SAND2023-06116C

Classifying topological nonlinear modes using local markers

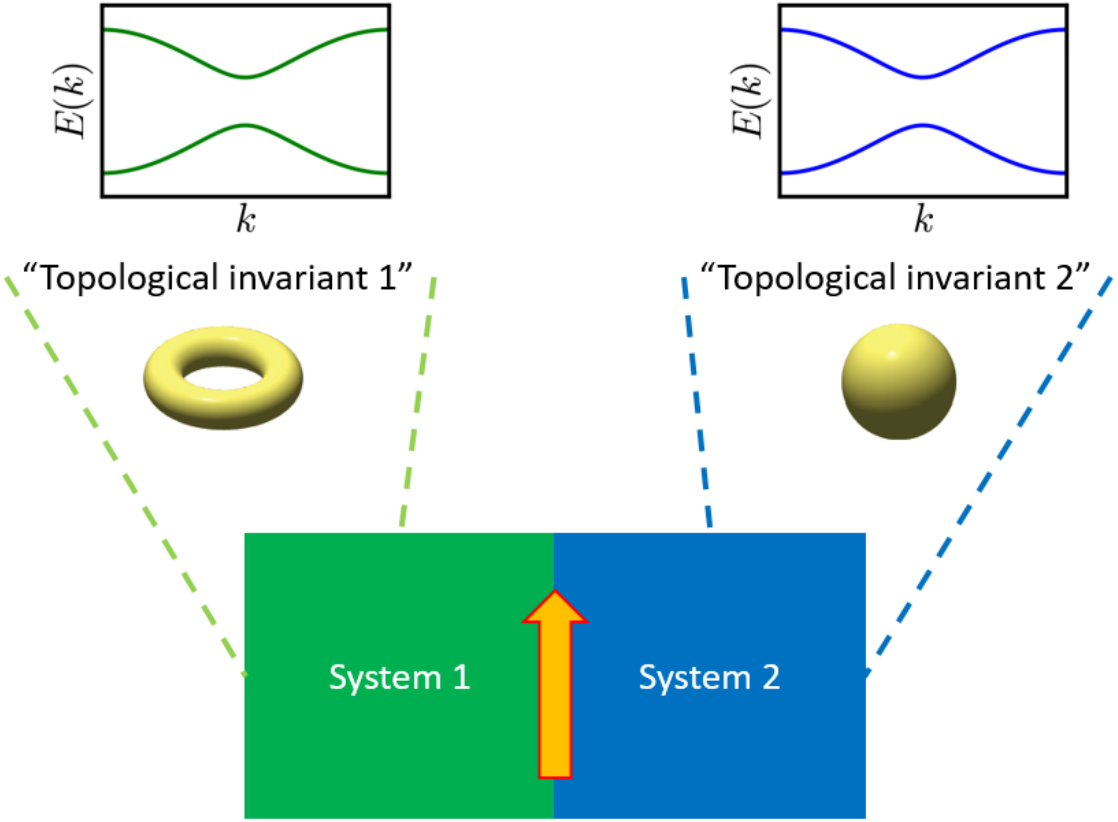
Stephan Wong, Terry A. Loring, Alexander Cerjan

META 2023

19th of July 2023

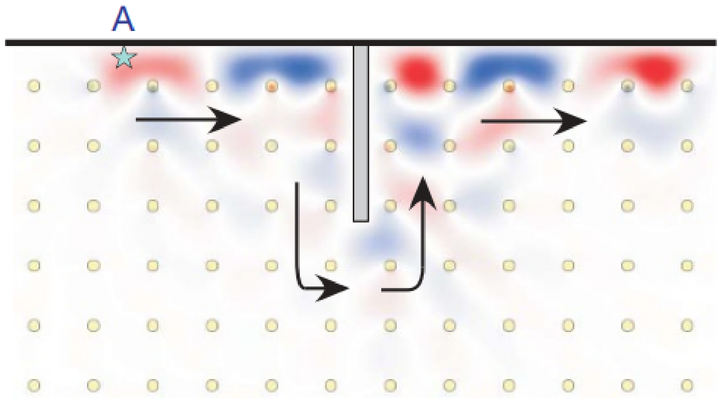
✓ Email: stewong@@sandia.gov

Topological insulators



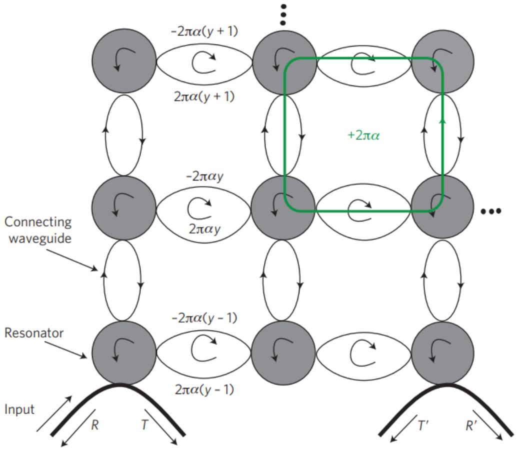
➔ **Topological edge state robust against disorders**

Gyromagnetic rods



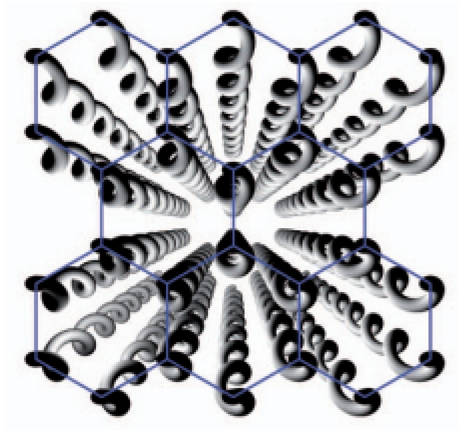
Z. Wang, M. Soljacic, Nature, 2009

Ring resonator array



M. Hafezi, J. Taylor et al., Nature, 2011

Helical waveguide array

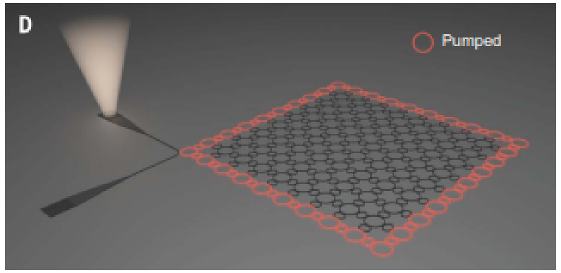


M. C. Rechtsman, A. Szameit et al., Nature, 2013

Nonlinear topological insulators

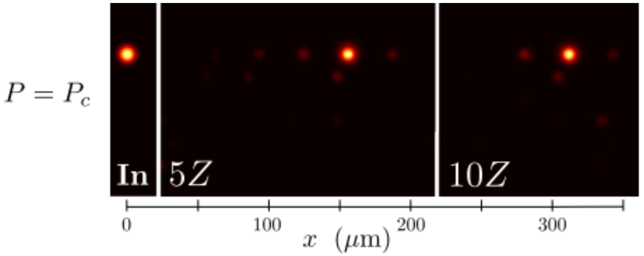
→ Natural in photonics

Lasers

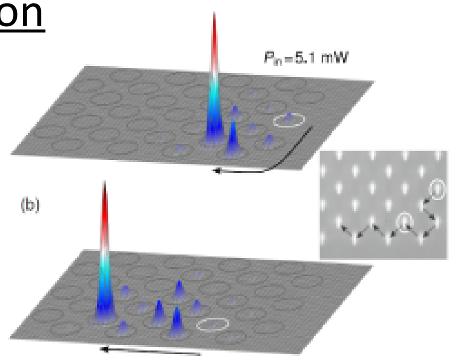


M. A. Bandres, M. Khajavikhan et al., Science, 2018

Topological edge soliton

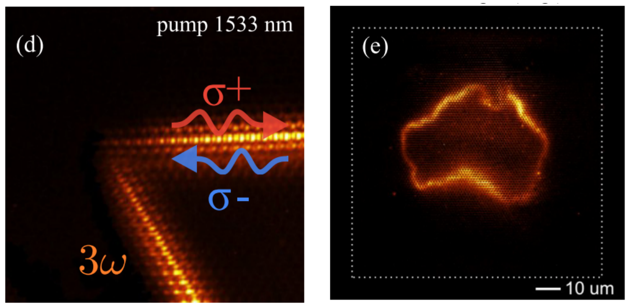


D. Leykam, Y. D. Chong, Phys. Rev. L, 2016



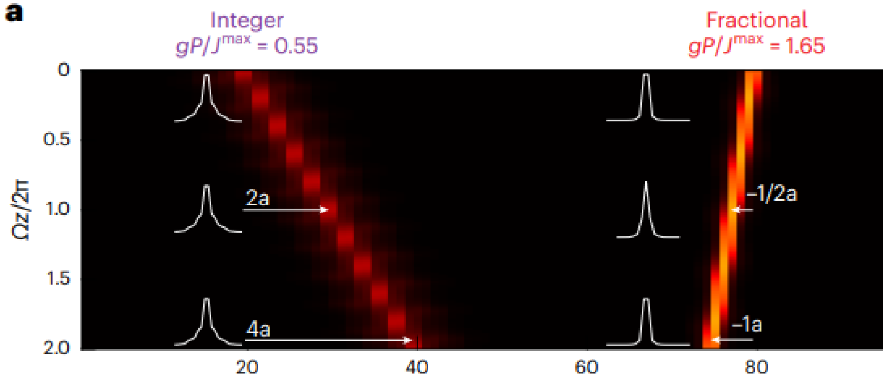
S. Mukherjee, M. Rechtsman et al., Phys. Rev. X, 2021

Frequency conversion



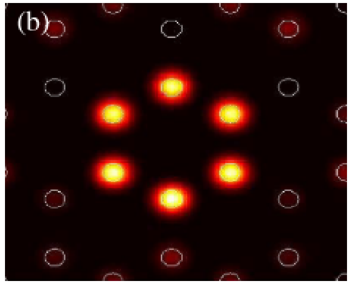
D. Smirnova, Y. Kivshar et al., Phys. Rev. L, 2019

