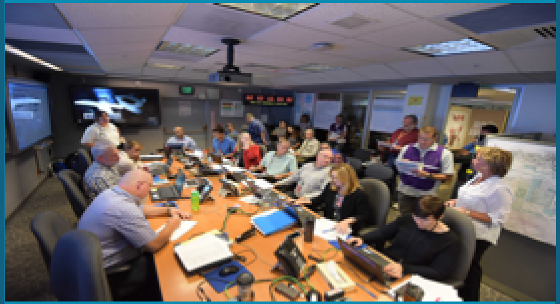
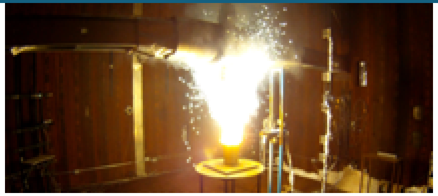
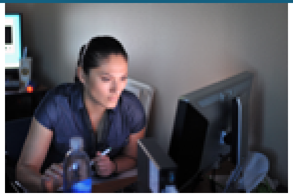
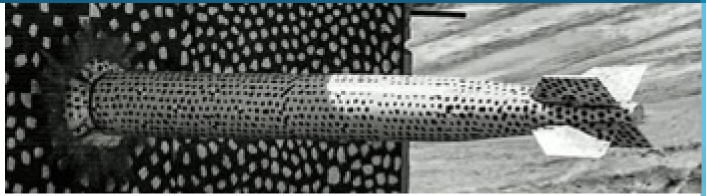
SAND2023-07683C

Meshing Ugly Geometry with Sculpt using Winding Numbers









17th USNCCM MS321 MeshTrends
July 2023

Presented by

Scott A. Mitchell
CCR Center for Computing Research
Sandia National Laboratories

Center for Computing Research

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Advanced Scientific Computing Research (ASCR), Applied Mathematics Program.

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Meshing Ugly Geometry

Traditional meshing algorithms need **perfect** geometry

- Perfecting it takes too long and drives scientists crazy

We develop meshing algorithms that work on **ugly** geometry

- Price is geometric and *topological* (new) fidelity to the input
- TetWild, etc., already do this, but without quantified topological accuracy

We propose math to

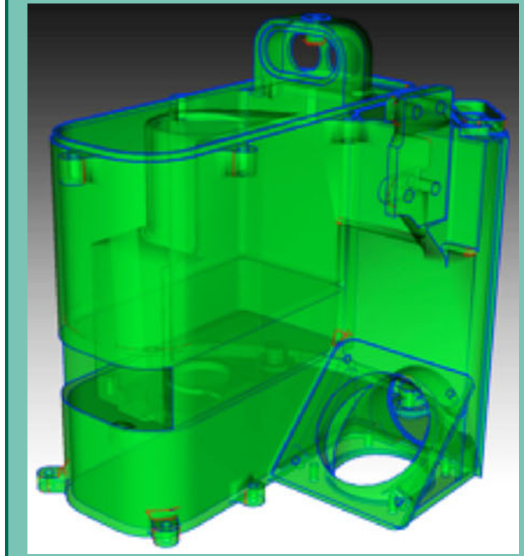
- **Measure** geometric and topological fidelity
  - By persistent homology
- **Prove** fidelity bounds
  - Parameterized by scientist-elected mesh size
  - Parameterized by measures of geometric ugliness

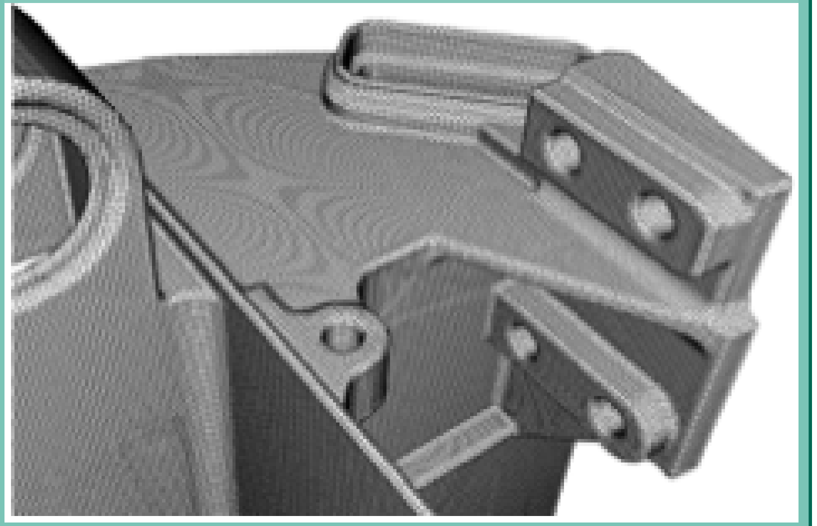
Sculpt

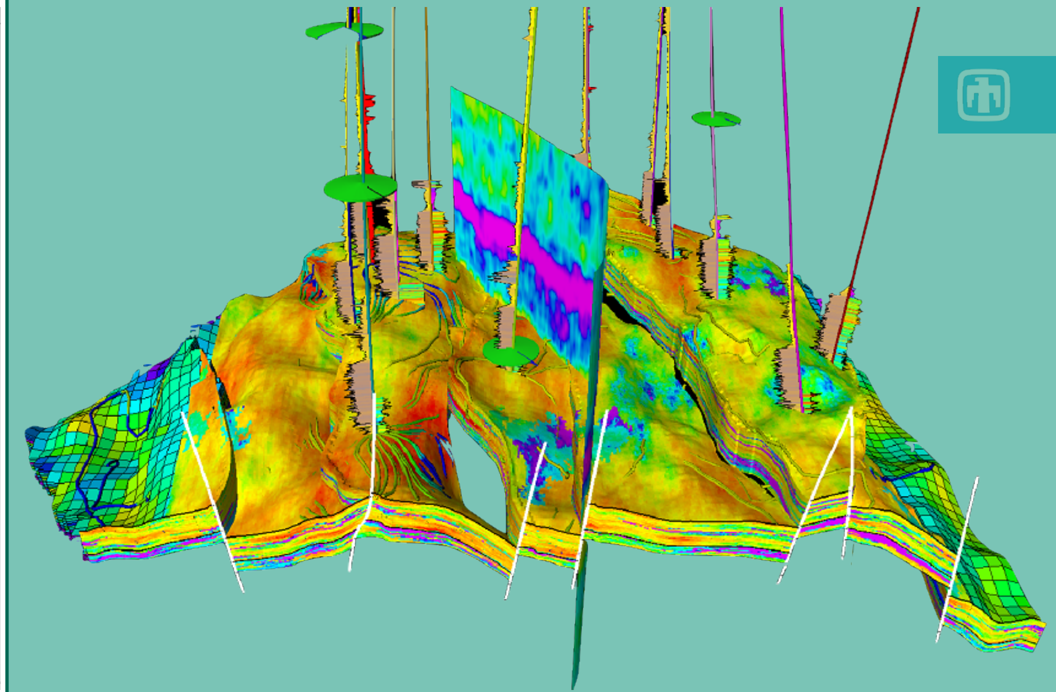
Fundamentally reconstructs the geometry

- Extend to ugly geometry
  - e.g. non-watertight geometry, gaps, overlaps, pinch points
  - by winding number in libigl for graceful inside/out

