



Principles underlying 2-D phononic pseudocrystal isolators

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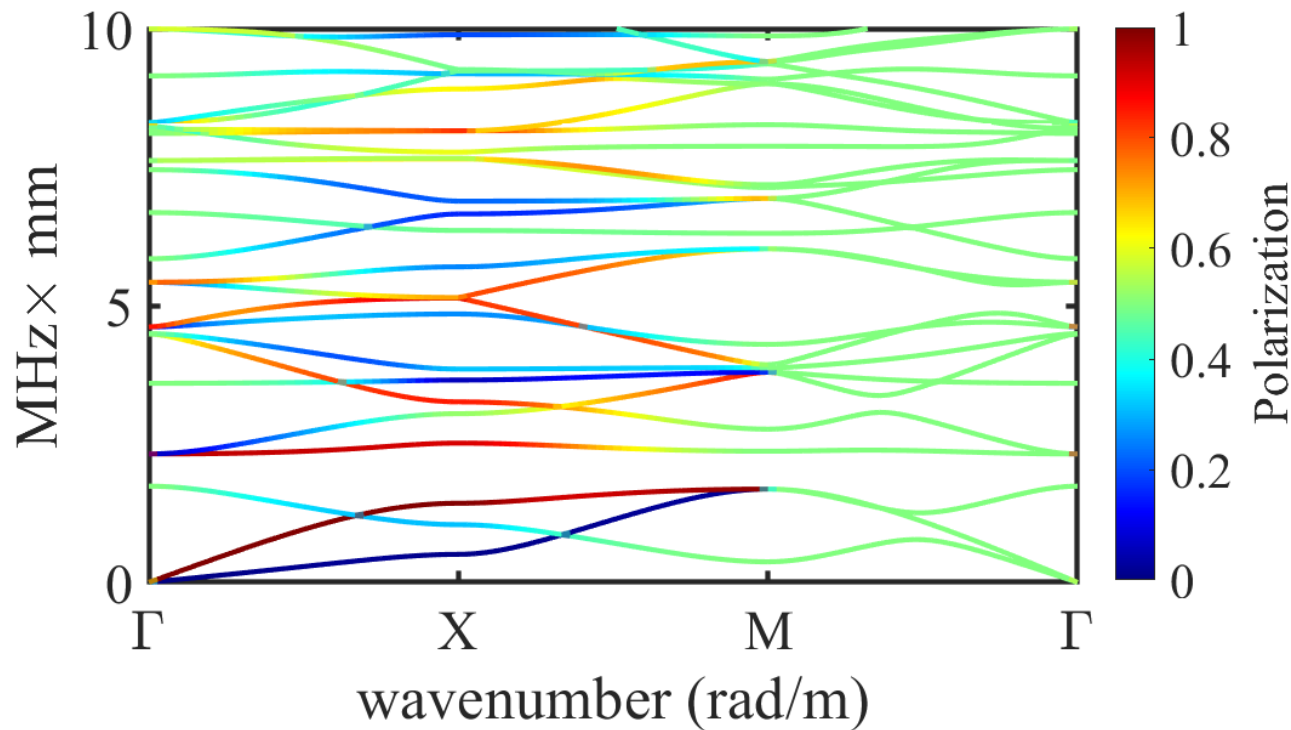
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6th International Conference on Phononic Crystals/Metamaterials/Metasurfaces, Phonon Transport, and Topological Phononics

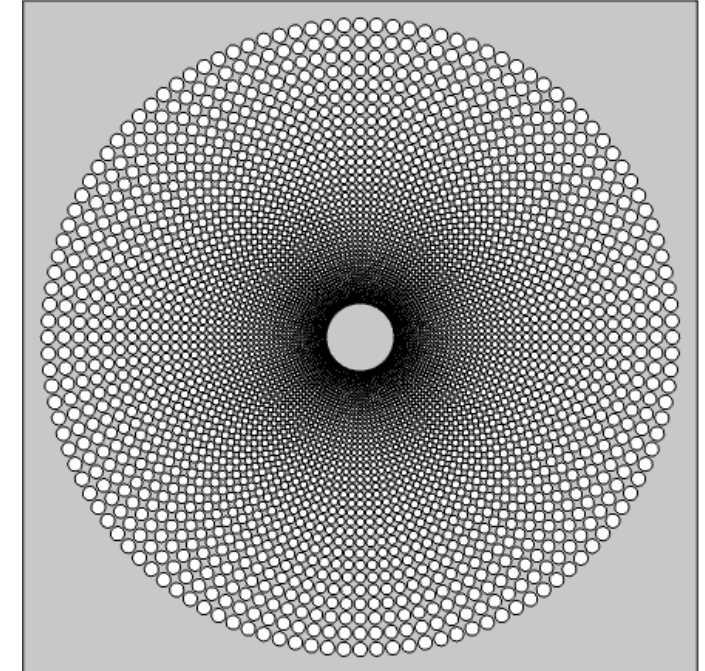
Phononic Pseudocrystal Isolators



- **Circumferential direction**: discrete symmetry
- **Radial direction**: self-similarity/linear channel shape/geometric growth by row
- **Filling fraction**: constant value of 0.4191 for isolator in current example
- **Local dispersion**: single underlying band structure scalable to all holes



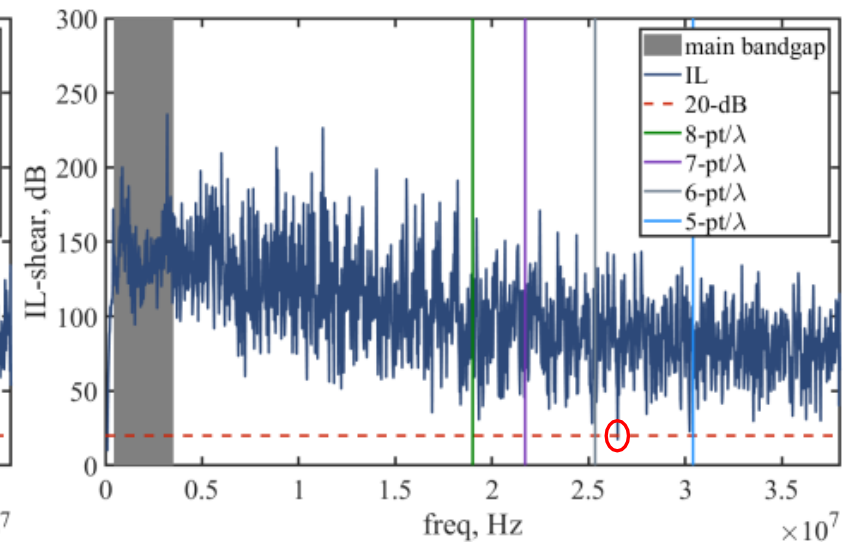
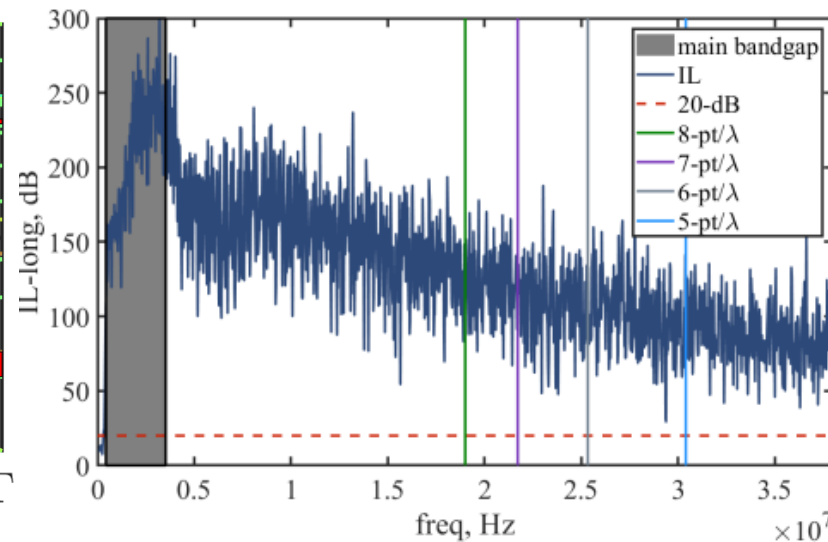
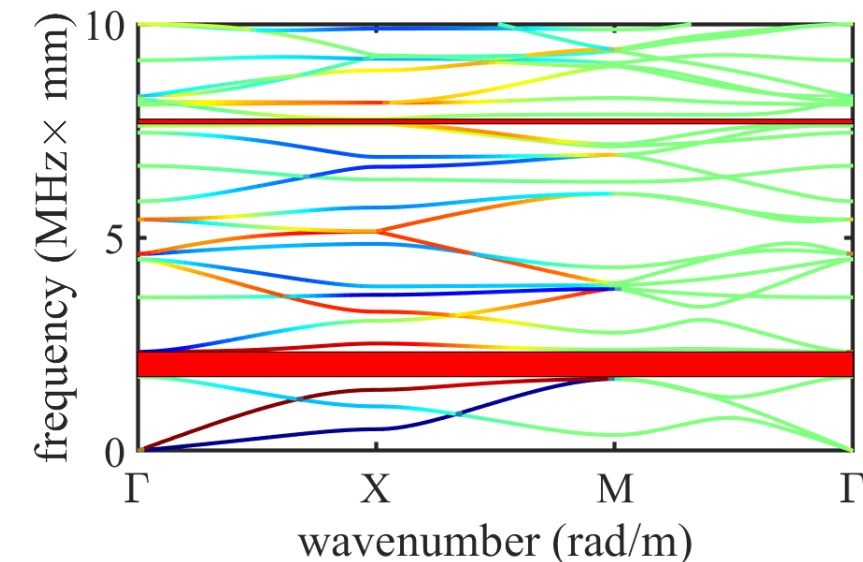
$$P = \frac{\int |u|^2 dV}{\int |u|^2 + |v|^2 dV}$$



