

1 Title:

2 Spherical Time-Encoded Radiation Imaging Simulations

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16

17 Abstract:

18 Radiation source localization is important for nuclear nonproliferation and can be obtained using

19 time-encoded imaging systems with unsegmented detectors. A scintillation crystal can be used

20 with a moving coded aperture mask to vary the detected count rate produced from radiation

21 sources in the far field. The modulation of observed counts over time can be used to reconstruct

22 an image with the known coded aperture mask pattern. Current time-encoded imaging systems

23 incorporate cylindrical coded aperture masks and have limits to their fully coded imaging field-
24 of-view. This work focuses on expanding the field-of-view to 4π by using a novel spherical coded
25 aperture mask. A regular icosahedron is used to approximate a spherical mask. This icosahedron
26 consists of 20 equilateral triangles; the faces of which are each subdivided into four equilateral
27 triangle-shaped voxels which are then projected onto a spherical surface, creating an 80-voxel
28 coded aperture mask. These polygonal voxels can be made from high-Z materials for gamma-ray
29 modulation and/or low-Z materials for neutron modulation. In this work, we present Monte Carlo
30 N-Particle (MCNP) simulations and simple models programmed in Mathematica to explore image
31 reconstruction capabilities of this 80-voxel coded aperture mask.

32

33 Keywords:

34 4π field-of-view, gamma-ray, nuclear nonproliferation, neutron, regular icosahedron, time-
35 encoded imaging.

36

37 Abbreviations:

38 cTEI – cylindrical time-encoded imaging

39 FOV – field-of-view

40 FWHM – full width at half max

41 GDA – Great Deluge Algorithm

42 MCNP – Monte Carlo N-Particle

43 sTEI – spherical time-encoded imaging

44 TEI – time-encoded imaging

45 1. Introduction

46 Imaging of radiation sources with a 4π field-of-view (FOV) has previously been achieved with
47 spherical arrays of detectors, both semiconductors and scintillators, and by Compton imaging
48 [1][2]. These methods utilize pixelated detectors and/or arrays of detectors, leading to increased
49 system cost, weight, and size. The use of radiation shielding material as the coded aperture array
50 with only a single non-pixelated scintillation crystal is relatively inexpensive and has a simple, fast
51 readout. This has previously been used with a rotating cylindrical coded aperture mask and an
52 unsegmented scintillation crystal for 1D [3] and 2D [4] imaging with a limited FOV. This technique
53 is referred to as cylindrical time-encoded imaging (cTEI). We are proposing the natural evolution
54 to spherical time-encoded imaging (sTEI), that will increase the FOV to 4π while retaining the
55 simplicity of the imaging modality.

56
57 Coded aperture masks can have a variety of static patterns or be made cyclic for time-encoded
58 imaging. For example, one ideal coded aperture pattern is the uniformly redundant array (URA)
59 [5], but there are multiple kinds of URAs, including modified [6] and mosaic [7] URAs, and under
60 ideal circumstances, the pattern autocorrelates a point source to a delta function. However,
61 more random coded aperture patterns can also be used, and we discuss next a technique to
62 choose random patterns that can perform comparably to URAs.

63
64 In previous work, we have designed cTEI masks [8] using the Great Deluge Algorithm (GDA) to
65 pseudo-optimize random coded aperture patterns. This algorithm creates a random mask
66 pattern of 0's and 1's and calculates a pattern quality metric [9]. It assigns the opposite value to

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a randomly chosen mask element and recalculates the quality metric, and the voxel change is accepted if the quality metric value is sufficiently greater than the previous value. The mask pattern converges to a pinhole camera for point sources and no background radiation, and it converges to a 50% open fraction for a nominal background level and/or extended sources.

2. Design and Methods

2.1 Coded Aperture Design

Two options for tiling a spherical surface are by orthogonal polar and azimuthal segmentation (i.e., longitude and latitude lines) or by an equilateral triangle segmentation with a regular icosahedron base as seen in Fig. 1. If the spherical system is stationary and only rotates around a single axis, the polar and azimuthal segmentation could be advantageous. The segmented voxels around the opposite poles of the system would be smaller than the voxels at the equator, allowing a variable modulation of sources depending on the choice of rotation axis. However, our system is ultimately envisioned to be a freely rolling sphere with the ability to rotate around any axis. For this application, it is advantageous to have coded aperture voxels of equal shape and size. The regular icosahedron has 60 rotational natural symmetries, allowing many axes of rotation useful for image reconstruction. These natural symmetries have been utilized in chemistry for topological mapping of chemicals [10] and are being revisited here for spherical axes of rotation.

