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Selecting Bases for Reduced Order Spectral Representations of Transient Temperature Profiles

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Outline

- Motivation/need for reduced order methods
- Taxonomy of reduced order methods
  - ROC
  - ROA
  - ROM
  - Use of spectral representations in reduced order methods
- Types of bases considered
  - Analytical eigenfunctions
  - Empirical eigenfunctions (POD)
- Demonstration problem
  - Problem overview
  - Selection/truncation criteria
- Conclusions
- Future Work

Motivation/Need for Reduced Order Methods

- Numerical and experimental studies produce lots of data
- These data
  - Require lots of storage space
  - Can be difficult to analyze and interpret
  - Are computationally expensive to produce iteratively
- Reduced order methods can help with these problems

Taxonomy

- Reduced order compression (ROC)
  - Compression of datasets for storage and quick retrieval
- Reduced order analysis (ROA)
  - Extraction of significant features from data
- Reduced order modeling (ROM)
  - Utilizing patterns in data to accelerate future modeling

Spectral Representations

- Most reduced order methods are built off spectral representations
- Spectral representations are composed of:
  - A basis of eigenfunctions
  - Expansion coefficients
- The same idea as a truncated, generalized Fourier Series:

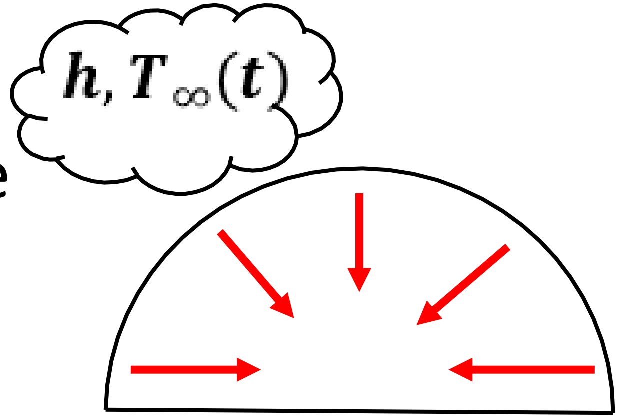
$$T(\vec{r},t) \approx \sum_{j=1}^{N} a_j(t) y_j(\vec{r}) \qquad \mathbf{T} \approx \Phi A$$

Types of Bases Considered

- Empirical eigenfunctions
  - Decomposition of data
  - Principal orthogonal decomposition (POD)
- Analytical eigenfunctions
  - Analysis or decomposition of differential operator
  - Sturm-Liouville Theorem