Quantized fractional Thouless pumping



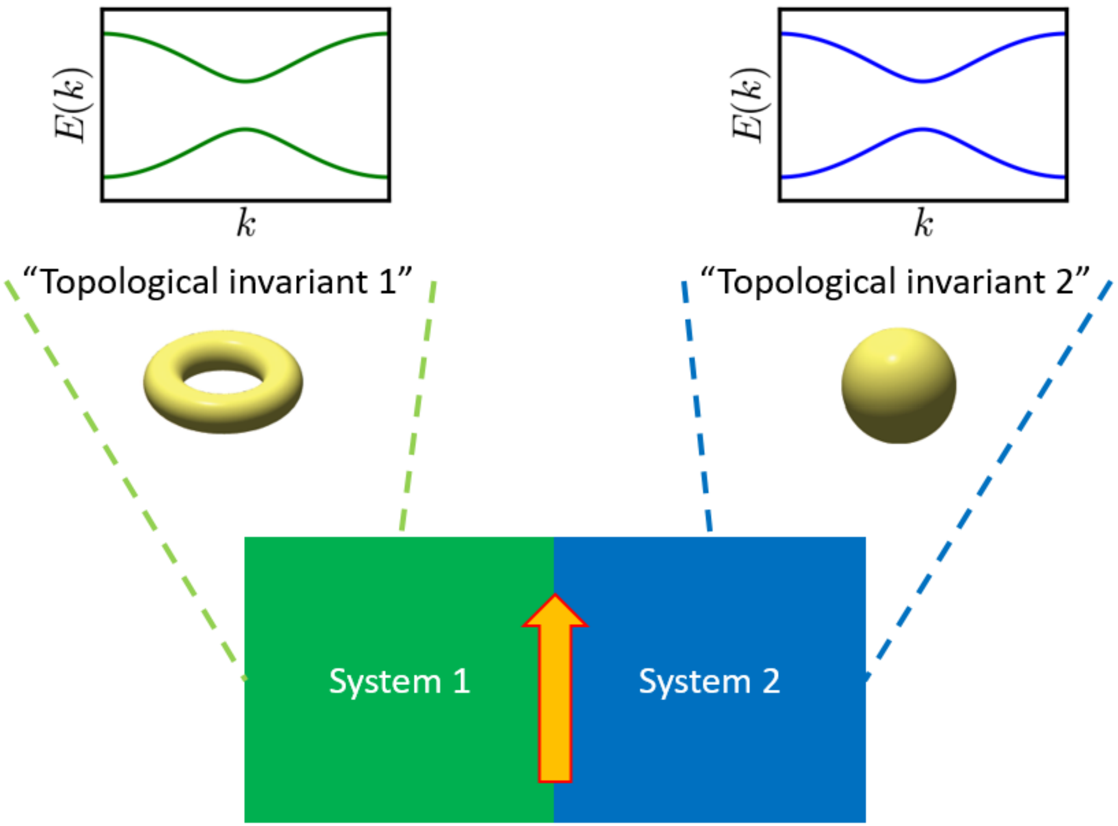
M. Jurgensen, M. Rechtsman et al., Nature Physics, 2022

Topological bulk soliton



Y. Lumer, M. Segev et al., Phys. Rev. L, 2013

Nonlinear topological insulators



➔ **Topological edge state robust against disorders**

In literature:

- Assume nonlinear effect as linear potential
- Assume periodicity of system

➔ significant approximation

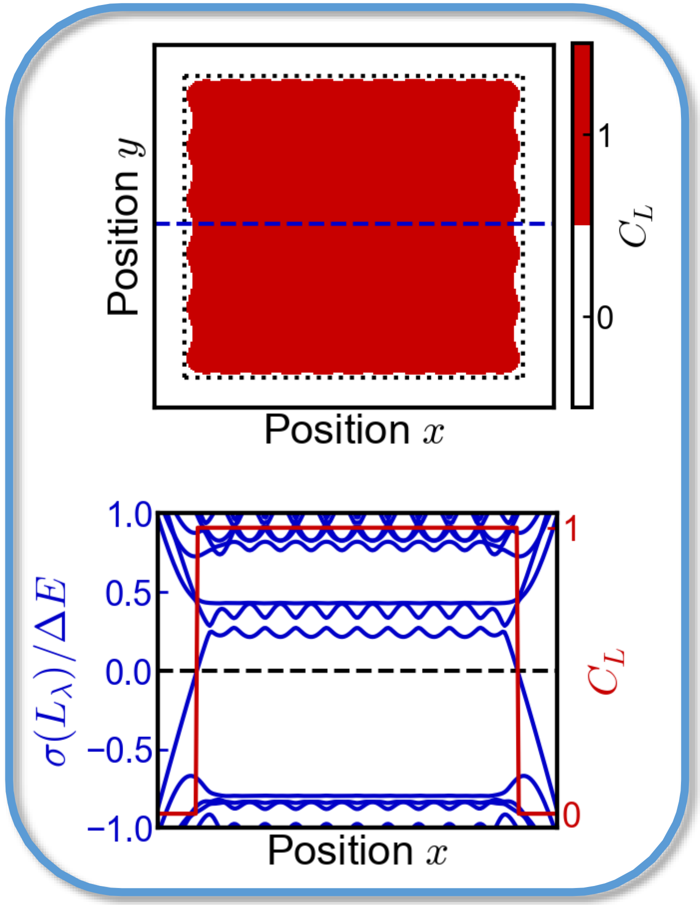
Nonlinearity are <u>local</u>

➔ **Nonlinear topological insulator fall outside scope of topological <u>band theory</u>**

