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National
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Bayesian Framework for Forecasting Dynamical Systems in the Presence of Corrupted Data



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Presented by:

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Sandia National Laboratories



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Motivation

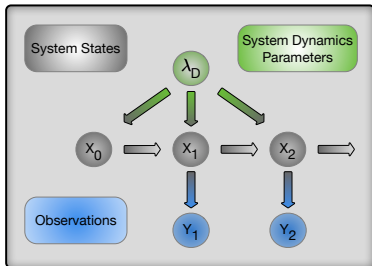
Bayesian System Identification

Results

- 2D Constant Velocity Vehicle
- Notional Balloon Trajectory

Summary

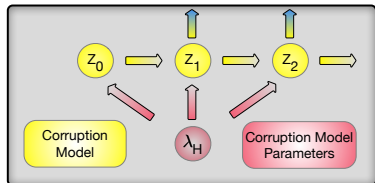
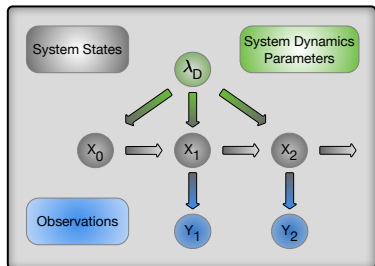
Traditional State Estimator



- Control methods that leverage traditional state-based estimation procedures (such as the Kalman filter depicted on the left) work well in benign environments but can be easily compromised by sensor errors.

- Monitoring properties of filters (e.g., Kalman innovations) can be used to detect some corruptions but these approaches cannot be used to infer information about faulty sensors and how to respond to it.

Novel State Estimator



- The Bayesian framework proposed here augments traditional state estimators with latent variables corresponding to model and measurement errors as well as corrupt observations.
- The Bayesian framework can handle limited data and embed additional information through priors.
- The posterior distribution of latent variables λ can be sampled with Markov Chain Monte Carlo techniques, to produce dynamics consistent with available information.



State

parameter uncertainty

model inadequacy

$$X_k = D_k(X_{k-1}, \lambda_D) + \delta_k(X_k, \lambda_\delta)$$

Observations

observations under benign conditions

corruption models

measurement error

$$Y_k = H_1(X_k, \lambda_H^{(1)}) + \sum_{i>1} H_i(X_k, \lambda_H^{(i)}) + \epsilon_k(X_k, \lambda_\epsilon)$$

Estimate λ in a Bayesian Framework

$$p(\lambda|\mathcal{D}) \propto p(\mathcal{D}|\lambda)p(\lambda)$$

- Likelihood $p(\mathcal{D}|\lambda)$ will be constructed using a sequence of innovations via a Kalman filter approach.

7 Bayesian System Identification - Likelihood Construction



Kalman Filter

- Prediction step

$$\begin{aligned}X_{k+1|k} &= F_k X_k \\P_{k+1|k} &= F_k P_k F_k^T + Q_k\end{aligned}$$

- Update step

$$\begin{aligned}Z_{k+1} &= Y_{k+1} - (H_{k+1} X_{k+1|k} + \sum_{i>1} H_k^{(i)}) \\S_{k+1} &= H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1} \\P_{k+1} &= (\mathbb{I} - P_{k+1|k} H_{k+1}^T S_{k+1}^{-1} H_{k+1}) P_{k+1|k} \\X_{k+1} &= X_{k+1|k} + P_{k+1|k} H_{k+1}^T S_{k+1}^{-1} Z_{k+1}\end{aligned}$$

- F/H : linearized dynamics/observation models; Q/R : corresponding covariances.
- Z is the innovation, and S is the innovation covariance.

Likelihood construction

- uses the sequence of innovations scaled by innovation covariances

$$p(\mathcal{D}|\boldsymbol{\lambda}) = \prod_{k=1}^{n_{obs}} N(Z_k|0, S_k)$$

- $\boldsymbol{\lambda} = \{\lambda_D, \lambda_\delta, \lambda_H^{(1)}, \lambda_\epsilon, \lambda_H^{(2)}, \dots\}$

parameter set used
in this presentation

8 2D Constant Velocity Vehicle



- State vector $\mathbf{x} = [x_1, x_2, v_{x_1}, v_{x_2}]$, observations $\mathbf{y} = [x_1, x_2]$:

$$\mathbf{x}_{k+1} = F_k \mathbf{x}_k + q_k$$

$$\mathbf{y}_{k+1} = H_k \mathbf{x}_{k+1} + r_{k+1} + b_{k+1}$$

with $q_k \sim N(0, Q)$, $r_{k+1} \sim N(0, R)$ mutually independent zero mean Gaussian noise processes and b_{k+1} sensor corruption model (doi:10.1002/0471221279).