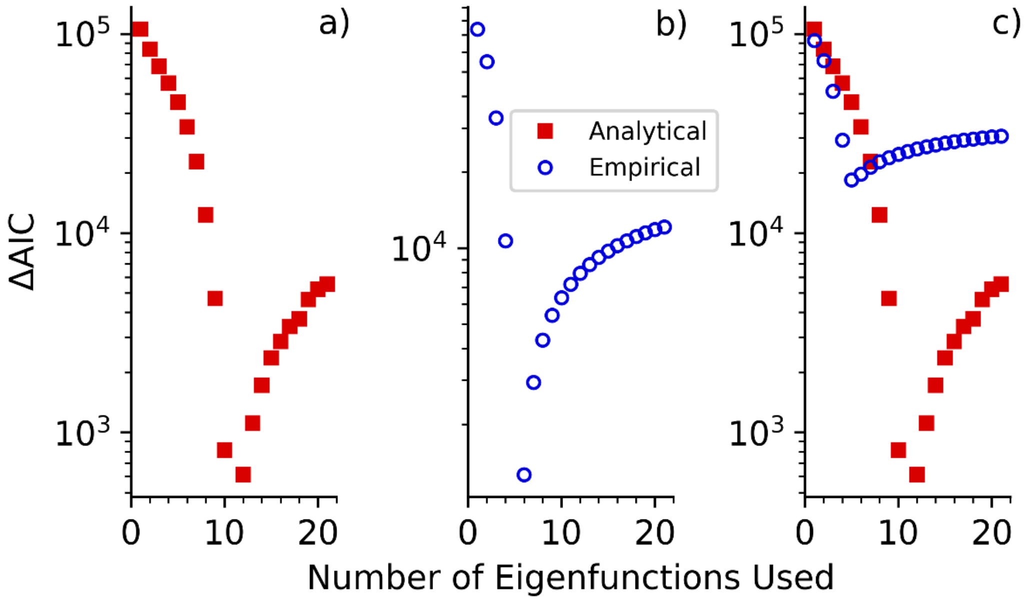
Problem Overview

- Hemisphere heated by convection with time varying ambient temperature
- Described by 1-D heat diffusion equation in spherical coordinates
- Exact analytical solution derived to generate data
- Analytical eigenfunctions derived using Sturm-Liouville
- Empirical eigenfunctions derived from data using POD
- Three tasks considered
  - Compression of the noiseless data
  - Analysis of the data in the presence of noise
  - Use of the bases for different thermal diffusivities



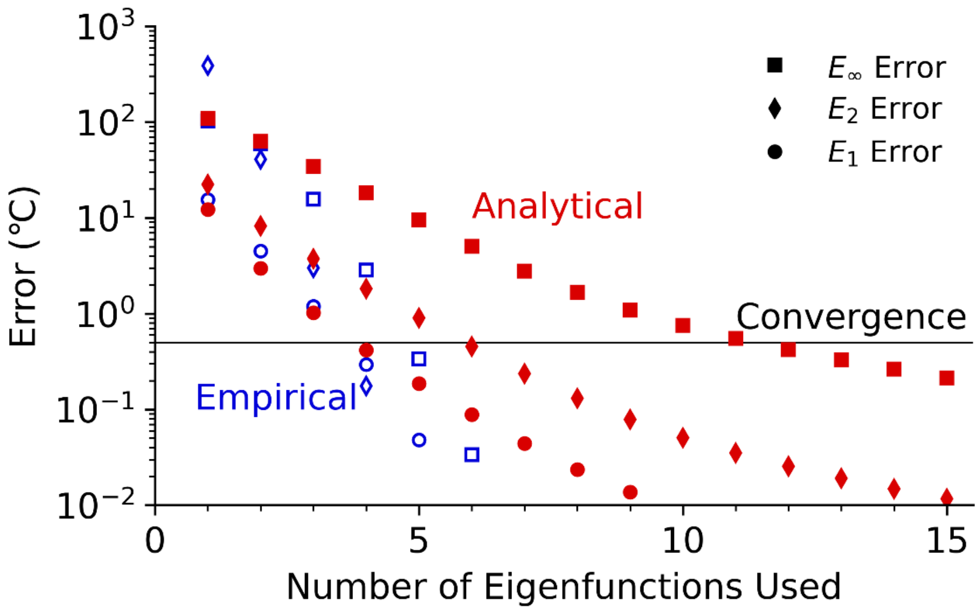
Selection/Truncation Criteria

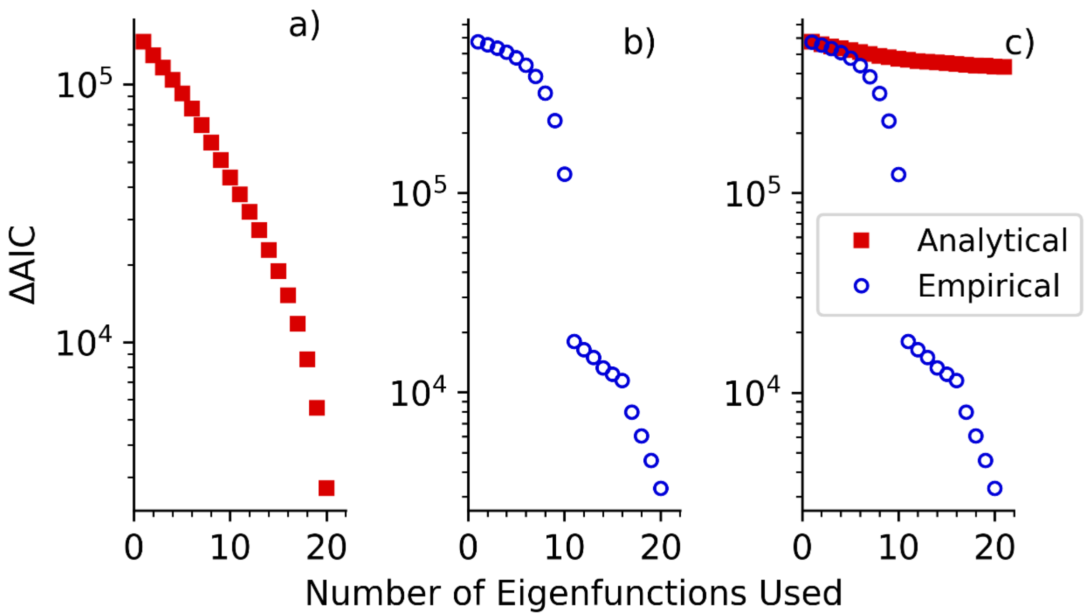
- Three standard error measures used
  - E_1 - average error
  - E_2 - root mean square error
  - E_∞ - max error
  - Convergence defined as an error $< 0.5°C$
- Akaike Information Criterion (AIC)
  - Estimate of information lost when a model is used
  - Accounts for error and dimensionality
  - Convergence defined when the AIC reaches a minimum