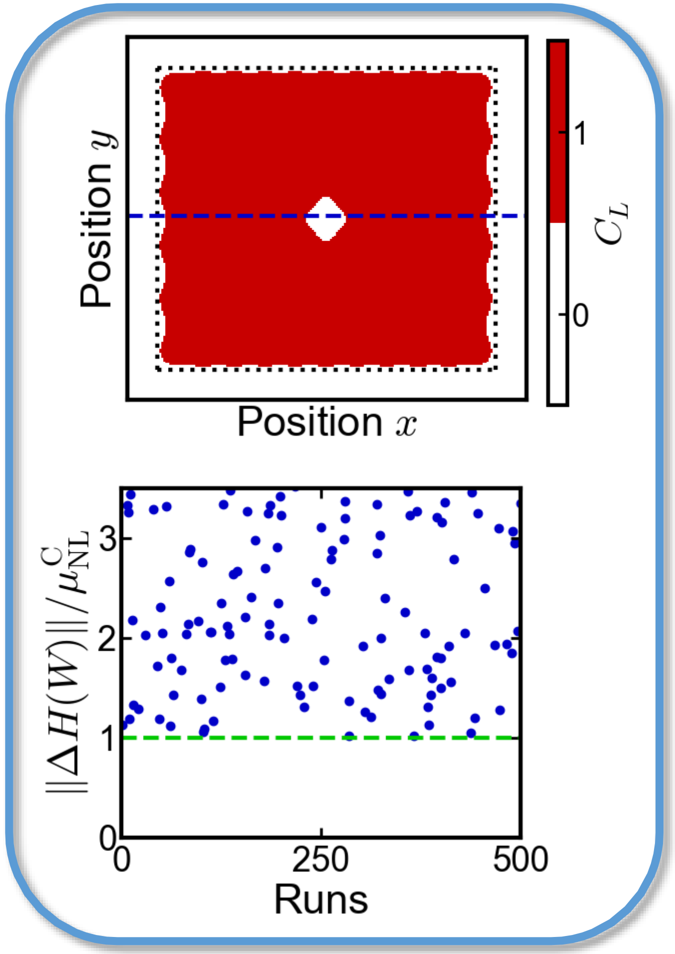
➔ **Need local approach to topology**

Outline

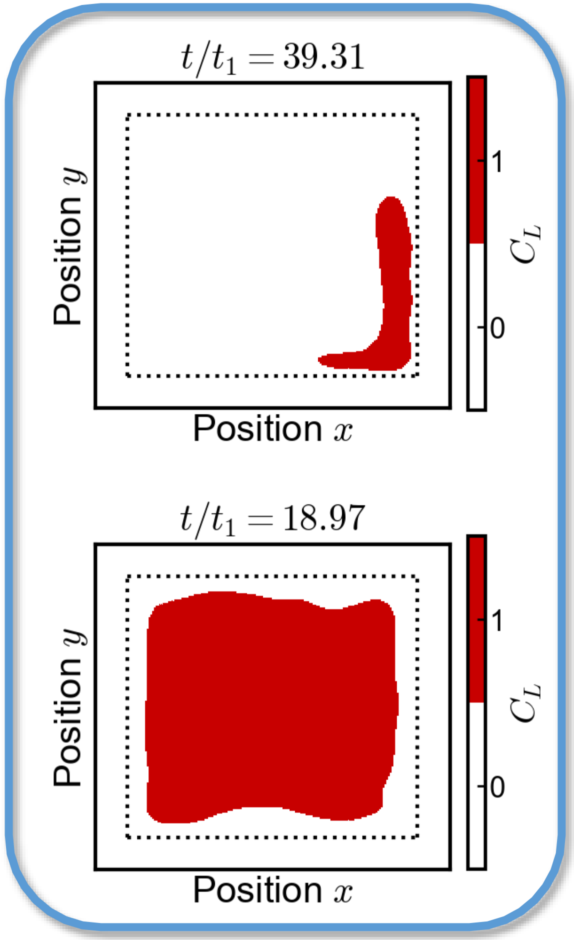
1. Local approach to topology



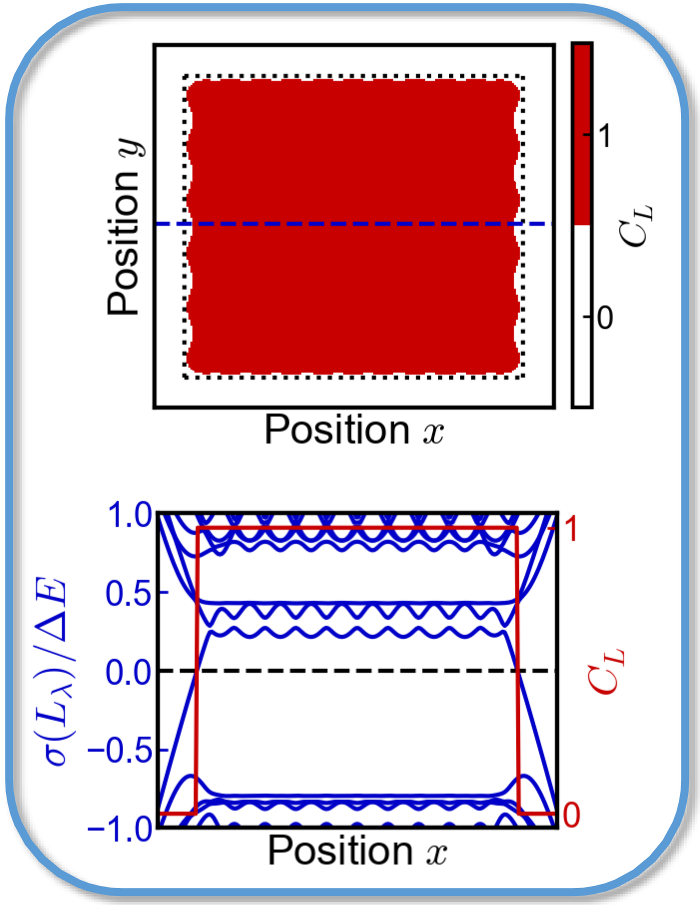
2. Topological nonlinear mode



3. Topological dynamics



1. Local approach to topology

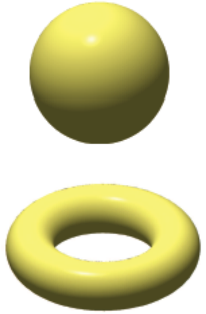


2. Topological nonlinear mode

3. Topological dynamics

Local approach to topology

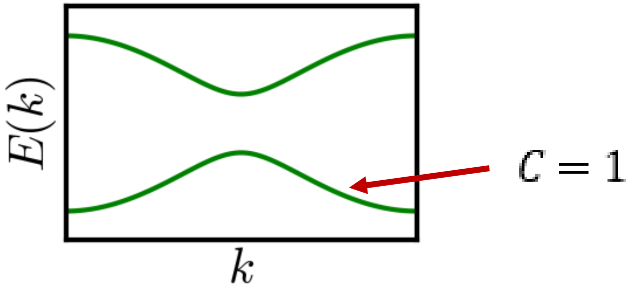
System, H



?

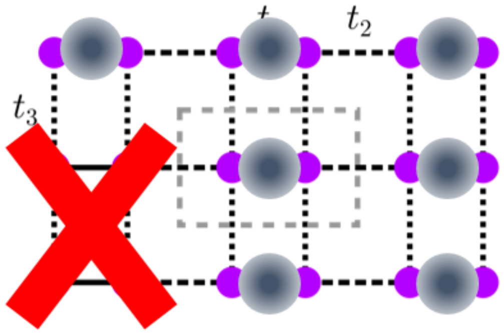
Topological non-trivial insulators
⇔ the system (Hamiltonian) cannot be continued to the **atomic limit**
$([\tilde{X},\tilde{H}]=0,[\tilde{Y},\tilde{H}]=0)$ without breaking symmetry class

Kitaev, AIP Conference Proceedings, 2009



❖ "Is the **Chern number non-trivial**" ?

M. Z. Hasan and C. L. Kane, Rev. Mod. Phys., 2010



❖ "Does the system have a **complete localized Wannier basis**" ?

B. Bradlyn, A. Bernevig et al., Nature, 2017

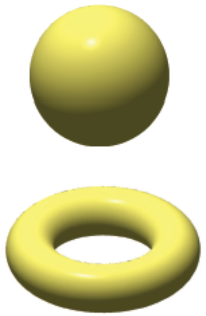
❖ "**Does the position X, Y and Hamiltonian H matrices can be modified to be commuting**" ? ← spectral localizer L_λ

T. A. Loring, Ann. Phys., 2015

Local approach to topology

Topological non-trivial insulators
⇔ the system (Hamiltonian) cannot be continued to the **atomic limit** $([\tilde{X},\tilde{H}]=0, [\tilde{Y},\tilde{H}]=0)$ without breaking symmetry class

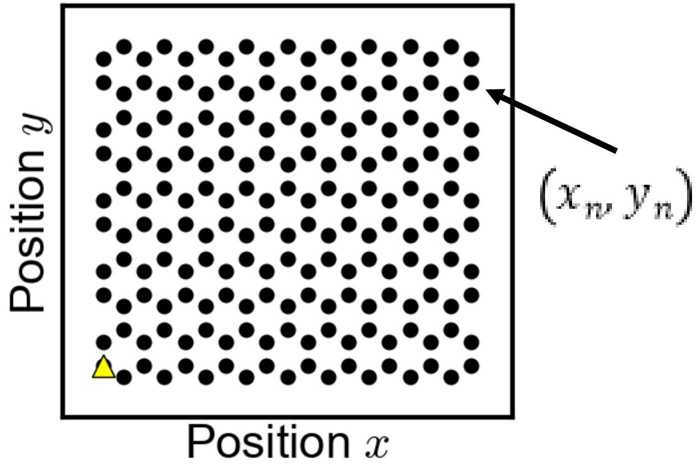
System, H



?

Kitaev, AIP Conference Proceedings, 2009

❖ **“Does the position X, Y and Hamiltonian H matrices can be modified to be commuting”** ?



$$X = \begin{pmatrix} \ddots & & \\ & x_n & \\ & & \ddots \end{pmatrix}, \quad Y = \begin{pmatrix} \ddots & & \\ & y_n & \\ & & \ddots \end{pmatrix}, \quad H = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

Spectral localizer:

$$L_{\lambda=(x,y,E)}(X,Y,H) = \begin{pmatrix} H-E & \kappa(X-x)-i\kappa(Y-y) \\ \kappa(X-x)+i\kappa(Y-y) & -(H-E) \end{pmatrix}$$

If $sig\big(L_\lambda(X,Y,H)\big) \neq 0$ ➔ X, Y, H cannot be modified to $\tilde{X},\tilde{Y},\tilde{H}$ such that $\underbrace{[\tilde{X},\tilde{H}]=0, [\tilde{Y},\tilde{H}]=0}_{\Leftrightarrow \text{ atomic limit}}$

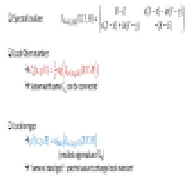
sig = #(positive eigenvalues) - #(negative eigenvalues)

T. A. Loring, Ann. Phys., 2015

Local approach to topology

Topological non-trivial insulators
⇔ the system (Hamiltonian) cannot be continued to the **atomic limit**
$([\tilde{X}, \tilde{H}] = 0, [\tilde{Y}, \tilde{H}] = 0)$ without breaking symmetry class

System, H



?

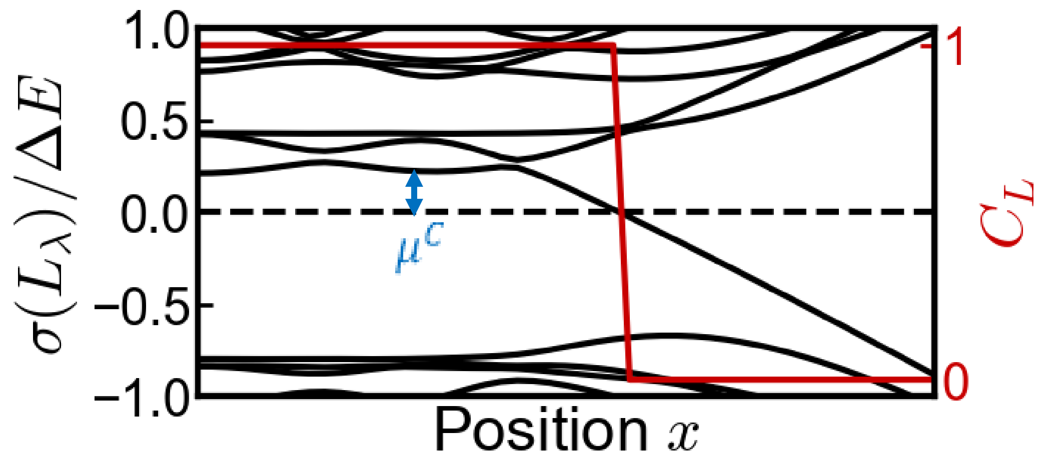
Kitaev, AIP Conference Proceedings, 2009

❖ **“Does the position X, Y and Hamiltonian H matrices can be modified to be commuting”** ?

- ❑ Spectral localizer: $$L_{\lambda=(x,y,E)}(X,Y,H) = \begin{pmatrix} H-E & \kappa(X-x) - i\kappa(Y-y) \\ \kappa(X-x) + i\kappa(Y-y) & -(H-E) \end{pmatrix}$$

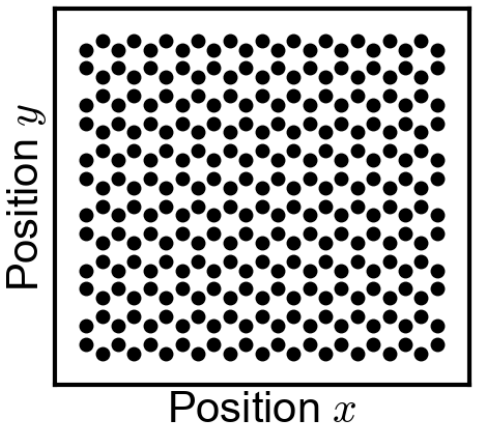
- ❑ Local Chern number:
  - → $C_L(x,y,E) = \frac{1}{2}\mathrm{sig}\left(L_{\lambda=(x,y,E)}(X,Y,H)\right)$
  - → System with same C_L can be connected

- ❑ Localizer gap:
  - → $\mu^C(x,y,E) = \sigma_{\min}\left[L_{\lambda=(x,y,E)}(X,Y,H)\right]$
    (smallest eigenvalue of L_λ)
  - → “same as band gap”: spectral value to change local invariant

