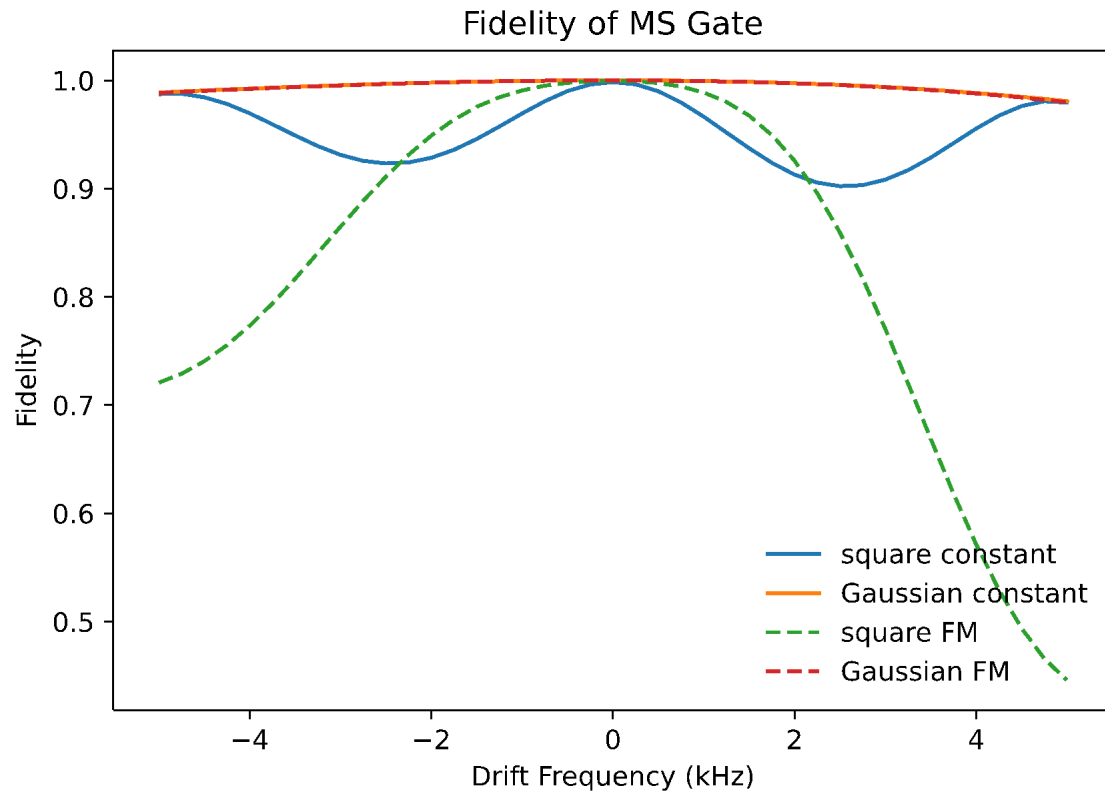
Closed loops are required for spin-motion disentanglement at the end of the gate.

FM and Constant Frequency Gaussians Are Comparable



- See essentially the same odd parity population near optimized detuning.
- Area enclosed still has roughly the same sensitivity (slope of 11/00 crossing) as constant frequency case.

Gaussian Pulse Shape Dominates Gate Performance (with Current Cost Functions)



Next steps:

- Further development of gate angle cost function
- Batch optimization
- Transfer function optimization
- Larger chains

Optimizer/Data Comparison For FM Gaussian

