SAND2023-07779C

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Ice Sheet Models of Different Fidelity for Uncertainty Quantification

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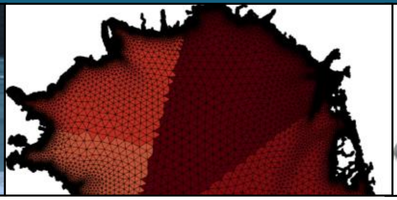
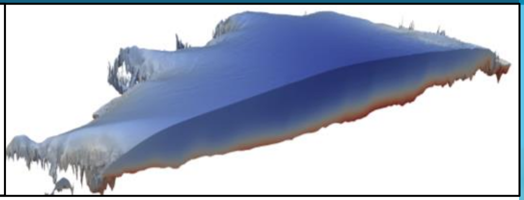
Center for Computing Research

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USNCCM17, Albuquerque, July 26, 2023

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Talk Outline

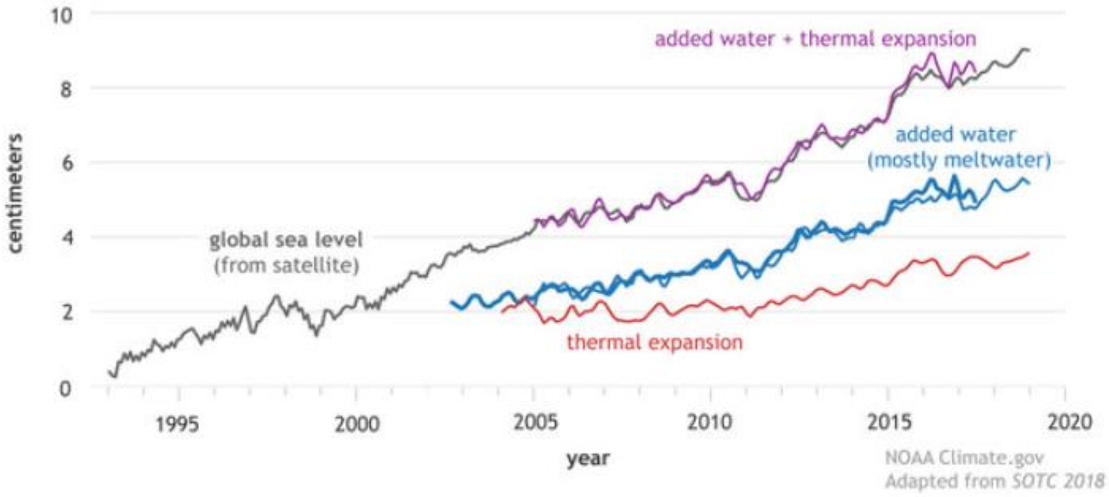
- Brief motivation and introduction to ice sheet equations
- Hierarchy of ice sheet models (high to low fidelity)
- Introduction to multi-fidelity methods
- Results on Humboldt glacier

Supported by US DOE Office of Science BER-SciDAC projects:
- ***FAnSSIE**: **F**ramework **F**or **A**ntarctic **S**ystem **S**cience **I**n **E**3SM*
- ***ProSPect**: **Pro**babilistic **S**ea-level **P**roj**ect**ion From Ice sheet And Earth System Models*

Brief Motivation an basic physics

- Modeling ice sheets (Greenland and Antarctica) dynamics is <u>essential to provide estimates for sea-level rise</u> in next decades to centuries.

Contributors to global sea level rise (1993-2018):



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- Ice behaves like a <u>very viscous shear-thinning fluid</u> driven by gravity.

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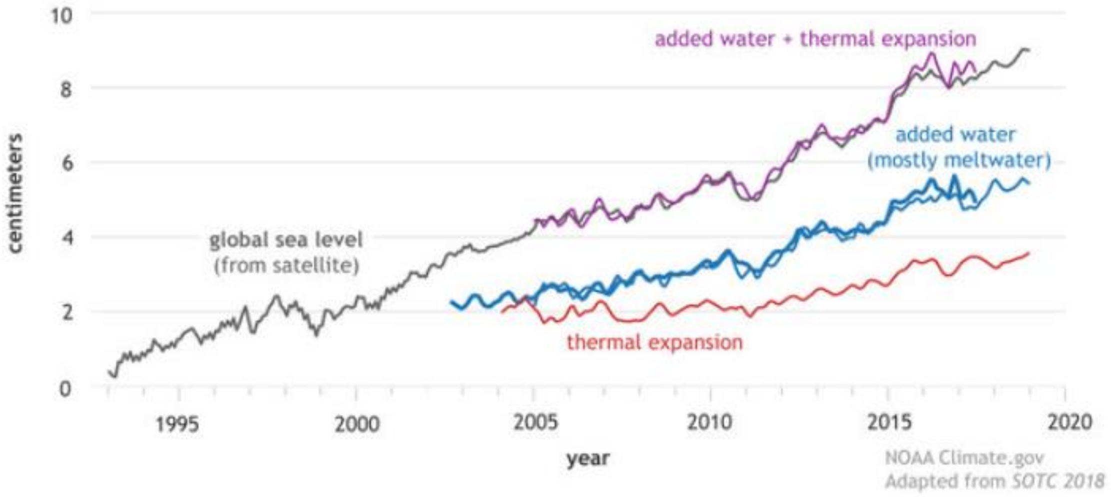

