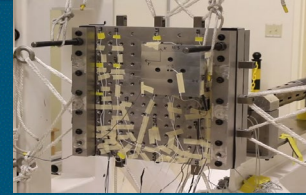




# Response Limiting & Optimizing Shaker Shocks



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## Introduction

## Shaker Shock Basics

- Sums of Decayed Sines

## Response Optimization & Response Limiting

- Theory
- Algorithms

## Results

- Test Article
- Pre-Test Analyses
- Shaker Shock Test

## Conclusions



# Introduction



The primary goal of any laboratory test is to expose the unit-under-test (UUT) to conservative, realistic representations of a field environment

- Random vibration, sine vibe, shock, thermal cycling, etc.

Satisfying this objective is not always straightforward due to laboratory test equipment constraints

- Mechanical shakers have nearly infinite impedance
- Over-testing and unrealistic failures can result when controlling the unit base acceleration

Force limiting and response limiting are relatively standard practices to reduce over-test risks in random vibration laboratory testing

Limiting in shaker shocks is done for the same reasons

- Force limiting can be done in shaker shock testing (in theory)
- **This presentation is about response limiting and optimizing single axis shaker shocks**



**At SNL we do response-limited testing much more than force limited testing**



# Objectives of Response Limiting



When a measured acceleration on the UUT exceeds a threshold, the control is reduced so that the response does not exceed the threshold (limit)

## PROS

- Response limiting can protect sensitive high-value subcomponents from over-test risk
  - Trade-off – Allow potential undertesting elsewhere
    - This is generally acceptable because test environments are conservative
- Response limits minimize the distortion of the input specification
  - Response limits are enforced at specific frequency bands of concern
    - Near resonances where response may be unacceptably amplified
- Response limits can be defined at multiple locations and in different axes simultaneously
- SRS based – Acceleration based on Sum of Decayed Sines (SDS)

## CONS

- Critical locations may be inaccessible for instrumenting
- Need pre-test response predictions/limits
- Must be pre-planned – not a closed-loop process like RV response limiting



# Objectives of Response Optimizing



Response optimizing seeks to define an input that minimizes the error between test measured responses and specified environments

- **SIMO (Single Input Multi Output) least-squares SRS optimization**

## PROS

- Better (best possible) match to known (field) environments at the defined response locations
  - Overall better test (more field-like environments)
  - Can be combined with response limiting

## CONS

- May heavily distort the input signal
- Need “field” SRS at response locations
- May over-test at some locations
  - Do not use with envelope response targets – use response limiting
- Critical locations may be inaccessible for instrumenting
- Open loop process – must be pre-planned



# Shaker Shock Basics



Shaker shocks are run open loop

- Shaker shocks are usually deterministic waveforms
- Shakers are driven by acceleration time histories

The test engineer can specify the input acceleration directly

- Waveform replication

The waveform can be created based on algorithms that create an acceleration history whose SRS matches the specified SRS

- Sum of Decayed Sines waveforms
- Random shock waveforms
- Zero Residual Displacement (ZERD) waveforms
- WAVSYN (half-sine modulated sinusoids)

The shaker shock synthesis tools used for this presentation are derived from David Smallwood's work

**This presentation is about SDS based shaker shocks based on Smallwood's SDS algorithm [1-6]**





Fundamental SDS waveform equation:

- $z(t) = \sum_{i=1}^N A_i e^{-2\pi f_i \zeta_i t} \sin(2\pi f_i t)$
- $A_i$  – Tonal amplitudes
  - Signs of  $A_i$  alternate – i.e.,  $A_i > 0$ ,  $A_{i+1} < 0$ , etc.
  - Gives smoother SRS and reduced compensating pulse
  - Initial guess –  $A_i = \frac{SRS(f_i)}{Q}$
- $f_i$  – Tonal frequencies
  - At least 5 tones/octave for 3% SRS damping
  - At least 3 tones/octave for  $\geq 5\%$  SRS damping
  - More tones/octave are better
    - Richer spectral content is probably more realistic and tends to produce lower peak acceleration
    - Avoid harmonics
- Duration ( $T$ ) is the greater of two values:
  - Real world waveform
  - 10 cycles of the lowest tonal frequency
- $\zeta_i$  – Tonal damping ratio
  - $\zeta_i = \frac{1}{2\pi\tau f_i}$  -  $\tau$  is the exponential time constant
    - Usually  $\tau$  is a constant –  $\tau < \frac{T}{3}$
    - Ensures adequate ringdown
    - Higher frequency tones ring for more cycles
- Sample rate
  - $SR \geq 10f_{max}$ ;  $5f_{max}$  may be OK for flat & declining SRS



Creating an acceleration history whose SRS matches a specified SRS is an iterative process

- The SRS is a nonlinear transformation

The Smallwood SDS algorithm [1] estimates the tonal amplitudes starting with the lowest frequency and proceeds to higher frequencies

- The amplitude at the  $i^{th}$  frequency,  $A_i$ , is adjusted to minimize the SRS error at that frequency:
  - $Err(f_i) = \frac{SRS_c(f_i) - SRS_{ref}(f_i)}{SRS_{ref}(f_i)}$
  - $SRS_c(f_i)$  is based on the current tone and all other tones computed to that point

Many shaker controllers (Spectral Dynamics, Data Physics, etc.) can make the input signals from SDS tables

- The SDS software in some controllers is based on the Smallwood algorithm
- Differences in how they minimize residual velocity and displacement
  - Compensating pulse or high-pass filter
- NASA GSFC (Dan Kaufman) has recently developed a different SDS approach – conceptually similar



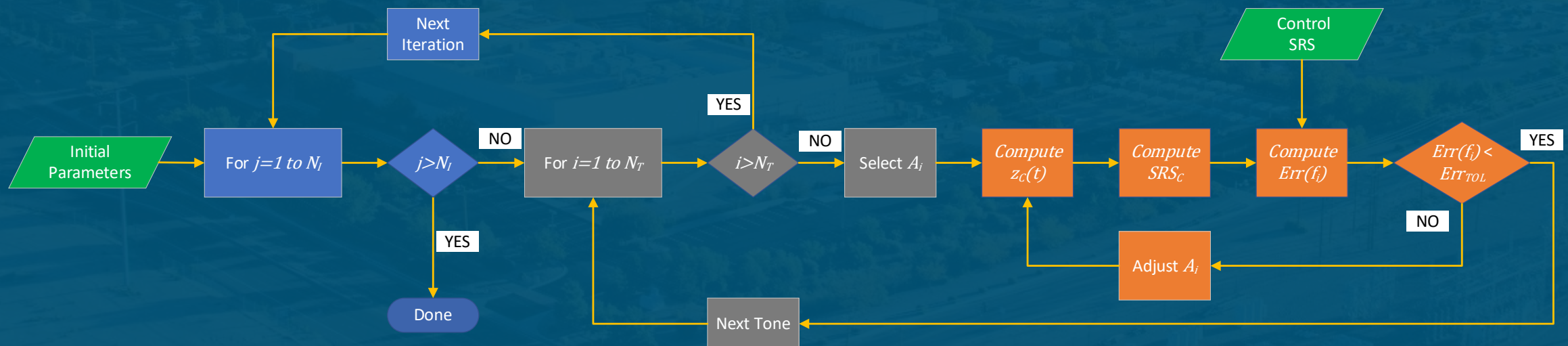
# Sum of Decayed Sines – Basic Algorithm Flowchart



## Three loops

- Inner loop – Adjust amplitude  $A_i$  to minimize error
  - Composite acceleration time history -  $z_c(t)$
  - SRS of composite acceleration –  $SRS_C$
- Intermediate loop – all tones from left to right
  - Number of Tones –  $N_T$
- Outer loop – Repeat intermediate loop  $N$  times (corrects for coupling of tones)
  - Number of iterations -  $N_I$

$$Err(f_i) = \frac{SRS_C(f_i) - SRS_{CTRL}(f_i)}{SRS_{CTRL}(f_i)}$$

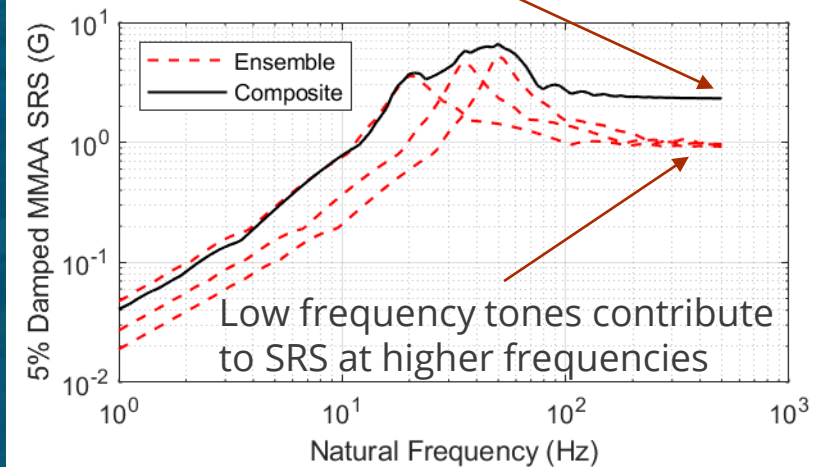
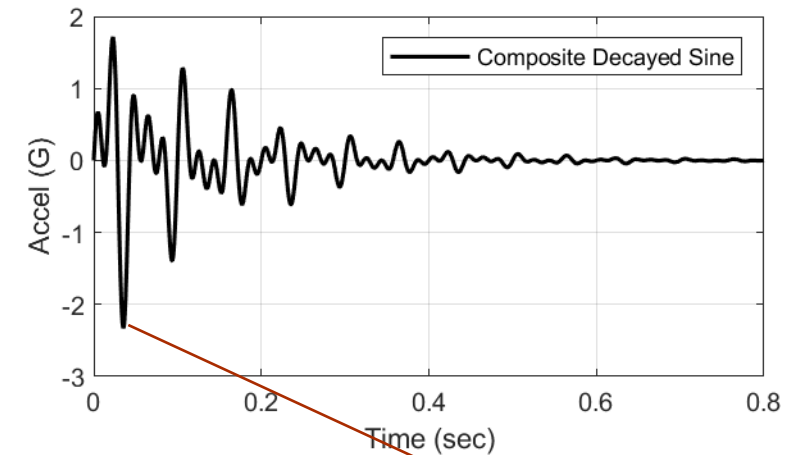
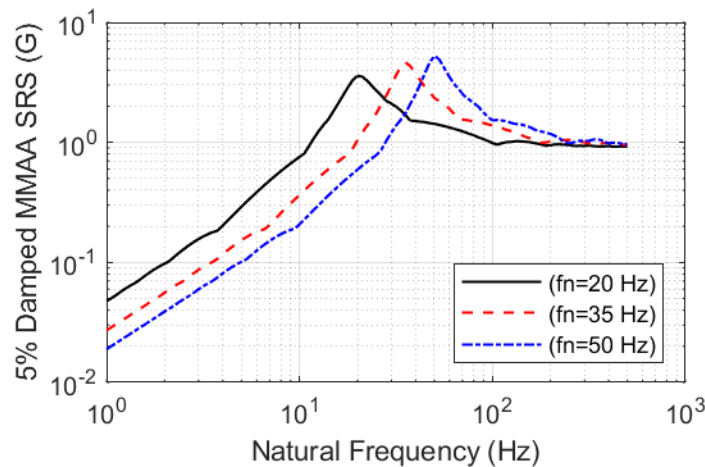
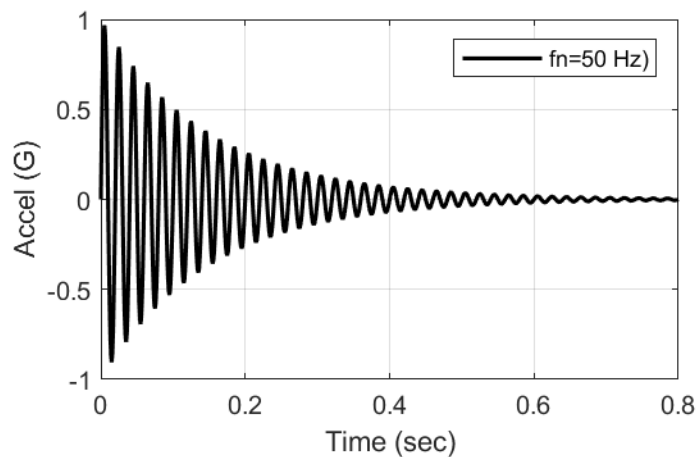
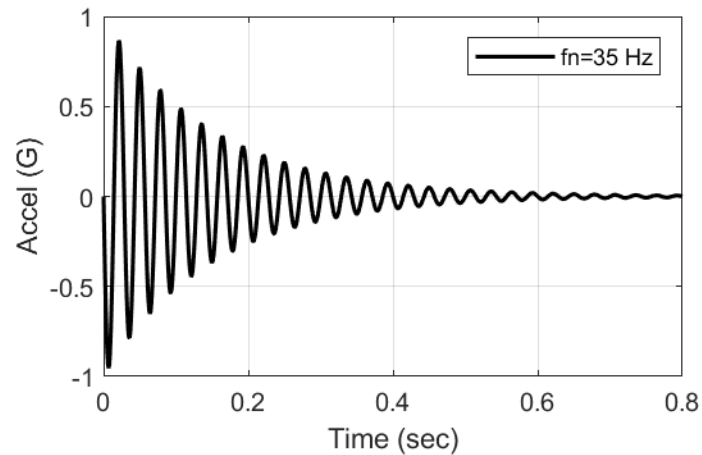
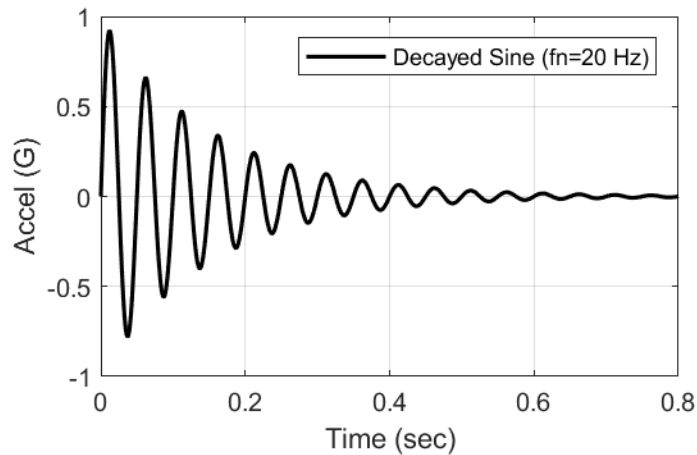




# Sum of Decayed Sines - Example



This example shows a 3 tone SDS generated shock







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## Objective

- Define an input that minimizes the SRS error between  $N$  test-measured responses and specified environments in a least-squares sense
  - The control channel can be included in the response optimization set
- The result is not a global minimum – it depends on the SDS equation parameters
  - Primarily the tonal frequencies

SIMO optimization algorithm has the same three loops as the basic SDS algorithm

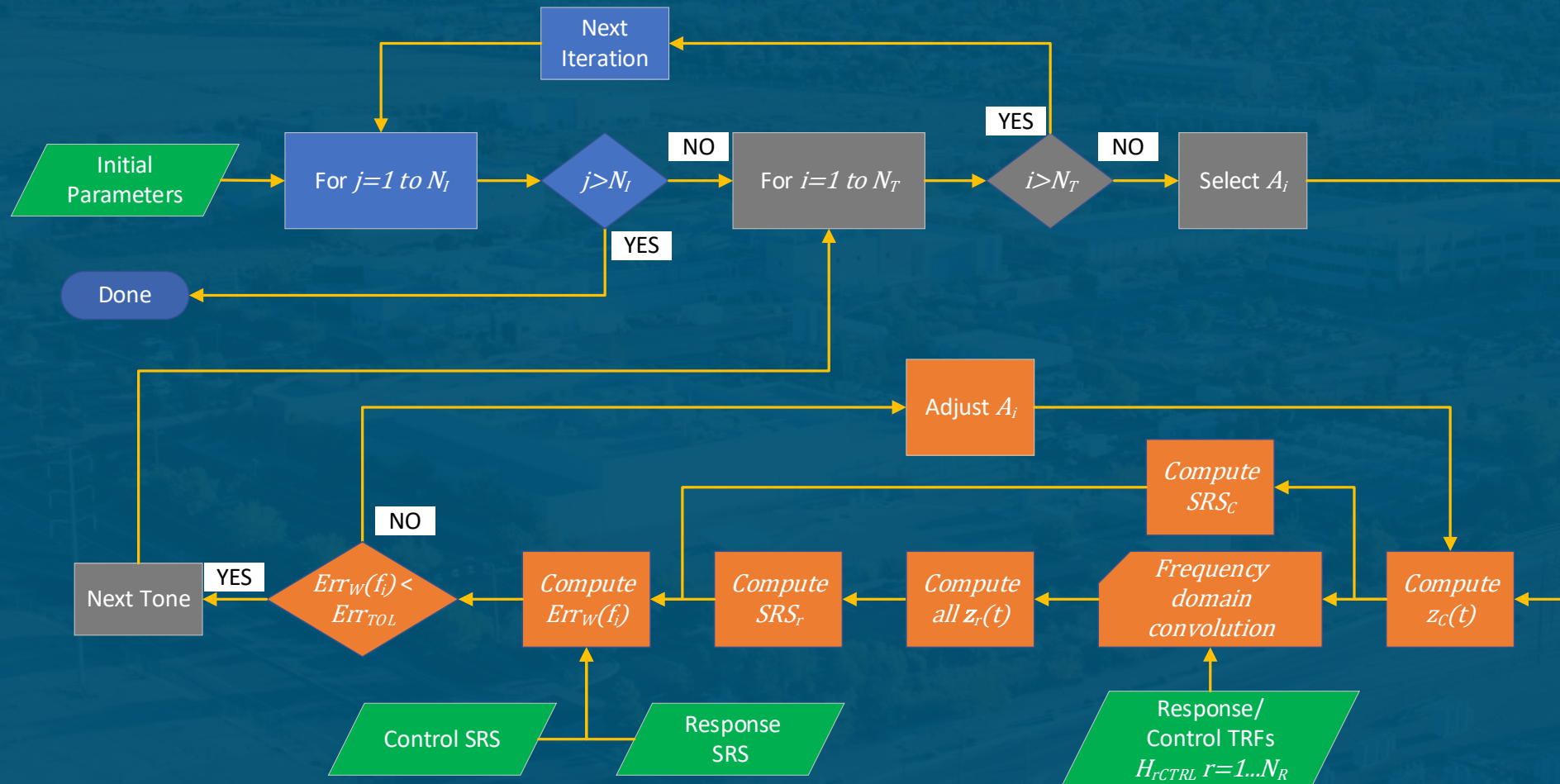
- Need Transmissibility Response Functions (TRFs) from control to response locations
  - One TRF per response location
  - If the control channel is included, TRF = 1 at all frequencies
- Different error function for the inner loop:
  - $$Err_W(f_i) = \frac{\sum_{r=1}^{N_R} \mu_r (SRS_C(f_i) - SRS_r(f_i))}{\sum_{r=1}^{N_R} \mu_r SRS_r(f_i)}$$
  - Inner loop error function is the weighted sum of errors for each response location



