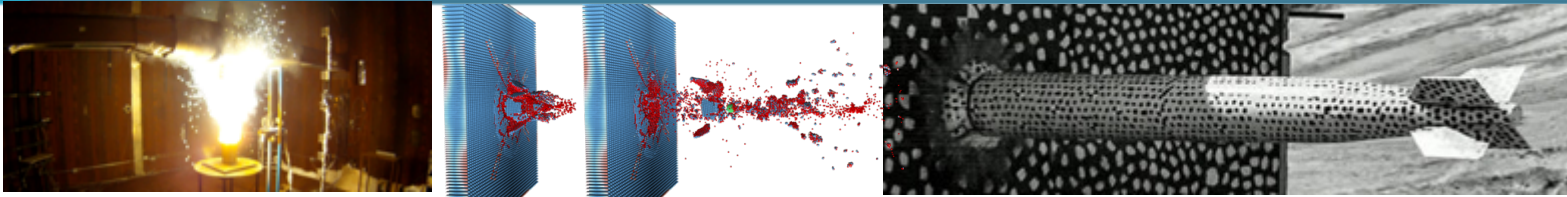




Meshfree modeling of coupled thermal-mechanical-chemical phenomena in energetic aggregates



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X International Congress of Computational Methods for Coupled Problems in Science and Engineering (COUPLED)

June 5-7, 2023, Chania, Crete, Greece

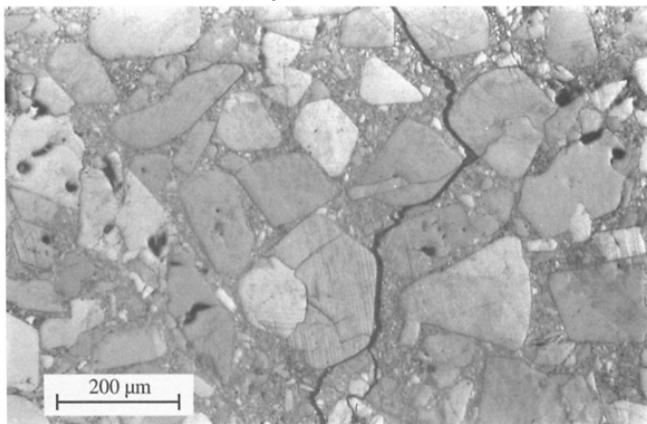
Energetic Materials: A Modeling Challenge



- ❖ Complex material structure
- ❖ Chemically reactive (fast, exothermic)
- ❖ **Everything** is a function of temperature

Multi-Physics!

Plastic Bonded Explosive [Rae, 2002]



Energetic Crystals [Yarrington, 2018]

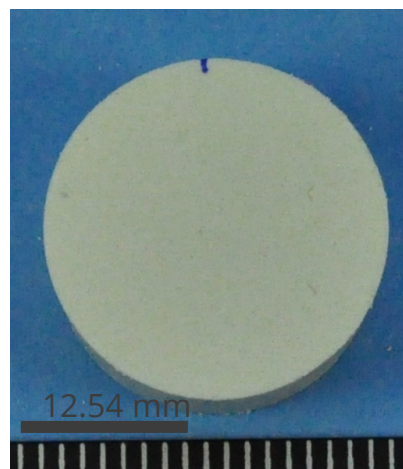
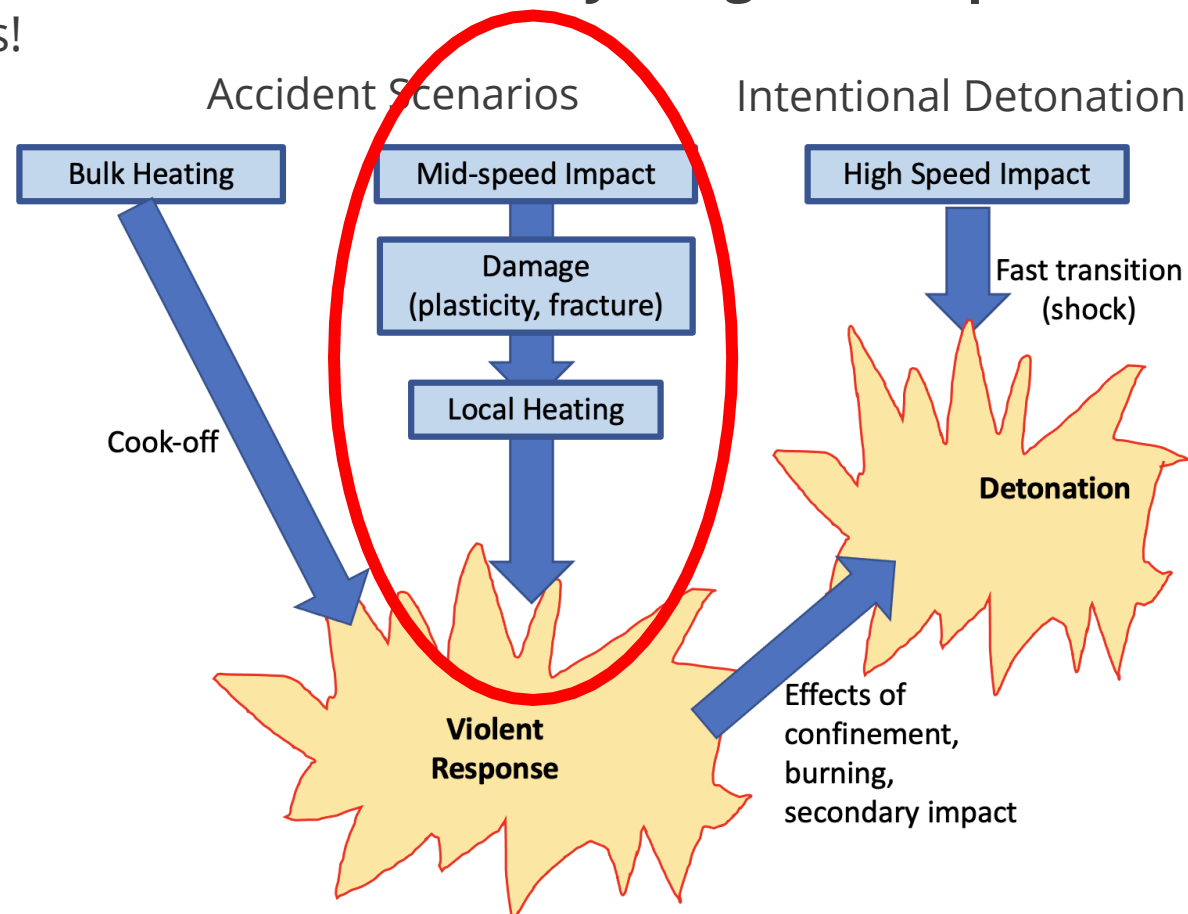


Image: courtesy Marcia Cooper

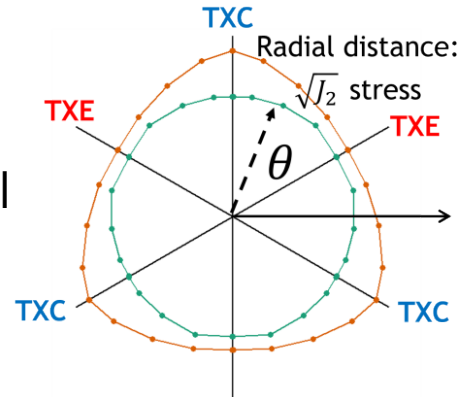
A few different ways to get an explosion...