The theory we started with...



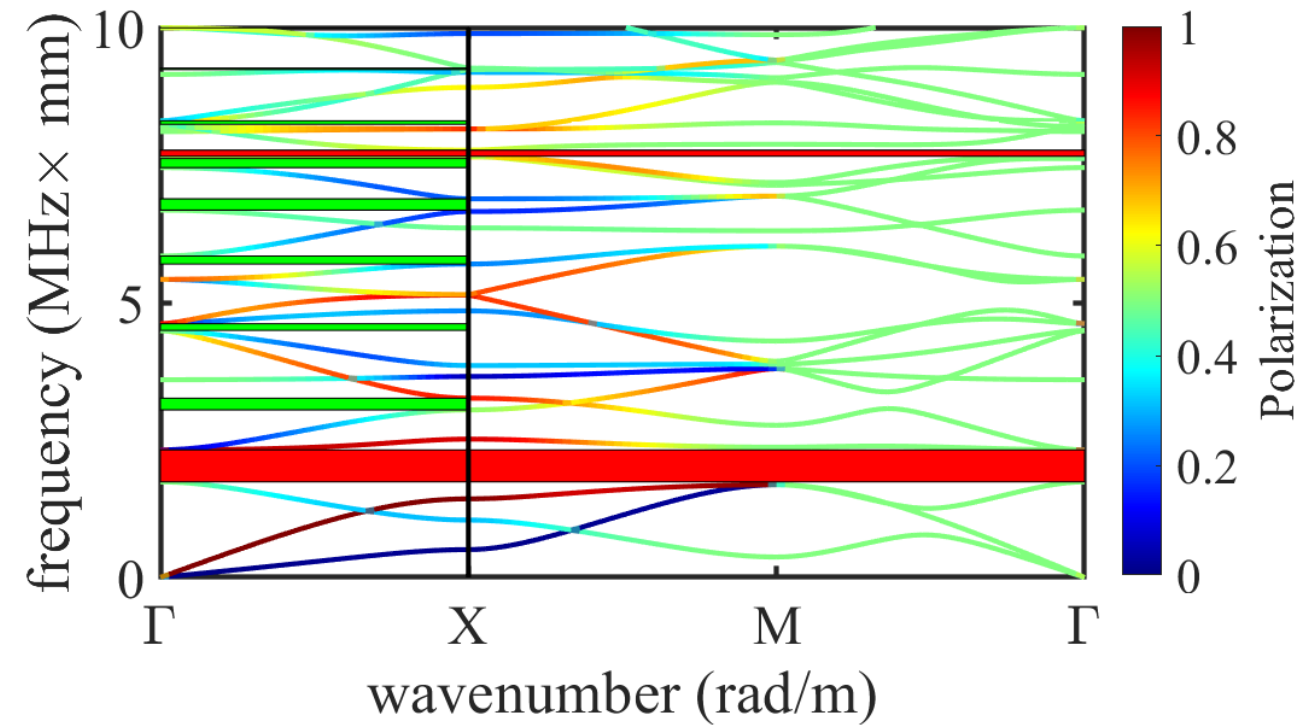
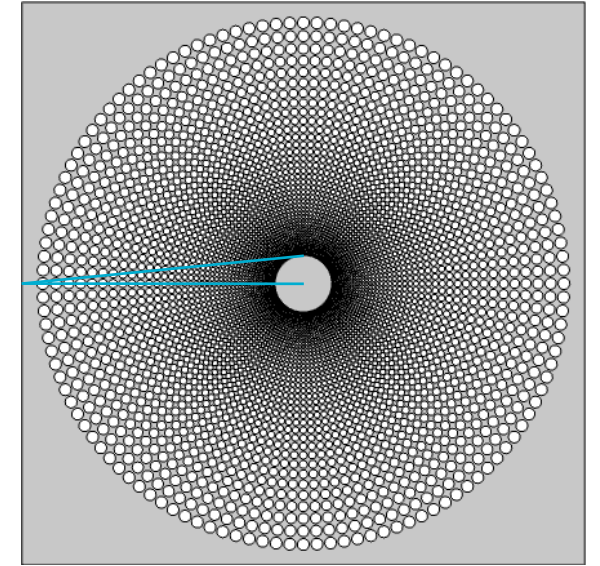
- Conventional phononic crystals require ~ 5 similar unit cells for efficacy (suppression in bandgap)
- Loosened the definition of *similar*: similar = "identical" \rightarrow similar = "overlapping bandgap"
- Larger suppression range than advertised: designed for $\frac{f_{up}}{f_{low}} = \sim 8$, got 88.3 or >126.7 .



What happened? Can we patch the theory back into working order by appealing to angular-limitations?



- Single-channel calculation limits dispersion to the x-direction
- But a similar angle limitation occurs naturally...
- However, additional band gaps are too small (need $\frac{f_{up}}{f_{low}} \geq \approx 1.28$ to satisfy “five similar cells” rule)

