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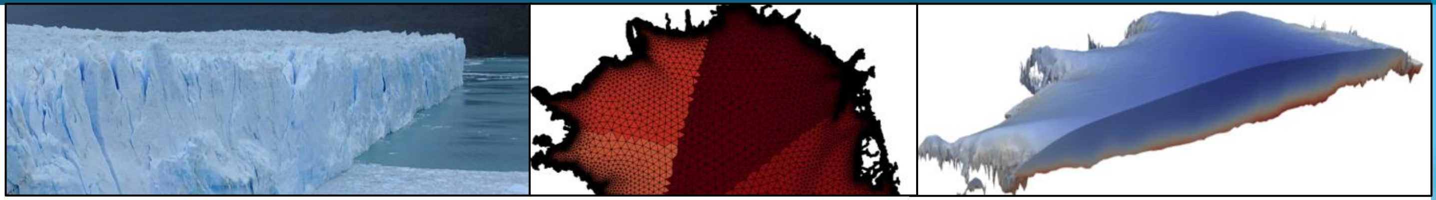
Sandia National Laboratories

A Hybrid Operator Network/Finite Element Method for Ice-Sheet Modeling

CCR
Center for Computing Research



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A. Howard, P. Stinis (PNNL), G. Karniadakis (Brown)

USNCCM17, Albuquerque, July 26, 2023

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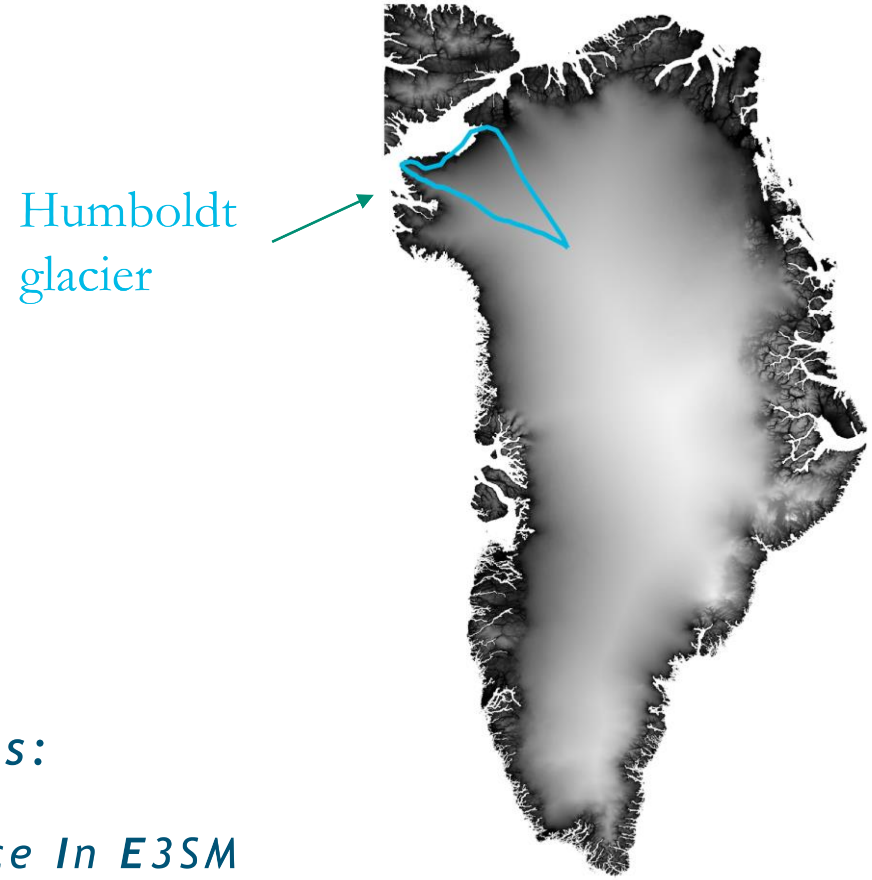
Talk Outline

- Brief motivation and introduction to ice sheet equations
- Introduction of the hybrid Finite Elements /Deep Operator Network model
- Results on Humboldt glacier, Greenland



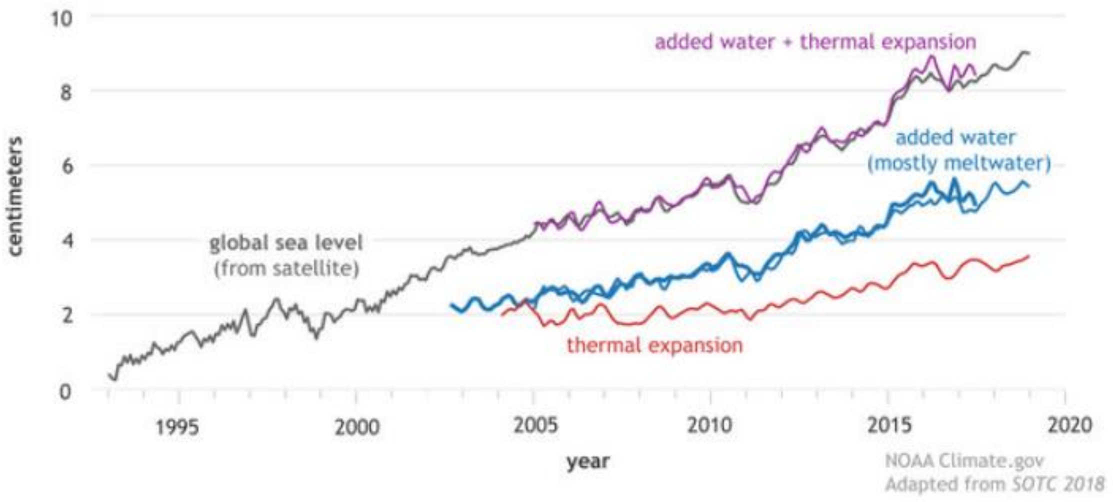
Supported by US DOE Office of Science projects:

- ***PhILMs**: **P**hysics **I**nformed **L**earning **M**achines*
- ***FAnSSIE**: **F**ramework **F**or **A**ntarctic **S**ystem **S**cience **I**n **E**3SM*
- ***ProSPect**: **Pro**babilistic **S**ea-level **P**roj**ect**ion From Ice sheet And Earth System Models*

Brief Motivation an basic physics

- Modeling ice sheets (Greenland and Antarctica) dynamics is essential to provide estimates for sea-level rise in next decades to centuries.

Contributors to global sea level rise (1993-2018):



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- Modeling ice sheets (Greenland and Antarctica) dynamics is essential to provide estimates for sea-level rise in next decades to centuries.

- Ice behaves like a very viscous shear-thinning fluid driven by gravity.