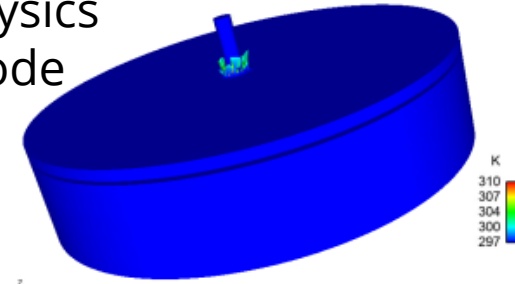
Overall Multiscale Approach



Continuum
Constitutive Model



Multiphysics
CRK code



Impact
Event

Mechanical Deformation
& Damage

Local Hotspot
Formation

Heat Conduction

Thermally Activated
Chemistry

Interface
Debonding

Plasticity

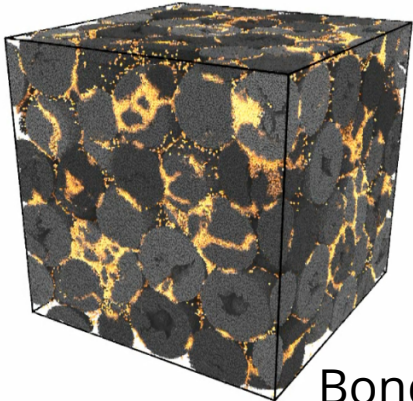
Crystal Fracture

Friction

Particle Rotation

Viscoelastic
Heating

Bonded Particle
Methods



Go

?

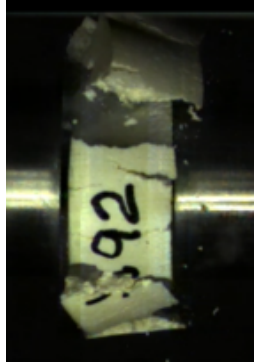
No Go

The Case for Meshfree Methods



Numerical Method Should Accurately Predict:

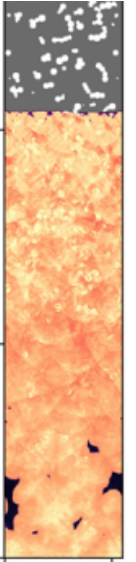
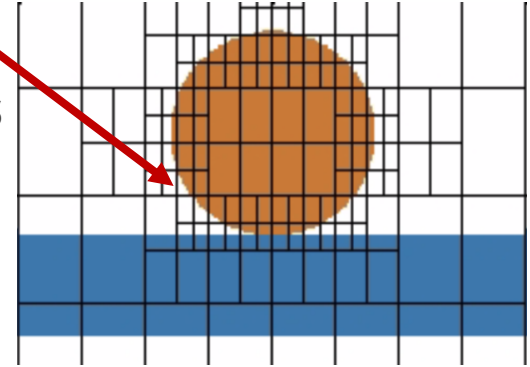
- Capture transition from solid to rubble
- Deformation-induced heating, chemistry



Example: Impact Test,
Marcia Cooper

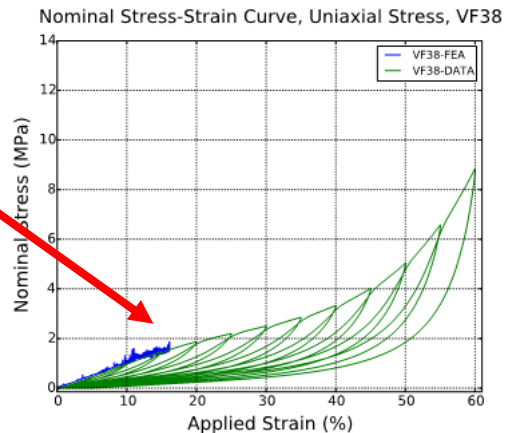
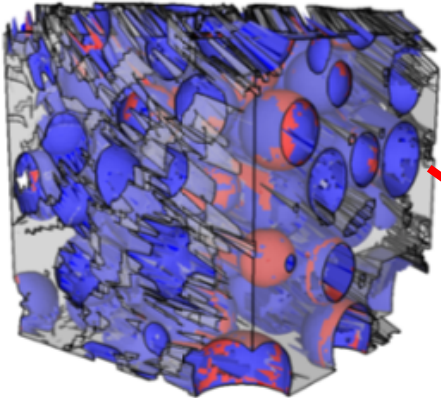
Problem: poorly resolved strain fields and interface physics, averaging in mixed material cells

Hydrocode Methods



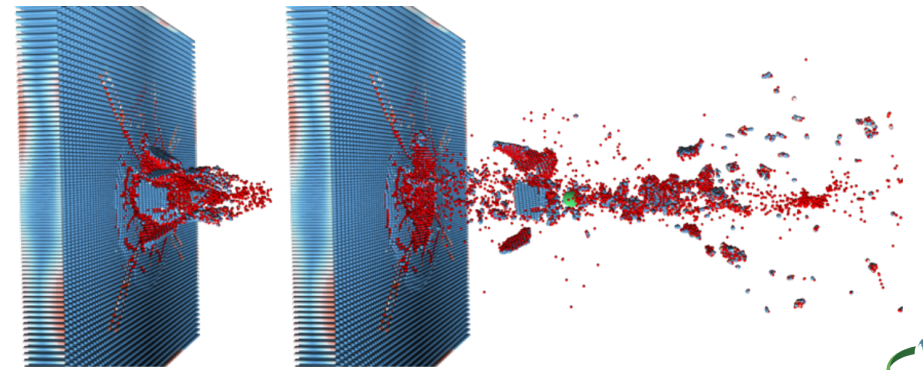
Mesh-based Methods (FEA)

Problem: Mesh entanglement at large deformations



Meshfree Methods

Show promise in overcoming these problems at both meso and macro scales



Meshfree Conforming Reproducing Kernel Method



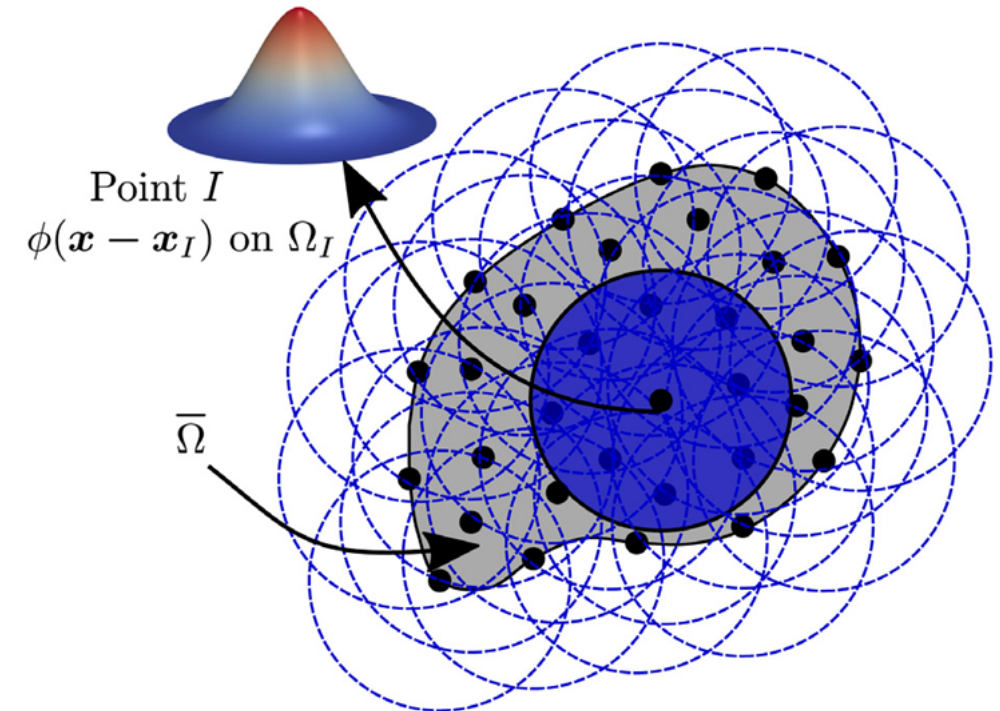
Reproducing Kernel Particle Method

- Galerkin-based variational method using the reproducing kernel discretization
- Shape functions are the product of a window/kernel function and correction function