## Algorithm Flowchart

- $SRS_r = SRS_{res} \cup SRS_{CTRL}$
- If the control channel is included, its TRF = 1 at all frequencies

$$Err_w(f_i) = \frac{\sum_{r=1}^{N_R} \frac{\mu_r(SRS_C(f_i) - SRS_r(f_i))}{SRS_r(f_i)}}{\sum_{r=1}^{N_R} \mu_r}$$





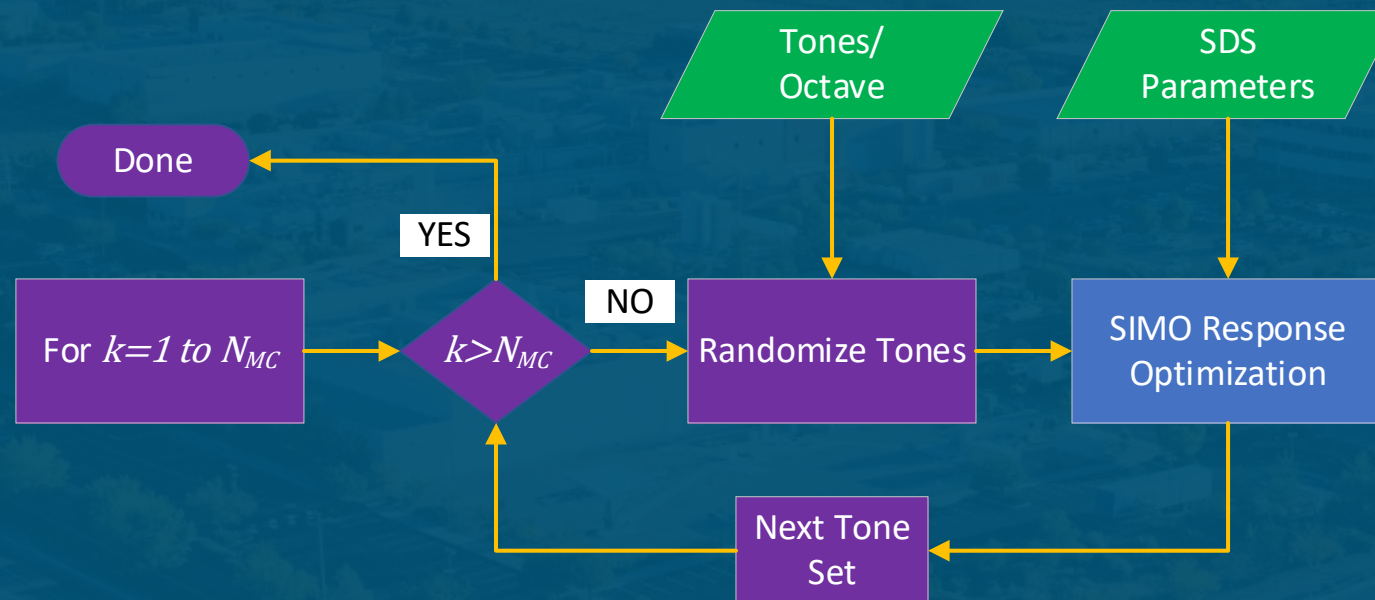
# Monte Carlo Response-Optimized Shaker Shocks



The SIMO optimized control acceleration is sensitive to the distribution of the tonal frequencies in the SDS equation

- The SIMO optimization problem is solved multiple times with different sets of tonal frequencies

## Monte Carlo Algorithm Flowchart



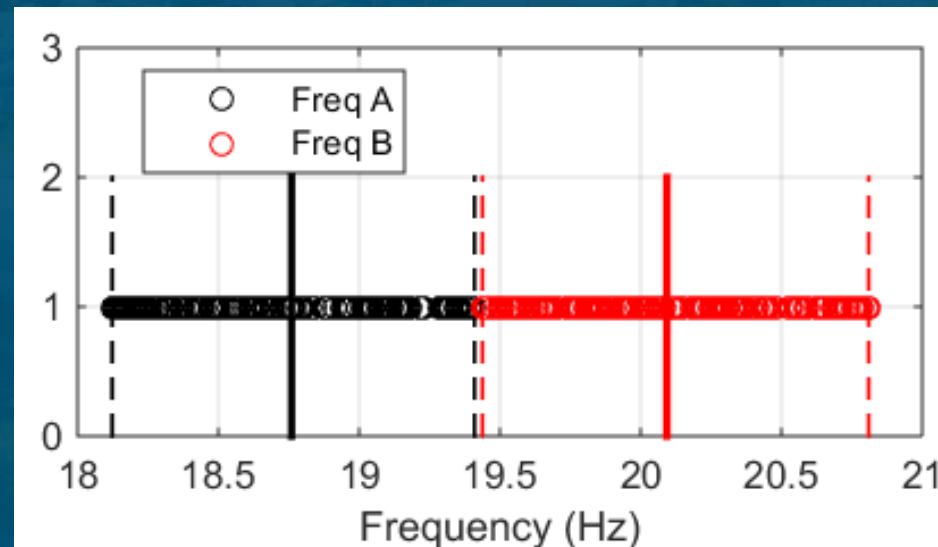


# Monte Carlo Response-Optimized Shaker Shocks



A Monte Carlo algorithm is used to generate the control SRS with various combinations of tonal frequencies

- Octal spacing of nominal tonal frequencies (e.g., 3 tones/octave)
- Tonal frequencies are defined from a uniform distribution around each nominal tone
  - Guard bands are imposed at the boundaries between adjacent tones to prevent a non-monotonic vector of frequencies



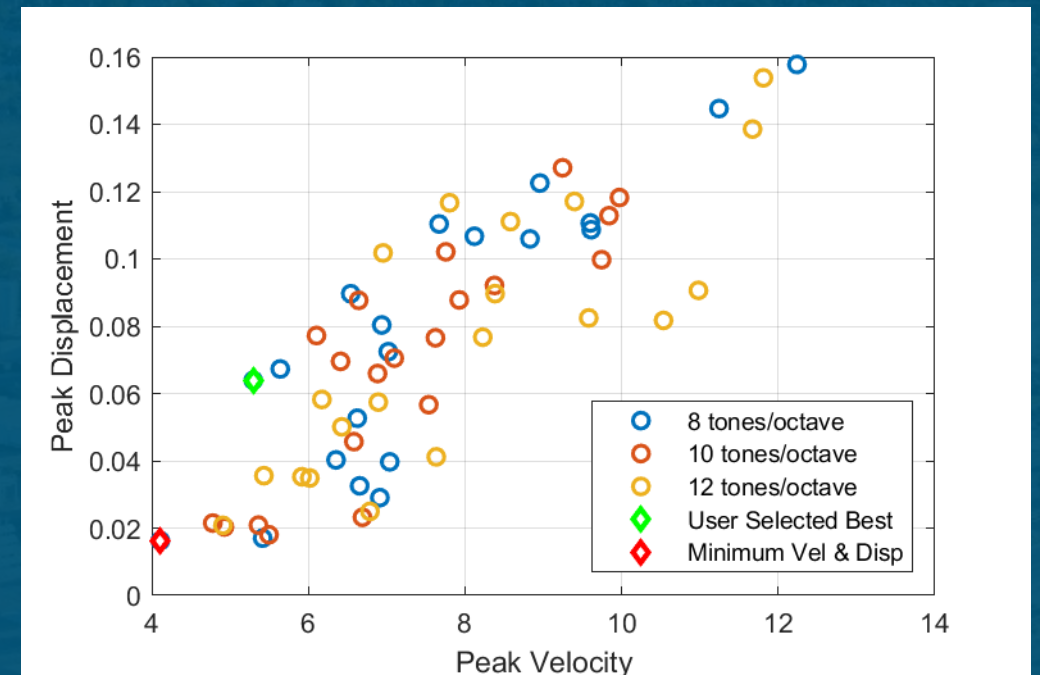
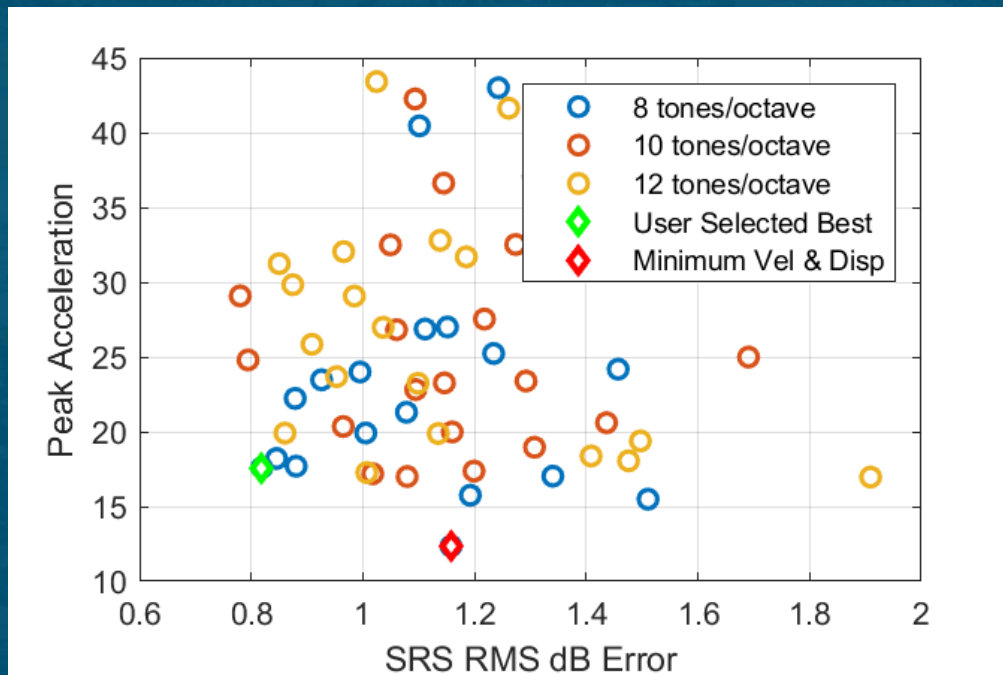


# Monte Carlo Response-Optimized Shaker Shocks



The “optimal” control acceleration history is chosen based on 2 or more scalar metrics

- Peak acceleration, Peak Vel, Peak Displacement, RMS dB Error, Energy
- “Optimal” is user selected from a scatter plot



$$E_{RMSDB} = \left( \frac{1}{N_R} \right) \sum_{r=1}^{N_R} \left( \sqrt{\frac{\sum_{i=1}^N \left[ \left( 20 \cdot \log \left( \frac{SRS_M(f_i)}{SRS_r(f_i)} \right) \right)^2 \Delta f(i) \right]}{f_B}} \right)$$



# Optimized Response & Limited Shaker Shocks



Response optimization generates a control SRS so that the SRS at response locations are a “best fit” in a least-squares sense

- No guarantee that SRS will not exceed target SRS at response locations
- Response optimization produces a “better fit” outside the limiting frequency bands

Response limiting creates a control shaker shock such that responses do not exceed target SRS in specified frequency bands

- Must include the “control” channel in the response set
  - This SRS will be matched if the limits do not engage
- Flowchart is the same as the response optimization flowchart
  - Different error equation
- Convergence is not guaranteed

**Optimal Response Limiting combines SIMO optimization with response limiting**

- **Can use Monte Carlo Optimization loop**

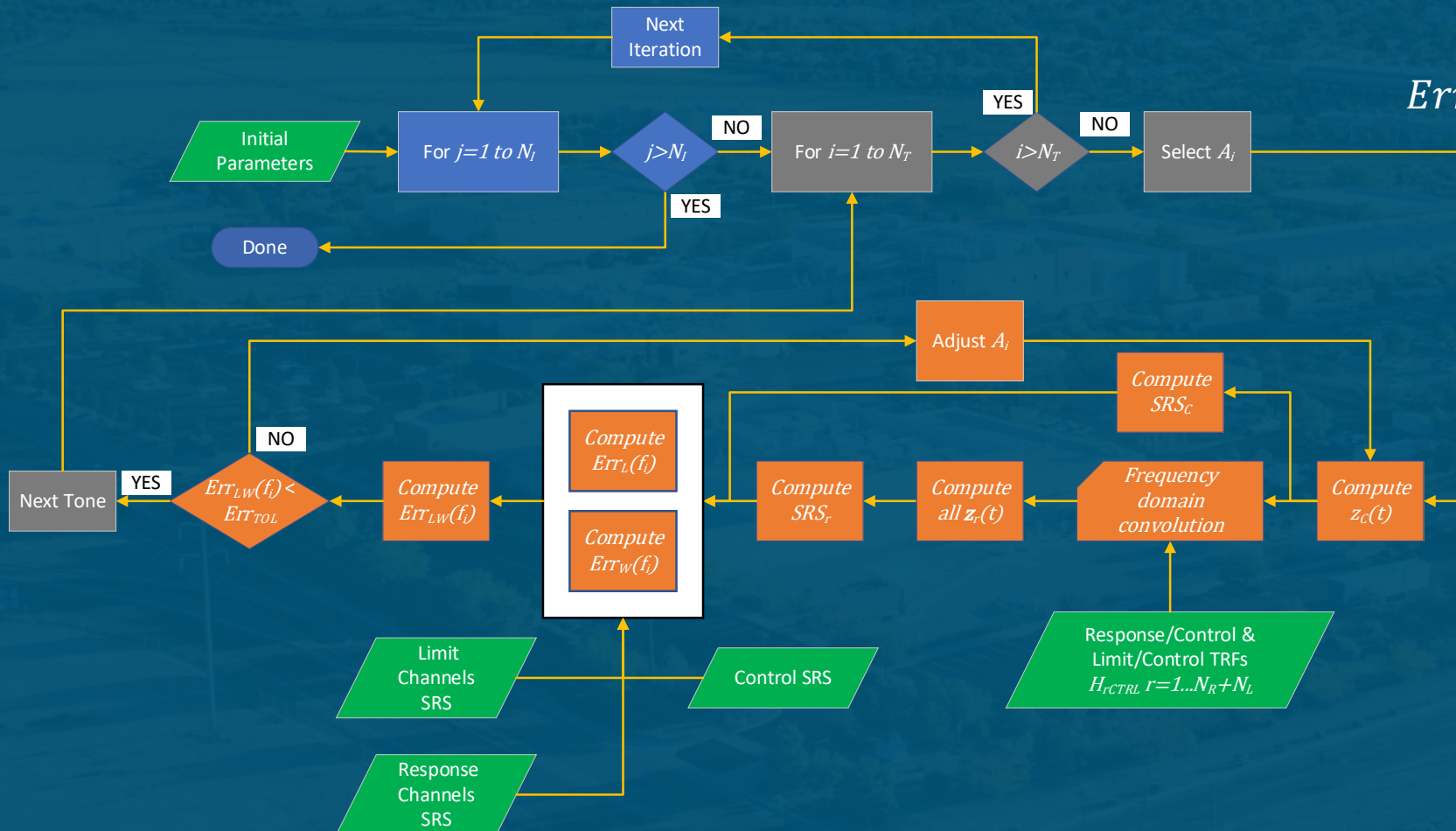


## Algorithm Flowchart

- Channels can be in the limit set and the optimization set
- The outer Monte Carlo loop can be added

$$Err_W(f_i) = \frac{\sum_{r=1}^{N_R} \frac{\mu_r (SRS_C(f_i) - SRS_r(f_i))}{SRS_r(f_i)}}{\sum_{r=1}^N \mu_r}$$

$$Err_L(f_i) = \max_{r=1 \dots N_L} \frac{(SRS_C(f_i) - SRS_r(f_i))}{SRS_r(f_i)}$$





# Optimized Response-Limited Shaker Shocks



The response-limiting algorithm uses the SIMO base algorithm with a set of conditional if statements

*if* ( $ERR_L > 0 \ \&\& \ ERR_W > 0$ ) | ( $ERR_L < 0 \ \&\& \ ERR_W < 0$ ) *then*

- $ERR_{LW} = \max[ERR_L, ERR_W]$

*elseif*  $ERR_L > 0 \ \&\& \ ERR_W < 0$  *then*

- $ERR_{LW} = ERR_L$

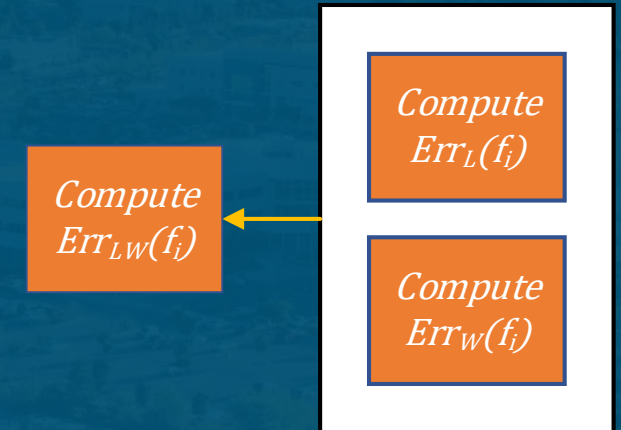
*elseif*  $ERR_L < 0 \ \&\& \ ERR_W > 0$  *then*

- $ERR_{LW} = ERR_W$

*end*

If the limits do not engage, the algorithm reverts to the basic SDS algorithm or the SIMO algorithm