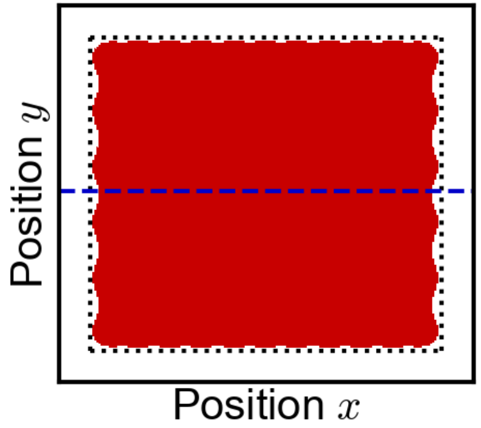


T. A. Loring, Ann. Phys., 2015

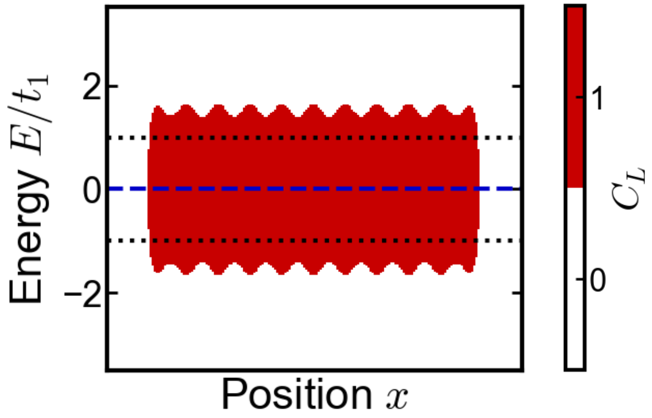
Example – Haldane lattice



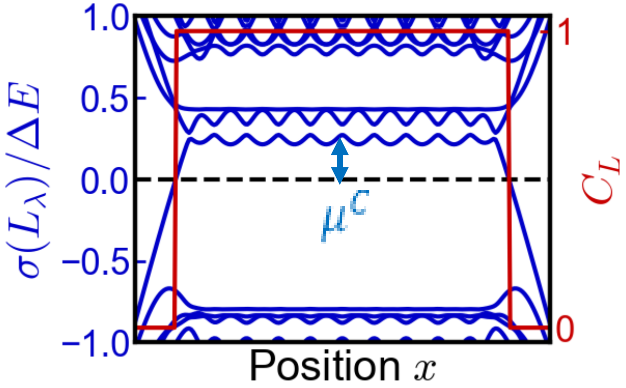
Probe topology in space

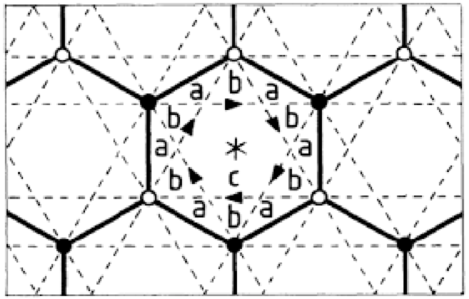


Probe topology in energy



Probe robustness in space-energy



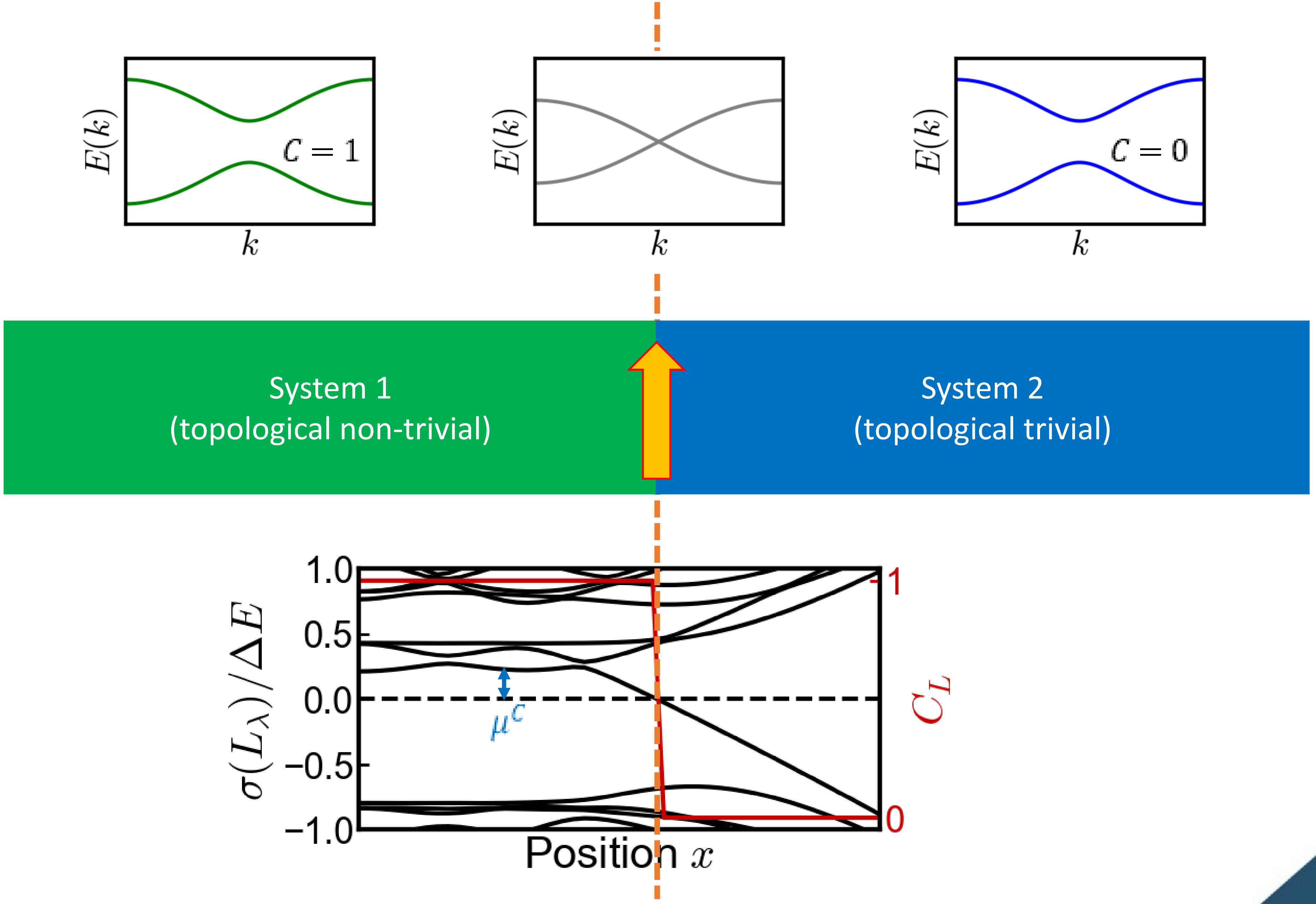


F. D. M. Haldane, PRL 1988

Local approach to topology

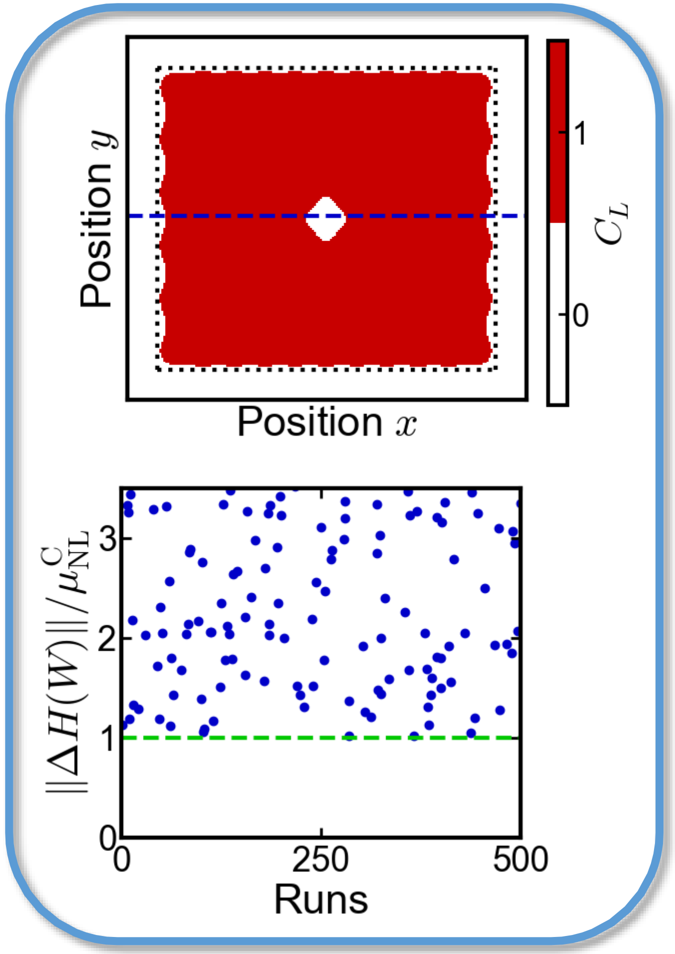
"Schematic picture"

Real-space picture



1. Local approach to topology

2. Topological nonlinear mode



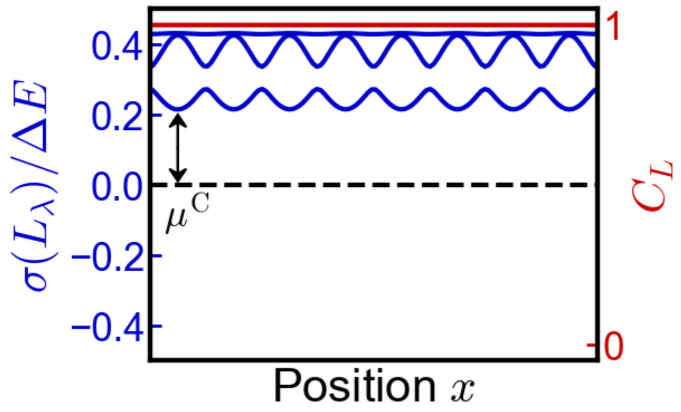
3. Topological dynamics

Nonlinear topological insulators – Topological profile

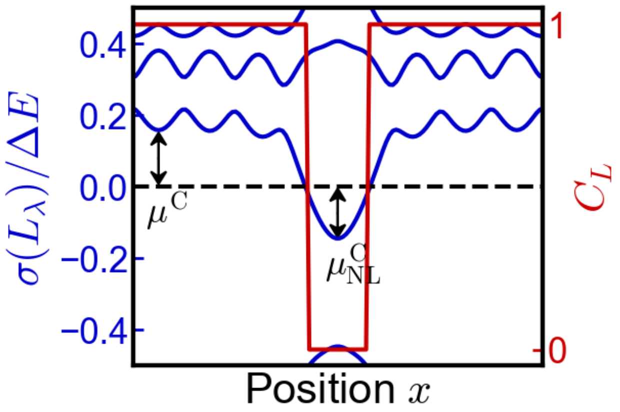
- ❑ Position operators: $X = \begin{pmatrix} \ddots & & \\ & x_n & \\ & & \ddots \end{pmatrix}, \qquad Y = \begin{pmatrix} \ddots & & \\ & y_n & \\ & & \ddots \end{pmatrix},$
- ❑ Hamiltonian: $H(\boldsymbol{\Psi}) = H_L + H_{NL}(\boldsymbol{\Psi}), \qquad H(\boldsymbol{\Psi})\boldsymbol{\Psi}_{\mathrm{NL}} = E_{NL}\boldsymbol{\Psi}_{\mathrm{NL}}$
- ❑ Localizer: $L_{\lambda=(x,y,E)}(X, Y, H(\boldsymbol{\Psi})) = \begin{pmatrix} H(\boldsymbol{\Psi}) - E & \kappa(X-x) - i\kappa(Y-y) \\ \kappa(X-x) + i\kappa(Y-y) & -(H(\boldsymbol{\Psi}) - E) \end{pmatrix}$
- ❑ Local Chern number: $C_L(x, y, E) = \frac{1}{2}\,\mathrm{sig}\left(L_{\lambda=(x,y,E)}(X, Y, H(\boldsymbol{\Psi}))\right)$

