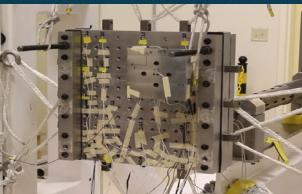


Response Limiting & Optimizing Shaker Shocks

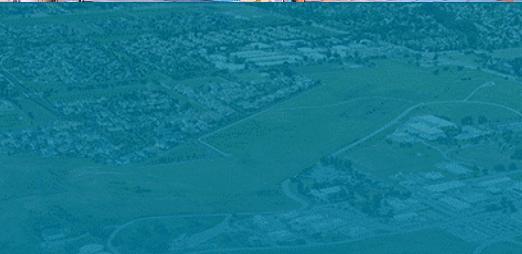


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June 26, 2023

Aerospace Shock Test Working Group Meeting

El Segundo, CA



Introduction



Introduction

Shaker Shock Basics

- Sums of Decayed Sines

Response Optimization & Response Limiting

- Theory
- Algorithms

Results

- Test Article
- Pre-Test Analyses
- Shaker Shock Test

Conclusions

Introduction



The primary goal of any laboratory test is to expose the unit-under-test (UUT) to conservative, realistic representations of a field environment

- Random vibration, sine vibe, shock, thermal cycling, etc.

Satisfying this objective is not always straightforward due to laboratory test equipment constraints

- Mechanical shakers have nearly infinite impedance
- Over-testing and unrealistic failures can result when controlling the unit base acceleration

Force limiting and response limiting are relatively standard practices to reduce over-test risks in random vibration laboratory testing

Limiting in shaker shocks is done for the same reasons

- Force limiting can be done in shaker shock testing (in theory)
- **This presentation is about response limiting and optimizing single axis shaker shocks**

At SNL we do response-limited testing much more than force limited testing



SAND2023-01844-O
Single Axis Shaker



SAND2023-01844-O
6-DOF Shaker



Objectives of Response Limiting

When a measured acceleration on the UUT exceeds a threshold, the control is reduced so that the response does not exceed the threshold (limit)

PROS

- Response limiting can protect sensitive high-value subcomponents from over-test risk
 - Trade-off – Allow potential undertesting elsewhere
 - This is generally acceptable because test environments are conservative
- Response limits minimize the distortion of the input specification
 - Response limits are enforced at specific frequency bands of concern
 - Near resonances where response may be unacceptably amplified
- Response limits can be defined at multiple locations and in different axes simultaneously
- SRS based – Acceleration based on Sum of Decayed Sines (SDS)

CONS

- Critical locations may be inaccessible for instrumenting
- Need pre-test response predictions/limits
- Must be pre-planned – not a closed-loop process like RV response limiting

Objectives of Response Optimizing



Response optimizing seeks to define an input that minimizes the error between test measured responses and specified environments

- **SIMO (Single Input Multi Output) least-squares SRS optimization**

PROS

- Better (best possible) match to known (field) environments at the defined response locations
 - Overall better test (more field-like environments)
 - Can be combined with response limiting

CONS

- May heavily distort the input signal
- Need “field” SRS at response locations
- May over-test at some locations
 - Do not use with envelope response targets – use response limiting
- Critical locations may be inaccessible for instrumenting
- Open loop process – must be pre-planned

Shaker Shock Basics



Shaker shocks are run open loop

- Shaker shocks are usually deterministic waveforms
- Shakers are driven by acceleration time histories

The test engineer can specify the input acceleration directly

- Waveform replication

The waveform can be created based on algorithms that create an acceleration history whose SRS matches the specified SRS

- Sum of Decayed Sines waveforms
- Random shock waveforms
- Zero Residual Displacement (ZERD) waveforms
- WAVSYN (half-sine modulated sinusoids)

The shaker shock synthesis tools used for this presentation are derived from David Smallwood's work

This presentation is about SDS based shaker shocks based on Smallwood's SDS algorithm [1-6]



Shaker Shock Basics – Sum of Decayed Sines



Fundamental SDS waveform equation:

- $z(t) = \sum_{i=1}^N A_i e^{-2\pi f_i \zeta_i t} \sin(2\pi f_i t)$
- A_i – Tonal amplitudes
 - Signs of A_i alternate – i.e., $A_i > 0, A_{i+1} < 0$, etc.
 - Gives smoother SRS and reduced compensating pulse
 - Initial guess – $A_i = \frac{SRS(f_i)}{Q}$
- f_i – Tonal frequencies
 - At least 5 tones/octave for 3% SRS damping
 - At least 3 tones/octave for $\geq 5\%$ SRS damping
 - More tones/octave are better
 - Richer spectral content is probably more realistic and tends to produce lower peak acceleration
 - Avoid harmonics
- Duration (T) is the greater of two values:
 - Real world waveform
 - 10 cycles of the lowest tonal frequency
- ζ_i – Tonal damping ratio
 - $\zeta_i = \frac{1}{2\pi\tau f_i}$ - τ is the exponential time constant
 - Usually τ is a constant – $\tau < \frac{T}{3}$
 - Ensures adequate ringdown
 - Higher frequency tones ring for more cycles
- Sample rate
 - $SR \geq 10f_{max}$; $5f_{max}$ may be OK for flat & declining SRS

Sum of Decayed Sines



Creating an acceleration history whose SRS matches a specified SRS is an iterative process

- The SRS is a nonlinear transformation

The Smallwood SDS algorithm [1] estimates the tonal amplitudes starting with the lowest frequency and proceeds to higher frequencies

- The amplitude at the i^{th} frequency, A_i , is adjusted to minimize the SRS error at that frequency:

$$\text{Err}(f_i) = \frac{SRS_C(f_i) - SRS_{ref}(f_i)}{SRS_{ref}(f_i)}$$

- $SRS_C(f_i)$ is based on the current tone and all other tones computed to that point

Many shaker controllers (Spectral Dynamics, Data Physics, etc.) can make the input signals from SDS tables

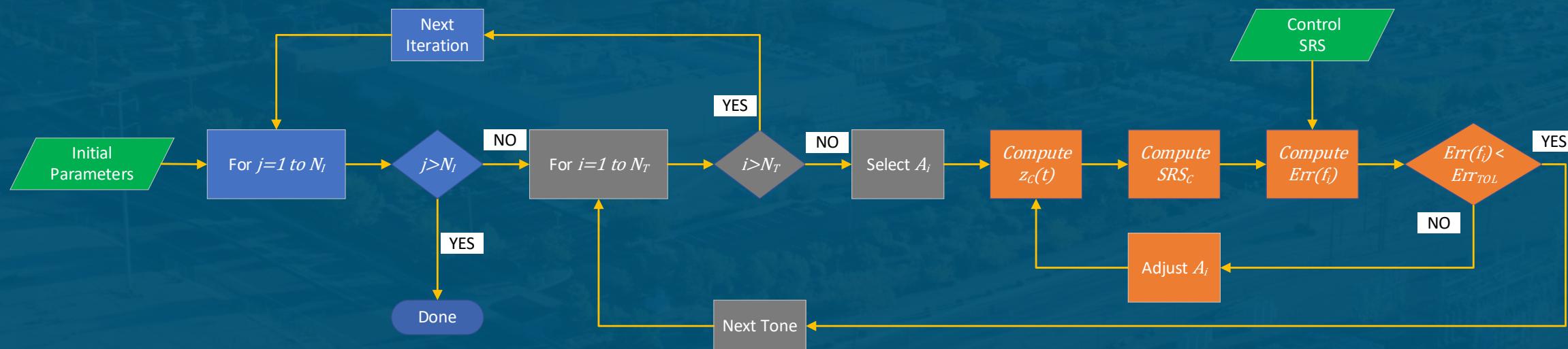
- The SDS software in some controllers is based on the Smallwood algorithm
- Differences in how they minimize residual velocity and displacement
 - Compensating pulse or high-pass filter
- NASA GSFC (Dan Kaufman) has recently developed a different SDS approach – conceptually similar

Sum of Decayed Sines – Basic Algorithm Flowchart



Three loops

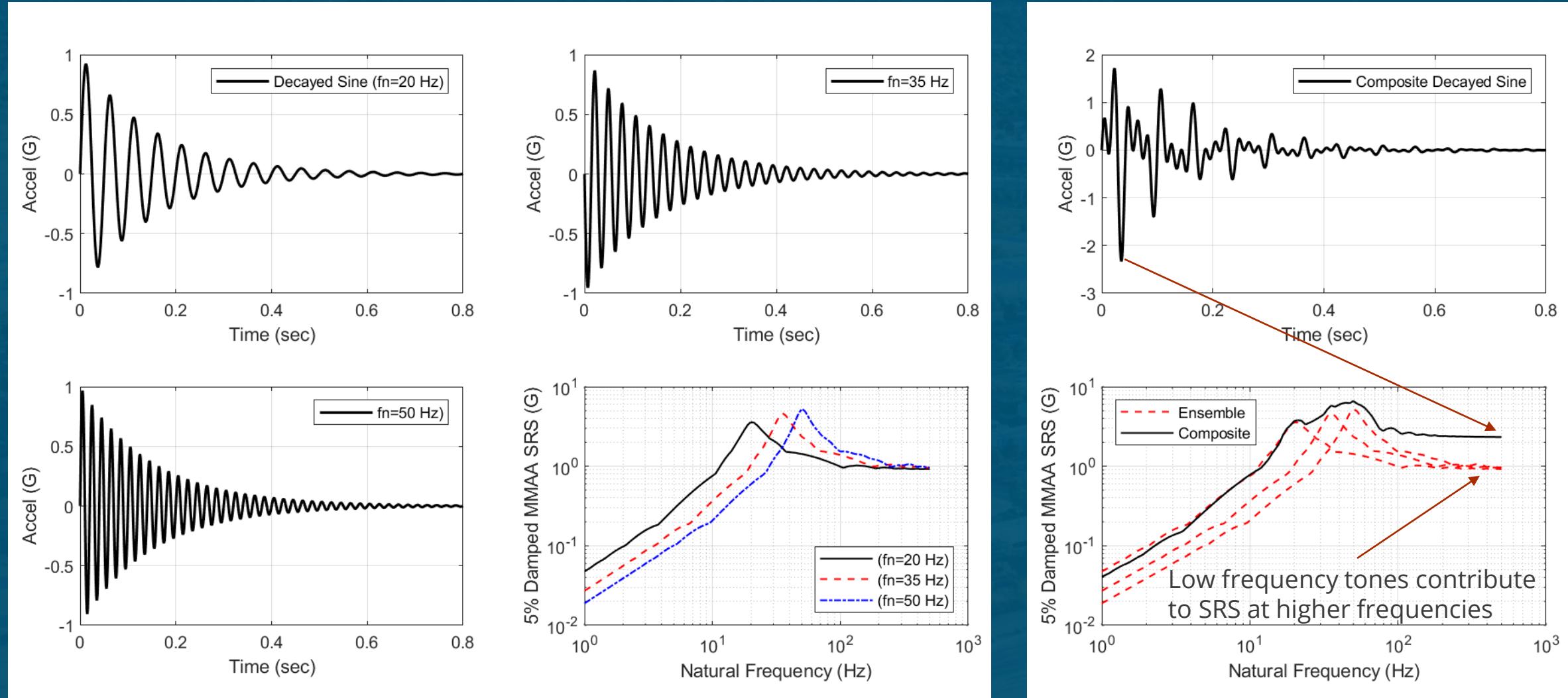
- Inner loop – Adjust amplitude A_i to minimize error
 - Composite acceleration time history - $z_c(t)$
 - SRS of composite acceleration – SRS_c
- Intermediate loop – all tones from left to right
 - Number of Tones – N_T
- Outer loop – Repeat intermediate loop N times (corrects for coupling of tones)
 - Number of iterations - N_I



$$Err(f_i) = \frac{SRS_c(f_i) - SRS_{CTRL}(f_i)}{SRS_{CTRL}(f_i)}$$

Sum of Decayed Sines - Example

This example shows a 3 tone SDS generated shock



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Response-Optimized Shaker Shocks (SIMO Optimization)



Objective

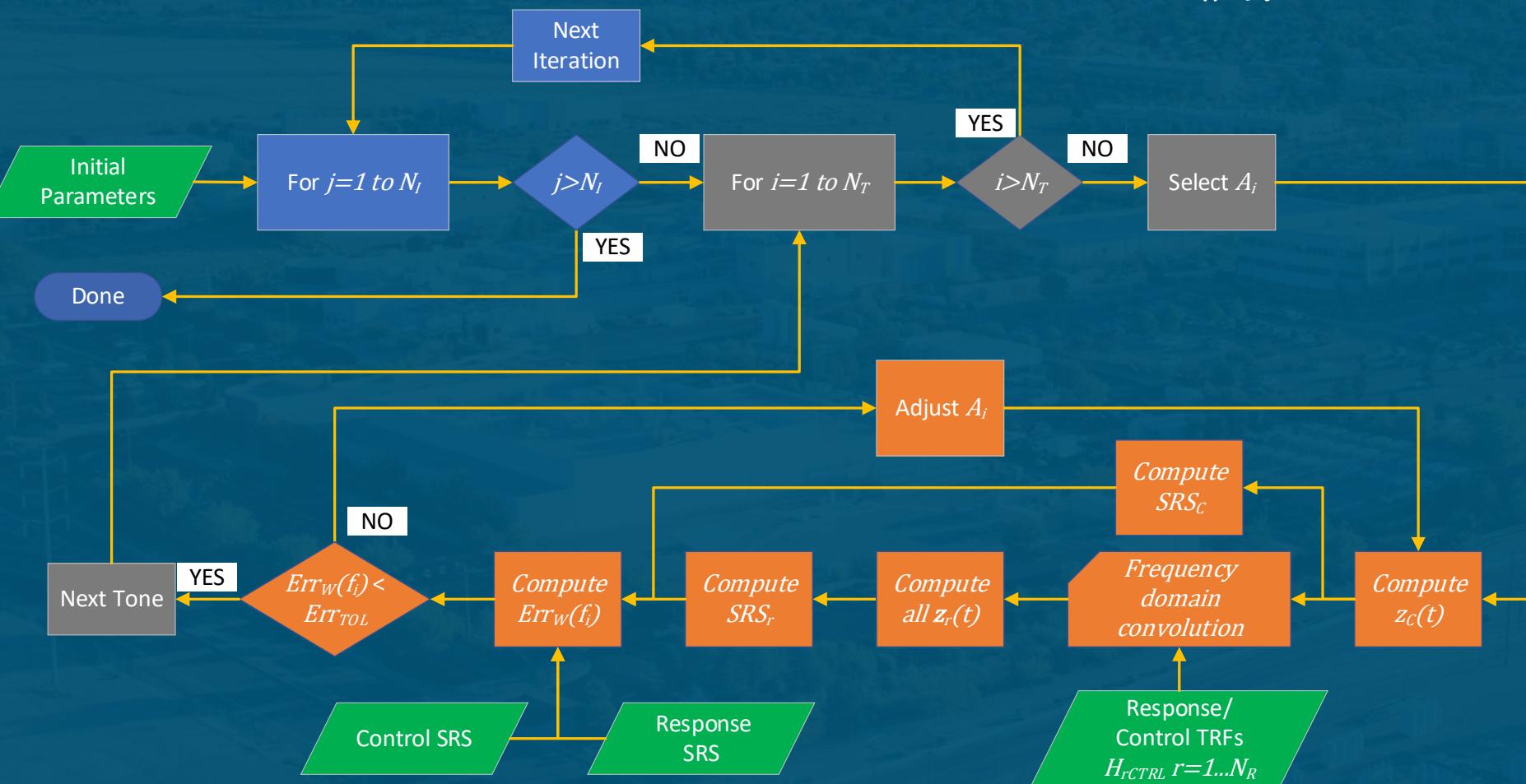
- Define an input that minimizes the SRS error between N test-measured responses and specified environments in a least-squares sense
 - The control channel can be included in the response optimization set
- The result is not a global minimum – it depends on the SDS equation parameters
 - Primarily the tonal frequencies

SIMO optimization algorithm has the same three loops as the basic SDS algorithm

- Need Transmissibility Response Functions (TRFs) from control to response locations
 - One TRF per response location
 - If the control channel is included, TRF = 1 at all frequencies
- Different error function for the inner loop:

$$\boxed{\mathbf{Err}_W(f_i) = \frac{\sum_{r=1}^{N_R} \mu_r (SRS_C(f_i) - SRS_r(f_i))}{\sum_{r=1}^{N_R} \mu_r}}$$

- Inner loop error function is the weighted sum of errors for each response location



$$Err_W(f_i) = \frac{\sum_{r=1}^{N_R} \frac{\mu_r (SRS_C(f_i) - SRS_r(f_i))}{SRS_r(f_i)}}{\sum_{r=1}^{N_R} \mu_r}$$

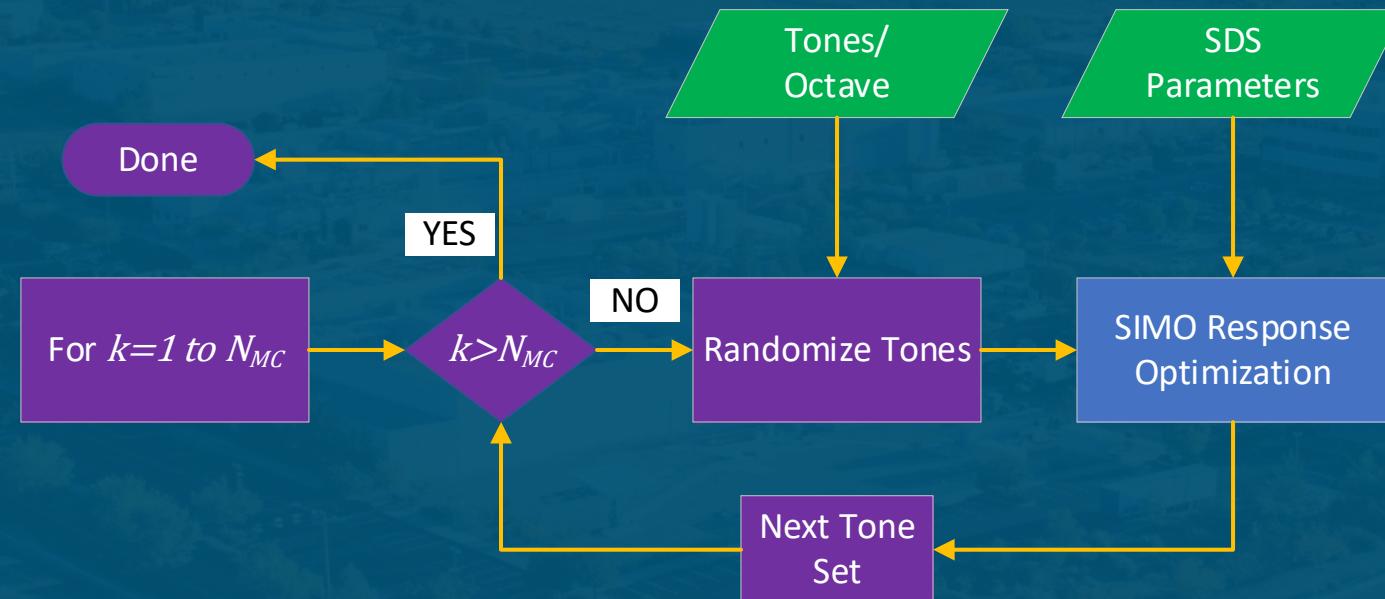
Monte Carlo Response-Optimized Shaker Shocks