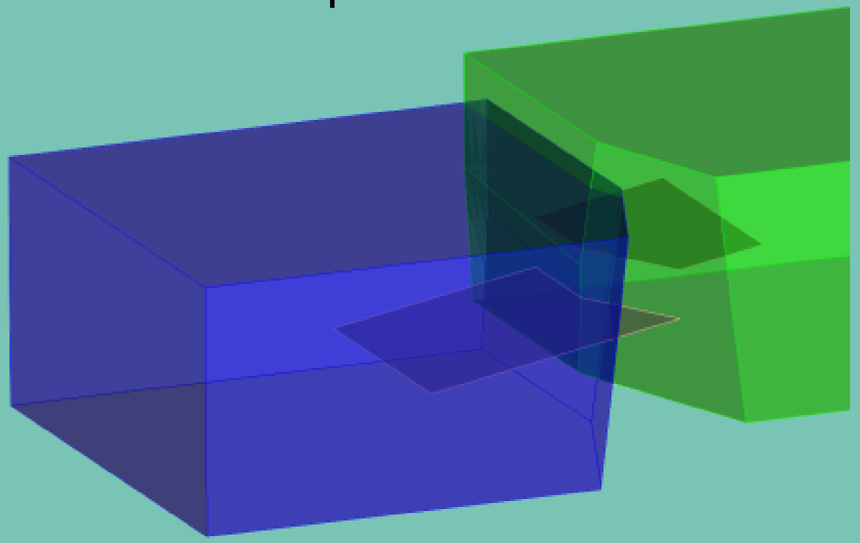
Sculpt already did engineering domains







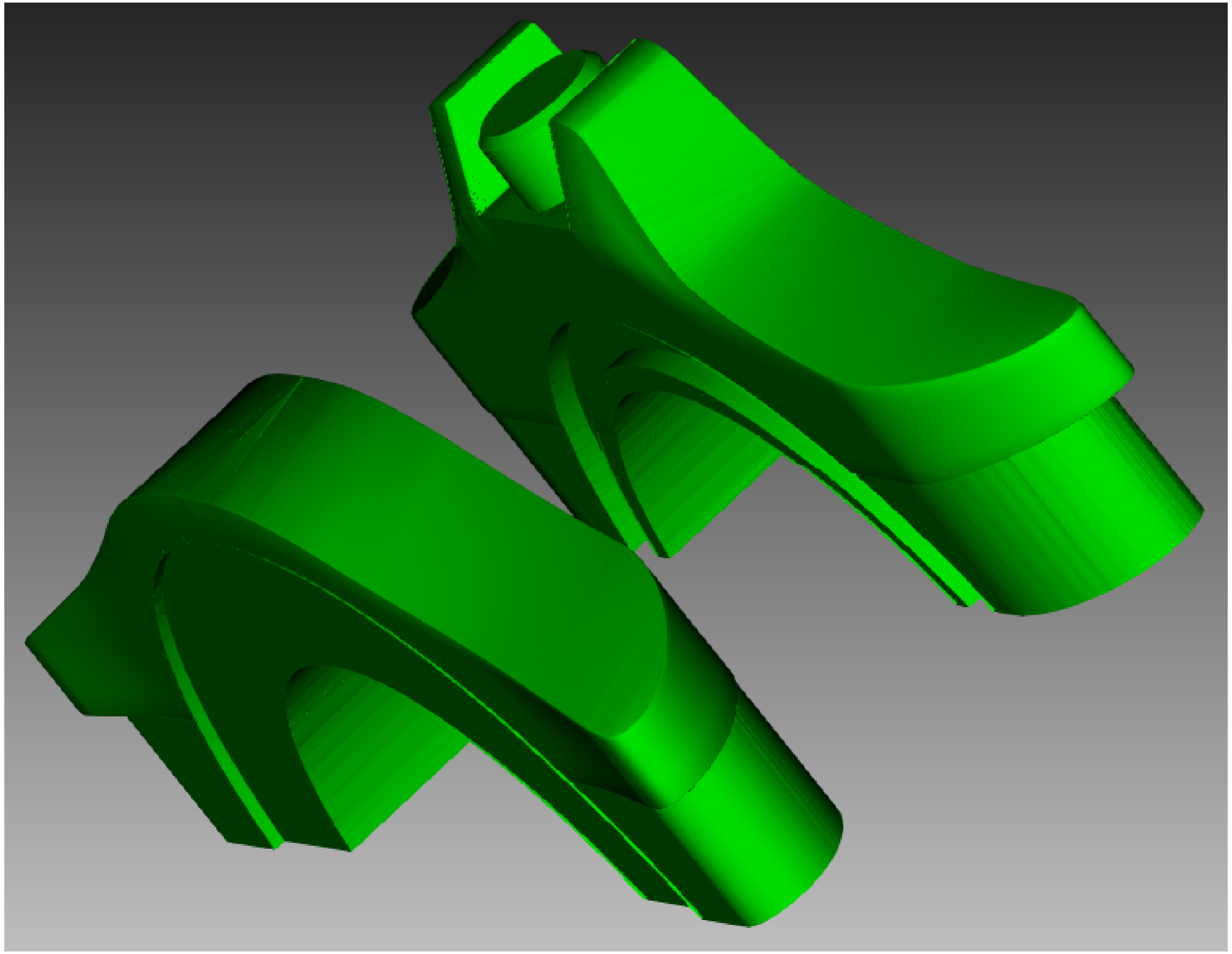
Extend Sculpt to Seismic Domains



Geometric Fantasy and Reality

Thingi10k watch model,
and motivation,
courtesy Jacobson, Panozzo et al.

Fantasy



Reality

Geometric Fantasy and Reality

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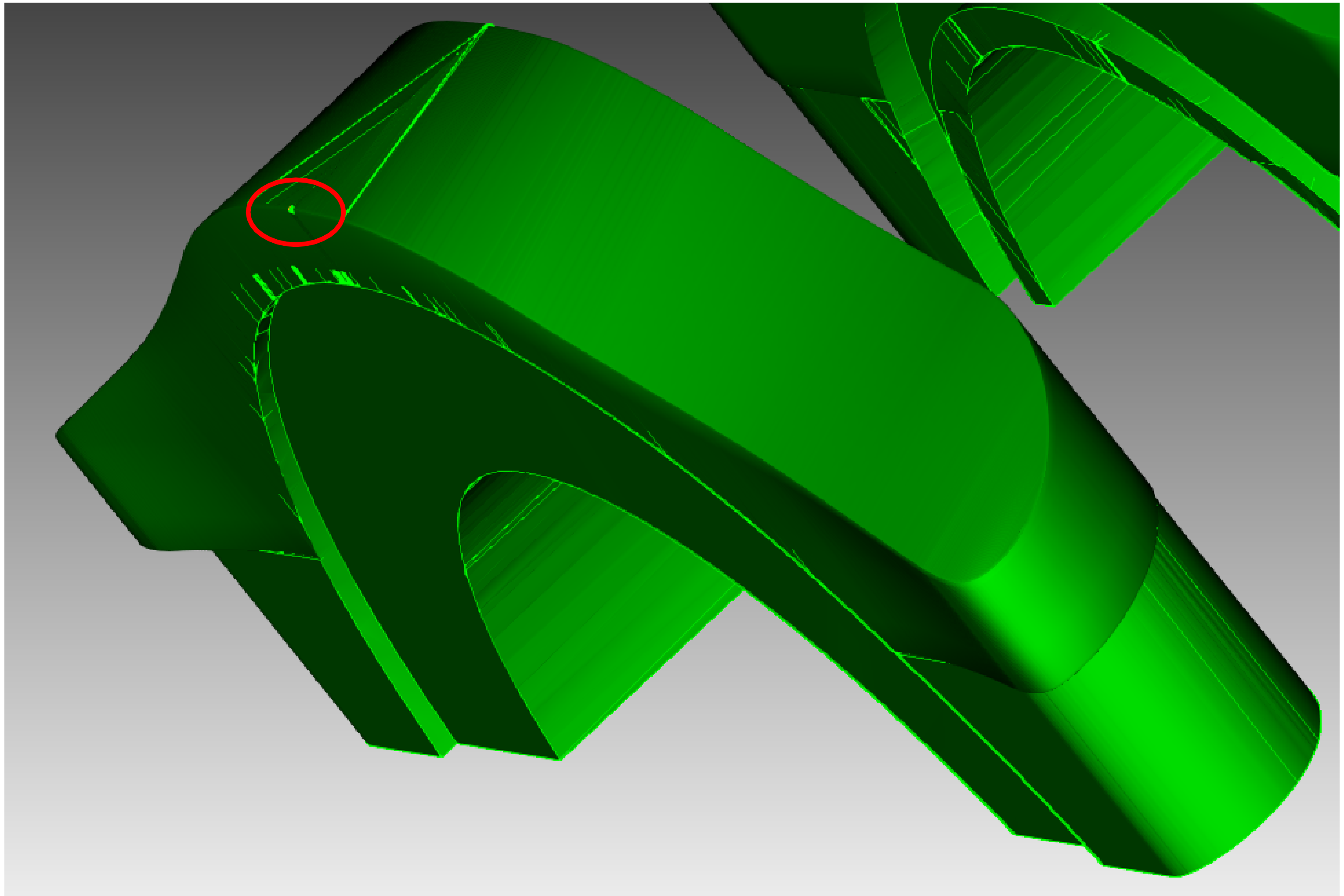
Fantasy



Reality

Geometric Fantasy and Reality

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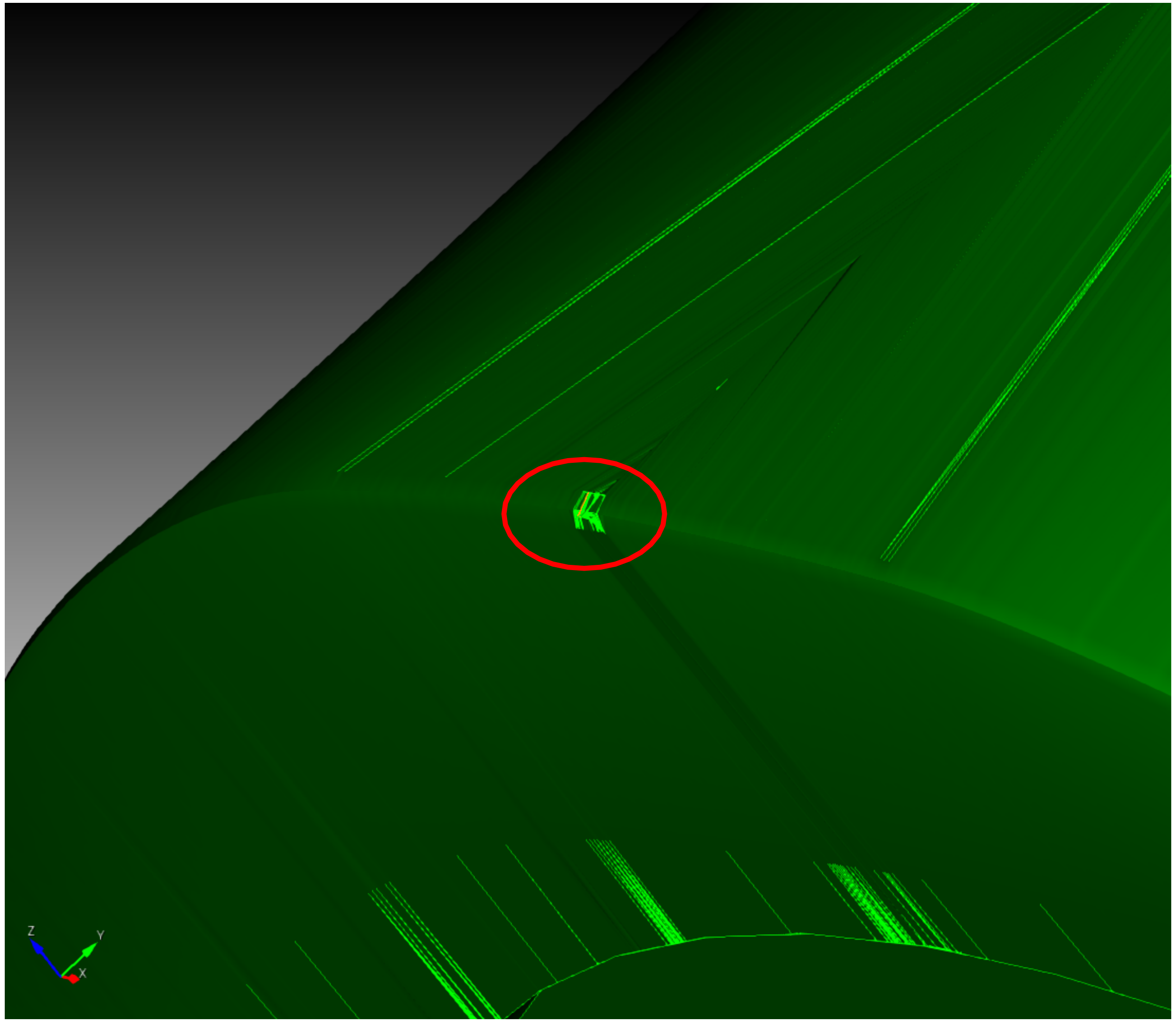
Fantasy