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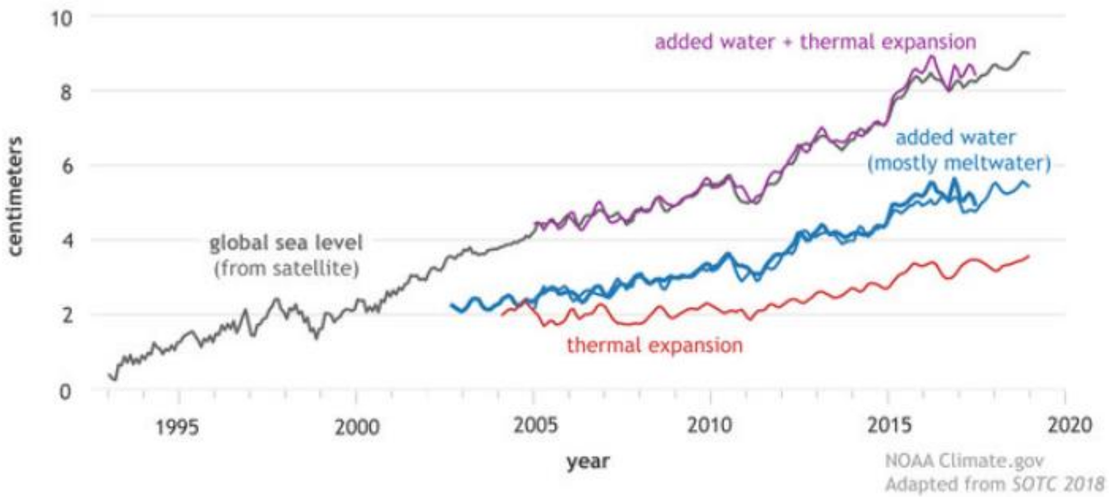

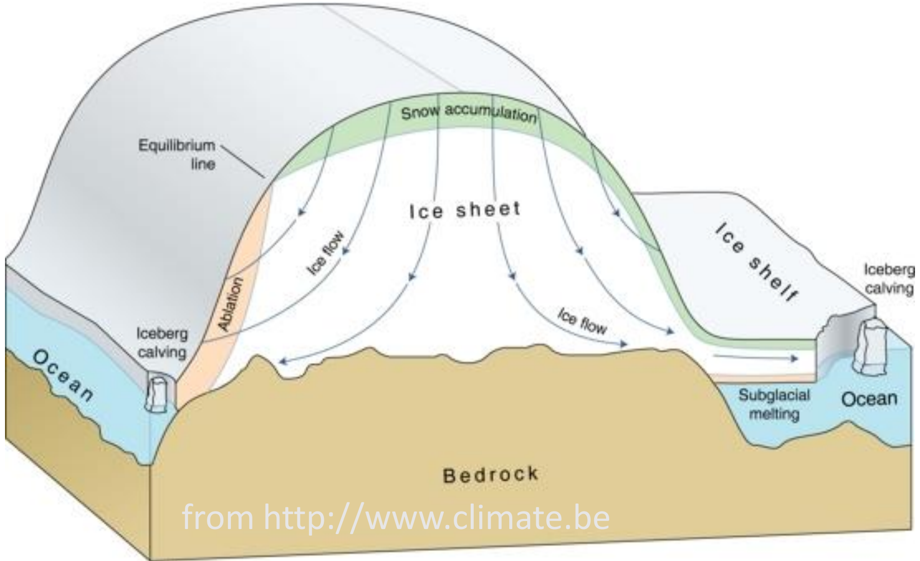

from http://www.climate.be

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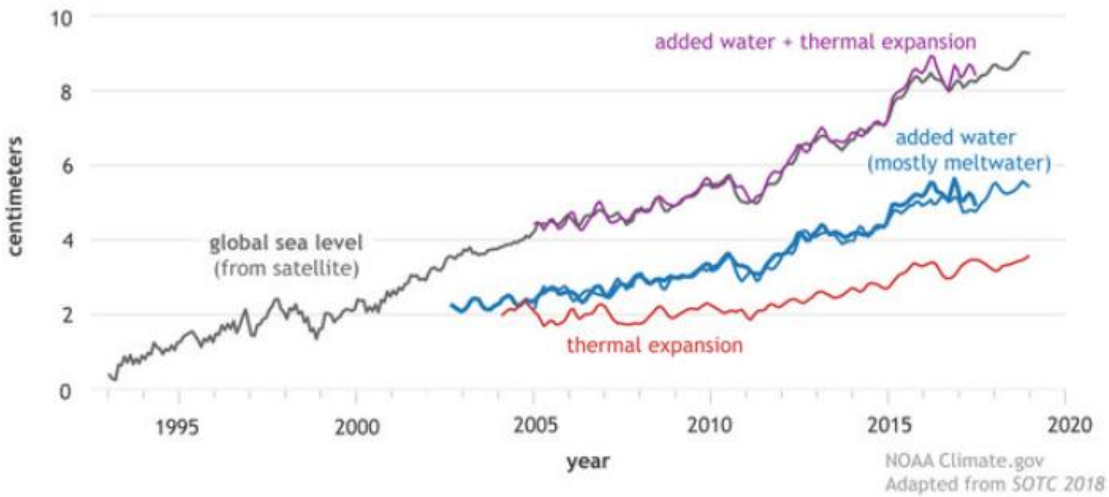


- There are several sources of uncertainties in an ice sheet model (e.g. uncertainties in sliding law, calving law, rheology) in addition to uncertainty in climate forcings.

- Quantifying the resulting uncertainty in the model prediction (e.g. sea-level rise) is a major challenges and computationally demanding as it requires the evaluation of the ice sheet model a large number of times.

- Here we explore the use of neural network models to accelerate the evaluation of the forward model.

Model: Ice velocity equations

Stokes equations:

$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

gravit. acceleration

ice velocity

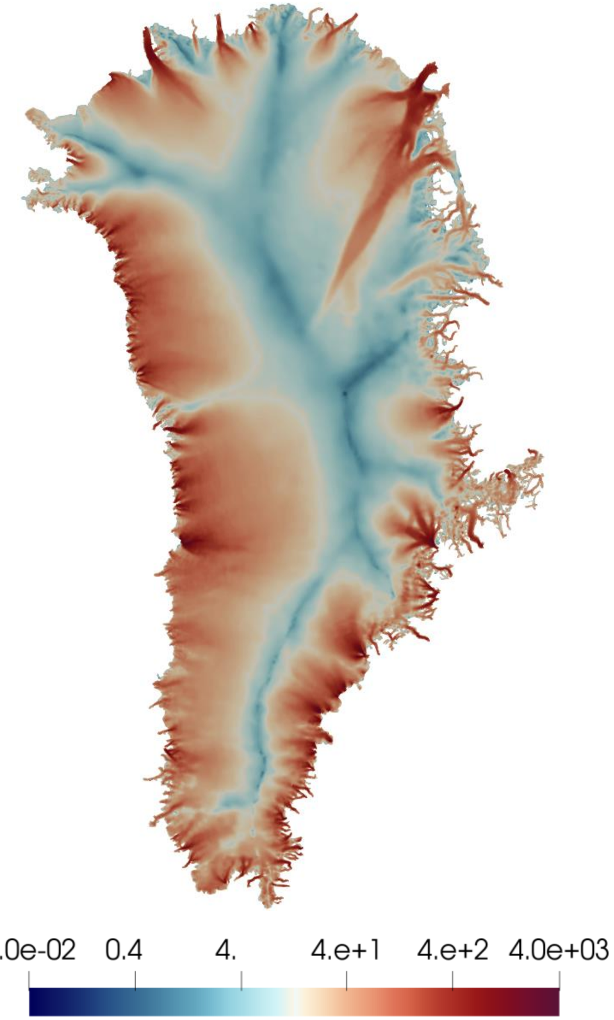
Stress tensor:

$$\sigma = 2\mu \mathbf{D} - pI, \qquad \mathbf{D}_{ij}(\mathbf{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Ice viscosity (dependent on temperature):

$$\mu = \frac{1}{2} A(T)\,|\mathbf{D}(\mathbf{u})|^{\frac{1}{n}-1}, \quad n \geq 1, \quad (\text{tipically } n \simeq 3)$$

Modeled surface ice speed [m/yr]
(Greenland ice sheet)



Model: Ice velocity equations

Stokes equations:

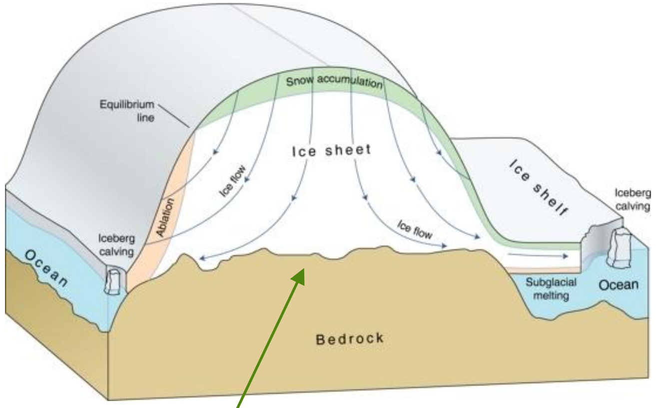
$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

Sliding boundary condition at ice bed:

$$\begin{cases} \mathbf{u} \cdot \mathbf{n} = 0, \quad \text{(impenetrablity)} \\ (\sigma \mathbf{n})_{\|} = \beta \mathbf{u} \end{cases}$$

Free slip: $\beta = 0$

No slip: $\beta = \infty$



bed

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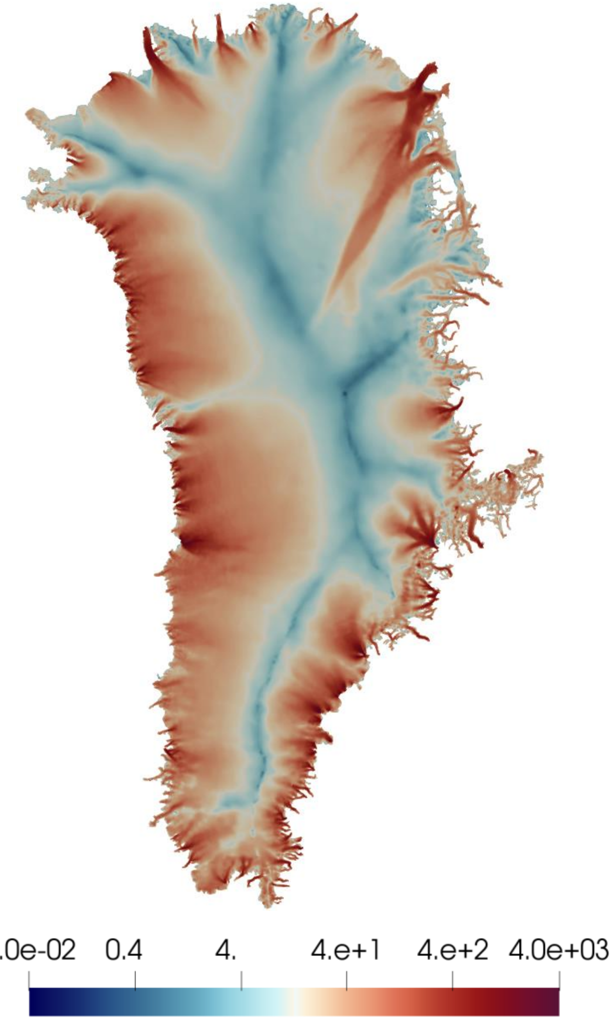


Neural Network surrogates

Thickness equation:

$$\partial_t H + \nabla \cdot (\bar{\mathrm{u}} H) = f_H$$

ice thickness — vertically avg. velocity — accumulation/ablation

Time discretization:

$$\frac{H_\beta^{n+1} - H_\beta^{n}}{\Delta t} + \nabla \cdot \left(\bar{\boldsymbol{u}}_\beta^n \, H_\beta^{n+1}\right) = F_H^{n+1}$$

$$\begin{cases} -\nabla \cdot \sigma(\boldsymbol{u}_\beta^n) = \rho \boldsymbol{g} & \text{in } \Omega_{H^n} \\ \nabla \cdot \boldsymbol{u}_\beta^n = 0 & \text{in } \Omega_{H^n} \end{cases}$$

$$\bar{\boldsymbol{u}}_\beta^n = \mathcal{G}(\beta, H^n)$$

Stokes equation maps the thickness and the basal friction into the velocity

Neural Network surrogates

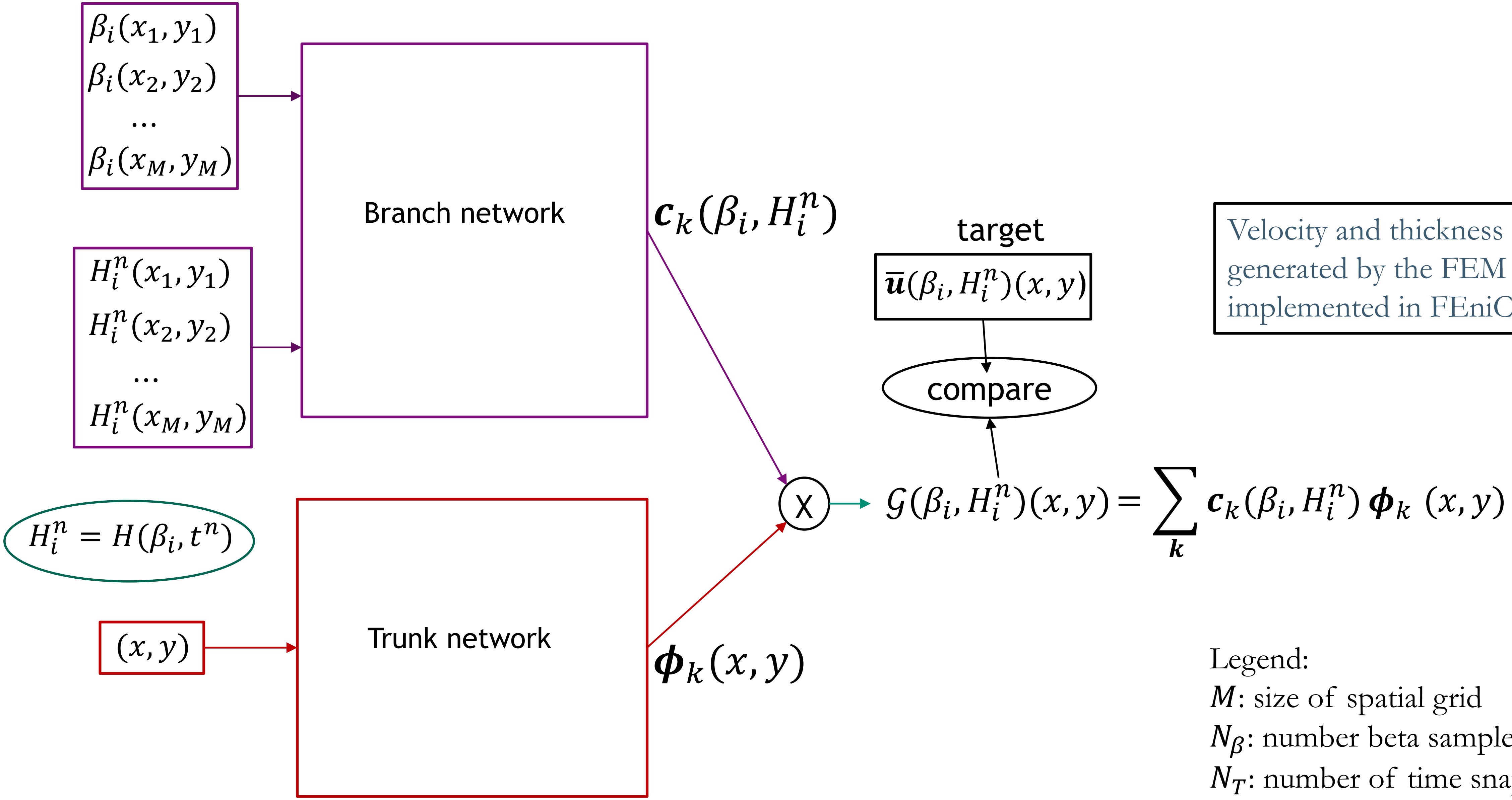
The velocity solver is the most expensive part of the model.
Idea[1]: replace the velocity solve with a Deep Operator Network [2]

$$\bar{\boldsymbol{u}}_\beta^n = \boxed{\mathcal{G}(\beta, H^n)} \quad \Longleftrightarrow \quad \boxed{\text{DeepONet}}$$

- Instead of approximating functions, DeepONet approximate *nonlinear* continuous operators.
- The universal approximation theorem provides a strong mathematical foundation of DeepONets

[1] G. Jouvet, G. Cordonnier, B. Kim, M. Lüthi, A. Vieli, A. Aschwanden, Deep learning speeds up ice flow modelling by several orders of magnitude, *Journal of Glaciology*, 2021
[2] Lu, L., Jin, P., Pang, G. et al. Learning nonlinear operators via DeepONet based on the universal approximation theorem of operators. *Nat Mach Intell* **3,** 218–229 (2021).

