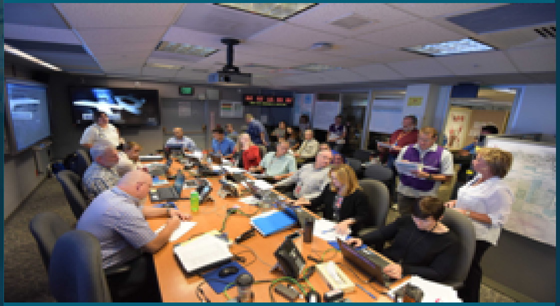
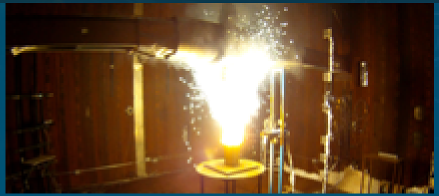
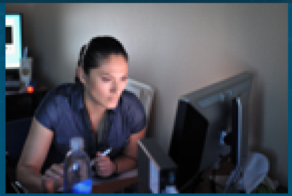
SAND2023-06959C

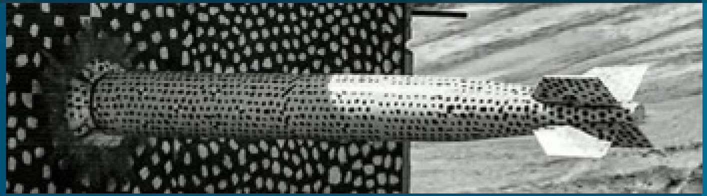
Sandia National Laboratories

Diodes and Sheaths: A Five-Moment Plasma Model Perspective









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U.S. DEPARTMENT OF ENERGY NNSA National Nuclear Security Administration

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Motivation and Outline

Motivation

- Developing Plasma simulation capabilities for Sandia mission work
- A V&V effort runs concurrently with the develop effort
- For verification, we seek to utilize analytic/semi-analytic plasma solutions that have bearing on the application space
- We also seek ever increasing physical complexity (e.g., elastic collisions)

Outline

- The Multifluid Plasma Models
- Energy exchange terms in Five-Moment models
- Equations of State
- Verification Examples: 1D Planar collisional **diodes** and floating collisional **sheaths**
  - Prototype Plasma Models
  - Semi-analytic versions of the models
  - Model differences and mitigation
  - Mesh refinement studies
- Summary and Perspectives

EMPIRE Five-Moment Multifluid Plasma Model with Collisions

EMPIRE-Fluid

- Use Maxwell Molecule collision model (analytic, collision rate is independent of relative velocity)
- Use simple ionization models
- Fluid equations discretized using discontinuous Galerkin FE
- IMEX: implicit EM and sources, explicit fluid transport
- Solves Maxwell equations for the electromagnetic fields using a compatible discretization based on edge-face elements

$$
\begin{aligned}
\frac{\partial \rho_\alpha}{\partial t} + \nabla\cdot[\rho_\alpha \mathbf{u}_\alpha] &= \mathbf{S}^{\mathcal{I}}_{\rho_{\alpha\beta}} \\
\frac{\partial \rho_\alpha \mathbf{u}_\alpha}{\partial t} + \nabla\cdot[\rho_\alpha \mathbf{u}_\alpha \otimes \mathbf{u}_\alpha + P_\alpha \mathbf{I}] &= \frac{q_\alpha \rho_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}) + \mathbf{R}_{\alpha\beta} + \mathbf{S}^{\mathcal{I}}_{\rho \mathbf{u}_{\alpha\beta}} \\
\frac{\partial \mathcal{E}_\alpha}{\partial t} + \nabla\cdot[(\mathcal{E}_\alpha + P_\alpha)\mathbf{u}_\alpha] &= \frac{q_\alpha \rho_\alpha}{m_\alpha}\mathbf{u}_\alpha\cdot\mathbf{E} + Q_{\alpha\beta} + \mathbf{u}_\alpha\cdot\mathbf{R}_{\alpha\beta} + \mathbf{S}^{\mathcal{I}}_{\mathcal{E}_{\alpha\beta}}
\end{aligned}
$$

$$P_\alpha = n_\alpha k_B T_\alpha \qquad \mathcal{E}_\alpha = \frac{P_\alpha}{\Gamma_\alpha - 1} + \frac{1}{2}\rho_\alpha ||\mathbf{u}_\alpha||^2$$

B(x) = 0, E is PEC on both walls

Multi-Fluid (ionization collisions)

$$
\begin{aligned}
<\sigma_{ion} v_e>_{const} &= \nu^{iz} = \text{constant} \\
\nu_{ion} &= \frac{n_{e0}}{n_{n0}} <\sigma_{ion} v_e>_{const} = K^{iz} n_{e0} \\
\nu_{ion} &= \frac{n_e}{n_{n0}} <\sigma_{ion} v_e>_{const} = K^{iz} n_e \\
S^{\mathcal{I}}_{\rho_e} &= \nu_{ion}\frac{m_e}{m_n}\rho_{n0} \\
S^{\mathcal{I}}_{\rho_i} &= \nu_{ion}\frac{m_i}{m_n}\rho_{n0} \\
S^{\mathcal{I}}_{\mathcal{E}_e} &= \nu_{ion}\frac{m_e}{m_n}\rho_{n0} P_{e0}/(\Gamma_e - 1) \\
S^{\mathcal{I}}_{\mathcal{E}_i} &= \nu_{ion}\frac{m_i}{m_n}\rho_{n0} P_{i0}/(\Gamma_i - 1)
\end{aligned}
$$

Five-Moment Maxwell Fluid Elastic collision frequency model

$$
\begin{aligned}
\sigma^{tr} &\equiv \sigma_0/g \\
\nu^M_{\alpha\beta} &= n_\beta \sigma_0 \\
\nu^M_{\beta\alpha} &= n_\alpha \sigma_0 \\
\nu^E_{\alpha\beta} &= \frac{m_{\alpha\beta}}{m_\alpha + m_\beta} n_\beta \sigma_0 \left[3 + \left(\frac{U_{\alpha\beta}}{v_{\alpha\beta}}\right)^2\right] \\
\nu^E_{\beta\alpha} &= \frac{m_{\beta\alpha}}{m_\beta + m_\alpha} n_\alpha \sigma_0 \left[3 + \left(\frac{U_{\beta\alpha}}{v_{\beta\alpha}}\right)^2\right] \\
\mathbf{U}_{\alpha\beta} &= (\mathbf{u}_\alpha - \mathbf{u}_\beta) \\
v_{\alpha\beta} &= \left(\frac{k_B T_\alpha}{m_\alpha} + \frac{k_B T_\beta}{m_\beta}\right)^{1/2} \qquad m_{\alpha\beta} = \frac{m_\alpha m_\beta}{m_\alpha + m_\beta}
\end{aligned}
$$

Maxwell Molecule Collision frequency is independent of velocity and analytic

$$
\begin{aligned}
\mathbf{R}_{\alpha\beta} &= -\nu^M_{\alpha\beta} m_{\alpha\beta} n_\alpha (\mathbf{u}_\alpha - \mathbf{u}_\beta) \\
\mathbf{R}_{\beta\alpha} &= -\nu^M_{\beta\alpha} m_{\beta\alpha} n_\beta (\mathbf{u}_\beta - \mathbf{u}_\alpha)
\end{aligned}
$$

$$
\begin{aligned}
Q_{\alpha\beta} &= n_\alpha \left[-\nu^E_{\alpha\beta} k_B (T_\alpha - T_\beta) + \frac{m_\beta}{T_\beta}\left(\frac{m_\alpha}{T_\alpha} + \frac{m_\beta}{T_\beta}\right)^{-1} \nu^M_{\alpha\beta} m_{\alpha\beta} (\mathbf{u}_\alpha - \mathbf{u}_\beta)^2\right] \\
Q_{\beta\alpha} &= n_\beta \left[-\nu^E_{\beta\alpha} k_B (T_\beta - T_\alpha) + \frac{m_\alpha}{T_\alpha}\left(\frac{m_\beta}{T_\beta} + \frac{m_\alpha}{T_\alpha}\right)^{-1} \nu^M_{\beta\alpha} m_{\beta\alpha} (\mathbf{u}_\beta - \mathbf{u}_\alpha)^2\right]
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{R}_{\beta\alpha} &= -\mathbf{R}_{\alpha\beta} \\
Q_{\beta\alpha} + \mathbf{u}_\beta\cdot\mathbf{R}_{\beta\alpha} &= -(Q_{\alpha\beta} + \mathbf{u}_\alpha\cdot\mathbf{R}_{\alpha\beta})
\end{aligned}
$$

SNL SAND2019-9621 (Shields et al.)

Surrogate Plasma Models

We use 1D surrogate models to explore algorithms, boundary conditions, source terms, etc.