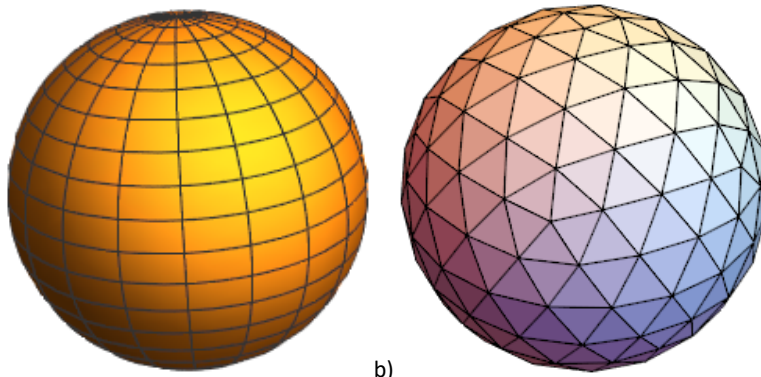


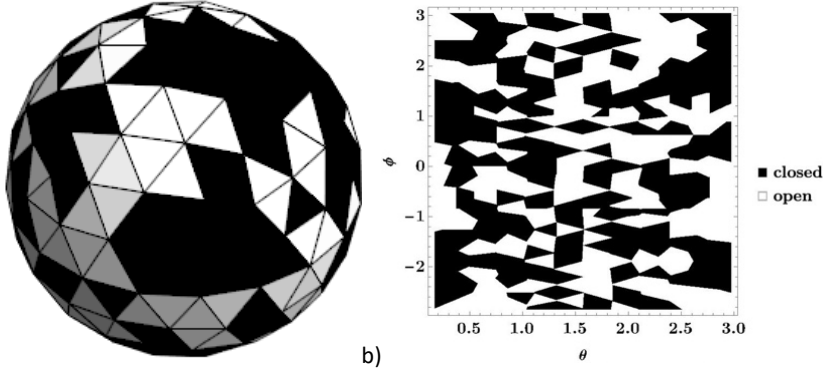
Fig. 1: The two spherical mask pattern shapes being considered: a) a polar and azimuthally segmented mask resembling a globe, b) a regular icosahedron split into smaller aperture voxels.

The GDA coded aperture optimization has been used by others to optimize the coded aperture of other 4π imaging systems; one used a rounded 2D-cross segmented into squares [1] and another utilized the regular icosahedron [2]. Both systems used spherical arrays of detectors, but the GDA can still be used to optimize a spherical coded aperture with a central detector.

2.2 Validation Modeling

To simulate a sTEI system, a polygonal approximation of a spherical mask pattern was first coded in Mathematica. A regular icosahedron was created, and each of the 20 triangular faces were then subdivided twice into four smaller triangles to make the 320-element mask shown in Fig. 2. These smaller triangles were then projected onto a spherical surface to create an approximately spherical mask. The mask was chosen to have an approximately 50% open fraction and the spatial distribution of open elements on the mask was randomly selected.

103



104 a) b)

105 Fig. 2: Random coded aperture pattern with a segmented regular icosahedral base a) projected
 106 onto a sphere and b) plotted as a polar and azimuthally segmented mask. Closed mask
 107 elements are black, open mask elements are white.