The SIMO optimized control acceleration is sensitive to the distribution of the tonal frequencies in the SDS equation

- The SIMO optimization problem is solved multiple times with different sets of tonal frequencies

Monte Carlo Algorithm Flowchart

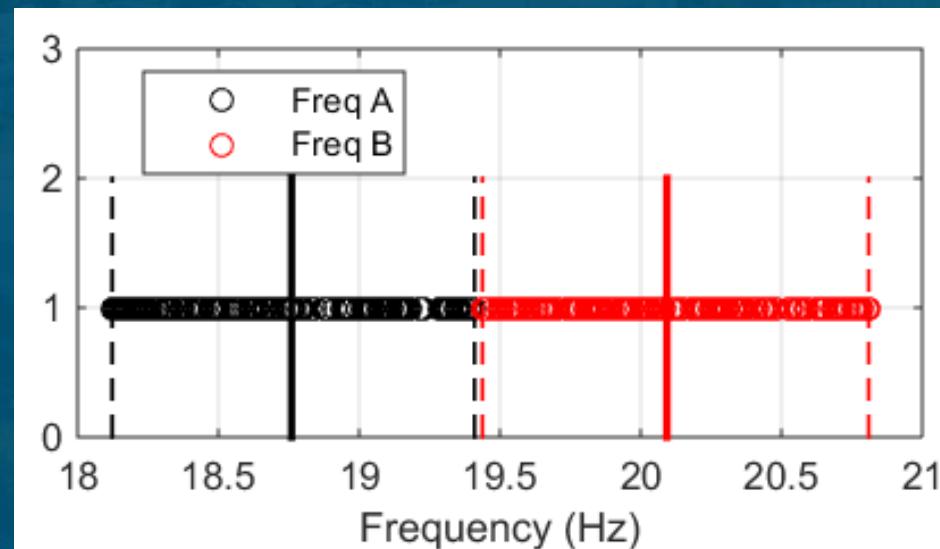


Monte Carlo Response-Optimized Shaker Shocks



A Monte Carlo algorithm is used to generate the control SRS with various combinations of tonal frequencies

- Octal spacing of nominal tonal frequencies (e.g., 3 tones/octave)
- Tonal frequencies are defined from a uniform distribution around each nominal tone
 - Guard bands are imposed at the boundaries between adjacent tones to prevent a non-monotonic vector of frequencies

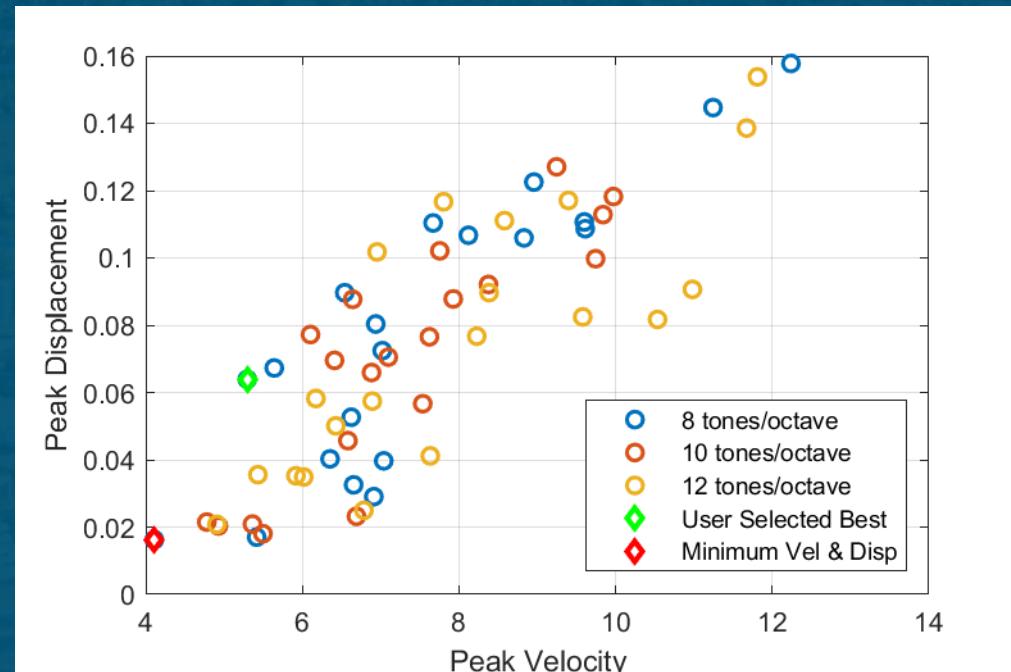
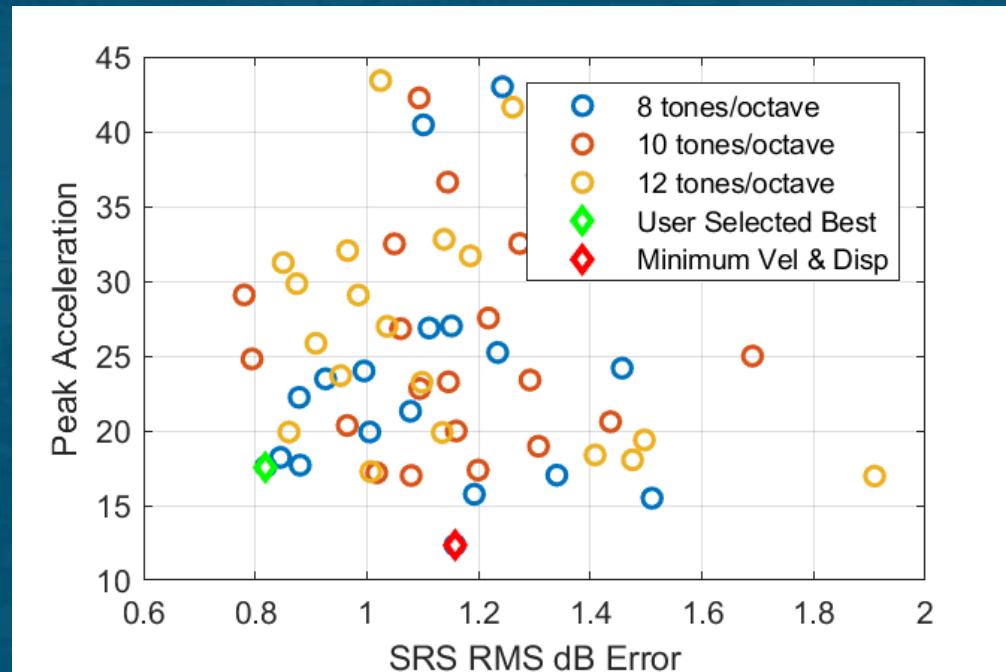


Monte Carlo Response-Optimized Shaker Shocks



The “optimal” control acceleration history is chosen based on 2 or more scalar metrics

- Peak acceleration, Peak Vel, Peak Displacement, RMS dB Error, Energy
- “Optimal” is user selected from a scatter plot



$$E_{RMSDB} = \left(\frac{1}{N_R} \right) \sum_{r=1}^{N_R} \left(\sqrt{\frac{\sum_{i=1}^N \left[\left(20 * \log \left(\frac{SRS_M(f_i)}{SRS_r(f_i)} \right) \right)^2 \Delta f(i) \right]}{f_B}} \right)$$



Optimized Response & Limited Shaker Shocks

Response optimization generates a control SRS so that the SRS at response locations are a “best fit” in a least-squares sense

- No guarantee that SRS will not exceed target SRS at response locations
- Response optimization produces a “better fit” outside the limiting frequency bands

Response limiting creates a control shaker shock such that responses do not exceed target SRS in specified frequency bands

- Must include the “control” channel in the response set
 - This SRS will be matched if the limits do not engage
- Flowchart is the same as the response optimization flowchart
 - Different error equation
- Convergence is not guaranteed

Optimal Response Limiting combines SIMO optimization with response limiting

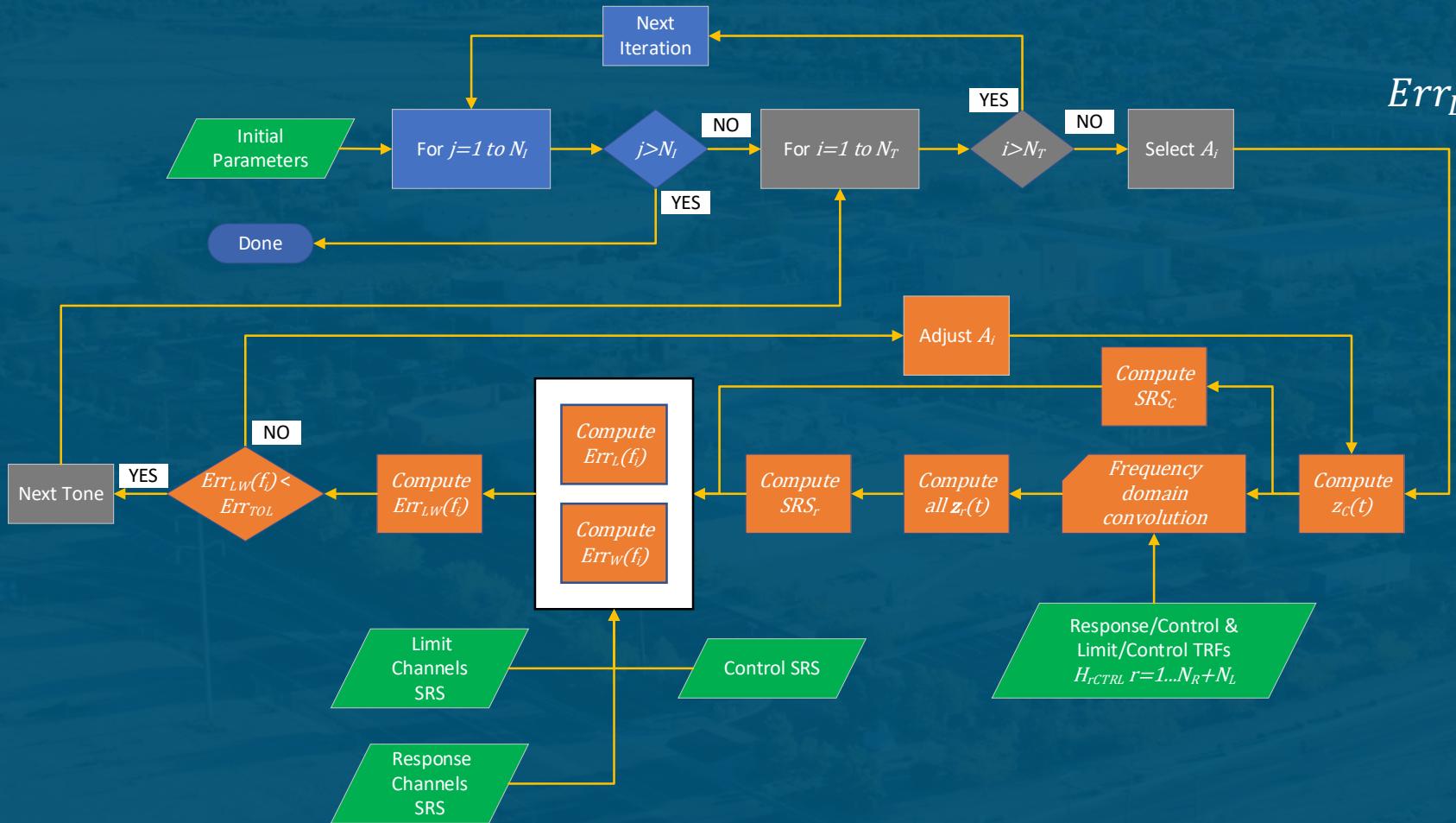
- **Can use Monte Carlo Optimization loop**

Optimized Response & Limited Shaker Shocks



Algorithm Flowchart

- Channels can be in the limit set and the optimization set
- The outer Monte Carlo loop can be added



$$Err_W(f_i) = \frac{\sum_{r=1}^{N_R} \frac{\mu_r (SRS_C(f_i) - SRS_r(f_i))}{SRS_r(f_i)}}{\sum_{r=1}^N \mu_r}$$

$$Err_L(f_i) = \max_{r=1 \dots N_L} \frac{(SRS_C(f_i) - SRS_r(f_i))}{SRS_r(f_i)}$$

Optimized Response-Limited Shaker Shocks

The response-limiting algorithm uses the SIMO base algorithm with a set of conditional if statements

if ($ERR_L > 0 \&\& ERR_W > 0$) | ($ERR_L < 0 \&\& ERR_W < 0$) *then*

- $ERR_{LW} = \max[ERR_L, ERR_W]$

elseif $ERR_L > 0 \&\& ERR_W < 0$ *then*

- $ERR_{LW} = ERR_L$

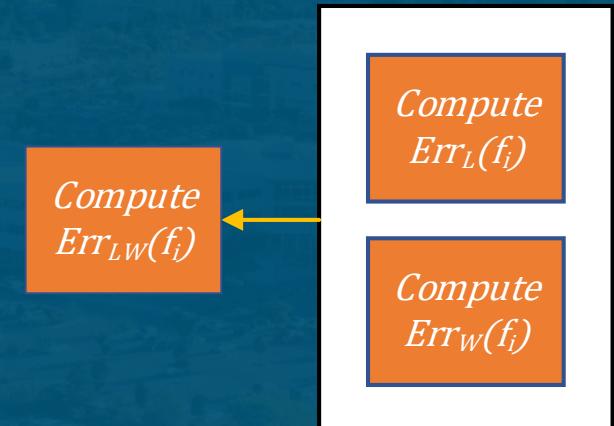
elseif $ERR_L < 0 \&\& ERR_W > 0$ *then*

- $ERR_{LW} = ERR_W$

end

If the limits do not engage, the algorithm reverts to the basic SDS algorithm or the SIMO algorithm

We also use the slope of the correction (not shown here) when updating A_i



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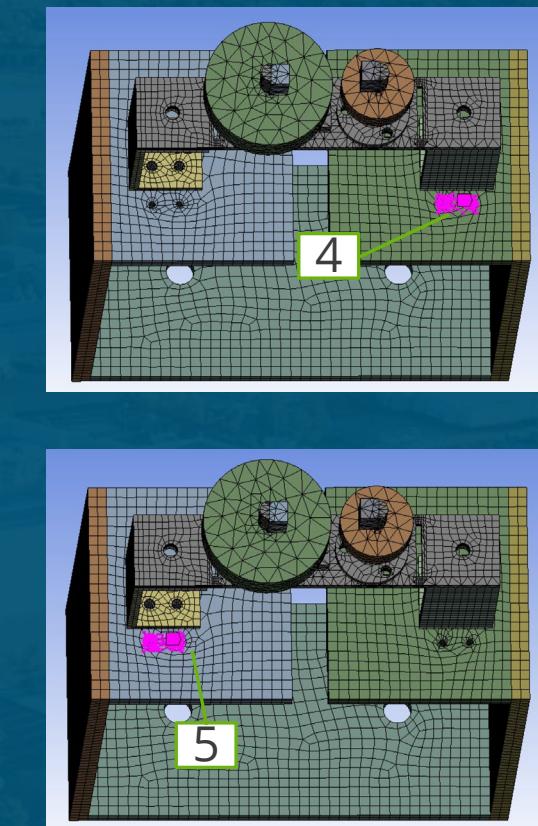
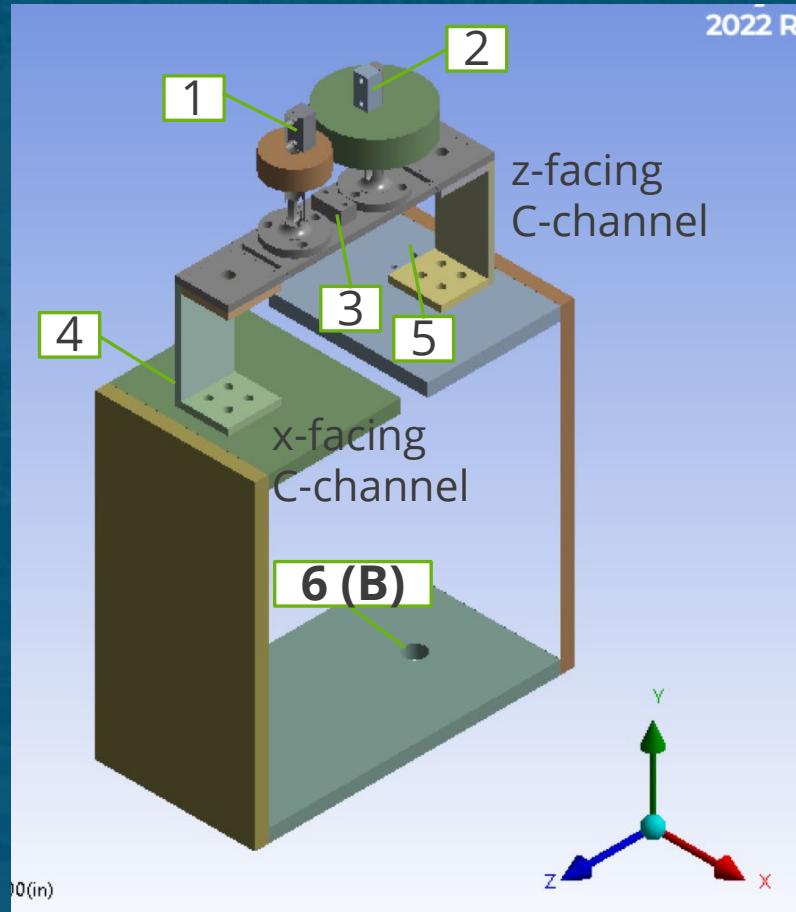
Test Article – BARBECUE