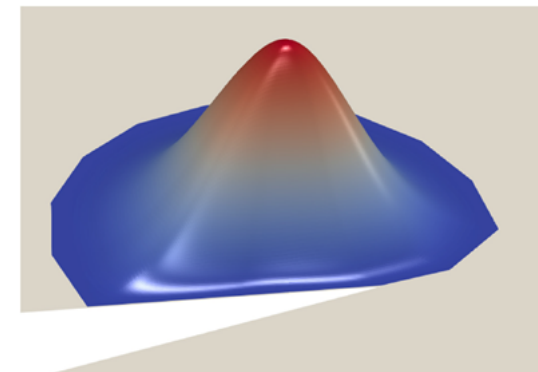
$$u^h(x) = \sum_{I=1}^{NP} \Psi_I d_I; \quad \Psi_I = C(x; x - x_I) \phi_a(x - x_I)$$

Conforming Reproducing Kernel

- Graph distance informed window/kernels replace traditional Euclidian kernels to provide improved accuracy and robustness for nonconvex geometries and essential boundary conditions



J. Koester, J.S. Chen, Comput. Methods Appl. Mech. Engrg. 347 (2019) 588–621





CRK-Thermal implemented to simultaneously solve momentum

$$\int_{\Omega} \mathbf{w} \cdot \rho \ddot{\mathbf{u}} d\Omega + \int_{\Omega} \mathbf{F}(\nabla \mathbf{w}) : \mathbf{P}(\nabla \mathbf{u}) d\Omega = \mathbf{f}^{\text{ext}}(\mathbf{u}),$$

and conservation of energy

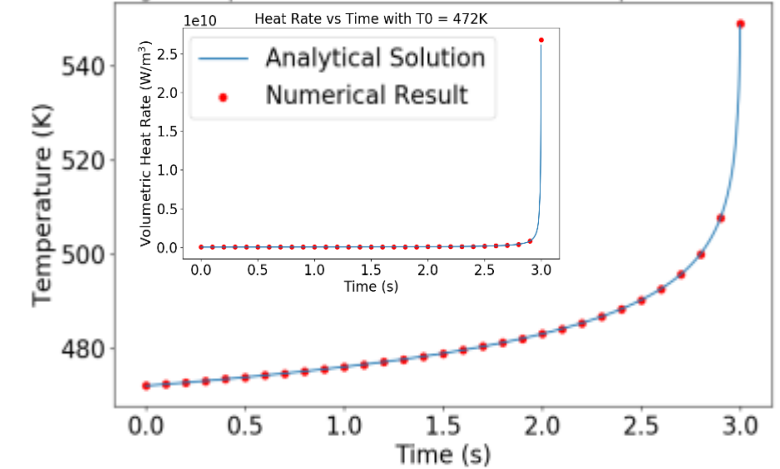
$$\int_{\Omega} w \rho C_P \dot{T} d\Omega + \int_{\Omega} \underbrace{\nabla w \cdot K \nabla T}_{\text{Thermal conduction}} d\Omega = \int_{\Omega} w \left(\underbrace{\dot{\epsilon}^P}_{\text{Adiabatic heating from material plasticity}} + \underbrace{\dot{q}^{\text{species}}}_{\text{Chemical heating}} \right) d\Omega + \int_{\partial\Omega} w h d\Gamma.$$

*Thermal
conduction*

*Adiabatic heating from
material plasticity*

Chemical heating

Average Temperature vs Time with initial Temperature = 472K



$$Q = \rho \Delta H Z e^{-E_a/RT}$$

➤ Viscoplastic-ViscoSCRAM constitutive model

- Viscoelasticity
- Cracking damage (Statistical Crack Mechanics)
- Pressure-dependent viscoplasticity with Drucker-Prager yield surface

- Chemical heating from exothermic decomposition
 - Currently restricted to Arrhenius rate
 - More sophisticated models in progress

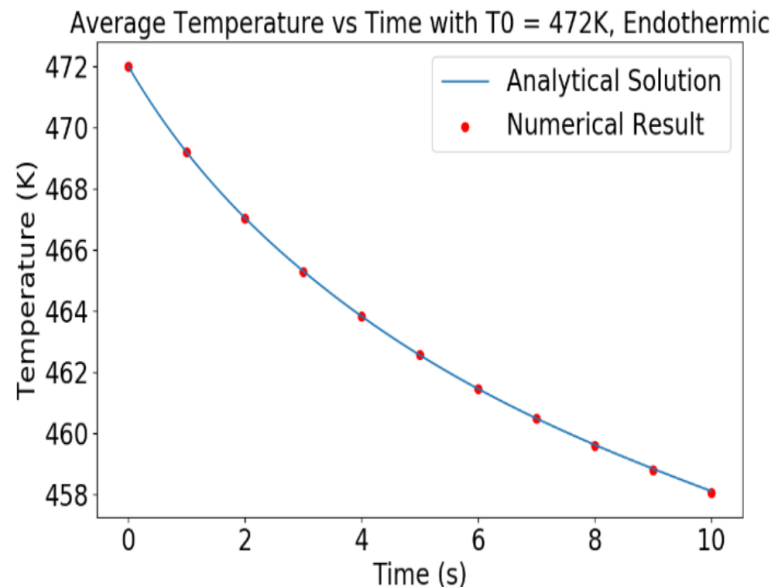
Thermo-chemical coupling verification



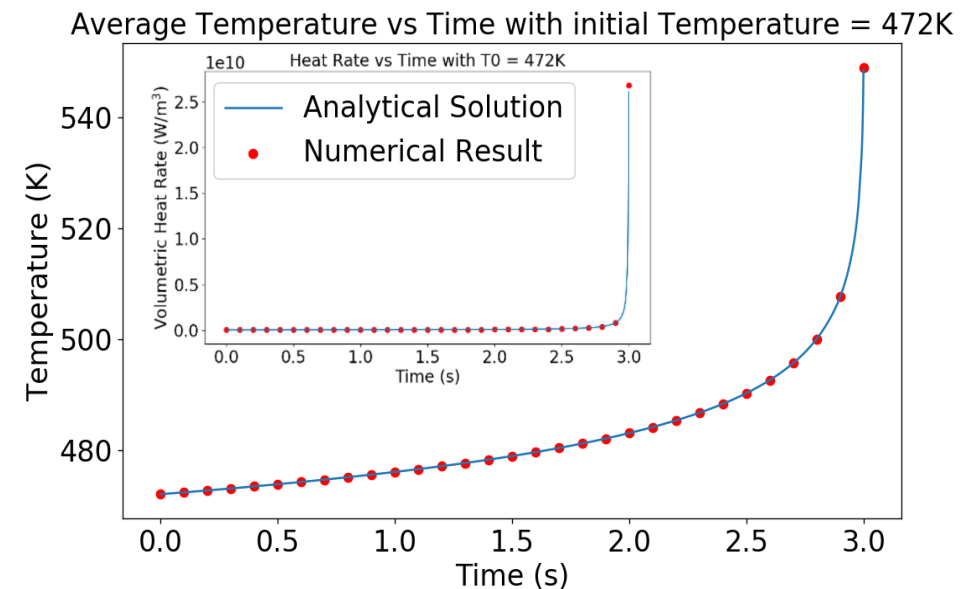
- Temporal verification based on Frank-Kamenetskii equation[1]:

$$-\lambda \nabla^2 T + \rho C \frac{dT}{dt} = \rho \Delta H Z e^{-E_a/RT} \xrightarrow{\text{Uniform temperature change (temporal variation only)}} \rho C \frac{dT}{dt} = \rho \Delta H Z e^{-E_a/RT}$$

Endothermic Process



Exothermic Process



¹Cooper, Paul W.. Explosives Engineering, John Wiley & Sons, Incorporated, 1996.