108

109 An imaging plane was created with 5,760 imaging pixels, 18 subpixels per coded aperture voxel.

110 The system response matrix for the spherical mask is made of 1's and 0's corresponding to the

111 open and closed elements closest to each imaging voxel. Each row the of the system response

112 matrix is a circulant rotation of the previous row, creating an m -by- m array where m is the

113 number of imaging pixels. The count modulation, $\vec{y}_0$, for a source distribution, $\vec{x}$, with a sensing

114 matrix, A , is the forward projection of $\vec{x}$ through the sensing matrix, as shown in Eqn. (1).

$$\vec{y}_0 = A \cdot \vec{x} \quad (1)$$

115

116

117

118 A source distribution was created with the same dimensions as the imaging plane. A single pixel

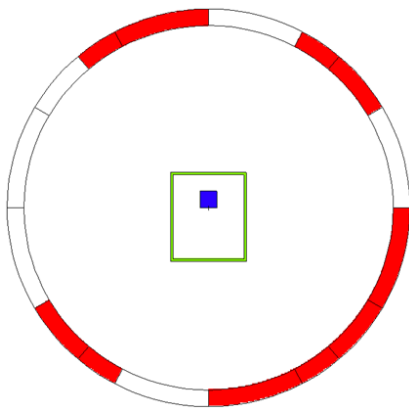
119 was assigned an arbitrary intensity value of 100 and the rest were assigned a value of 10,

120 corresponding to a nominal background level. Maximum likelihood expectation maximization
121 (MLEM) was used to reconstruct spatial images from the data generated by Mathematica [11].
122 The results will be shown in Section 3.1.

123

124 2.3 MCNP Simulation

125 MCNP6.2 was used to further simulate the radiation response of the system. MCNP is a more
126 realistic simulation tool for radiation transport and allows for radiation scatter in surroundings
127 to be tracked. The regular icosahedron was projected onto a sphere with more fidelity using
128 MCNP versus the polygonal structure in Mathematica. This was made possible with ARB
129 (arbitrary) macrobodies defining the vertices of the coded aperture mask, and then two SO
130 (origin-centered sphere) macrobodies cut through the ARB. The intersection of all three
131 macrobodies defines a spherically wedged coded aperture voxel. A vertical slice of the geometry
132 can be seen in Fig. 3. Two geometries were tested for validation of the sTEI system.



134

135 Fig. 3: Side slice view of a sample sTEI coded aperture mask with filled voxels (red), a
136 polycarbonate light-tight box (green), and a 1-inch CLLBC scintillation crystal (blue).

137
138 MCNP includes the count rate artifacts produced when comparing sources in the near- and far-
139 field. The simulations assumed a 35 cm diameter spherical mask of 80 tessellated equilateral
140 triangles. The mask pattern used was randomly generated with a 50% chance for a voxel to be
141 open or closed. A 1-inch CLLBC scintillator [12] was simulated at the center of the mask inside
142 and inside a polycarbonate light-tight box. An isotropic Cs-137 point source was simulated at 3.40
143 m and 8.08 m from the sTEI mask to compare near- and far-field count rate artifacts. The source
144 was simulated at each point corresponding to a projection through the centroid of each triangle.
145 This permits simulating the sphere rotated around arbitrary axes and covering the 4π image
146 space.

147
148 Another simulation was run for a further source distance of 8.5 m and directed as a cone at the
149 sTEI coded aperture system. Four projected source positions were used per triangle for a total
150 of 320 sources positions at the centroids of a further segmented 320-sided regular icosahedron.
151 This simulation was reconstructed using 25 iterations of MLEM to further test the imaging
152 capabilities.

153
154 To explore the advantage of multiple axes of rotation, the sTEI mask was simulated as having a
155 70 cm diameter and a 2.54 cm thickness of tungsten. A 1 MeV monoenergetic gamma-ray source
156 at -2-meters along the Z-axis was chosen, with a 1-inch CLLBC scintillator as the central non-

157 pixelated detector. The detector cell was kept static while the random pattern spherical mask
158 was rotated around the X-, Y-, and then Z-axis. The regular icosahedron is symmetrical around
159 these axes, and although more axes can be simulated, there is diminishing return on image
160 quality improvements with each additional axis.

161
162 The MCNP6.2 simulations did not include the effects of background radiation, room return, pulse
163 processing, or detector efficiency. The results in Section 3.2 only use total detector count rate,
164 although the use of spectroscopy would further improve the simulated angular resolution.