We also use the slope of the correction (not shown here) when updating  $A_i$







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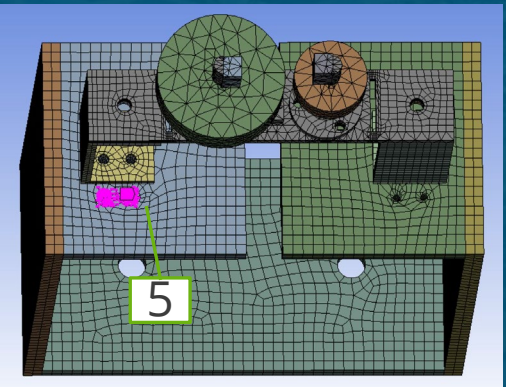
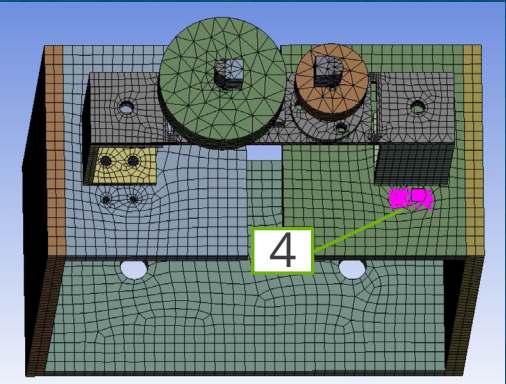
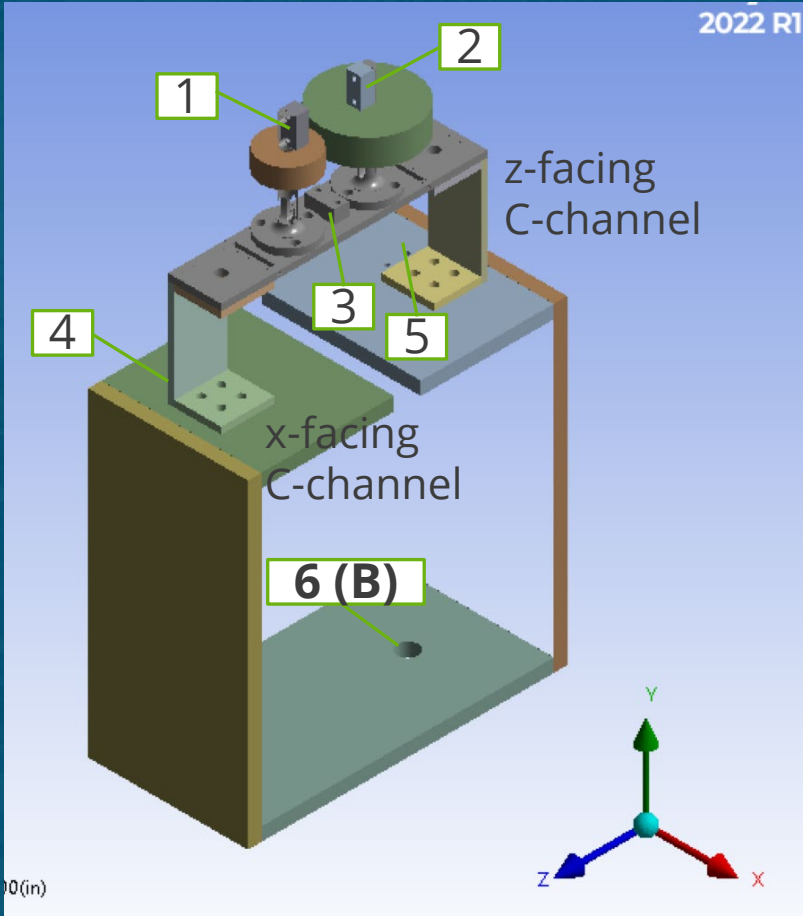


# Test Article – BARBECUE

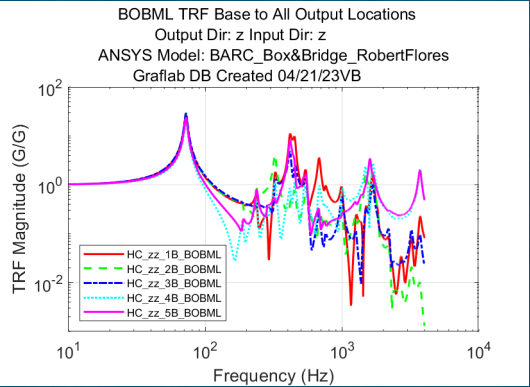
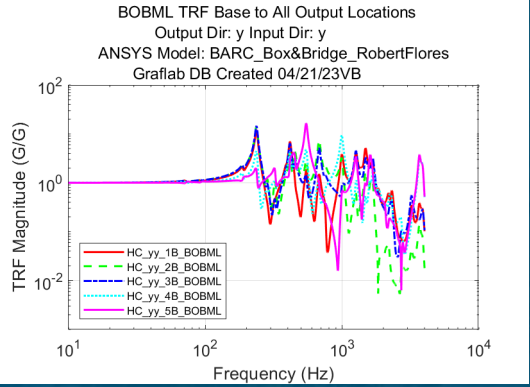
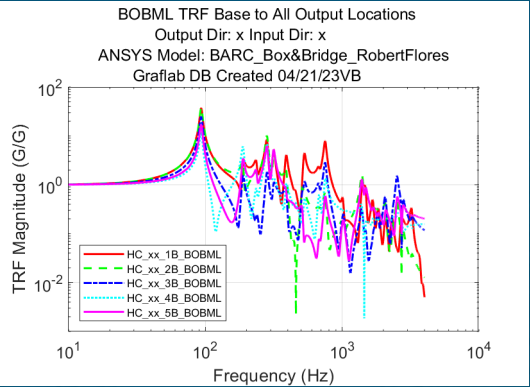
## Box And Removable Bridge with External Components Under Evaluation

### BOBML configuration

- Bridge On Box Medium mass & Large mass



Mode	Frequency (Hz)
1	72.132
2	93.162
3	188.06
4	236.45
5	281.71
6	314.68
7	322.64
8	389.17
9	415.25
10	449.53
11	510.73
12	527.88
13	545.96



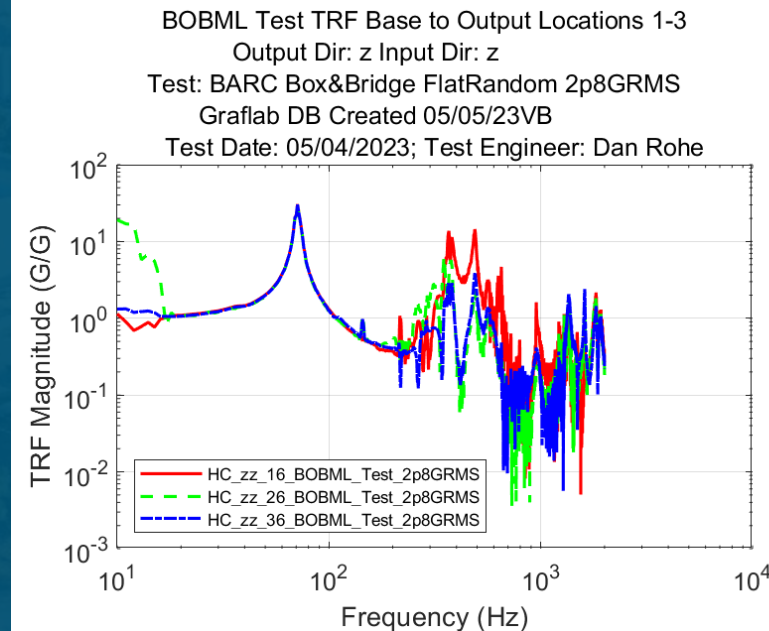
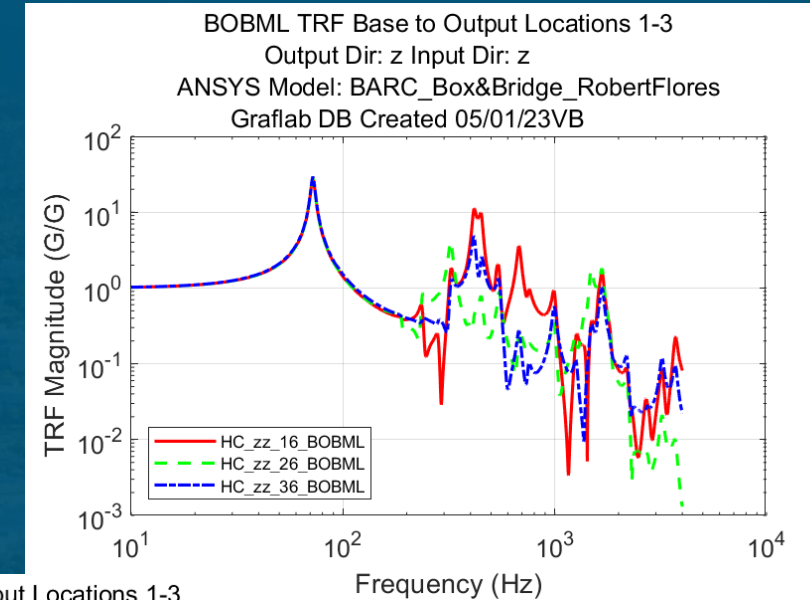
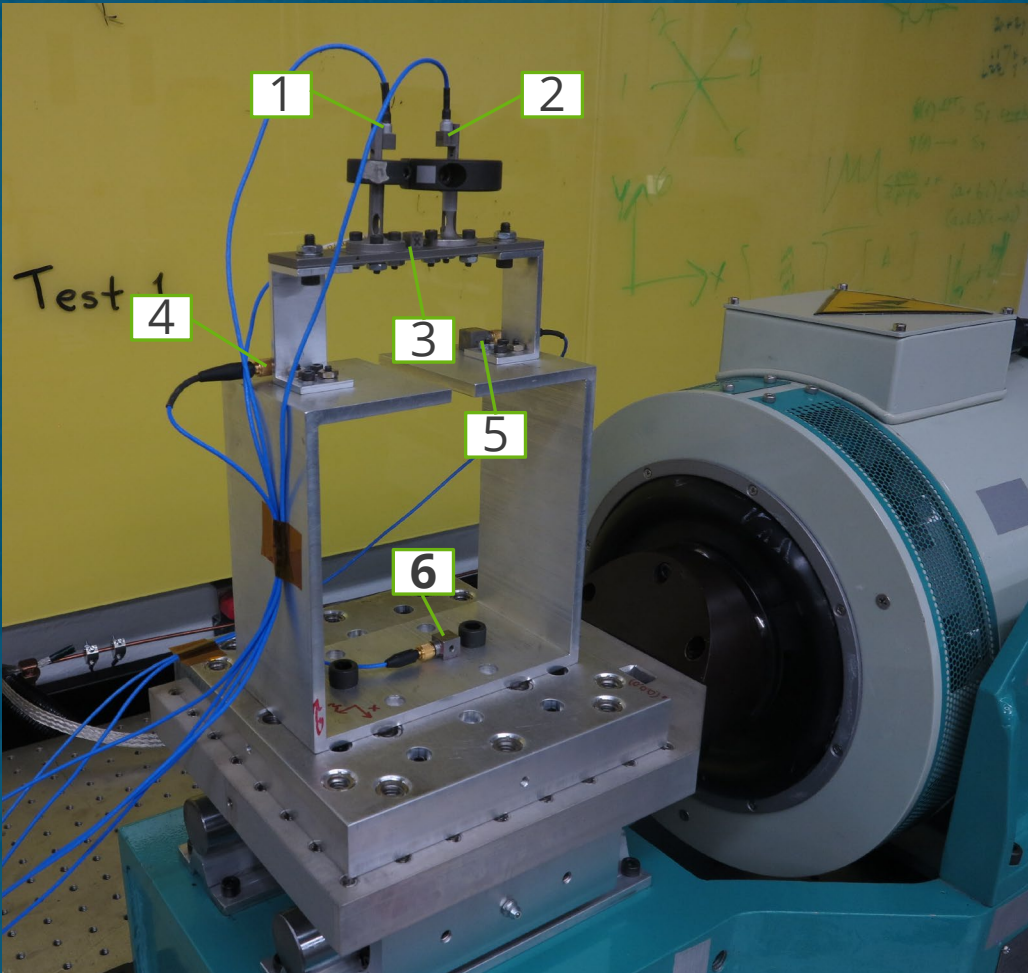


# BARBECUE Test



Testing was conducted in the Z axis

- The tests were controlled at the base
- Location 6





# BARBECUE Response Optimization & Limiting



Two resonances are used in this study

- Fundamental resonance at 71 Hz
- Resonance at 370 Hz

Field environment

- Measured responses at locations 1,2,3 and 6
- Represents what would have been measured on the BARBECUE in service

Test environment

- A straight-line laboratory test spec that represents the field environment
  - SRS at the base made using the best practice of “filling-in” dips in the field SRS
  - Straight-line response limits at locations 2 & 3

Measured TRFs and test spec are used to pre-compute the test base input for different objectives

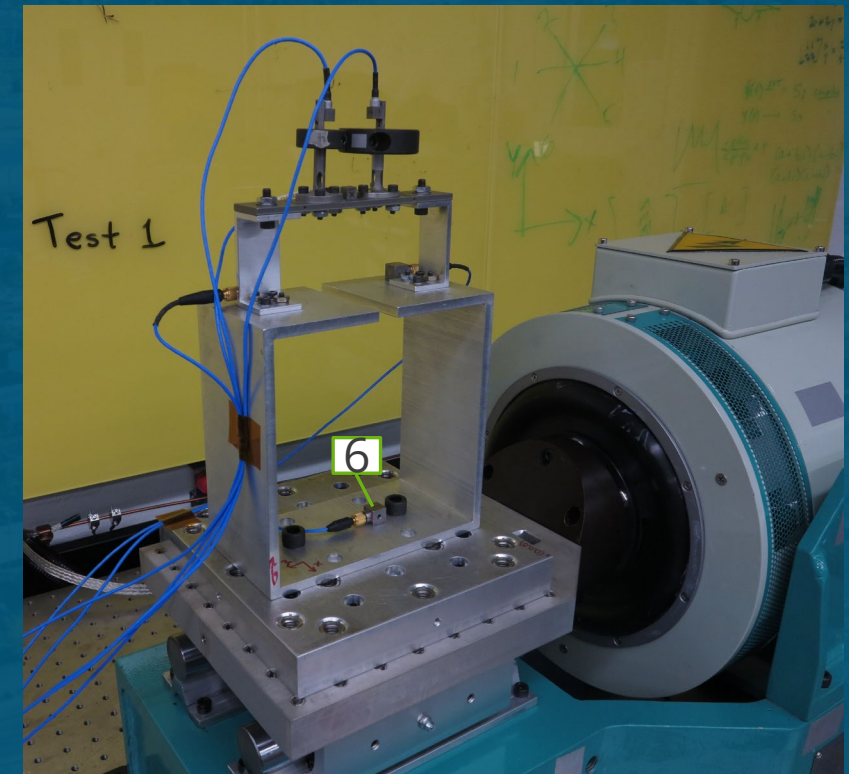
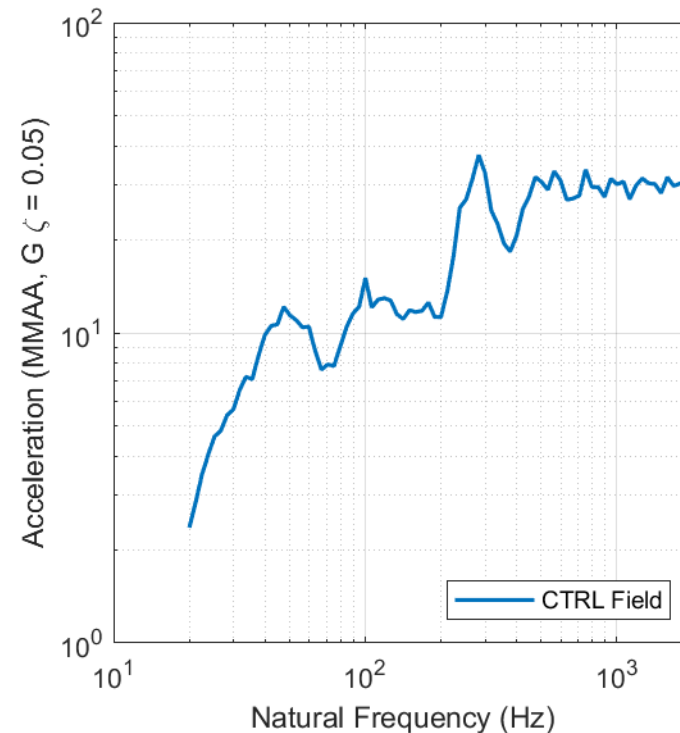
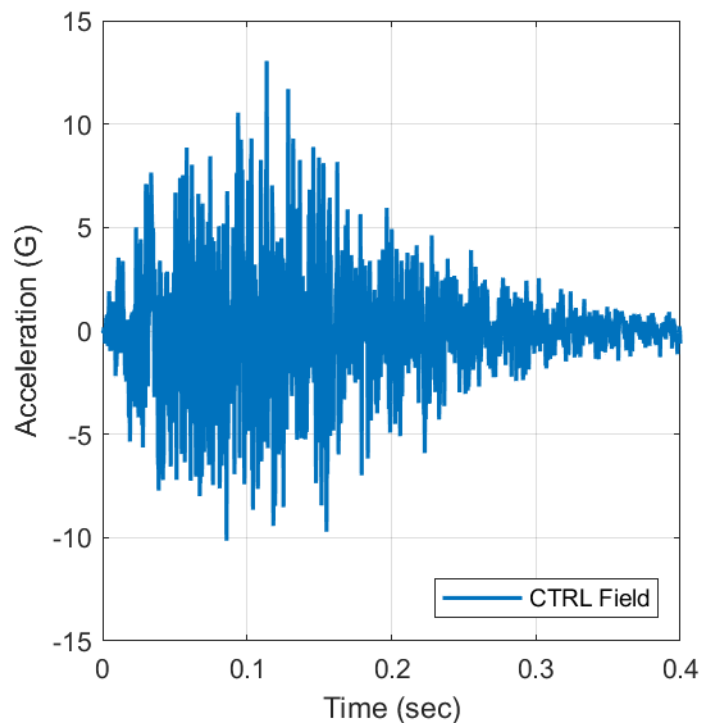
- Experimental validation