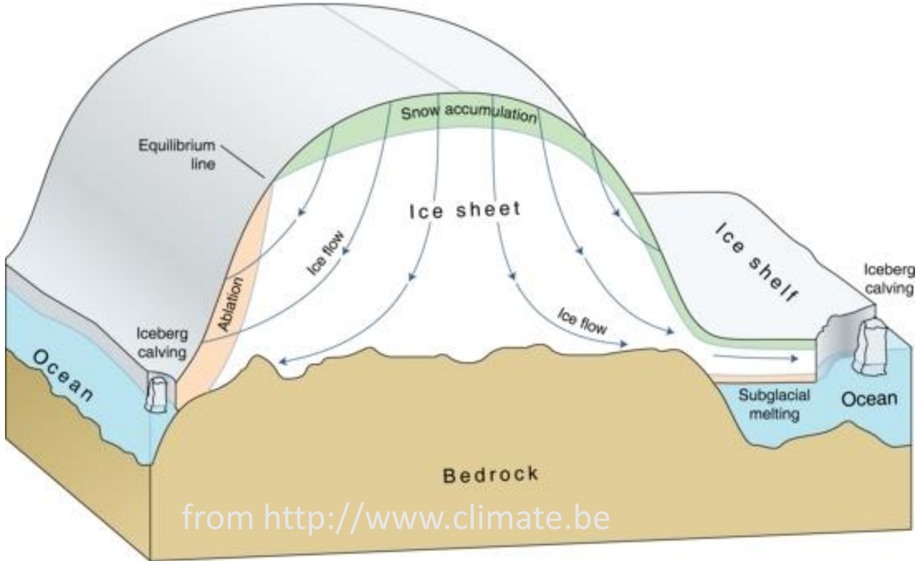

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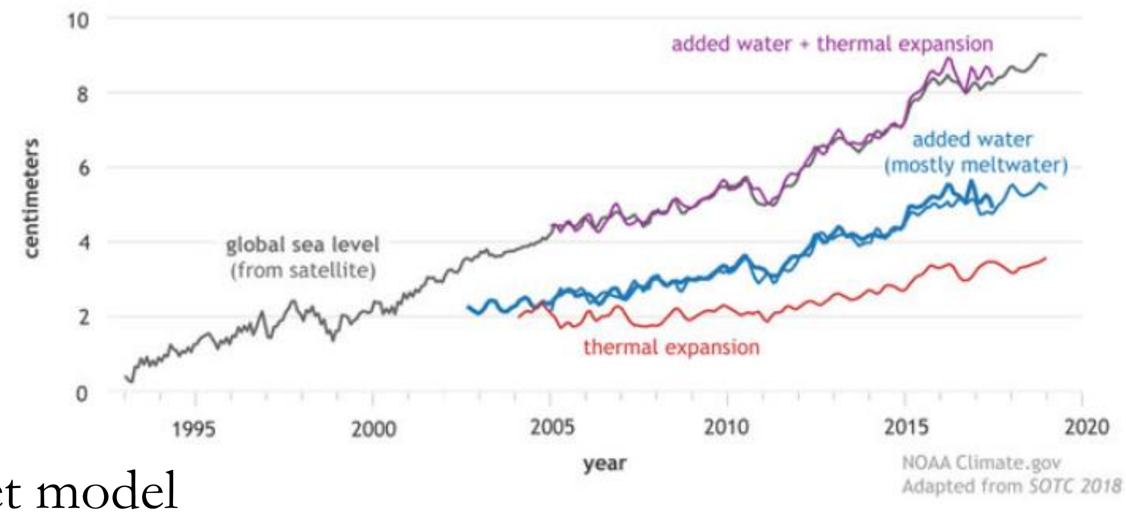


- Modeling ice sheets (Greenland and Antarctica) dynamics is essential to provide estimates for sea-level rise in next decades to centuries.

- Ice behaves like a very viscous shear-thinning fluid driven by gravity.

- There are several sources of uncertainties in an ice sheet model (e.g. uncertainties in sliding law, calving law, rheology) in addition to uncertainty in climate forcings.

- Quantifying the resulting uncertainty in the model prediction (e.g. sea-level rise) is a major challenges and computationally demanding as it requires the evaluation of the ice sheet model a large number of times.

- Here we explore the use of multi-fidelity approaches to accelerate the uncertainty quantification (UQ) analysis: we consider a hierarchy of model with different fidelity and cost, and develop a strategy to favor sampling of less expensive models over expensive ones, while maintaining a target accuracy.

Model: Ice velocity equations

Stokes equations:

$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

gravit. acceleration

ice velocity

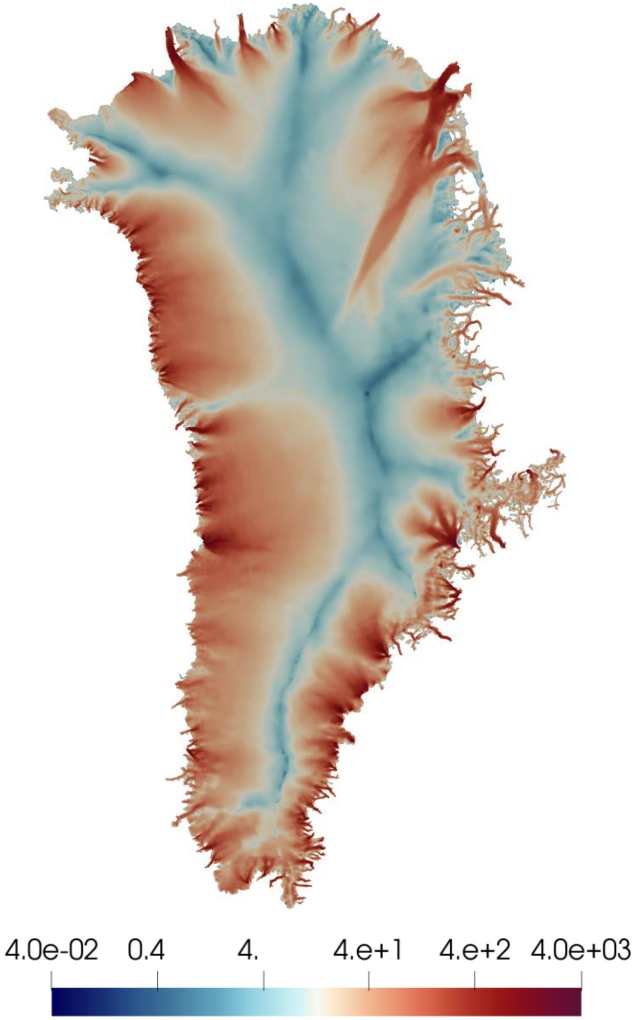
Stress tensor:

$$\sigma = 2\mu \mathbf{D} - pI, \qquad \mathbf{D}_{ij}(\mathbf{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Ice viscosity (dependent on temperature):

$$\mu = \frac{1}{2} A(T)\, |\mathbf{D}(\mathbf{u})|^{\frac{1}{n}-1}, \quad n \geq 1, \quad (\text{tipically } n \simeq 3)$$

Modeled surface ice speed [m/yr]
(Greenland ice sheet)



Model: Ice velocity equations

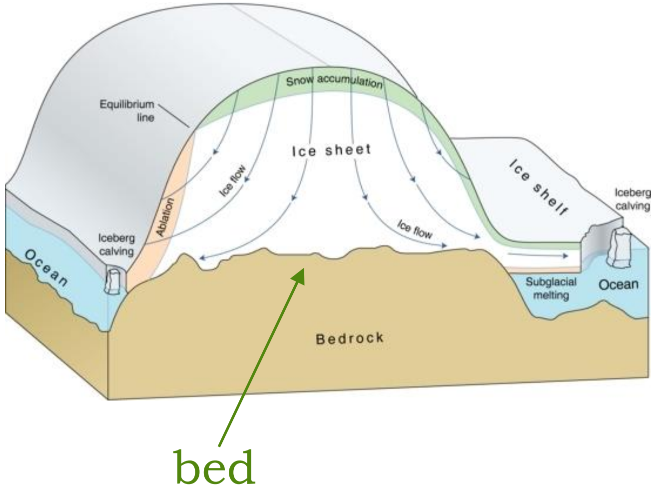
Stokes equations:

$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

Sliding boundary condition at ice bed:

$$\begin{cases} \mathbf{u} \cdot \mathbf{n} = 0, \quad \text{(impenetrablity)} \\ (\sigma \mathbf{n})_{\parallel} = \beta \mathbf{u} \end{cases}$$

Free slip:	$\beta = 0$
No slip:	$\beta = \infty$



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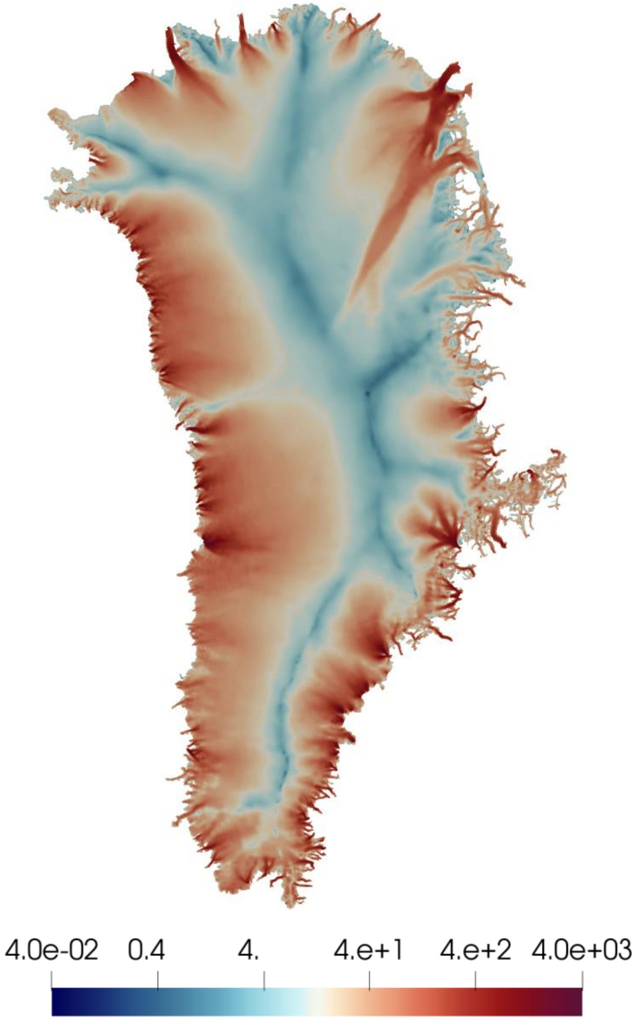


Model: Ice velocity equations

Stokes equations:

$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

Thickness evolution equation:

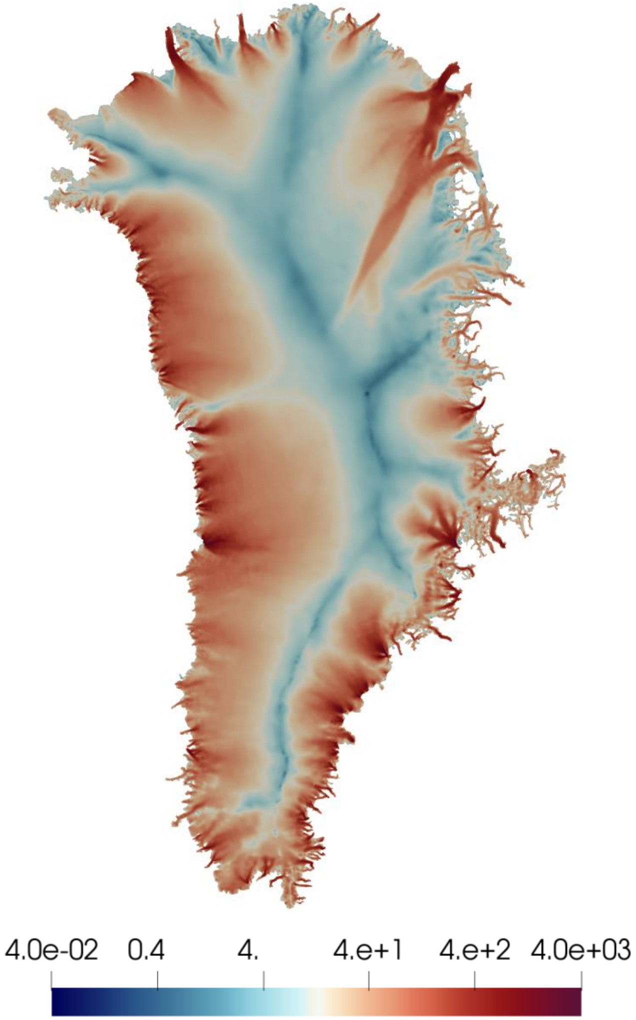
$$\partial_t H + \nabla \cdot (\bar{\mathbf{u}} H) = f_H$$

ice thickness

depth-averaged velocity

accumulation/ablation

Modeled surface ice speed [m/yr]
(Greenland ice sheet)



Multi-fidelity Models

Hierarchy of approximations of Stokes model, based on the fact that ice sheets are shallow

Increasing fidelity and cost

First Order (FO) model
(3d PDE)

$$-\nabla \cdot \left(2\mu\tilde{\mathbf{D}}\right) - \partial_z(\mu\,\partial_z \mathbf{u}) = -\rho g \nabla s$$
$$2\mu\tilde{\mathbf{D}}\mathbf{n} = \beta\mathbf{u}, \quad \text{on bed}$$

Mono-Layer Higher-order (MOLHO) model
(two 2d PDEs)

Solve FO with trial function

$$\mathbf{u} = \bar{\mathbf{u}}(x,y) + \mathbf{u}_{\text{def}}(x,y)\,\varphi(z)$$

Shallow Shelf Approx. (SSA)
(2d PDE, for floating fast-flowing ice)

$$-\nabla \cdot \left(2\mu H\tilde{\mathbf{D}}(\bar{\mathbf{u}})\right) + \beta\bar{\mathbf{u}} = -\rho g H \nabla s$$

Shallow Ice Approx. (SIA)
(for grounded slow-flowing ice)

$$\bar{\mathbf{u}} = -\left(\frac{2A\rho^3 g^3}{5} H^4 |\nabla s|^2 + \frac{\rho g}{\beta} H\right)\nabla s$$

Problem setup
(approximation and assumptions)

- Ice geometry is fixed (ice front can retreat but cannot advance, ice flux through margin allowed). No calving.
- Ice thickness and velocity model are solved implicitly (monolithic coupling), with backward Euler in time
- Problem discretized with piece-wise linear continuous finite elements on triangles.
- Thickness positivity is guaranteed using two methods:
  - Nonconservative: At each time step the thickness is updated at each node so that it is greater than 1m
  - Conservative: Thickness is constrained to be larger than 1m with a optimization-base[1] procedure that guarantees that mass changes are consistent with forcing terms and boundary fluxes

Ice-sheet models implemented in FEniCS[2]. The non-conservative methods are indicated with a "star" (SSA – conservative, SSA* non conservative)

1. P. Bochev et al., CMAME, 2020
2. FEniCS code, developed by C. Sockwell and M. Perego from an original implementation by D. Brinkerhoff

Set up of uncertainty quantification problem

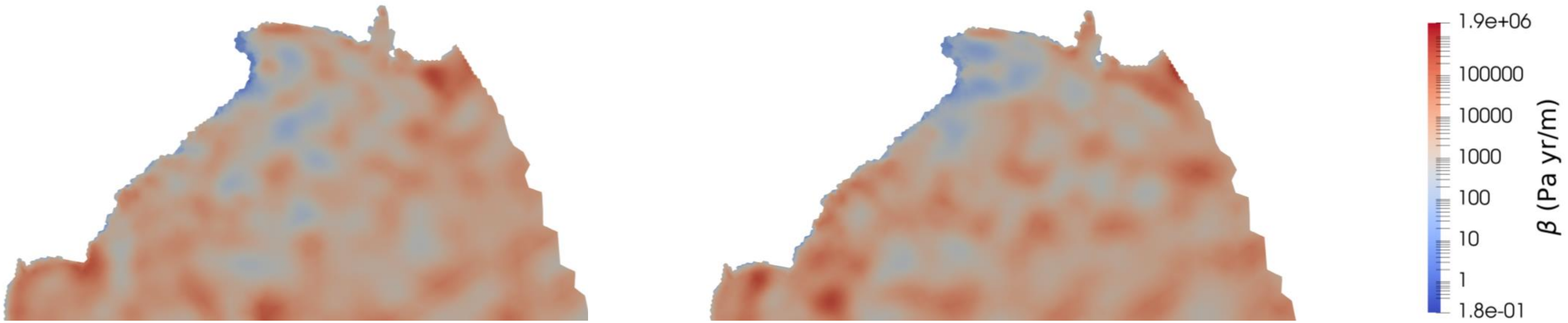
- We are interested in computing uncertainty in the ***total ice mass loss***, our Quantity of Interest (**QoI**), due to the uncertainty in the ***basal friction***.
- We assume that the basal friction distribution is lognormal, centered on the value β_{opt} obtained solving an inverse problem to match observations.