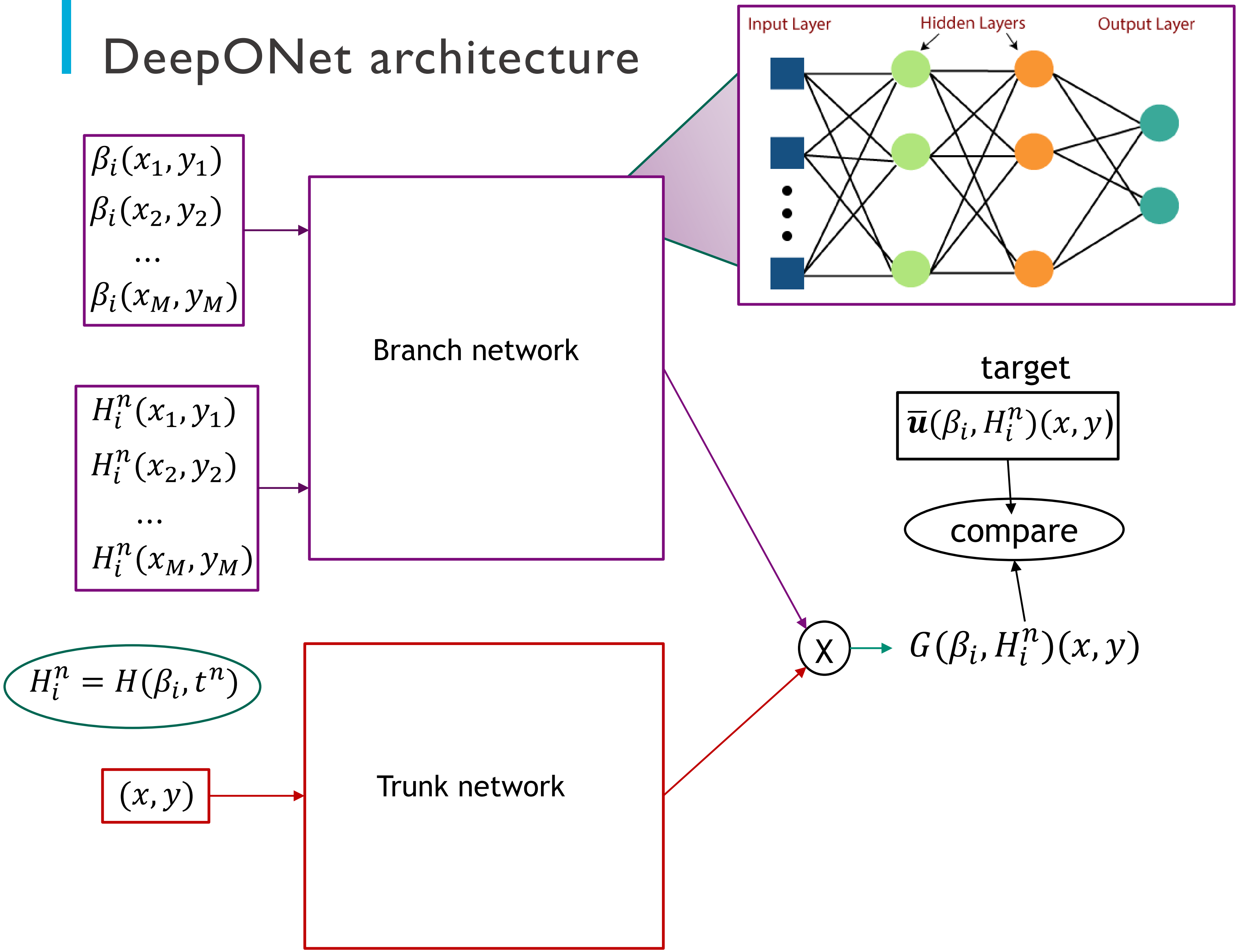
DeepONet architecture



Velocity and thickness data are generated by the FEM code, implemented in FEniCS

Legend:
M: size of spatial grid
N_β: number beta samples
N_T: number of time snapshots

DeepONet architecture



Velocity and thickness data are generated by the FEM code, implemented in FEniCS

Input/Output:

Branch input size: $(N_\beta N_T, 1, \mathbf{2M})$

Trunk input size: $(N_\beta N_T, M, \mathbf{2})$

Target size: $(N_\beta N_T, M, \mathbf{2})$

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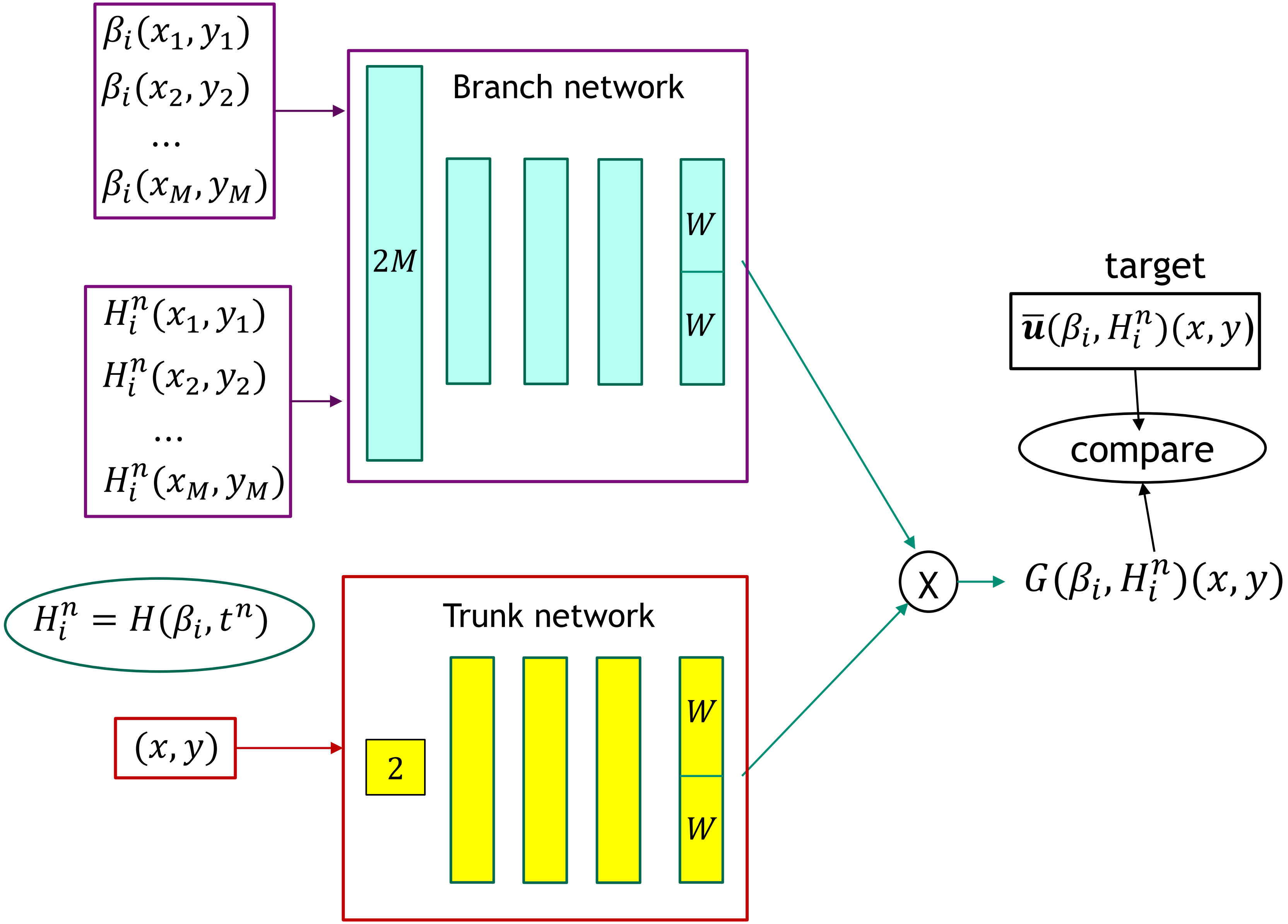
DeepONet architecture