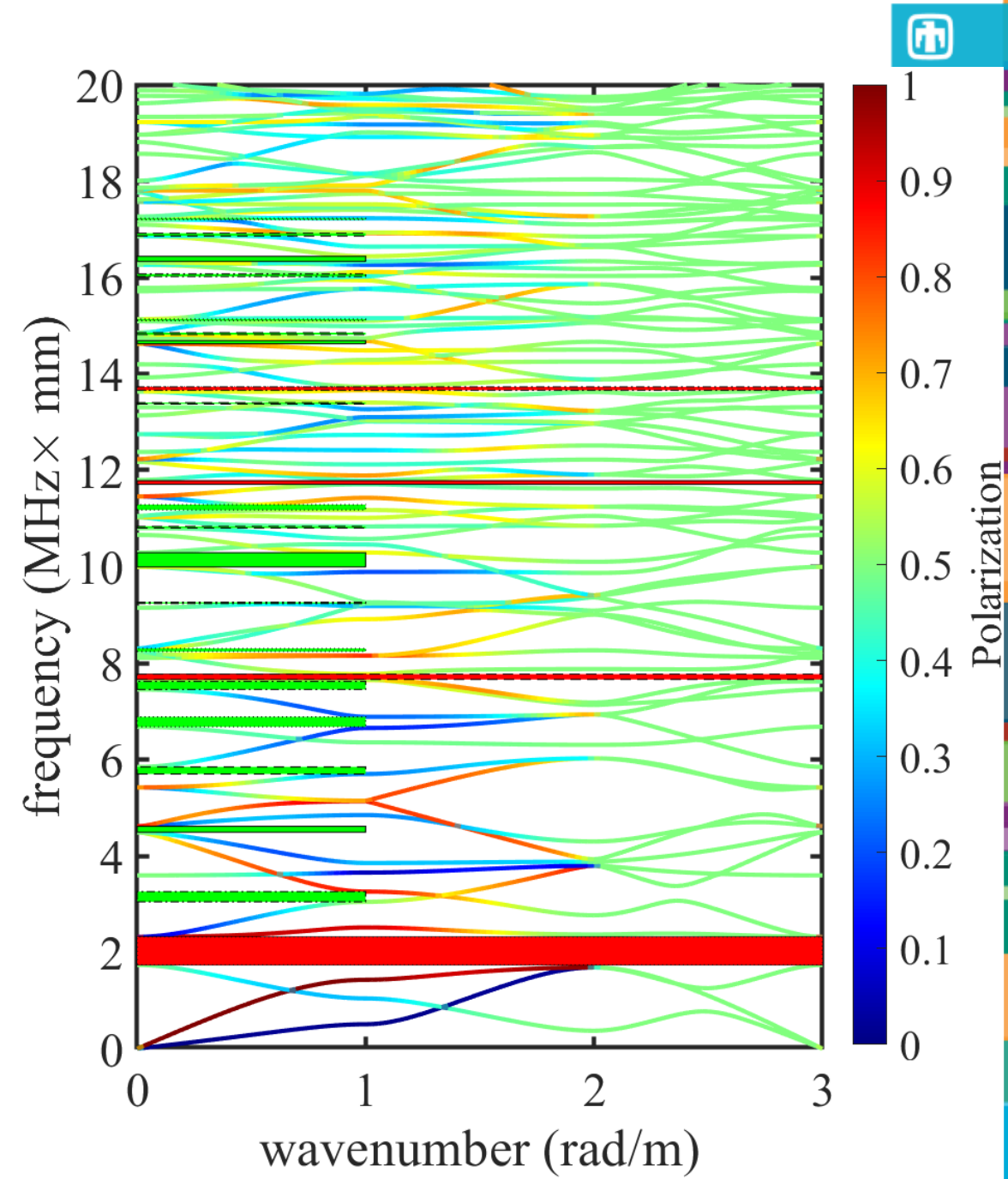


- At least one other thing is going on....

Limited angle propagation

Multiple additional bandgaps open when restricted to the radial direction

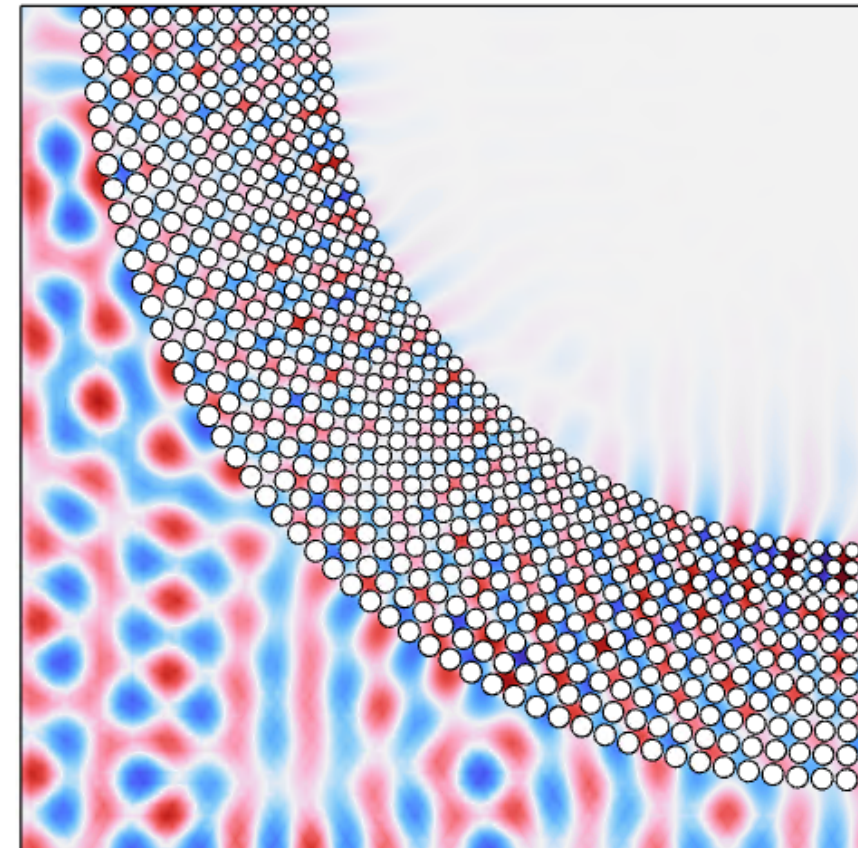
- Number of gaps increases from 4 \rightarrow 21
- Gaps increase from 4.19% \rightarrow 13.13% of the space between $f \in (0, 20)$ MHz \times mm



Boundary coupling/wavelength reduction

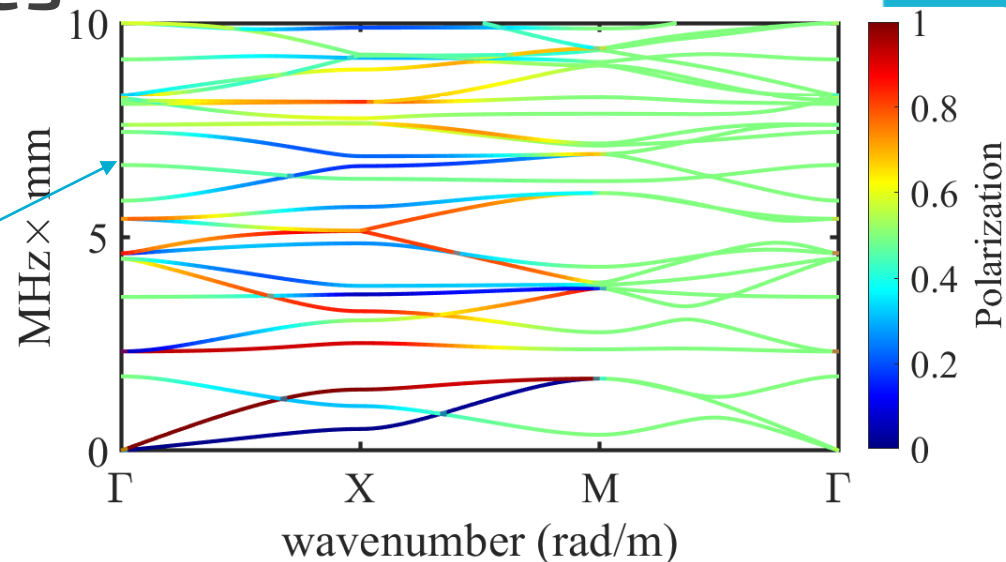
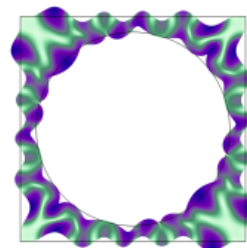


- Very long wavelengths behave as though the local sound speed has changed resulting in refractive effects (lower sound speed within interposer) at boundaries
- Holes channel some waves reducing $\lambda_{circ.}$ \rightarrow wavelength is too short for frequency \rightarrow evanescences!
- Appears to contribute to low frequency suppression in acoustic simulations

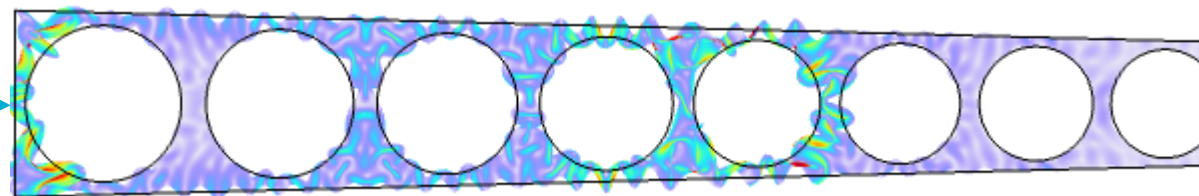
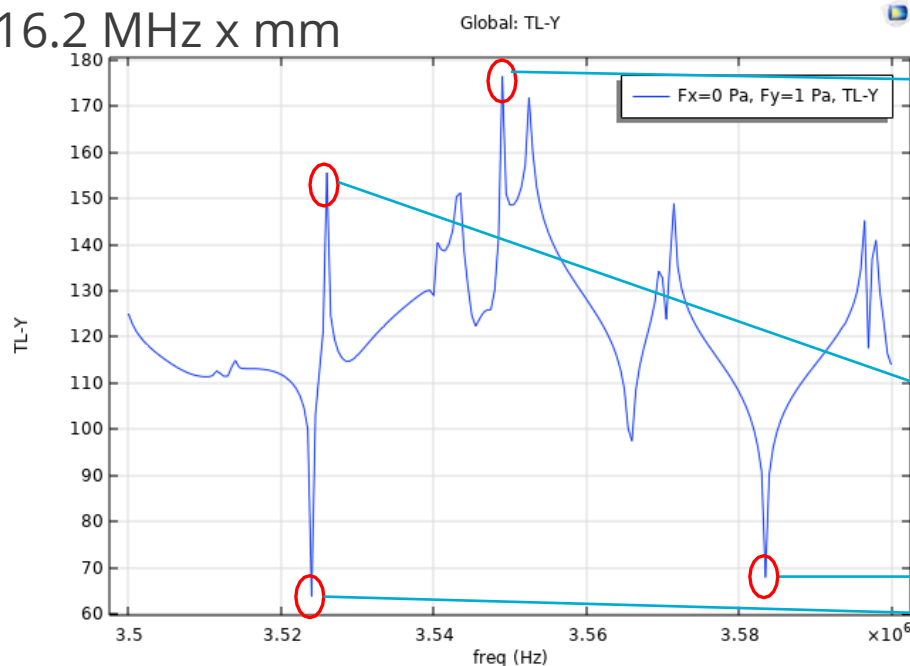