Greatly Speeds up discovery

1D FV Multi-Fluid (**MF**): Euler-Poisson

- Third-order upwind bias, TVD limiter
- HLLC flux function

1D FD (**eulerCL**): Euler-Poisson

- Solves the electron species only
- Third-order upwind bias finite-difference

1D FD (**eulerPT**): Pressure-Temperature Poisson

- Solves the electron species only
- Third-order upwind bias

Explicit RK3 TVD time integration

Poisson equation solved with Thomas Algorithm

EoS: Energy, isentropic, isothermal

1D1V 5-Moment Model (**eulerCL, MF**)

$$
\begin{aligned}
\frac{\partial \rho_e}{\partial t} + \frac{\partial}{\partial x}\left[\rho_e u_e\right] &= 0 \\
\frac{\partial \rho_e u_e}{\partial t} + \frac{\partial}{\partial x}\left[\rho_e u_e u_e + P_e\right] &= \frac{q_e \rho_e}{m_e}\mathbf{E} + R_{en} \\
\frac{\partial \mathcal{E}_e}{\partial t} + \frac{\partial}{\partial x}\left[\left(\mathcal{E}_e + P_e\right) u_e\right] &= \frac{q_e \rho_e}{m_e} u_e \mathbf{E} + u_e R_{en} + Q_{en} \\
\mathcal{E}_e &= \frac{P_e}{(\Gamma_e - 1)} + \frac{1}{2}\rho_e u_e^2 \\
P_e &= n_e k_B T_e \\
\frac{\partial^2 \phi}{\partial x^2} &= \frac{n_e e}{\epsilon_0}
\end{aligned}
$$

1D1V Internal Energy Model (**eulerPT**)

$$
\begin{aligned}
\frac{\partial n_e}{\partial t} + \frac{\partial (n_e v_e)}{\partial x} &= 0 \\
\frac{\partial v_e}{\partial t} + v_e \frac{\partial v_e}{\partial x} &= -\left(\frac{1}{m_e n_e}\right)\frac{\partial P_e}{\partial x} + \frac{q_e}{m_e} E + R_{en} \\
\frac{\partial T_e}{\partial t} + v_e \frac{\partial T_e}{\partial x} &= -\frac{\Gamma_e - 1}{n_e k_B} P_e \frac{\partial v_e}{\partial x} + \frac{\Gamma_e - 1}{n_e k_B} Q_{en} \\
P_e &= n_e k_B T_e \\
\frac{\partial^2 \phi}{\partial x^2} &= -\frac{n_e q_e}{\epsilon_0}
\end{aligned}
$$

Model Differences: Energy Exchange

power from electric field to fluid

Total energy (1D1V2S)

$$\frac{\partial \mathcal{E}_e}{\partial t} + \frac{\partial}{\partial x}\left[(\mathcal{E}_e + P_e)\, u_e\right] = \frac{q_e \rho_e}{m_e} u_e \mathbf{E} + u_e R_{en} + Q_{en}$$

internal energy xc between species

Langevin Equation: constant: E, ν , $u_n \approx 0, m_e \ll m_n$

$$n_e m_e \dot{u} = -eEn_e - m_e n_e \nu (u_e - u_n)$$

Steady solution for velocity

$$u_e = -\frac{eE}{m_e \nu}$$

Power per unit volume from electric field to the fluid

$$u_e(-eE)n_e = u_e(u_e m_e \nu) n_e$$

Power per unit volume converted to heat

$$\mathcal{P}_e = \eta J_e^2 \qquad \eta = \frac{m_e \nu}{e^2 n_e} \qquad J_e = -e n_e u_e$$

Energy equation (1D) power sources due to E

$$-e n_e u_e E, \quad u_e R_{en}$$

Energy exchange between species

$$Q_{en} \approx n_e \nu\left[-k_B (T_e - T_n) + m_e (u_e - u_n)^2\right]$$

(Tanenbaum 1967, Freidberg, 20

Equations of State Approximations

Five-Moment model solves total energy equation

Classic models typically reduce complexity by choosing a simplified EOS

- Isentropic, pressure proportional to density raised to gamma
- Isothermal, T=constant

Compressibility is reduced by choosing $\Gamma = 1 + \varepsilon$

Five-Moment Energy Transport and EOS

$$
\begin{aligned}
\frac{\partial \mathcal{E}_\alpha}{\partial t} + \nabla \cdot \left[\left(\mathcal{E}_\alpha + P_\alpha \right) \mathbf{u}_\alpha \right] &= \frac{q_\alpha \rho_\alpha}{m_\alpha} \mathbf{u}_\alpha \cdot \mathbf{E} + Q_{\alpha\beta} + \mathbf{u}_\alpha \cdot \mathbf{R}_{\alpha\beta} \\
P_\alpha &= (\Gamma_\alpha - 1)(\mathcal{E}_\alpha - \frac{1}{2}\rho_\alpha ||\mathbf{u}_\alpha||^2) \\
T_\alpha &= \frac{P_\alpha}{n_\alpha k_B} \\
a_\alpha^2 &= \Gamma_\alpha \frac{P_\alpha}{\rho_\alpha}
\end{aligned}
$$

Isentropic (adiabatic, reversible, diode)

$$
\begin{aligned}
\frac{P}{P_0} &= \left(\frac{\rho}{\rho_0}\right)^\Gamma = \left(\frac{n}{n_0}\right)^\Gamma \\
A_0 &= \frac{n_0 k_B T_0}{(m n_0)^\Gamma} \\
P &= A_0 \rho^\Gamma \\
T &= \frac{P}{n k_B} \\
a^2 &= \Gamma \frac{P}{\rho}
\end{aligned}
$$

Isothermal (sheath)

$$
\begin{aligned}
T &= T_0 = \text{constant} \\
P &= n k_B T \\
a^2 &= \frac{P}{\rho}
\end{aligned}
$$

Γ and Compressibility

$$
\begin{aligned}
T &= T_0 \left(\frac{\rho}{\rho_0}\right)^{\Gamma - 1} \\
P &= n k_B T \\
a^2 &= \Gamma \frac{P}{\rho}
\end{aligned}
$$

1D Planar Diode Theory and Prototype Models

Child-Langmuir Law for space charge limit current density (SCL) Child,1911, Langmuir, 1913)

- The drift velocity v_e=0 and the number density n_e is infinite

$$J^{CL} = \frac{4}{9}\epsilon_0 \left(\frac{2|q|}{m_e}\right)^{1/2} \frac{|V|^{3/2}}{d^2}$$

Cold Diode solution (Jaffe, 1944)

- Cold electron beam with finite drift and density
- Semi-analytic, well suited for Particle-in-Cell kinetic solvers (Smith et al. 2019)

$$J_0 = \frac{16}{9}\epsilon_0 \left(\frac{2|q|}{m_e}\right)^{1/2} \frac{|W/q|^{3/2}}{d^2}$$

$$J^J = \frac{J_0}{4}\left[1 + \left(1 - \frac{qV}{W}\right)^{1/2}\right]^3$$

Warm Diode (Rokhlenko&Lebowitz, Oliver, Hamlin et al., 2022)

- Finite temperature, drift velocity and number density
- Includes hydrodynamic force due to finite temperature through the pressure gradient and Lorentz force
- Assumes an isentropic eos

Mott-Gurney Diode (Akimov & Schamel, 2002)

- Cold fluid, includes friction force due to collisions and Lorentz force
- Recovers the Mott-Gurney SCL formula

$$J^{MG} = \frac{9}{8}\epsilon_0 \left(\frac{|q|}{m_e \nu}\right) \frac{|V|^2}{d^3}$$

Warm Mott-Gurney Diode

- Includes **hydrodynamic**, **Lorentz** and **friction** forces due to collisions
- In the limit of P→0, recovers the the Mott-Gurney solution
- In the limit as nu→0, recovers the warm diode solution
- In the limit as P →0 and nu→0, recovers the Jaffe solution
- Parameterize collisions by Knudsen number

$$K_n = \frac{\lambda mfp}{d} = \frac{v_{e0}}{\nu d}$$

Semi-Analytic Diode Models

Physical Models

Warm Diode (Rokhlenko&Lebowitz, Oliver, Hamlin et al., **Hamlin2022**)

$$
\begin{aligned}
\frac{\partial n_e}{\partial t} + \frac{\partial (n_e v_e)}{\partial x} &= 0 \\
\frac{\partial v_e}{\partial t} + v_e \frac{\partial v_e}{\partial x} &= -\frac{e}{m_e} E - \frac{1}{m_e n_e} \frac{\partial P_e}{\partial x} \\
\frac{\partial^2 \phi}{\partial x^2} &= \frac{e}{\epsilon_0} n_e \\
P_e &= A n_e^\Gamma \\
E &= -\frac{\partial \phi}{\partial x}
\end{aligned}
$$

Mott-Gurney Warm Diode (Akimov & Schamel, **AK2002**)

$$
\begin{aligned}
\frac{\partial n_e}{\partial t} + \frac{\partial (n_e v_e)}{\partial x} &= 0 \\
\frac{\partial v_e}{\partial t} + v_e \frac{\partial v_e}{\partial x} &= -\frac{e}{m_e} E - \frac{1}{m_e n_e} \frac{\partial P_e}{\partial x} - \nu v_e \\
\frac{\partial^2 \phi}{\partial x^2} &= \frac{e}{\epsilon_0} n_e \\
P_e &= A n_e^\Gamma \\
E &= -\frac{\partial \phi}{\partial x}
\end{aligned}
$$