$$\beta = \exp(\gamma)\,,\ \text{where}\ \gamma \sim \mathcal{N}\left(\log(\beta_{opt}), k\right),\ \text{and}\ k(\boldsymbol{x_1}, \boldsymbol{x_2}) = \sigma^2 \exp\left(-\frac{|\boldsymbol{x_1}-\boldsymbol{x_2}|^2}{2\,l^2}\right)$$

variance (→ σ^2)

correlation length (→ l)

Samples of basal friction β near Humboldt glacier outlet ($\sigma^2 = 1$, $l = 10$ km):



Multi-fidelity Approach

Multi-level Monte Carlo

Models of different fidelity for the QoI (total mass change): $f_0, f_1, \dots, f_M$

high-fidelity

$$E[f_0] = E[f_1] + E[f_0 - f_1]$$

$$E[f_0] \approx \hat{Q}(\boldsymbol{z}) = \frac{1}{N_1}\sum_{k=1}^{N_1} f_1(\boldsymbol{z}_k^1) + \frac{1}{N_0}\sum_{k=1}^{N_0} f_0(\boldsymbol{z}_k^0) - f_1(\boldsymbol{z}_k^0)$$

If $\boldsymbol{z}^0 \cap \boldsymbol{z}^1 = \emptyset$

$$Var[\hat{Q}(\boldsymbol{z})] = \frac{1}{N_1} Var[f_1] + \frac{1}{N_0} Var[f_0 - f_1]$$

Model is cheap, can evaluate a large number of times

If models are well correlated this variance is small

Multi-fidelity Approach

Generalized approximate control variate

Models of different fidelity for the QoI (total mass change): $f_0, f_1, \dots, f_M$

high-fidelity

$$\hat{Q}_i(\boldsymbol{z}_i) := \frac{1}{N}\sum_{k=1}^{N} f_i(z_{i,k})$$

$$\hat{Q}_0(\boldsymbol{\alpha}, \boldsymbol{z}) = \hat{Q}_0(\boldsymbol{z}_0) + \sum_{k=1}^{M} \alpha_i \left(\hat{Q}_i(\boldsymbol{z}_i^1) - \hat{Q}_i(\boldsymbol{z}_i^2)\right)$$

$$= \hat{Q}_0(\boldsymbol{z}_0) + \sum_{k=1}^{M} \alpha_i \, \Delta_i(\boldsymbol{z}_i) = \hat{Q}_0(\boldsymbol{z}_0) + \boldsymbol{\alpha}^T \boldsymbol{\Delta}$$

Minimize variance of estimator by

1. Selecting what models to use
2. Selecting the sampling strategy

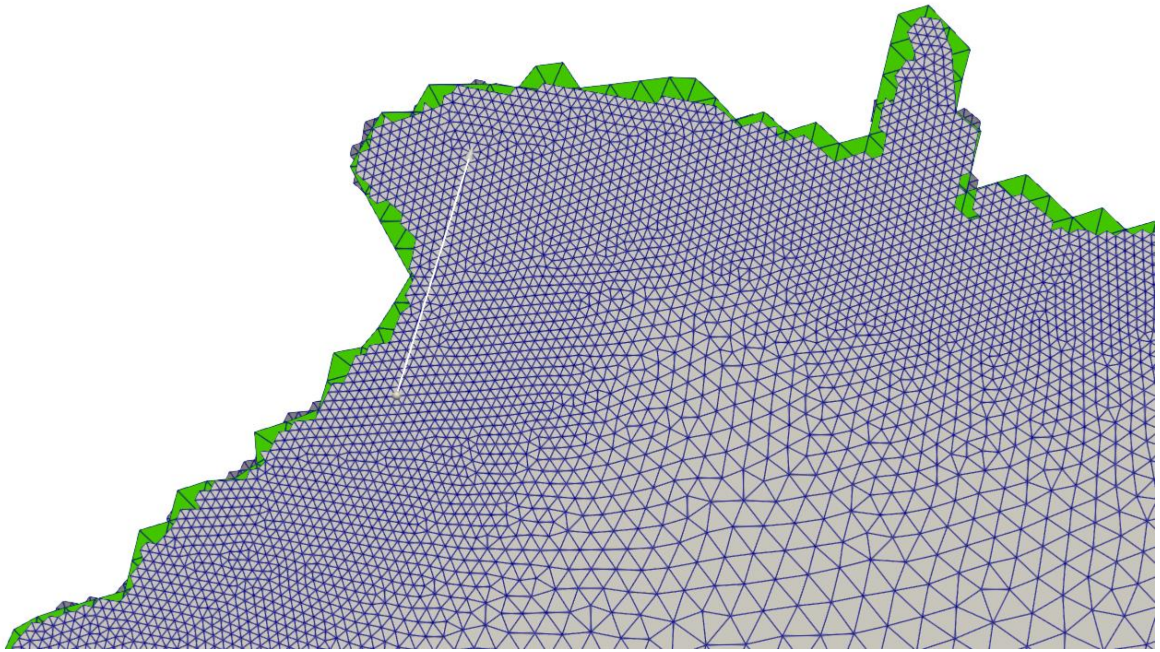
Optimal weights that minimize variance of estimator