Coupling with hole interior resonances

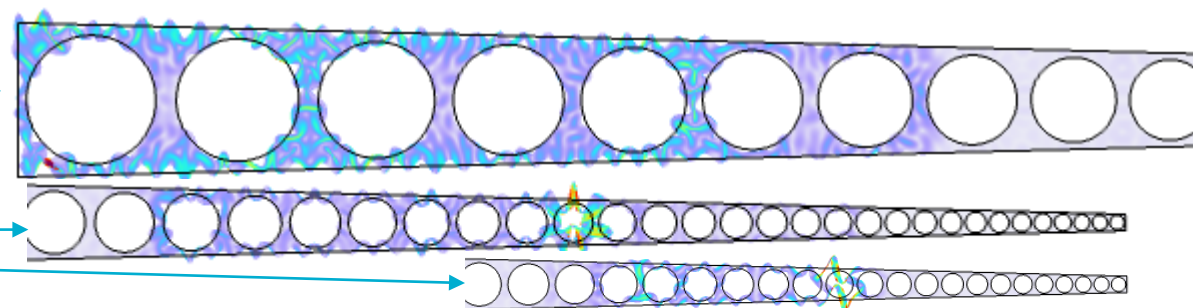
- Hole resonances suppress transmission
- Holes vary widely in scale
- Each hole supports many resonances
- → Large number of sharp narrow resonances



$$3.526 \text{ MHz} \times 4.6 \text{ mm} \\ = 16.2 \text{ MHz} \times \text{mm}$$



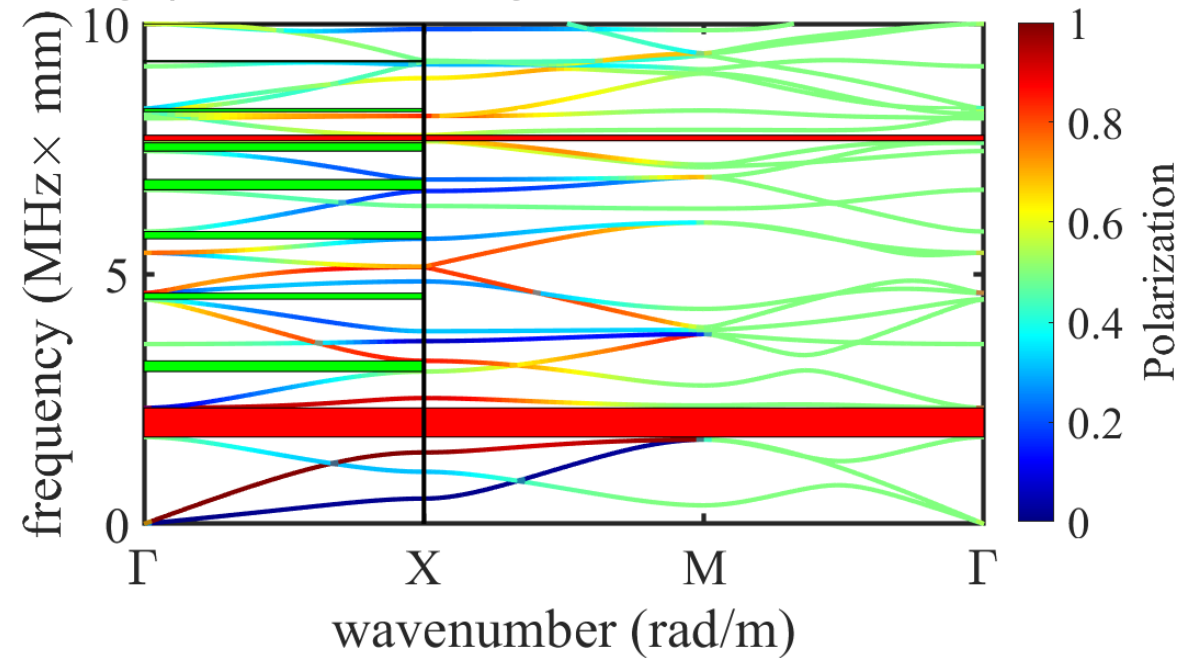
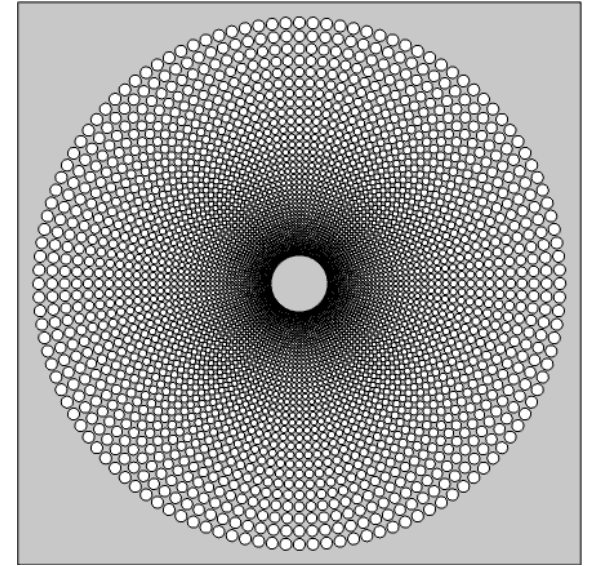
Shear wave excitation shown



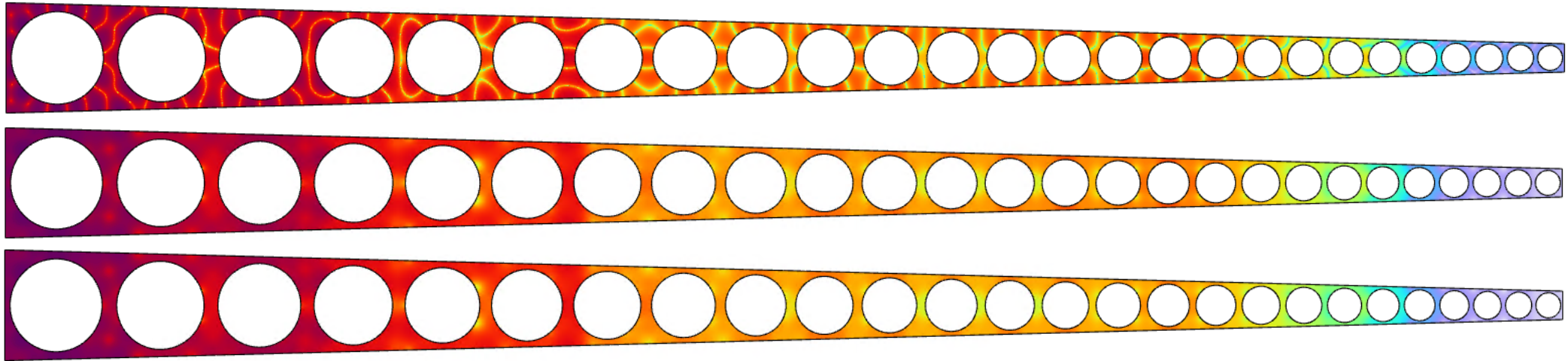
Stranded propagation?



- Within a regular phononic crystal, mode conversion only happens at specific values k , f , where indicated in the band structure
- Apply the “adiabatic assumption” within the isolator
- Could open up substantial bandgaps (if correct):
 - Any mode with local minimum trapped eventually
 - Any band separations would become bandgaps under scaling
 - Could also enlarge existing bandgaps
- But does it actually work?

