



Meshfree modeling of coupled thermal-mechanical-chemical phenomena in energetic aggregates



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Energetic Materials: A Modeling Challenge



- ❖ Complex material structure
- ❖ Chemically reactive (fast, exothermic)
- ❖ **Everything** is a function of temperature

Multi-
Physics!

Plastic Bonded Explosive [Rae, 2002]

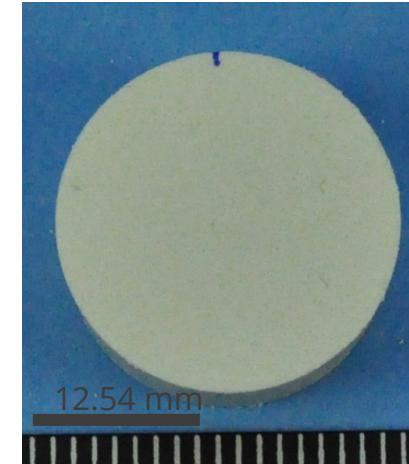
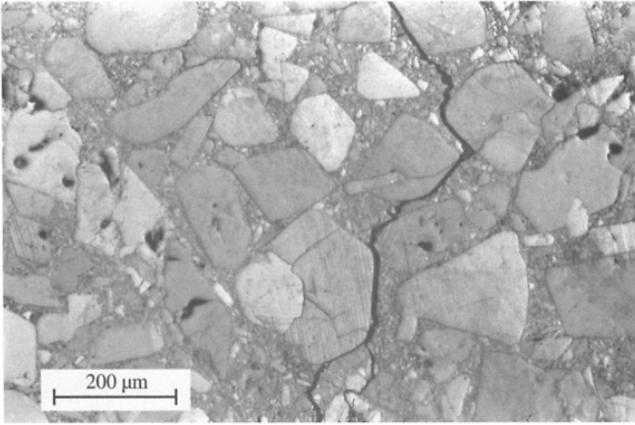
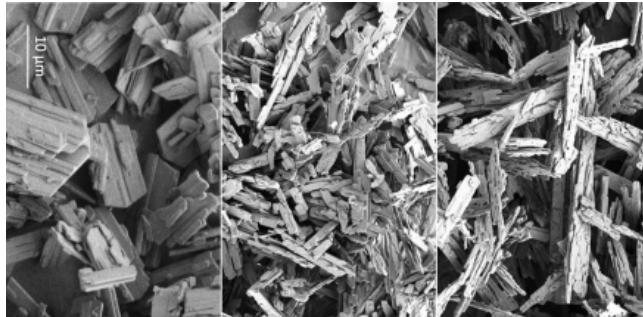
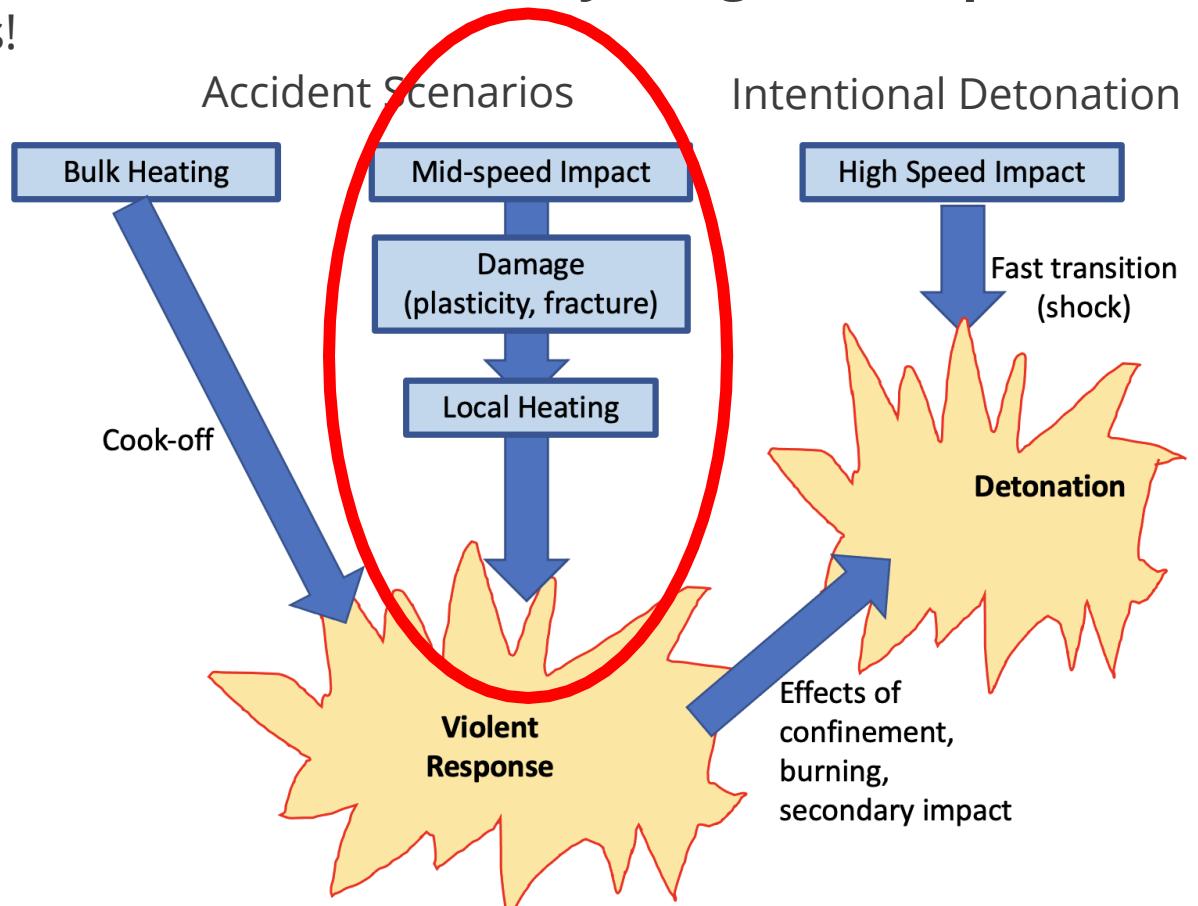


Image: courtesy Marcia Cooper

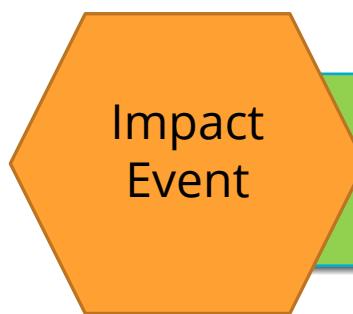


Energetic Crystals [Yarrington, 2018]

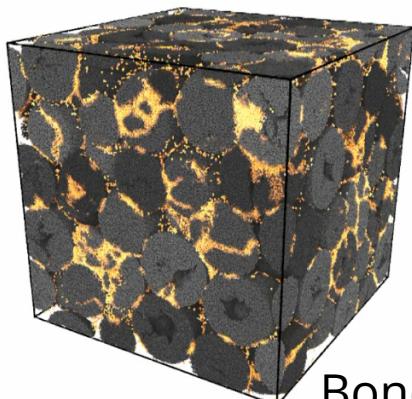
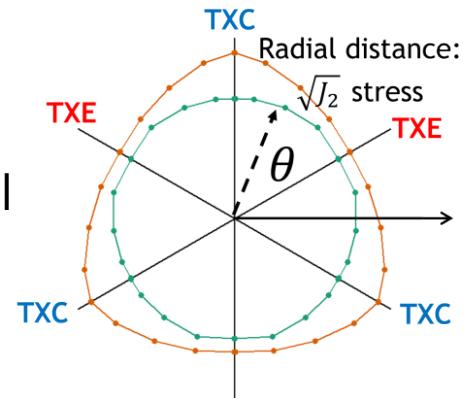
A few different ways to get an explosion...