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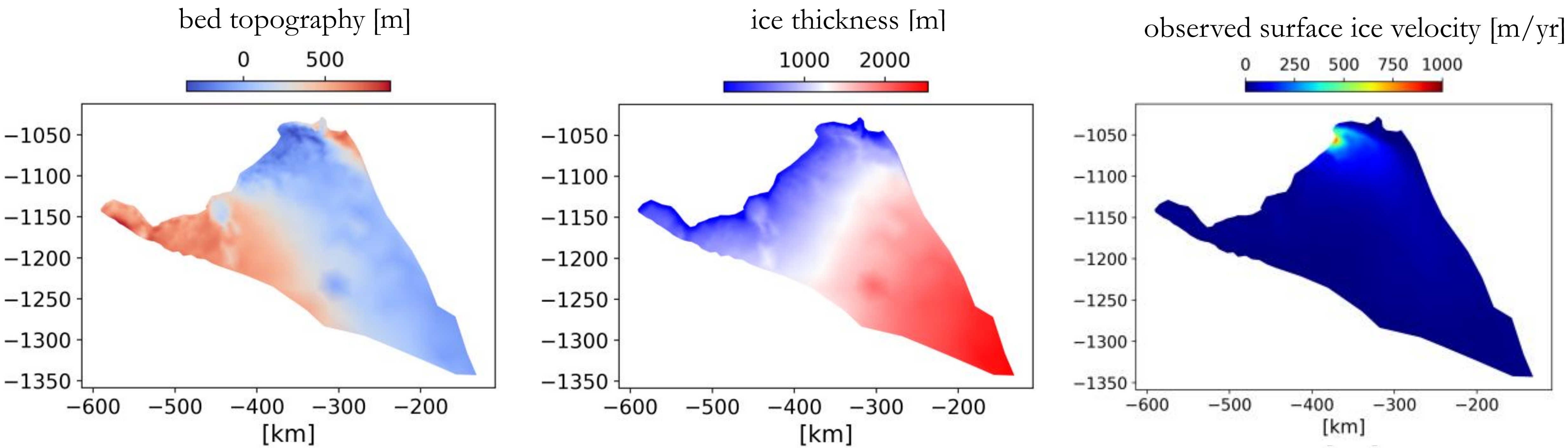
Problem setup (approximation and assumptions)

- Ice geometry is fixed (ice front can retreat but cannot advance, ice flux through margin allowed). No calving.
- Ice thickness and velocity model are coupled in a staggered fashion, with backward Euler discretization in time
- Problem discretized with piece-wise linear continuous finite elements on triangles.
- We use a simplification of the Stokes model, **MOLHO**[2], that relies on the fact that ice-sheet are shallow and solves two 2d equations for the depth-averaged velocity $\bar{\boldsymbol{u}}(x, y)$ and a corrective velocity $\bar{\boldsymbol{u}}_{def}(x, y)(1 - \zeta^4)$ varying in the normalized vertical component ζ.

Ice-sheet models implemented in FEniCS[3]. DeepONet implemented in JAX[4]

1. P. Bochev et al., CMAME, 2020
2. T. D. dos Santos, M. Morlighem, D. Brinkerhoff, The Cryosphere, 2022
3. FEniCS code, developed by C. Sockwell and M. Perego from an original implementation by D. Brinkerhoff
4. Jax code developed by Q. He and A. Howard

Humboldt glacier
(north-west Greenland)



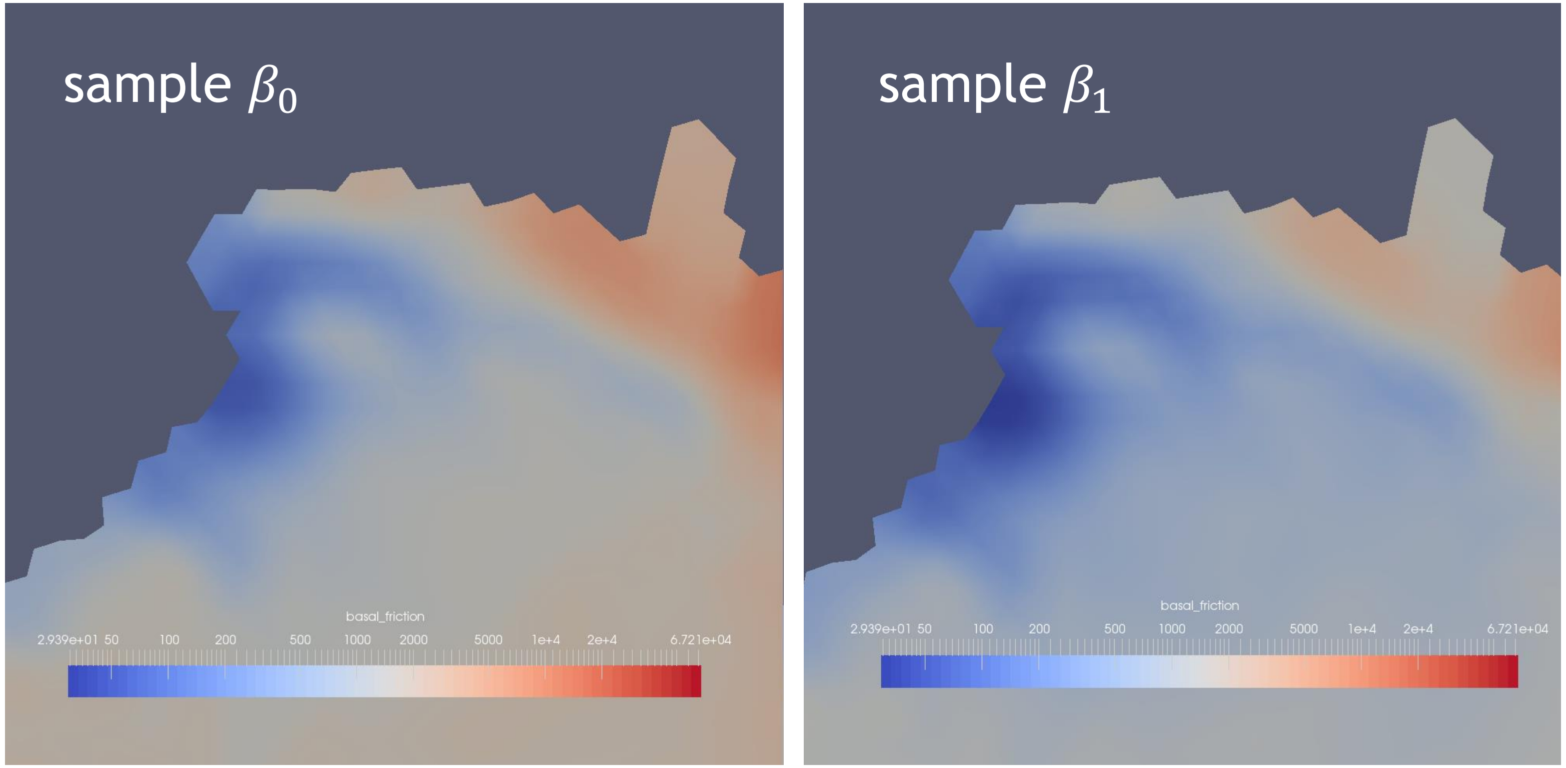
Humboldt (basal friction samples)