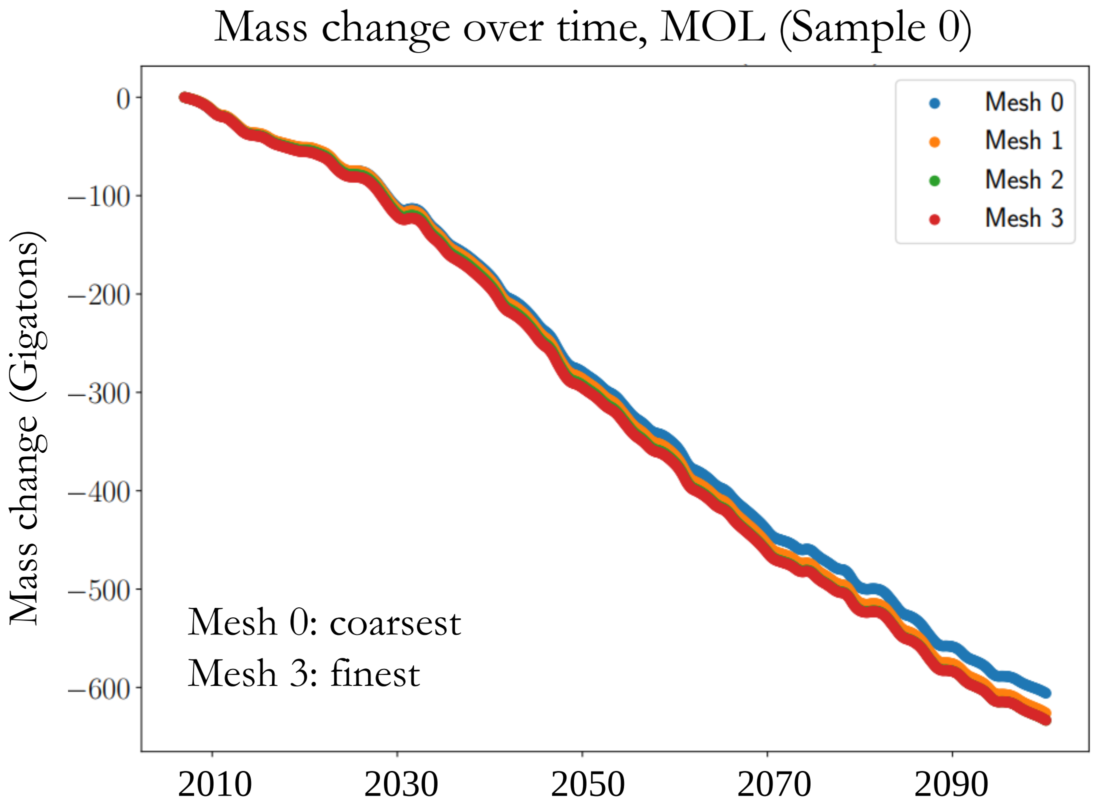
$$\boldsymbol{\alpha}^{ACV} = -\,\mathrm{Cov}[\boldsymbol{\Delta}, \boldsymbol{\Delta}]^{-1}\,\mathrm{Cov}[\boldsymbol{\Delta}, \hat{Q}_0]$$

Methods implemented in **PyApprox** by J. Jakeman
https://sandialabs.github.io/pyapprox/intro.html

We consider *four different mesh resolutions* and *three different formulations*: **MOL**HO, **SSA**, **SIA**.

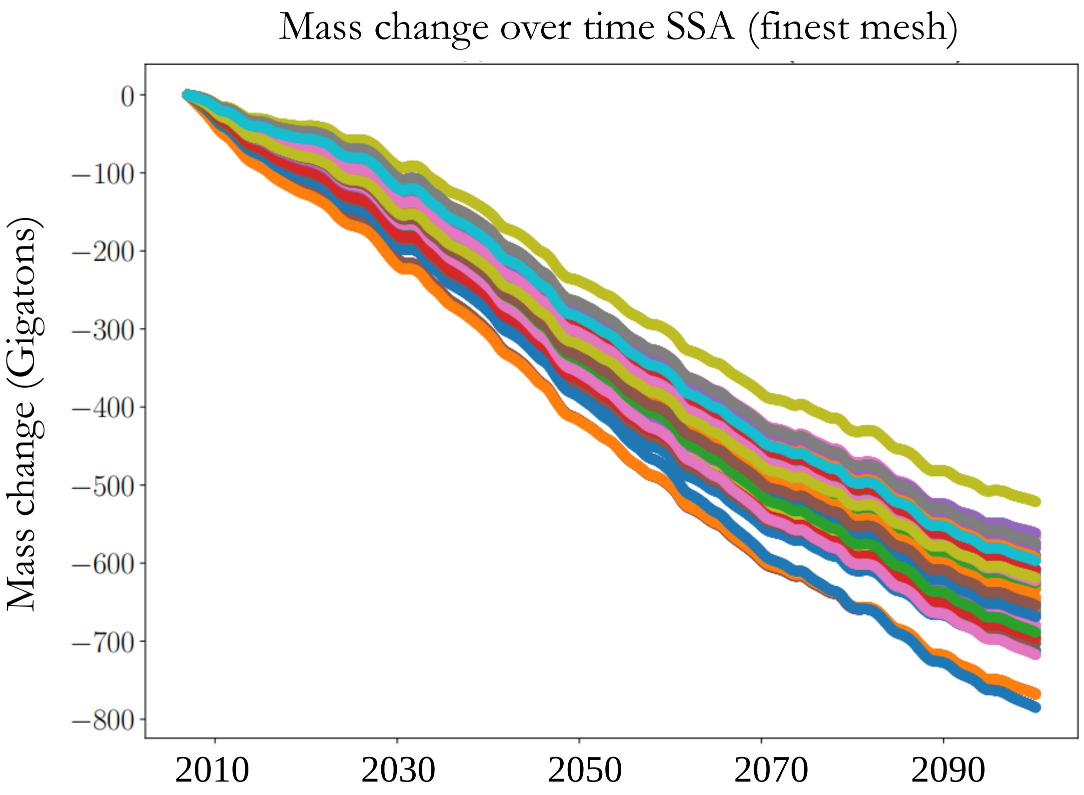
Models
(different mesh resolutions)