Overall Multiscale Approach



Continuum
Constitutive Model



Bonded Particle
Methods

Mechanical Deformation
& Damage

Local Hotspot
Formation

Interface
Debonding

Plasticity

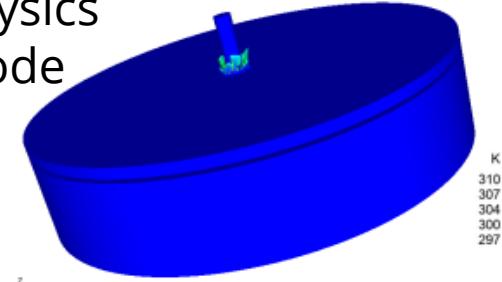
Crystal Fracture

Friction

Particle Rotation

Viscoelastic
Heating

Multiphysics
CRK code



Heat Conduction

Thermally Activated
Chemistry

Go

?

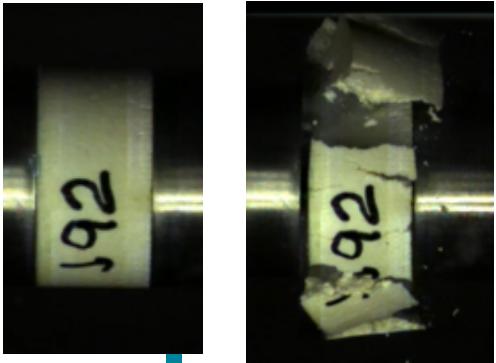
No Go

The Case for Meshfree Methods



Numerical Method Should Accurately Predict:

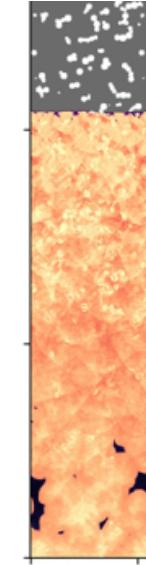
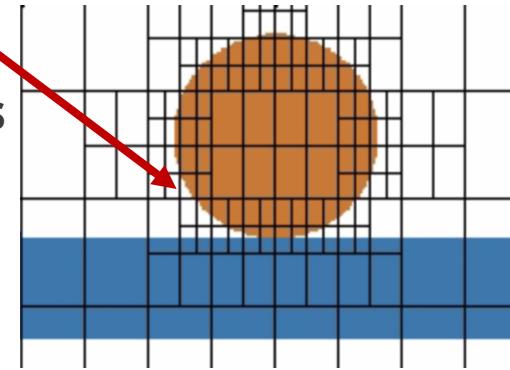
- Capture transition from solid to rubble
- Deformation-induced heating, chemistry



Example: Impact Test,
Marcia Cooper

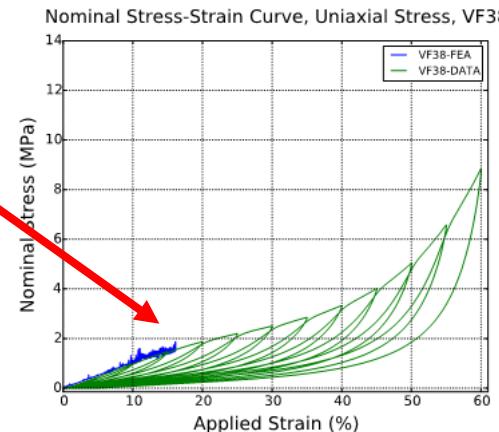
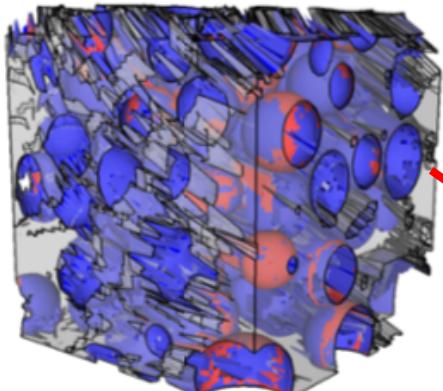
Problem: poorly resolved strain fields and interface physics, averaging in mixed material cells

Hydrocode Methods



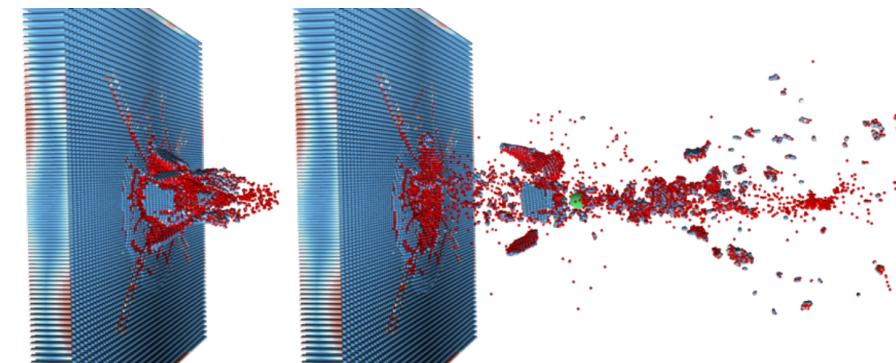
Mesh-based Methods (FEA)

Problem: Mesh entanglement at large deformations



Meshfree Methods

Show promise in overcoming these problems at both meso and macro scales



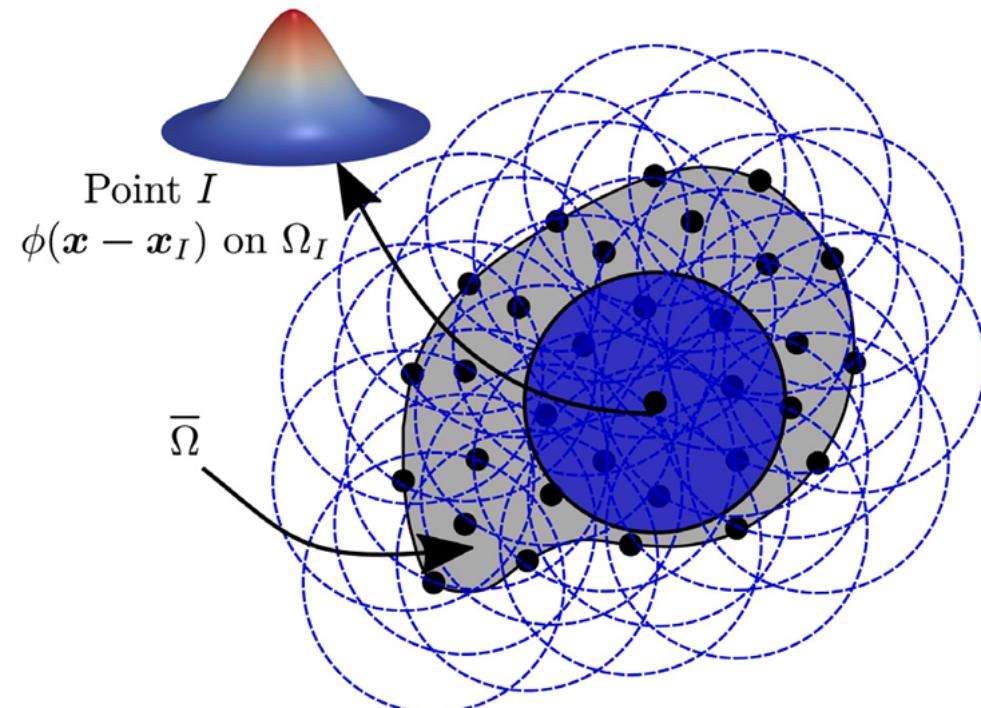
Meshfree Conforming Reproducing Kernel Method