Basal friction sampled from a log-normal distribution:

$$\beta = \exp(\gamma)\,,\ \text{where}\ \gamma \sim \mathcal{N}\left(\log(\beta_{\text{opt}}), k\right),\ \text{and}\ k(\boldsymbol{x_1}, \boldsymbol{x_2}) = \sigma^2 \exp\left(-\frac{|\boldsymbol{x_1} - \boldsymbol{x_2}|^2}{2\, l^2}\right)$$

Workflow:

- Generated beta samples
- Generate thickness and velocity data for different beta samples using Finite Elements (FEM) code
- Train the DeepONet w/ velocity data



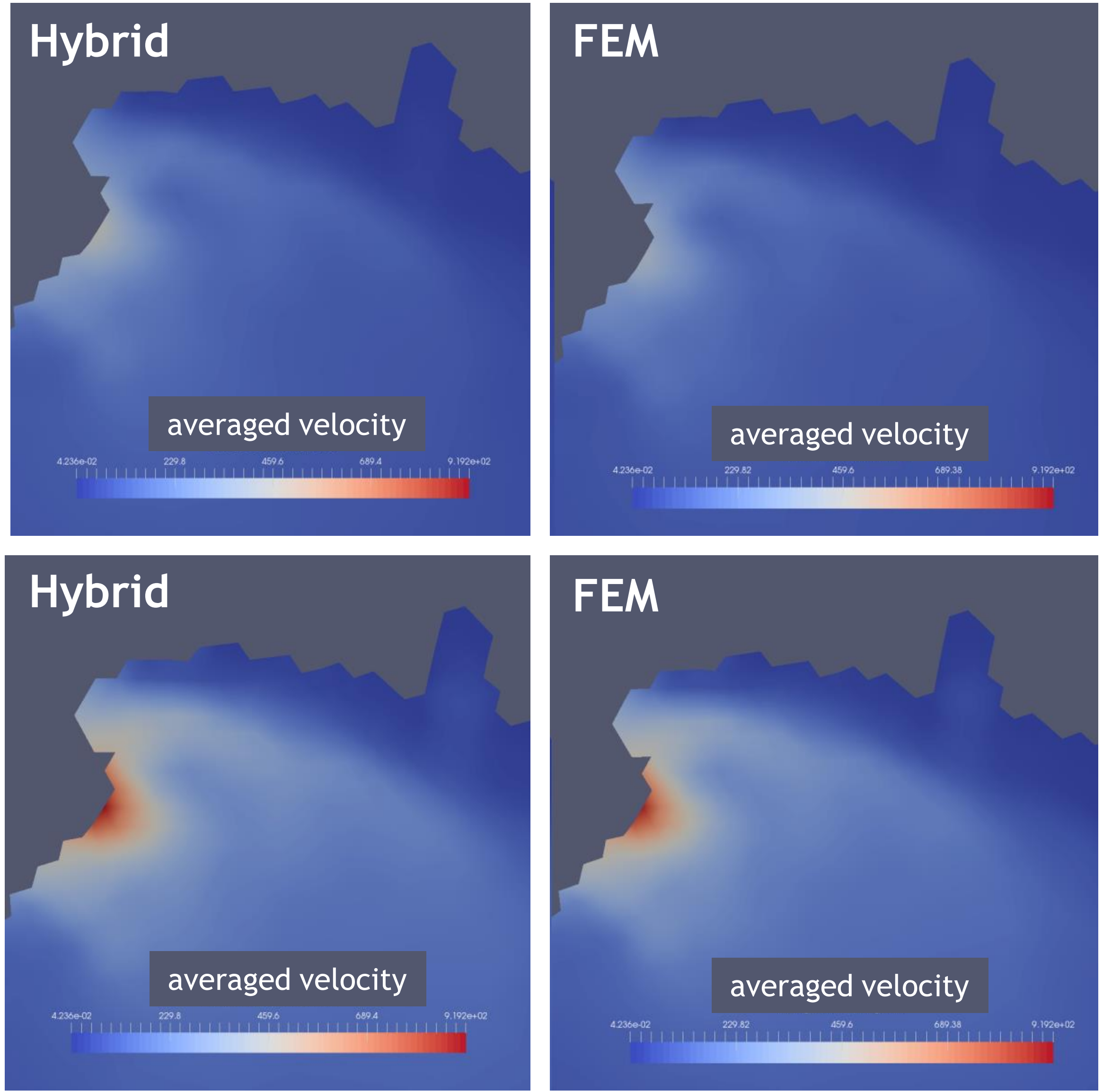
Basal friction samples

$\sigma^2 = 0.2, \mathrm{l} = 50\mathrm{km}$

Humboldt (computing averaged velocity w/ DeepONets)