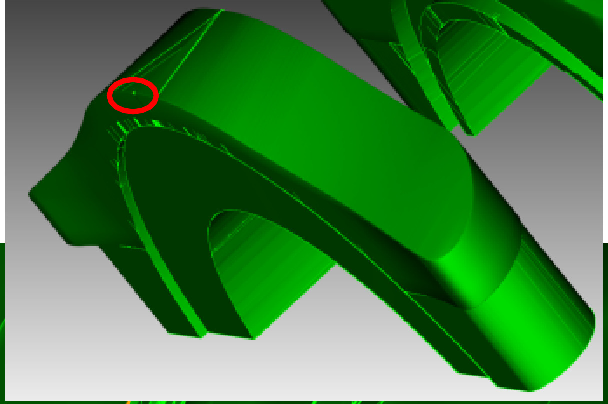
Reality

Geometric Fantasy and Reality

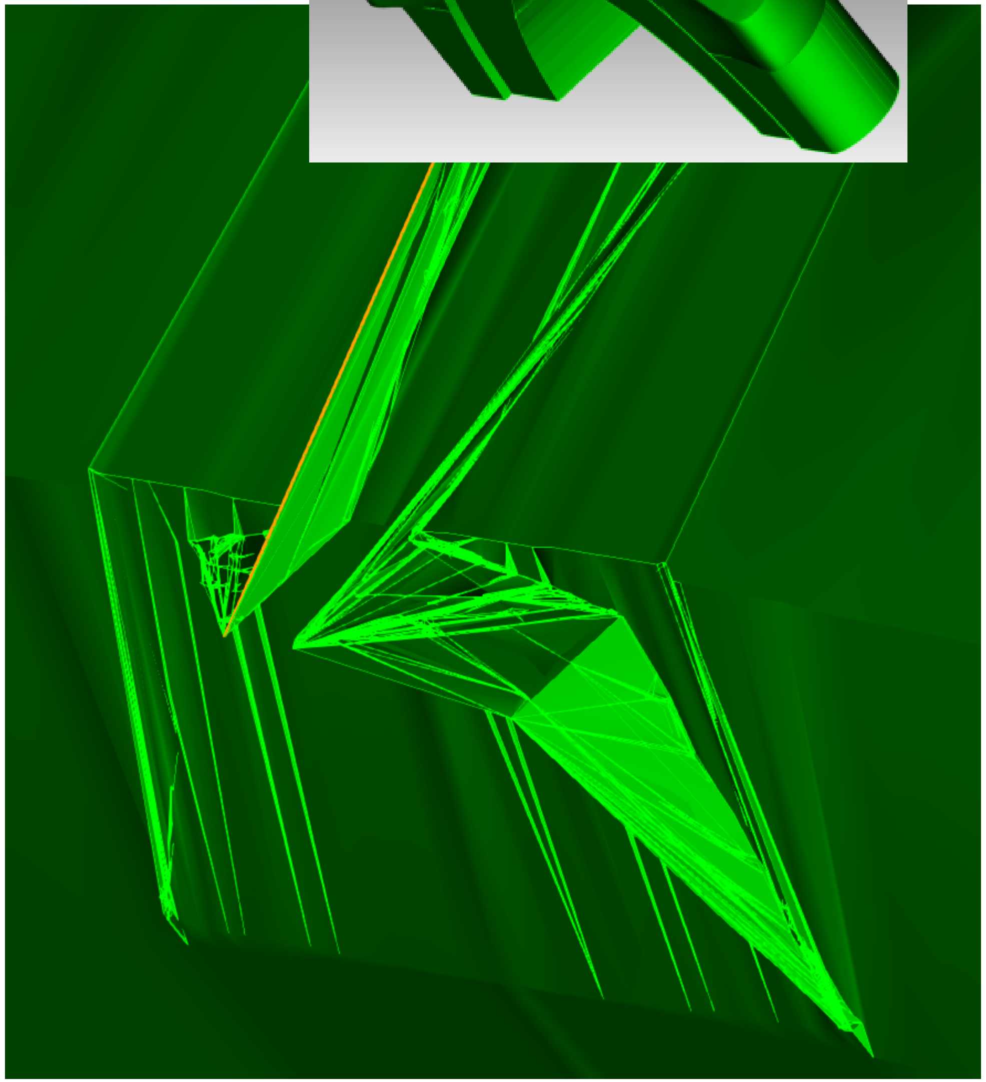
Venus:
You're fantasizing again.
Why would we want to?

Fantasy

Co-author:
We must make VoroCrust reconstruct not just the samples, but all input triangles and edges exactly!



Thingi10k watch model, and motivation, Jacobson, Panozzo et al.



Reality

Research Objectives

Dominant philosophy of practice and theory change:

Process geometry ⟷ mesh → discretize

meshing ugly geometry

Goals: Free scientist to

- use any geometry available
- select any mesh size regardless of geometry
  - Mesh is discrete approximation to geometry
  - **Why require geometry perfection at scale smaller than the mesh resolution?**

Research objectives to achieve goals

- Fidelity measures, math definitions
- Algorithms with quantified fidelity described by mathematical theorems

Geometry Input

Old School	New School
Perfect	Inexplicable
◦ Smooth manifolds or	◦ Point clouds
◦ Watertight CAD models	◦ Triangle soups
Defines desired mesh size	Independent of mesh size
◦ Scientist can select finer, not coarser	◦ Scientist can select any mesh size
Defines mesh geometry	Approx. mesh geometry
Defines mesh topology	Approx. mesh topology

Research Objectives

Choose your mesh: pick any two, the third is dependent!

1. Element Quality
2. Geometric & Topological Fidelity
3. Local Size (# elements)

Old School

1 + 2 exactly reconstruct input geometry & topology

→ 3 small elements near small and sharp features

New School

1 + 3 mesh size is what the scientist needs

→ 2 Geometric and topological fidelity is approximate

Geometry Input

Old School

- Perfect
  - Smooth manifolds or
  - Watertight CAD models
- Defines desired mesh size
  - Scientist can select finer, not coarser
- Defines mesh geometry
- Defines mesh topology

New School

- Inexplicable
  - Point clouds
  - Triangle soups
- Independent of mesh size
  - Scientist can select any mesh size
- Approx. mesh geometry
- Approx. mesh topology

Local feature size, lfs

- Traditional measure of input geometry
- Limits max local mesh size
- Predicts geometric fidelity
  - Distance between mesh and geometry

Persistent Homology

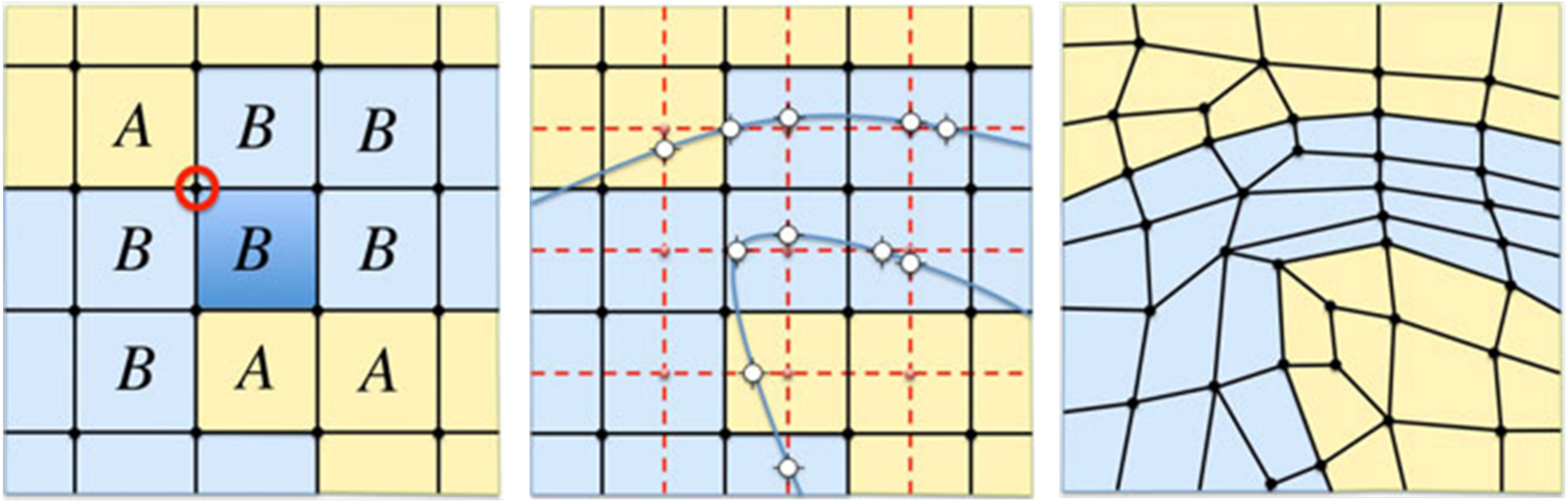
- Measures how topology changes with geometric resolution
- If mesh size > lfs, can we predict the topological fidelity?
  - Input geometry topology vs. output mesh topology

Algorithms we are going to prove things about.
1. Sculpt: Marching-Cubes Hex Meshing 3D