Reproducing Kernel Particle Method

- Galerkin-based variational method using the reproducing kernel discretization
- Shape functions are the product of a window/kernel function and correction function

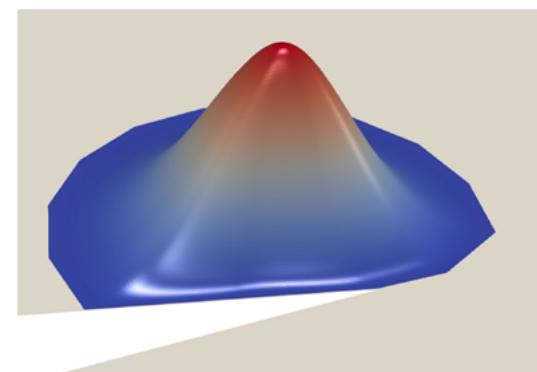
$$u^h(x) = \sum_{I=1}^{NP} \Psi_I d_I; \quad \Psi_I = C(x; x - x_I) \phi_a(x - x_I)$$



J. Koester, J.S. Chen, Comput. Methods Appl. Mech. Engrg. 347 (2019) 588-621

Conforming Reproducing Kernel

- Graph distance informed window/kernels replace traditional Euclidian kernels to provide improved accuracy and robustness for nonconvex geometries and essential boundary conditions



Thermo-mechanical-chemical coupling in CRK Multiphysics



CRK-Thermal implemented to simultaneously solve momentum

$$\int_{\Omega} \mathbf{w} \cdot \rho \ddot{\mathbf{u}} d\Omega + \int_{\Omega} \mathbf{F}(\nabla \mathbf{w}) : \mathbf{P}(\nabla \mathbf{u}) d\Omega = \mathbf{f}^{\text{ext}}(\mathbf{u}),$$

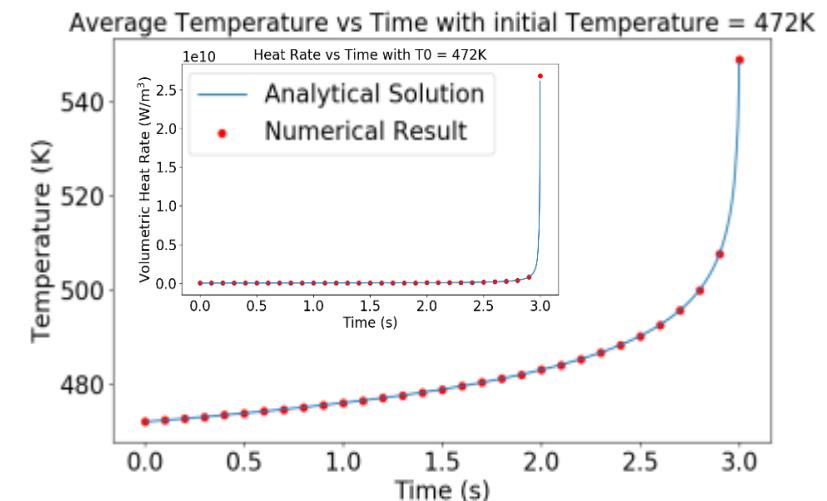
and conservation of energy

$$\int_{\Omega} w \rho C_P \dot{T} d\Omega + \int_{\Omega} \nabla w \cdot K \nabla T d\Omega = \int_{\Omega} w \left(\dot{q}^{\epsilon^P} + \dot{q}^{\text{species}} \right) d\Omega + \int_{\partial\Omega} w h d\Gamma.$$

*Thermal
conduction*

*Adiabatic heating from
material plasticity*

Chemical heating



$$Q = \rho \Delta H Z e^{-E_a/RT}$$

➤ Viscoplastic-ViscoSCRAM constitutive model

- Viscoelasticity
- Cracking damage (Statistical Crack Mechanics)
- Pressure-dependent viscoplasticity with Drucker-Prager yield surface

- Chemical heating from exothermic decomposition
 - Currently restricted to Arrhenius rate
 - More sophisticated models in progress

Thermo-chemical coupling verification



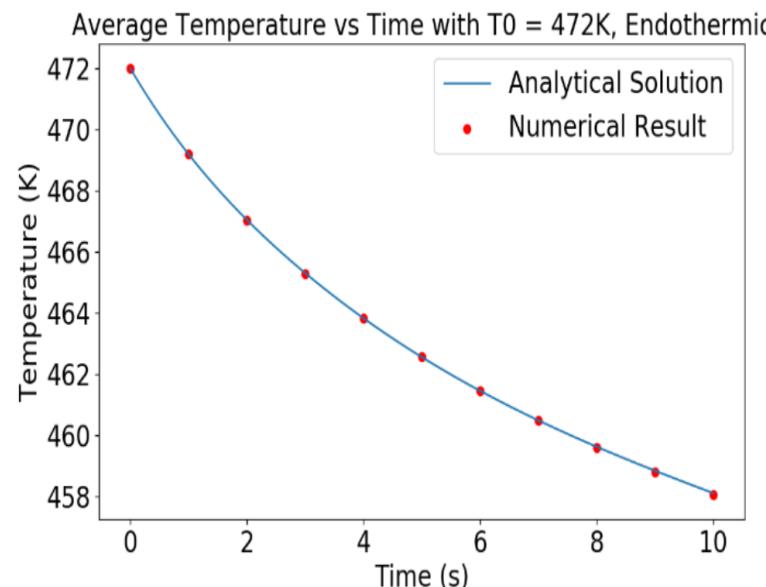
- Temporal verification based on Frank-Kamenetskii equation[1]:

$$-\lambda \nabla^2 T + \rho C \frac{dT}{dt} = \rho \Delta H Z e^{-E_a/RT}$$

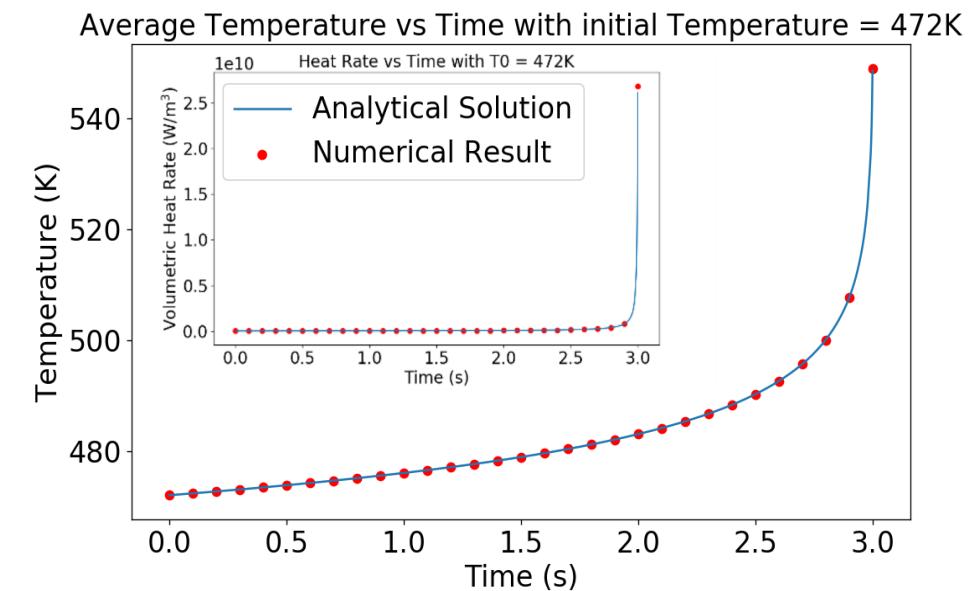
Uniform temperature change
(temporal variation only) →

$$\rho C \frac{dT}{dt} = \rho \Delta H Z e^{-E_a/RT}$$

Endothermic Process



Exothermic Process



Thermo-chemical coupling verification



- Verification test based on Frank-Kamenetskii critical temperature[1]:

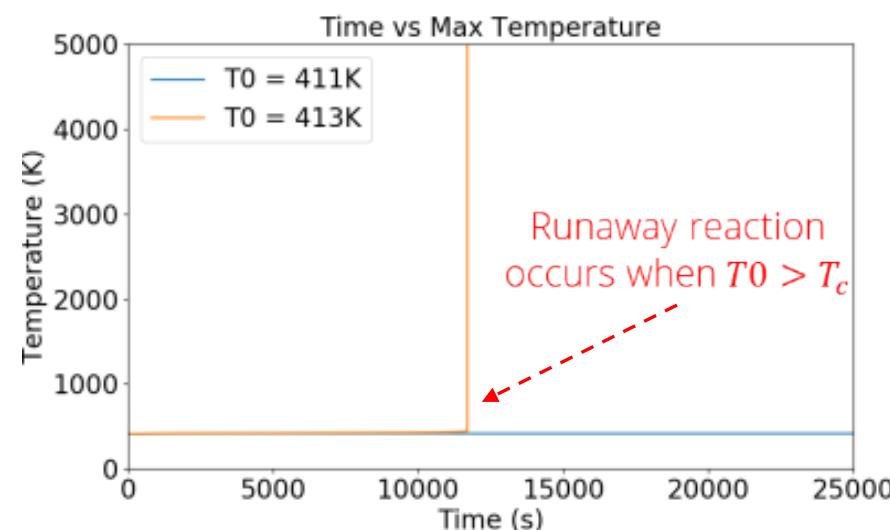
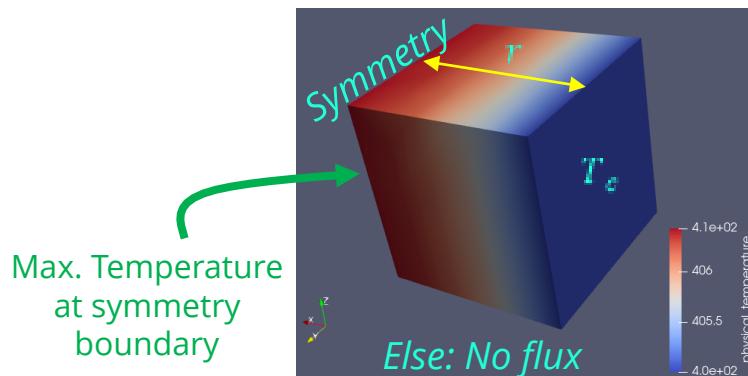
$$-\lambda\nabla^2T + \rho C \frac{dT}{dt} = \rho\Delta HZe^{-E_a/RT}$$

Steady state solution (spatial variation only)

$$-\lambda\nabla^2T = \rho\Delta HZe^{-E_a/RT}$$

$$\frac{E_a}{T_c} = R \ln\left(\frac{r^2\rho\Delta HZE_a}{T_c^2\lambda\delta R}\right)$$

Cubic domain with no flux boundary except one surface with constant T_c (infinite slab)



Solve for critical ambient temperature (T_c) before runaway reaction¹

Analytical Critical Temperature:
~412K

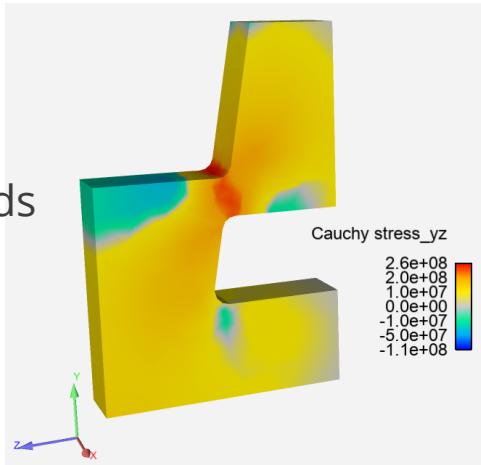
Shear-Induced Heating in Steel



- Top-hat geometry designed to induce high localized shear
- Material properties: steel

Shear Stress

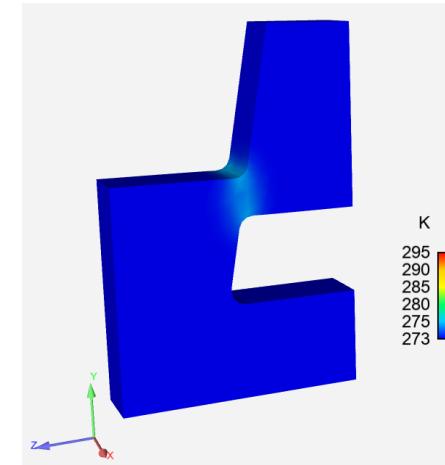
Time = 65
microseconds



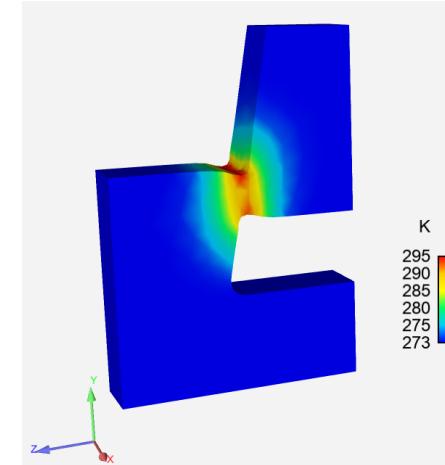
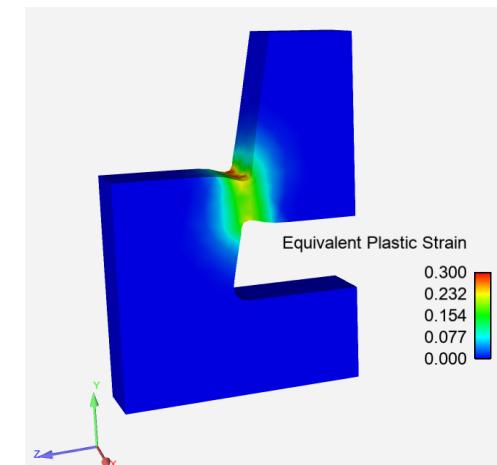
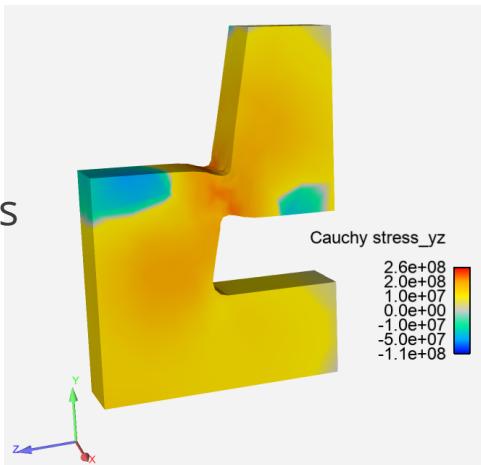
Equivalent Plastic Strain