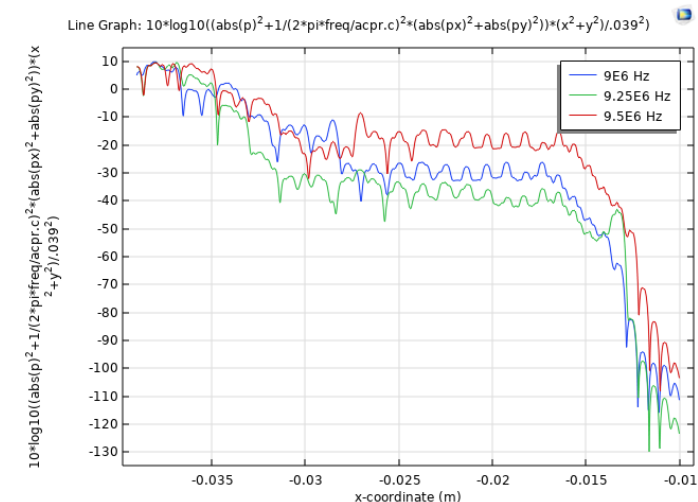


Channel extinction

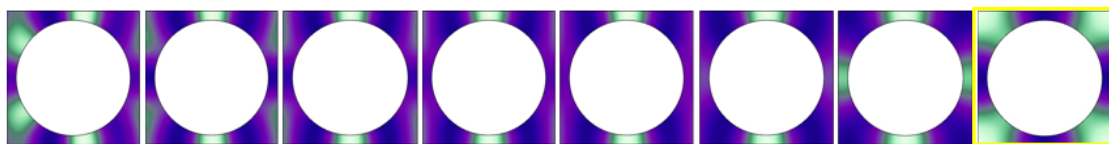
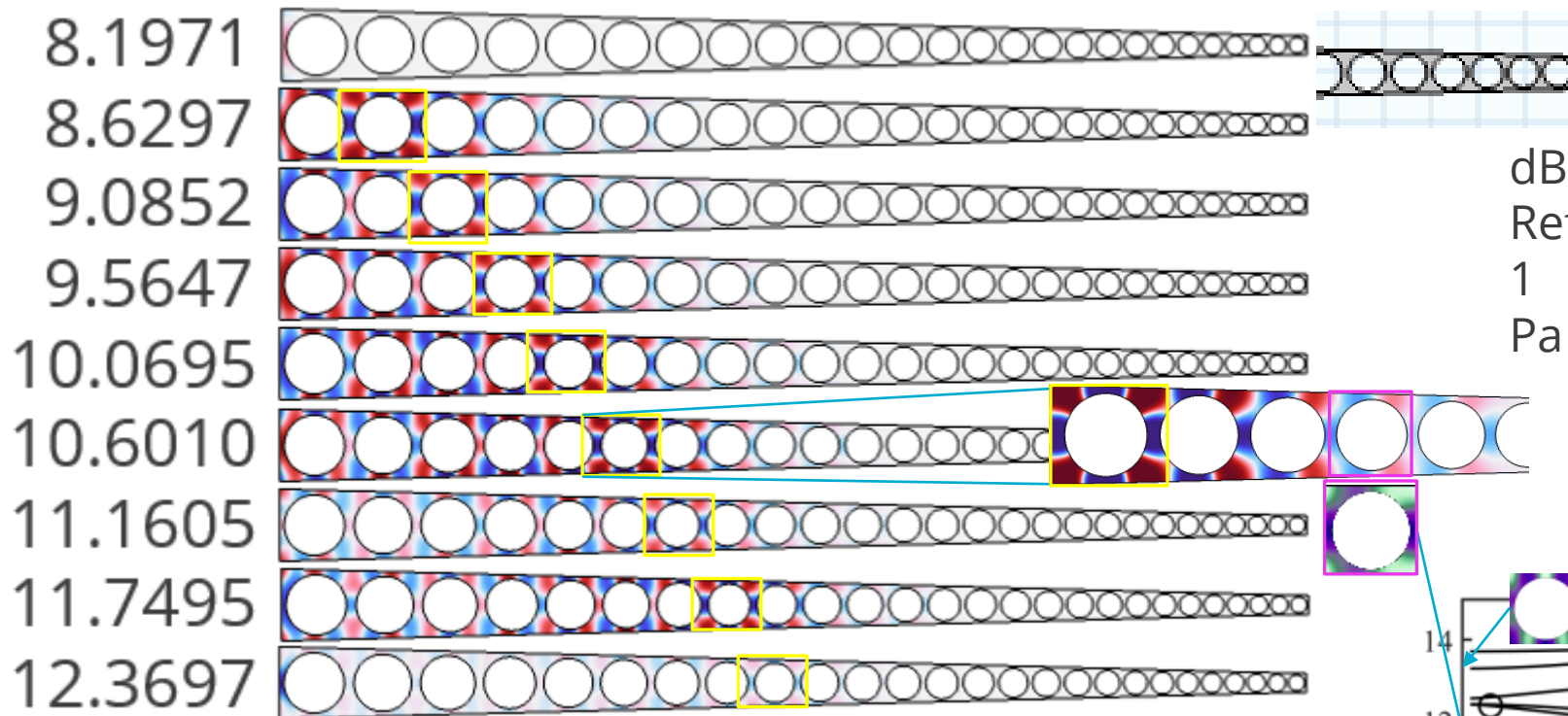


- Initially plotted $10\log_{10}(|p|^2)$, which was rough because of the nodal planes
- Used acoustic energy density (and later) with radial scaling term to clarify extinction pattern (get rid of nodal mess)

$$10\log_{10}\left(\left[|p|^2 + \frac{1}{k}\left(\left|\frac{\partial p}{\partial x}\right|^2 + \left|\frac{\partial p}{\partial y}\right|^2\right)\right]\frac{r_0}{r}\right)$$

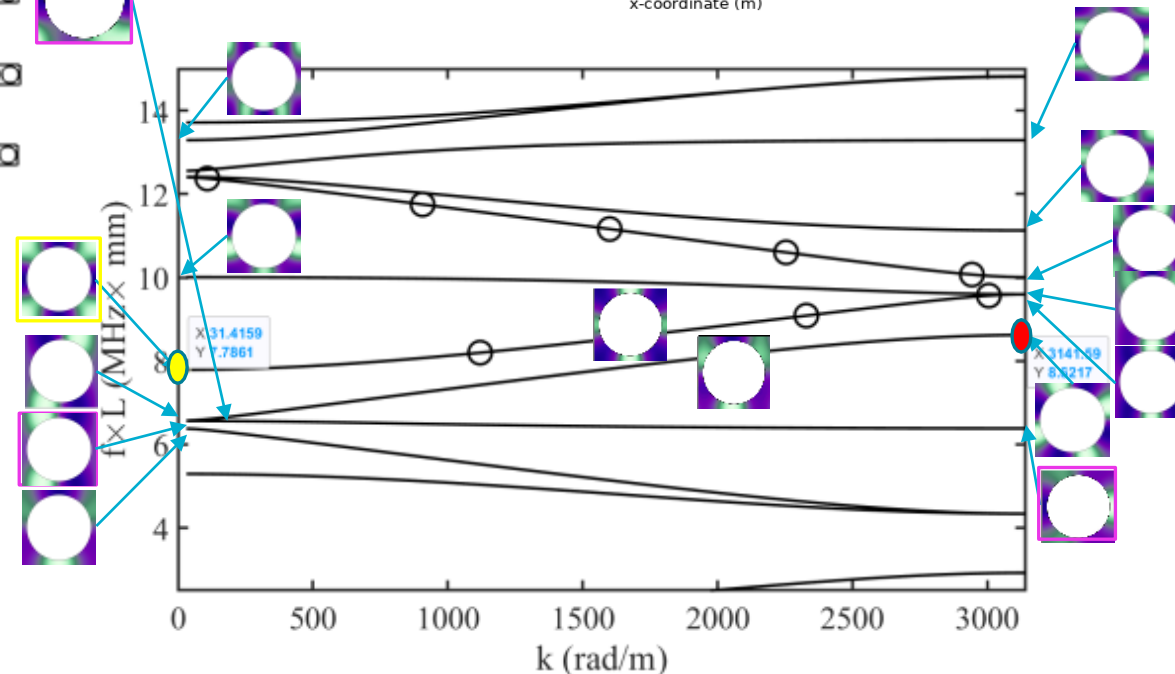
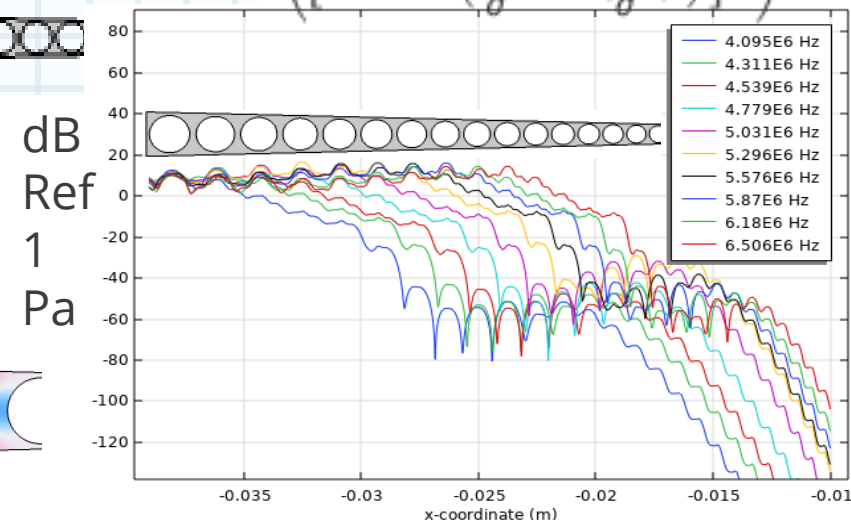