Transformed Models

Warm Diode SST

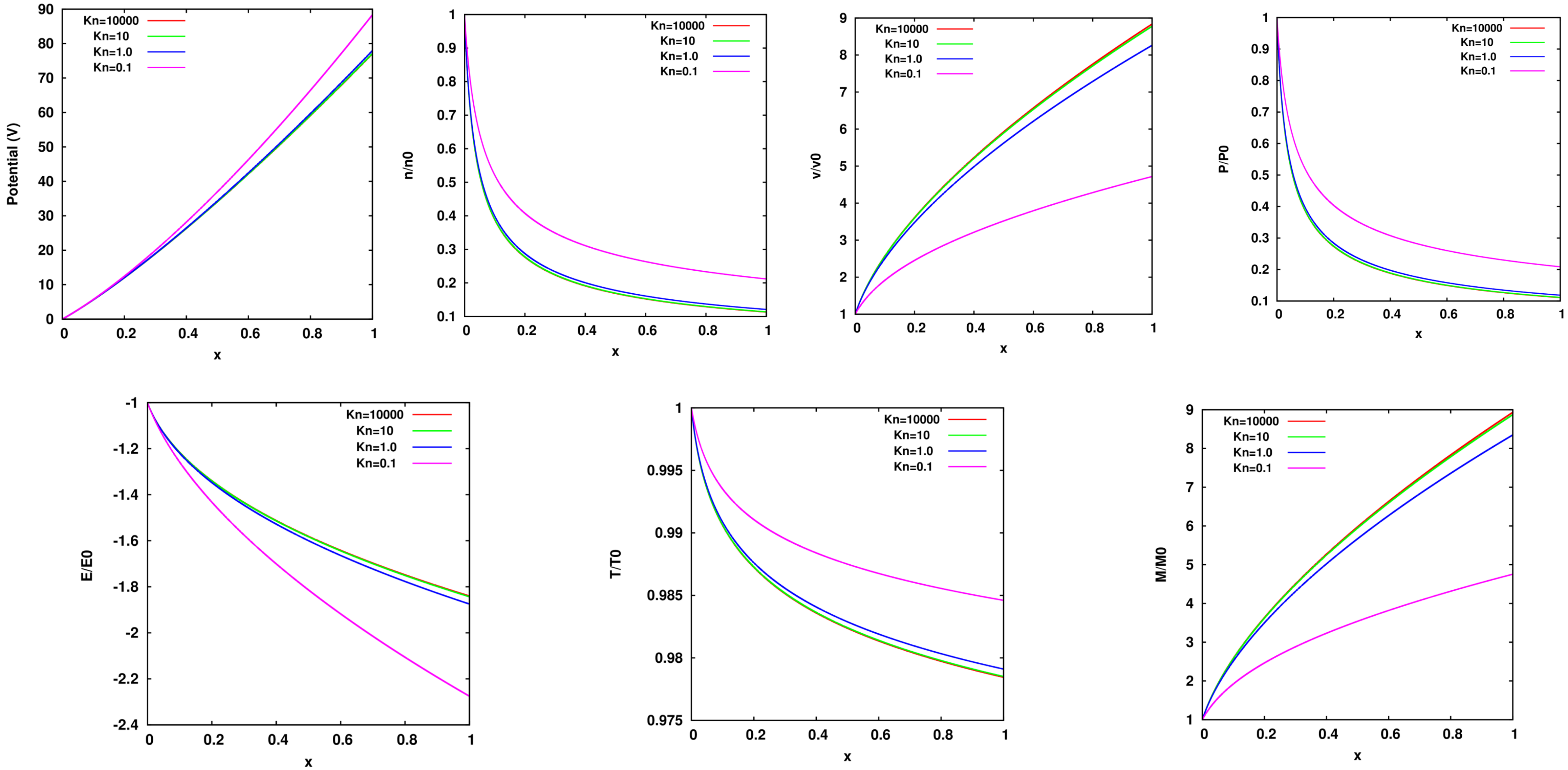
$$
\begin{aligned}
J_{e0} &= n_e e v_e = \text{constant} \\
v_e \frac{\partial v_e}{\partial x} &= -\frac{e}{m_e} E - \frac{1}{m_e n_e} \frac{\partial P_e}{\partial x} \\
\frac{\partial E}{\partial x} &= -\frac{e}{\epsilon_0} n_e \\
P_e &= A n_e^\Gamma
\end{aligned}
$$

Mott-Gurney Warm Diode SST

$$
\begin{aligned}
J_{e0} &= n_e e v_e = \text{constant} \\
v_e \frac{\partial v_e}{\partial x} &= -\frac{e}{m_e} E - \frac{1}{m_e n_e} \frac{\partial P_e}{\partial x} - \nu v_e \\
\frac{\partial E}{\partial x} &= -\frac{e}{\epsilon_0} n_e \\
P_e &= A n_e^\Gamma
\end{aligned}
$$

Warm Mott-Gurney Diode: Knudsen Number Sweep

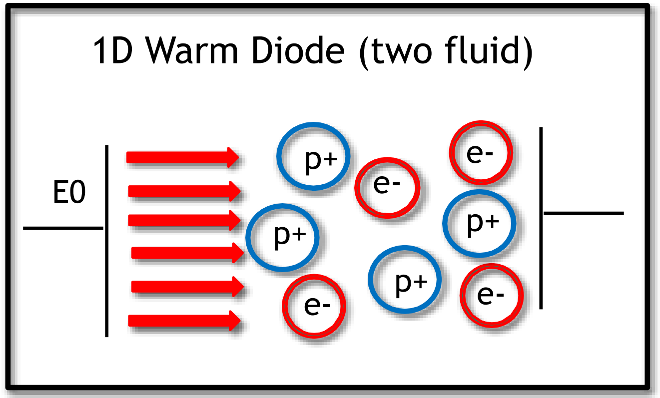
(Γ=1.01, W=1eV, Te=0.01eV, d=0.01m, I0=10A, E0=-5000V/m)

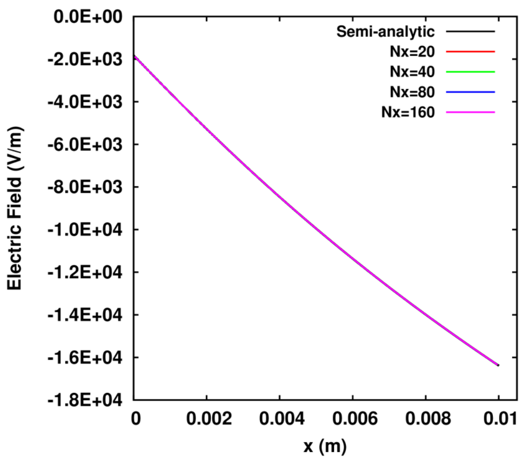


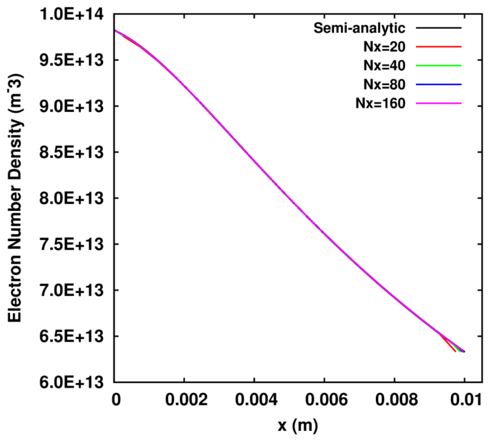
Warm Diode: EMPIRE Mesh Refinement Study
(Γ=1.01, Te=100eV, W=100eV, J0=93.35A, E0=-1821)

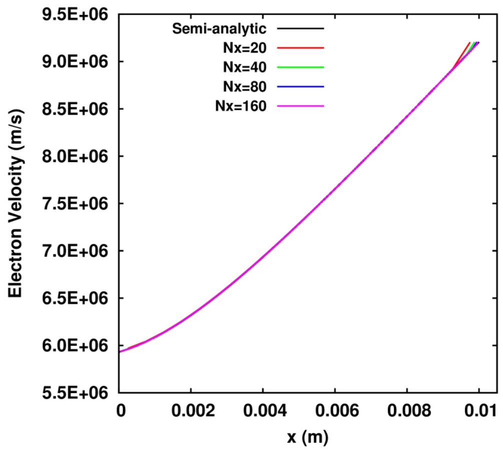
Details

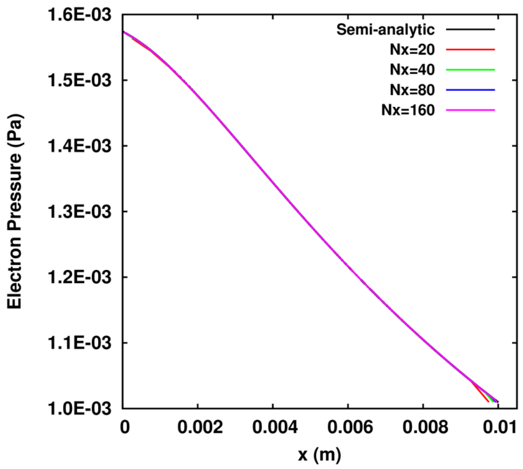
- Obtain a solution for electron fluid
- Approximately isentropic
- Supersonic inflow and outflow
- No collisions