Temperature



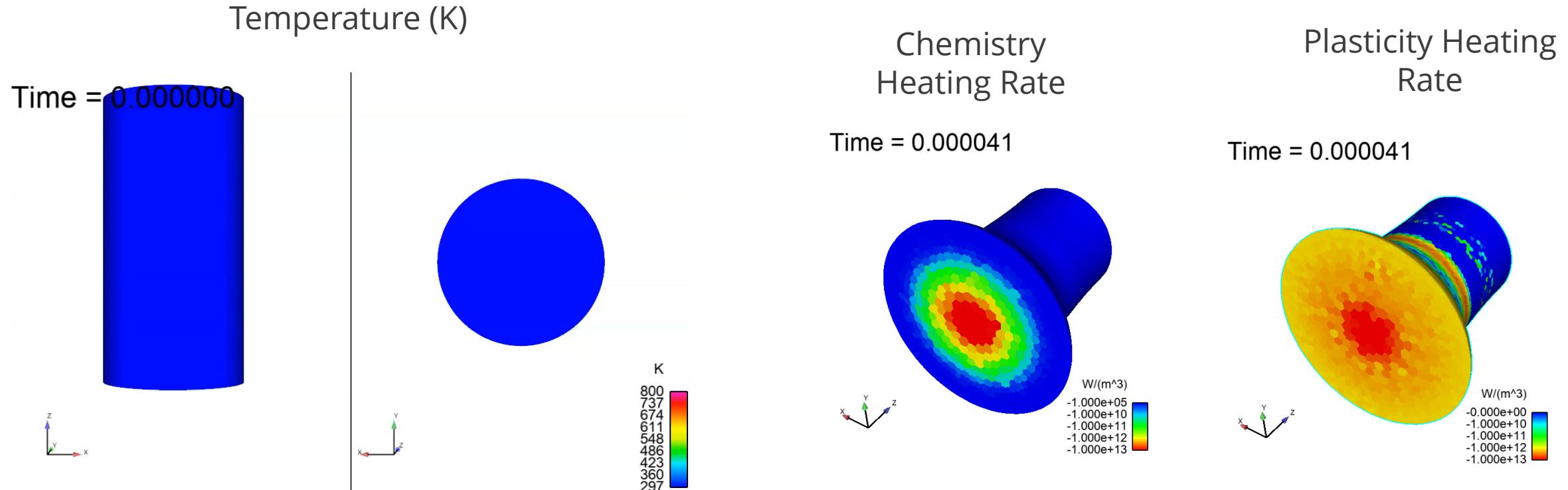
Time = 250
microseconds



Thermal Runaway in Taylor Bar Impact: 450 m/s



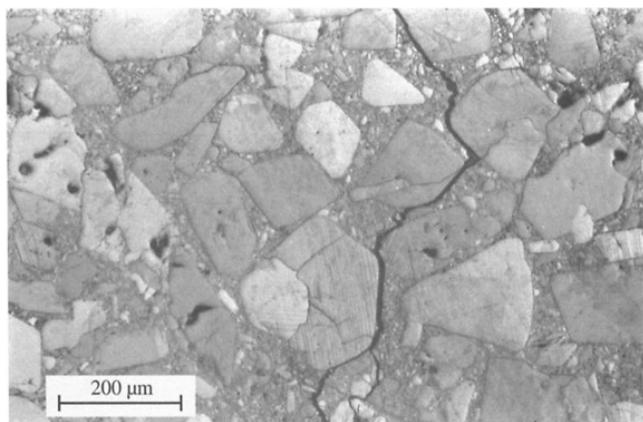
- Material properties: pure energetic crystals
- Energy dissipated due to plastic deformation raises temperature enough to start runaway chemistry



Now we consider energetic aggregate materials



- Composite mechanical behavior is complex:
 - Strain rate dependent
 - Temperature dependent
 - Pressure dependence
 - Tension-Compression Asymmetry
 - Many inelastic deformation mechanisms: Viscoelasticity (binders), Cracking (intra- and inter-granular), Porosity opening, dislocation slip, twinning (some energetic crystals)



Plastic Bonded Explosive [Rae, 2002]

How to represent various inelastic mechanisms in a macroscale model?

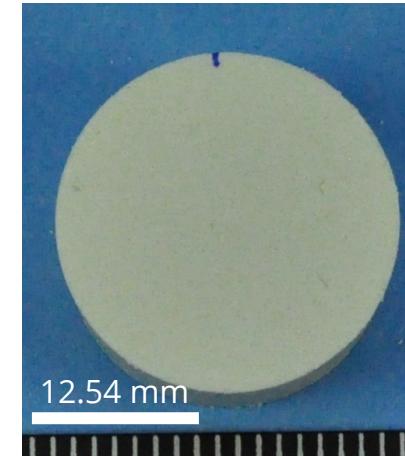
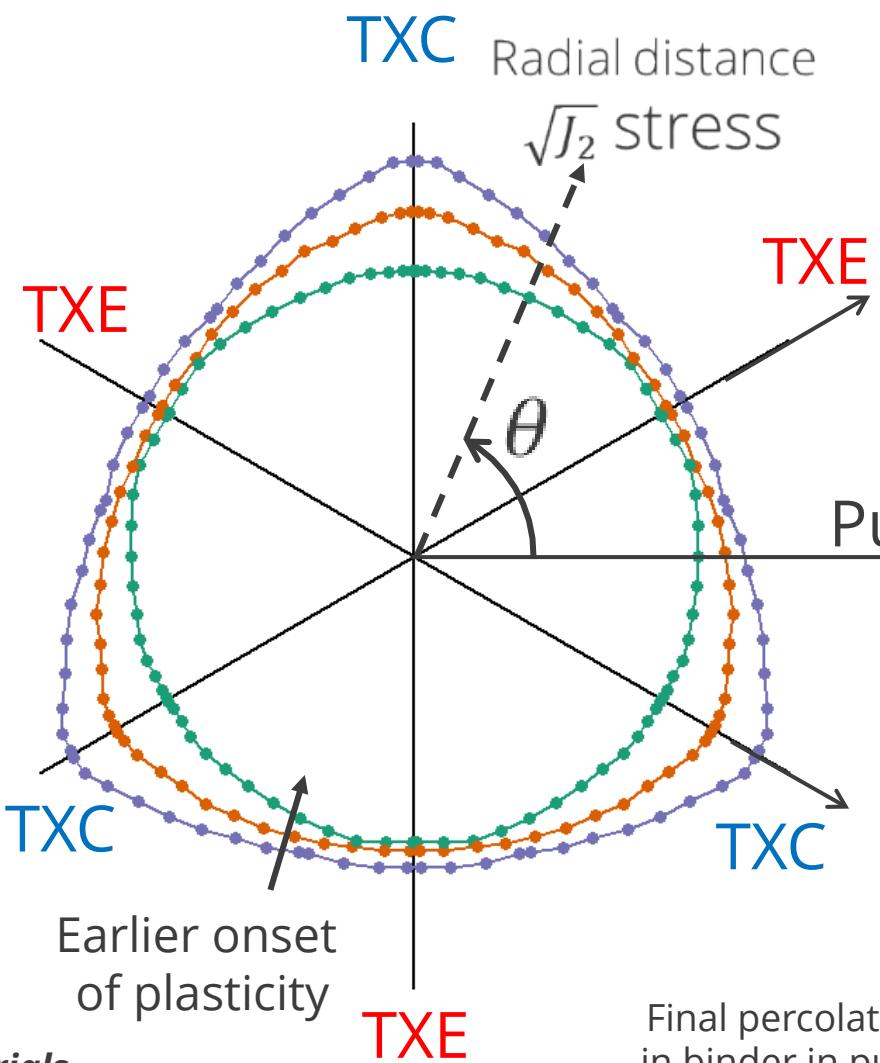
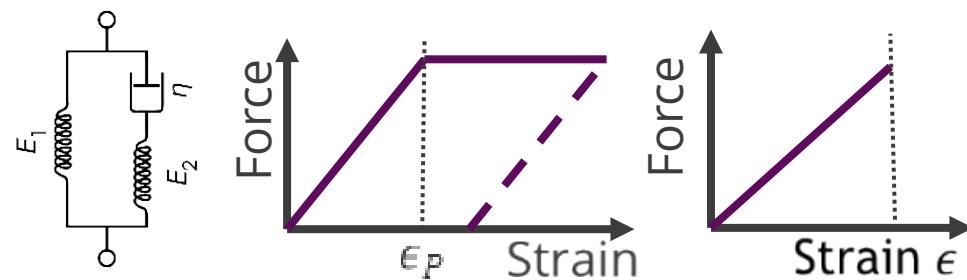