Example mode separation @ 8 MHz x mm



- Not all couple well
- All end with the same way with the +/-x mode
- Preceded by l-mode, etc
- Partial reprieve from tunneling to a branch with appropriate symmetry properties

$$10\log_{10}\left(\left|p\right|^2 + \frac{1}{k}\left(\left|\frac{\partial p}{\partial x}\right|^2 + \left|\frac{\partial p}{\partial y}\right|^2\right)\right)\frac{r_0}{r}$$



Conclusions

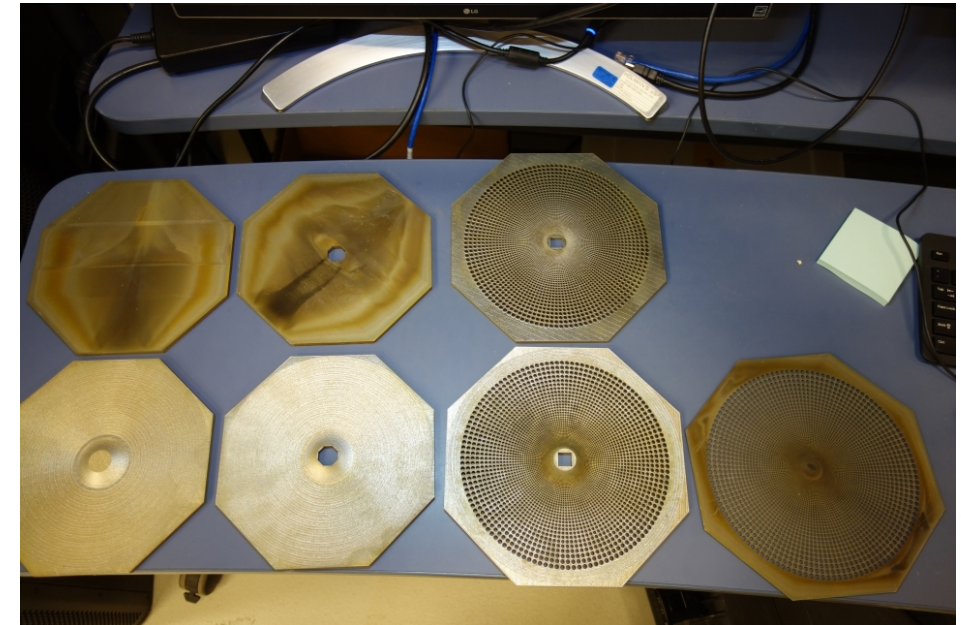


A variety of effects appear to contribute to the extreme wideband attenuation in phononic pseudocrystals, including:

- Angle-limited propagation
- A multitude of hole interior resonances
- Wavelength shortening leading to evanescence at exit
- Stranded propagation

Next time:

- Experiments!



ASA Meeting Applause! and Interrogation....



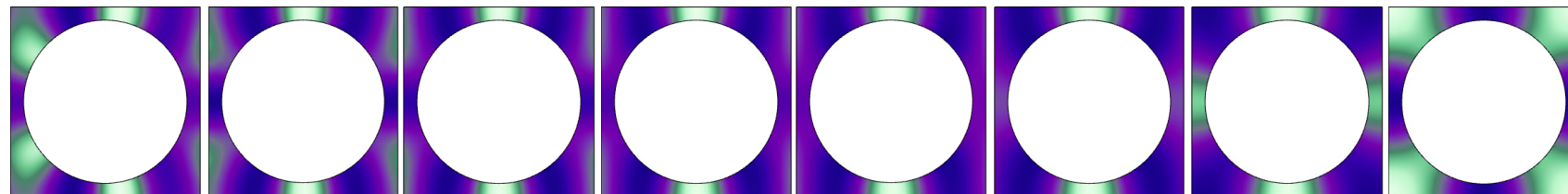
Lest I forget: I am (co-)/organizing two Phononics sessions for the Ottawa **Acoustic Society of America Meeting**:

- The phononic dispersion relations: Calculation, interpretation, and applications in phononics and metamaterials
- Developments and applications in phononic crystals

Contact me at shswift@sandia.gov if interested!

Questions?

11.16, $k=1592.8$



Dispersion perspective on suppression

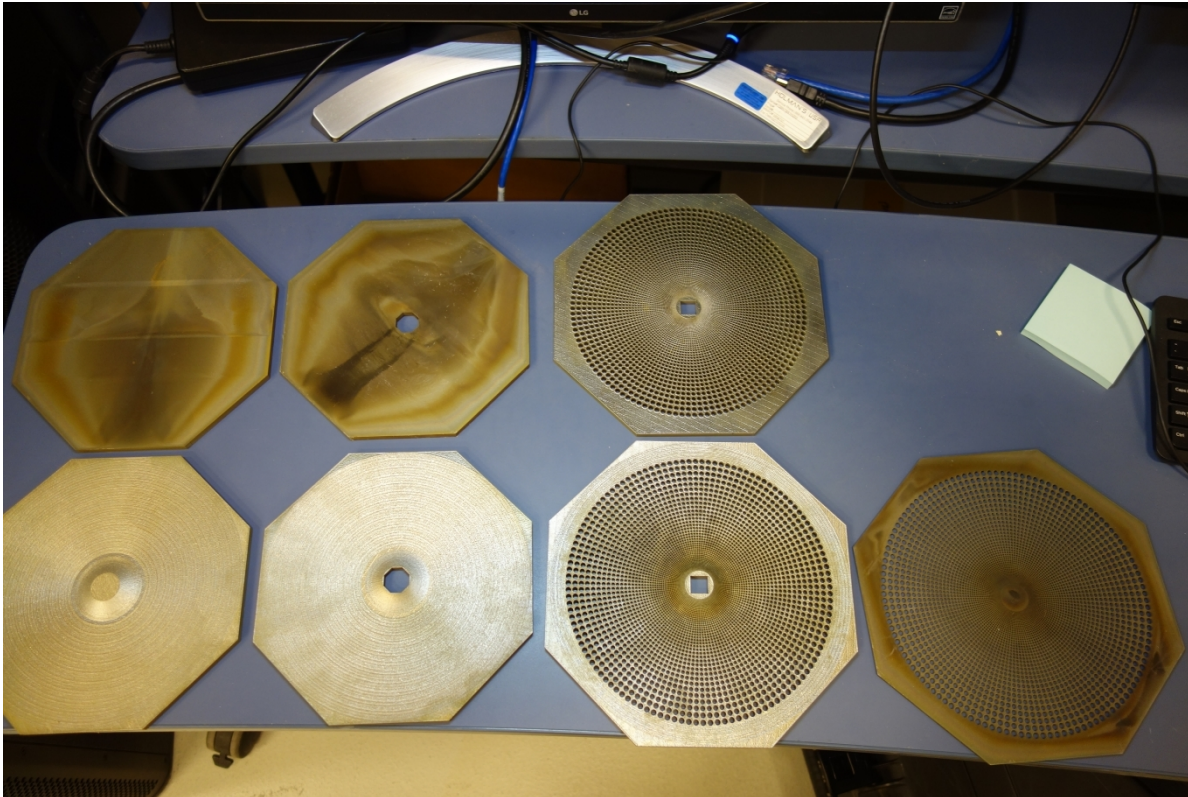


Numerous total bandgaps, numerous flat bands

Next time: Experimental results! (Assuming it worked)



