SAND2023-07464C

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EQUIVALENCE OF CONDITIONAL POINT SAMPLING WITH THE ATOMIC MIX APPROXIMATION AND CHORD LENGTH SAMPLING WHEN USING SIMPLE USER OPTIONS

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M&C 2023, August 15, 2023

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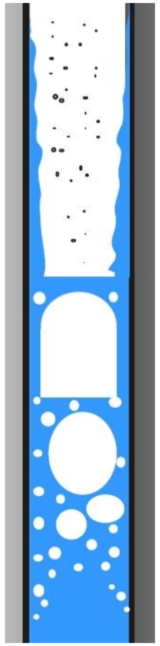
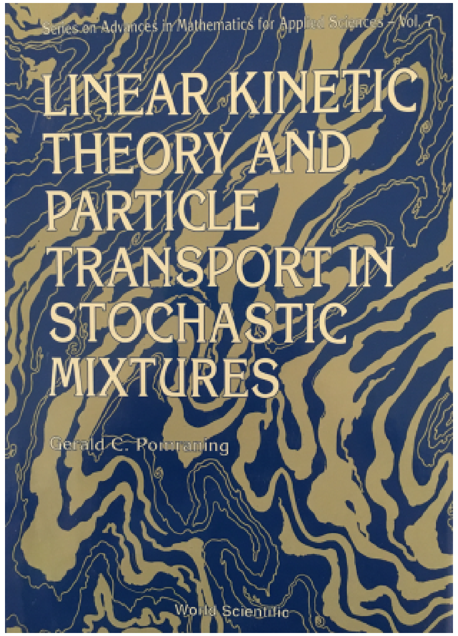
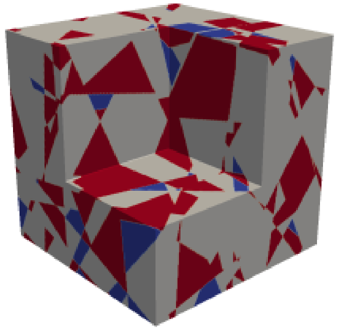
STOCHASTIC MEDIA TRANSPORT







In stochastic media (SM) transport,

We want

- Accuracy
- Efficiency

We also want

- Higher-order results (e.g., variance, PDFs)
- Trade-offs (accuracy and efficiency)
- Simplicity (for developers and analysts)