Image: courtesy Marcia Cooper

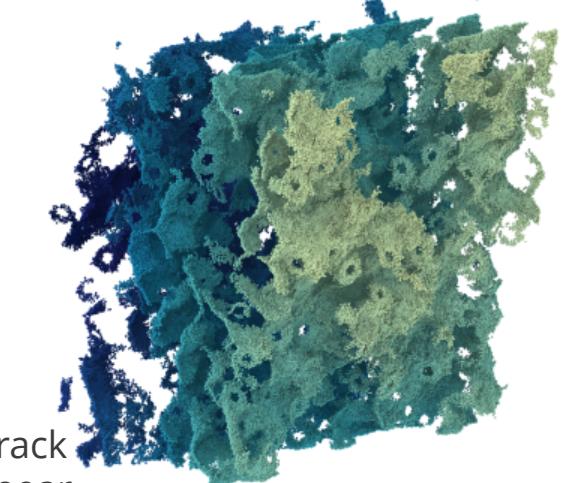
Mesoscale studies to inform continuum models



- Map out yield surfaces, often assumed to have simple shape (e.g. Drucker-Prager)
- Testing how changes in binder's material properties impact yield surface
 - better understanding of inelastic yielding for continuum models



Pure Shear

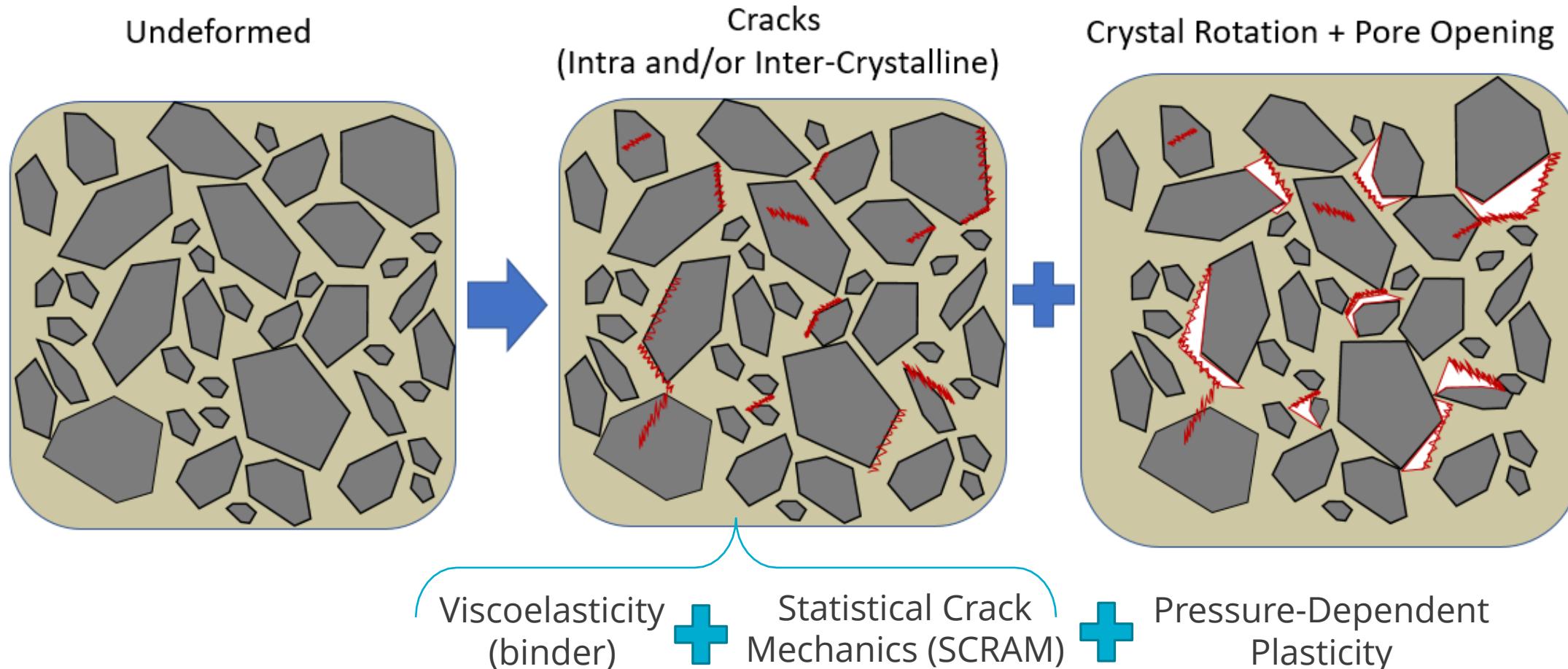


Final percolating crack in binder in pure shear

Hypothesized Mesoscale Deformation Mechanisms



- Simplified view of mesoscale processes and macroscale interpretation



ViscoPlastic-ViscoSCRAM Model Theory



➤ Kinematics:

$$\boldsymbol{\epsilon} = \boldsymbol{e} + \frac{1}{3} \epsilon_{\text{vol}} \mathbf{I}$$

$$\sigma_m = K \epsilon_{\text{vol}}$$

➤ Viscoelasticity

$$\dot{\boldsymbol{s}} = 2G^\infty \dot{\boldsymbol{e}}^{ve} + \sum_{\kappa=1}^N \left(2G^{(\kappa)} \dot{\boldsymbol{e}}^{ve} - \frac{\boldsymbol{s}^{(\kappa)}}{\tau^{(\kappa)}} \right)$$

Prony series of shear moduli and relaxation times

$$\dot{\boldsymbol{s}}^{(\kappa)} = 2G^{(\kappa)}(\dot{\boldsymbol{e}} - \dot{\boldsymbol{e}}^p) - \frac{\boldsymbol{s}^{(\kappa)}}{\tau^{(\kappa)}} - \frac{G^{(\kappa)}}{G_0} \left[\frac{3}{a} \left(\frac{c}{a} \right)^2 \dot{c} s + \left(\frac{c}{a} \right)^3 \dot{\boldsymbol{s}} \right]$$

$$\boldsymbol{e} = (\boldsymbol{e}^{ve} + \boldsymbol{e}^D) + \boldsymbol{e}^p$$

➤ SCRAM Damage

$$e^D = \frac{1}{2G_0} \left(\frac{c}{a} \right)^3 s$$

$$\dot{c} = \begin{cases} v_{\text{res}} \left(\frac{K_I}{K_1} \right)^m & \text{for } K_I < K' \\ v_{\text{res}} \left[1 - \left(\frac{K_0 \mu}{K_I} \right)^2 \right] & \text{otherwise} \end{cases}$$