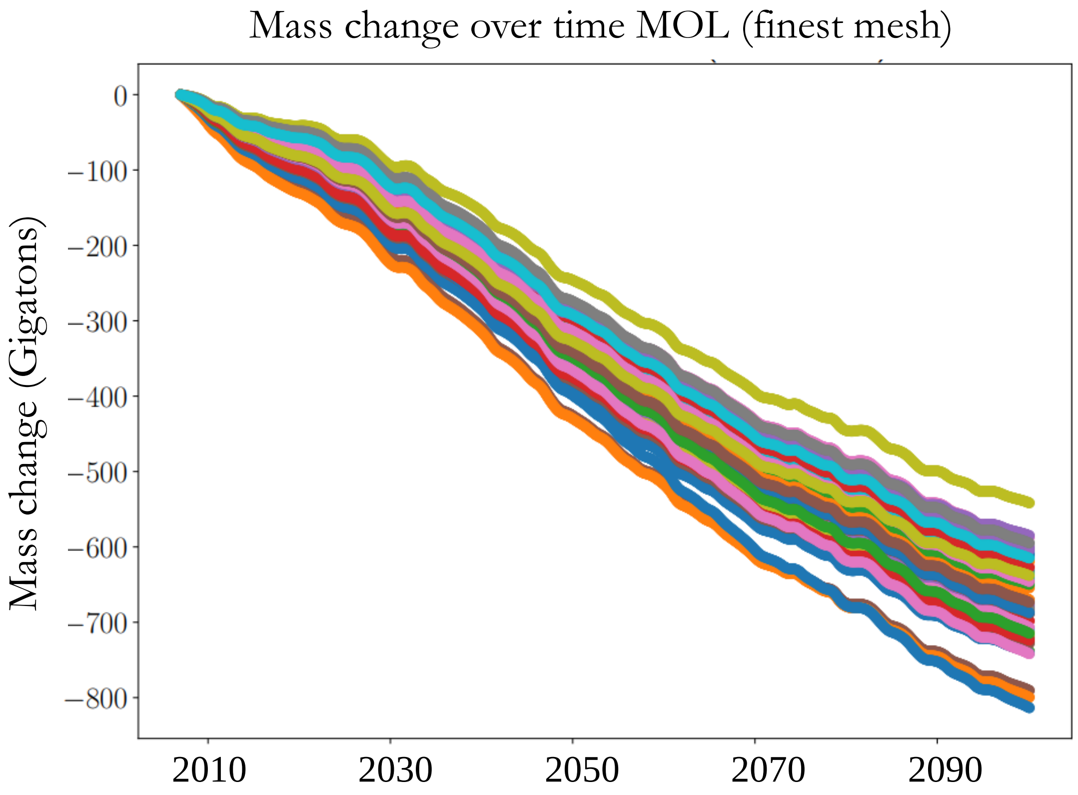




We consider *four different mesh resolutions.*

Models
(different formulations)





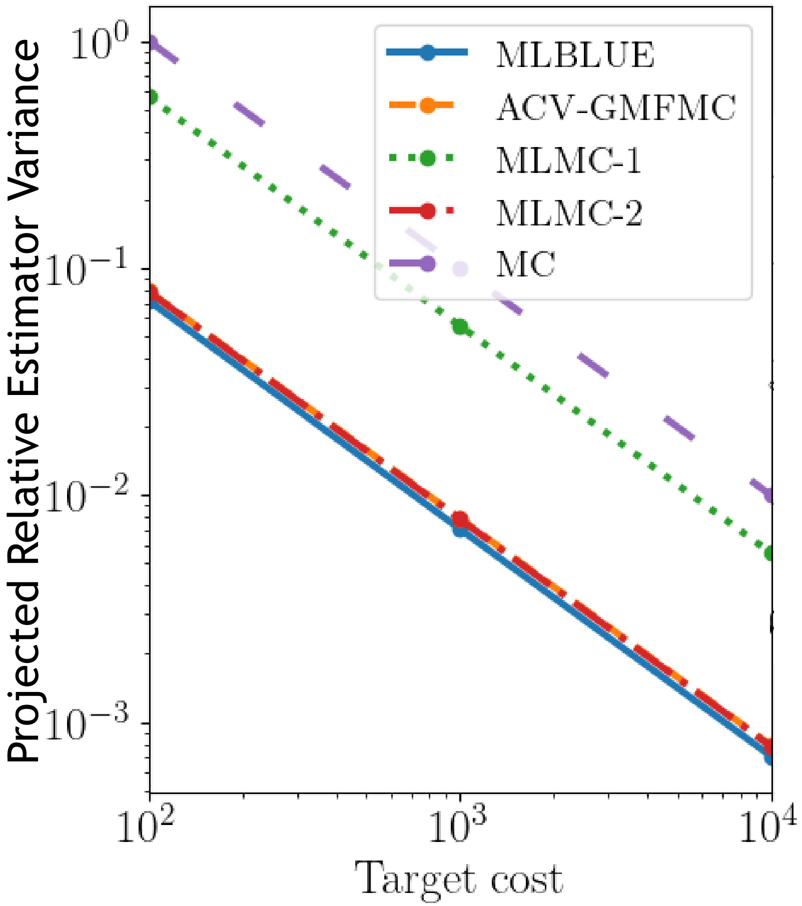
Mass change over time for different samples

Correlation of different models

Correlation between models, based on 100 samples of basal friction

Multi-fidelity Results

We compare different multi-fidelity approaches with the vanilla Monte Carlo approach.
The best approach is projected to be about **14x** faster than the Monte Carlo approach and **7x** times faster than the classic Multi-level Monte Carlo approach.