Topological non-trivial nonlinear mode:
creation of nonlinearity-induced topological interface (at E_{NL})

Linear regime



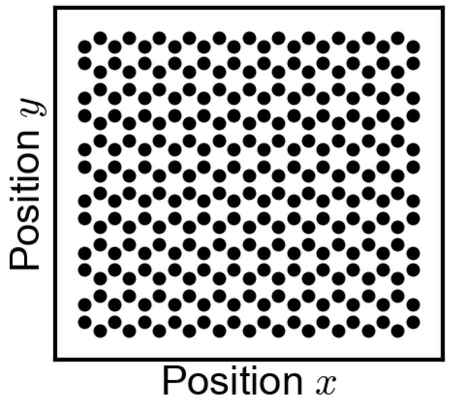
Nonlinear regime

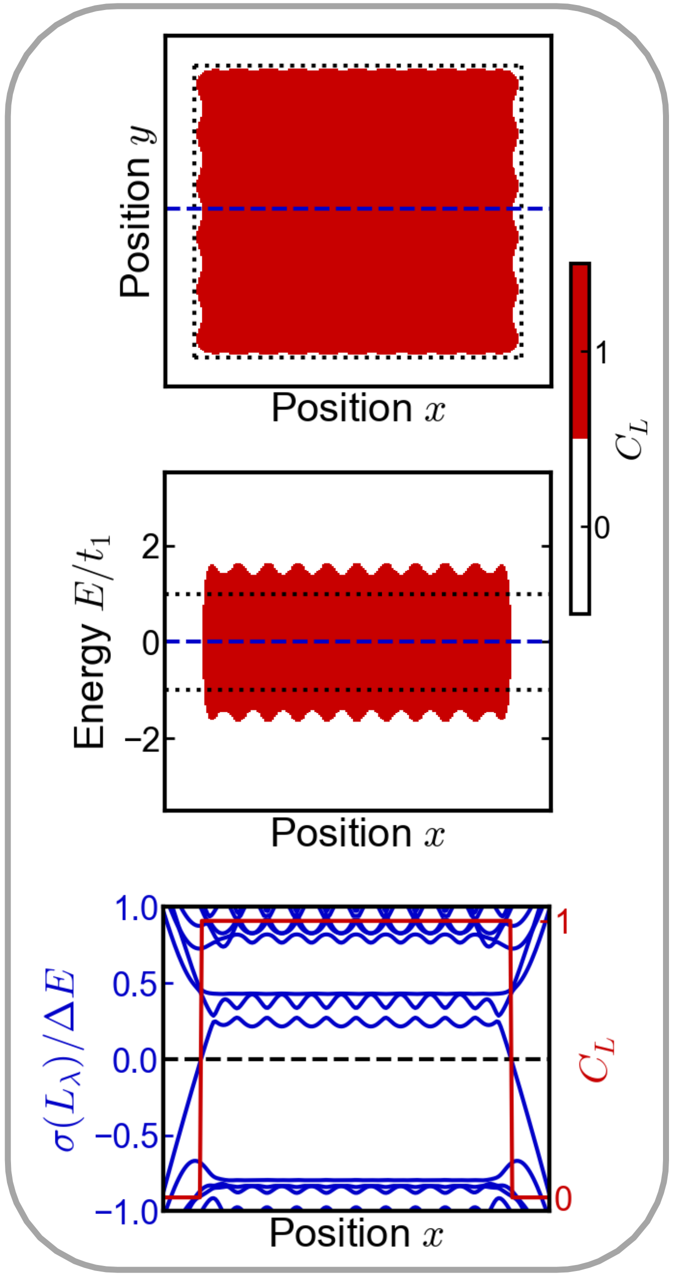


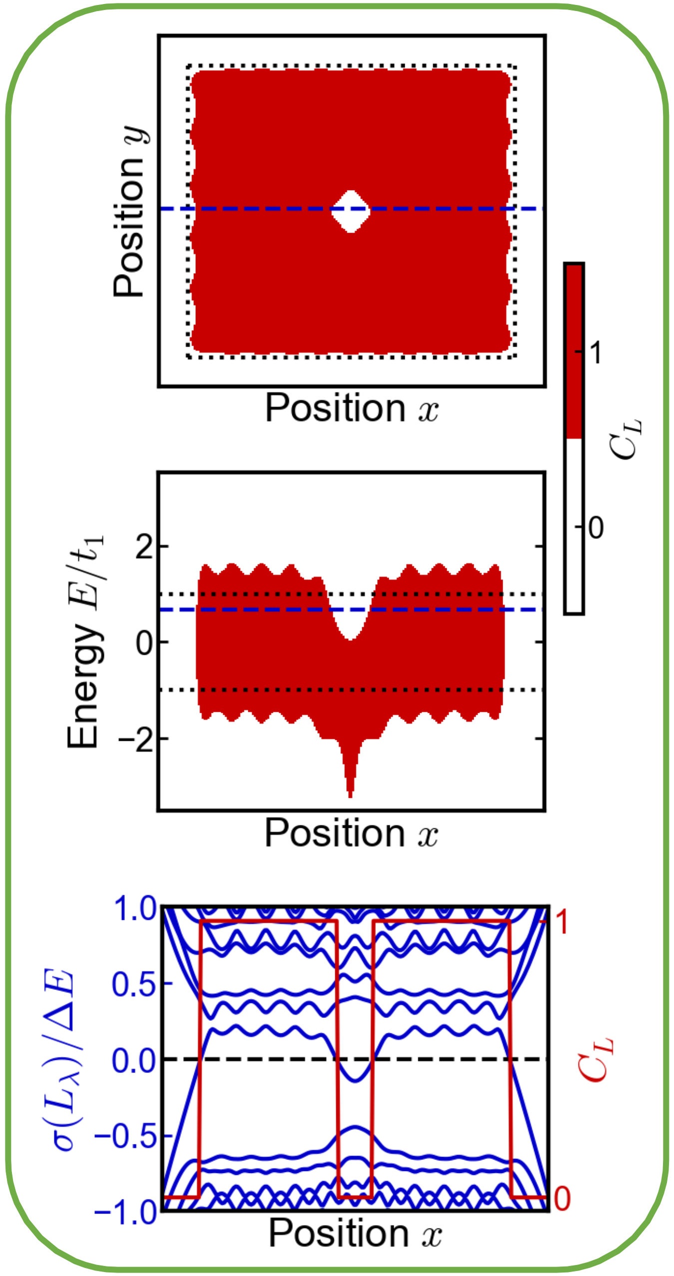
Topological non-trivial nonlinear mode

$$[H_{NL}(\mathbf{\Psi})]_{nm} = g|\Psi_n|^2\delta_{nm}$$

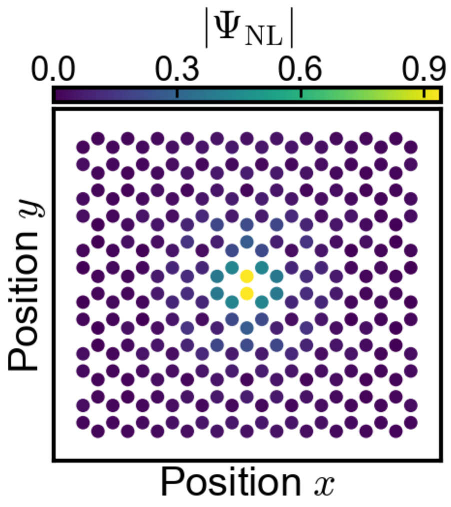
Linear regime







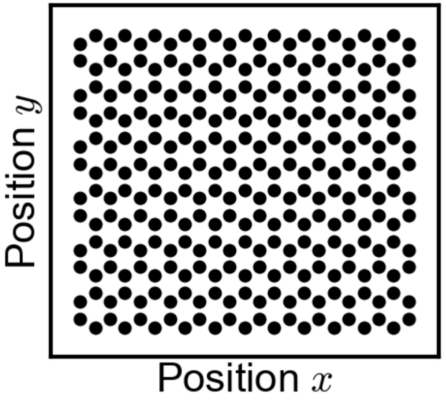
Nonlinear regime

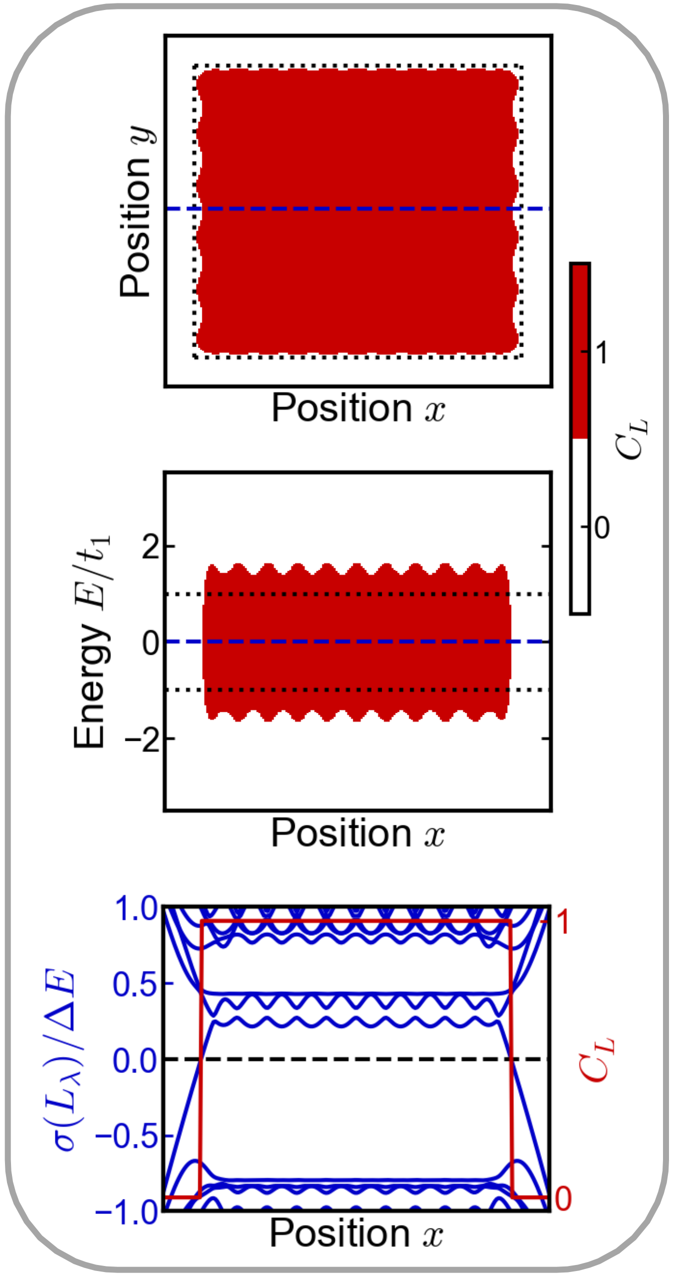


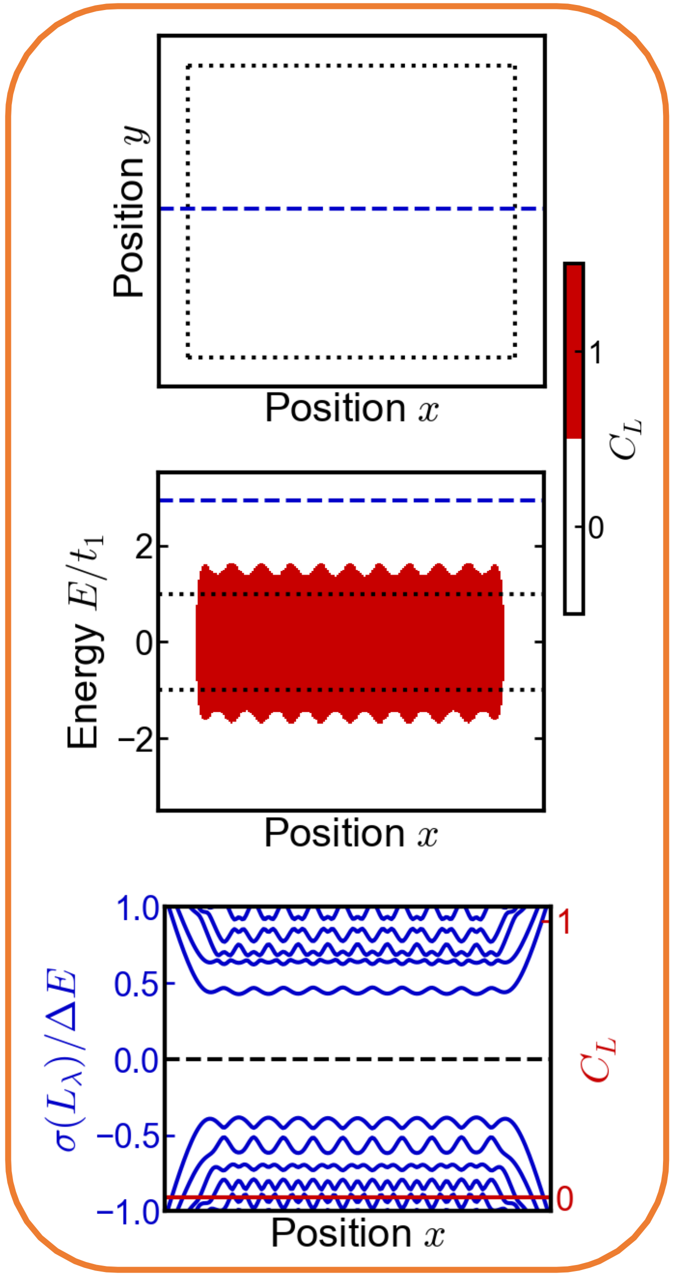
Topological trivial nonlinear mode