$$D = \frac{\left(\frac{c}{a} \right)^3}{1 + \left(\frac{c}{a} \right)^3}$$

➤ Drucker-Prager Plasticity

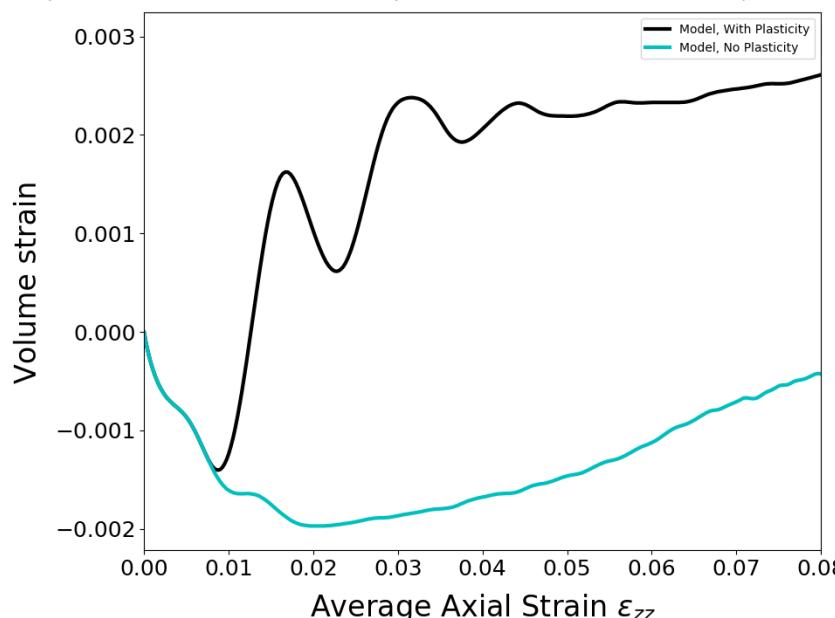
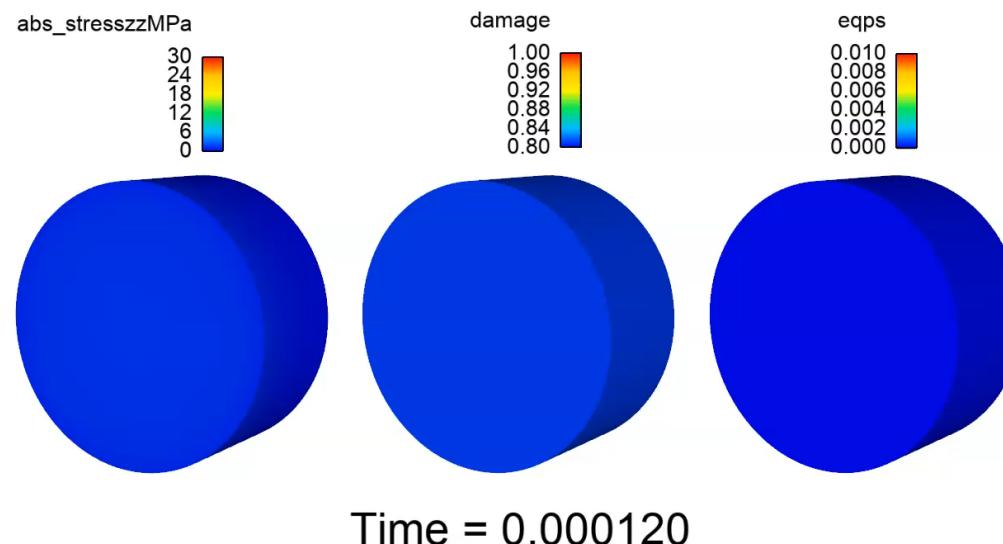
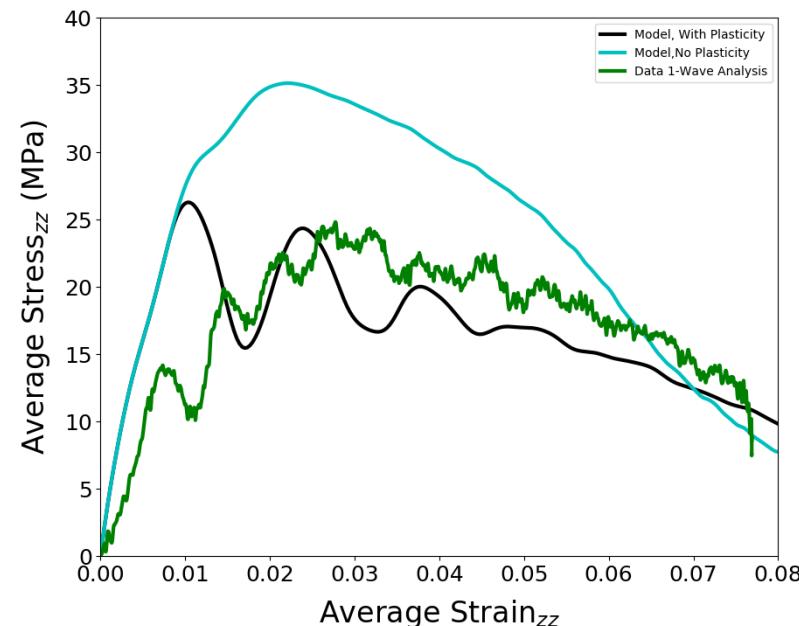
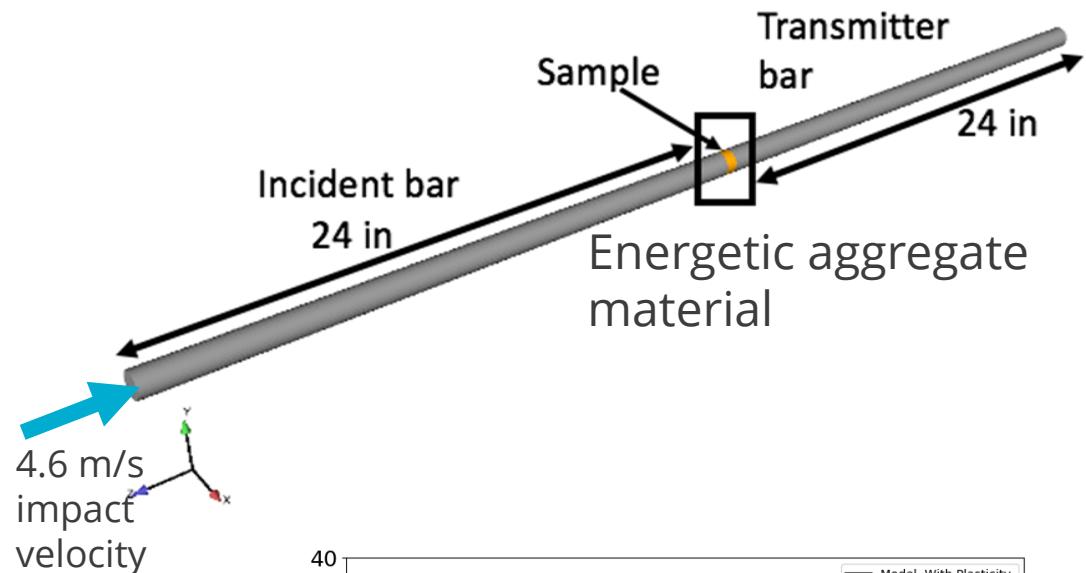
$$f(\sigma_{ij}) = \sigma_e + A \cdot \sigma_m - \sigma_y$$