Method	Description	Models
MC	Monte Carlo	MOL_3 - highest fidelity model
MLMC-1	Multi-level Monte Carlo	MOL_3, MOL_2, MOL_1
MLMC-2	Multi-level Monte Carlo	MOL_3, SSA^*_2, SSA^*_0
ACV-GMFMC	Generalized Approximate Control Variate[1]	MOL_3, MOL^*_0, SSA^*_2, SSA^*_0
MLBLUE	Multilevel Best Linear Unbiased Estimators[2]	MOL_3, MOL^*_0, SSA^*_2, SSA^*_0

➢ SSA^*_0 is **30x** faster than MOL_3

1. G. Bomarito, P. Leser, J. Warner, W. Leser, *JCP*, 2022
2. D Schaden, E Ullmann, SIAM/ASA J. Uncertainty Quantification 8 (2), 601 - 635, 2020

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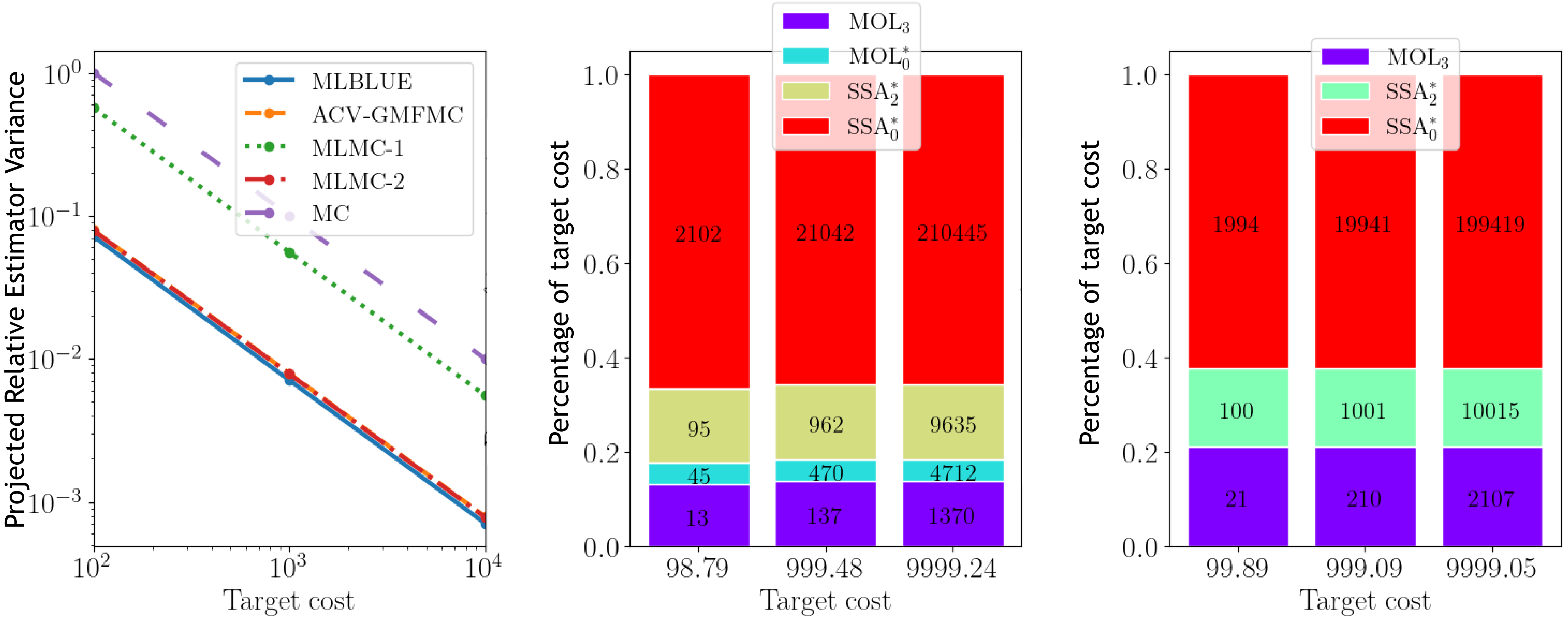

Final Remarks

- Demonstrated effectiveness of multi fidelity approach in ice-sheet application.
- Correlations between models is very high in our example. Correlation will likely be lower when considering high-order model for velocity (e.g. FO), more physics (e.g. calving) and when allowing the geometry to change.
- TODO: Use the multi-fidelity approach on different glaciers with improved accuracy for high-fidelity model.
- TODO: Include NN surrogate in our multi-fidelity strategy.

	MOL_3	MOL_2	MOL_1	MOL_0	MOL_3^*	MOL_2^*	MOL_1^*	MOL_0^*	SSA_3
Realtive Cost	1.0e+00	6.3e-01	4.3e-01	1.2e-01	8.1e-01	5.1e-01	3.5e-01	9.7e-02	4.7e-01
	SSA_2	SSA_1	SSA_0	SSA_3^*	SSA_2^*	SSA_1^*	SSA_0^*	SIA_0	SIA_0^*
Realtive Cost	2.9e-01	1.9e-01	5.6e-02	2.7e-01	1.6e-01	1.1e-01	3.1e-02	4.7e-02	2.3e-02