Sculpt images courtesy

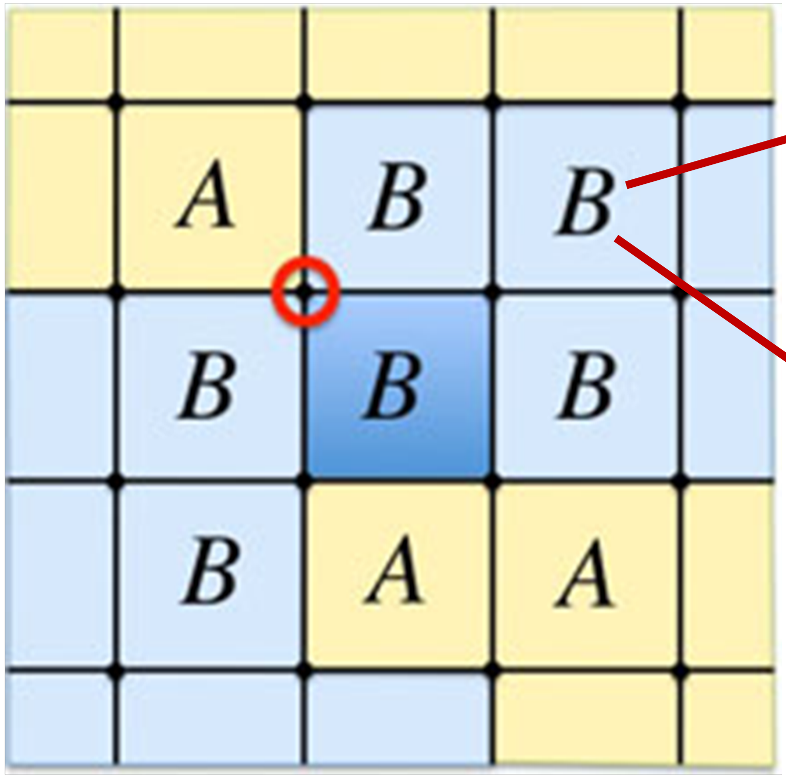
Steven J. Owen, Matthew L. Staten, and Marguerite C. Sorensen. Parallel hex meshing from volume fractions. In William Roshan Quadros, editor, *International Meshing Roundtable*, pages 161–178, Berlin, Heidelberg, 2012. Springer Berlin Heidelberg.

- Background grid
- Assign volume fractions
- Assign materials to nodes
- Construct hex mesh with consistent global topology



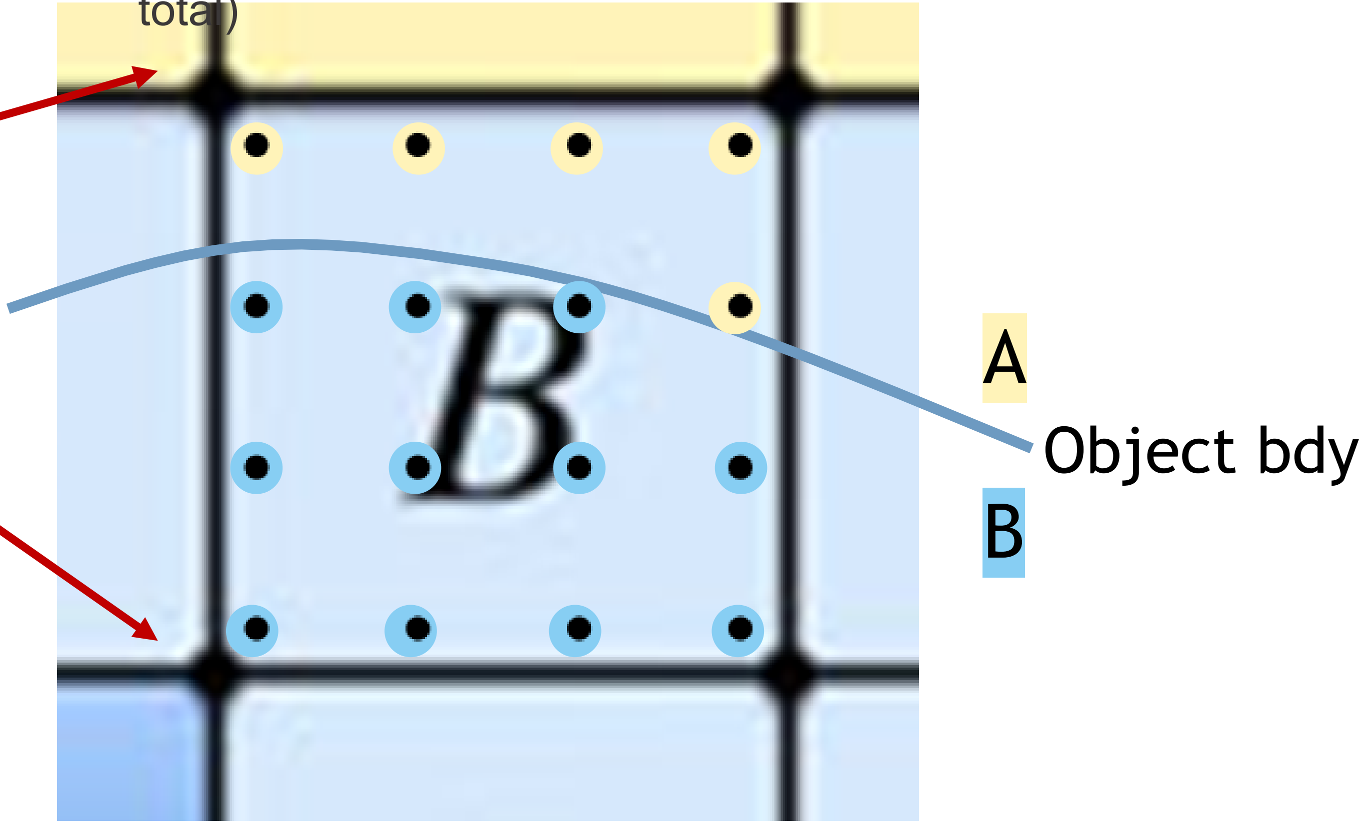
Algorithms we are going to prove things about.
1. Sculpt: Marching-Cubes Hex Meshing 3D

- Background grid
- **Assign volume fractions**



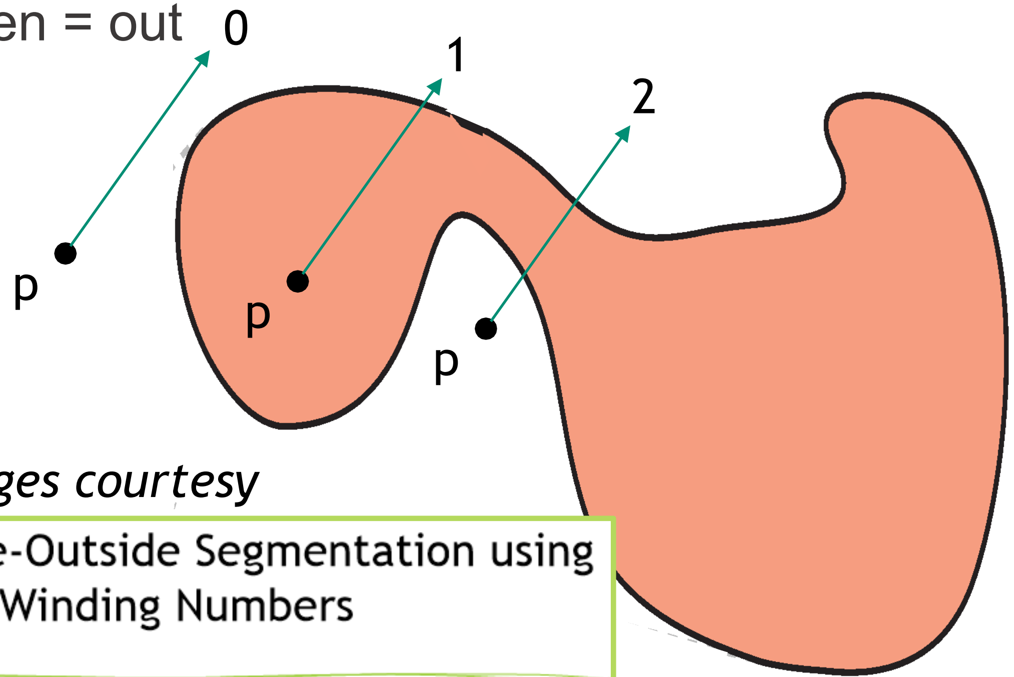
Assign volume fractions

- Discrete query points in each cell
- Determine which volume, A or B, it is inside
  - **Is p inside A or out?**
  - Is p inside B or out?
- Volume fraction A is (number in A) / (number total)



Ray Casting for Inside/Out

Def. p Inside? Cast (random) ray from p to infinity
Count number of times ray crosses object boundary
if odd = inside
even = out



Base images courtesy

Robust Inside-Outside Segmentation using Generalized Winding Numbers

Alec Jacobson ETH Zurich
Ladislav Kavan University of Pennsylvania
Olga Sorkine-Hornung ETH Zurich

igl INTERACTIVE GEOMETRY LAB

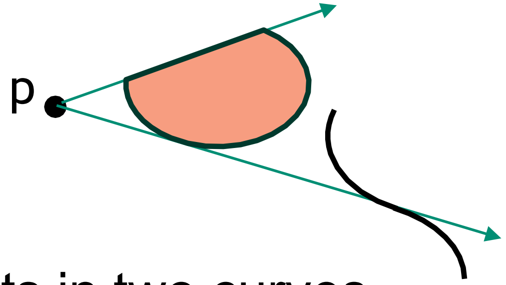
October 9, 2013

ETH Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich

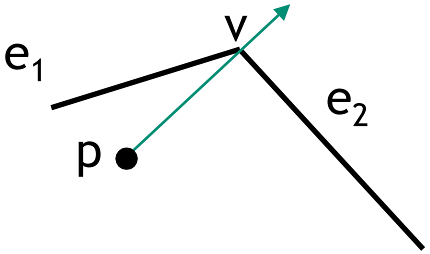
Challenges:

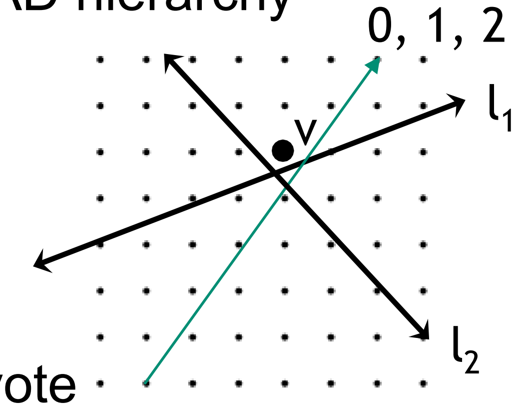
- non-linear surfaces
  - root finding

- tangencies



- points in two curves
  - floating point coordinates
  - trimmed CAD hierarchy



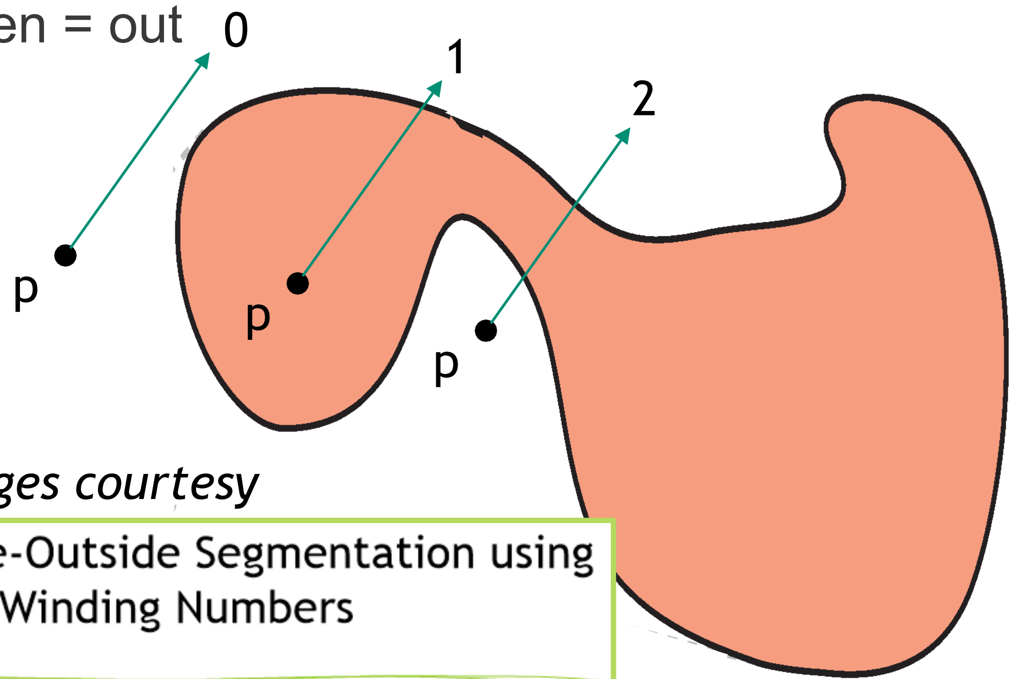


$e_1 = \{u, v, l_1\}$

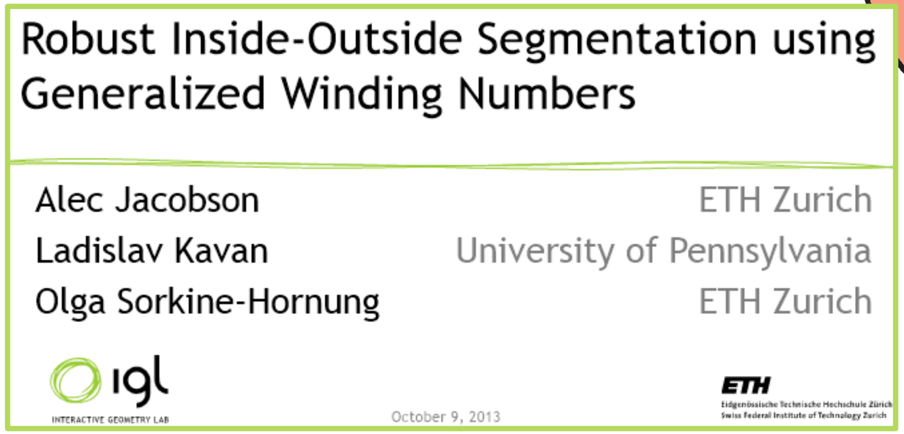
Solutions: re-cast and popular vote

Ray Casting for Inside/Out

Def. p Inside? Cast (random) ray from p to infinity
Count number of times ray **crosses** object boundary
if odd = inside
even = out



Base images courtesy



Fundamental issue:
- "**Does it cross?**" is discrete decision based on imprecise floating point calculation **where it crosses**

Any discrete **error is large, 1,**
- regardless of how small the error is in the floating point calculation

Re-cast and popular vote
- reduces the final probability of error, but not its magnitude.

Solutions: re-cast and popular vote

Winding Number for Inside/Out

Winding number images courtesy

Robust Inside-Outside Segmentation using Generalized Winding Numbers

Alec Jacobson — ETH Zurich
Ladislav Kavan — University of Pennsylvania
Olga Sorkine-Hornung — ETH Zurich

igl INTERACTIVE GEOMETRY LAB — October 9, 2013 — ETH Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich

Def. Winding Number: for query point p,
traverse boundary of object,
counting the *number* of times it goes (*winds*) around p

$$w(\mathbf{p}) = \frac{1}{2\pi}\oint_{\mathcal{C}} d\theta$$

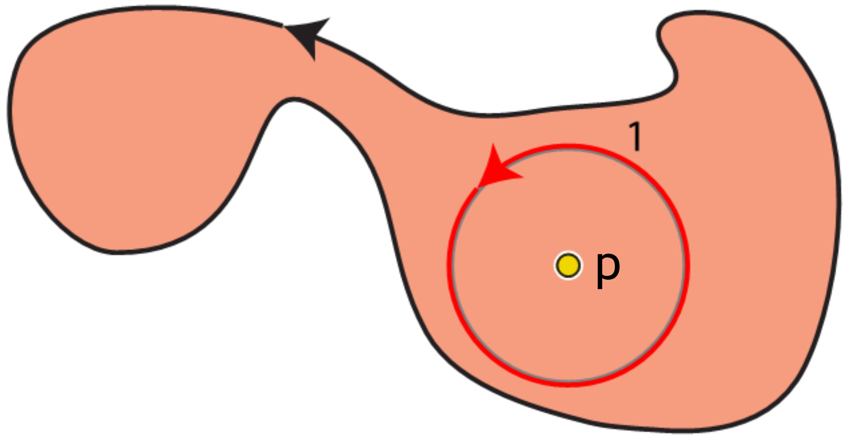
Fundamental improvement:

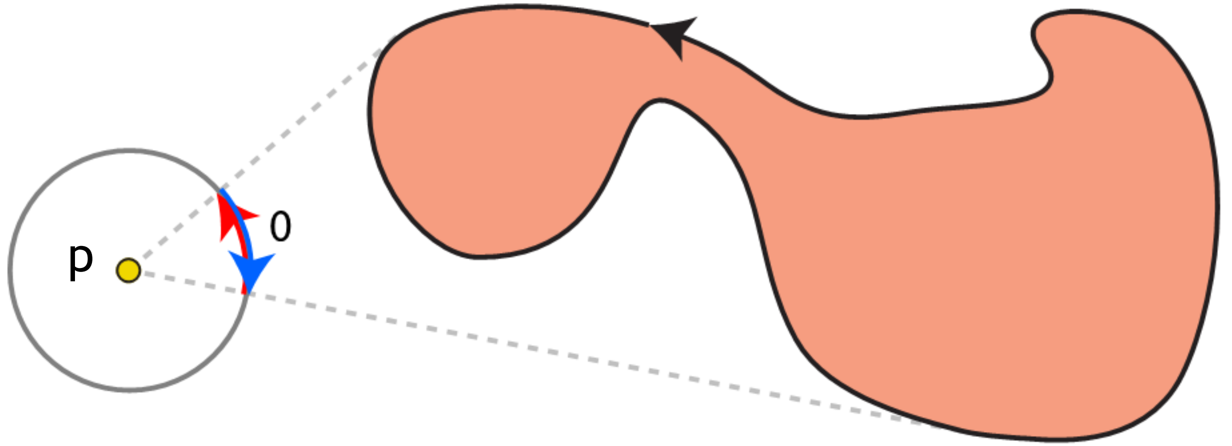
Answer is a floating point calculation from entire boundary.