165
166 3. Results and Discussion

167 3.1 Validation Modeling Results

168 Counts were simulated in Mathematica by forward projecting the source distribution through
169 the sensing matrix and then Poisson sampling the counts. From the data, the image shown in Fig.
170 4 was reconstructed using MLEM. The reconstruction pinpointed the maximum intensity pixel in
171 the source space, validating this method as a preliminary technique for sTEI simulation. The
172 Mathematica results are treated as qualitative, and the image resolution is constrained by the
173 imaging plane segmentation. This method was sufficient to show MLEM images of forward
174 projections.

175

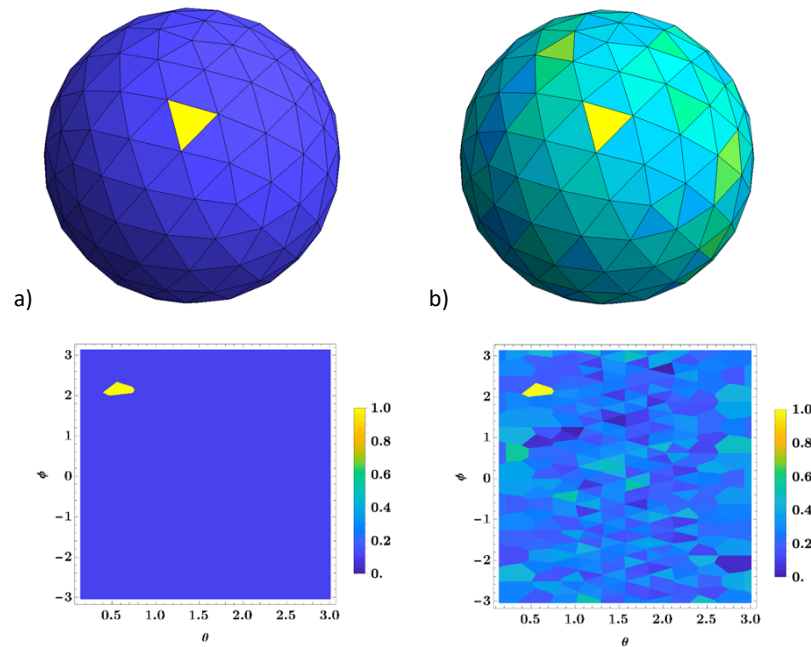


Fig. 4: Mathematica simulation displayed in spherical coordinates with a) the single true source pixel and b) the MLEM reconstruction, both normalized for qualitative view.

3.2 MCNP Simulation Results

MLEM was used to reconstruct an image for a source in the far field, 8.5-m from the centroid detector. This simulated source was an anisotropic Cs-137 gamma-ray cone source directed at the sTEI system. The MLEM reconstruction is seen in Fig. 5 projected onto a sphere and plotted in azimuthal and polar coordinates. The imaging resolution of this simulation is $\sim 7^\circ$ in azimuth and $\sim 6^\circ$ in the polar direction at the FWHM. This could be improved by optimizing the coded aperture mask pattern, thickness, and radius.

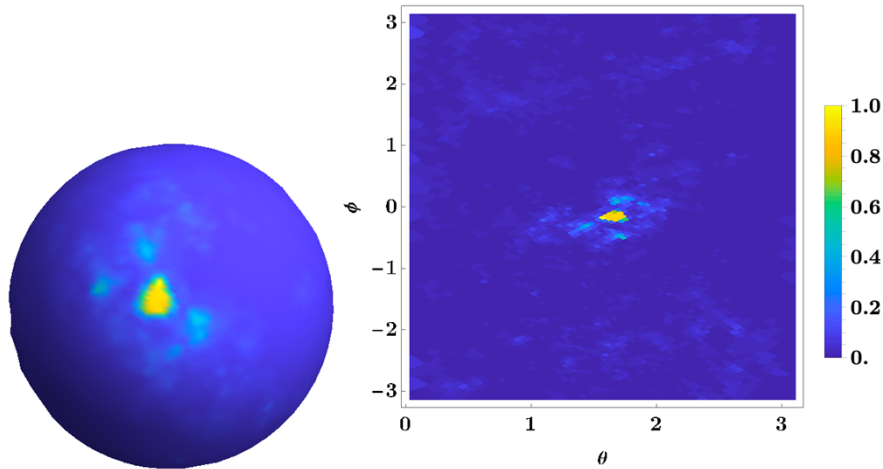


Fig. 5: A MCNP6.2 simulated Cs-137 far field source directed at the sTEI system, reconstructed using MLEM with 20 iterations.