Box And Removable Bridge with External Components Under Evaluation

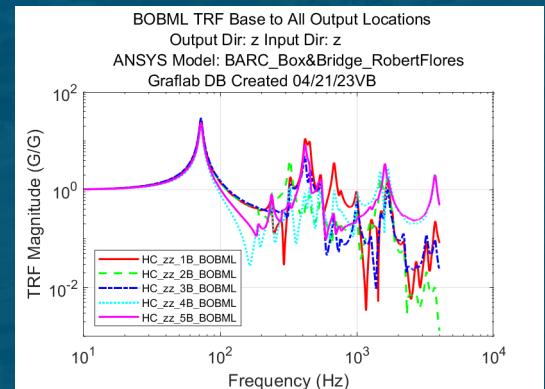
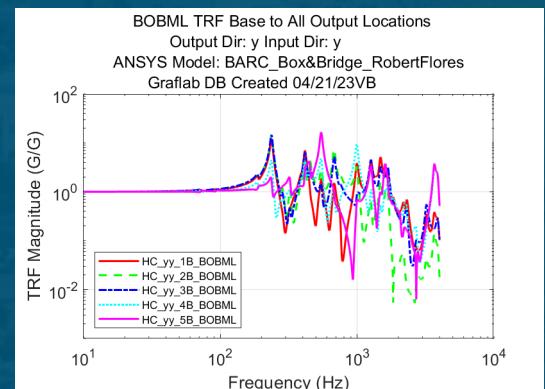
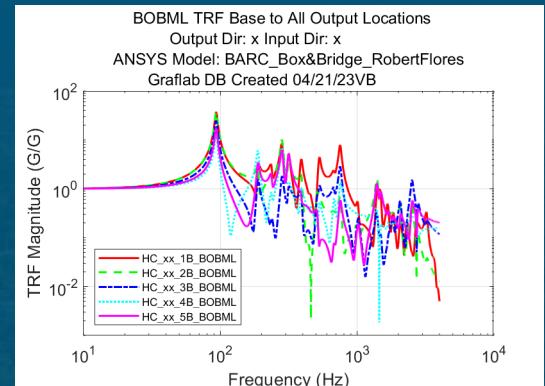


BOBML configuration

- Bridge On Box Medium mass & Large mass



Mode	Frequency (Hz)
1	72.132
2	93.162
3	188.06
4	236.45
5	281.71
6	314.68
7	322.64
8	389.17
9	415.25
10	449.53
11	510.73
12	527.88
13	545.96

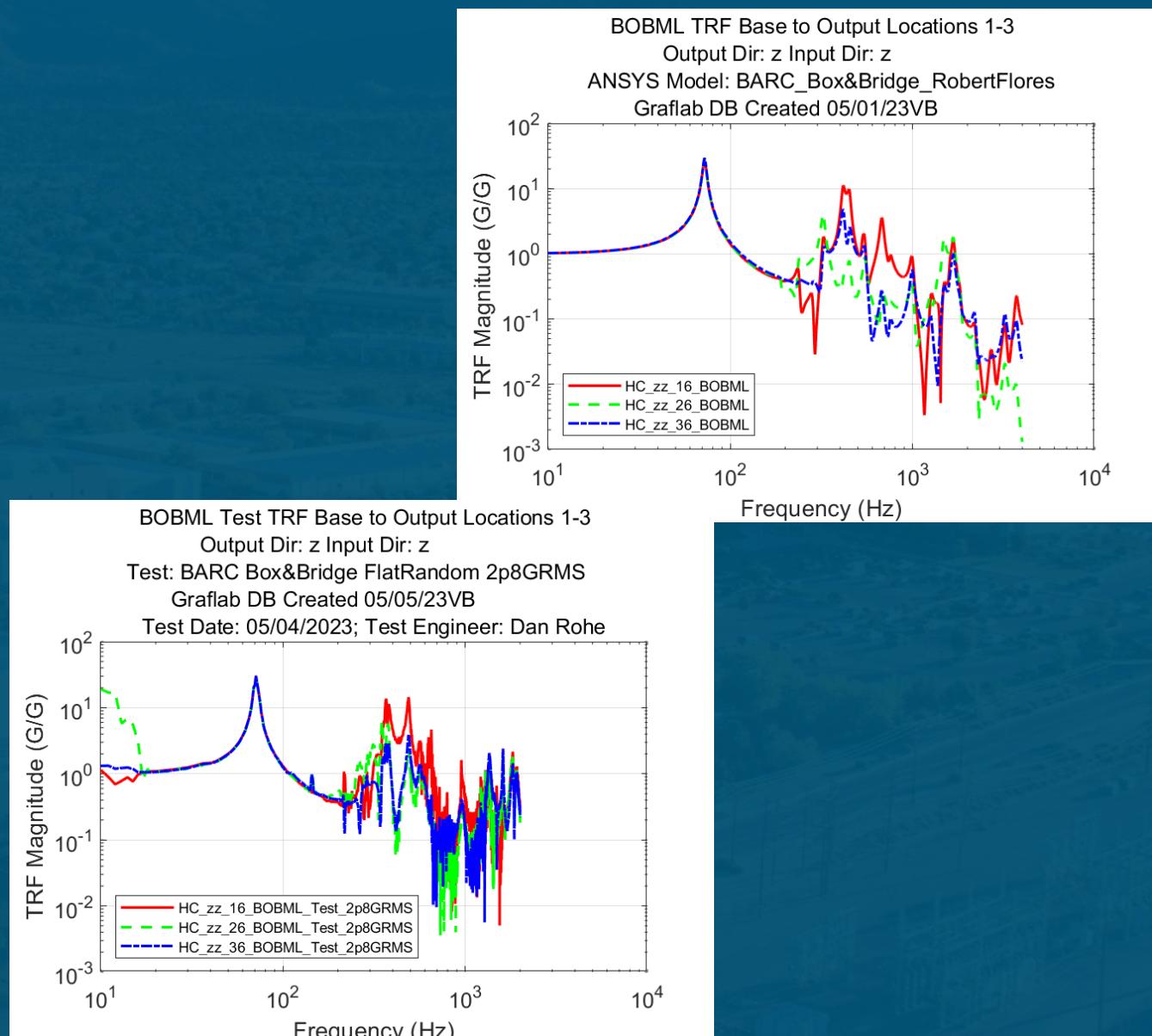
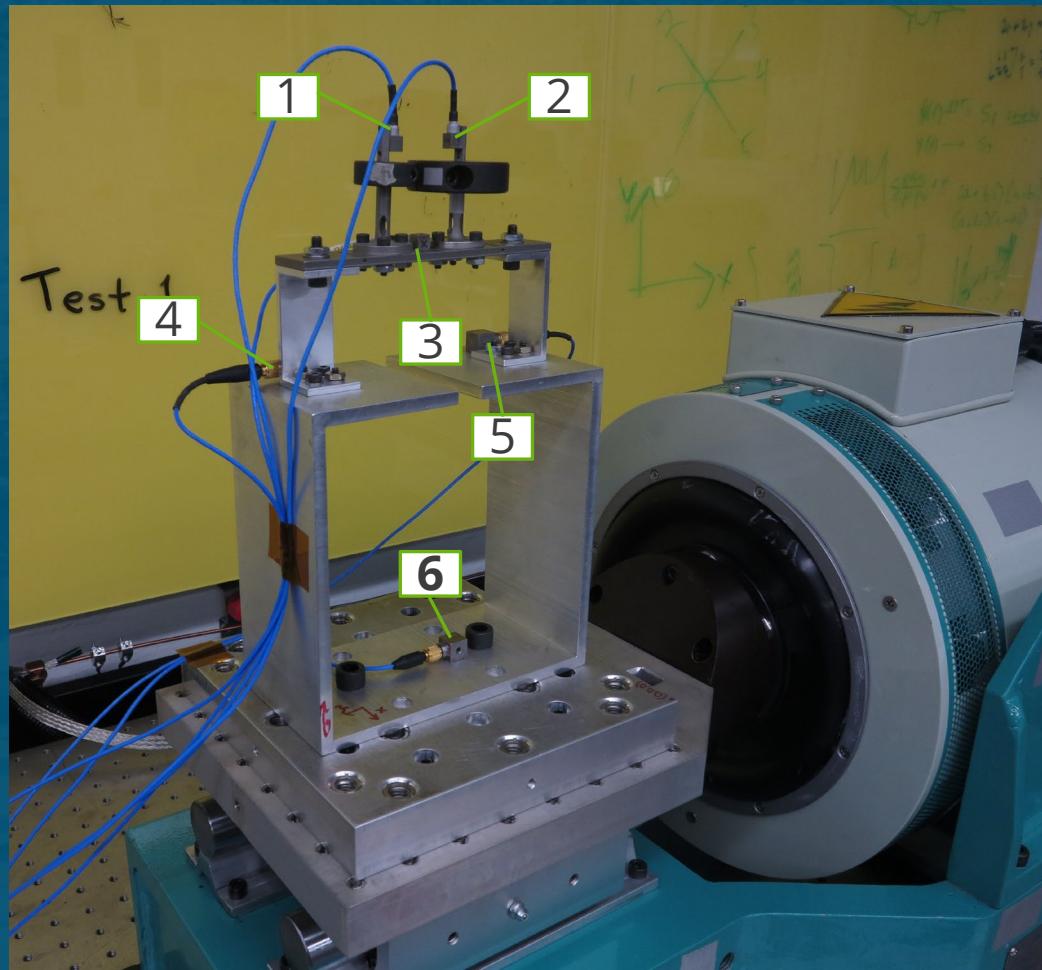


BARBECUE Test



Testing was conducted in the Z axis

- The tests were controlled at the base
 - Location 6



BARBECUE Response Optimization & Limiting



Two resonances are used in this study

- Fundamental resonance at 71 Hz
- Resonance at 370 Hz

Field environment

- Measured responses at locations 1,2,3 and 6
- Represents what would have been measured on the BARBECUE in service

Test environment

- A straight-line laboratory test spec that represents the field environment
 - SRS at the base made using the best practice of “filling-in” dips in the field SRS
 - Straight-line response limits at locations 2 & 3

Measured TRFs and test spec are used to pre-compute the test base input for different objectives

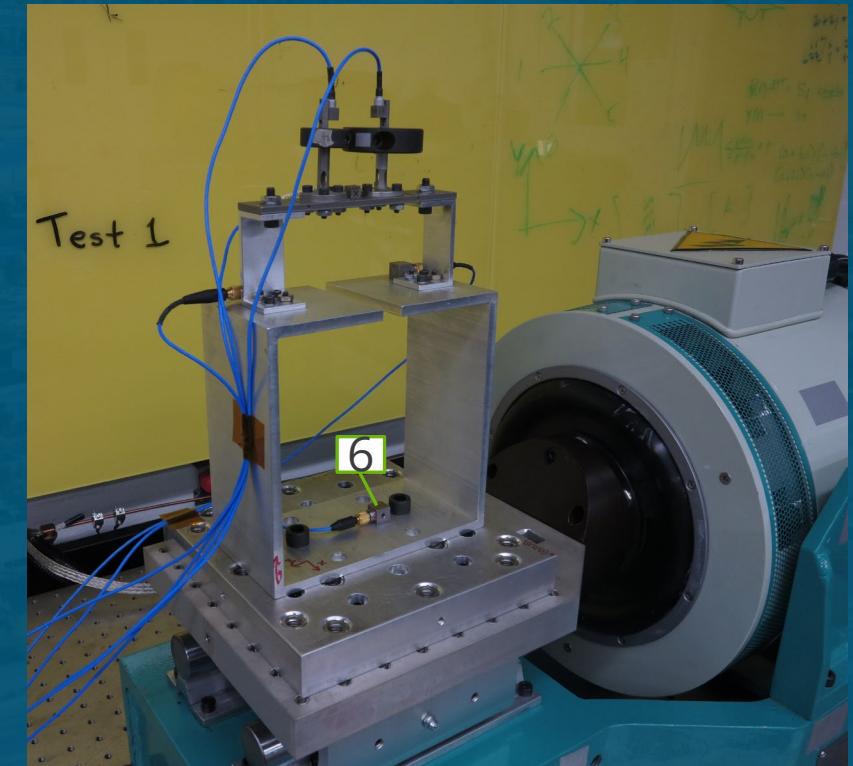
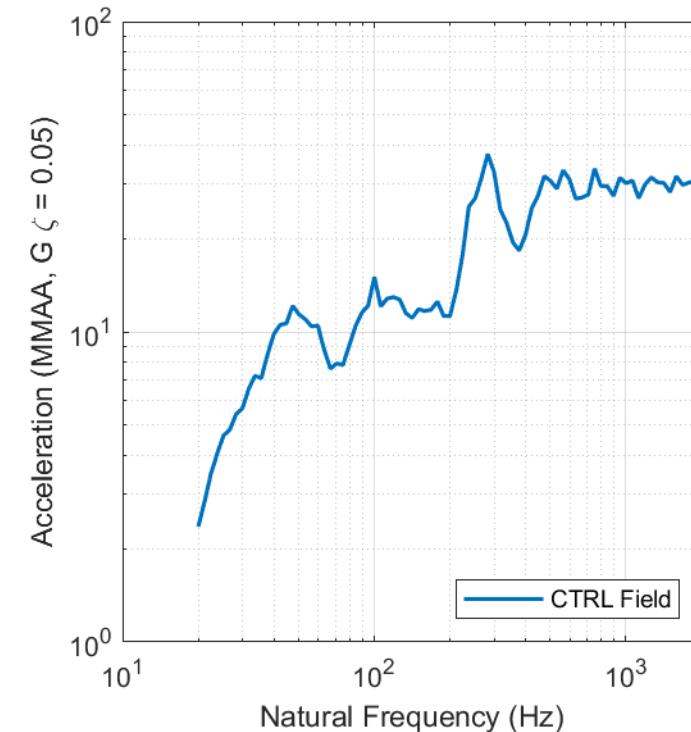
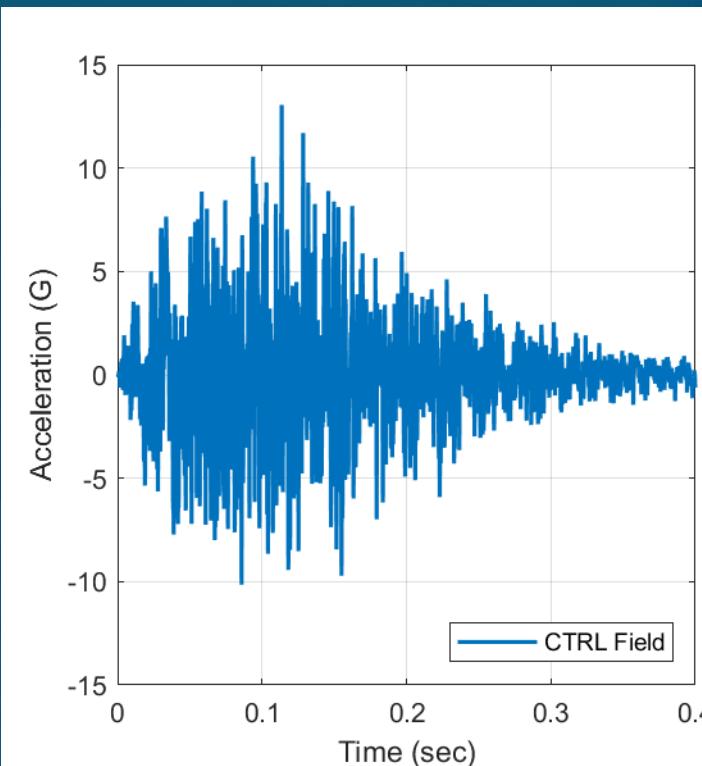
- Experimental validation

Field Environments

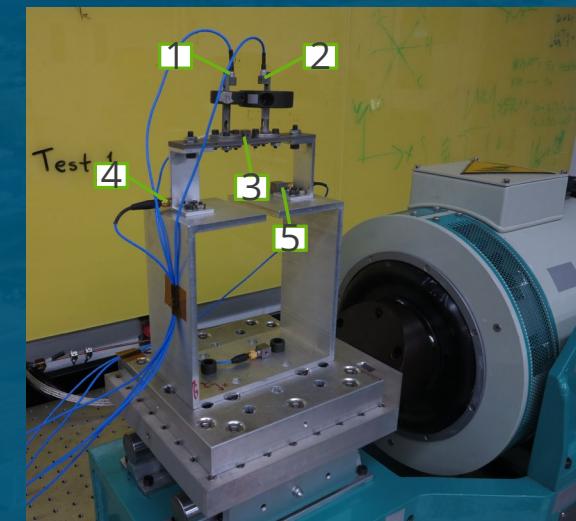
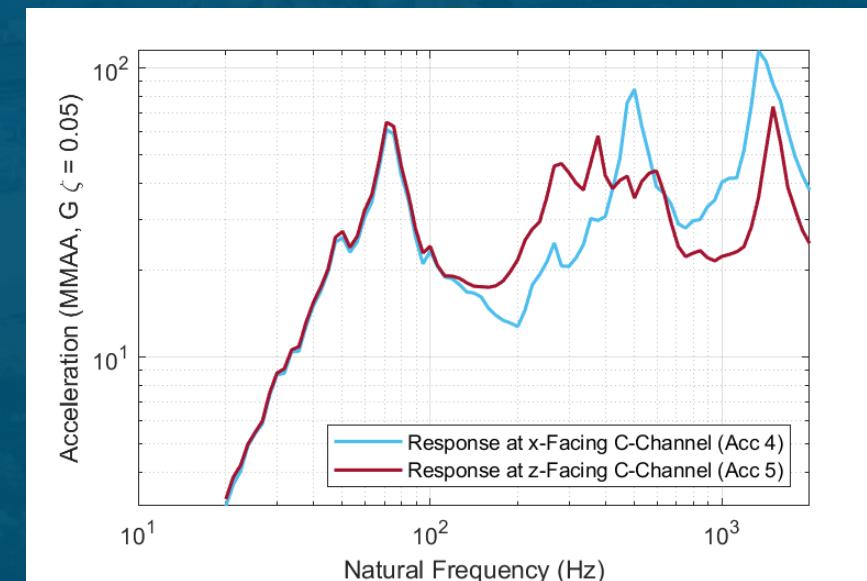
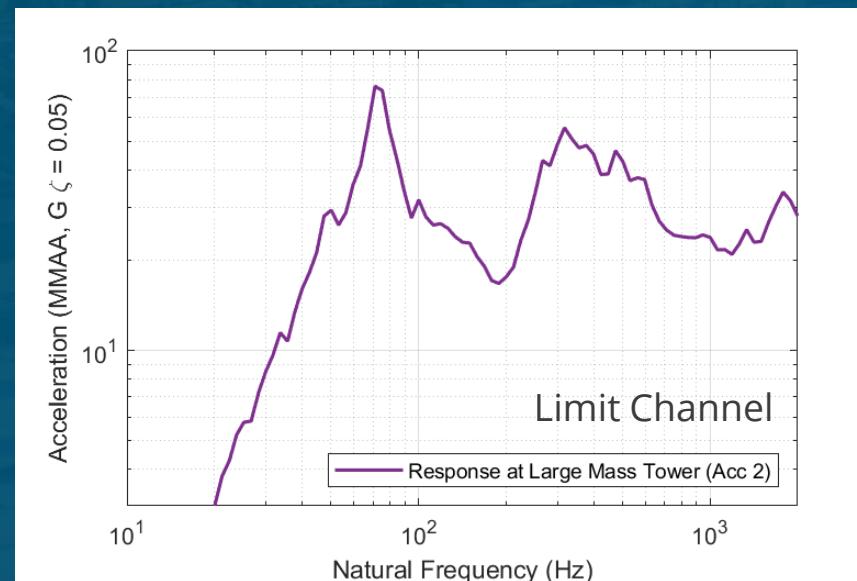
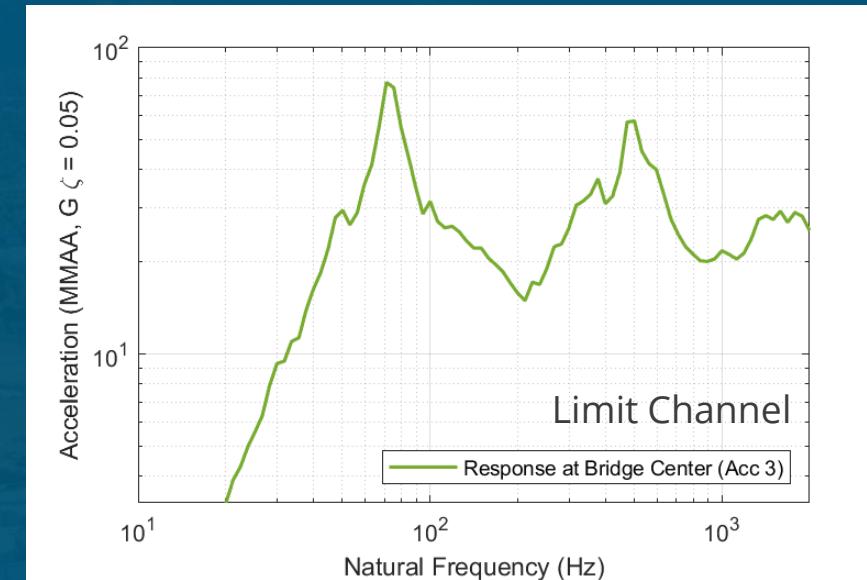
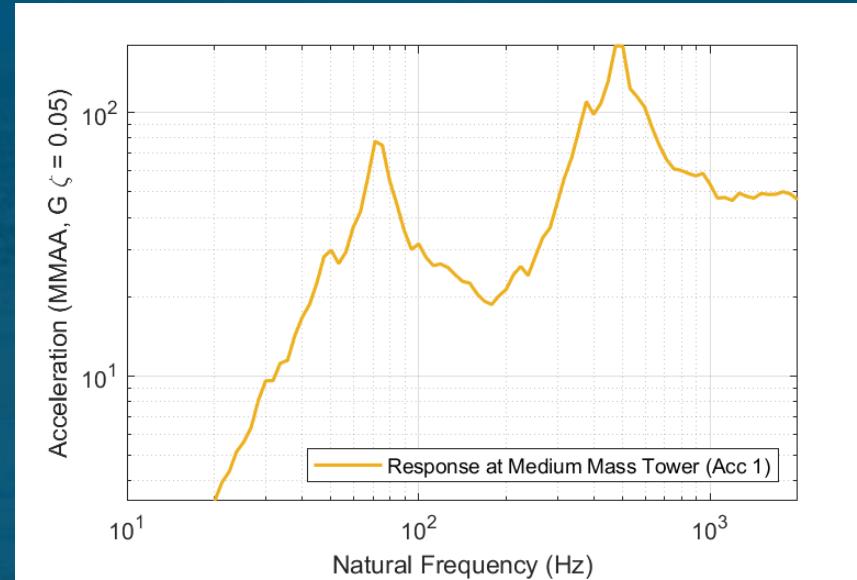
“Field measured” acceleration at the BARBECUE base