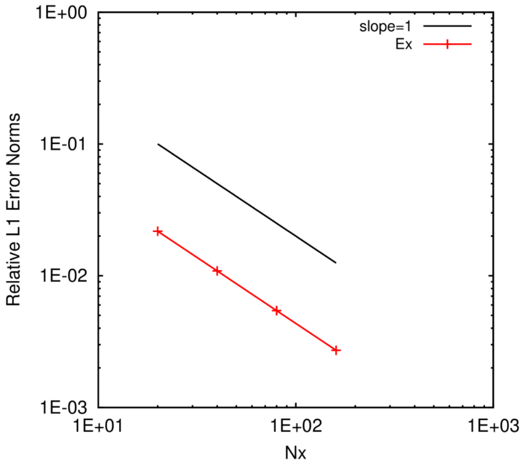


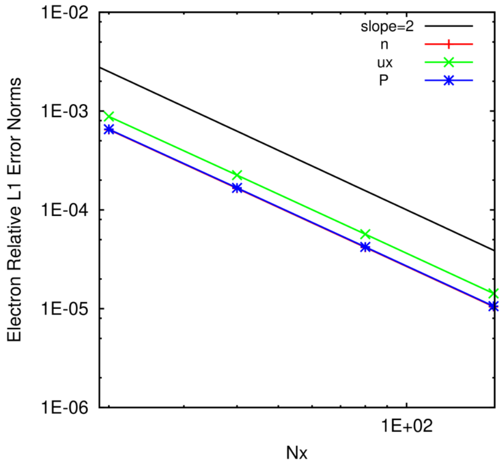








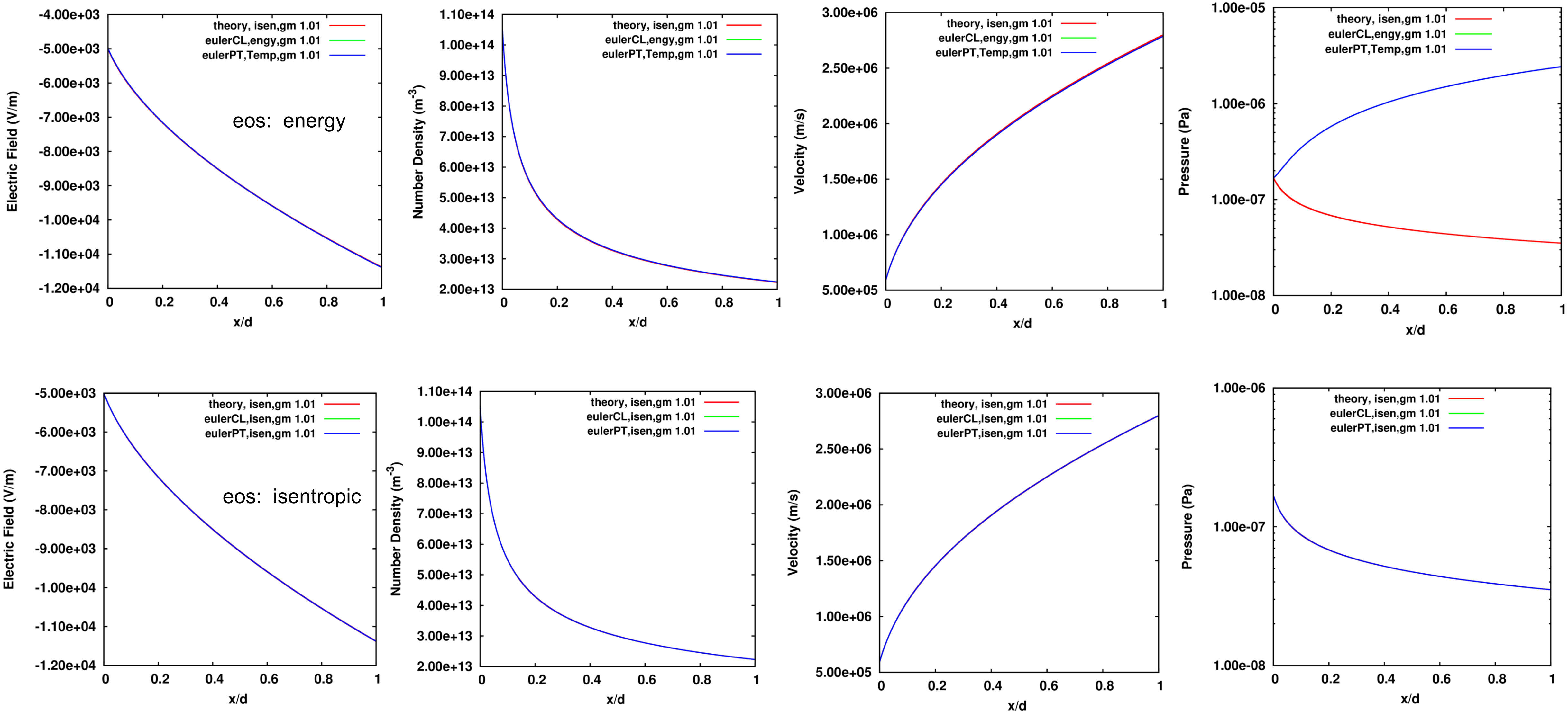




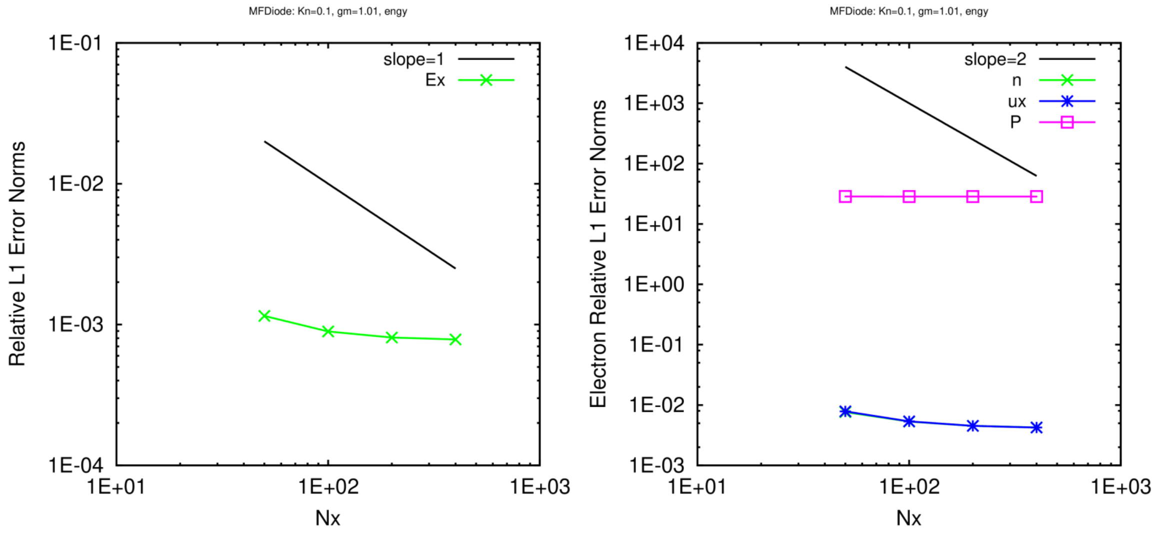
Achieve expected order of accuracy

Mott-Gurney Warm Diode: eulerCL, eulerPT

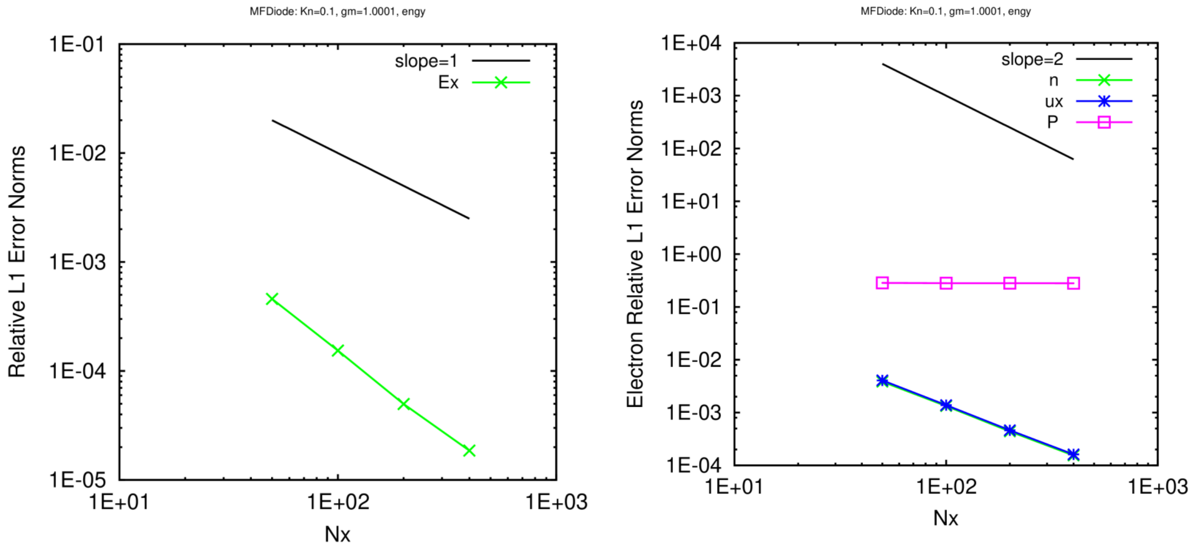
(Kn=0.1, Γ=1.01, J0=10, W=1eV, T0=0.1eV)



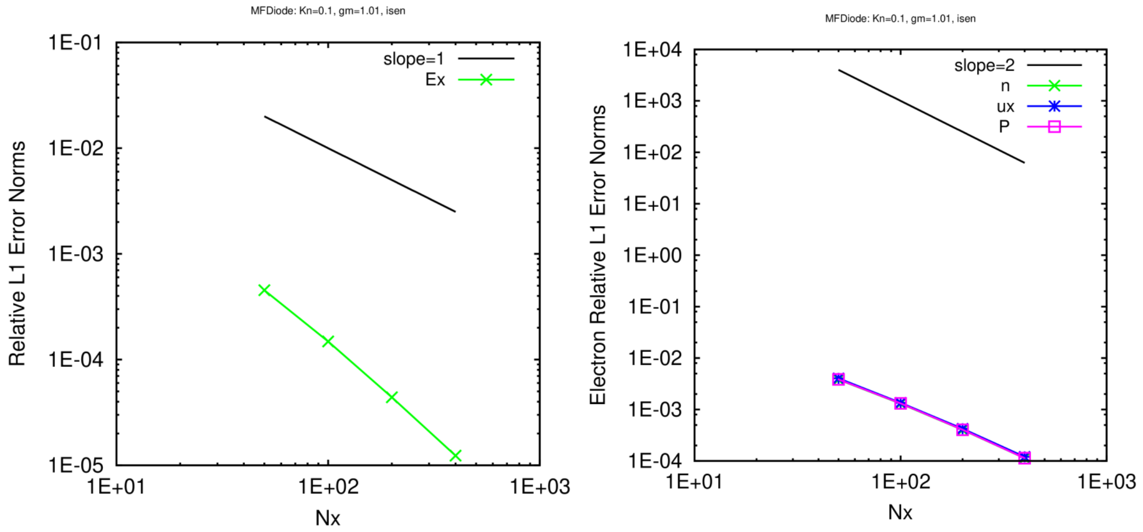
Mott-Gurney Diode MF: (Kn=0.1, EoS Sensitivity to Γ)