The final sTEI simulation was designed to validate the need for multiple axes of rotation and the use of the regular icosahedron as the base geometry for sTEI. The total detector count rate modulation can be seen in Fig. 6 for the three axes of rotation with a 1-MeV monoenergetic gamma-ray source in the negative-Z direction. As expected, the tungsten mask heavily modulates the source when rotated about the X- and Y-axis, while rotation around the Z-axis should not result in the count rate changing. The MCNP6.2 output tally error is greater than the modulation bounds for the Z-axis rotation, concluding that the count rate is constant for a Z-axis rotation when the source is along the Z-axis of the system. If the system were only rotated around the Z-axis, it would be incorrectly concluded that there is no source present. The difference in count rate between the rotations around the X- and Y-axis determines that there is a source and could potentially improve imaging quality when combining the data between the two instead of using

one or the other. This is because the coded aperture mask pattern seen by the source is different for each axis of rotation. Exploiting this effect should be done in future work.

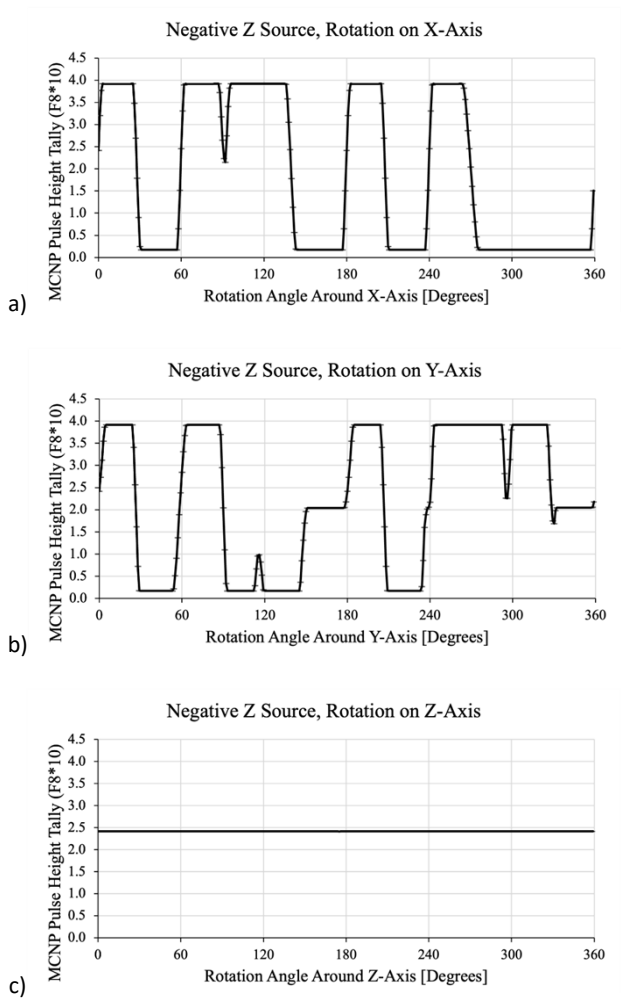


Fig. 6: The modulation of detector count rate for rotation of the STEI system around the a) X-axis, b) Y-axis, and c) Z-axis.

212

213 3 Conclusion

214 Spherical time-encoded imaging has been validated with simulation as a potential evolution from
215 cylindrical time-encoded imaging. The spherical coded aperture mask should be designed as a
216 regular icosahedron due to its natural symmetries and for its ability to evenly segment each
217 tessellated equilateral triangle. As multiple axes of rotation become desirable, a tessellation
218 coded aperture becomes more important for simplification of image reconstruction and should
219 even be advantageous. The three choices of rotation around the X-, Y-, and Z-axis allow for no
220 data to be lost along any one axis.

221

222 This simulation work should inspire new ideas and designs for 4π imaging systems. Future work
223 could be designing a free moving sphere with a regular icosahedral coded aperture shell for
224 advanced and adaptive imaging of radiation in the environment.

225

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