“Field measurements” at locations 1 – 3

- Accelerations and SRS at locations 1 – 3 were simulated using frequency domain convolution with TRFs and the base acceleration



Field Environments – Response Locations



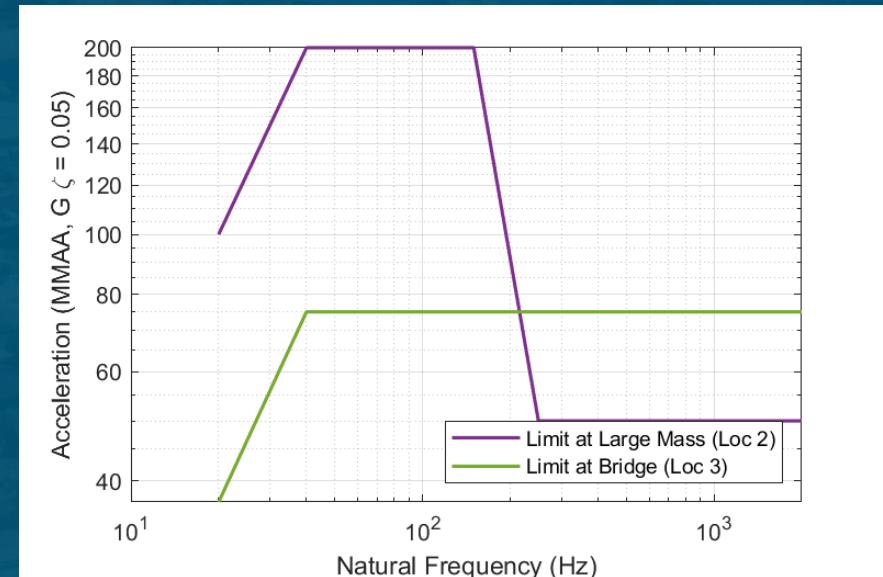
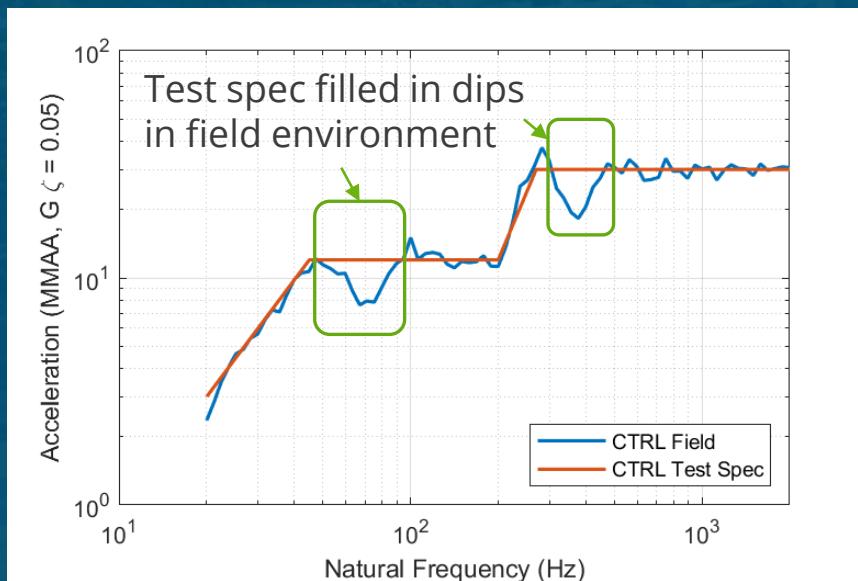
Test Environment

Laboratory test specification

- Straight-line SRS at base
- Straight-line SRS response limits at locations 1 & 2

Assumptions

- Field-measured SRS at response locations are known
- Field-measured input at the base is unknown or not available

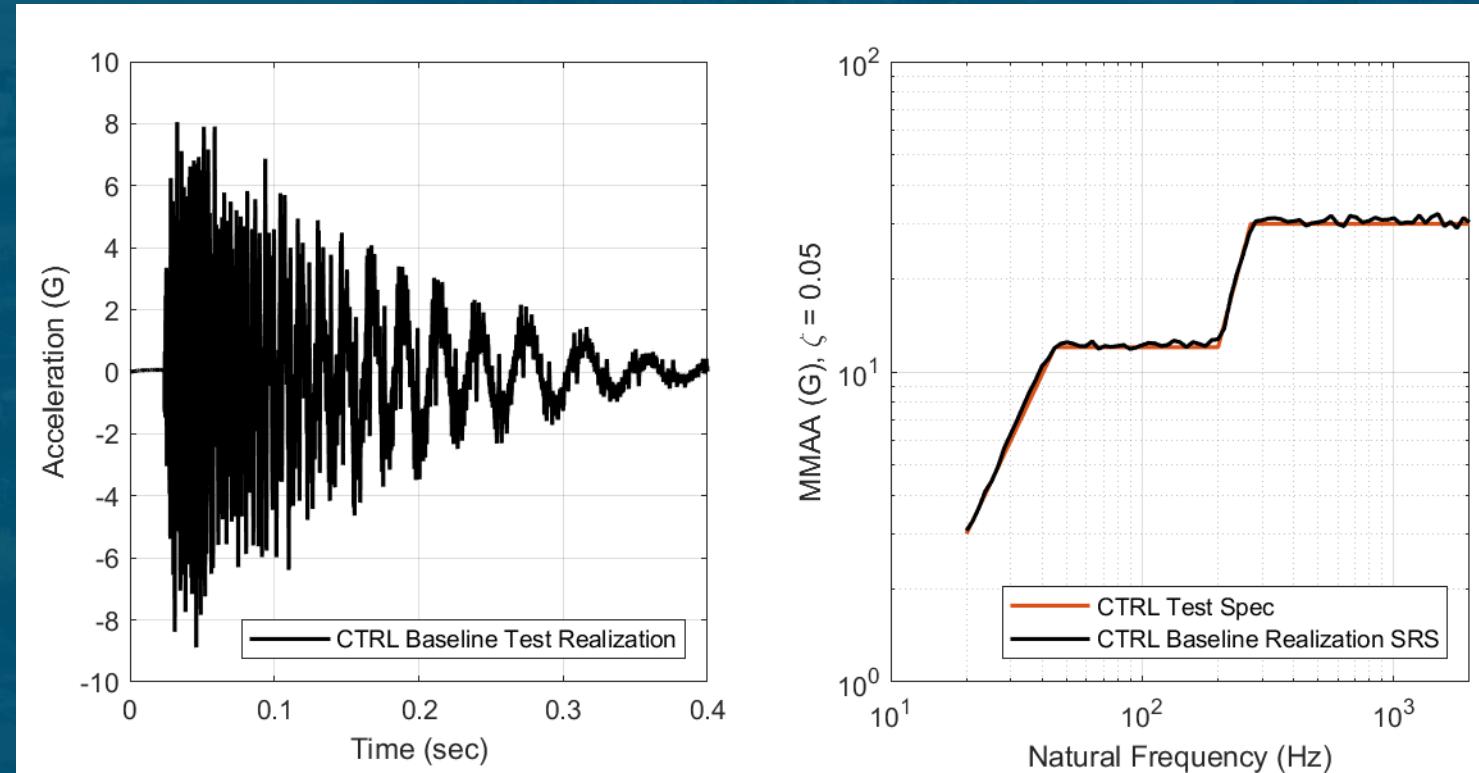


Baseline – No Response Optimization or Limiting

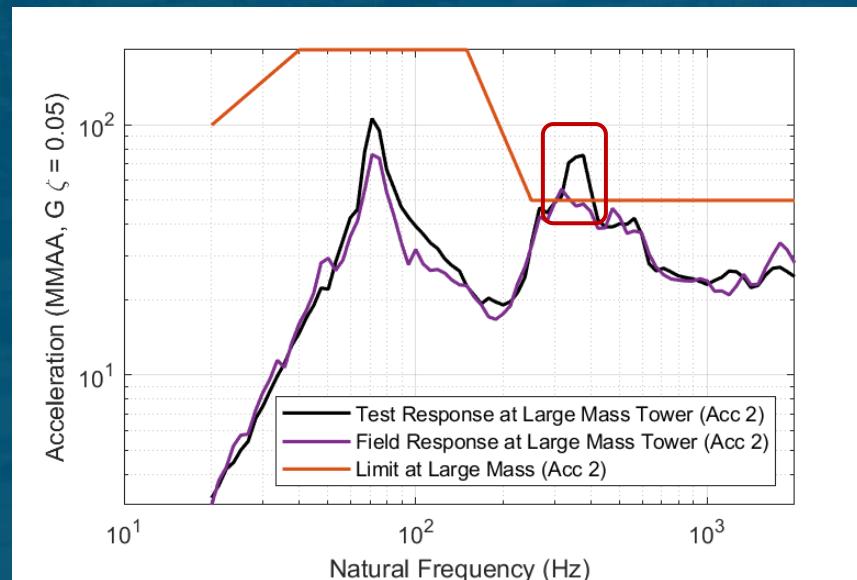
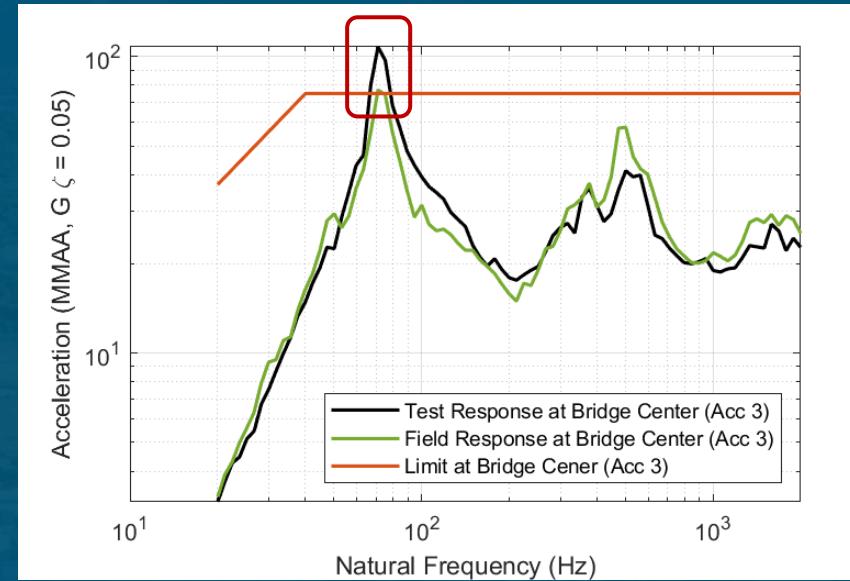
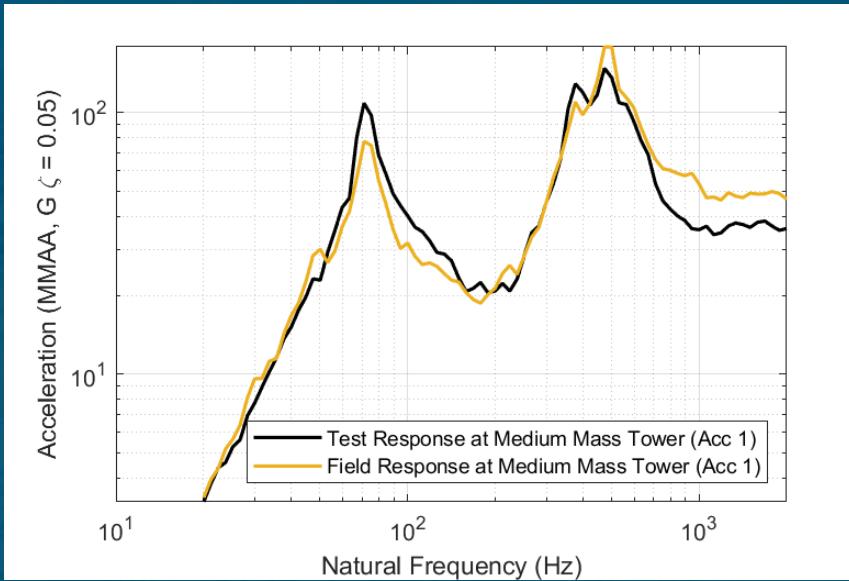


Computed a SDS acceleration history at the base

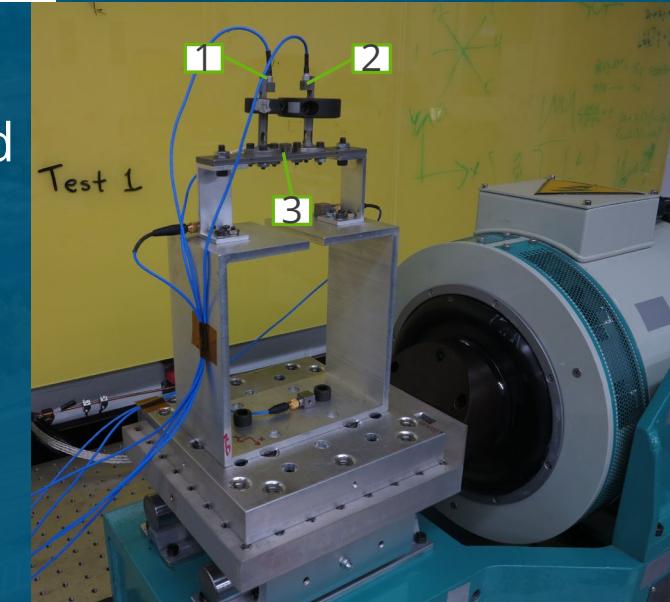
Propagated that through the TRFs to the response channels



Baseline – No Response Optimization or Limiting



Without limiting, the responses to the test input exceed the field environments



BARBECUE Response Optimization & Limiting Objectives



Case 1 – SISO Response limiting (One control location)

- Modify base excitation so that responses at locations 2 & 3 do not exceed response limits
 - If response limits are not active, Case 1 reduces to the basic SDS problem
- Response at location 1 uncontrolled