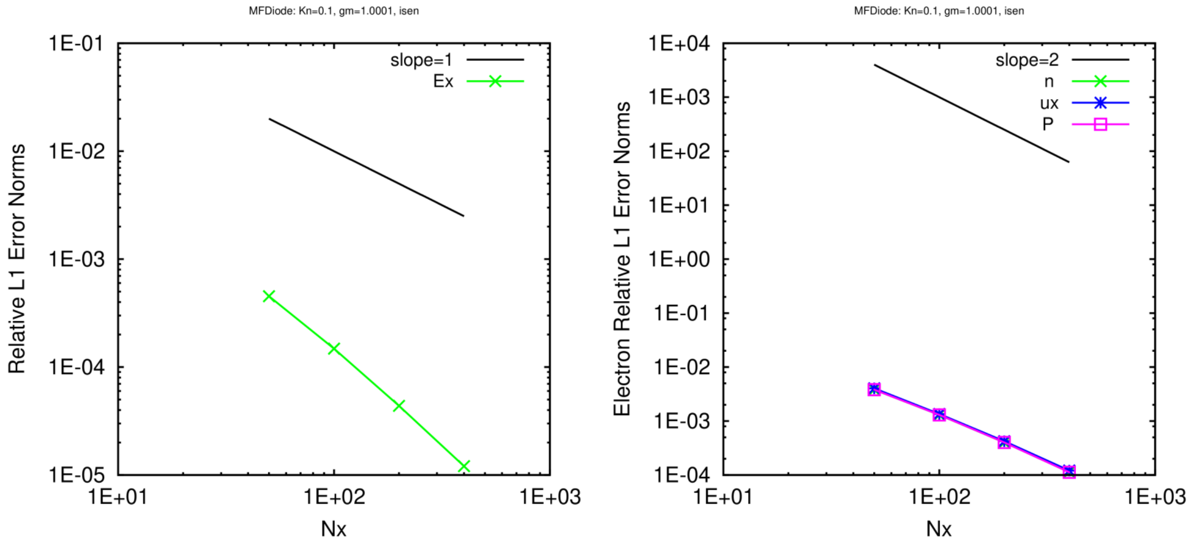
Γ=1.01, energy eos



Γ=1.0001, energy eos

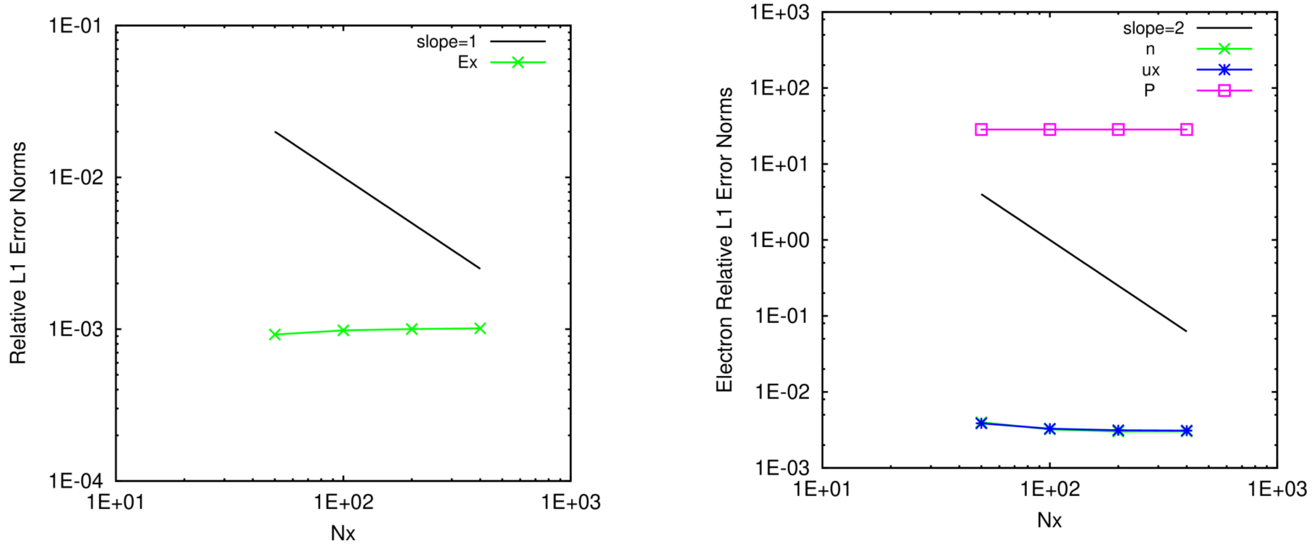


Γ=1.01, isentropic eos

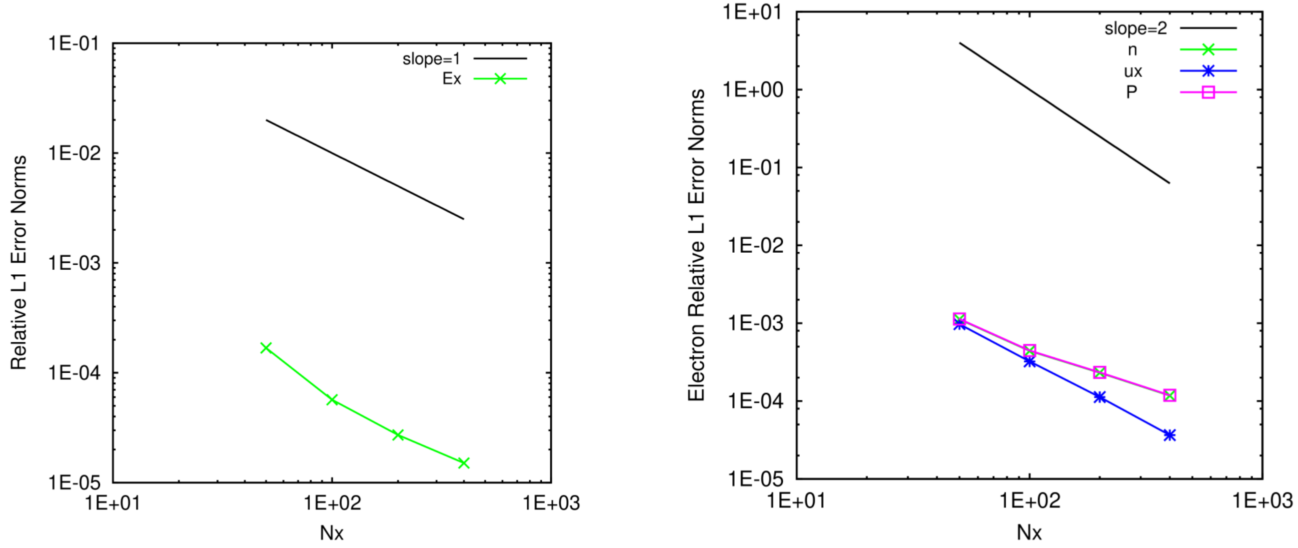


Γ=1.0001, isentropic eos

Mott-Gurney Diode: EMPIRE Energy Equation and Isentropic EoS

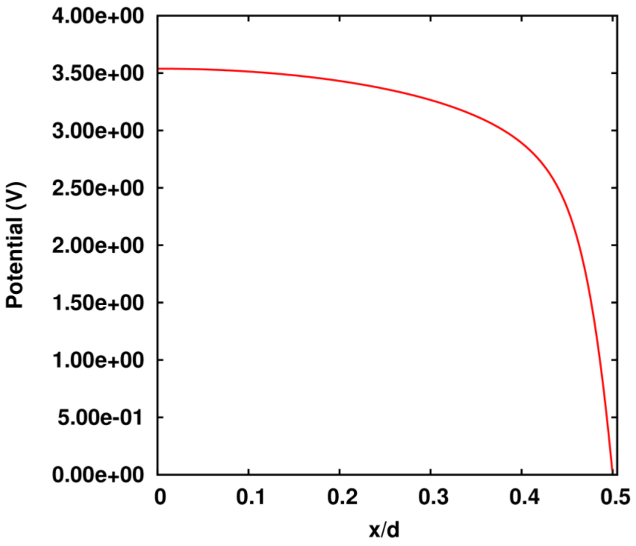


Energy, Kn=0.1, Γ=1.01

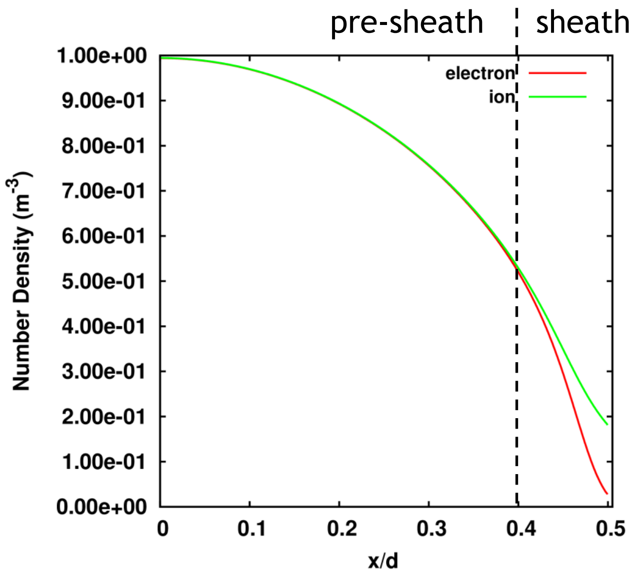


Isentropic, Kn=0.1, Γ=1.01

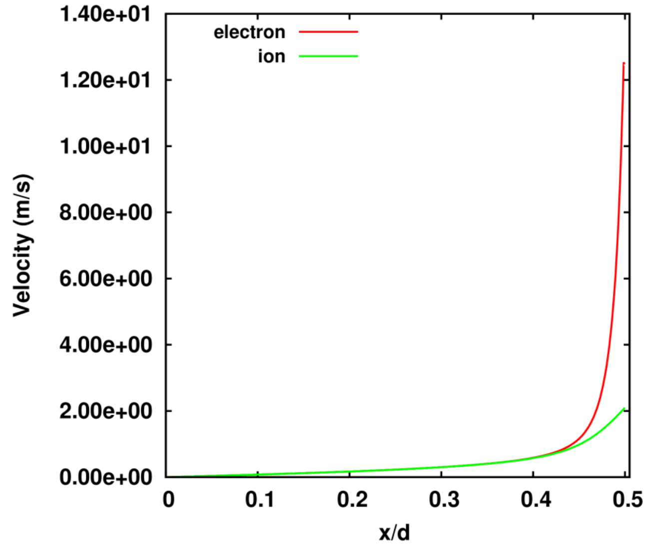
Characteristics of the 1D SST Symmetric Planar Sheath