Any error is small, so rounding to 0,1 usually OK

Errors arise in human-ambiguous cases

If shape is watertight,
winding number is perfect measure of inside



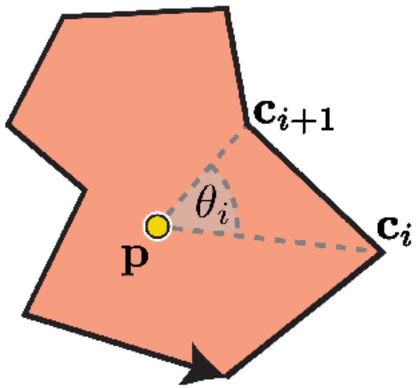


Winding Number for Inside/Out – how to calculate

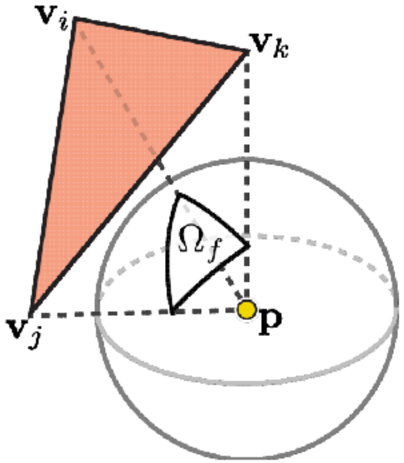
Naive discretization is simple and exact

$$w(p) = \frac{1}{2\pi}\oint_C d\theta$$

$$w(p) = \frac{1}{2\pi}\sum_{i=1}^{n} \theta_i$$



Generalizes elegantly to 3D via solid angle



$$w(p) = \frac{1}{4\pi}\iint_S \sin(\varphi)\, d\theta\, d\varphi$$

$$w(p) = \frac{1}{4\pi}\sum_{f=1}^{n} \Omega_f$$

Angle subtended by each boundary piece (triangle)

- Order-independent calculation → easy parallel

Approximate groups of far-away triangles by large triangles, hierarchical octree

- “Fast Winding Numbers for Soups and Clouds”

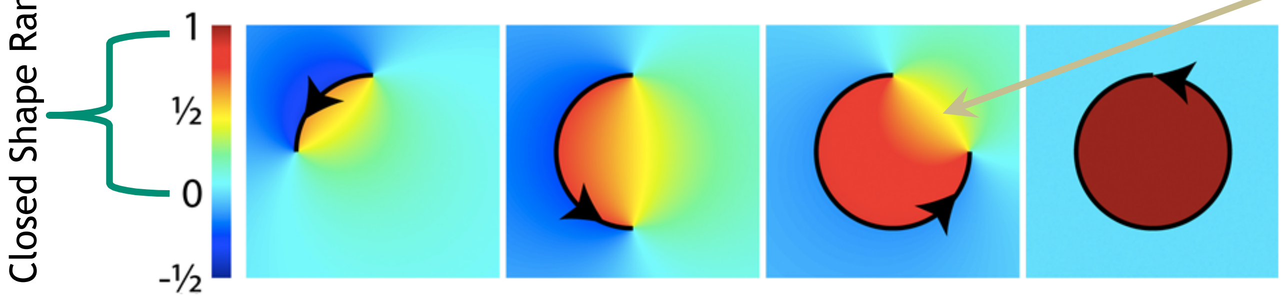
Winding Number for Inside/Out of Ugly Geometry

Types of ugliness:

Ambiguous cases for humans and winding numbers (≈ 0.5) coincide

What happens if the shape is open?

$$w(\mathbf{p}) = \frac{1}{2\pi}\oint_{\mathcal{C}} d\theta$$

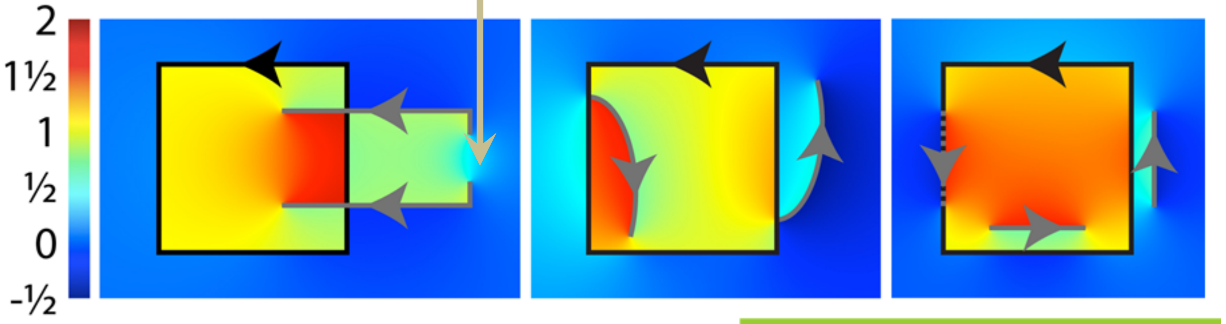


Gracefully tends toward perfect indicator as shape tends towards watertight

Ugly ← Beautiful

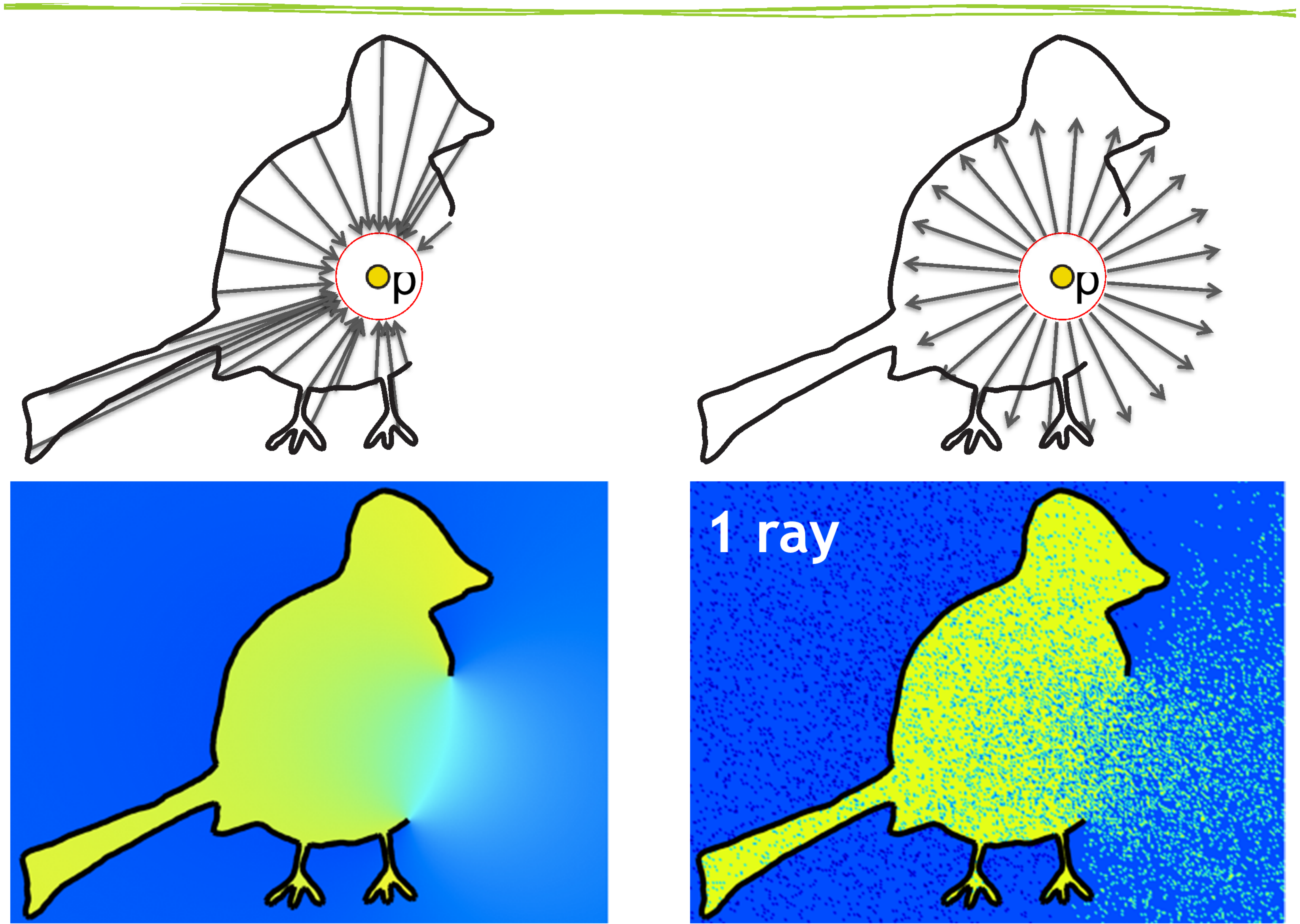
What if shape is self-intersecting? Non-manifold?