Thermo-chemical coupling verification



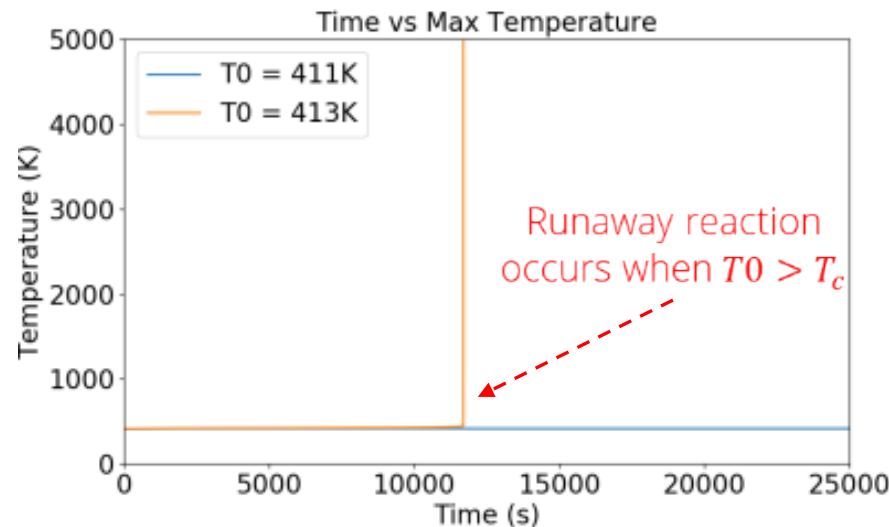
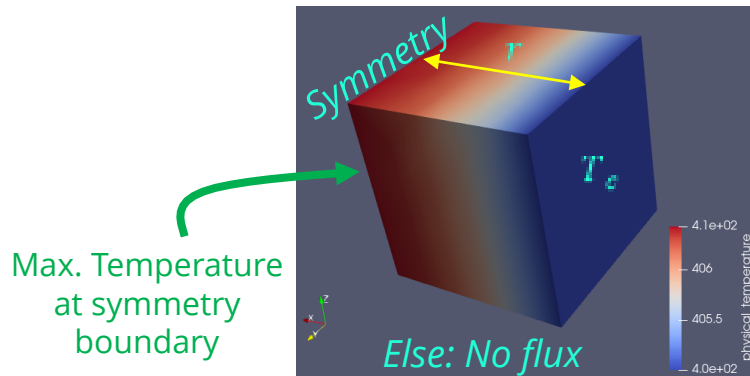
- Verification test based on Frank-Kamenetskii critical temperature[1]:

Steady state solution (spatial variation only)

$$-\lambda \nabla^2 T + \rho C \frac{dT}{dt} = \rho \Delta H Z e^{-E_a/RT} \quad \longrightarrow \quad -\lambda \nabla^2 T = \rho \Delta H Z e^{-E_a/RT}$$

$$\frac{E_a}{T_c} = R \ln \left(\frac{r^2 \rho \Delta H Z E_a}{T_c^2 \lambda \delta R} \right)$$

Cubic domain with no flux boundary except one surface with constant T_c (infinite slab)



Solve for critical ambient temperature (T_c) before runaway reaction¹

Analytical
Critical
Temperature:
~412K

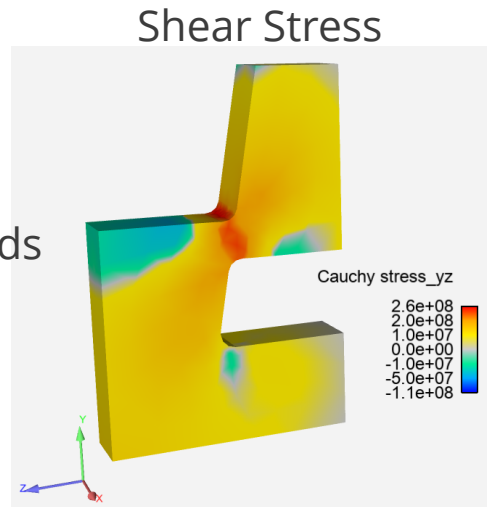
¹Cooper, Paul W.. Explosives Engineering, John Wiley & Sons, Incorporated, 1996.

Shear-Induced Heating in Steel

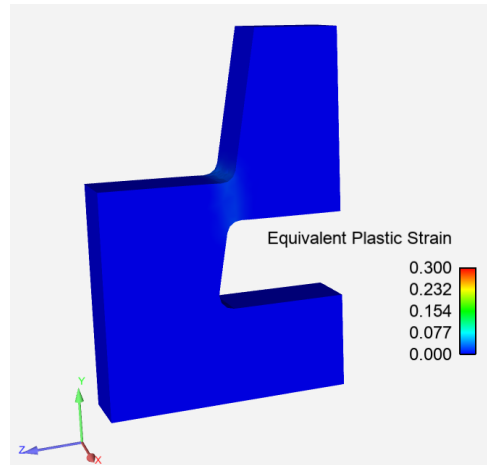


- Top-hat geometry designed to induce high localized shear
- Material properties: steel

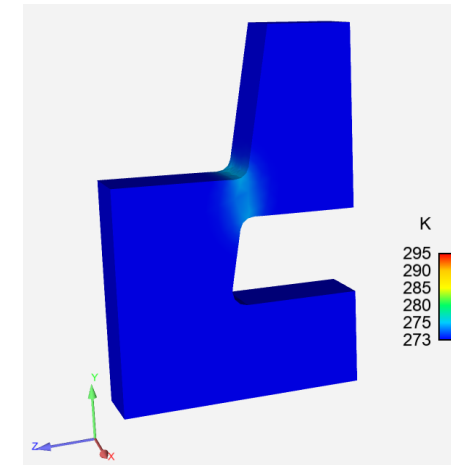
Time = 65
microseconds



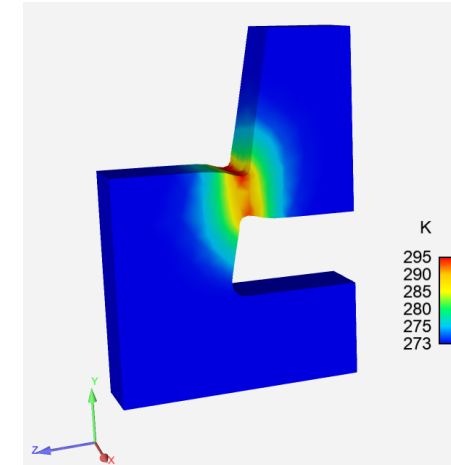
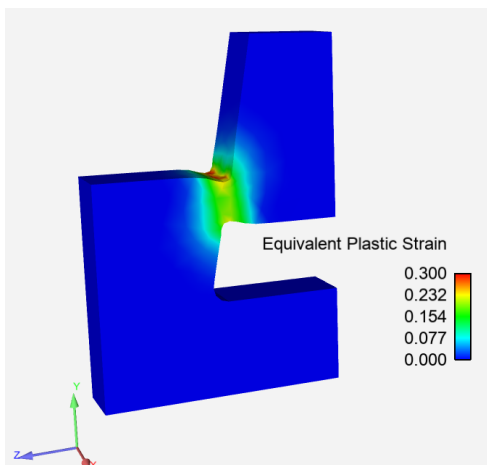
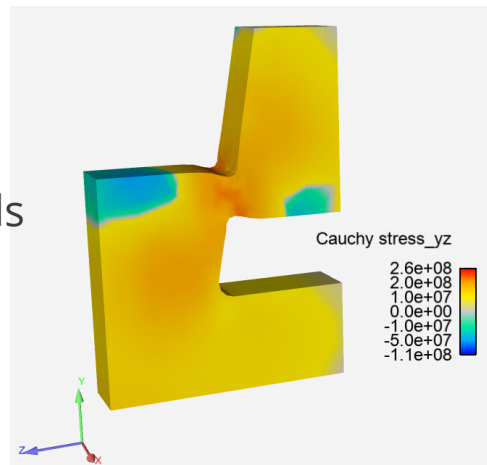
Equivalent Plastic Strain