sample β_0

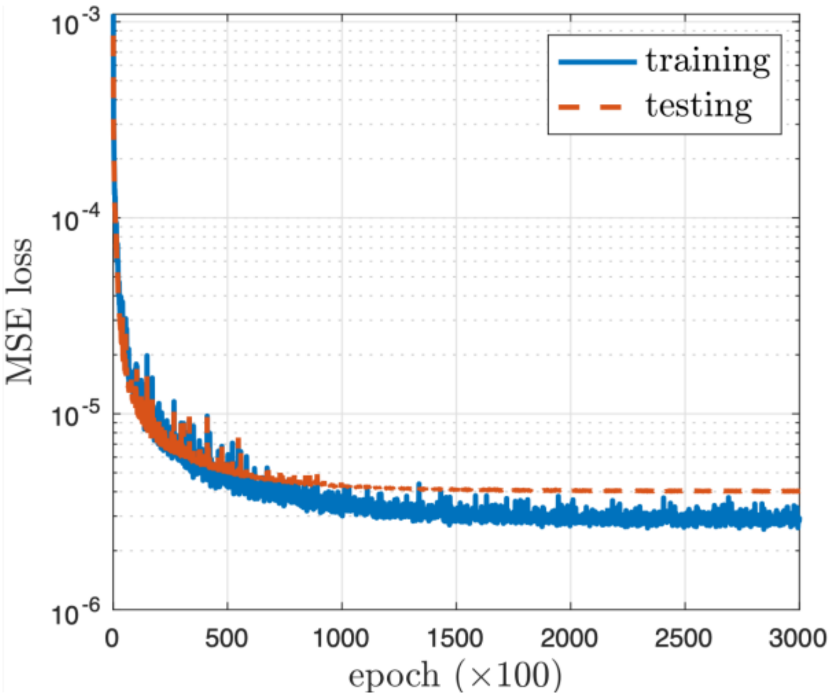
sample β_1



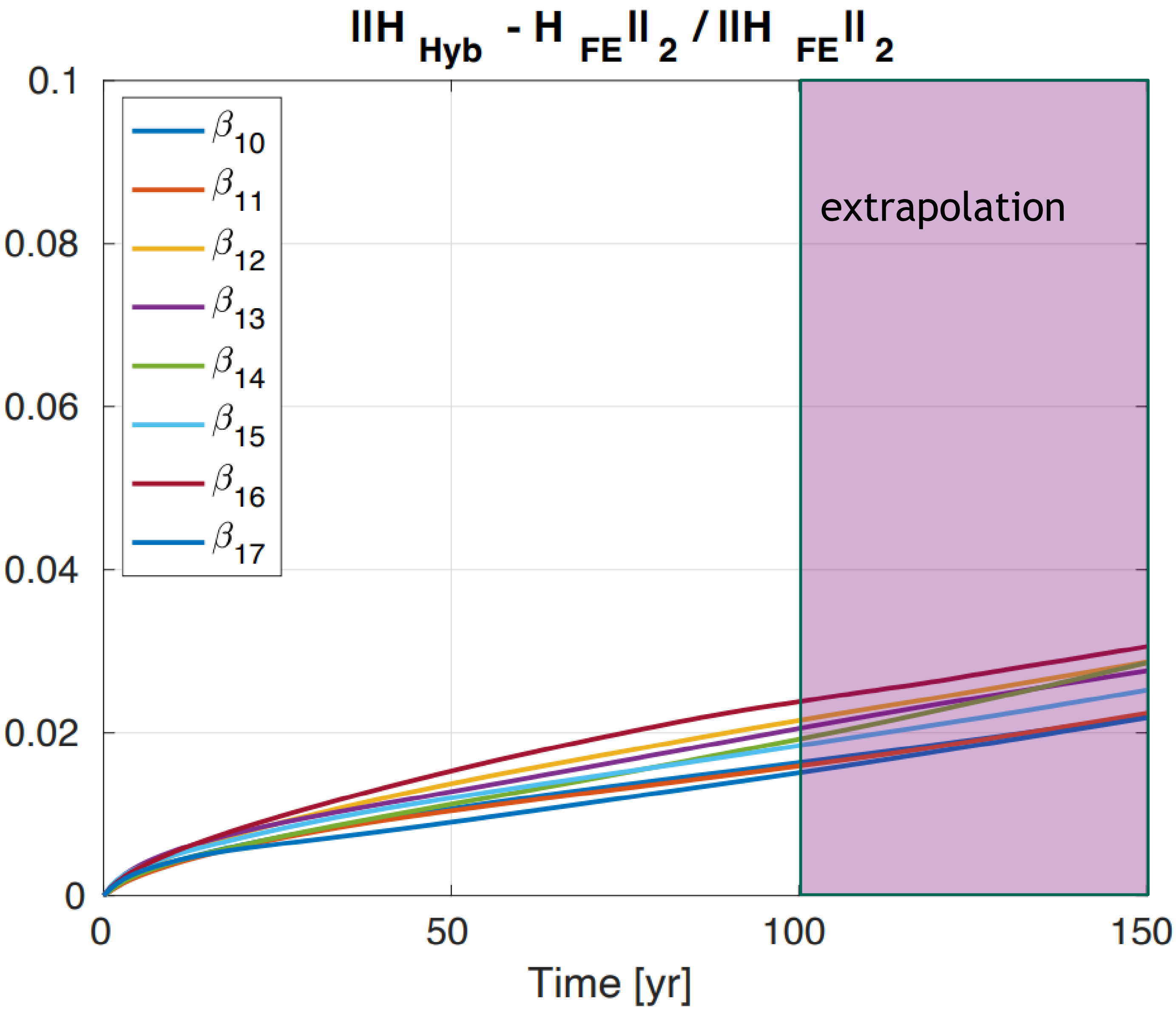
Hybrid: thickness solved w/ ***FEM*** calling the ***DeepONet*** at each time step to compute velocity

FEM: thickness and velocity models solved with FEM

Left: Averaged velocity at T=100 yr for *test* beta samples (***NOT*** used for training)



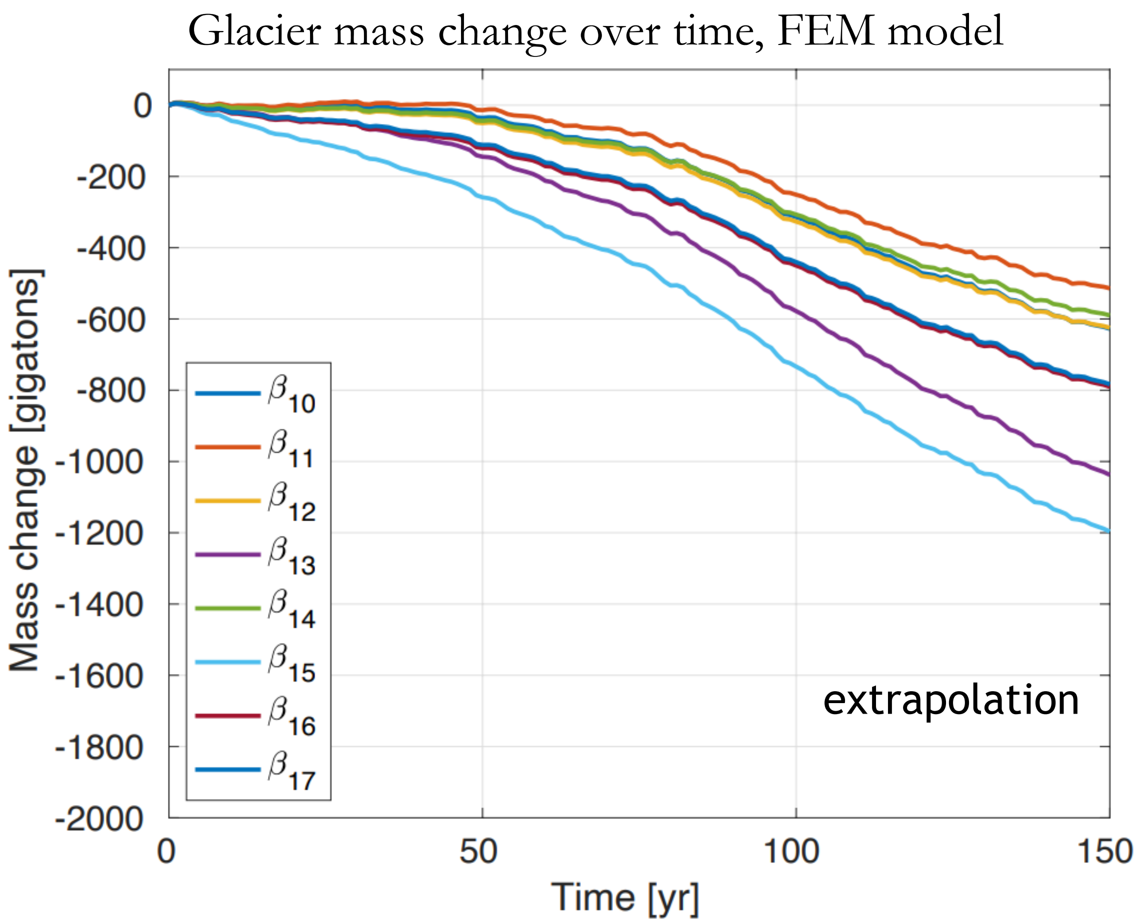
Humboldt (relative error as a function of time for different samples)