“Field measured” acceleration at the BARBECUE base

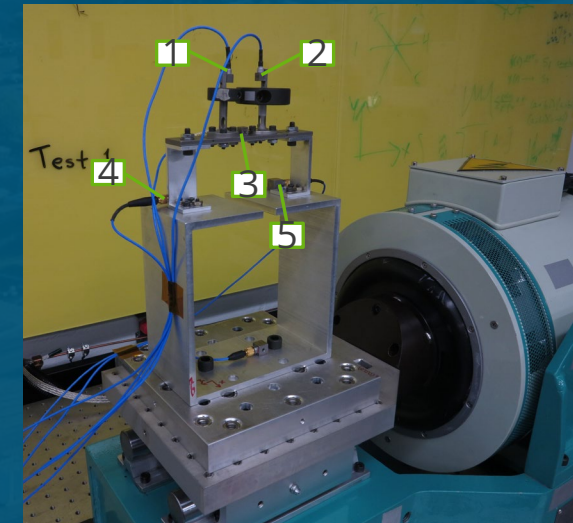
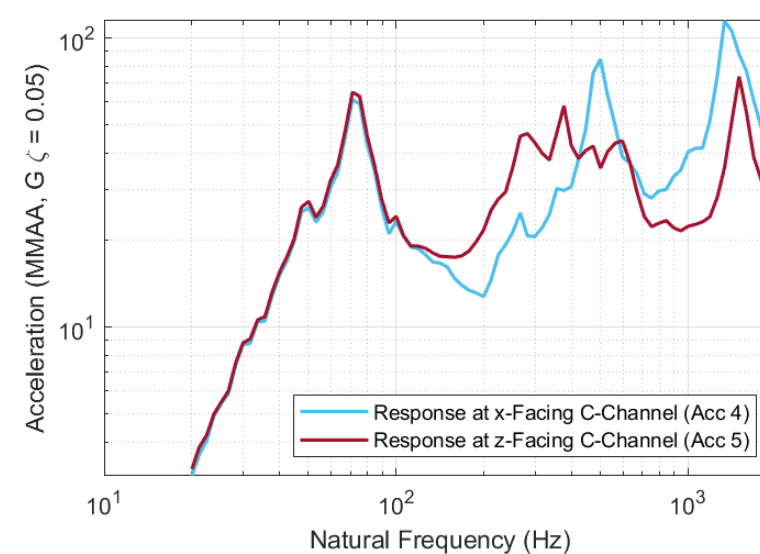
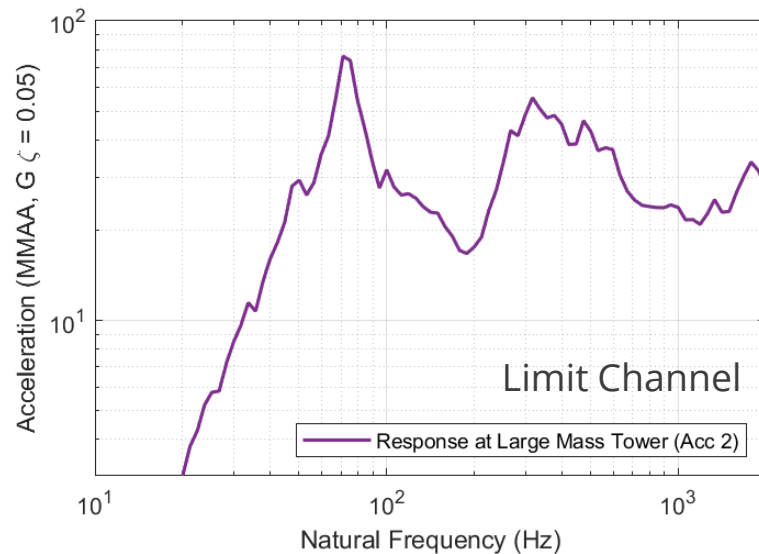
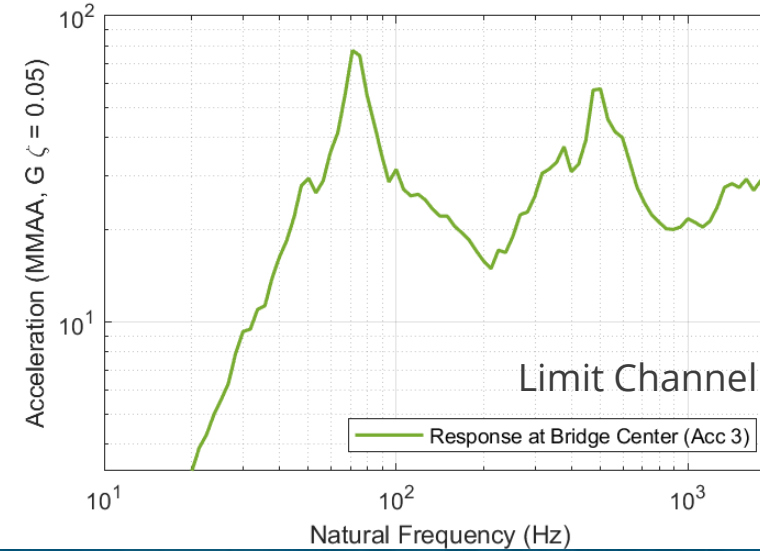
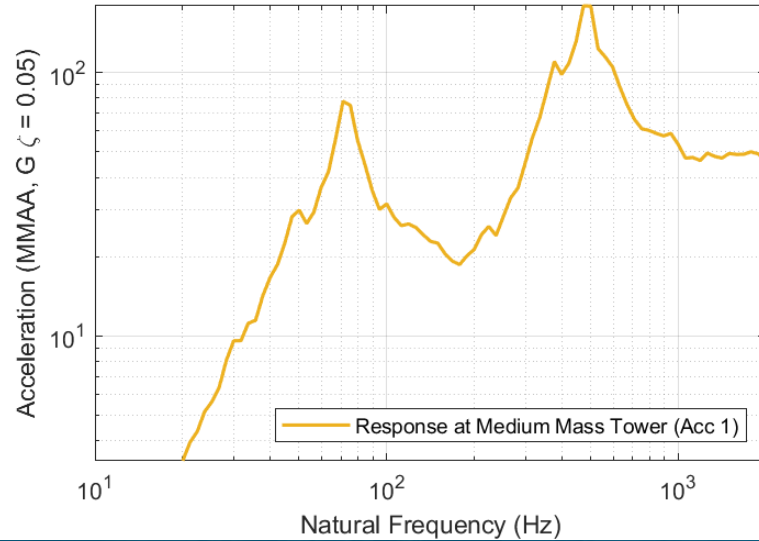
“Field measurements” at locations 1 – 3

- Accelerations and SRS at locations 1 – 3 were simulated using frequency domain convolution with TRFs and the base acceleration





# Field Environments – Response Locations



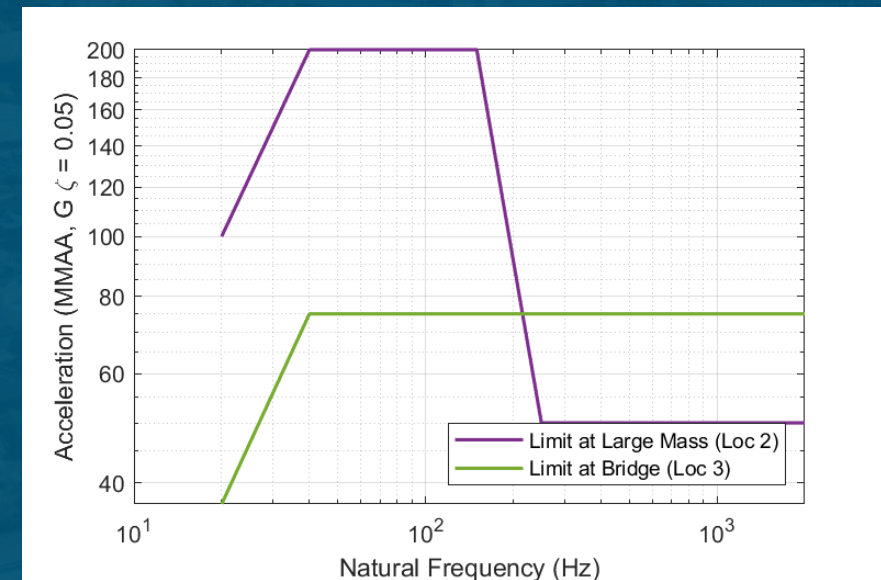
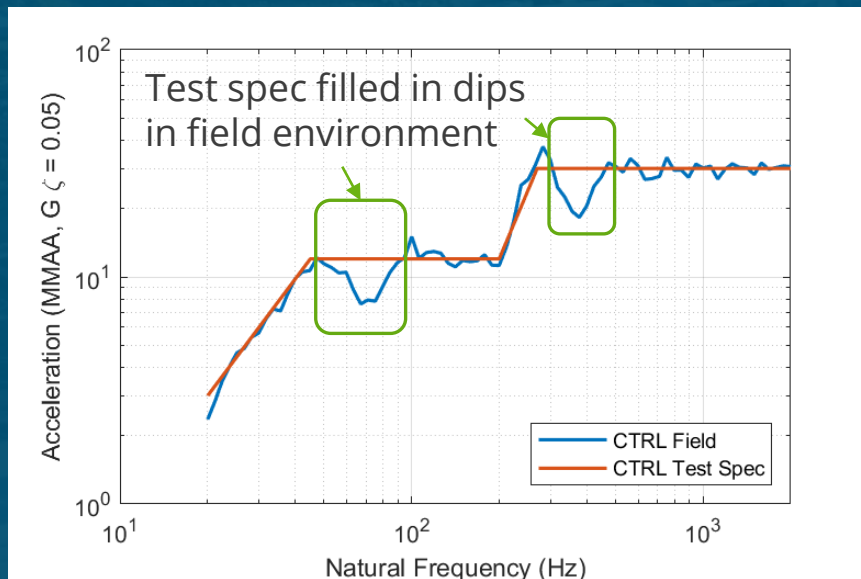


## Laboratory test specification

- Straight-line SRS at base
- Straight-line SRS response limits at locations 1 & 2

## Assumptions

- Field-measured SRS at response locations are known
- Field-measured input at the base is unknown or not available



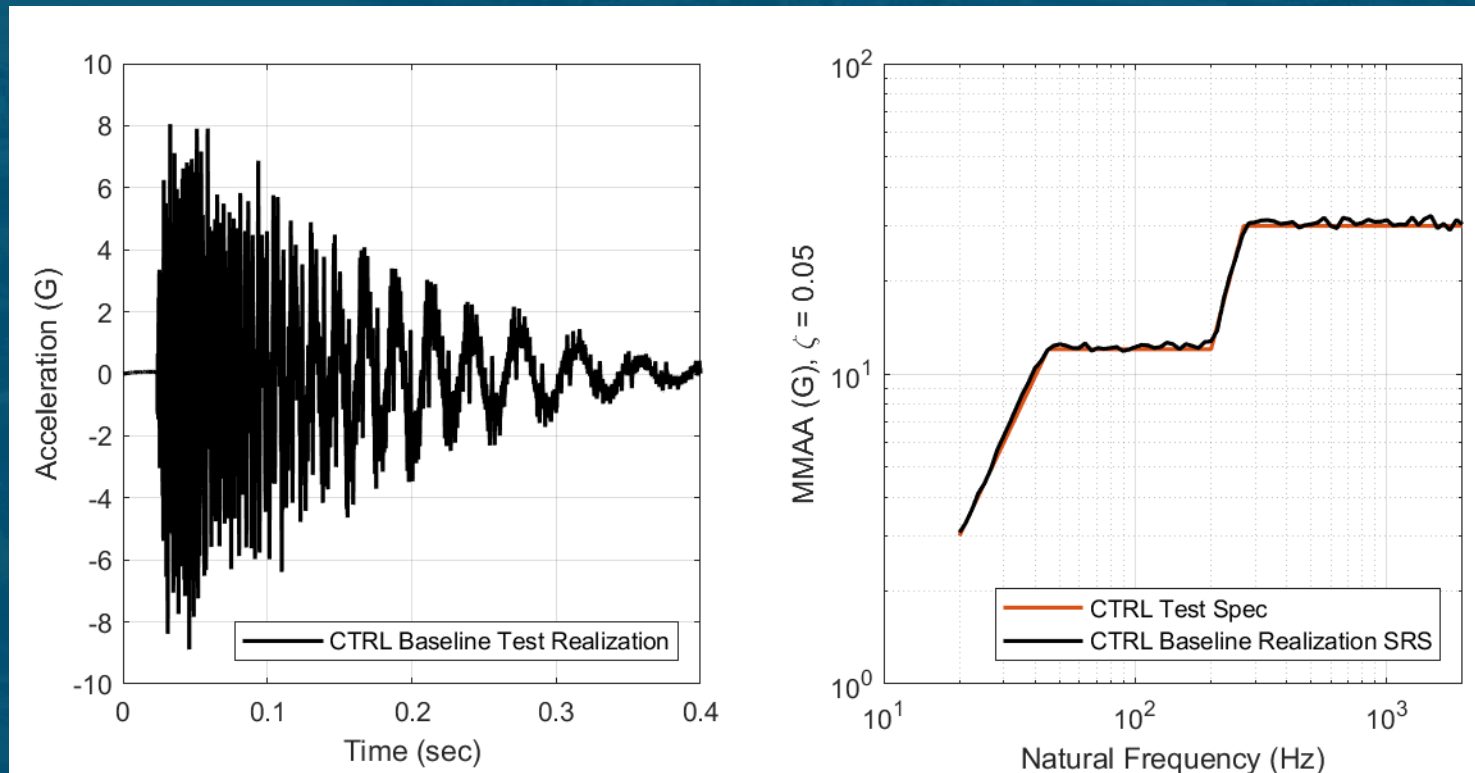


# Baseline – No Response Optimization or Limiting



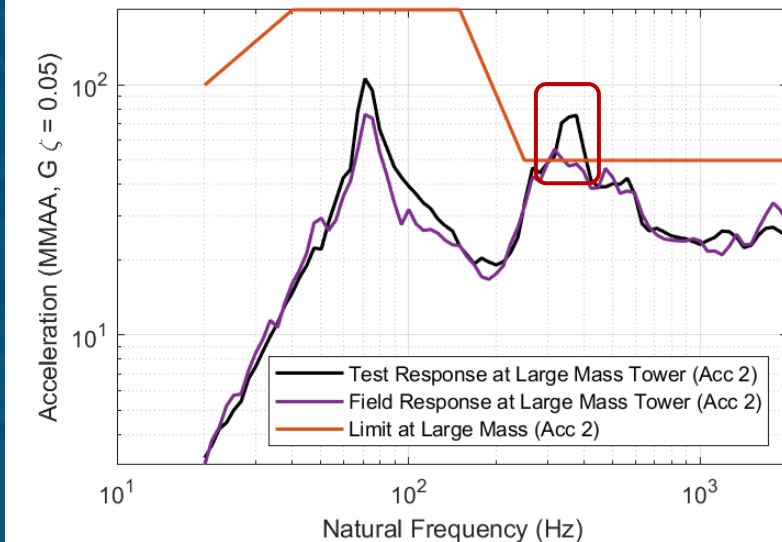
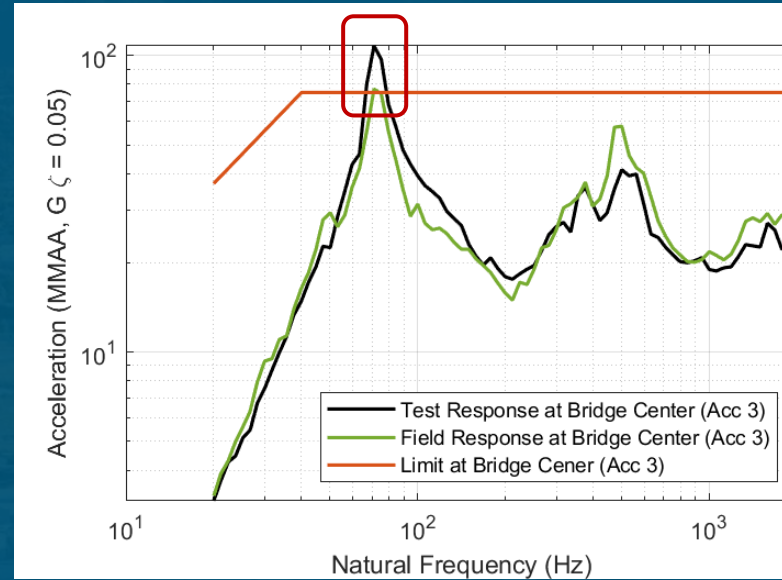
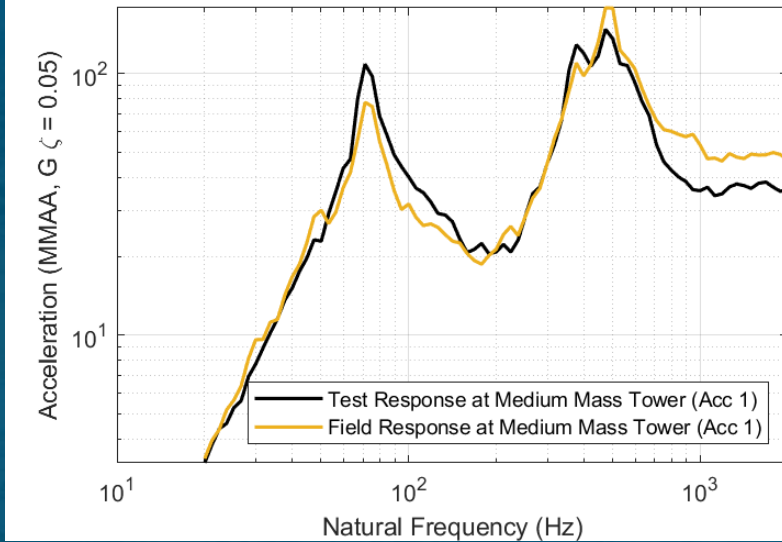
Computed a SDS acceleration history at the base

Propagated that through the TRFs to the response channels

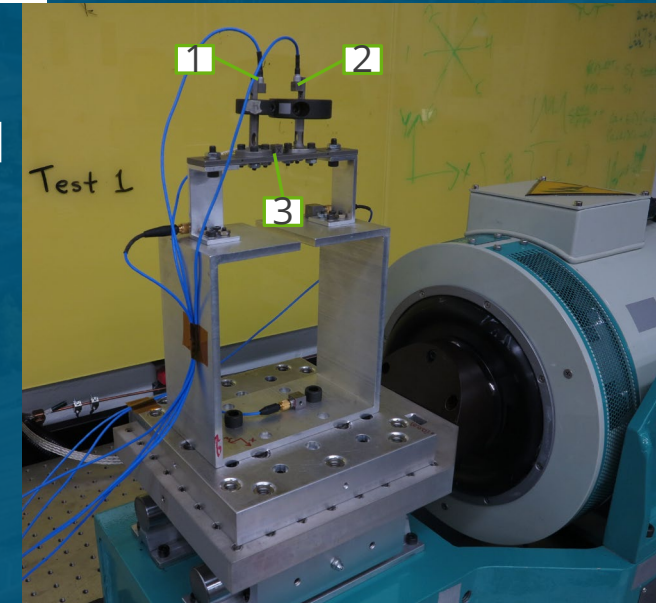




# Baseline – No Response Optimization or Limiting



Without limiting, the responses to the test input exceed the field environments





# BARBECUE Response Optimization & Limiting Objectives



## Case 1 – SISO Response limiting (One control location)

- Modify base excitation so that responses at locations 2 & 3 do not exceed response limits
  - If response limits are not active, Case 1 reduces to the basic SDS problem
- Response at location 1 uncontrolled