Temperature



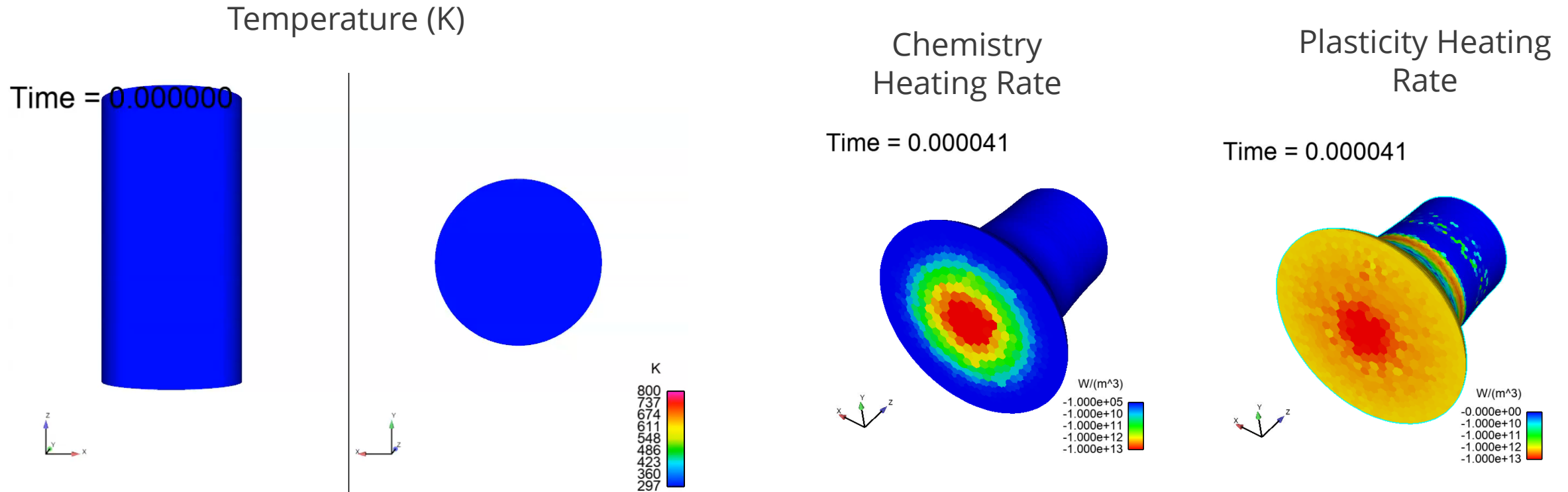
Time = 250
microseconds



Thermal Runaway in Taylor Bar Impact: 450 m/s



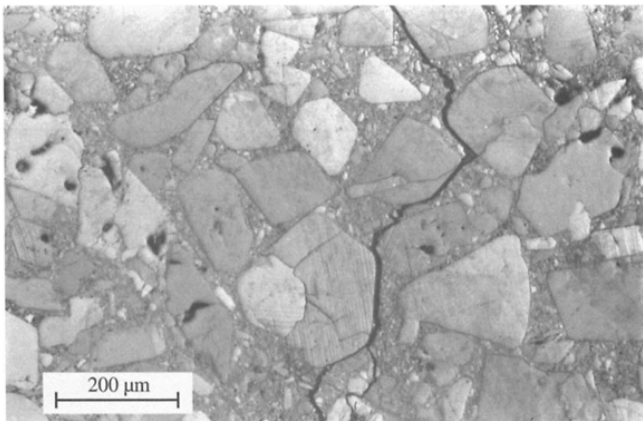
- Material properties: pure energetic crystals
- Energy dissipated due to plastic deformation raises temperature enough to start runaway chemistry



Now we consider energetic aggregate materials



- Composite mechanical behavior is complex:
 - Strain rate dependent
 - Temperature dependent
 - Pressure dependence
 - Tension-Compression Asymmetry
 - Many inelastic deformation mechanisms: Viscoelasticity (binders), Cracking (intra- and inter-granular), Porosity opening, dislocation slip, twinning (some energetic crystals)



Plastic Bonded Explosive [Rae, 2002]

How to represent various inelastic mechanisms in a macroscale model?