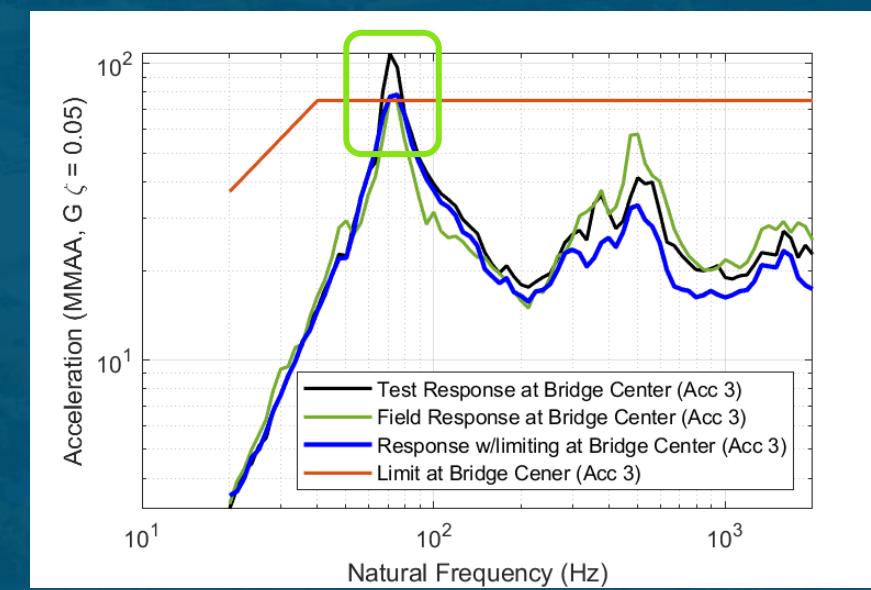
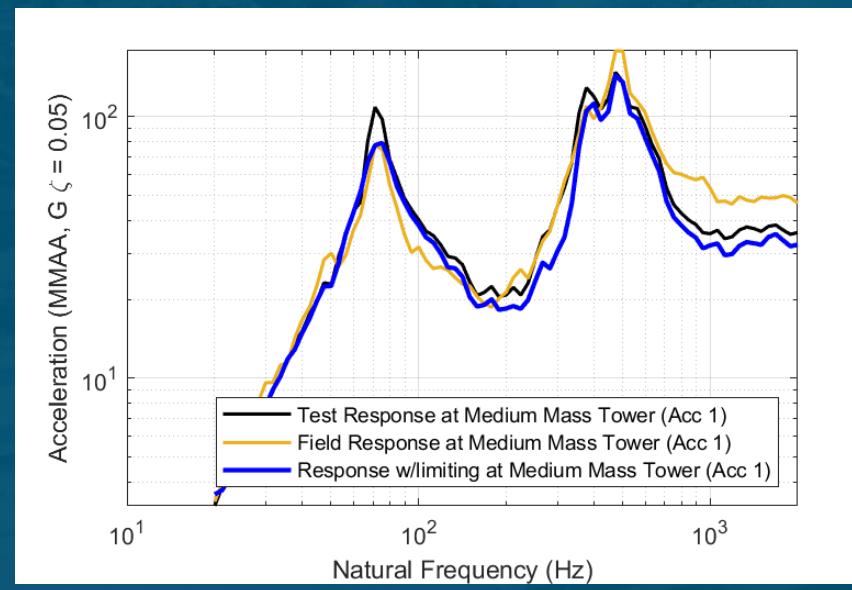
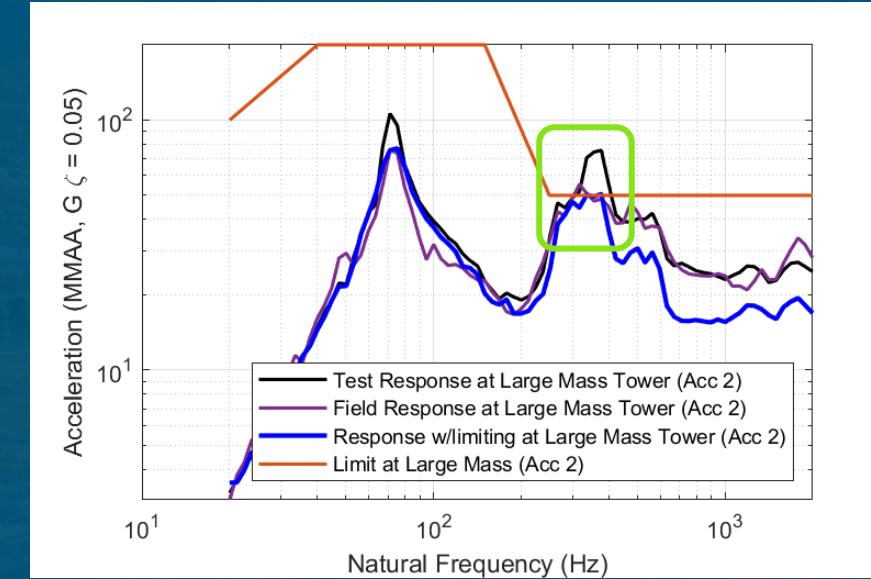
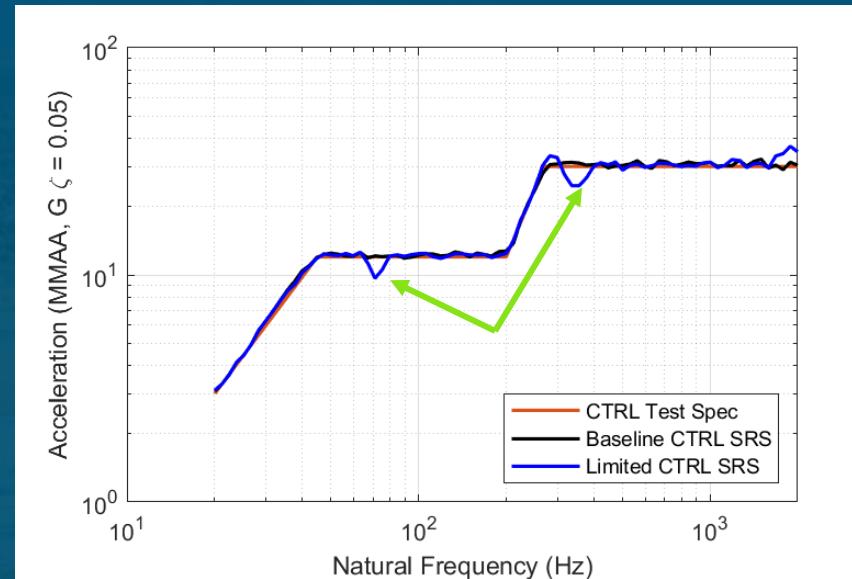
Case 2 – SIMO Response optimization & limiting (Multiple response locations)

- Modify base excitation so that responses at locations 2 & 3 do not exceed response limits AND
- Minimize the SRS error at locations 1,2, & 3

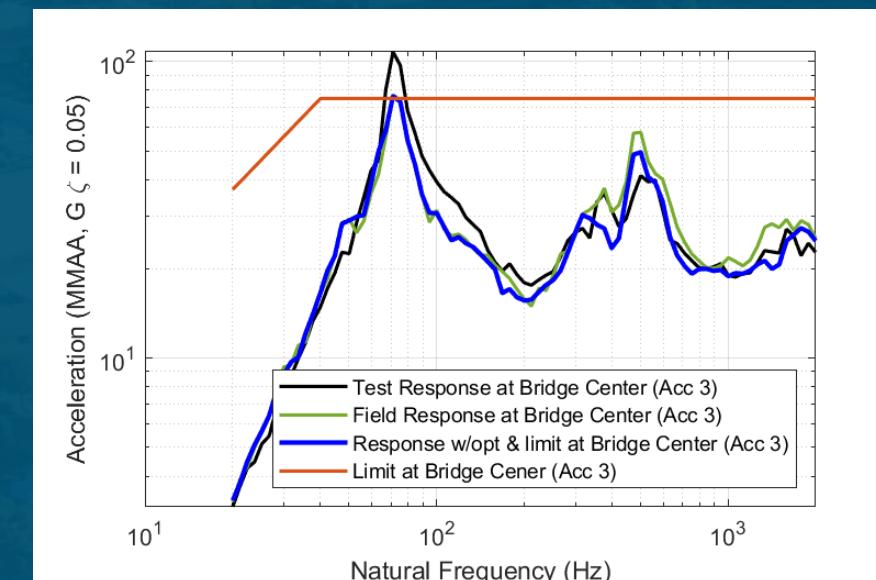
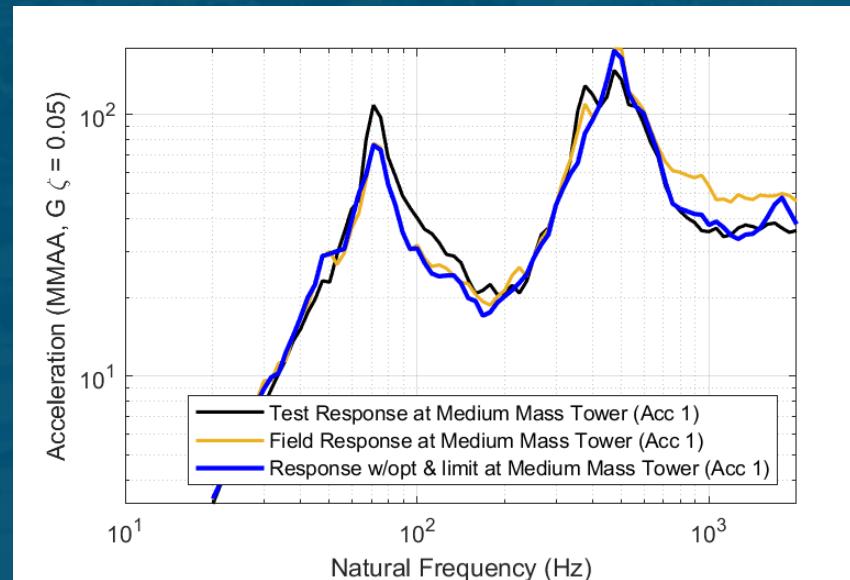
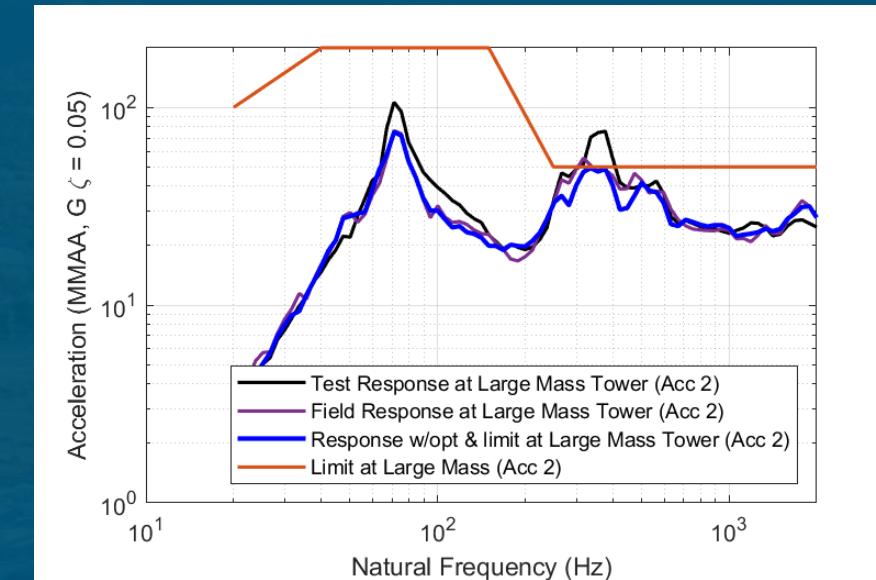
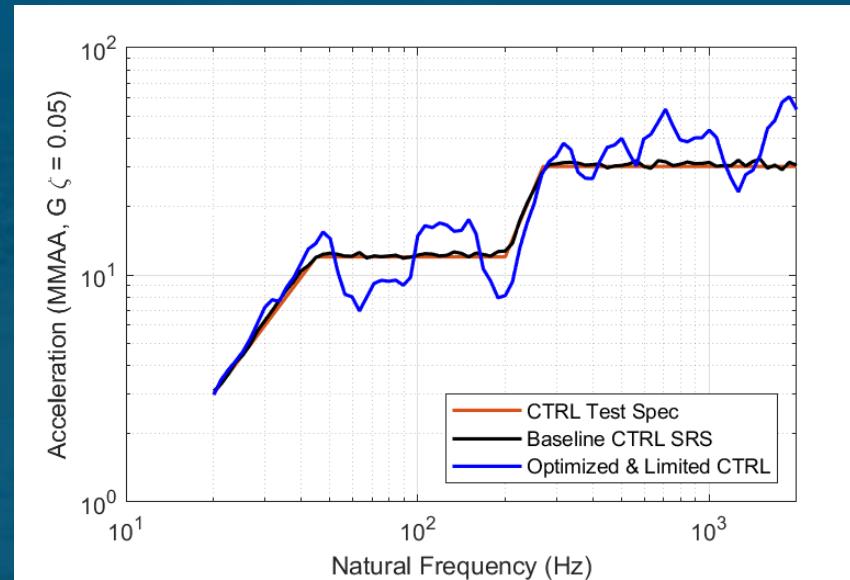
Case 3 – Monte Carlo SIMO response optimization

- Minimize the SRS error at locations 1,2, & 3 AND
- “Optimize” scalar metrics

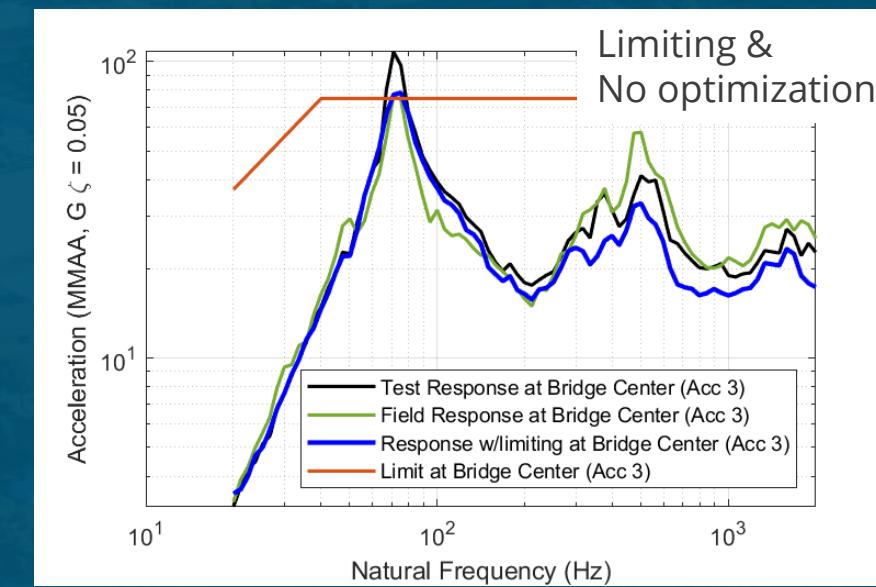
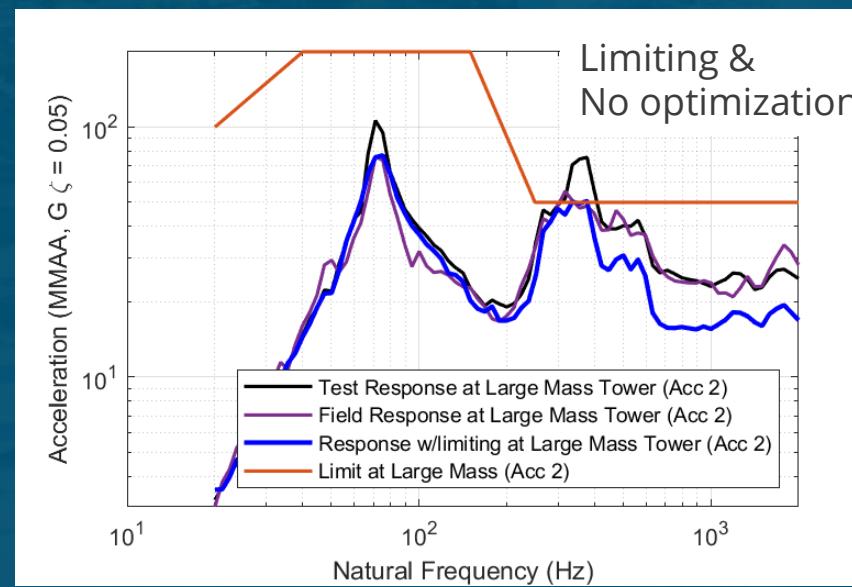
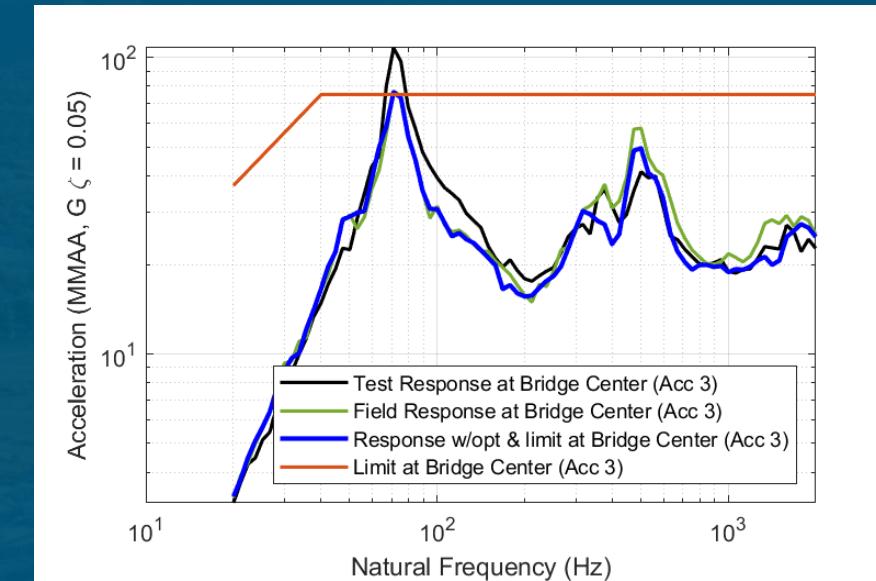
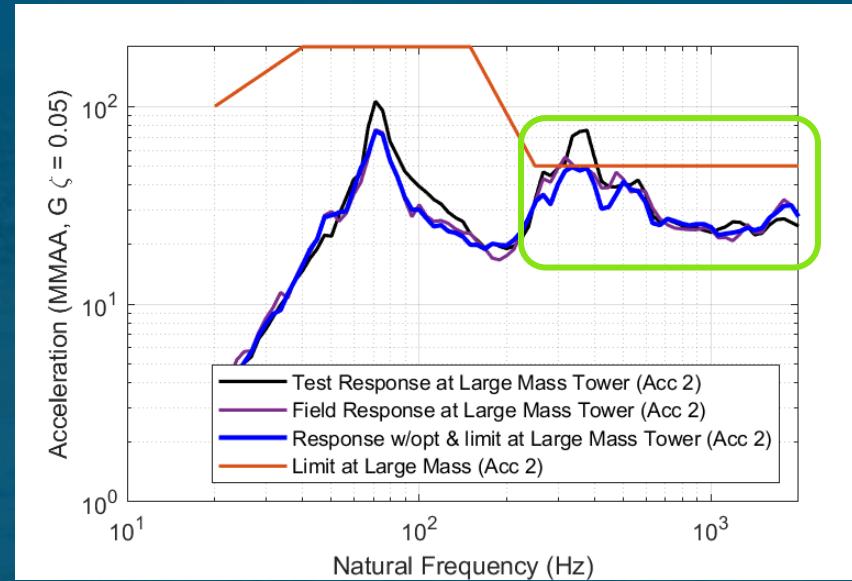
Case 1 – SISO Response-Limiting