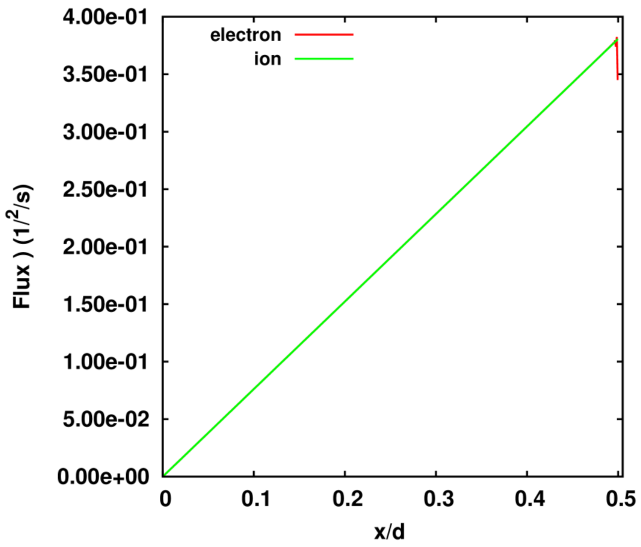
Floating potential, phi_w=0



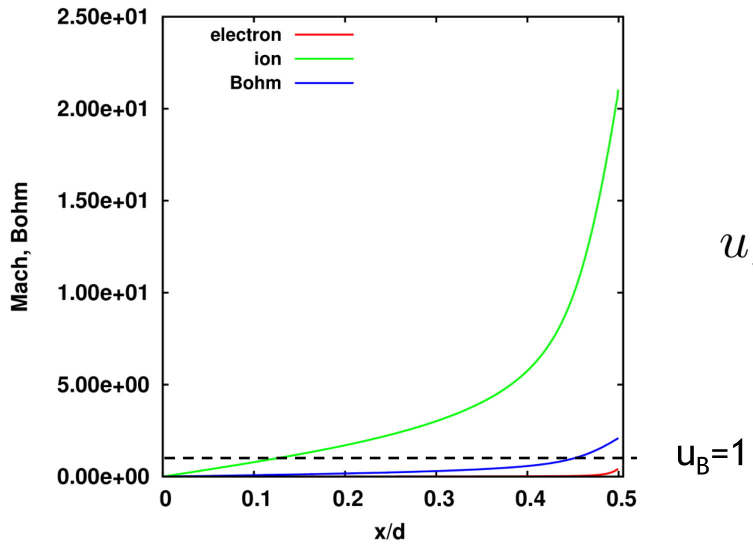
Absorbing walls for electrons and ions



Electron and ion velocity



Constant ionization rate



Sheath location defined at the Bohm velocity

$$u_B = \sqrt{k_B T_e / m_i}$$

Semi-Analytic Sheath models of Alvarez-Laguna et al. and Chabert

AL2020 model (Alvarez-Laguna et al., **AL2020**)

$$
\begin{aligned}
\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e u_e) &= n_e \nu^{iz} \\
\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) &= n_e \nu^{iz} \\
m_e \frac{\partial n_e u_e}{\partial t} + \frac{\partial}{\partial x}(m_e n_e u_e^2 + P_e) &= n_e e \frac{\partial \phi}{\partial x} - m_e u_e n_e \nu_{en} \\
m_i \frac{\partial n_i u_i}{\partial t} + \frac{\partial}{\partial x}(m_i n_i u_i^2 + P_i) &= -n_i e \frac{\partial \phi}{\partial x} - m_i u_i n_i \nu_{in} \\
\frac{\partial^2 \phi}{\partial x^2} &= \frac{(n_e - n_i)}{\epsilon_0} e
\end{aligned}
$$

$$P_e = n_e k_B T_e \quad P_i = n_i k_B T_i$$

AL2020 Transformed model

$$
\begin{aligned}
\frac{\partial \bar{n}_e \bar{u}_e}{\partial \bar{x}} &= \frac{\partial \bar{n}_i \bar{u}_i}{\partial \bar{x}} = \bar{K}^{iz} \\
\left[\frac{\bar{n}_e^2}{\epsilon} - (\bar{n}_e \bar{u}_e)^2\right] \frac{\partial \bar{n}_e}{\partial \bar{x}} &= -2\bar{K}^{iz} \bar{u}_e \bar{n}_e^2 - \bar{\nu}_{en} \bar{u}_e \bar{n}_e^3 + \frac{\bar{n}_e^3}{\epsilon} \frac{\partial \bar{\phi}}{\partial \bar{x}} \\
\left[\kappa \bar{n}_i^2 - (\bar{n}_i \bar{u}_i)^2\right] \frac{\partial \bar{n}_i}{\partial \bar{x}} &= -2\bar{K}^{iz} \bar{u}_i \bar{n}_i^2 - \bar{\nu}_{in} \bar{u}_i \bar{n}_i^3 - \bar{n}_i^3 \frac{\partial \bar{\phi}}{\partial \bar{x}} \\
\frac{\partial^2 \bar{\phi}}{\partial \bar{x}} &= \frac{\bar{n}_e - \bar{n}_i}{\bar{\lambda}^2}
\end{aligned}
$$

$$n_0 = n_e = n_i, \quad \epsilon = m_e/m_i, \quad \kappa = T_i/T_e = 0, \quad \phi_0 = k_B T_e/e, \quad T_e = \text{constant}$$

$$u_0 = u_B = \sqrt{k_B T_e/m_i}, \quad t_0 = L_0/u_0, \quad \lambda_{D0} = (\varepsilon_0 k_B T_e/e^2 n_0)^{1/2}$$

$$\bar{\nu}^{iz} = t_0 \nu^{iz}, \quad \bar{\nu}_{en} = t_0 \nu_{en}, \quad \bar{\nu}_{in} = t_0 \nu_{in}, \quad \bar{\lambda} = \lambda_{D0}/L_0$$

$$
\begin{aligned}
\bar{K}^{iz} &= \text{constant} = \bar{\nu}^{iz} \\
\bar{K}^{iz} &= \bar{n}_e \text{ constant} = \bar{n}_e \bar{\nu}^{iz}
\end{aligned}
$$

Chabert model (Chabert, 2014, **Chabert**)

$$
\begin{aligned}
\frac{d}{dx}(n_i u_i) &= \nu_I n_e \\
n_i m_i u_i \frac{du_i}{dx} &= n_i e E - m_i u_i (\nu_I n_e + \nu_{mom} n_i) \\
n_e &= n_{e0} \exp\left(\frac{e\phi}{k_B T_e}\right) \\
\frac{dE}{dx} &= \frac{e}{\epsilon_0}(n_i - n_e)
\end{aligned}
$$

$$T_e = \text{constant}, \quad T_i = 0, \quad m_e = 0, \quad \nu_I = \text{constant}$$

Chabert Transformed model

$$
\begin{aligned}
(nu)' &= e^{\eta} \\
(nuu)' &= n\epsilon - \beta n u \\
q^2 \epsilon' &= n - e^{\eta} \\
\eta' &= -\epsilon
\end{aligned}
\qquad
\begin{aligned}
\xi &= x/\lambda_I \\
n &= n_i/n_0 \\
u &= u_i/u_B \\
\eta &= e\phi/k_B T_e \\
\epsilon &= eE\lambda_I/k_B T_e
\end{aligned}
\qquad
\begin{aligned}
q &= \frac{\lambda_D}{\lambda_I} \\
\lambda_I &= \frac{u_B}{\nu_I} \\
\beta &= \nu_M/\nu_I
\end{aligned}
$$

Alvarez-Laguna et al., Plasma Sources Sci. and Tech. 29, 2020.
Chabert, Plasma Sources Sci. and Tech. 23, 2014.

Semi-Analytic Solution Comparison: Chabert and AL2020

Chabert, Plasma Sources Sci. and Tech. 23, 2014.
Alvarez-Laguna et al., Plasma Sources Sci. and Tech. 29, 2020.