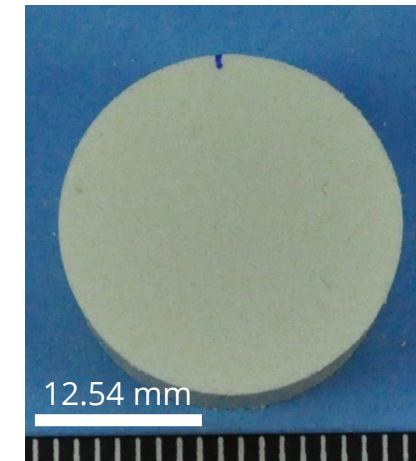
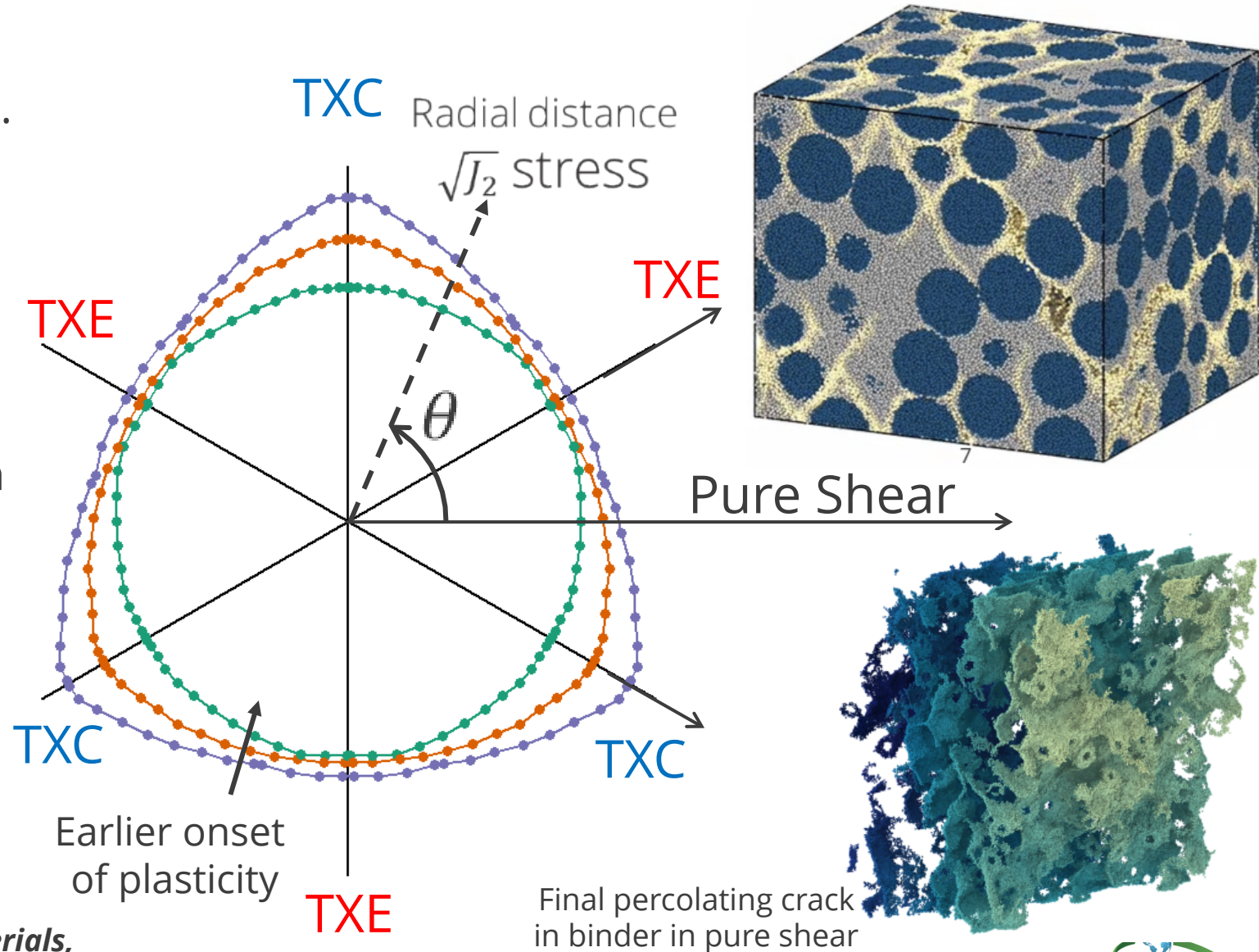
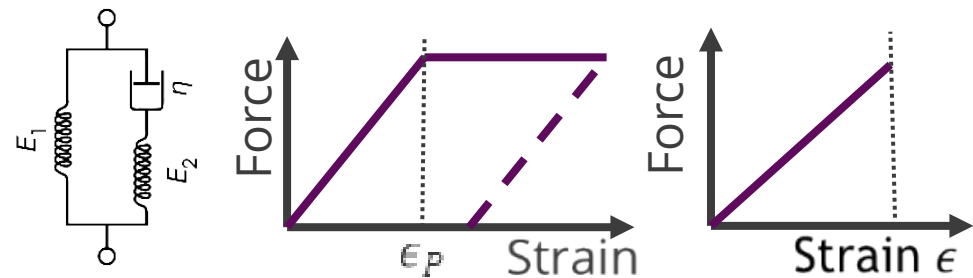


Image: courtesy Marcia Cooper

Mesoscale studies to inform continuum models



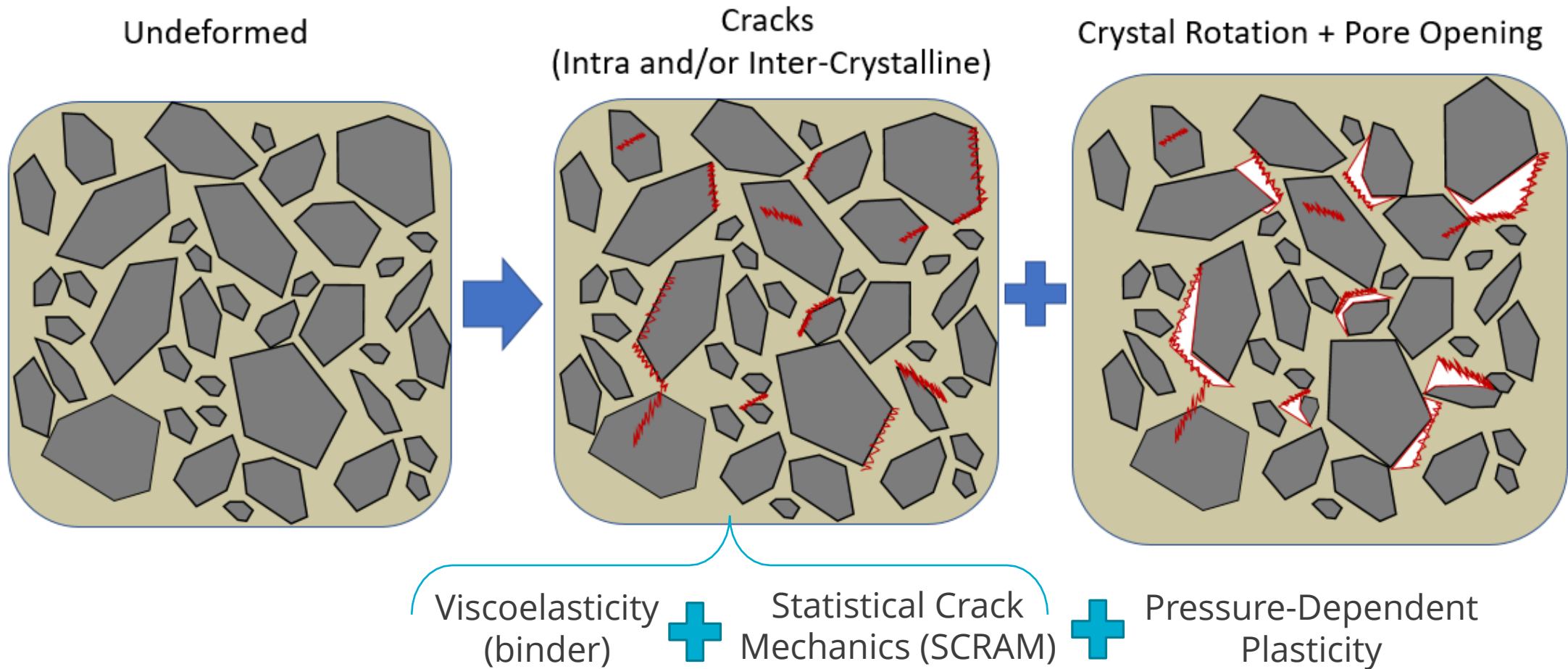
- Map out yield surfaces, often assumed to have simple shape (e.g. Drucker-Prager)
- Testing how changes in binder's material properties impact yield surface
 - better understanding of inelastic yielding for continuum models



Hypothesized Mesoscale Deformation Mechanisms



- Simplified view of mesoscale processes and macroscale interpretation



➤ Kinematics:

$$\epsilon = e + \frac{1}{3} \epsilon_{\text{vol}} \mathbf{I}$$

$$\sigma_m = K \epsilon_{\text{vol}}$$

$$e = (e^{ve} + e^D) + e^p$$

➤ Viscoelasticity

$$\dot{s} = 2G^\infty \dot{e}^{ve} + \sum_{\kappa=1}^N \left(2G^{(\kappa)} \dot{e}^{ve} - \frac{s^{(\kappa)}}{\tau^{(\kappa)}} \right)$$

Prony series of shear moduli and relaxation times

$$\dot{s}^{(\kappa)} = 2G^{(\kappa)}(\dot{e} - \dot{e}^p) - \frac{s^{(\kappa)}}{\tau^{(\kappa)}} - \frac{G^{(\kappa)}}{G_0} \left[\frac{3}{a} \left(\frac{c}{a} \right)^2 \dot{c} s + \left(\frac{c}{a} \right)^3 \dot{s} \right]$$