$$w(\mathbf{p}) = \frac{1}{2\pi}\oint_{\mathcal{C}} d\theta$$



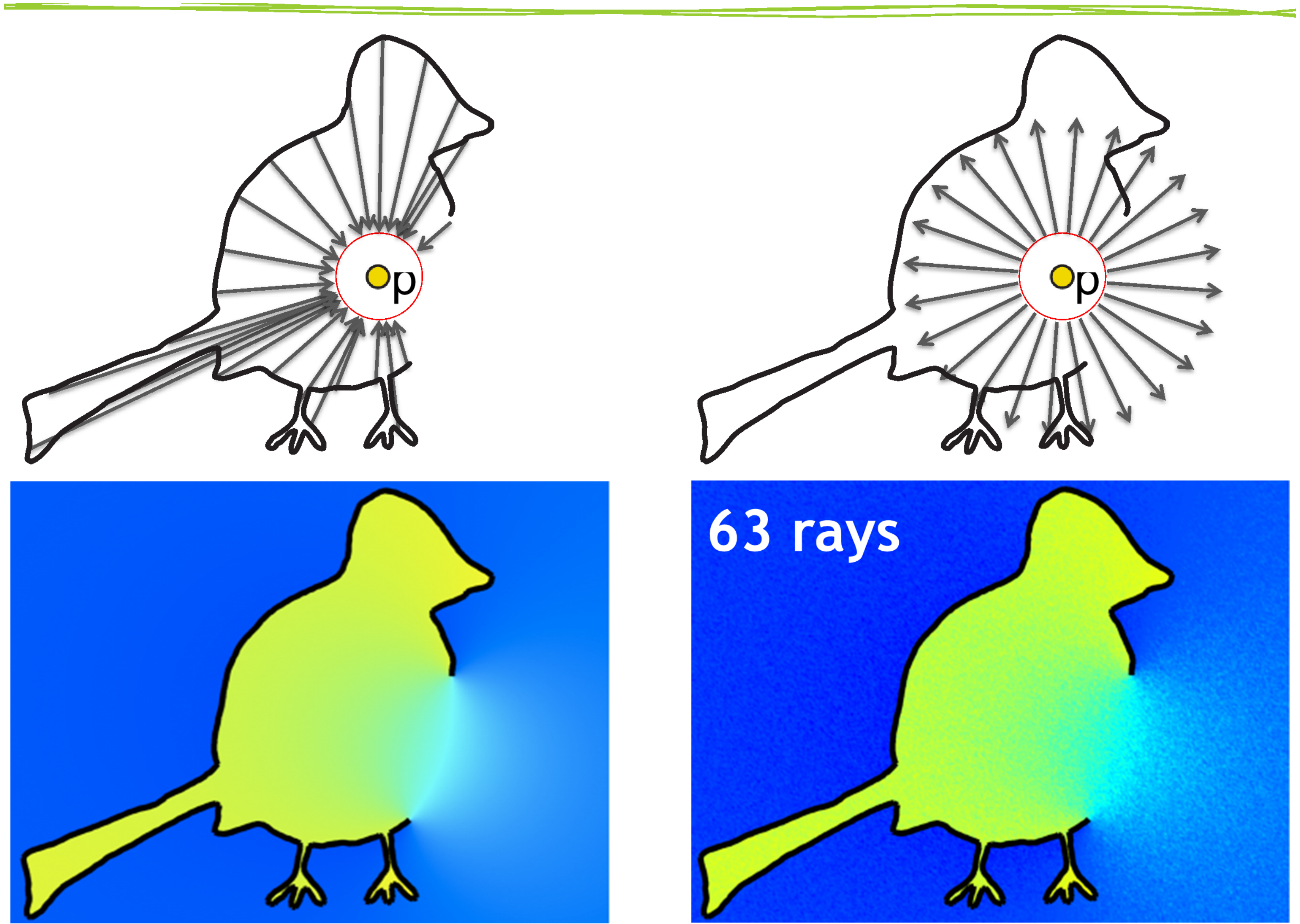
Jumps by ±1 across input facets

Winding Number Ambiguity vs. Ray Casting Errors



Causes serious issues for algorithms that mesh convex hull and discard elements outside

Winding Number Ambiguity vs. Ray Casting Errors



Sculpt Volume Fractions from Winding Number from libigl

igl open-source

Assign volume fractions

- Discrete query points in each cell
- Determine which volume, A or B, it is inside
  - **Is p inside A or out?**
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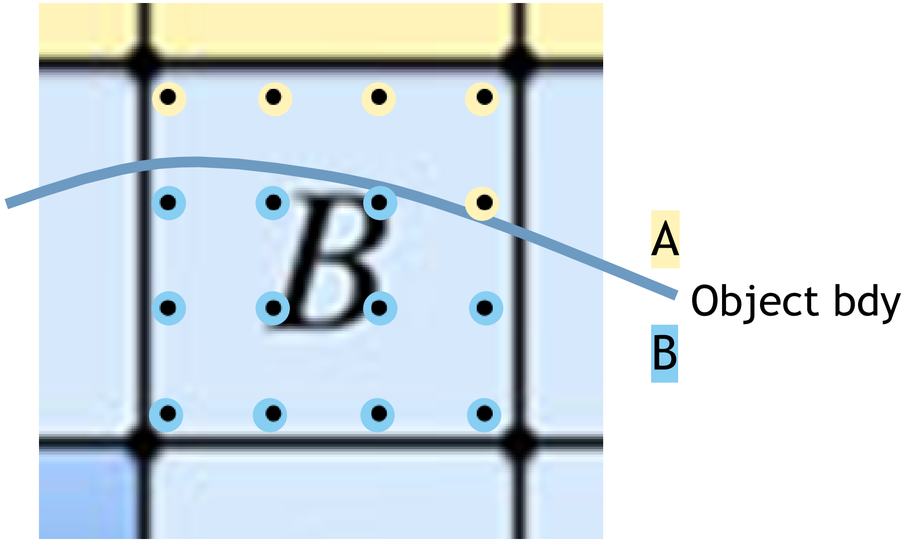
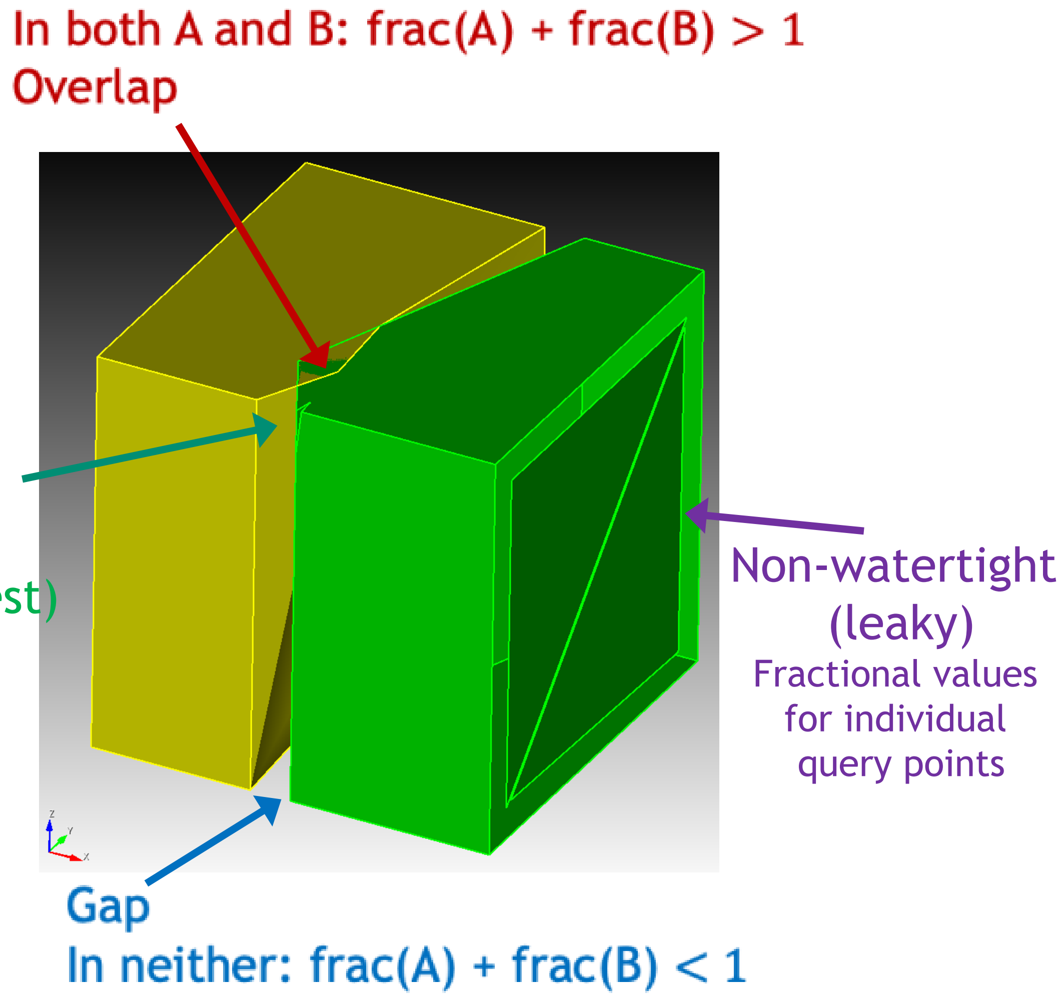
- Volume fraction A is (number in A / (number total)