$$g(\sigma_{ij}) = \sigma_e + B \cdot \sigma_m - \sigma_y$$

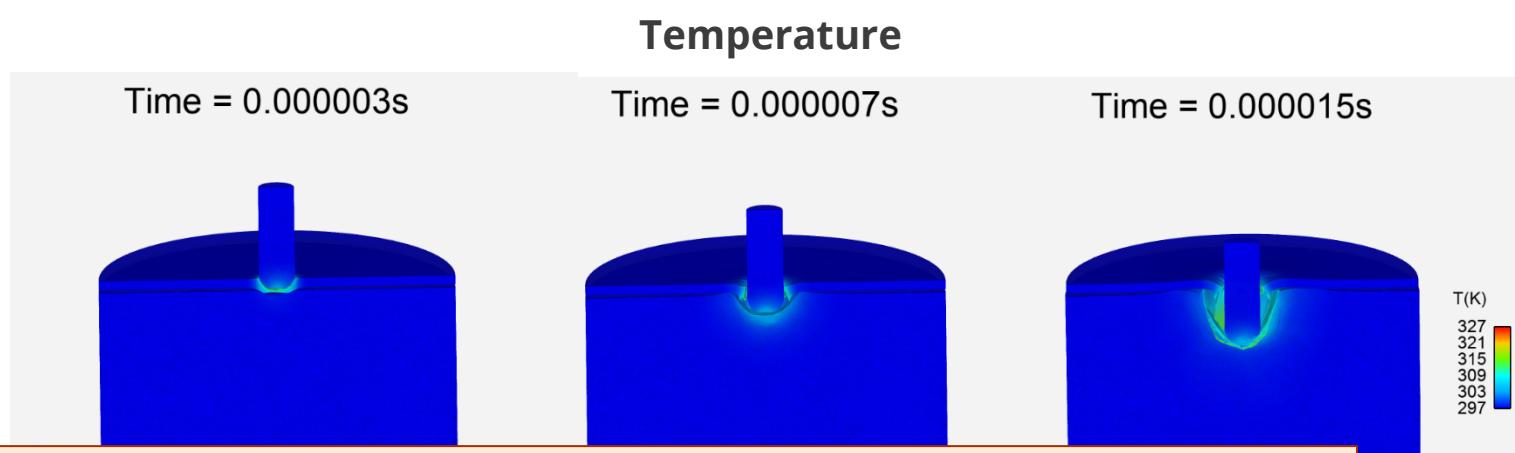
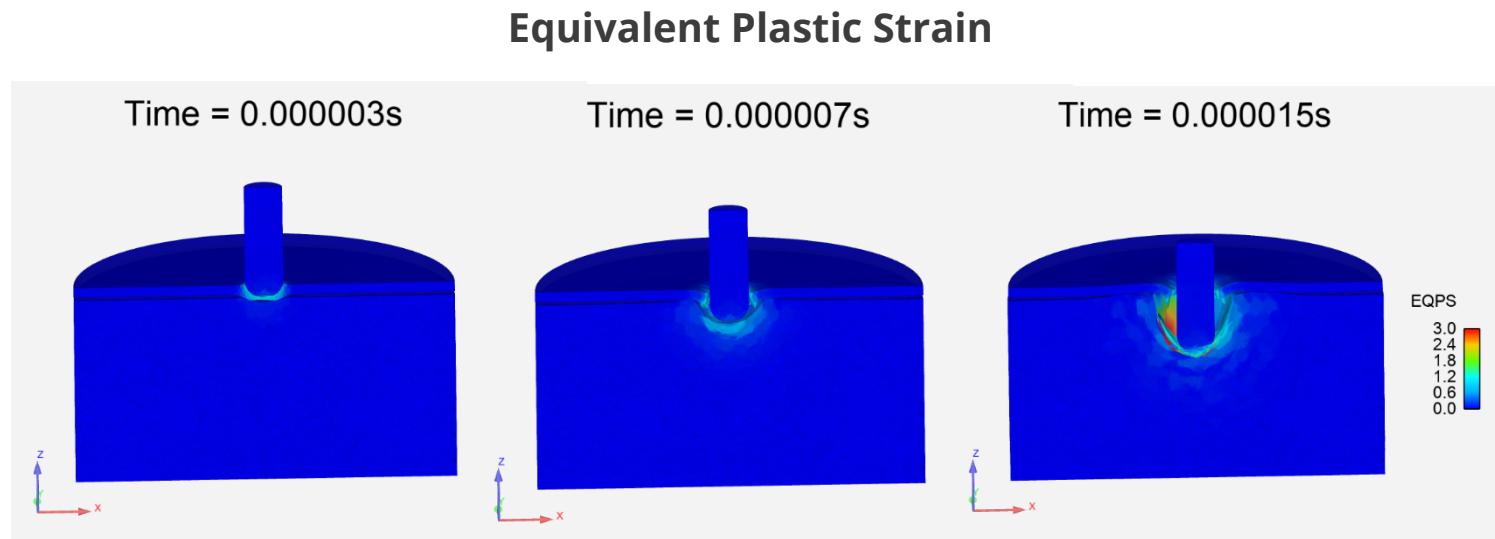
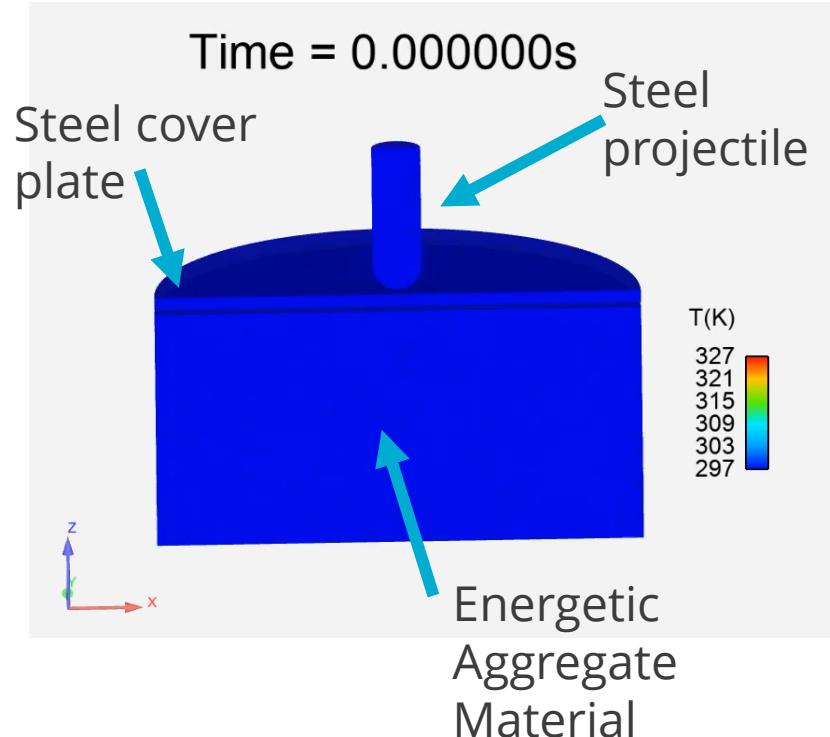
$$\dot{\boldsymbol{\epsilon}}^p = \lambda \frac{\partial g}{\partial \boldsymbol{\sigma}}$$

$$\lambda = \frac{1}{\tilde{\tau}} \langle \frac{f(\sigma_{ij})}{\sigma_0} \rangle^{\tilde{m}} \quad \sigma_e = \sqrt{\frac{3}{2} s_{ij} s_{ij}}$$

Mechanical Model Validation: Split Hopkinson Pressure Bar



Multi-material impact example



Extreme numerical robustness coupled with multiphysics enables new simulation capability

Conclusions & Ongoing Work



- Multifaceted effort to understand and model mechanically induced reaction in energetic materials
- Meshfree numerical methods (continuum and mesoscale)
- Understanding damage and heat generation mechanisms
- Upscaling to macroscale constitutive models (yield surface, damage evolution)
- Macroscale, Multiphysics predictions of impact-induced runaway temperatures
- Conforming Reproducing Kernel (CRK) Method as a powerful tool for *Lagrangian* continuum mechanics at massive strains and coupled physics

Thank You!