➤ SCRAM Damage

$$e^D = \frac{1}{2G_0} \left(\frac{c}{a} \right)^3 s$$

$$\dot{c} = \begin{cases} v_{res} \left(\frac{K_I}{K_1} \right)^m & \text{for } K_I < K' \\ v_{res} \left[1 - \left(\frac{K_{0\mu}}{K_I} \right)^2 \right] & \text{otherwise} \end{cases}$$

$$D = \frac{\left(\frac{c}{a} \right)^3}{1 + \left(\frac{c}{a} \right)^3}$$

➤ Drucker-Prager Plasticity

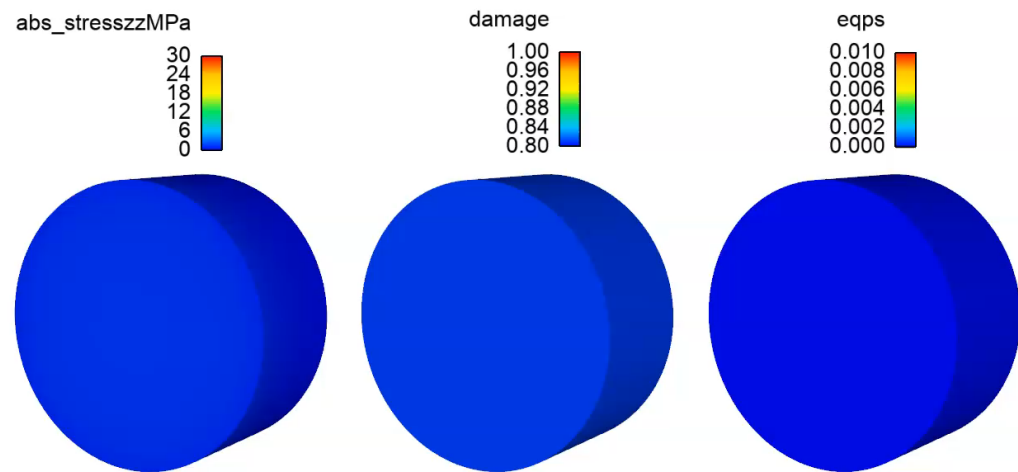
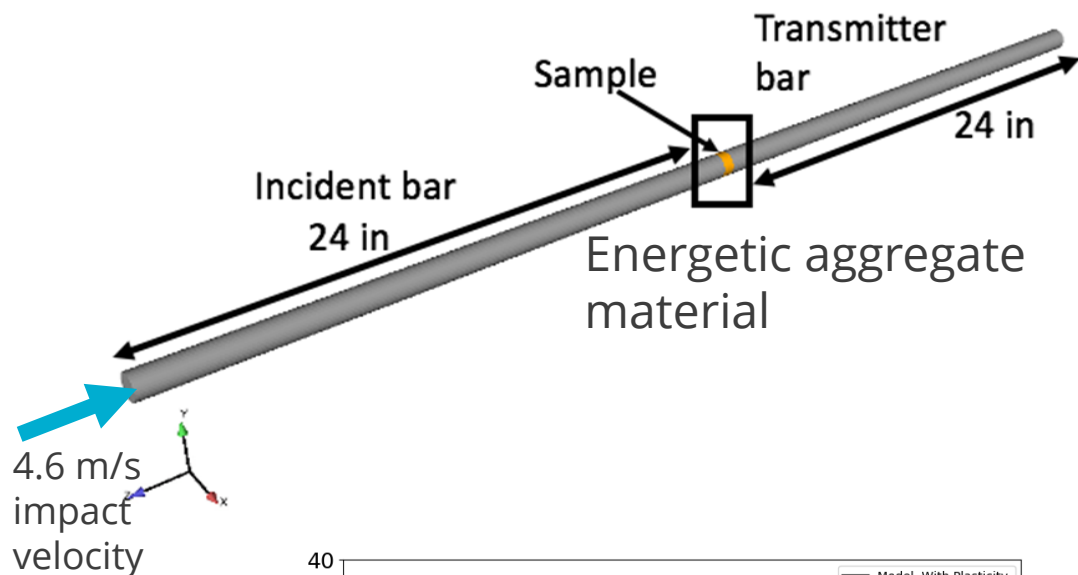
$$f(\sigma_{ij}) = \sigma_e + A \cdot \sigma_m - \sigma_y$$

$$g(\sigma_{ij}) = \sigma_e + B \cdot \sigma_m - \sigma_y$$

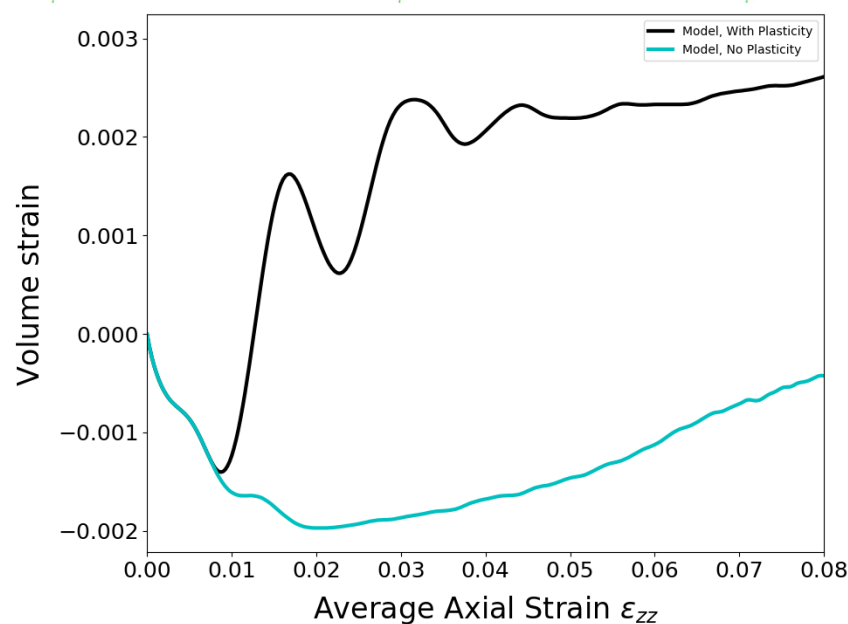
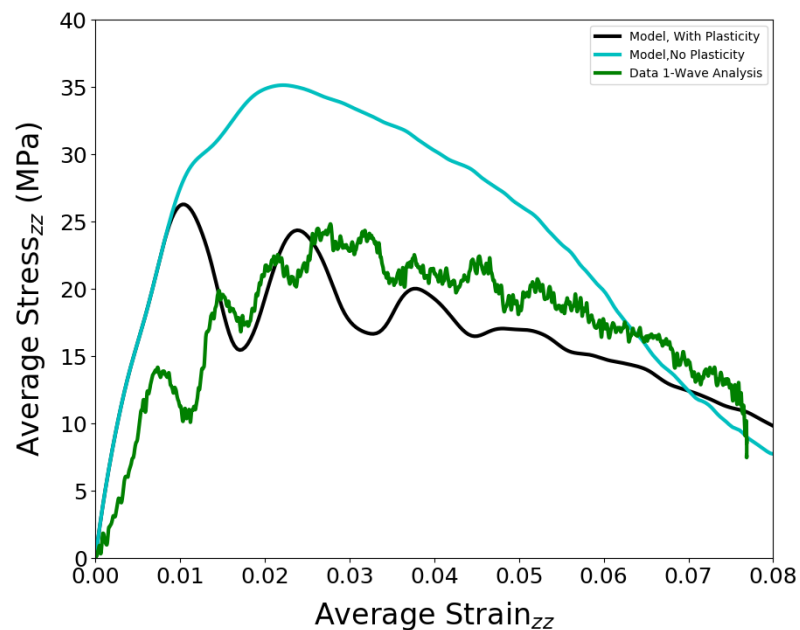
$$\dot{\epsilon}^p = \lambda \frac{\partial g}{\partial \sigma}$$

$$\lambda = \frac{1}{\tilde{\tau}} \left\langle \frac{f(\sigma_{ij})}{\sigma_0} \right\rangle_{\tilde{m}} \quad \sigma_e = \sqrt{\frac{3}{2} s_{ij} s_{ij}}$$