$$F_k = \begin{bmatrix} \mathbb{I}_2 & \mathbb{I}_2 \\ 0_2 & \mathbb{I}_2 \end{bmatrix}, \quad Q = \sigma^2 \begin{bmatrix} \frac{1}{3}\mathbb{I}_2 & \frac{1}{2}\mathbb{I}_2 \\ \frac{1}{2}\mathbb{I}_2 & \mathbb{I}_2 \end{bmatrix}, \quad H_k = [\mathbb{I}_2 \quad 0_2], \quad R = \lambda_\epsilon \begin{bmatrix} 7^2 & 3^2 \\ 3^2 & 8^2 \end{bmatrix}$$

and

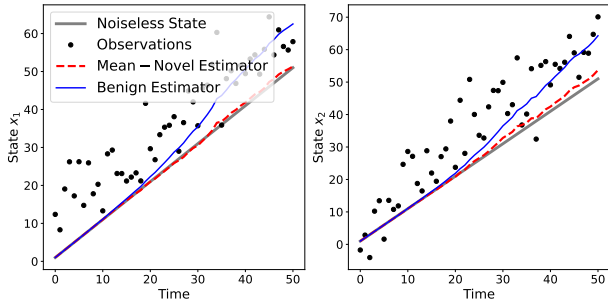
$$b_{k+1} = \begin{bmatrix} \lambda_H^{(2)} & \lambda_H^{(2)} \end{bmatrix}^T$$

9 2D Constant Velocity Vehicle

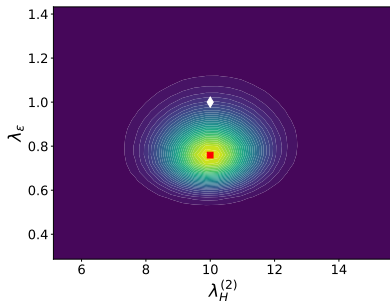


- “True values”: $\lambda_H^{(2)} = 10$ and $\lambda_\epsilon = 1$; known process scaling factor $\sigma = 0.02$
- Benign estimator starts to deviate from the true dynamics about half-way through the trajectory

Trajectory

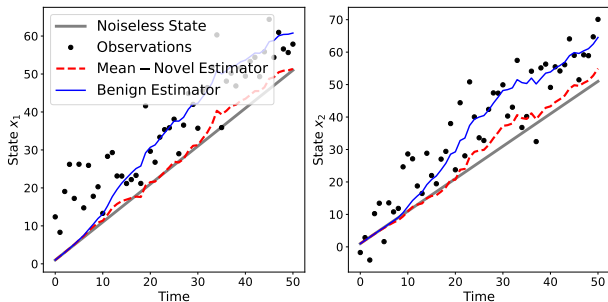
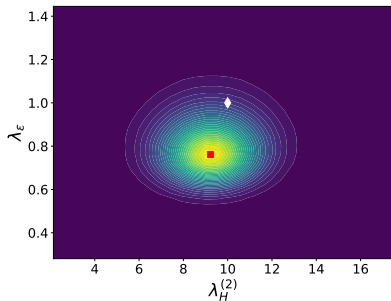


Posterior Distribution $p(\lambda_H^{(2)}, \lambda_\epsilon | \mathcal{D})$



- “True values”: $\lambda_H^{(2)} = 10$ and $\lambda_\epsilon = 1$; known process scaling factor $\underline{\sigma} = 0.1$
- Larger process uncertainties lead to benign estimator results that start to deviate earlier from the true dynamics while the novel estimator accounts for signal errors.

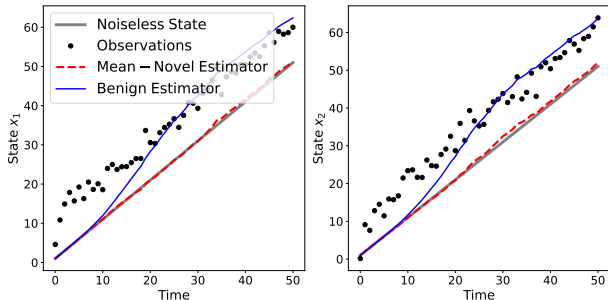
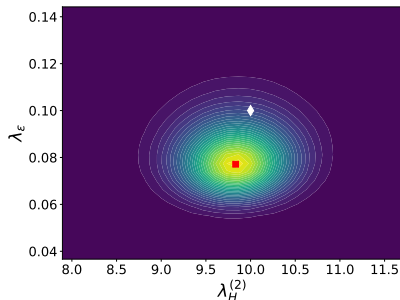
Trajectory

Posterior Distribution $p(\lambda_H^{(2)}, \lambda_\epsilon | \mathcal{D})$ 



- “True values”: $\lambda_H^{(2)} = 10$ and $\lambda_\epsilon = 0.1$; known process scaling factor $\sigma = 0.02$
- The benign estimator results start to deviate from the true trajectory earlier as the corruption model now dominate the observation process.