## Case 2 – SIMO Response optimization & limiting (Multiple response locations)

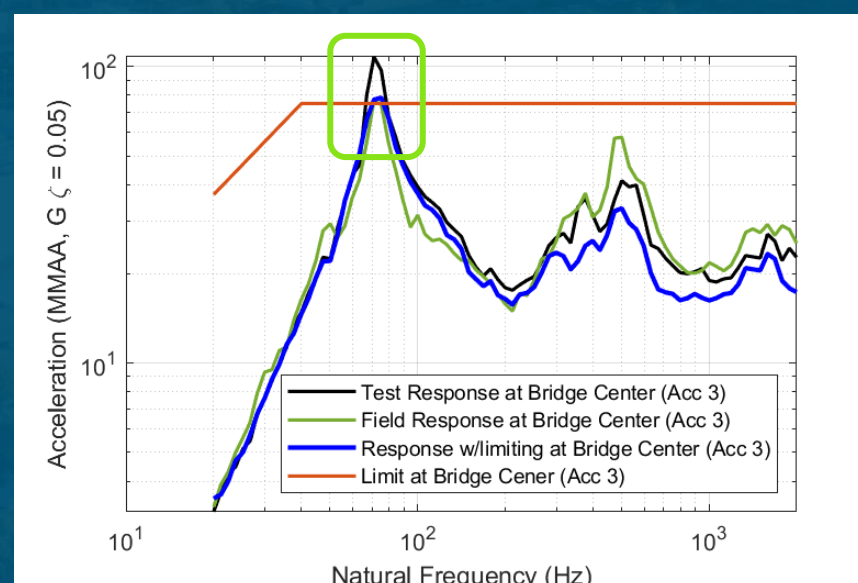
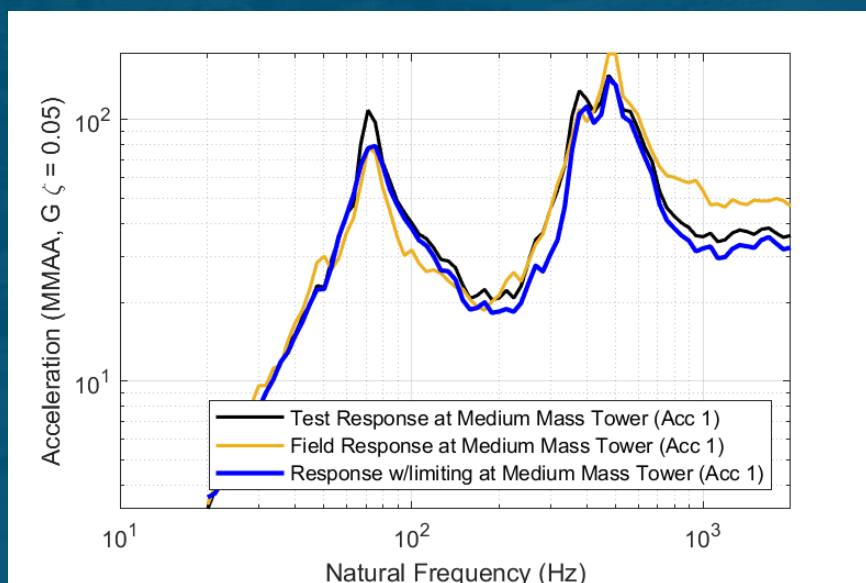
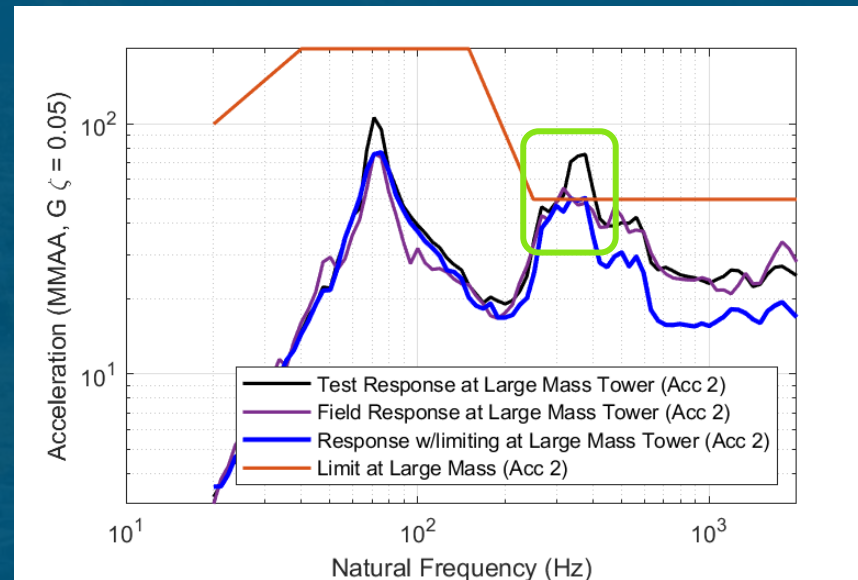
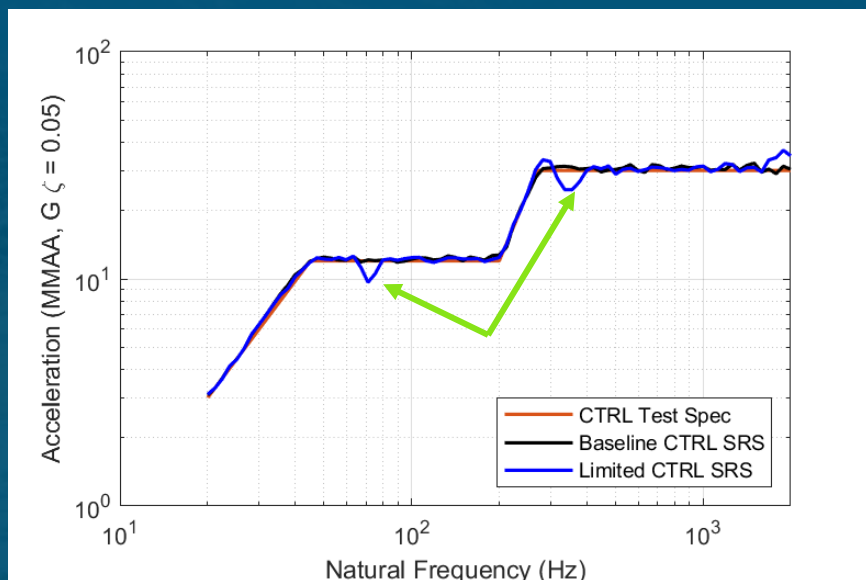
- Modify base excitation so that responses at locations 2 & 3 do not exceed response limits AND
- Minimize the SRS error at locations 1,2, & 3

## Case 3 – Monte Carlo SIMO response optimization

- Minimize the SRS error at locations 1,2, & 3 AND
- “Optimize” scalar metrics

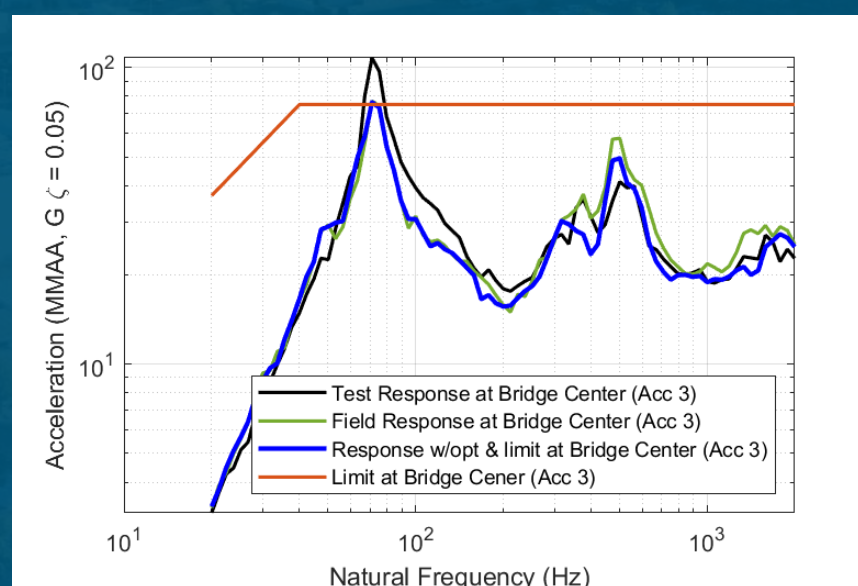
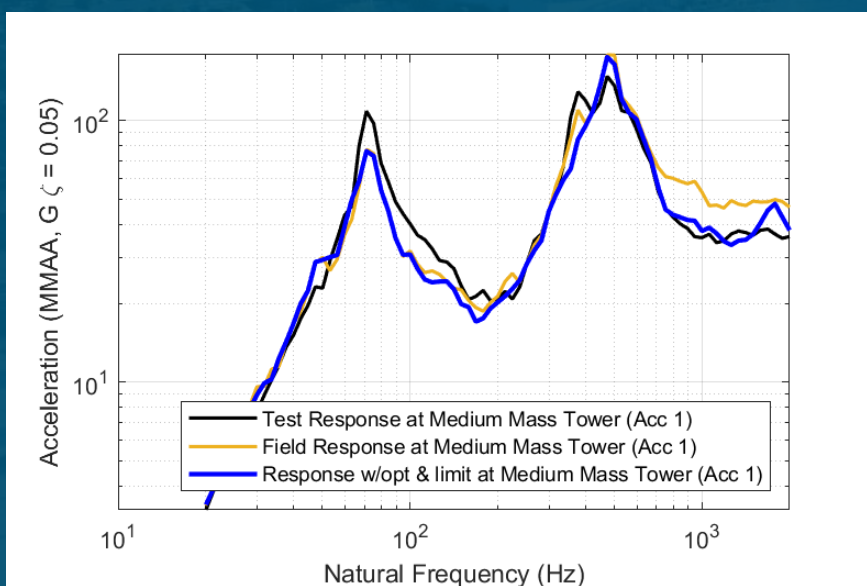
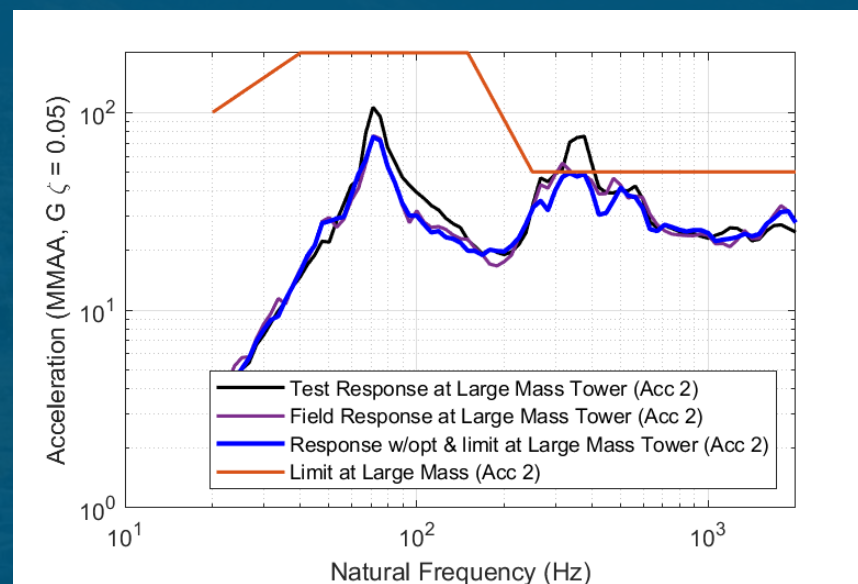
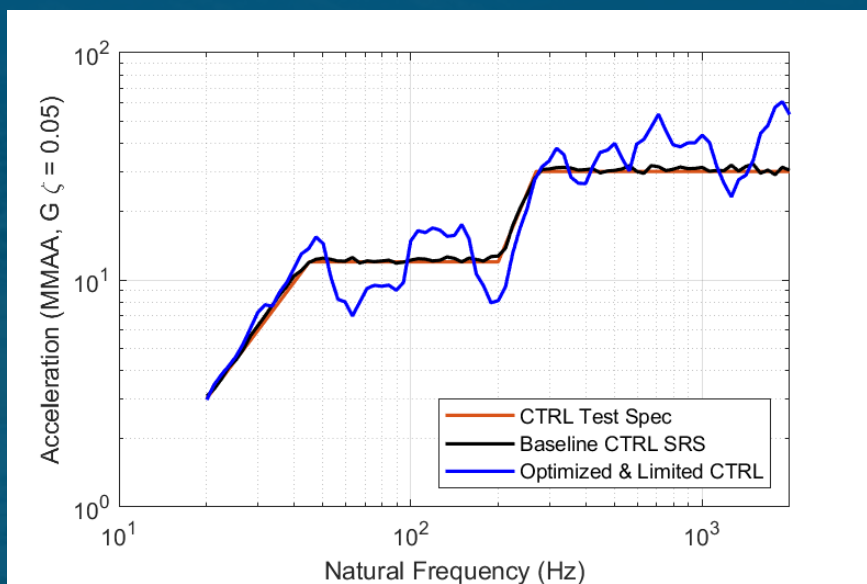


# Case 1 – SISO Response-Limiting



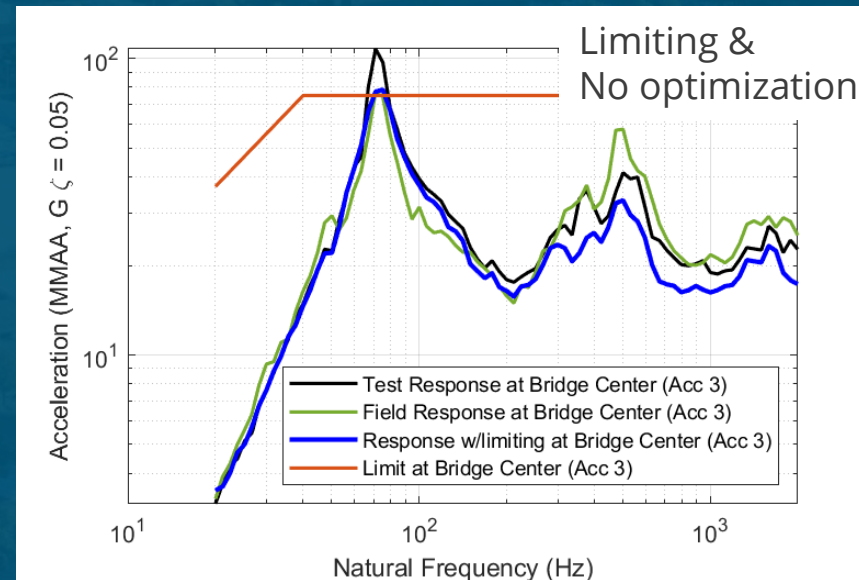
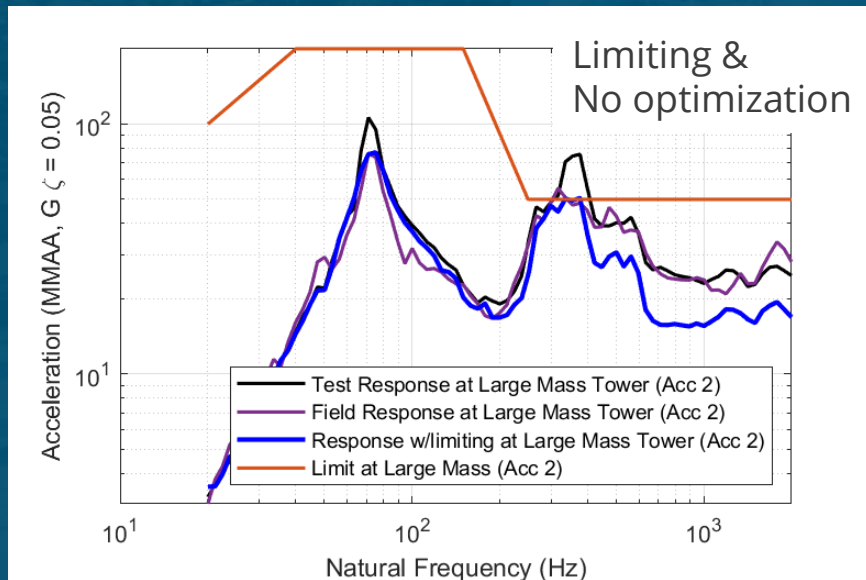
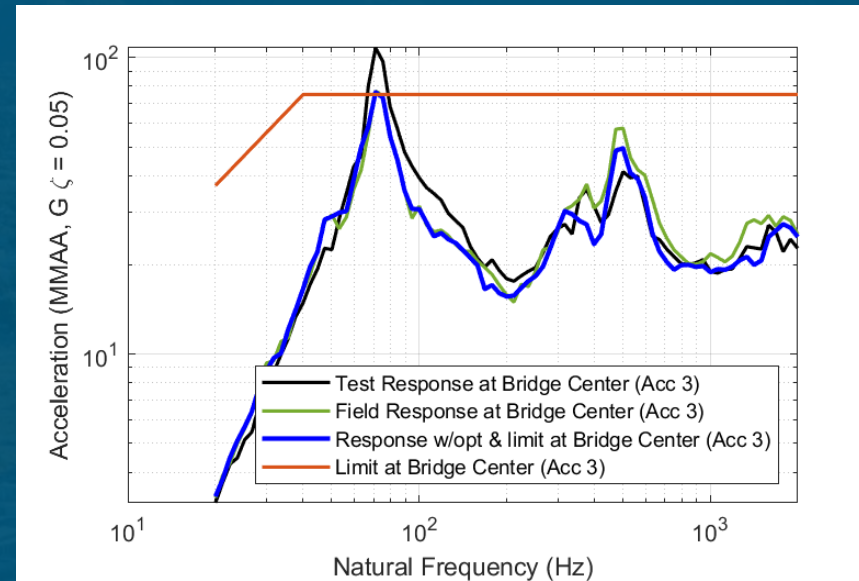
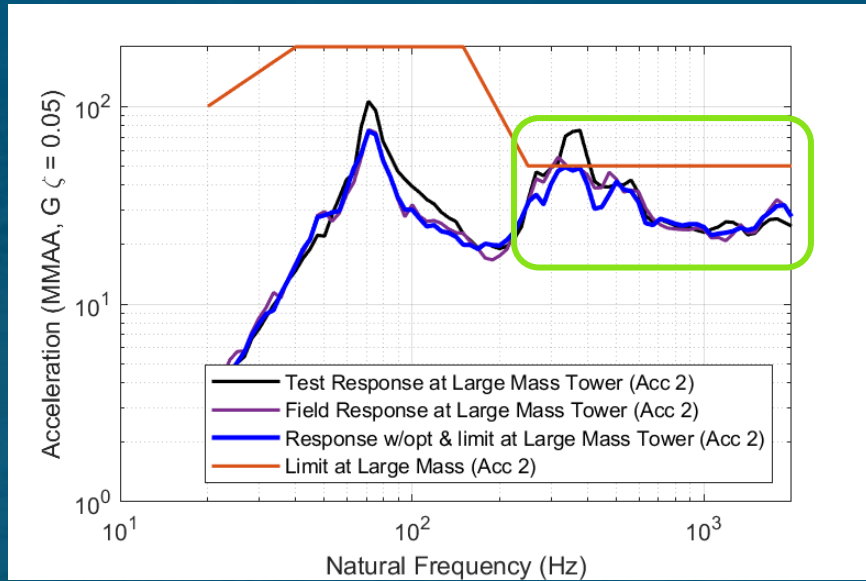