Compression

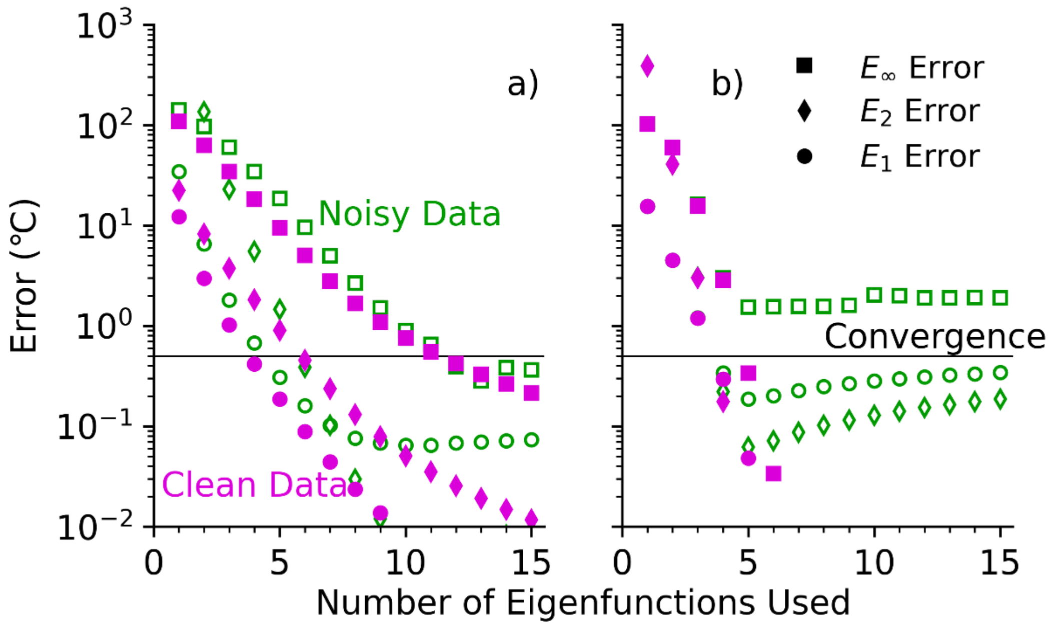
- Empirical eigenfunctions derived from noiseless data
- Noiseless data projected onto both bases