- Ionization source is S=nu_iz*n_e(x)
- **AL2020** – Mathematica IVP solution to the AL2020 model (T_i=0)
- **Chabert** – Mathematica IVP solution to Chabert model (T_i=0)
- With no elastic collisions, sheath solution is a function of one parameter; q
- Neither model specifies wall BCs
- Flux is non-zero at centerline for symmetric case
- nu_iz=constant
- Obtain solutions for electrons and ions

$$q = \frac{\lambda_D}{\lambda_I} = \frac{\lambda_D \nu_I}{u_B}$$

$$\lambda_D = \left(\epsilon_0 k_B T_e / e^2 n_0\right)^{1/2}$$

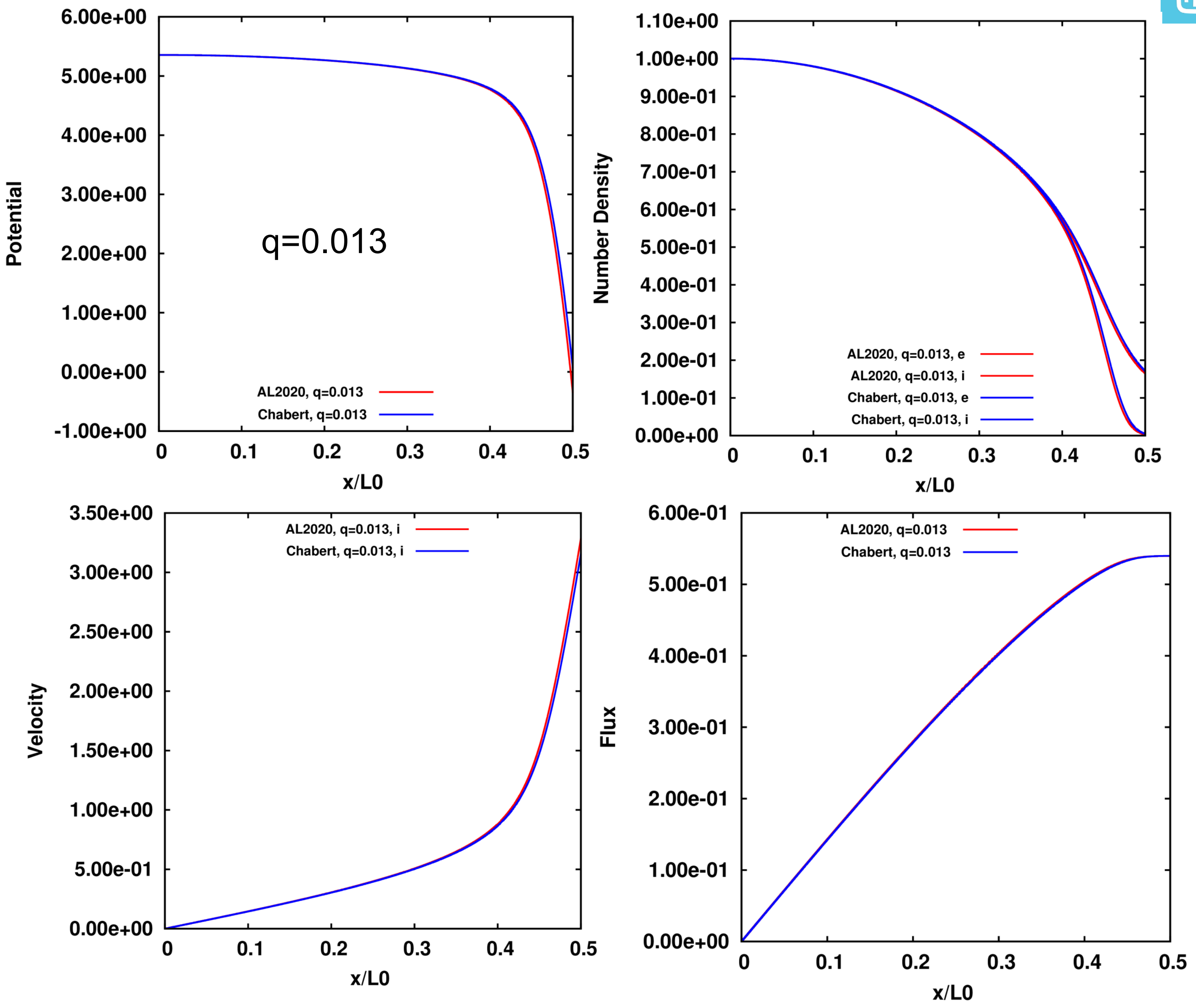
$$u_B = \sqrt{k_B T_e / m_i}$$

$$\beta = \nu_M / \nu_I$$

$$K_n = \frac{\lambda_{mfp}}{L_0} = \frac{u_B}{L_0 \nu} = \frac{u_B}{\sigma_0 n_n L_0}$$

$$\lambda_{mfp} = \frac{u_B}{\nu}$$

$$\nu = \sigma_0 n_n$$



Absorbing Wall BCs: Vacuum and Thermal Wall Flux

Vacuum BCs

$$\mathcal{F}_{ew}^{HLLC}(\mathbf{U}_e^I, \mathbf{U}_{vac}^g)$$

$$\mathbf{U}_{vac}^g = \left\{ \begin{array}{lcl} \rho_{vac} & = & \text{const.} \\ u_{vac} & = & \text{const.} \\ P_{vac} & = & \text{const.} \end{array} \right\}$$

P. Cagas et al., A boundary value "reservoir problem" and boundary conditions for multi-component multi-fluid simulations of sheaths, Phys. of Plasmas, 28, 2021.

Thermal flux BCs (const. Tw)

$$\mathcal{F}_{ew}^{HLLC}(\mathbf{U}_e^I, \mathbf{U}_{th}^g)$$

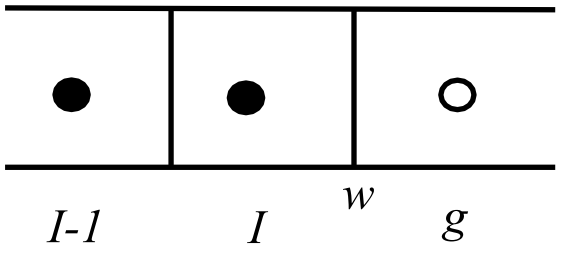
$$\mathbf{U}_{th}^g = \left\{ \begin{array}{c} \rho_e^g \\ u_e^g \\ P_e^g \end{array} \right\}$$

given: T_{ew}

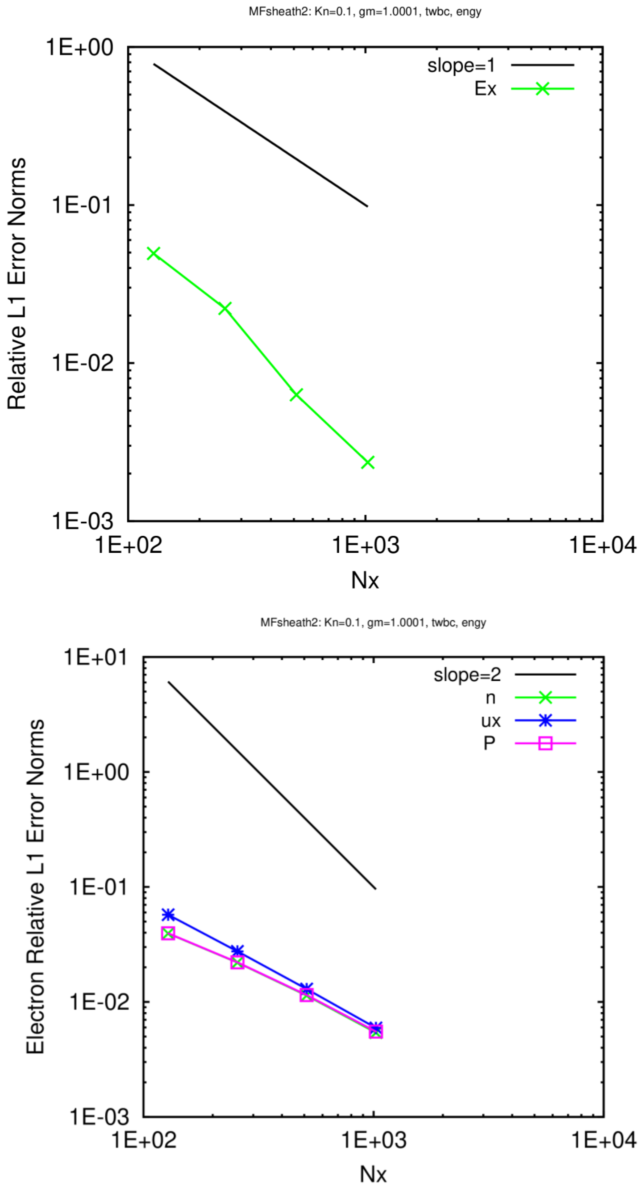
$$\begin{array}{lcl} u_{ew} & = & \sqrt{\dfrac{k_B T_{ew}}{2\pi m_e}} \\ \Gamma_e^g & = & 2\Gamma_e|_I - \Gamma_e|_{I-1} \\ & \text{or} & \\ \Gamma_e^g & = & \Gamma_e|_I \\ u_e^g & = & 2u_{ew} - u_e|_I \\ \rho_e^g & = & \Gamma_e^g / u_e^g \\ T_e^g & = & 2T_{ew} - T_e|_I \\ P_e^g & = & \rho_e^g k_B T_e^g / m_e \end{array}$$

Alvarez Laguna et al., Plasma-sheath transition in multi-fluid models with inertial terms under low pressure conditions: comparison with the classical kinetic theory. Plasma Src. Sci. and Tech., 29, 2020.