# Case 2 – SIMO Response Optimization & Limiting

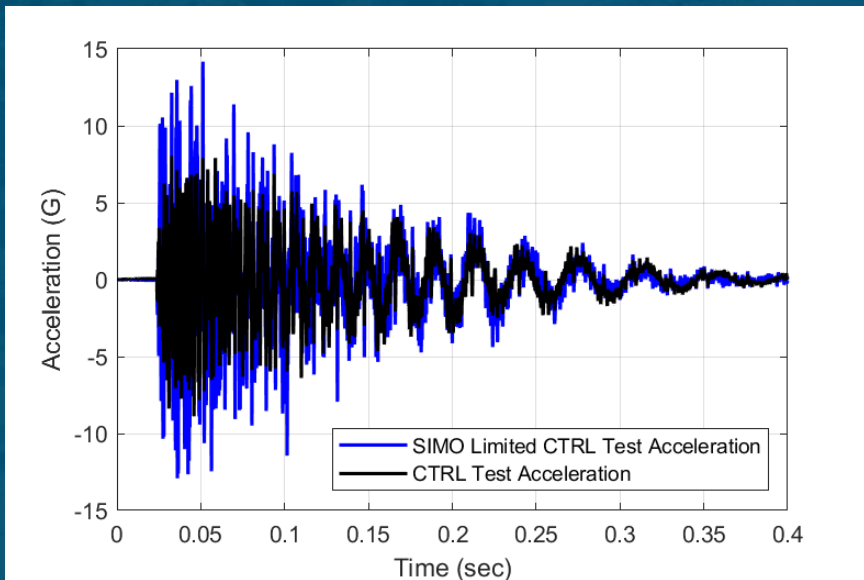
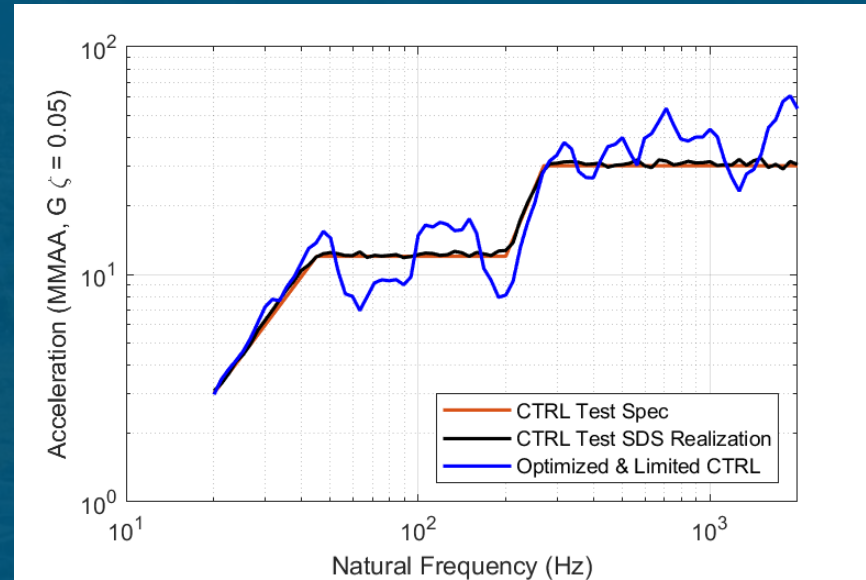
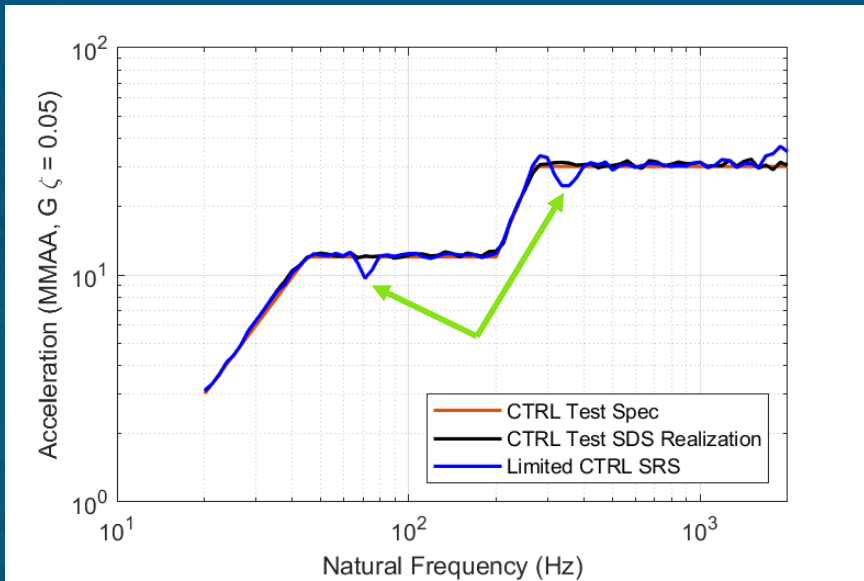




# Case 2 - Response Optimization & Limiting



# Case 2 - Response Optimization & Limiting



The control SRS is more distorted when response optimization is used

- Peak acceleration is higher

Response limiting alone affected the acceleration in the time domain very little

- Response limiting was not very deep or broad



## Case 3 – Monte Carlo SIMO Response Optimization



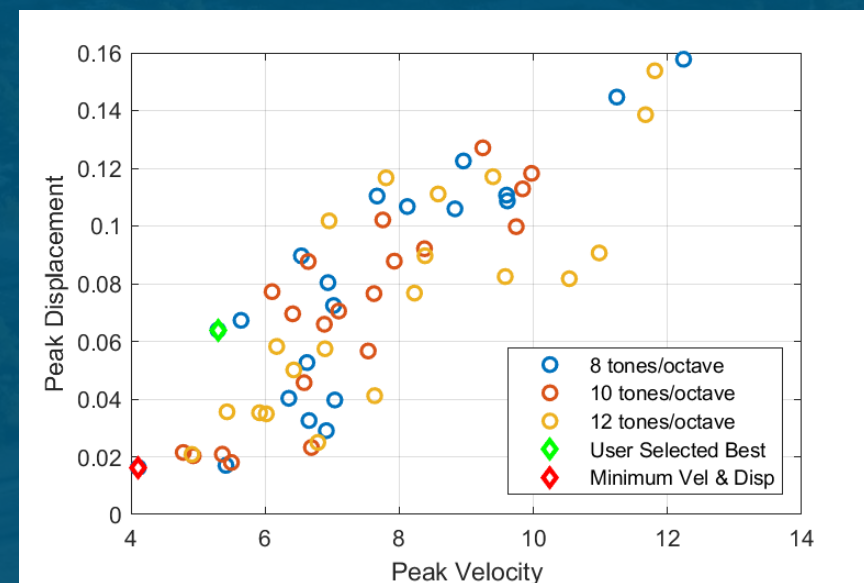
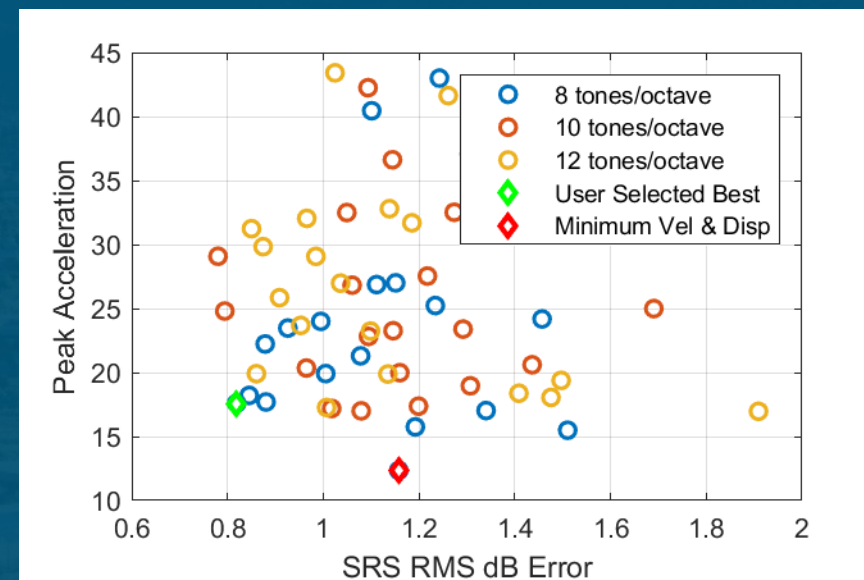
60 Monte Carlo simulations were run

- 20 each for 8, 10, and 12 tones per octave

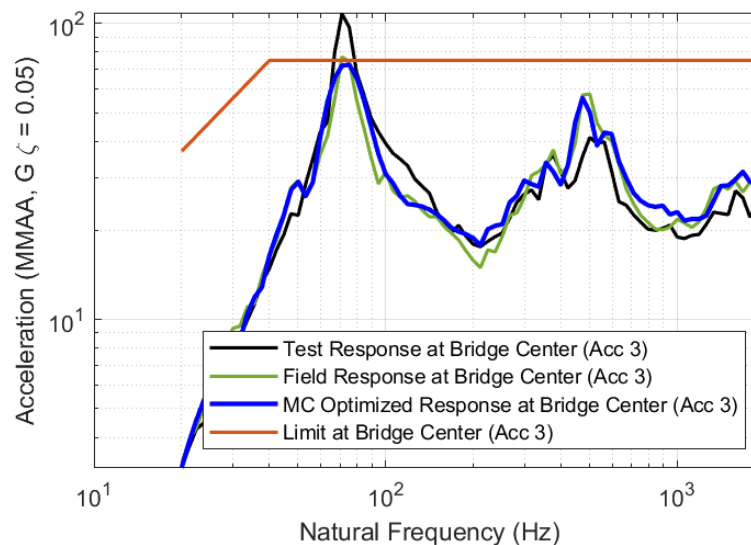
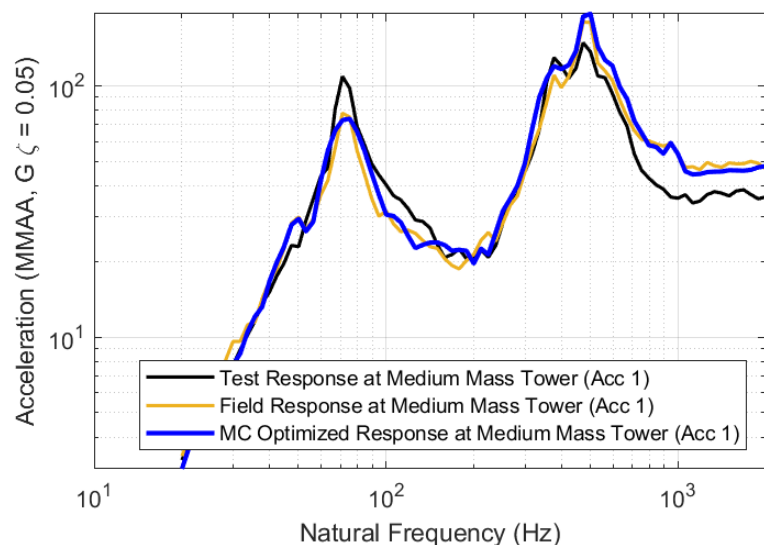
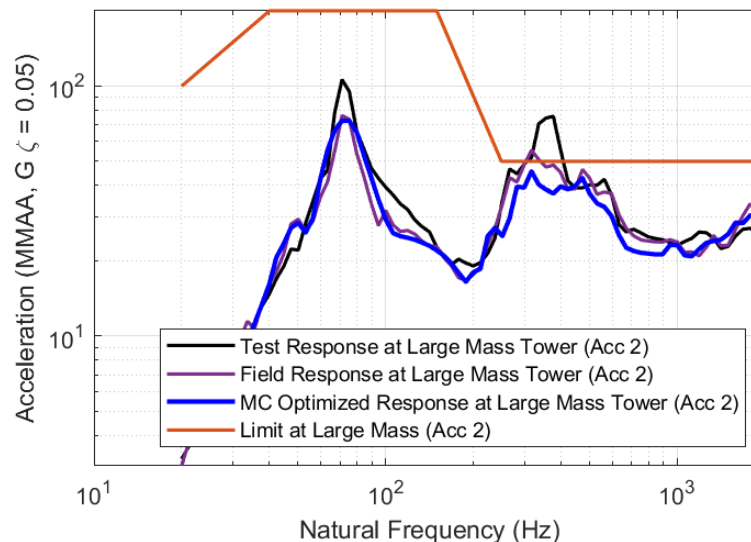
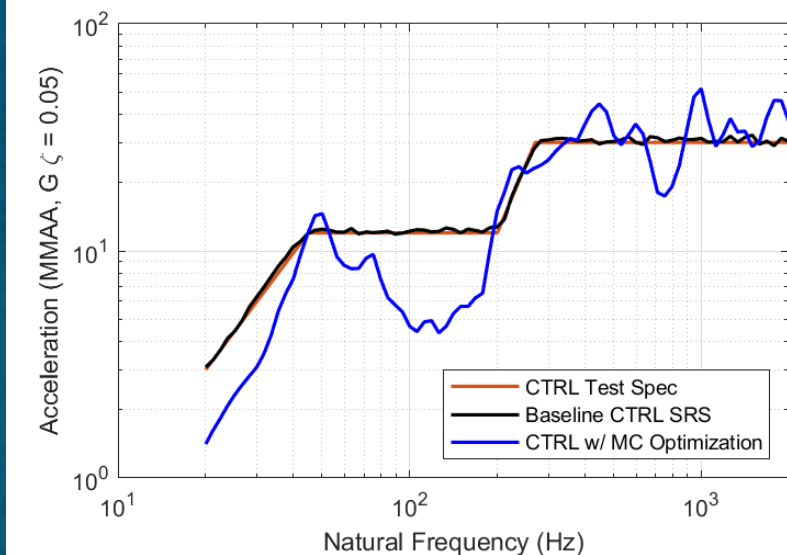
User-selected realization based on RMS dB error and peak acceleration

Peak displacement vs peak velocity scatter plot shows the “best” (user selected) realization in terms of those parameters

- Useful if shaker capability limits must be considered

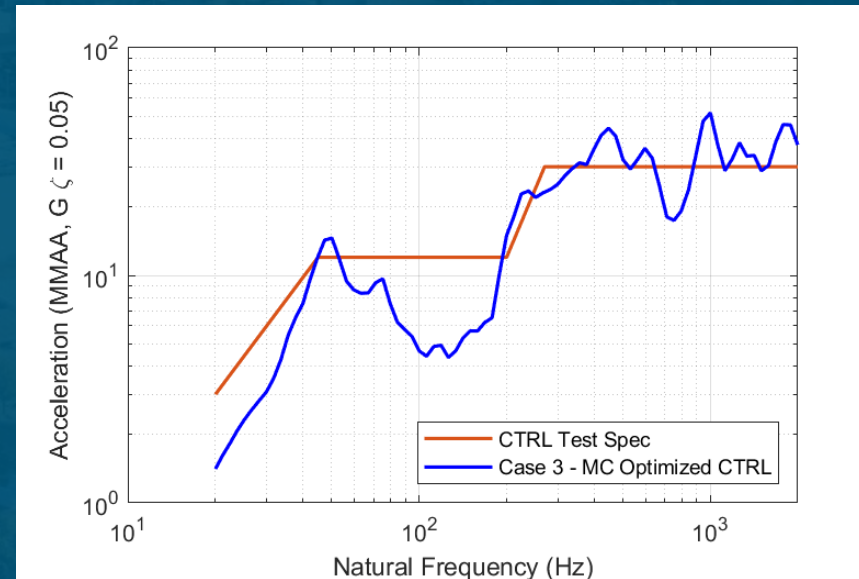
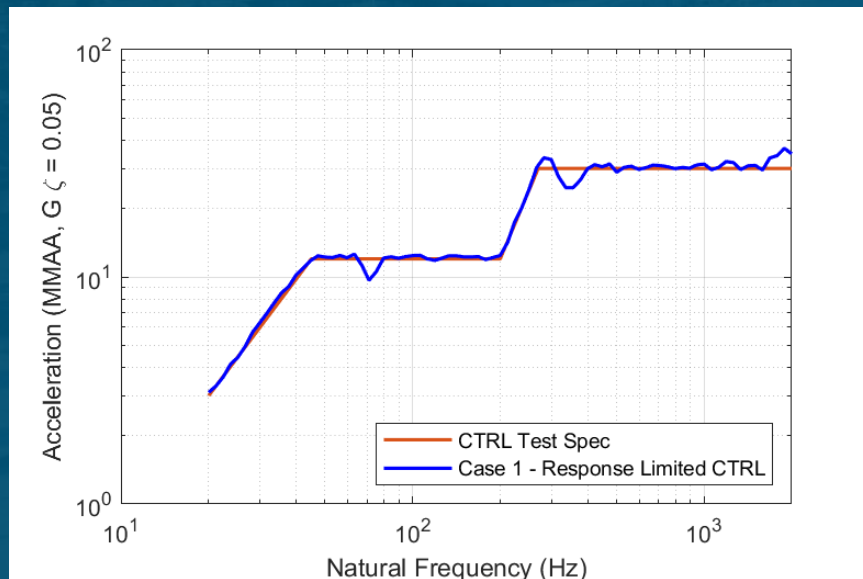
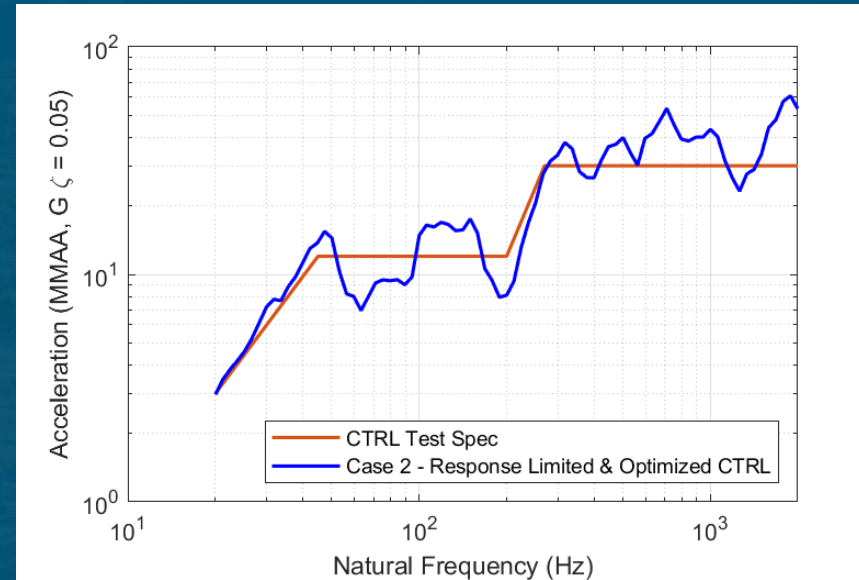
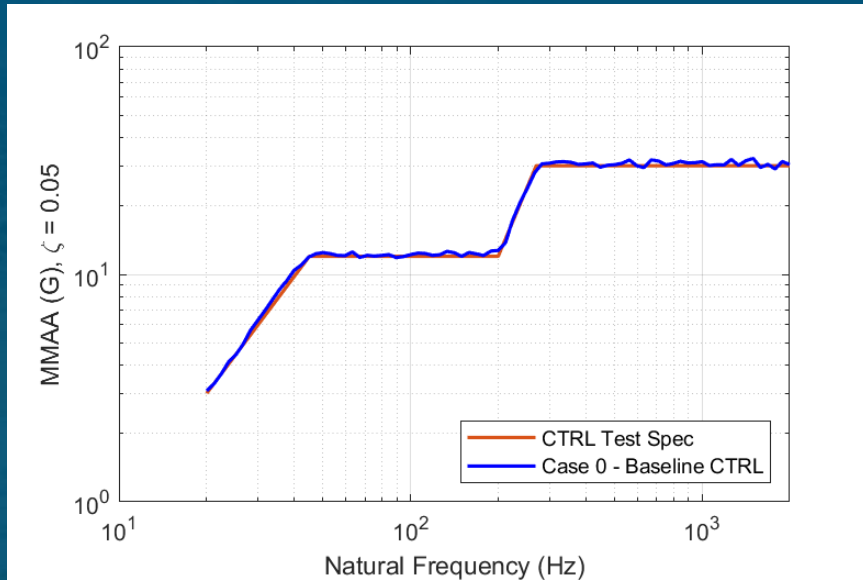