3D-printed metal



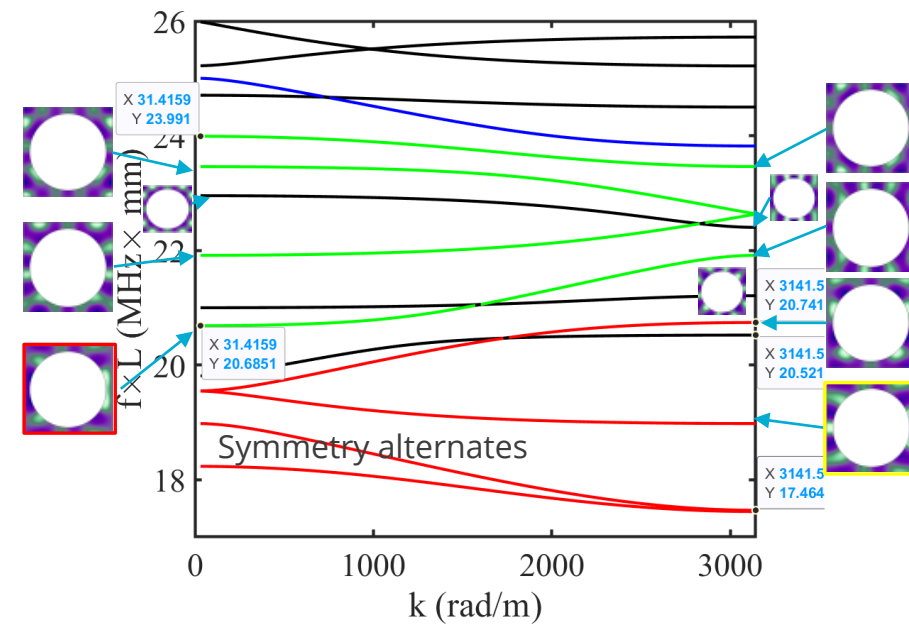
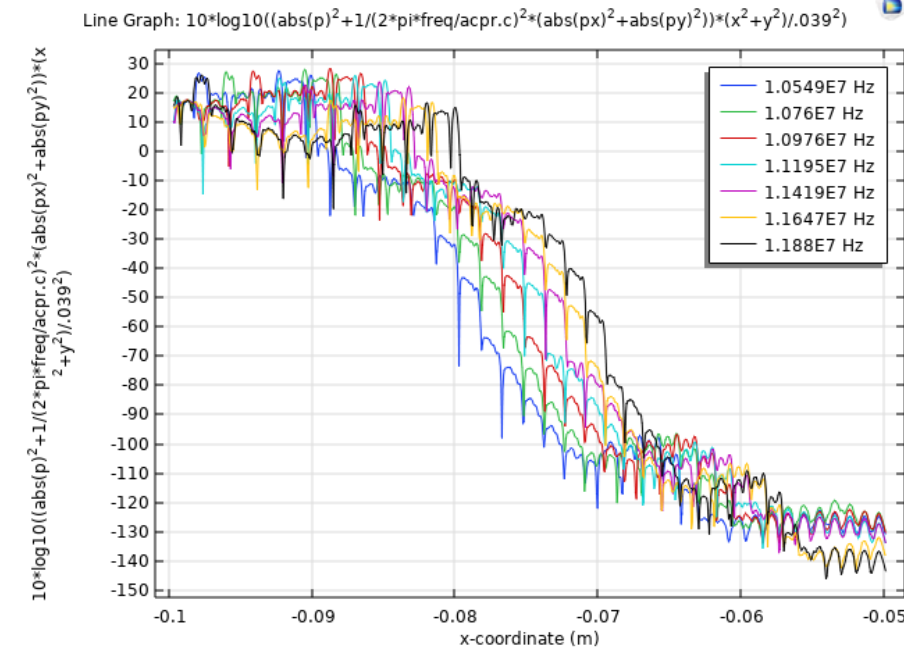
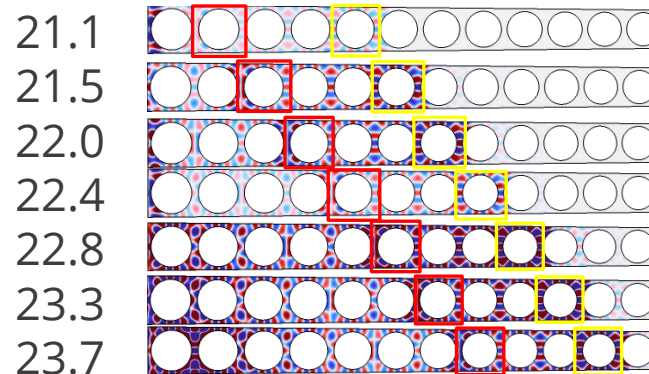
Mode tracking



Mode tracking can be accomplished by stipulating the frequency at which it occurs, solving the dispersion for the exact values of k corresponding to that frequency as an easy root finding problem, and then expanding the excitation as a linear combination of the associated eigenmodes.

Example 2

- $20.6851 \cdot 1.02^{(1:7)} = 21.1, 21.5, 22.0, 22.4, 22.8, 23.3, 23.7$, we expect that the lowest frequencies probably tunnel through; after a certain point, suppression should begin, and the higher the frequency, the longer it should take to initiate
- $20.6851 / 1.02^5 = 18.7351$
- 18-peak feature present in all channels