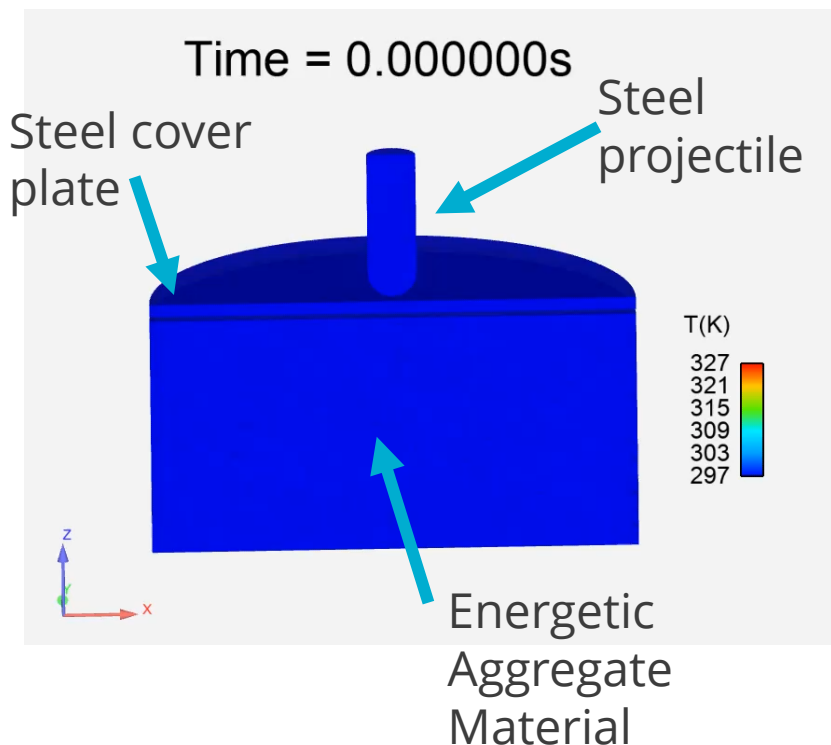
Mechanical Model Validation: Split Hopkinson Pressure Bar



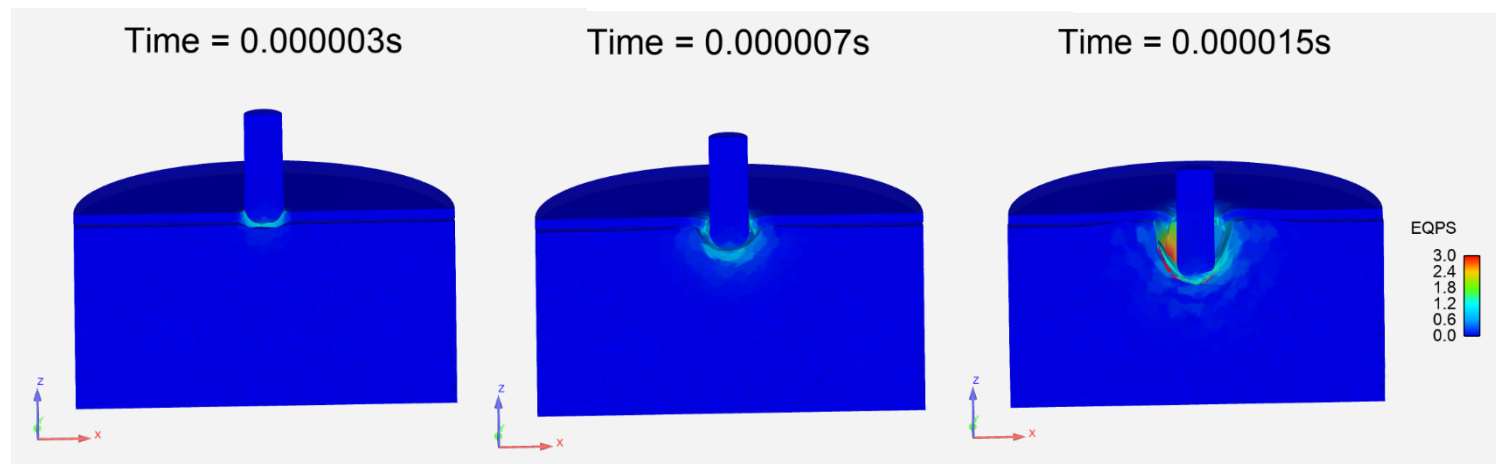
Time = 0.000120



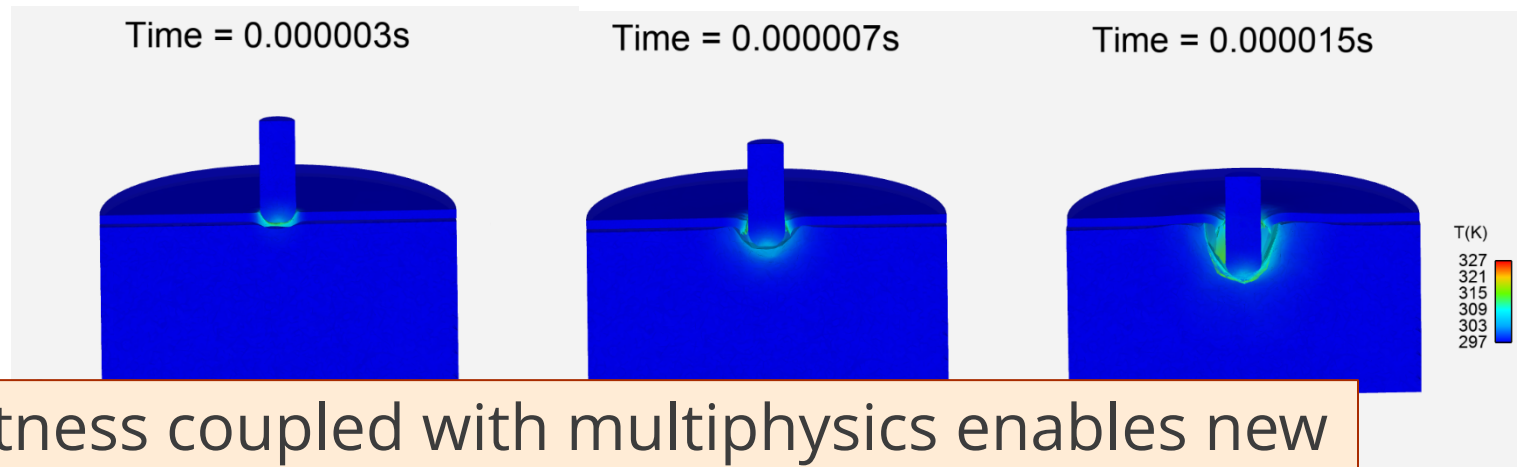
Multi-material impact example



Equivalent Plastic Strain



Temperature



Extreme numerical robustness coupled with multiphysics enables new simulation capability

Conclusions & Ongoing Work



- Multifaceted effort to understand and model mechanically induced reaction in energetic materials
- Meshfree numerical methods (continuum and mesoscale)
- Understanding damage and heat generation mechanisms
- Upscaling to macroscale constitutive models (yield surface, damage evolution)
- Macroscale, Multiphysics predictions of impact-induced runaway temperatures
- Conforming Reproducing Kernel (CRK) Method as a powerful tool for *Lagrangian* continuum mechanics at massive strains and coupled physics

Thank You!