$$[H_{NL}(\mathbf{\Psi})]_{nm} = g|\Psi_n|^2\delta_{nm}$$

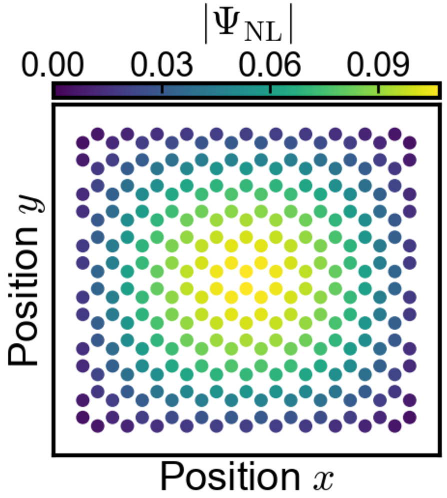
Linear regime







Nonlinear regime

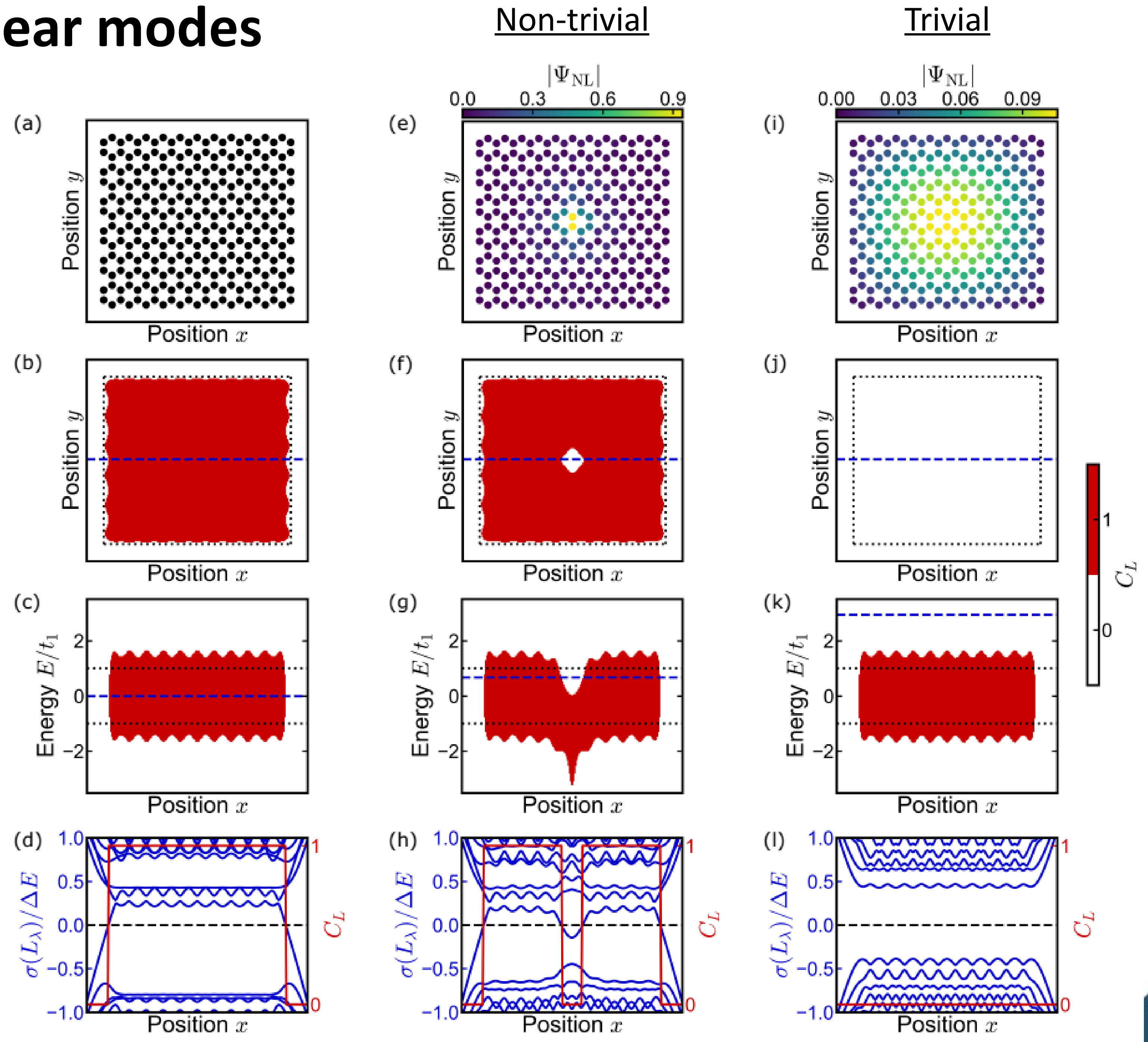


Topological nonlinear modes

On-site e_0 Kerr term:

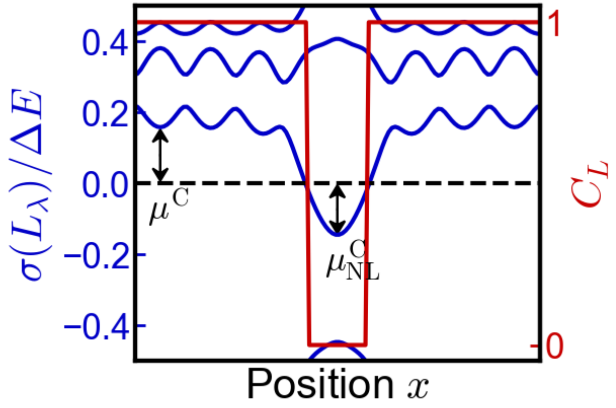
$$[H_{NL}(\mathbf{\Psi})]_{nm} = g|\Psi_n|^2\delta_{nm}$$

Topological non-trivial nonlinear mode:
creation of nonlinearity-induced topological interface (at E_{NL})



Protection against disorders W

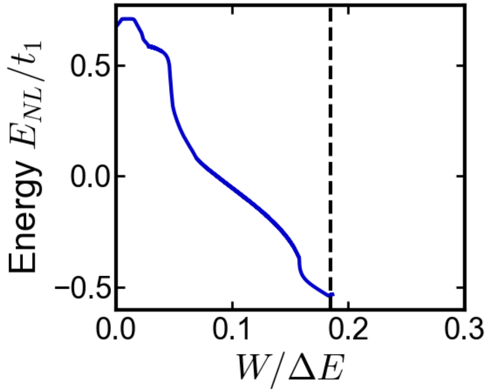
Change in matrix Hamiltonian: $\Delta H(W) = H_{pert}(W) - H_{pert}(0)$