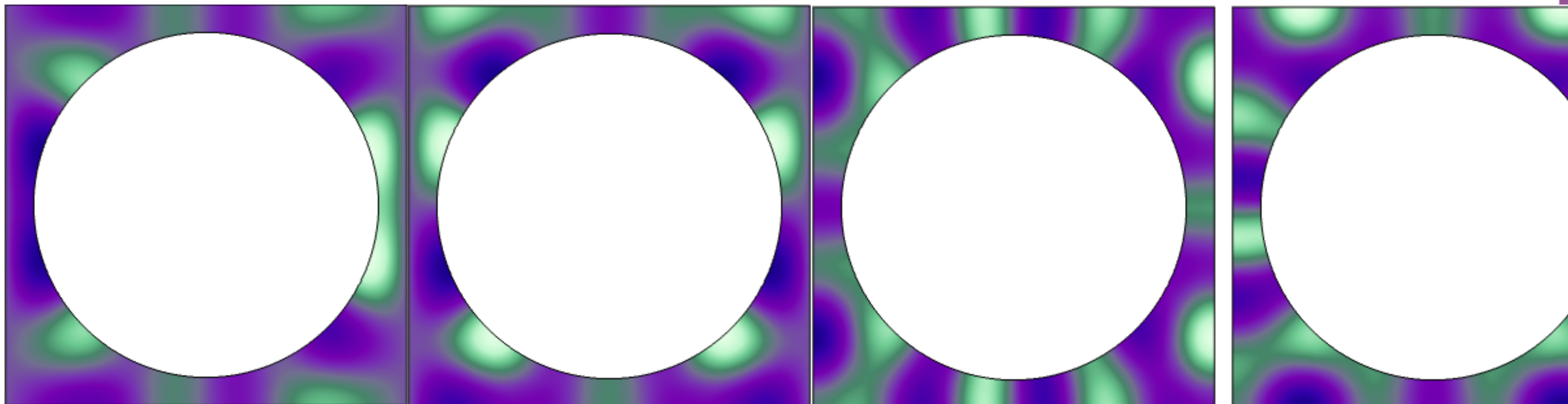


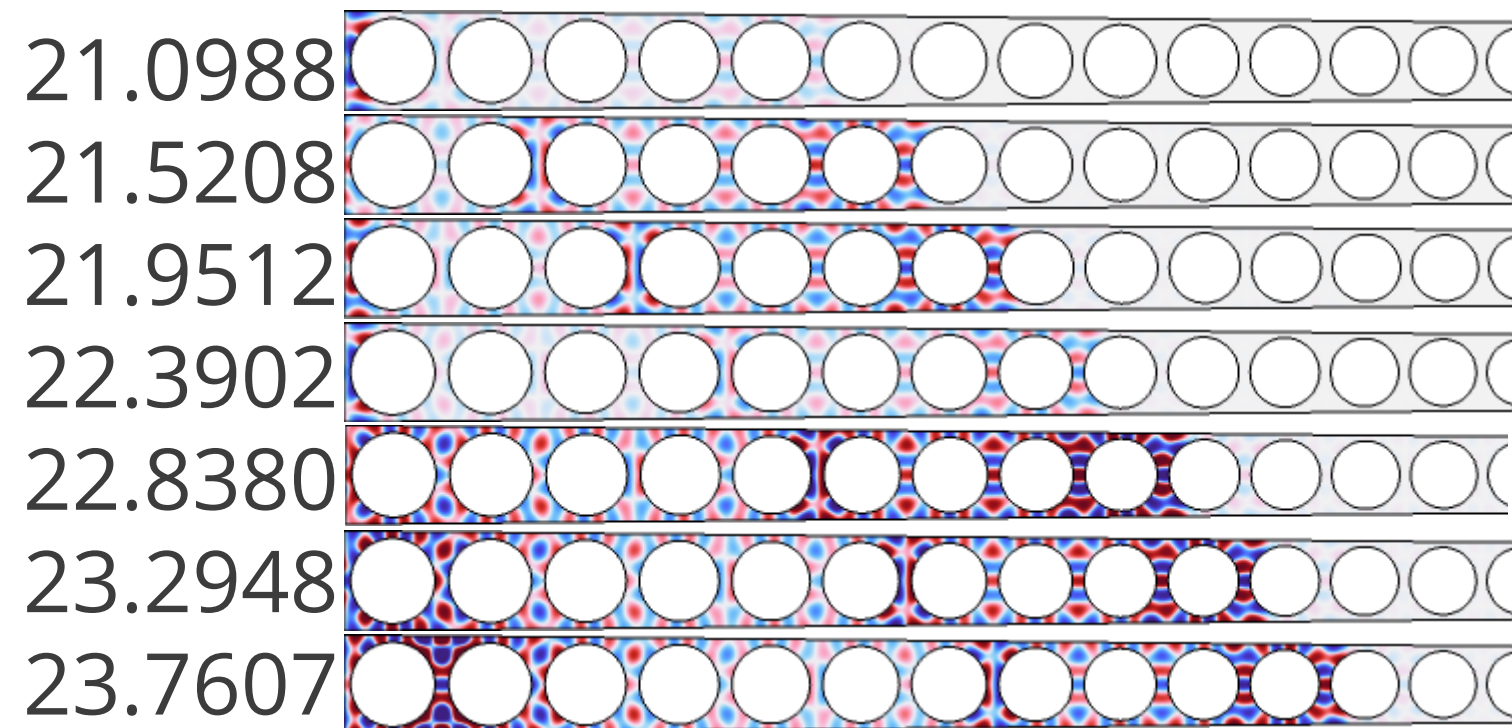
$K = 3.1416, F = 20.685e6$

$K = 3141.6, F = 20.741e6$

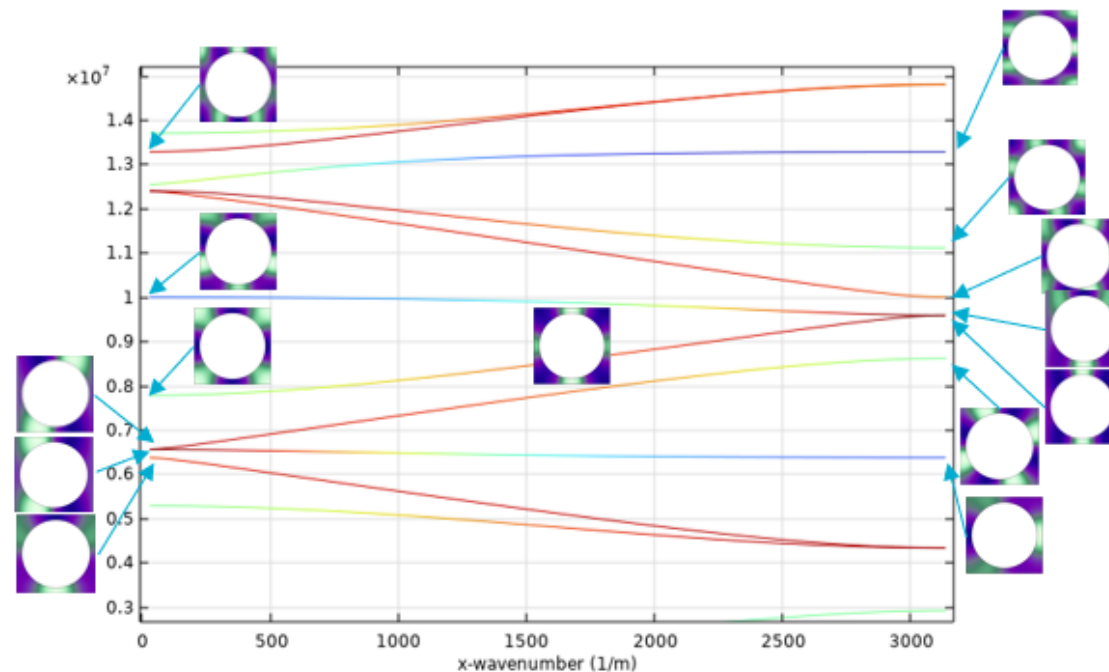
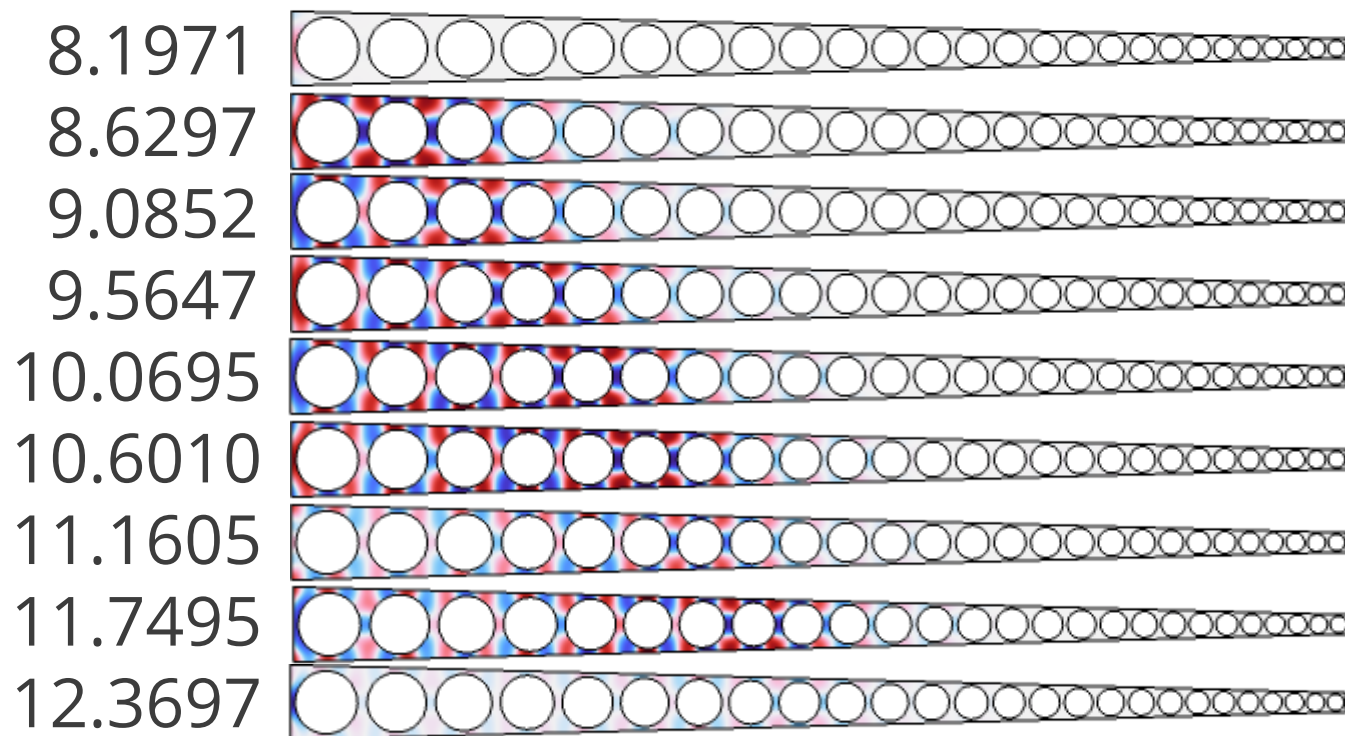
$K = 3141.6, F = 21.913e6$

$K = 3.1416, F = 21.913e6$





Example mode separation @ 8 MHz x mm, continued



Numerical experimental verification of stranded propagation



- Several possible routes:
 - Look for various features in the dispersion that would be expected to develop into bandgaps under variation of scale, then evaluate dispersion on whole article and see if they materialize
 - Excite a wave at one end of a channel; at each pseudocell expand the excitation as a sum of modes with their associated k-values (solid mechanics or acoustics)
 - Launch a wave down a channel and show that it is reflected/evanesces when expected

Acoustic demonstration opportunities



Acoustics provides a less dense band structure (1-mm x 1-mm cell, with sound speed given by aluminum longitudinal wave speed)

- 8 MHz has two available modes that are separated from one another in wavenumber with the upper and lower bands having distinct min and max, respectively
- 24 MHz has two available modes as well as regions where both are the only mode available at their frequency
- 25.5 MHz has a pair of dispersion lines crossing each of which has an independent frequency range, allowing us to test whether crossover is allowed
- 29.96 MHz, 30.3, 33.25 MHz each provide accessible mode pairs
- 32.7 intersection of isolated modes