# Case 3 – Monte Carlo Response Optimization





# Comparison of Control SRS for Cases 1 - 3



Baseline SRS matches the test spec

More optimization causes more warping of the CTRL input

# Comparison to the Field Environment



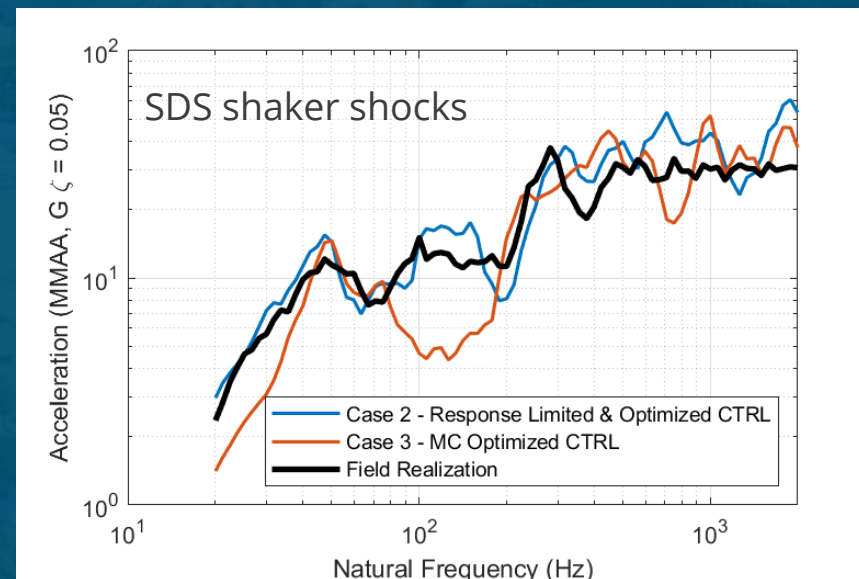
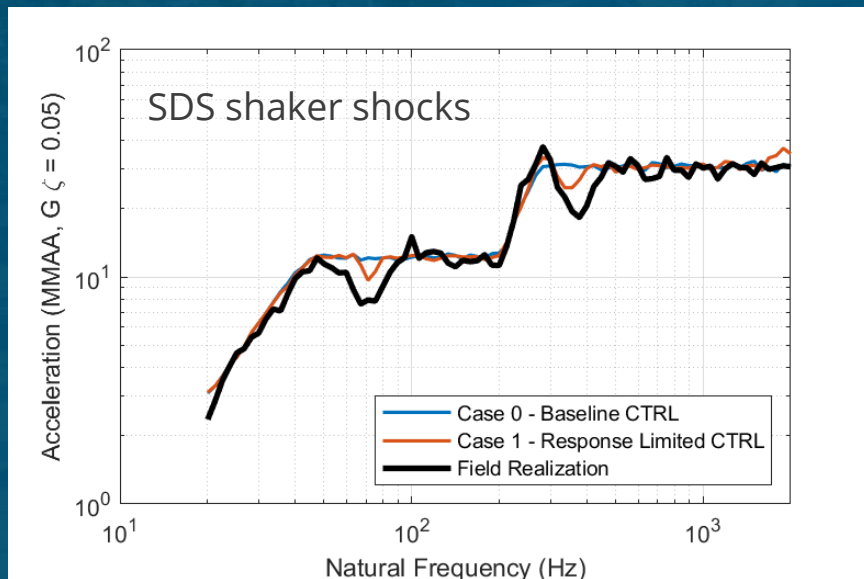
The field input realization was made from a notched straight-line SRS

- Test spec was the straight-line SRS without notches

Response-limited CTRL approximated the field environment reasonably well

Optimization warped the input SRS

- May be an issue at unmeasured response locations







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## Response Optimization & Response Limiting

- Theory
- Algorithms

## Results

- Test Article
- Pre-Test Analyses
- **Shaker Shock Test**

## Conclusions



The predicted inputs were implemented on the shaker

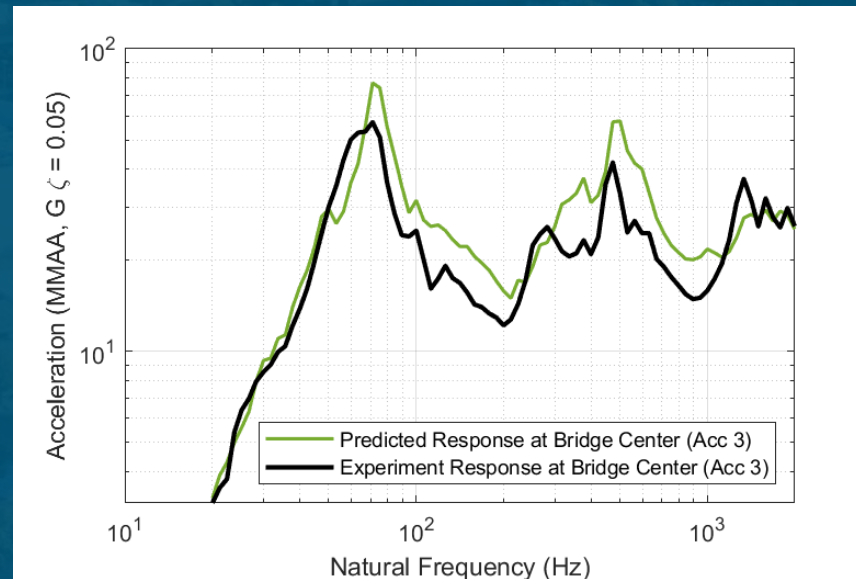
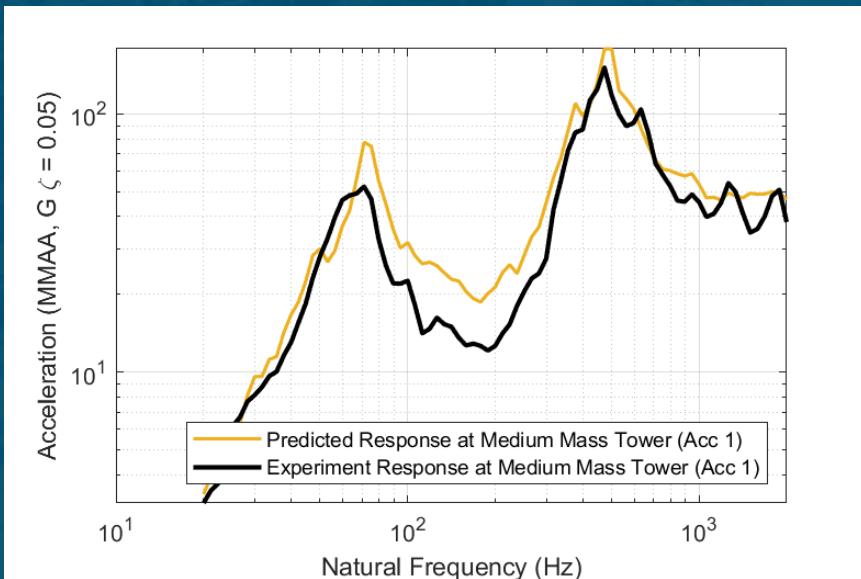
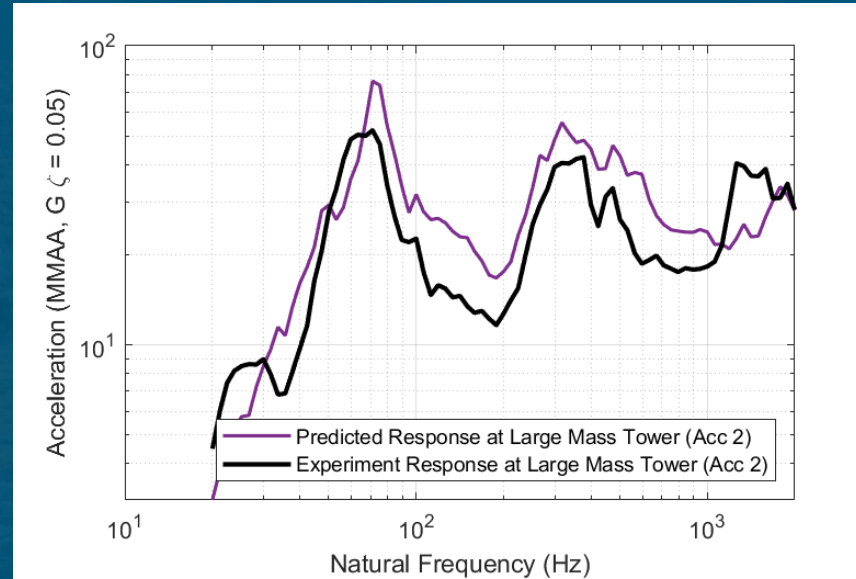
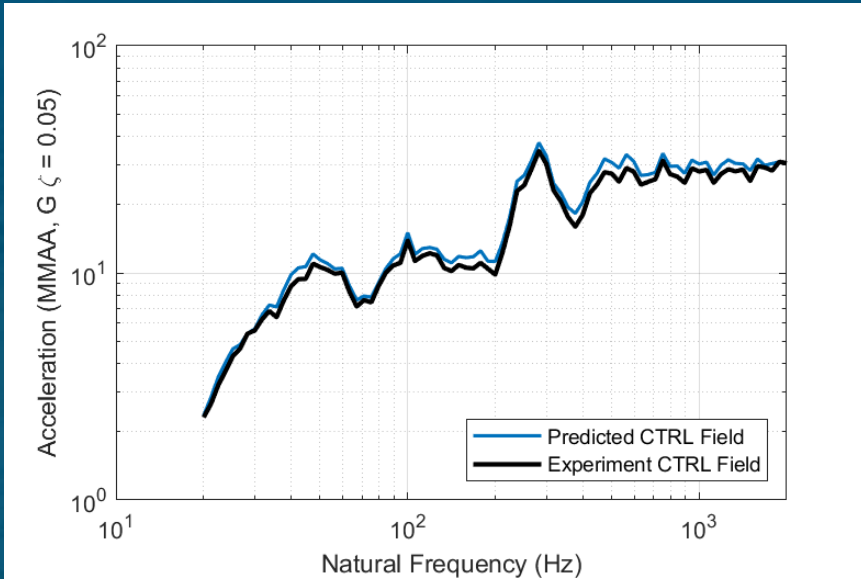
The inputs were replicated accurately

- The field SRS was replicated well
- The predicted responses to the field input were not as good
  - The difference may be an artifact of the field input being a transient random environment

The responses were not as accurate as expected

- The predicted responses to the field input were not as good
  - The error may be an artifact of the field input being a transient random environment
- More so for the field test, which may be an artifact of this being a transient random environment
- Perhaps the BARBECUE is not totally linear



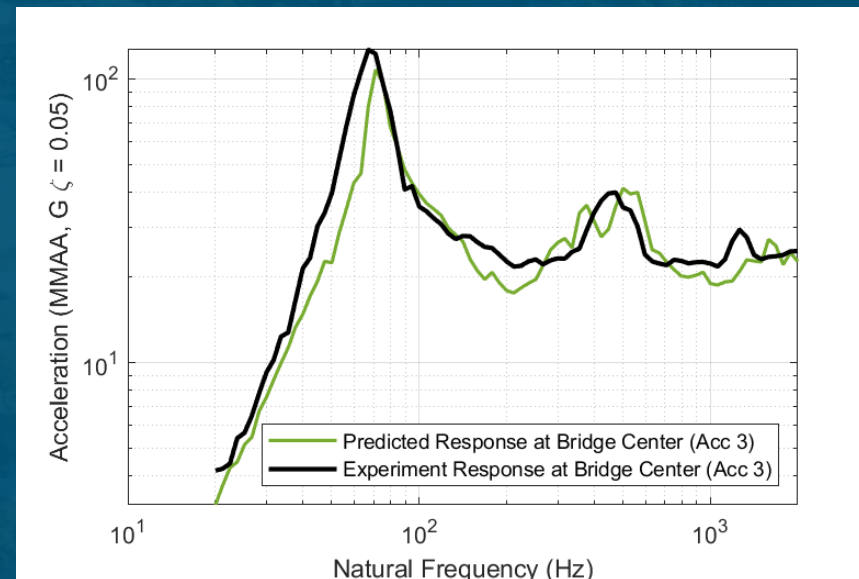
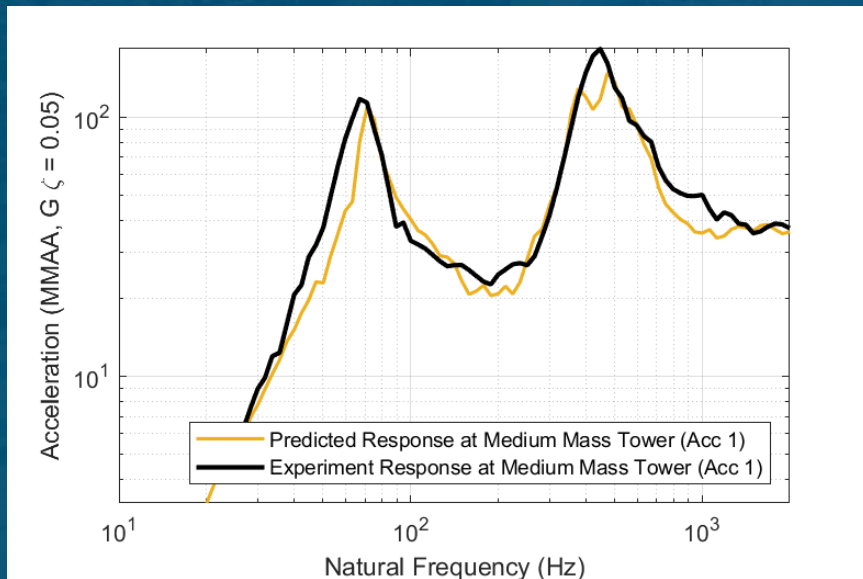
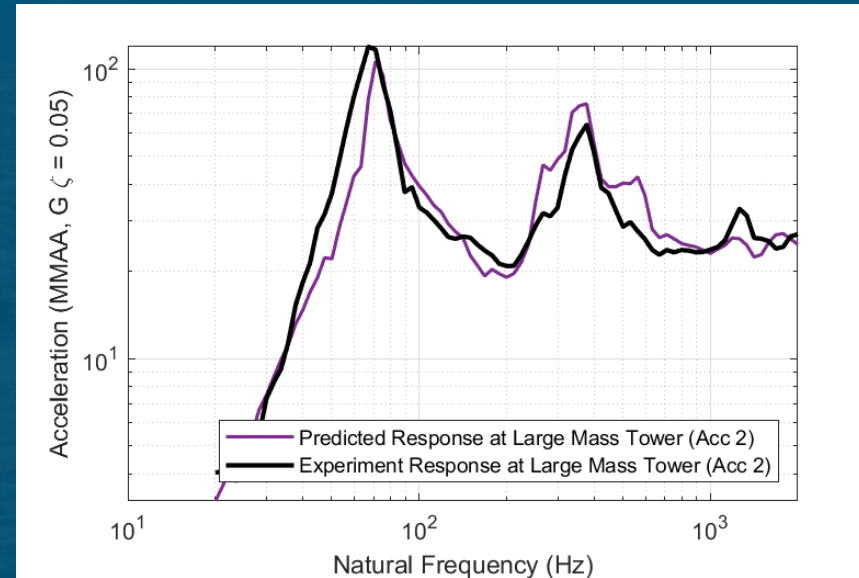
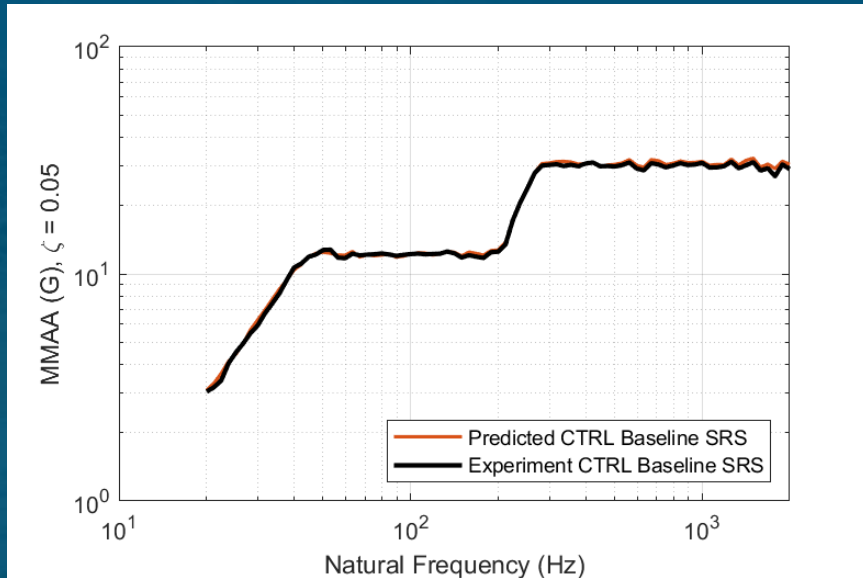


Experiment field input SRS matches the predicted SRS

Experiment response location SRS are not as good

- SRS at response locations is sensitive to the random realization of the field input