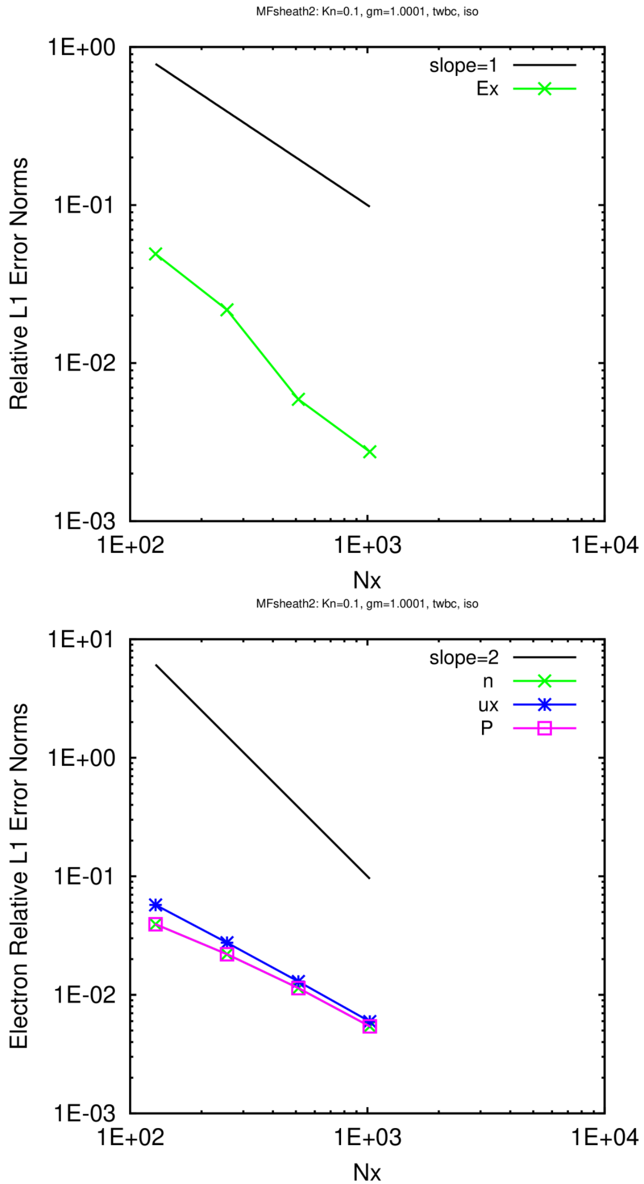
Finite volume stencil at right wall



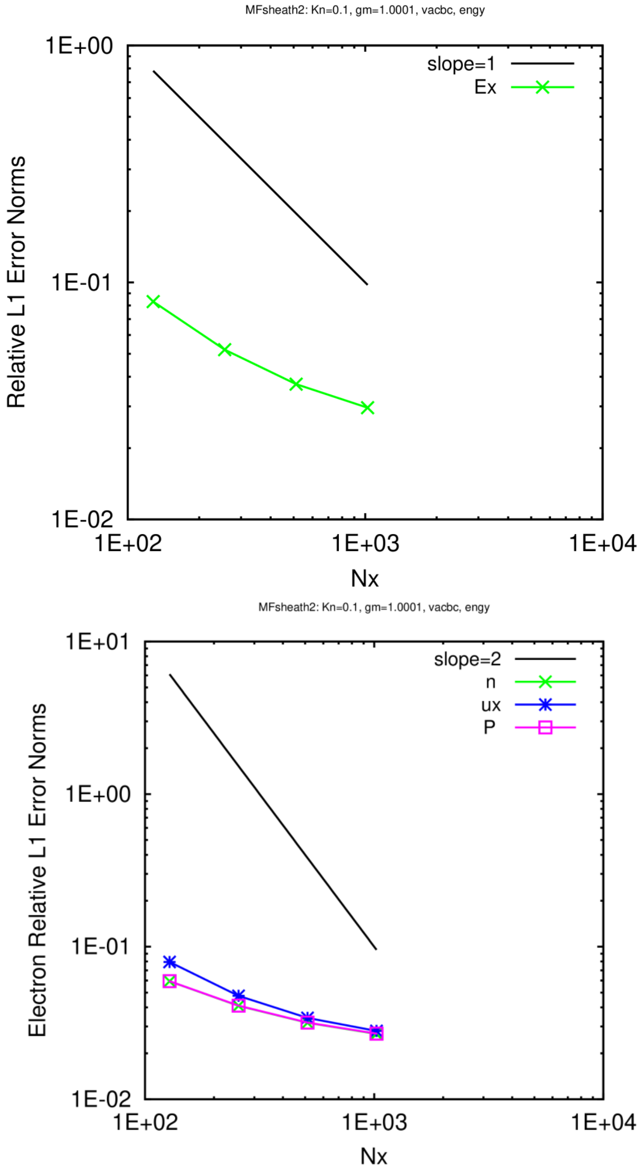
Planar Sheath Example: AL2020, MF (Γ=1.0001, Kn=0.1)



Thermal flux wall BC, energy



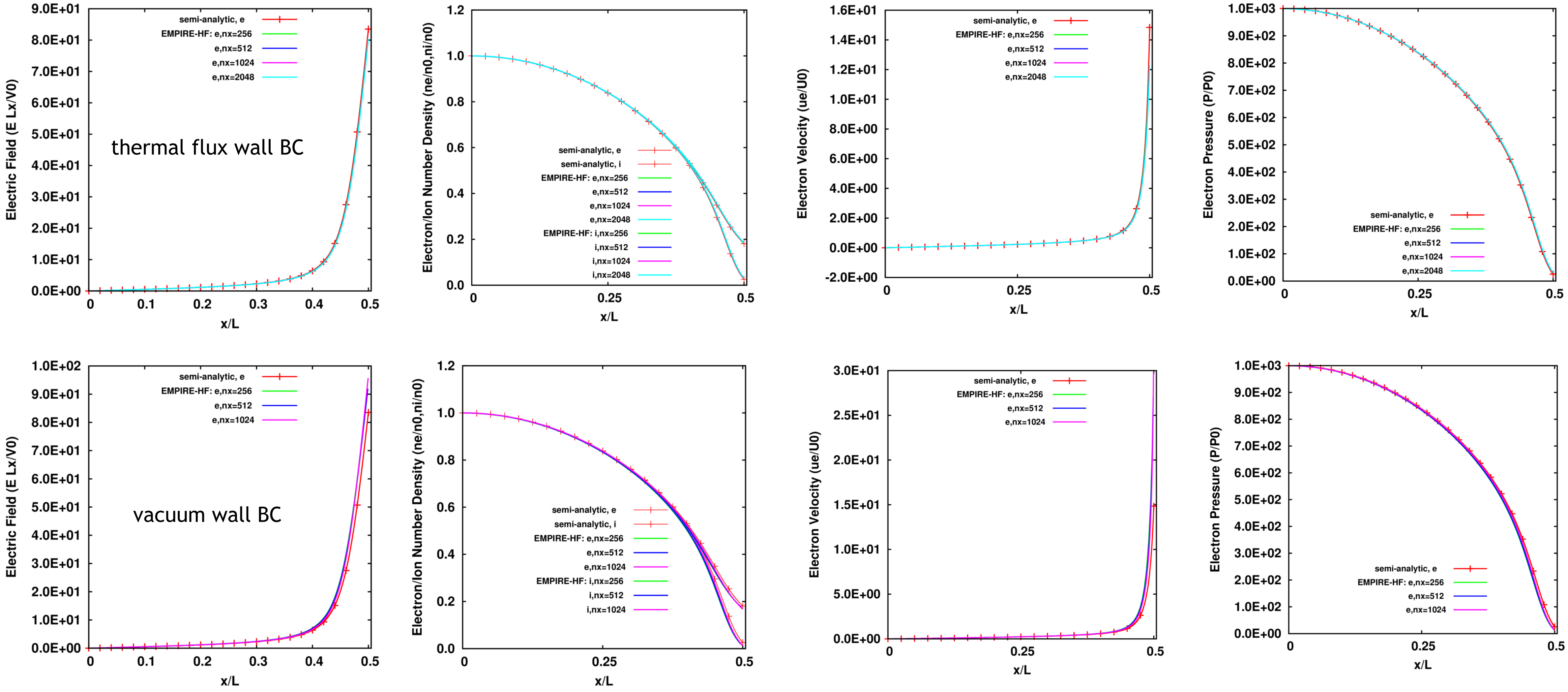
Thermal flux wall BC, isothermal



Vacuum wall BC, energy

Planar Sheath Results: EMPIRE (Γ=1.0001, Kn=0.1, Energy Eqn.)

These results are in *good qualitative agreement* with the semi-analytic models



Summary and Perspectives

We discussed difficulties associated with using classic plasma models for verification

- Energy exchange terms in the 5-moment model are important when collisions are present
- Zero ion temperature in Chabert and AL2020 sheath models can only be approximated

Despite these difficulties, partial verification of the 5-moment model has be achieved:

- Warm diode: hydrodynamic and Lorentz force coupling using energy eos
- Warm Mott-Gurney diode: hydrodynamic, Lorentz and friction force coupling using isentropic eos
- EMPIRE Sheath: shows good qualitative agreement using energy eos and small ε
- MF Sheath: hydrodynamic, Lorentz and friction forces with charge separation using energy and isothermal eos and small ε

Mitigation strategies

- Reducing $\Gamma = 1 + \varepsilon$ can reduce the magnitude of the energy exchange differences
- Using isentropic eos instead of total energy to reduce model differences
- 0D thermalization tests can be used to verify collision source terms
- Use of high resolution reference solutions instead of semi-analytic solutions

Absorbing wall BCs

- Thermal and vacuum wall BCs give comparable accuracy
- Vacuum BCs tend to be more robust for non-isothermal cases

$$\frac{\partial \rho_\alpha \mathbf{u}_\alpha}{\partial t} = \mathbf{R}_{\alpha\beta}$$
$$\frac{\partial \mathcal{E}_\alpha}{\partial t} = Q_{\alpha\beta} + \mathbf{u}_\alpha \cdot \mathbf{R}_{\alpha\beta}$$

0D ODEs for evolution of source terms

Thank you!