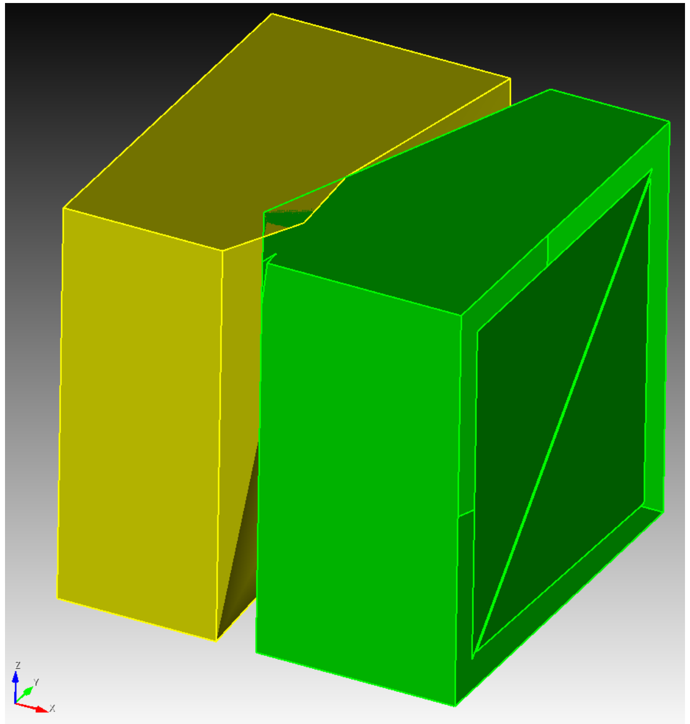


Regular background grid of cells and query points

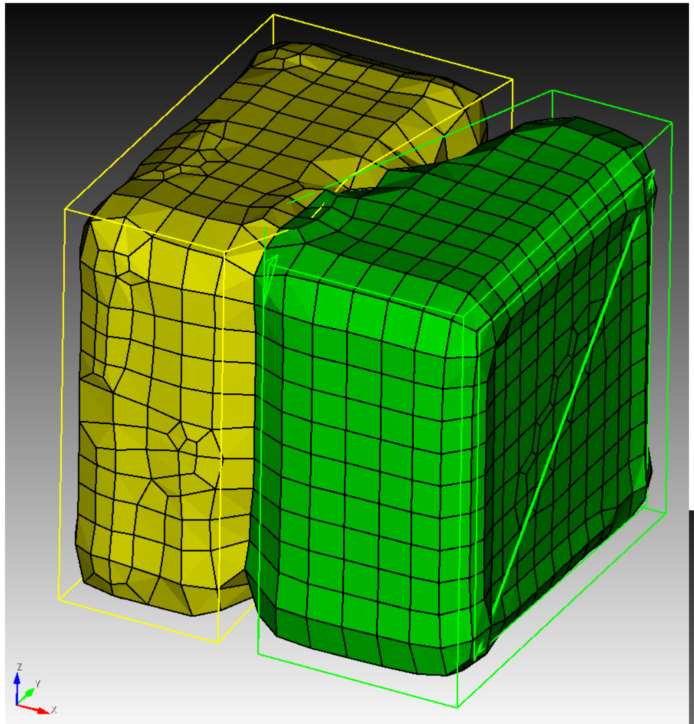
- Normalize if sum fractions >1 or <1

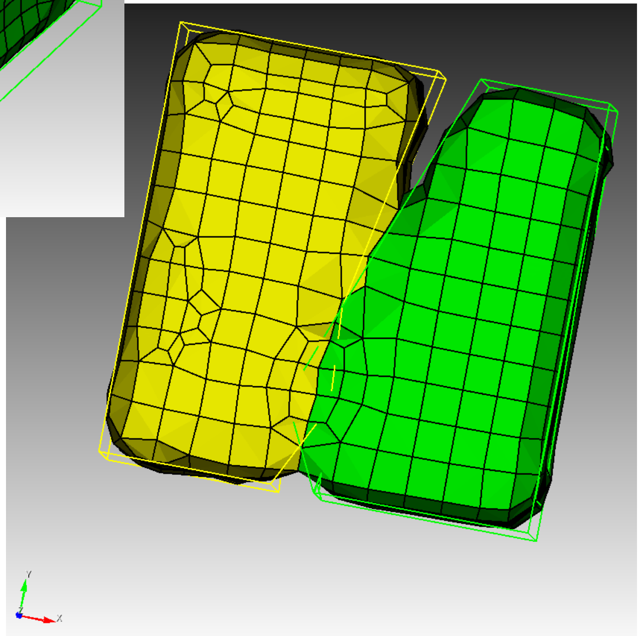


Sculpt Mesh from Uniform Background Grid



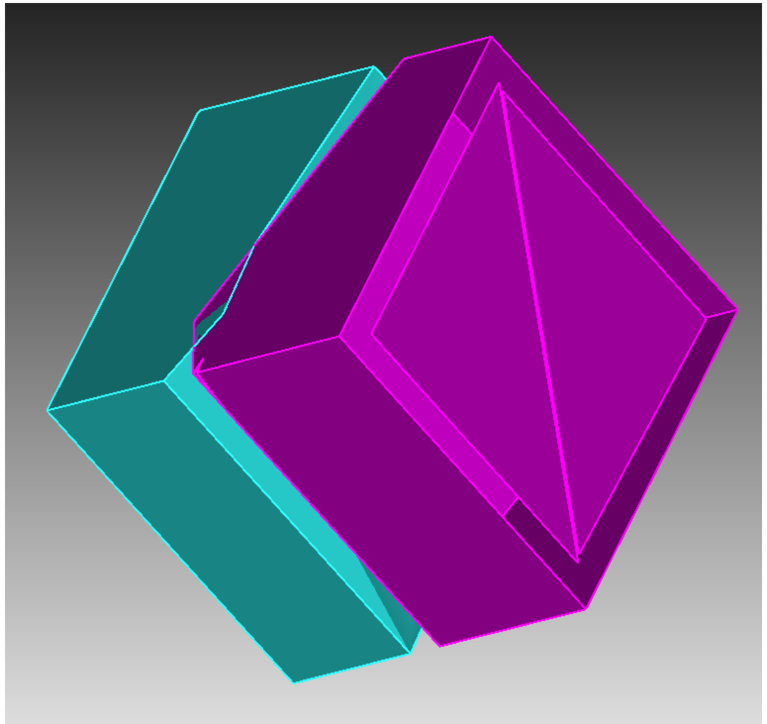
Input





Output

Sculpt Mesh from Custom Background Grid



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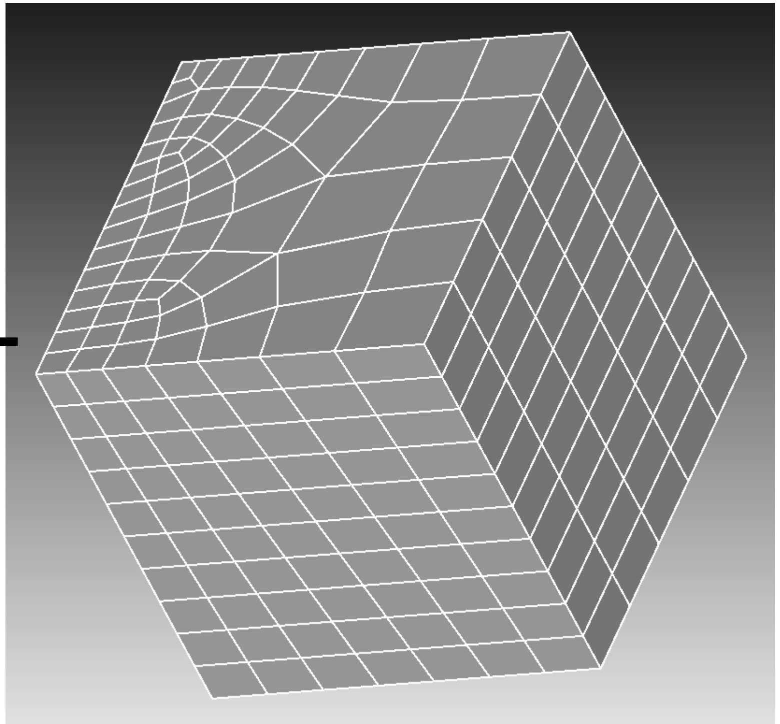

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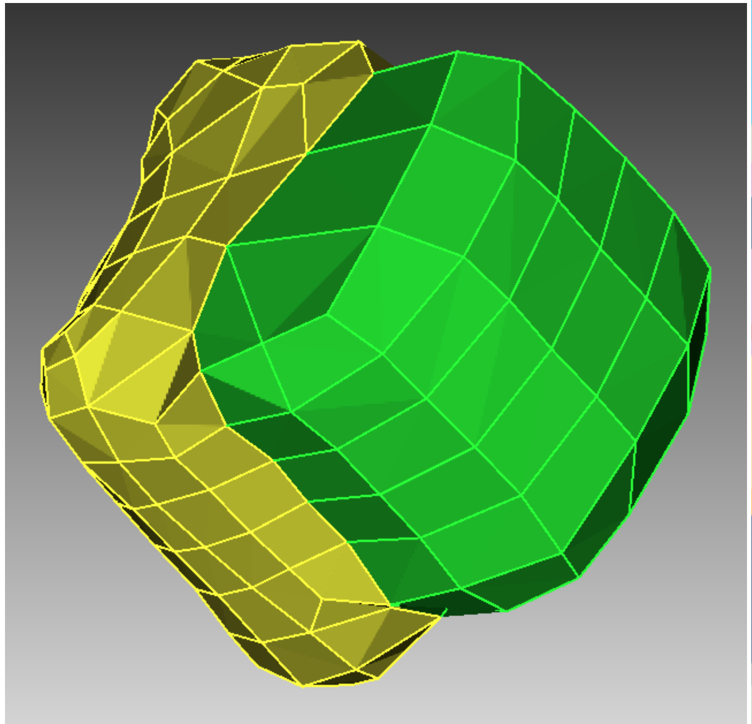


Input
Geom

Input
Grid

Output
Mesh

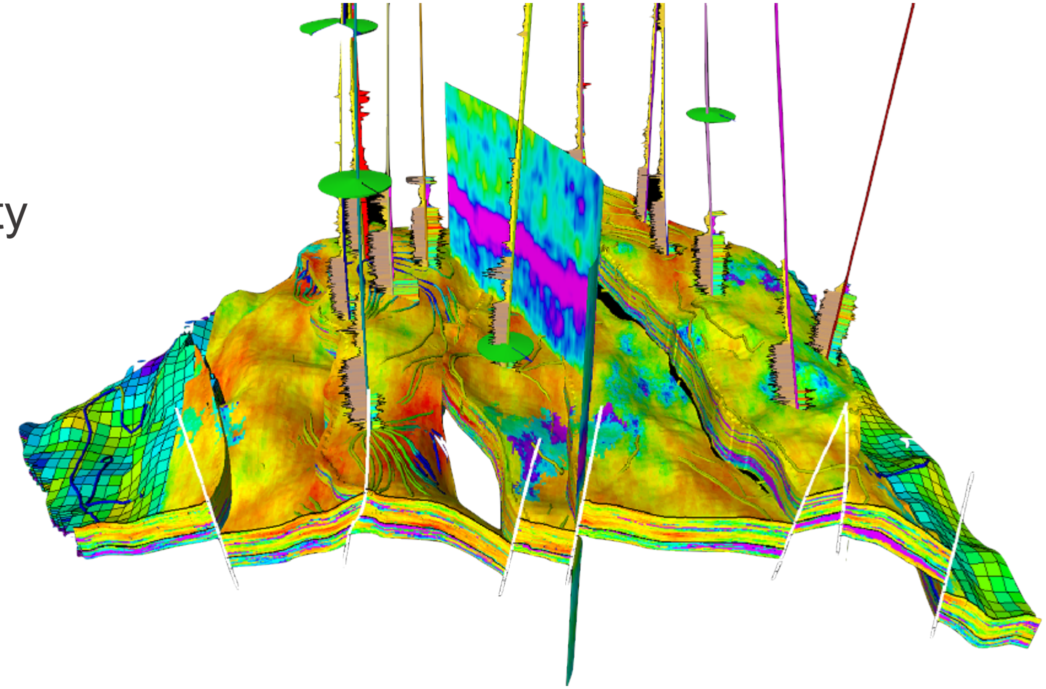
Future work

Use persistent homology of ugly geometry to

- Quantify tradeoffs between topological and geometric fidelity
- Predict mesh topology for given background grid
- Choose background grid size to
  achieve given user-desired topology
- Aleph — **A** **L**ibrary for **E**xploring **P**ersistent **H**omology
  א

Use shrink wrapping to

- Mesh finer than geometric fidelity
- Non-noisy mesh despite noisy geometry



More complex domains

Thank you

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Advanced Scientific Computing Research (ASCR), Applied Mathematics Program.

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