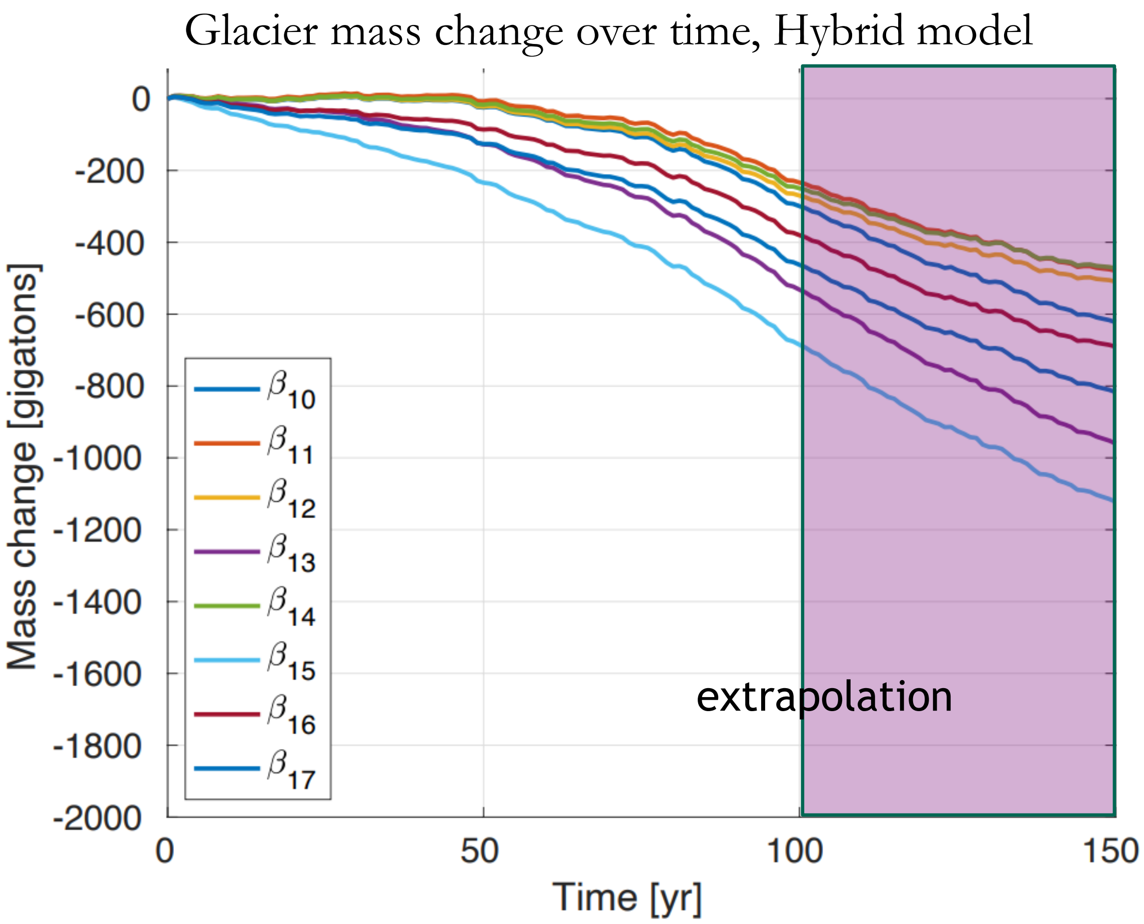


Settings:

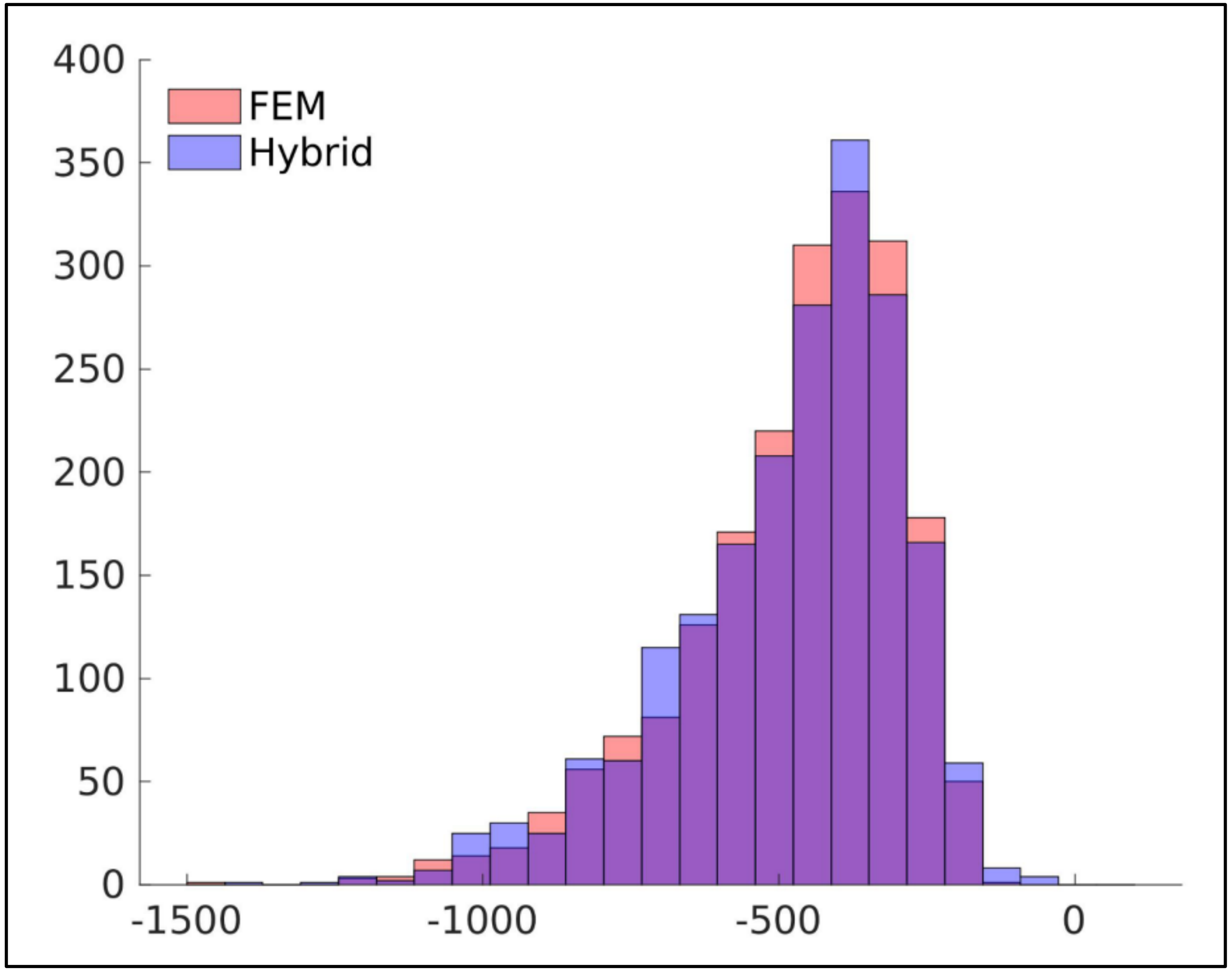
training beta samples: 300
#epochs: 300,000
4 layers of width W: 300
time snapshots: 1, 2, …, 100

Humboldt (glacier mass loss)





Humboldt – SSA model (glacier mass loss)



Histogram of glacier mass change in gigatons. Comparison between Hybrid and FEM for 2000 samples

Times per sample (s)	Total	Solve only
Finite element model	123.30	105.20
Hybrid model	24.15	9.46
Ratio	19.59 %	8.99%

Including “off-line” training costs, Hybrid model becomes advantageous over traditional model for more than 500 samples.

- Fast evaluation of forward model will enable the quantification of uncertainty on of sea level rise

Final Remarks

- We introduced a hybrid model based a DeepONet surrogate of the velocity equation
- We demonstrated its effectiveness in computing statistics of glacier mass loss
- Computational savings will be potentially greater when considering more expensive models
- TODO: scale this up to larger/higher resolution glaciers
- TODO: use resNet for thickness evolution, to further speed-up model
- TODO: explore use of DeepONet surrogate in multi-fidelity framework (see talk tomorrow afternoon in 705.1)