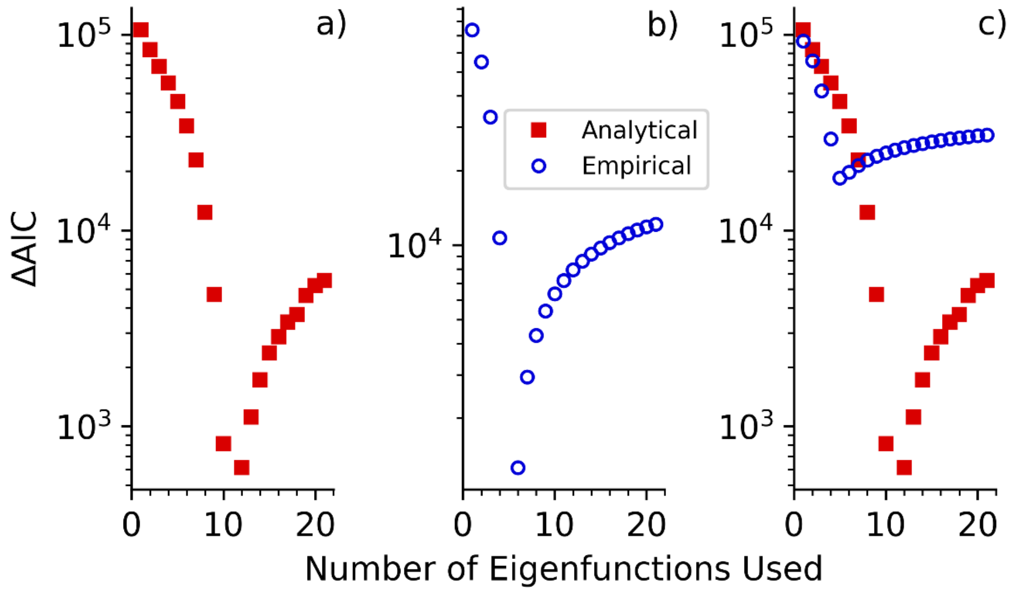




Analysis in the Presence of Noise

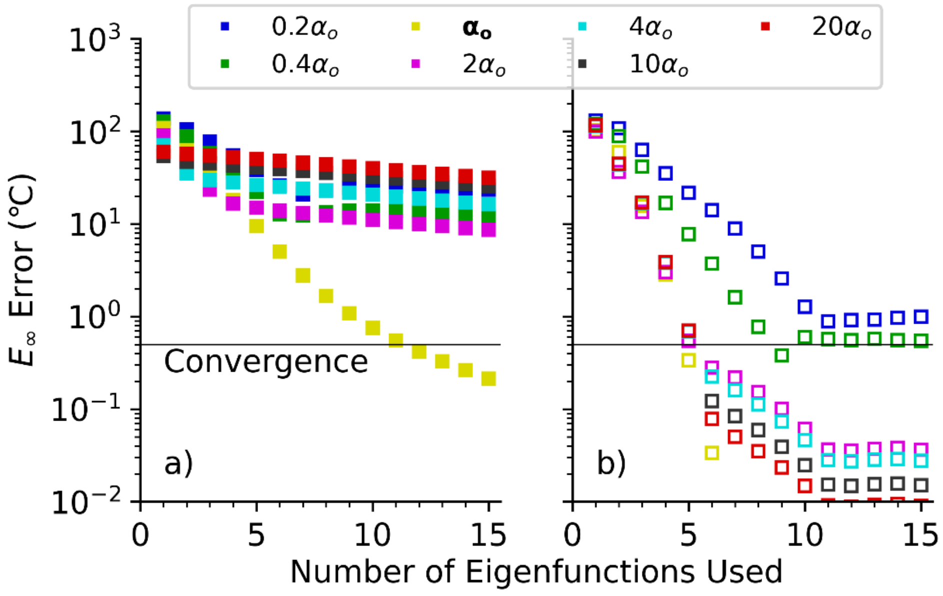
- Empirical eigenfunctions derived from noisy data
- Noisy data projected onto both bases
- Errors, AIC calculated relative to noiseless data





Varying Diffusivity

- Empirical eigenfunctions derived from data with nominal diffusivity
- Analytical eigenfunctions from nominal problem used
- Data derived with different diffusivities projected onto bases



Conclusions

- Analytical and empirical eigenfunctions are both viable options for representing noiseless data
- Analytical eigenfunctions are better at extracting the dynamics from noisy data
- Empirical eigenfunctions can be used as effective bases for representations for a wide range of thermal diffusivities
- The AIC can be used as an effective comparison and optimal truncation criterion, and can assist in the task of selecting a basis for reduced order spectral representations

Future Work

- Explore use of empirical eigenfunctions while varying other thermophysical properties or boundary conditions
- Use empirical eigenfunctions to perform ROM for UQ