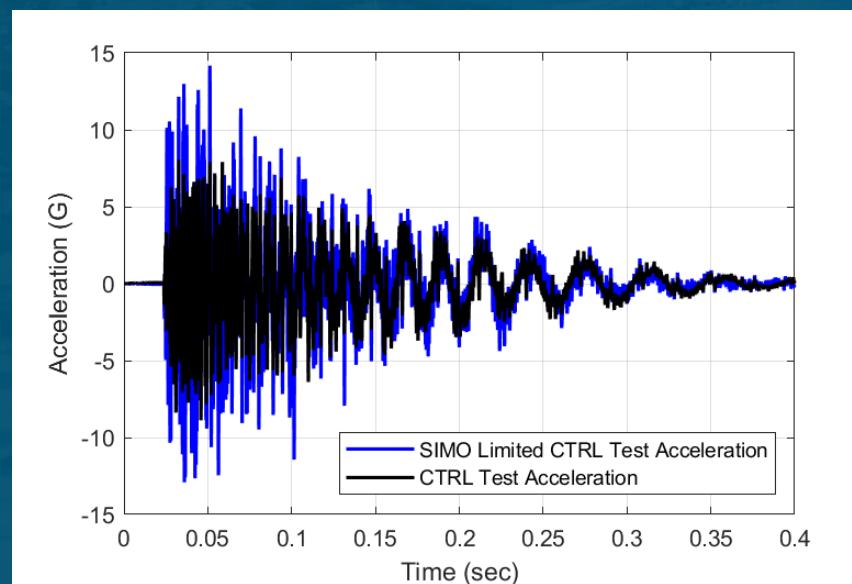
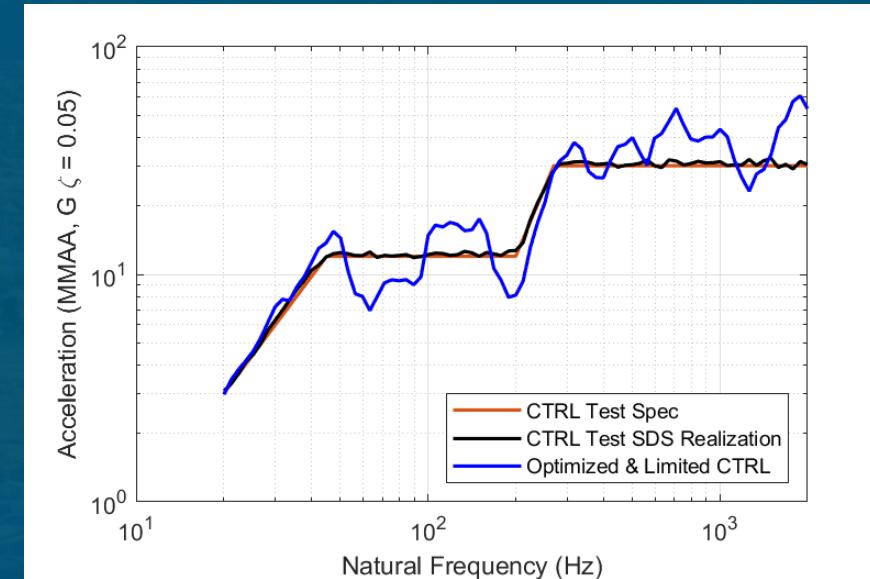
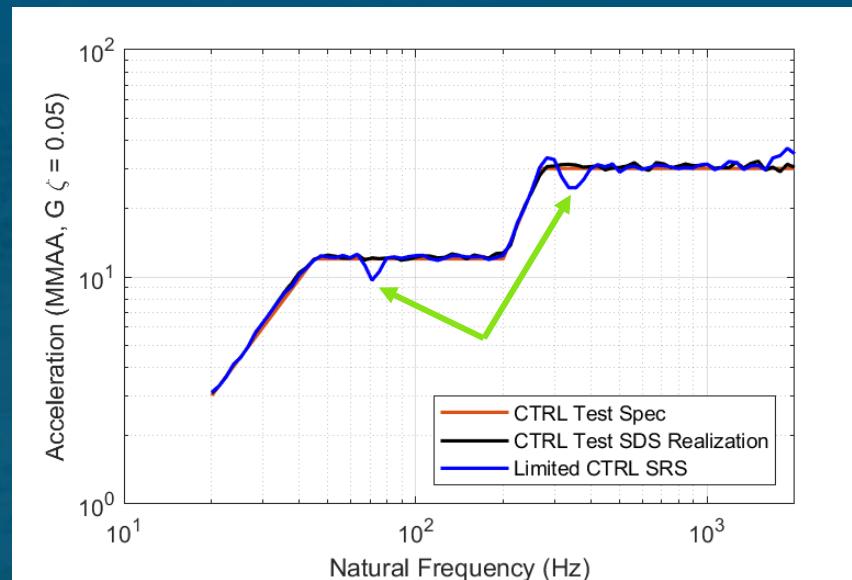
Case 2 – SIMO Response Optimization & Limiting



Case 2 - Response Optimization & Limiting



Case 2 - Response Optimization & Limiting



The control SRS is more distorted when response optimization is used

- Peak acceleration is higher

Response limiting alone affected the acceleration in the time domain very little

- Response limiting was not very deep or broad

Case 3 – Monte Carlo SIMO Response Optimization

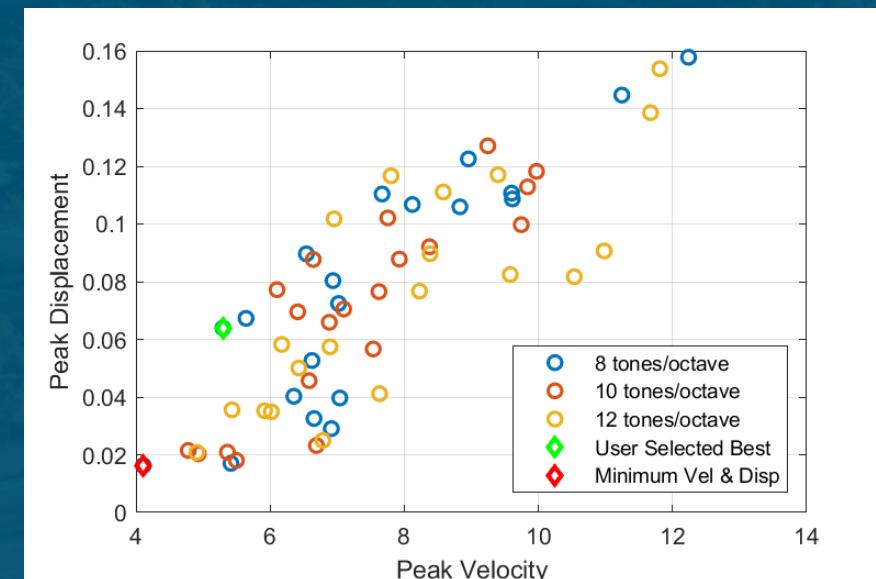
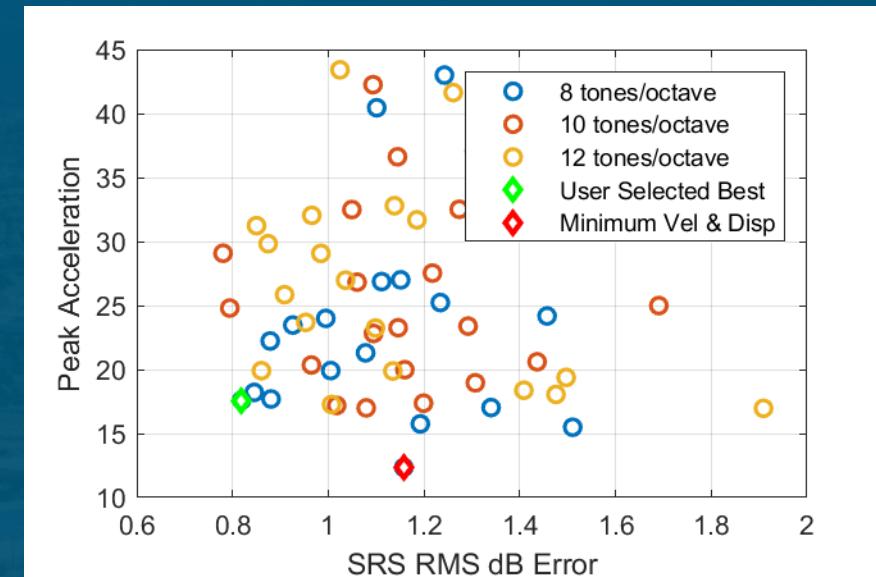
60 Monte Carlo simulations were run

- 20 each for 8, 10, and 12 tones per octave

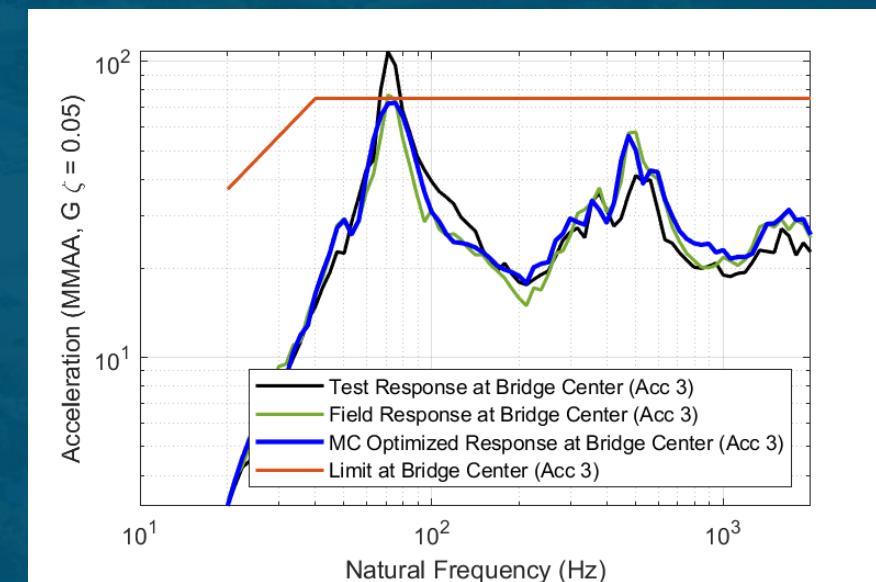
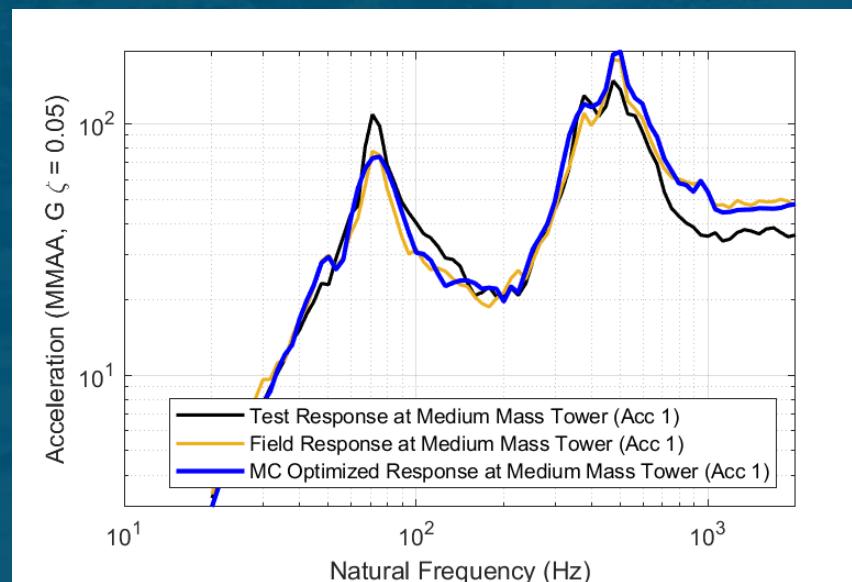
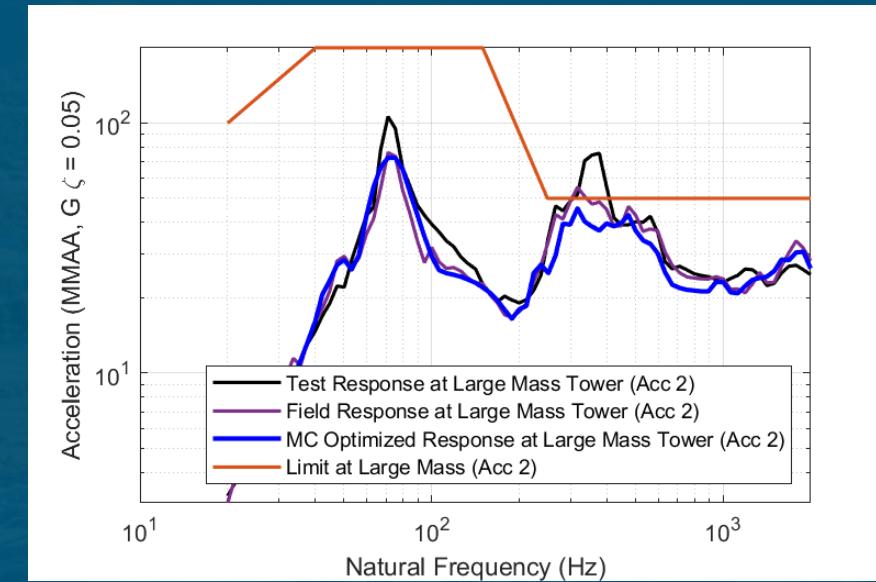
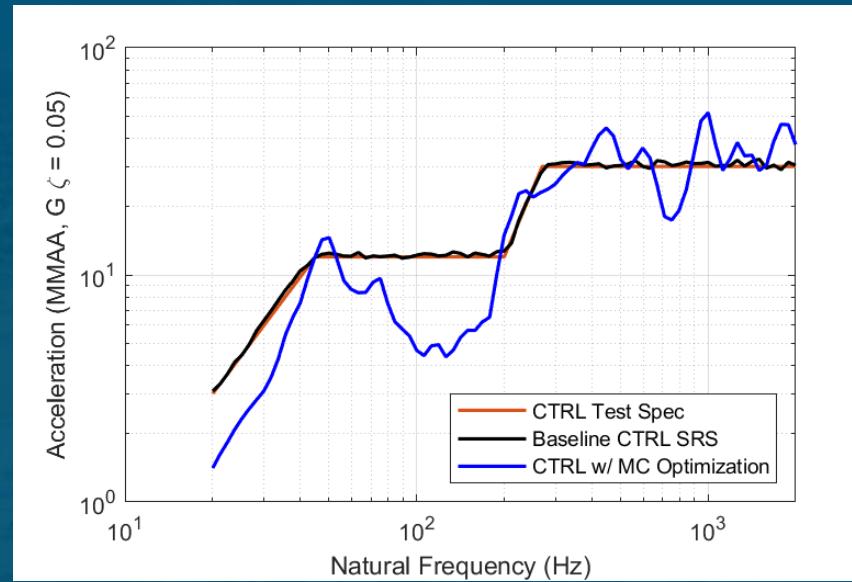
User-selected realization based on RMS dB error and peak acceleration

Peak displacement vs peak velocity scatter plot shows the “best” (user selected) realization in terms of those parameters

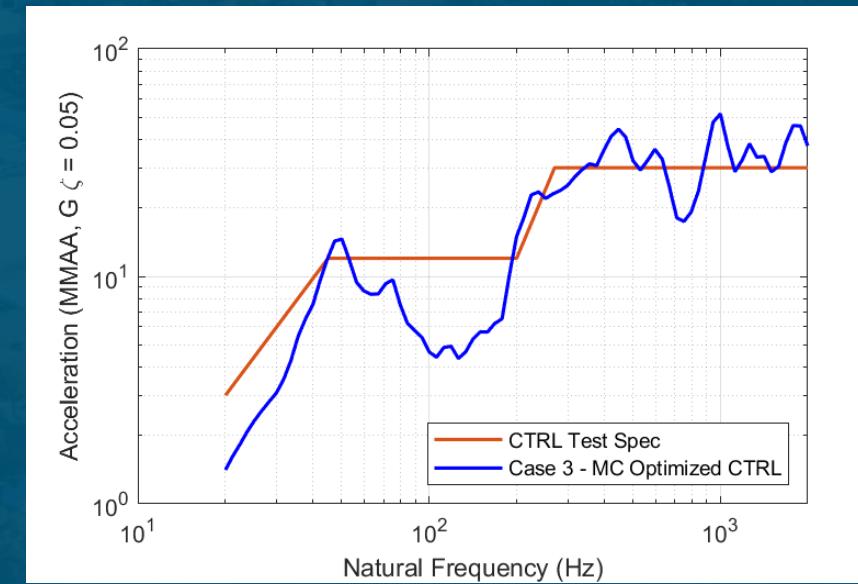
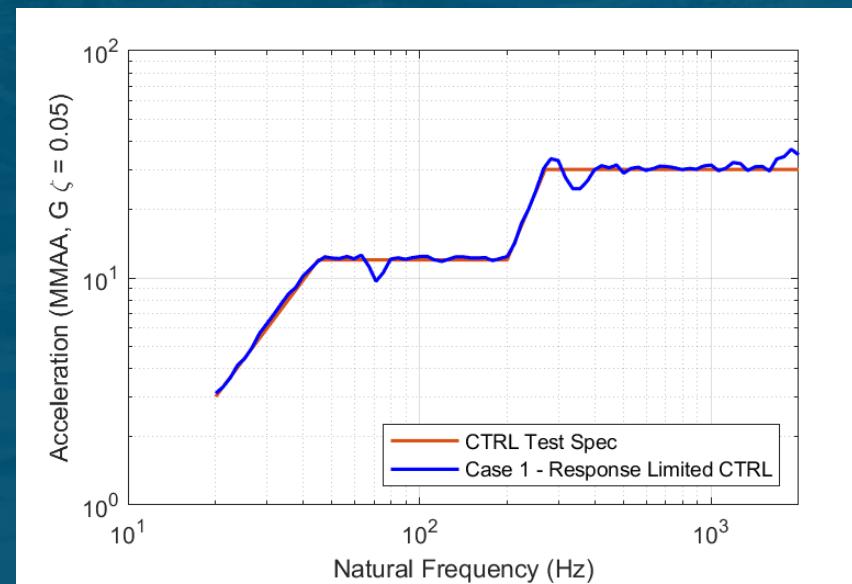
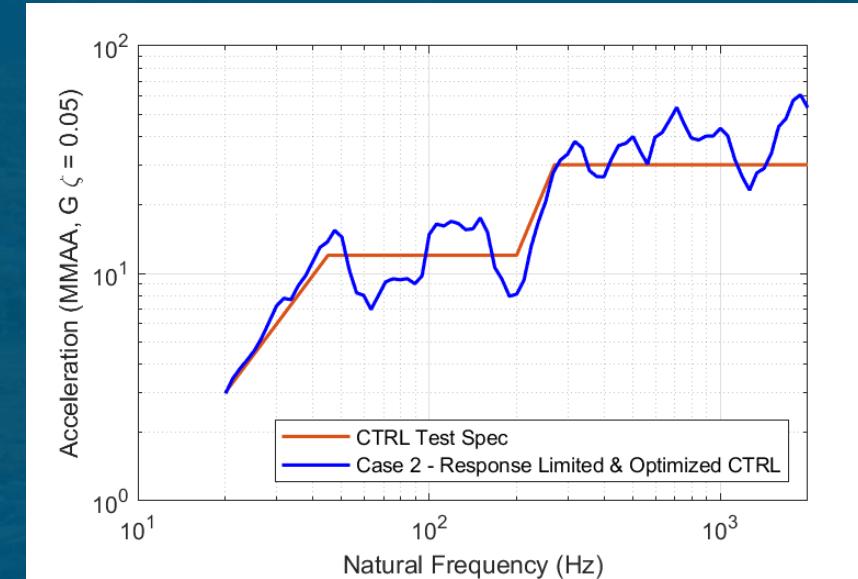
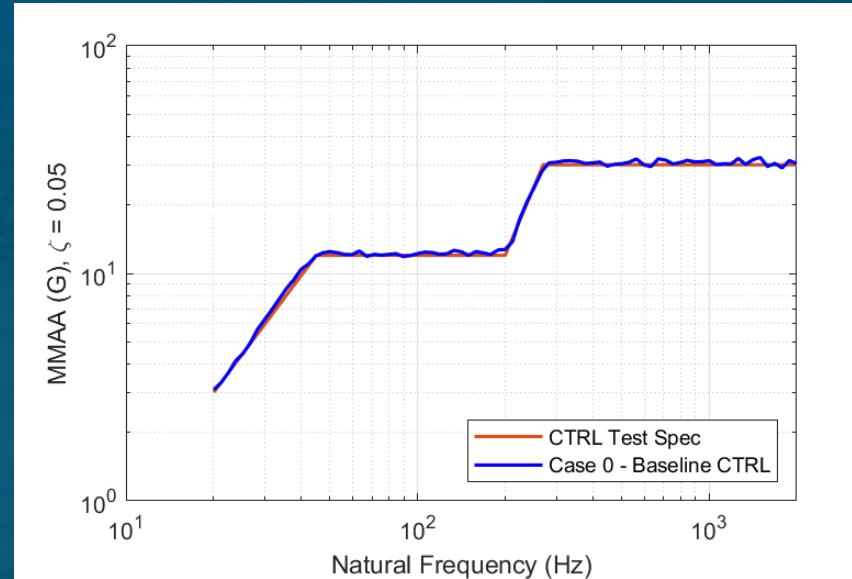
- Useful if shaker capability limits must be considered