Trajectory

Posterior Distribution $p(\lambda_H^{(2)}, \lambda_\epsilon | \mathcal{D})$ 

- Assume a horizontal trajectory and zero-acceleration kinematics (balloon follows wind velocity)

$$\phi_{k+1} = \phi_k + v_\phi(t, \phi_k, \lambda_k) \Delta_t \text{ (longitude)}$$

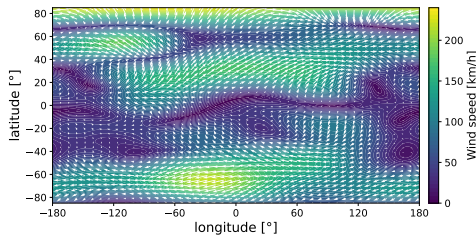
$$\lambda_{k+1} = \lambda_k + v_\lambda(t, \phi_k, \lambda_k) \Delta_t \text{ (latitude)}$$

- Observation and corruption models

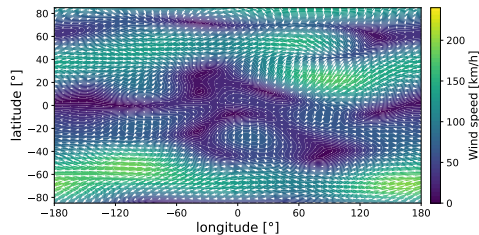
$$R = \lambda_\epsilon \mathbb{I}_2, \quad \mathbf{b} = [\pm \lambda_H^{(2)}, \pm \lambda_H^{(2)}]^T$$

- Velocity field generated via the Horizontal Wind Model version 2014 (doi:10.1002/2014EA000089)
- Example snapshots of the wind speed at 80km

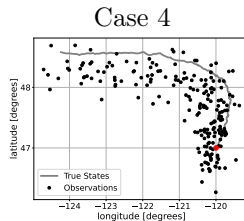
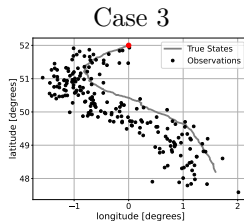
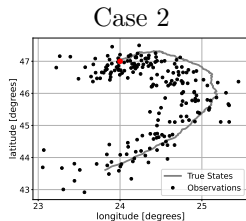
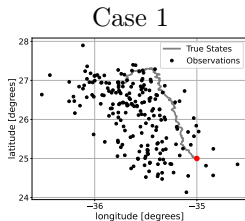
DoY 120, UT 8



DoY 120, UT 20



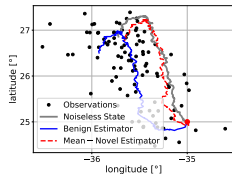
- Start from several initial locations.
- Same bias ($\lambda_H^{(2)} = 0.03^\circ$) applied to all location measurements and same observation covariance scaling factor ($\lambda_\epsilon = 0.06^\circ$).



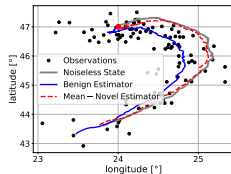


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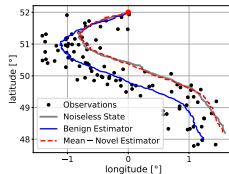
Case 1



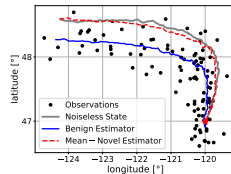
Case 2



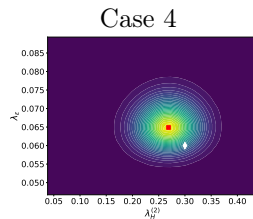
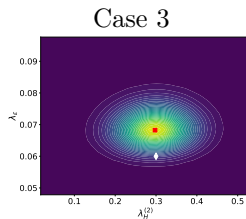
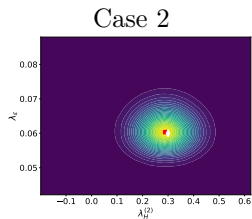
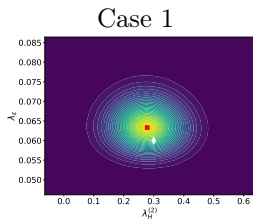
Case 3



Case 4



- Start from several initial locations.
- Same bias ($\lambda_H^{(2)} = 0.03^\circ$) applied to all location measurements and same observation covariance scaling factor ($\lambda_\epsilon = 0.06^\circ$).





- We proposed a Bayesian system identification framework that can learn system dynamics in the presence of sensor errors.
 - The joint posterior distribution for the model, process, and corruption parameters are based on a likelihood construction that employs the sequence of innovations along the trajectory.
- Parametric sweeps for 2D test models examined the interplay between process and observation uncertainties.
 - Results show the framework is able to learn canonical sensor corruption models.
- Future work will increase dynamical model complexity and the range of sensor error scenarios.