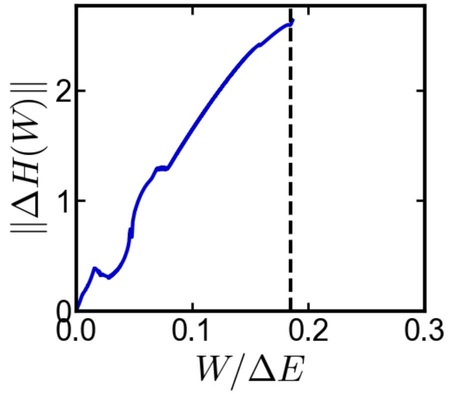


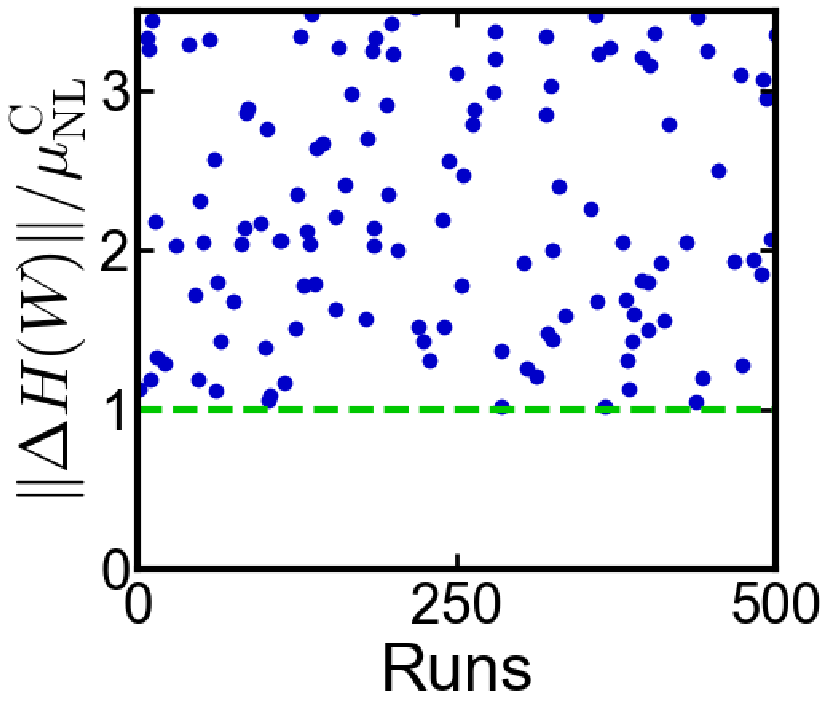
- ➢ <u>Linear regime:</u> topological protection until gap closes ⇔ $\|\Delta H(W)\| \sim \mu^C$
- ➢ <u>Nonlinear regime:</u> topological protection until gap closes ⇔ $\|\Delta H(W)\| \sim \mu^C_{\mathrm{NL}}$

Look at solution curve of unperturbed soliton $\mathbf{\Psi_0}$:

- Start from $\mathbf{\Psi_0}$
- $W = 0 \rightarrow 0.05\ \Delta E$
- Look $\|\Delta H(W_c)\|$, where W_c is when nonlinear solver stops converging





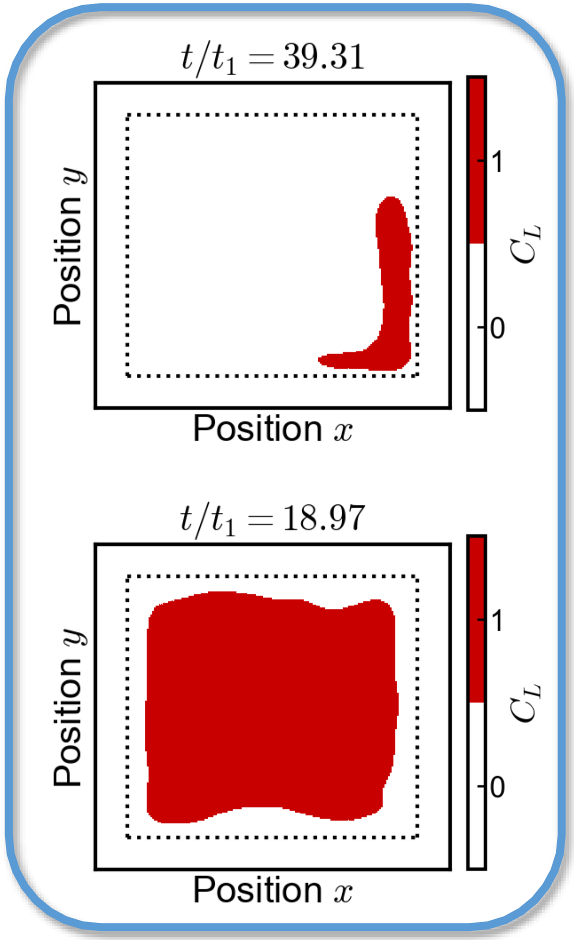


- ➢ **“Protection”: Guaranteed solution curve as long as $\|\boldsymbol{\Delta H(W)}\| \lesssim \boldsymbol{\mu^C_{NL}}$**

1. Local approach to topology

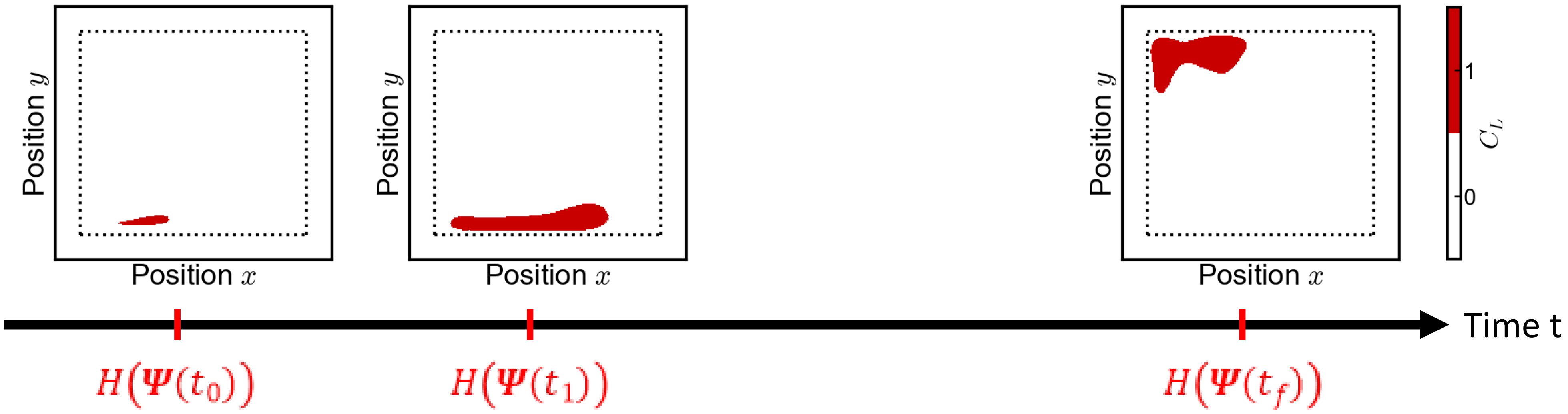
2. Topological nonlinear mode

3. Topological dynamics



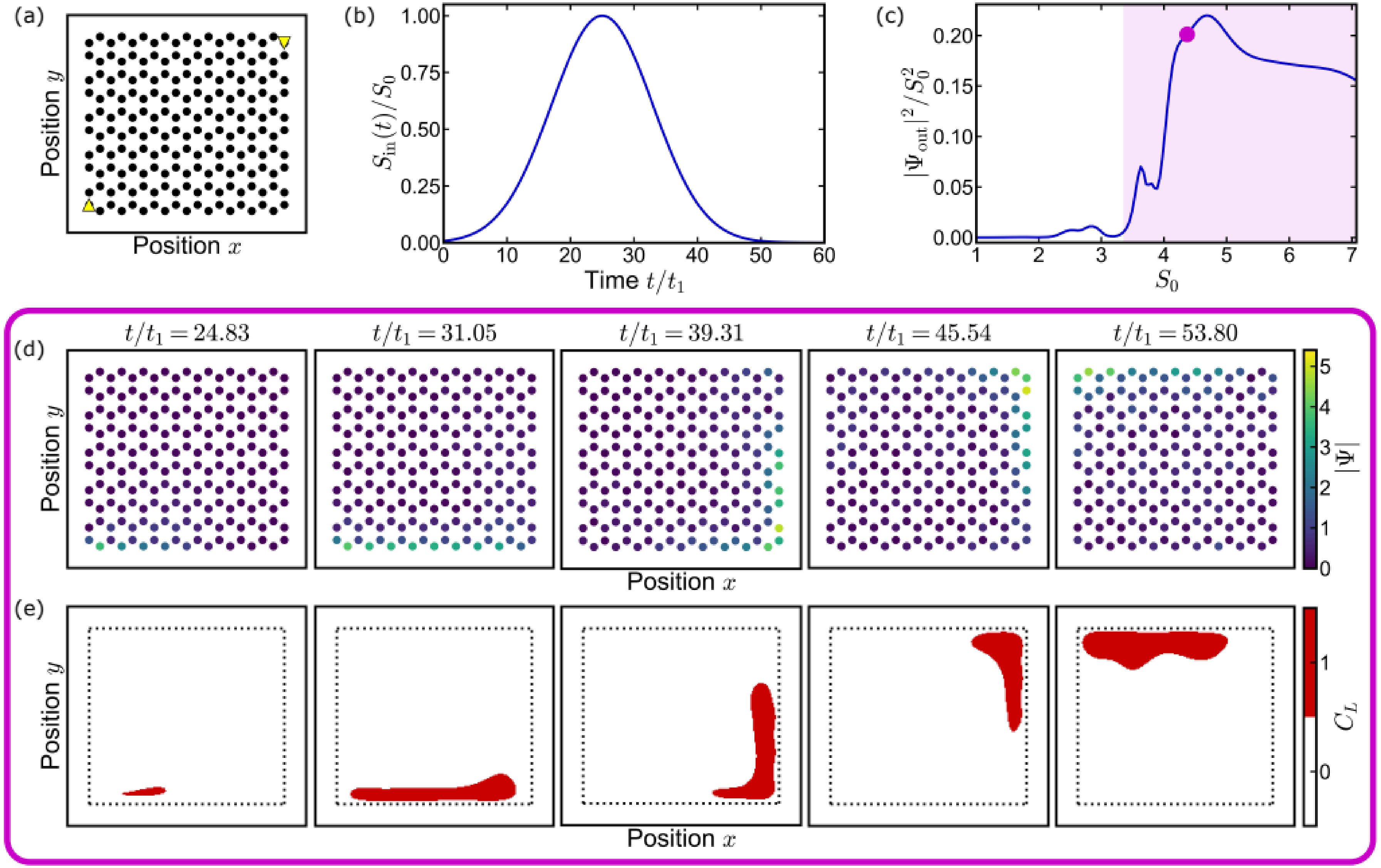
Nonlinear topological insulators – Topological dynamics