Case 3 – Monte Carlo Response Optimization



Comparison of Control SRS for Cases 1 - 3



Baseline SRS
matches the test
spec

More optimization
causes more
warping of the CTRL
input

Comparison to the Field Environment

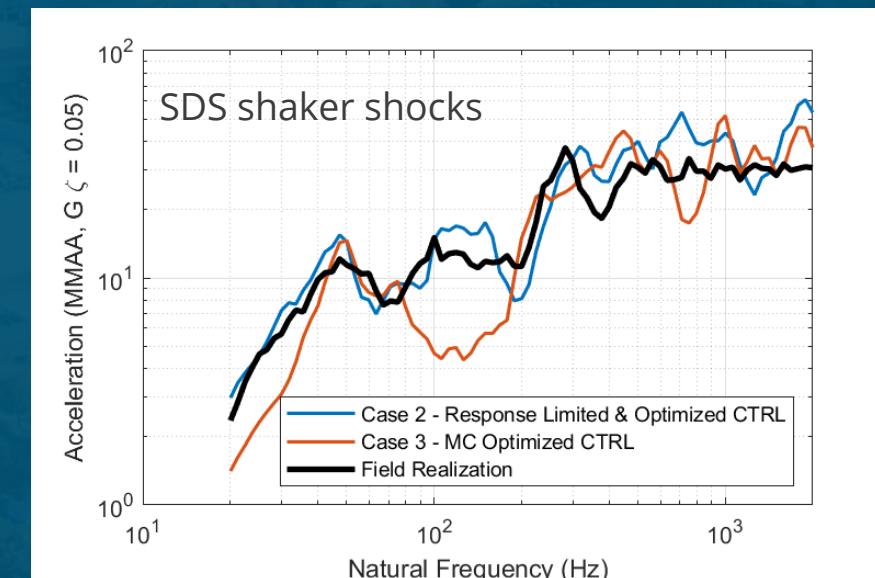
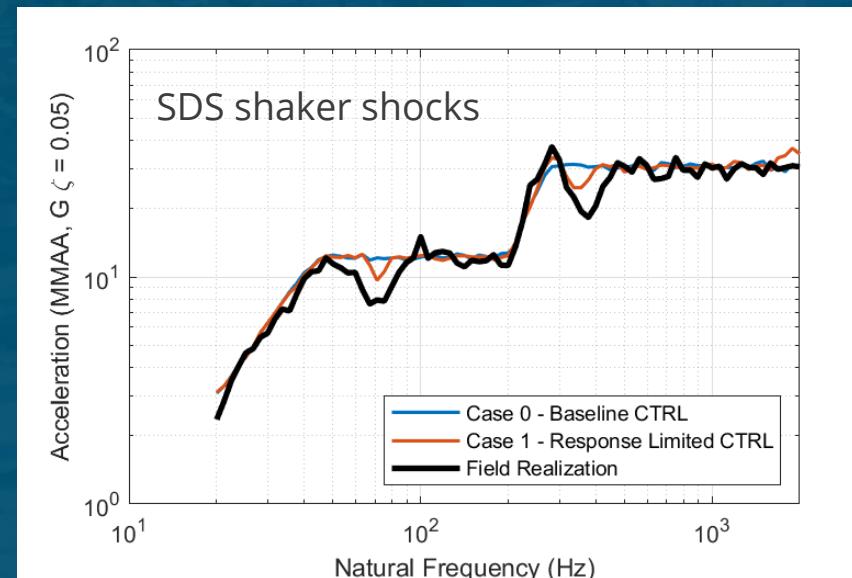
The field input realization was made from a notched straight-line SRS

- Test spec was the straight-line SRS without notches

Response-limited CTRL approximated the field environment reasonably well

Optimization warped the input SRS

- May be an issue at unmeasured response locations



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- Test Article
- Pre-Test Analyses
- **Shaker Shock Test**

Conclusions

Experimental Checks

The predicted inputs were implemented on the shaker

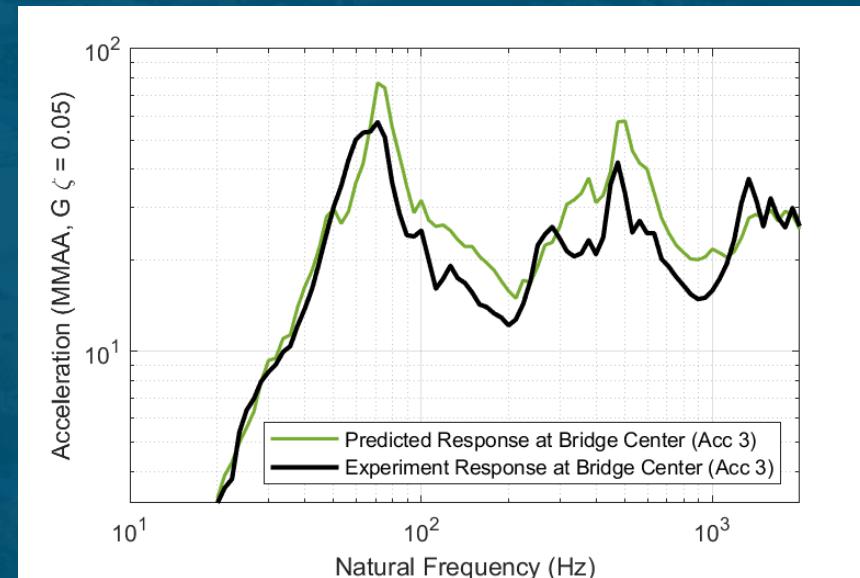
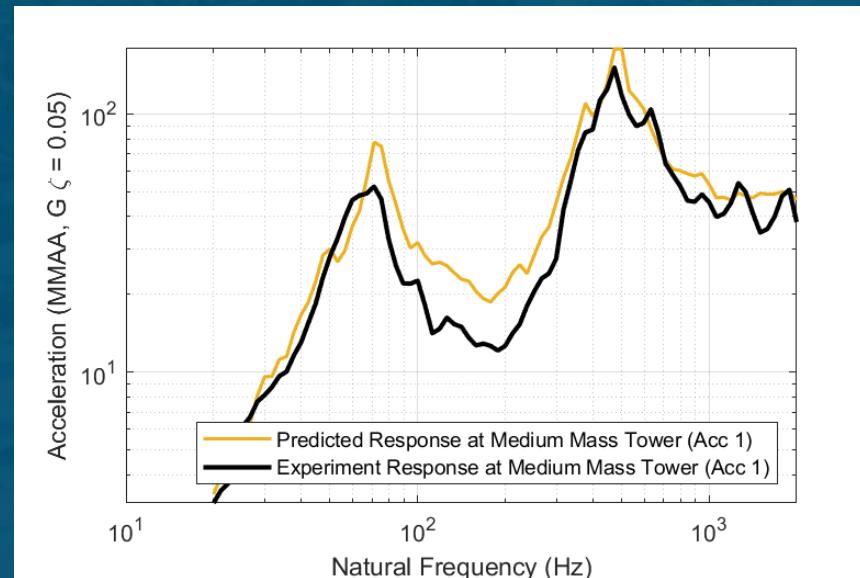
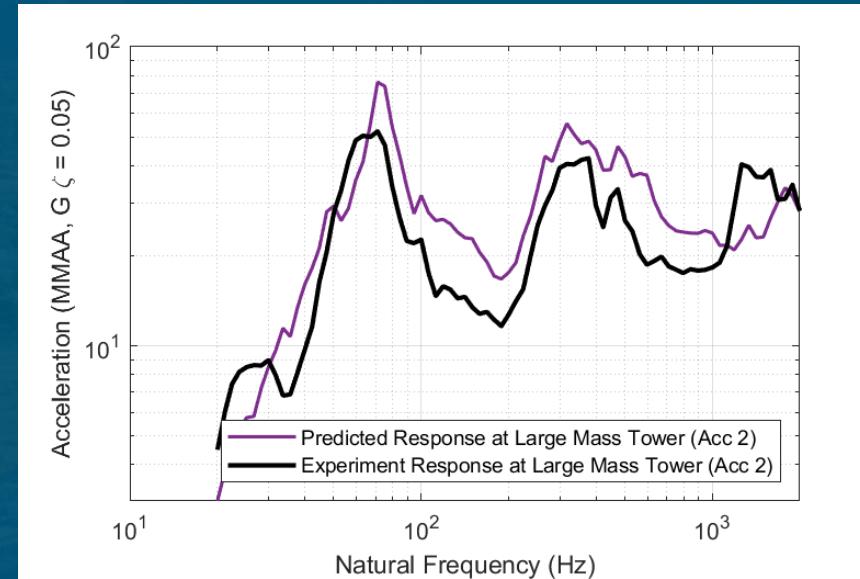
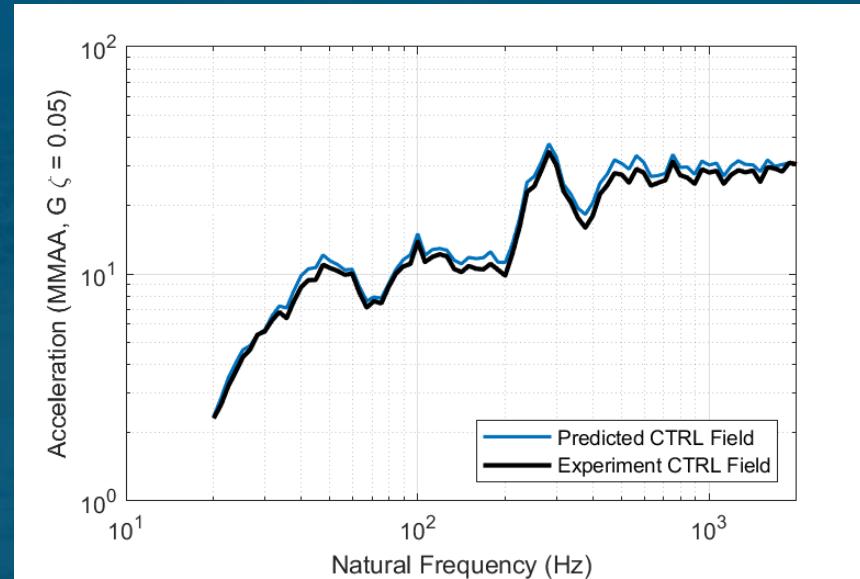
The inputs were replicated accurately

- The field SRS was replicated well
- The predicted responses to the field input were not as good
 - The difference may be an artifact of the field input being a transient random environment

The responses were not as accurate as expected

- The predicted responses to the field input were not as good
 - The error may be an artifact of the field input being a transient random environment
- More so for the field test, which may be an artifact of this being a transient random environment
- Perhaps the BARBECUE is not totally linear

Experimental Validation – Field Environments

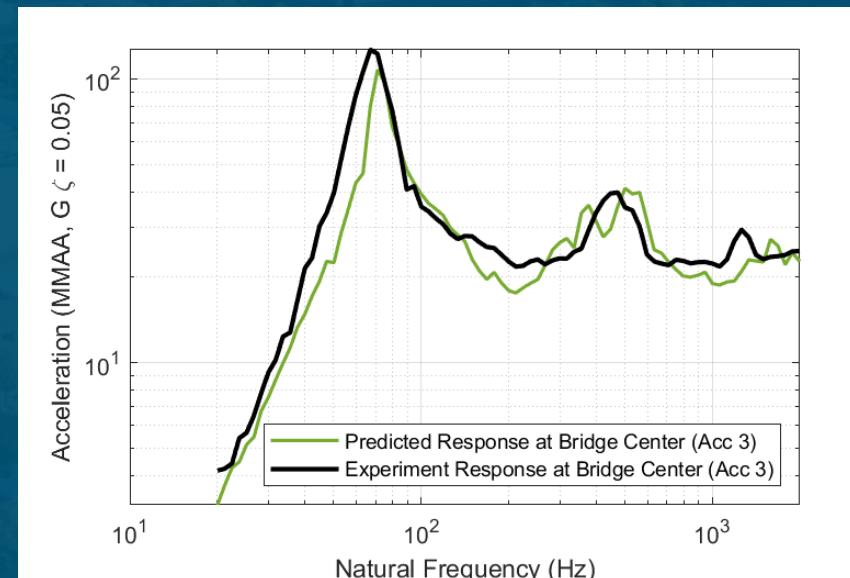
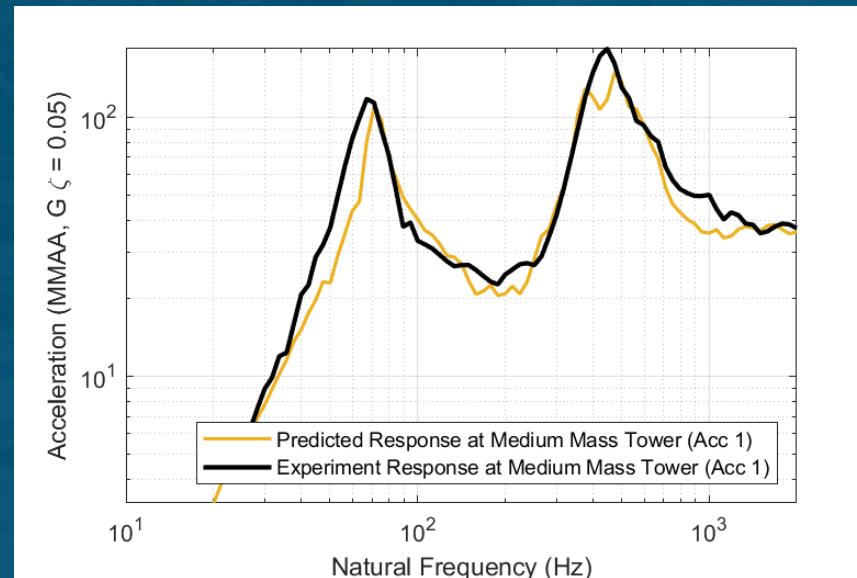
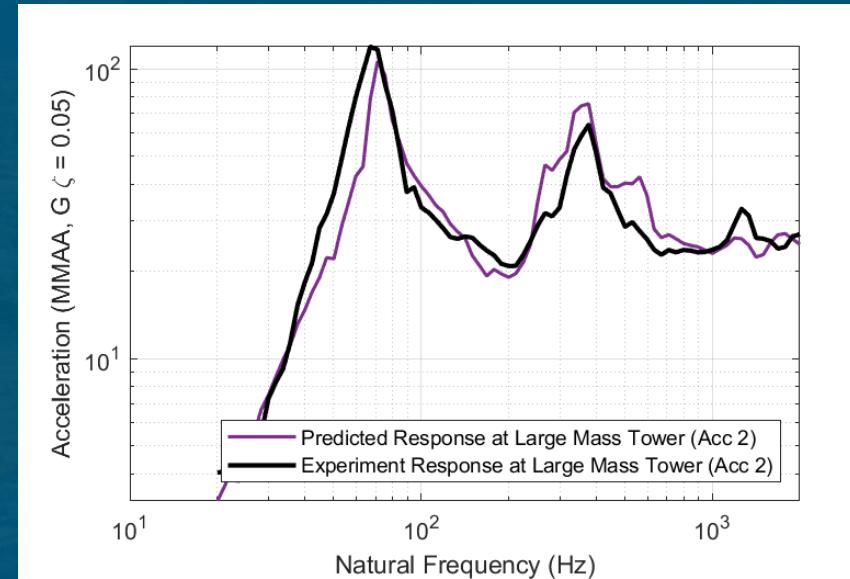
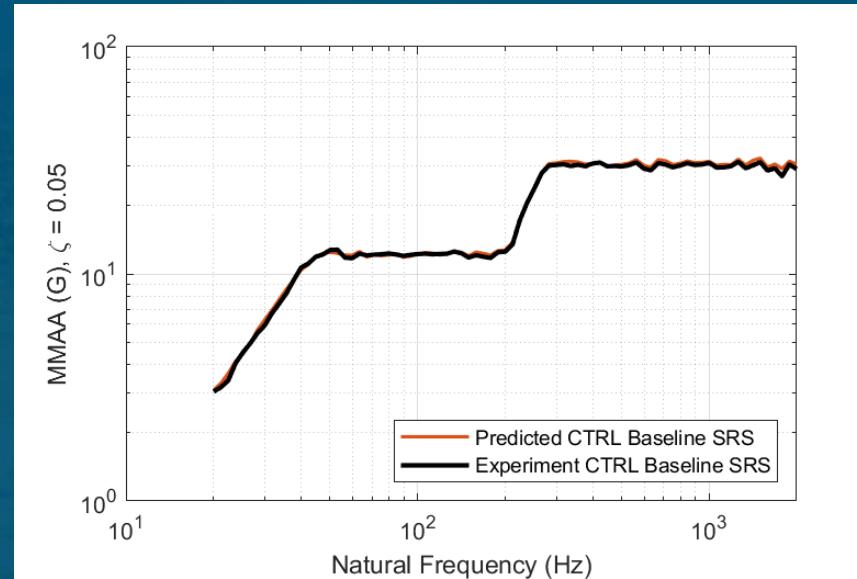


Experiment field input SRS matches the predicted SRS

Experiment response location SRS are not as good

- SRS at response locations is sensitive to the random realization of the field input