# Experimental Validation – Case 0 Baseline SDS



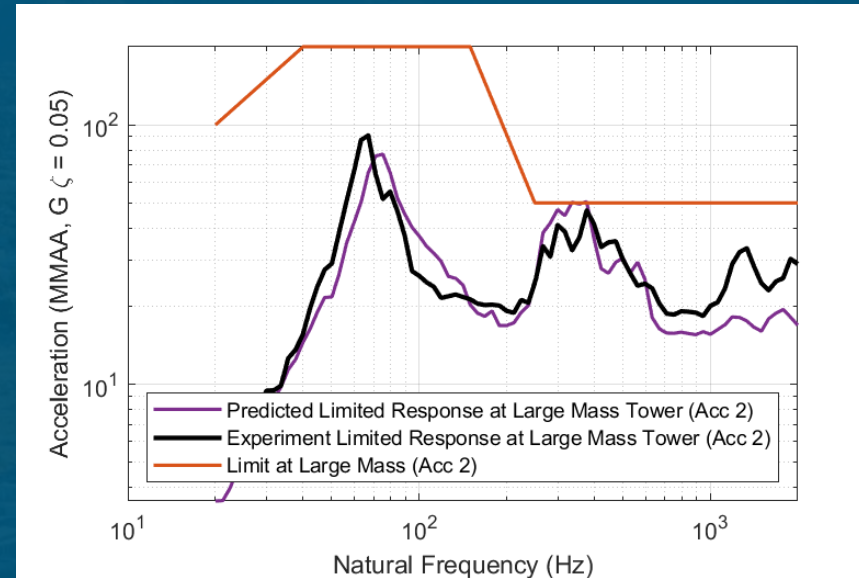
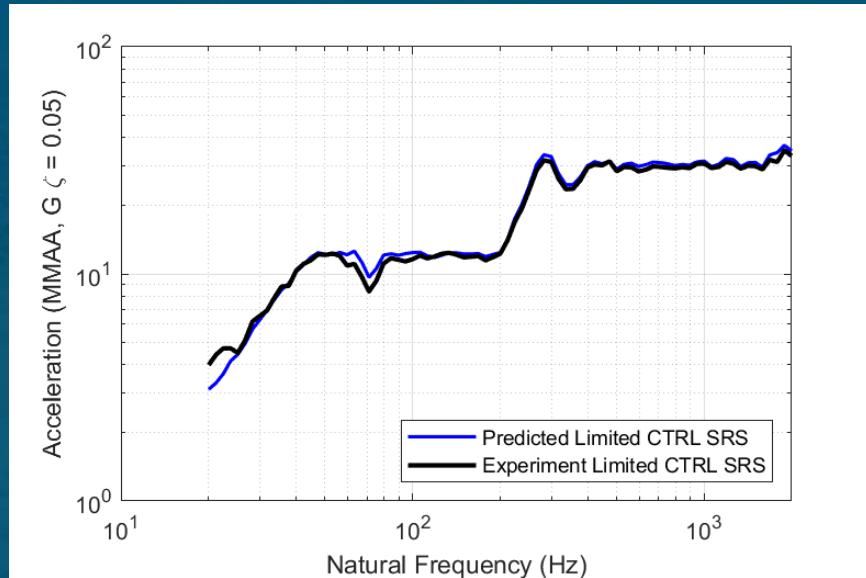
Experiment test spec SRS matches predicted test spec very well

- SDS acceleration is a deterministic waveform

Differences at response locations may be due to BARBECUE non-linearities

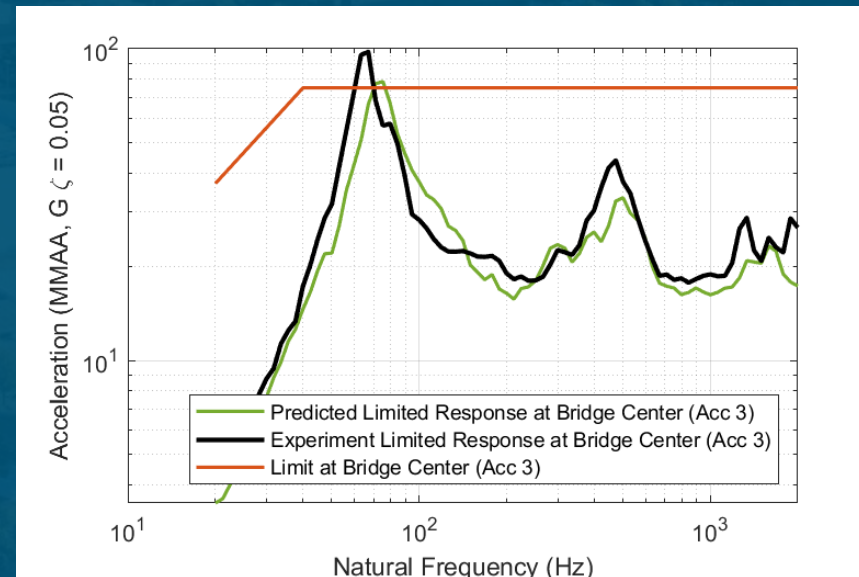
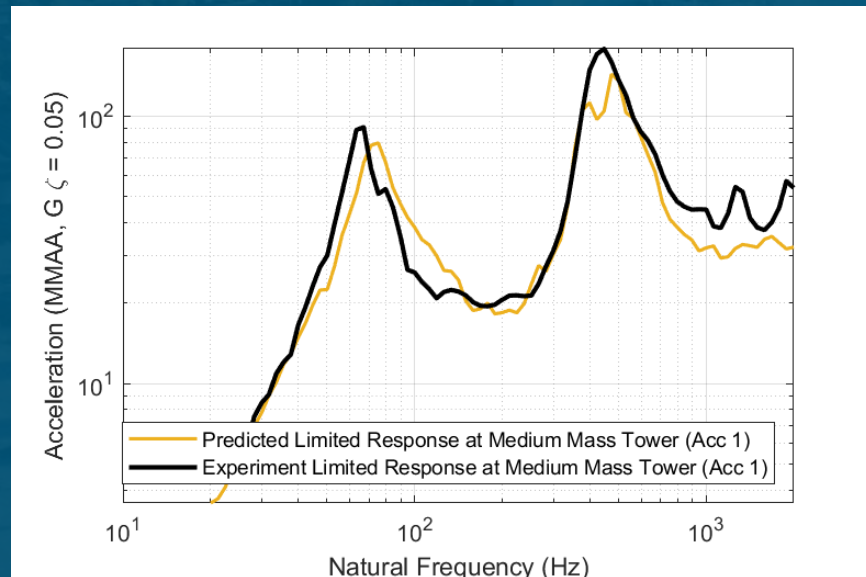


# Experimental Validation – Case 1 Response Limiting



Experiment response limited CTRL SRS matches predicted SRS very well

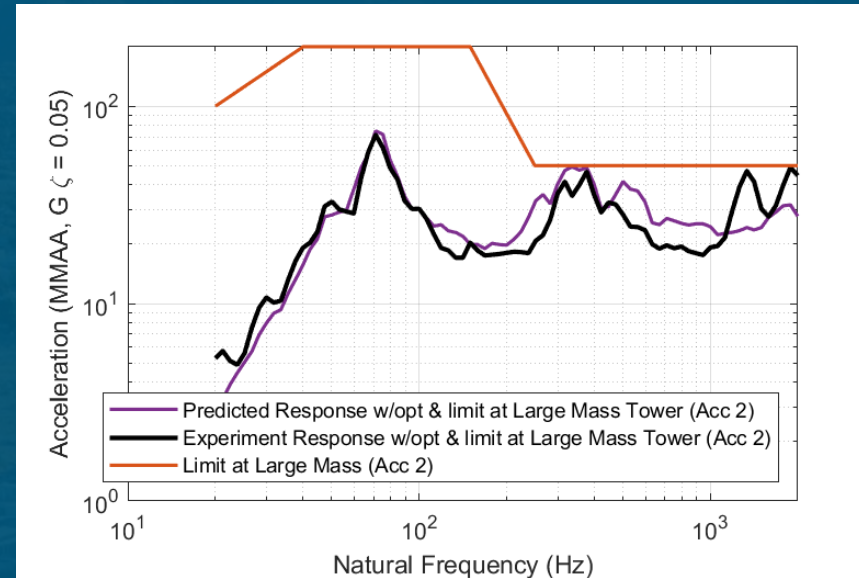
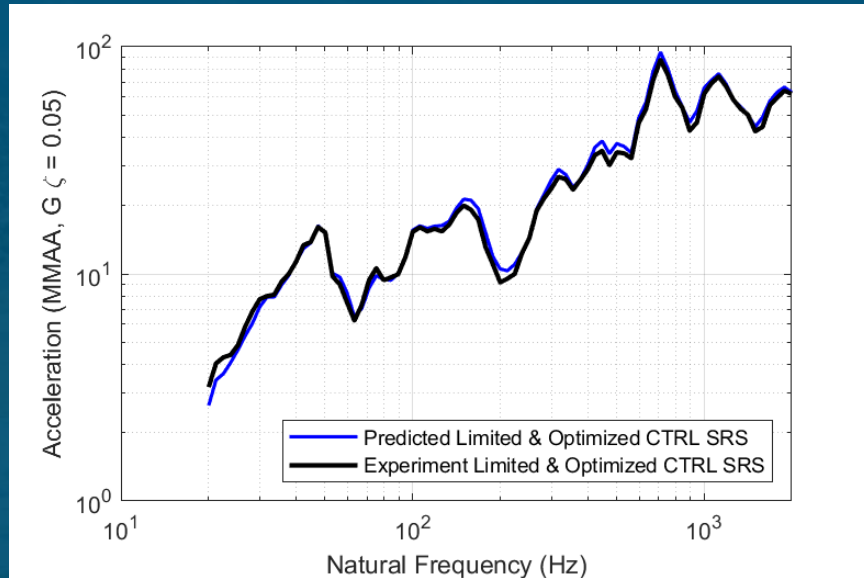
- SDS acceleration is a deterministic waveform



Response limit at location 3 (Bridge) violated in the experiment

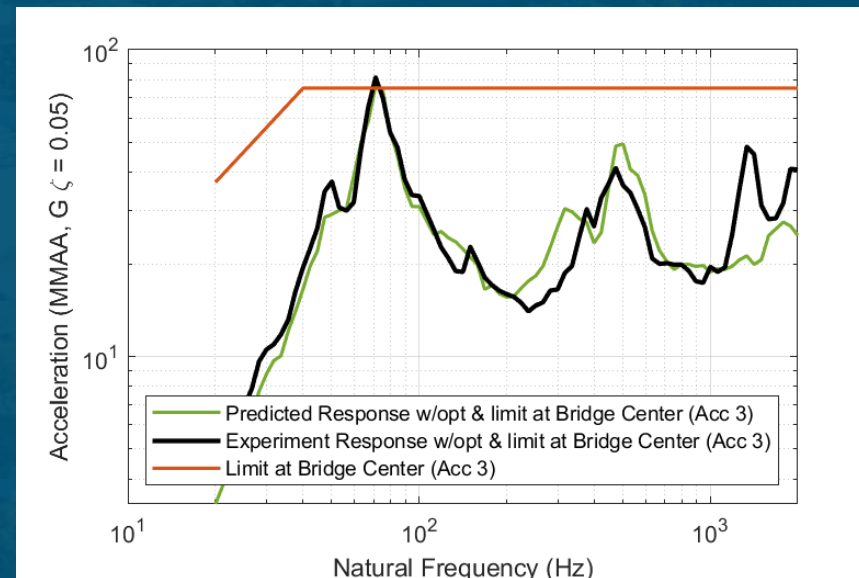
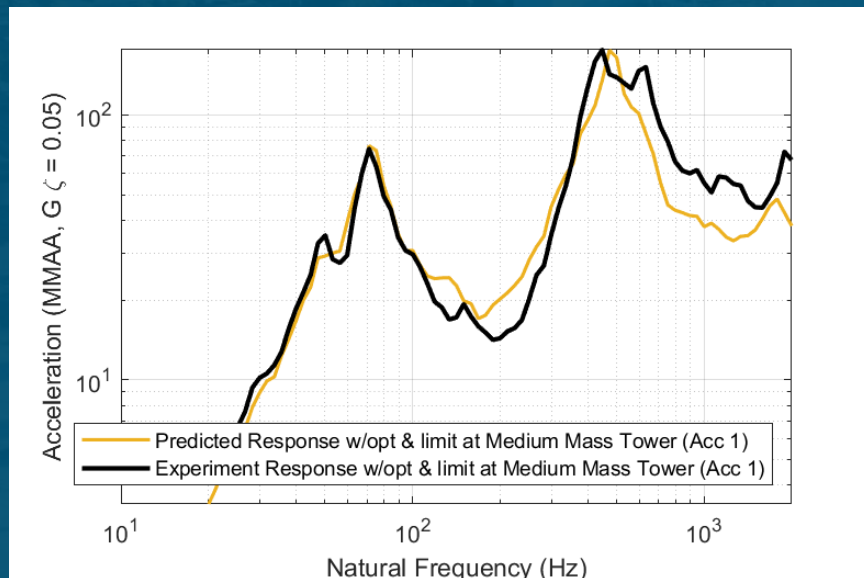
- Open-loop prediction based process

# Experimental Validation – Case 2 Response Optimization & Limiting



Experiment CTRL SRS matches predicted SRS very well at

Responses limit are satisfied at both locations 2 & 3







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# Summary & Conclusions



Response optimization and limiting are based on our baseline Sums of Decayed sines algorithm for generating an acceleration realization from a SRS

Unlike random vibration, shaker shock response limiting & optimization is pre-planned

- Requires transmissibilities from the control to the response locations

Response limiting did properly restrict the resonant responses

Response limiting by itself did not match the broadband field SRS at all 3 response locations as well as the other methods

- Least distortion of the control SRS

The Monte Carlo response optimization matched the field SRS at response locations 1 – 3 the best

We hope to extend these algorithms to MIMO environments

- 6-DOF shaker
- NEWT Method





- [1] D. O. Smallwood; "Methods used to Match Shock Response Spectra using Oscillatory Transients, Proceedings of the Institute of Environmental Sciences, 1974, pp409-420.
- [2] D. O. Smallwood and A.R. Nord, "Matching Shock Spectra with Sums of Decaying Sinusoids Compensated for Shaker Velocity and Displacement Limitations," 44<sup>th</sup> Shock and Vibration Symposium, Paper U-20, Houston, TX, Dec. 1973
- [3] C. Heitman, V. Babuška, J. Cap, and J. Reid, "Development of a Single Input Multiple Output (SIMO) Input Derivation Algorithm for Oscillatory Decaying Shocks; Proceedings of the 86th Shock and Vibration Symposium; October 5-8, 2015, Orlando, FL.
- [4] C. Heitman, J. Cap, and D. Murphy, "Monte Carlo Optimization of a Single Input Multiple Output (SIMO) Input Derivation for an Oscillatory Decaying Shock", Proceedings of the 87th Shock and Vibration Symposium, 2016, New Orleans, LA.
- [5] J. Cap, "The Application of Force Limiting Techniques to the Derivation of an Equivalent Static Load and to Transient Vibration Testing"; **oral presentation at the 1995 Spacecraft and Launch Vehicle Structural Dynamic Environments Technical Interchange Meeting; June 6-8, 1995, El Segundo CA.**
- [6] J. Cap, T. Hunt, C. Heitman, and M. Raymer, "Monte Carlo Optimization of a Hybrid Spectral / Temporal Single Input Multiple Output (SIMO) Input Derivation for an Oscillatory Decaying Shock"; Proceedings of the 88th Shock and Vibration Symposium; October 16-19, 2017, Jacksonville, FL.



# Acknowledgements



The following people provided input to make this briefing possible:

- Angela Patterson – ANSYS modeling
- Dan Rohe – Test engineer
- David Smallwood – Algorithms



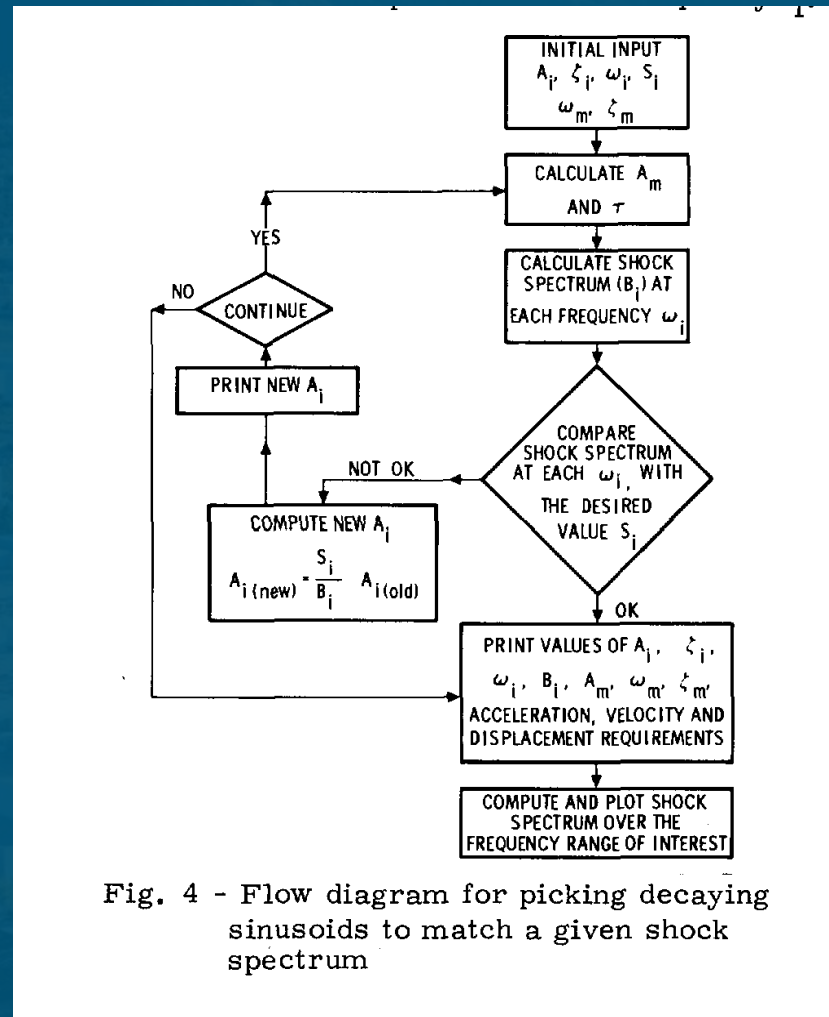


Fig. 4 - Flow diagram for picking decaying sinusoids to match a given shock spectrum

# Thank You Questions?



# Response Optimized Shaker Shocks (SIMO Optimization)



In addition to optimizing a set of desired SRS, one can optimize on other scalar parameters of interest

- Synthesize multiple “inputs”
- Obtain TRFs between input acceleration and parameter of interest
- Use convolution to synthesize waveforms
- Compile scalar parameter
- Choose optimal value

**The method has been successfully used to minimize amplifier voltage and current**  
**[6]**

# Comparison of gwinrand