- Position operators: $X = \begin{pmatrix} \ddots & & \\ & x_n & \\ & & \ddots \end{pmatrix}, \qquad Y = \begin{pmatrix} \ddots & & \\ & y_n & \\ & & \ddots \end{pmatrix},$
- Hamiltonian: $H(\mathbf{\Psi}) = H_L + H_{NL}(\mathbf{\Psi}), \qquad i\frac{d}{dt}\mathbf{\Psi} = H(\mathbf{\Psi})\mathbf{\Psi}$
- Localizer: $L_{\lambda=(x,y,E)}(X,Y,H(\mathbf{\Psi})) = \begin{pmatrix} H(\mathbf{\Psi}) - E & \kappa(X-x) - i\kappa(Y-y) \\ \kappa(X-x) + i\kappa(Y-y) & -(H(\mathbf{\Psi}) - E) \end{pmatrix}$
- Local Chern number: $C_L(x,y,E) = \frac{1}{2}\operatorname{sig}\left(L_{\lambda=(x,y,E)}(X,Y,H(\mathbf{\Psi}))\right)$



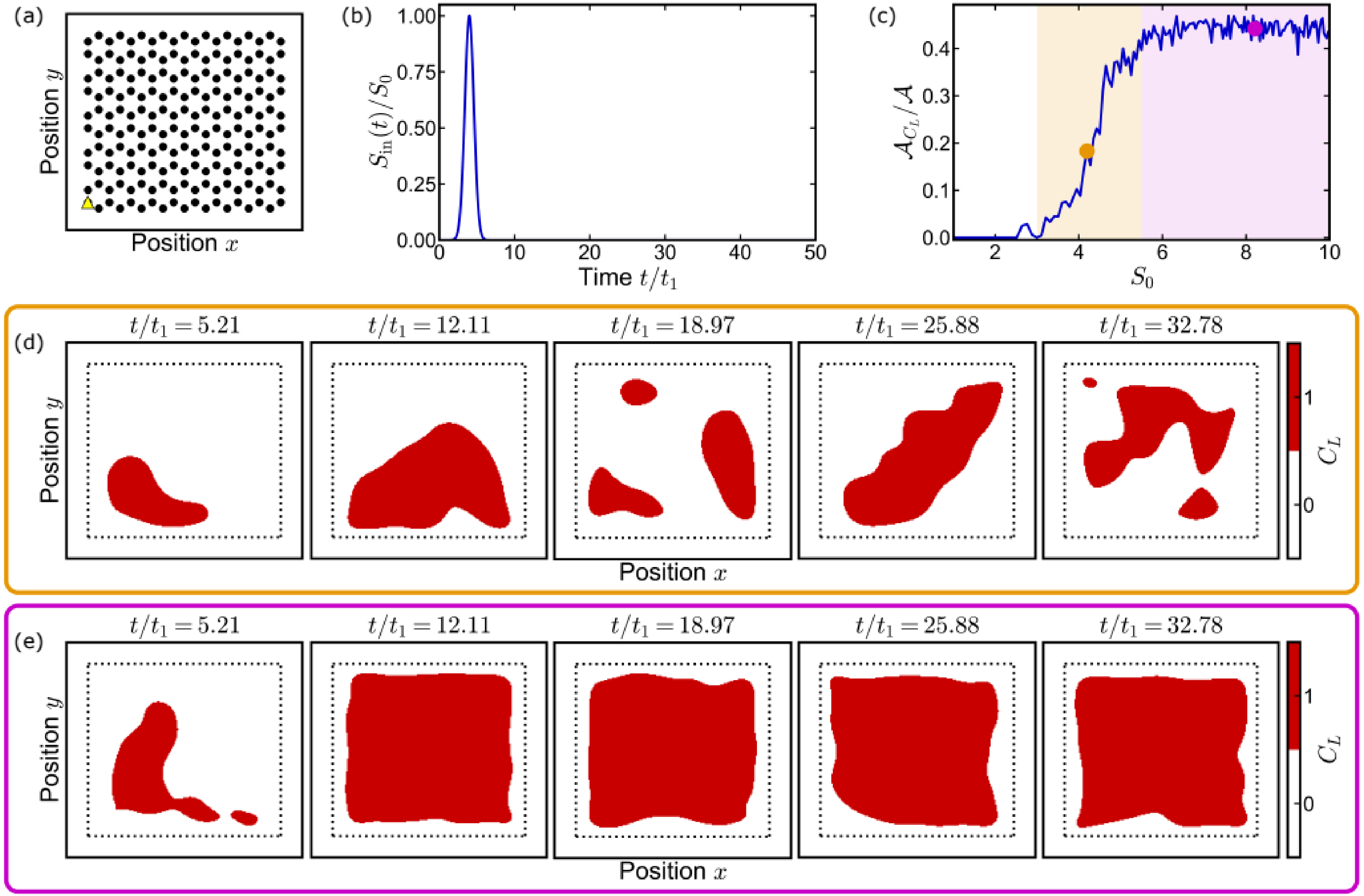
Topological dynamics – Topological nonlinear moving mode

On-site e_0 Kerr term: $[H_{NL}(\mathbf{\Psi})]_{nm} = g|\Psi_n|^2\delta_{nm}$



Topological dynamics – Different dynamical regime

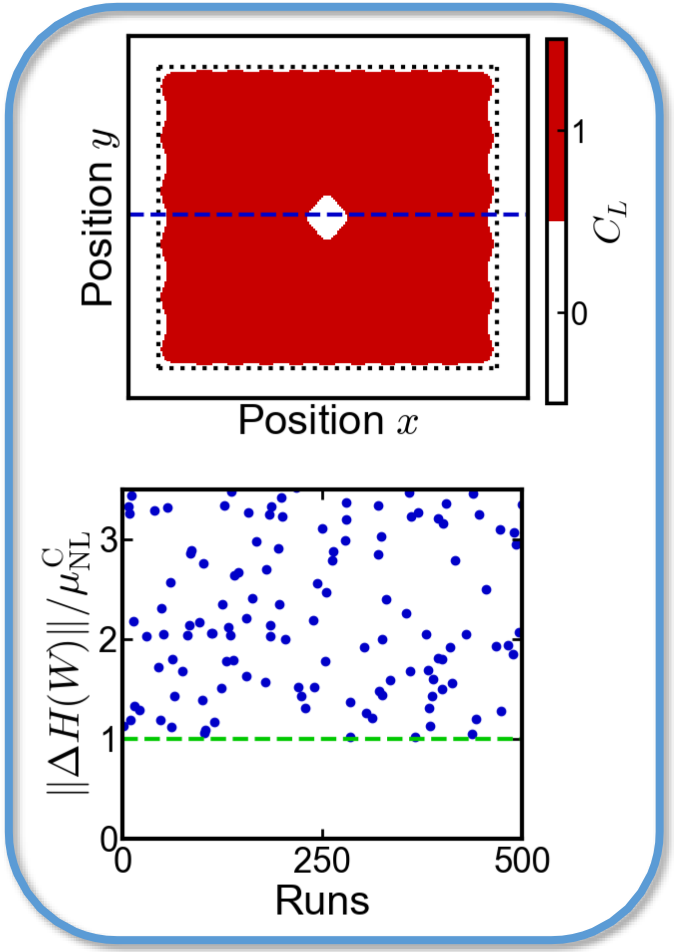
Coupling t_2 Kerr term: $[H_{NL}(\mathbf{\Psi})]_{nm} = g \sum_m (|\Psi_n|^2 + |\Psi_m|^2)\, e^{i\phi_{nm}}$



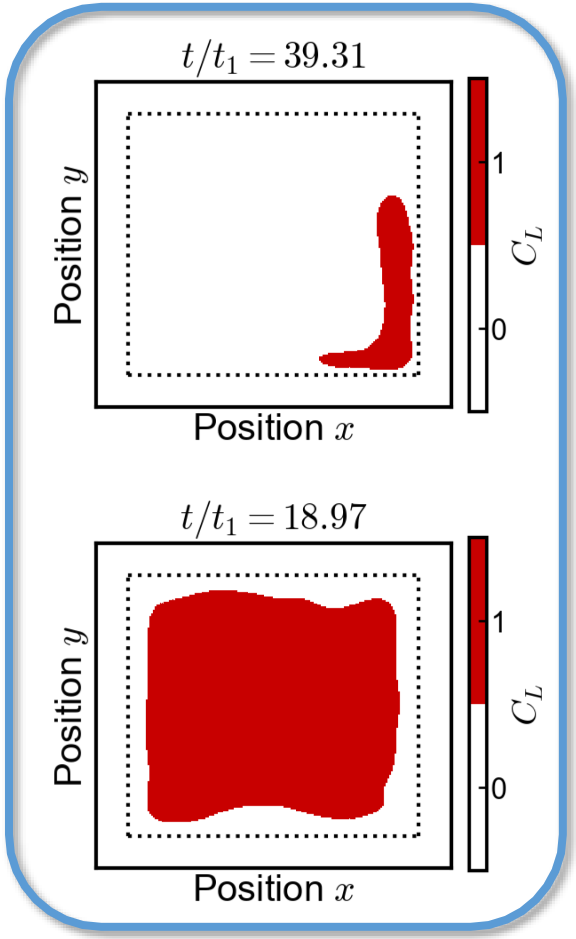
Conclusions

✓ Email: stewong@@sandia.gov

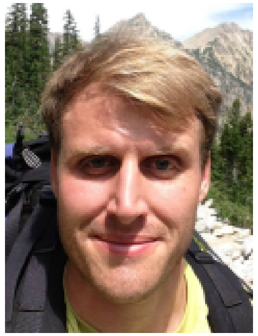
<u>Classify</u>
<u>topological nonlinear mode</u>



<u>Probe</u>
<u>topological dynamics</u>



Thank for your attention



Alexander Cerjan
Sandia / CINT

Terry A. Loring
University of New Mexico