Experimental Validation – Case 0 Baseline SDS

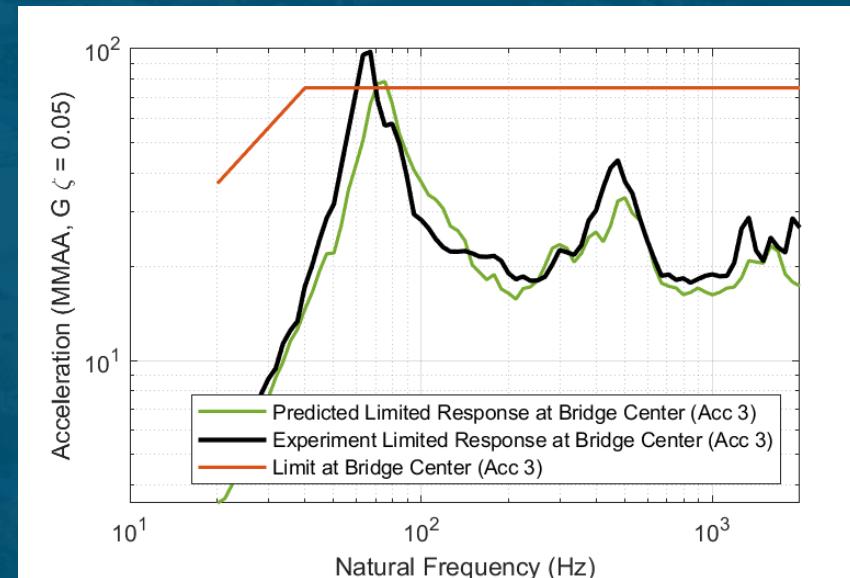
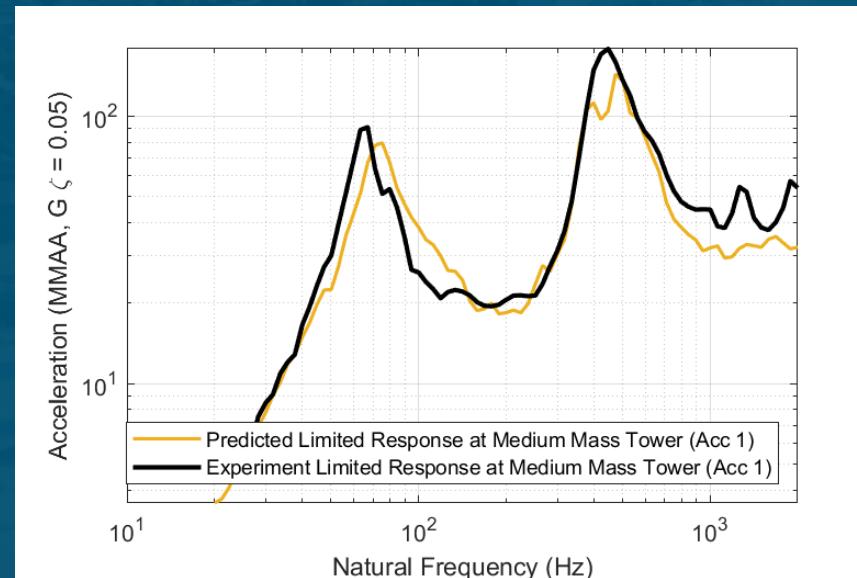
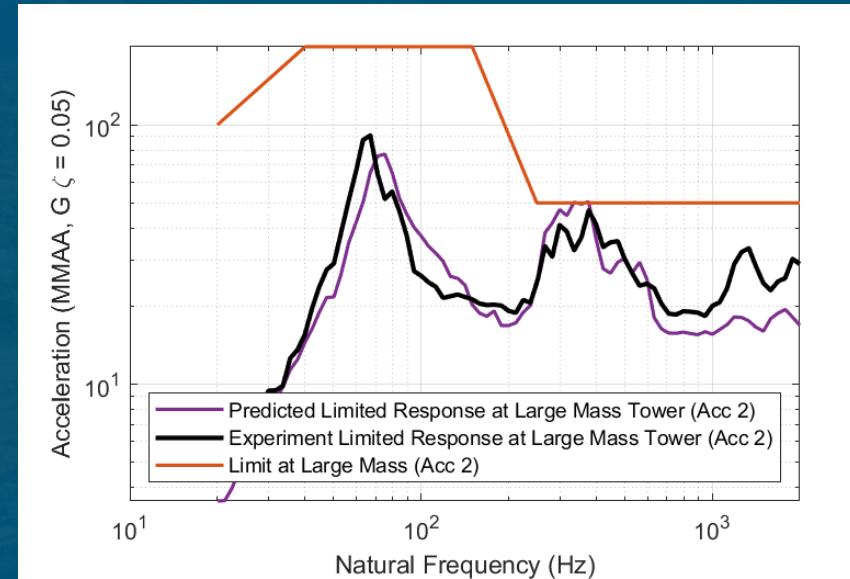
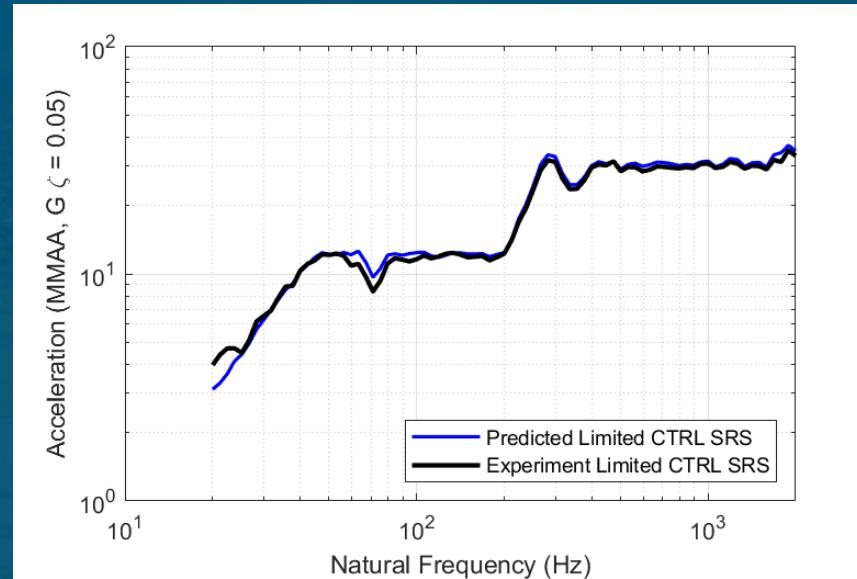


Experiment test spec SRS matches predicted test spec very well

- SDS acceleration is a deterministic waveform

Differences at response locations may be due to BARBECUE non-linearities

Experimental Validation – Case 1 Response Limiting



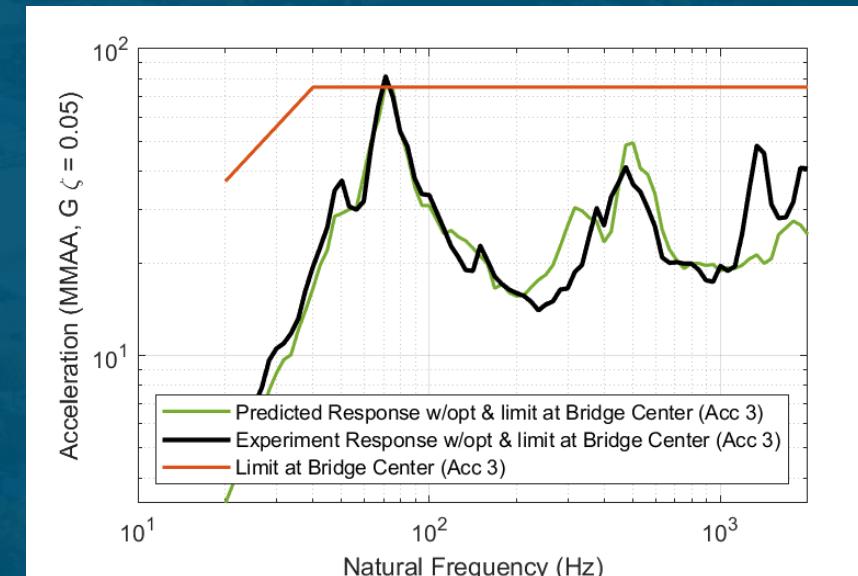
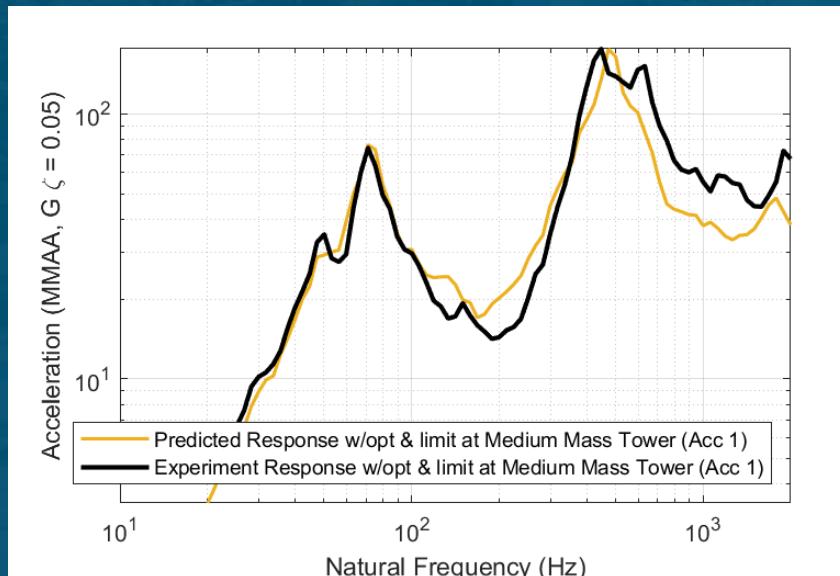
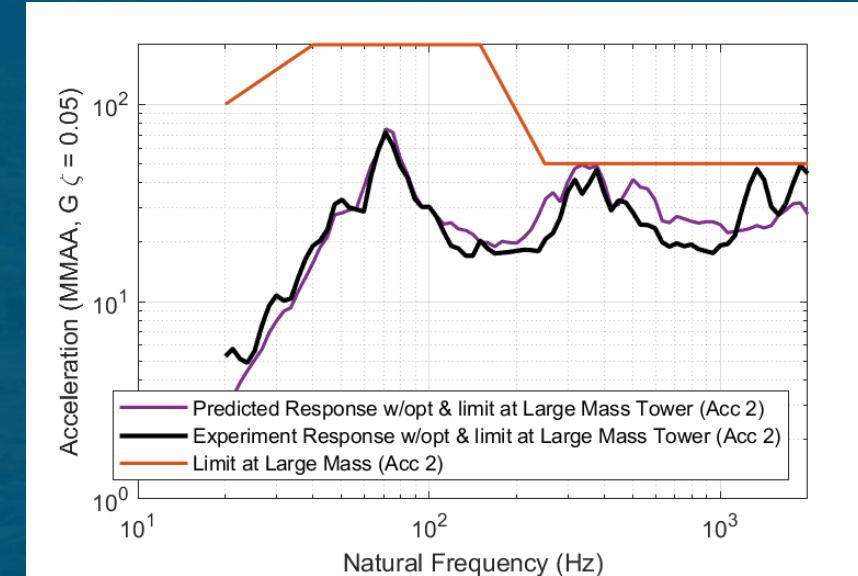
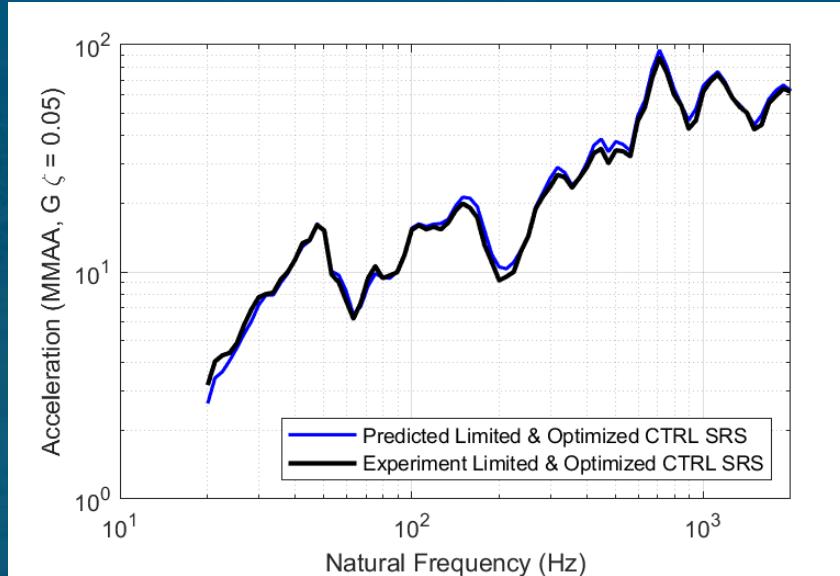
Experiment response limited CTRL SRS matches predicted SRS very well

- SDS acceleration is a deterministic waveform

Response limit at location 3 (Bridge) violated in the experiment

- Open-loop prediction based process

Experimental Validation – Case 2 Response Optimization & Limiting



Experiment CTRL SRS matches predicted SRS very well at
Responses limit are satisfied at both locations 2 & 3

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Summary & Conclusions

Response optimization and limiting are based on our baseline Sums of Decayed sines algorithm for generating an acceleration realization from a SRS

Unlike random vibration, shaker shock response limiting & optimization is pre-planned

- Requires transmissibilities from the control to the response locations

Response limiting did properly restrict the resonant responses

Response limiting by itself did not match the broadband field SRS at all 3 response locations as well as the other methods

- Least distortion of the control SRS

The Monte Carlo response optimization matched the field SRS at response locations 1 – 3 the best

We hope to extend these algorithms to MIMO environments

- 6-DOF shaker
- NEWT Method

References

- [1] D. O. Smallwood; "Methods used to Match Shock Response Spectra using Oscillatory Transients, Proceedings of the Institute of Environmental Sciences, 1974, pp409-420.
- [2] D. O. Smallwood and A.R. Nord, "Matching Shock Spectra with Sums of Decaying Sinusoids Compensated for Shaker Velocity and Displacement Limitations," 44th Shock and Vibration Symposium, Paper U-20, Houston, TX, Dec. 1973
- [3] C. Heitman, V. Babuška, J. Cap, and J. Reid, "Development of a Single Input Multiple Output (SIMO) Input Derivation Algorithm for Oscillatory Decaying Shocks; Proceedings of the 86th Shock and Vibration Symposium; October 5-8, 2015, Orlando, FL.
- [4] C. Heitman, J. Cap, and D. Murphy, "Monte Carlo Optimization of a Single Input Multiple Output (SIMO) Input Derivation for an Oscillatory Decaying Shock", Proceedings of the 87th Shock and Vibration Symposium, 2016, New Orleans, LA.
- [5] J. Cap, "The Application of Force Limiting Techniques to the Derivation of an Equivalent Static Load and to Transient Vibration Testing"; oral presentation at the 1995 Spacecraft and Launch Vehicle Structural Dynamic Environments Technical Interchange Meeting; June 6-8, 1995, El Segundo CA.
- [6] J. Cap, T. Hunt, C. Heitman, and M. Raymer, "Monte Carlo Optimization of a Hybrid Spectral / Temporal Single Input Multiple Output (SIMO) Input Derivation for an Oscillatory Decaying Shock"; Proceedings of the 88th Shock and Vibration Symposium; October 16-19, 2017, Jacksonville, FL.

Acknowledgements



The following people provided input to make this briefing possible:

- Angela Patterson – ANSYS modeling
- Dan Rohe – Test engineer
- David Smallwood – Algorithms

Flowchart from Smallwood's 1973 SAVIAC paper

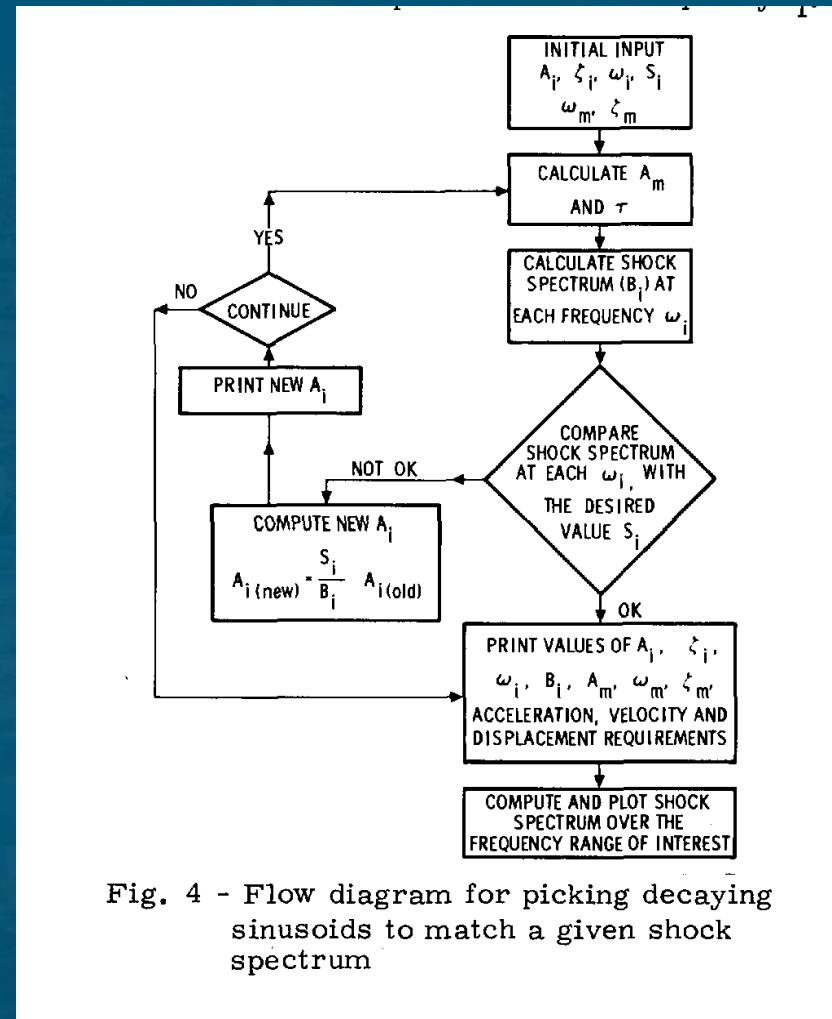


Fig. 4 - Flow diagram for picking decaying sinusoids to match a given shock spectrum

D. O. Smallwood and A.R. Nord, "Matching Shock Spectra with Sums of Decaying Sinusoids Compensated for Shaker Velocity and Displacement Limitations," 44th Shock and Vibration Symposium, Paper U-20, Houston, TX, Dec. 1973

Thank You
Questions?

Response Optimized Shaker Shocks (SIMO Optimization)



In addition to optimizing a set of desired SRS, one can optimize on other scalar parameters of interest

- Synthesize multiple “inputs”
- Obtain TRFs between input acceleraton and parameter of interest
- Use convolution to synthesize waveforms
- Compile scalar parameter
- Choose opimal value

The method has been successfully used to minimize amplifier voltage and current
[6]

Comparison of gw inrand