LANDSCAPE OF SM TRANSPORT MODELS – PRE-COPS

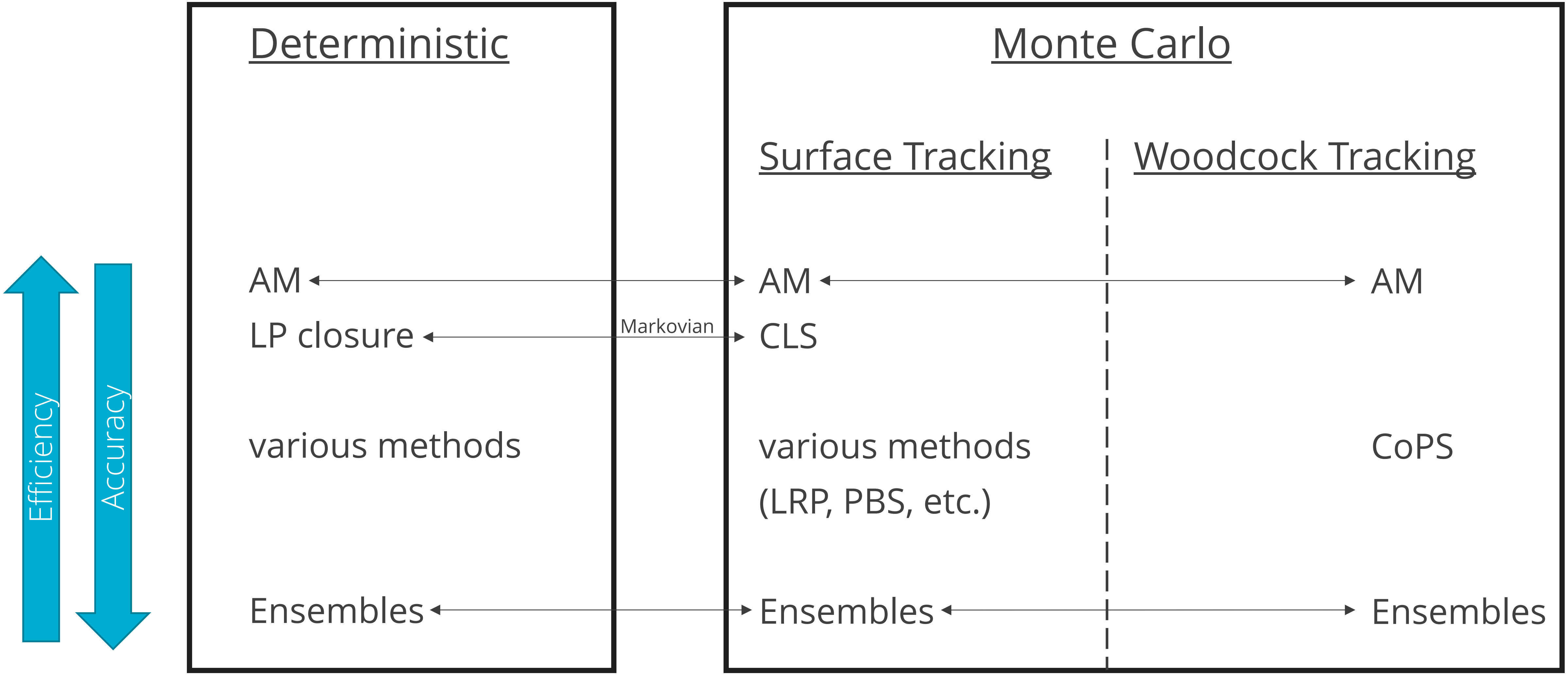
Efficiency

Accuracy

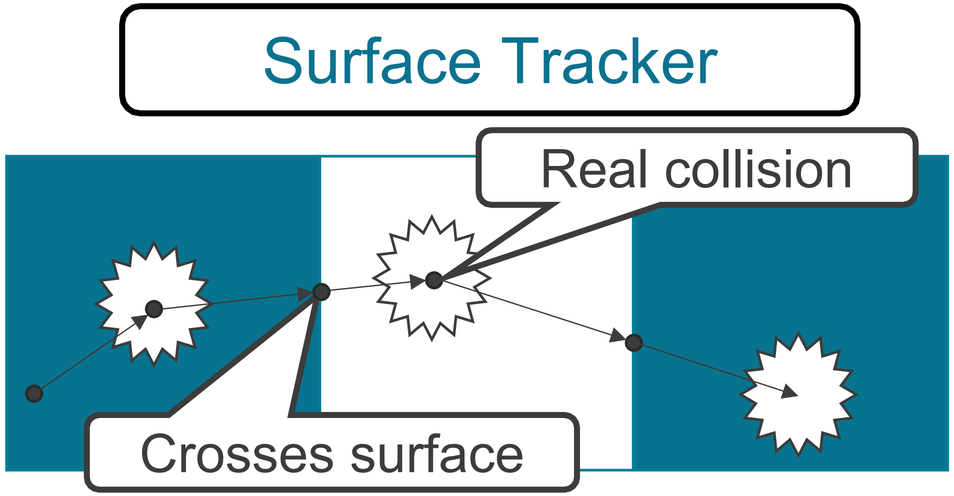
Deterministic		Monte Carlo
AM	↔	AM
LP closure	↔ Markovian	CLS
various methods		various methods (LRP, PBS, etc.)
Ensembles	↔	Ensembles

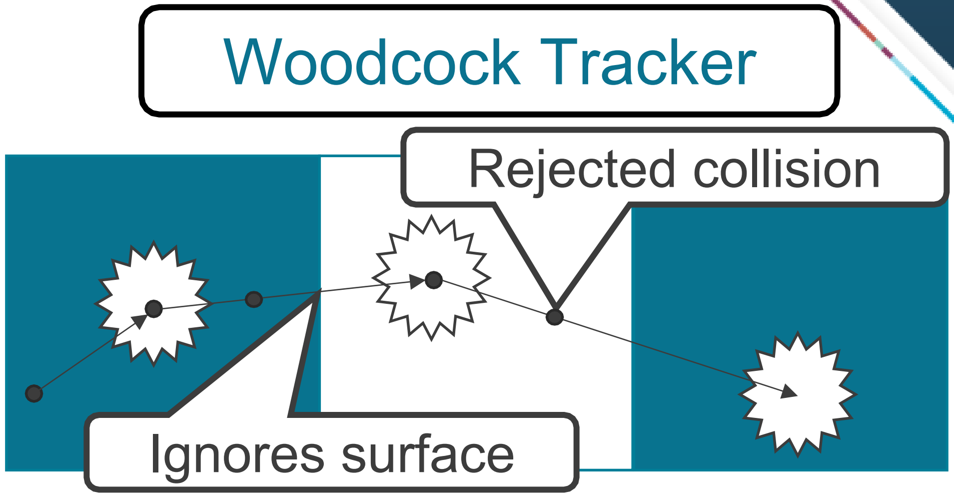
LANDSCAPE OF SM TRANSPORT MODELS - W/ COPS

CoPS



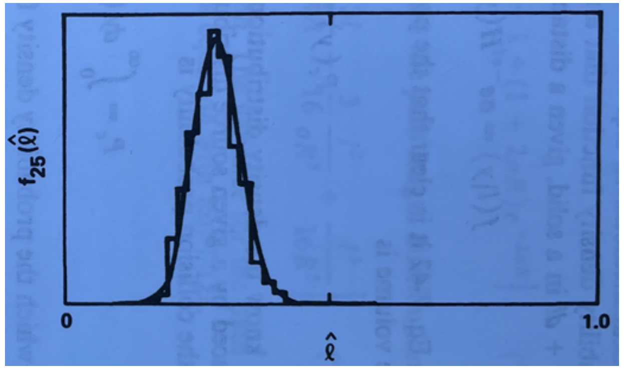
WHY HAVE A WOODCOCK-BASED METHOD?

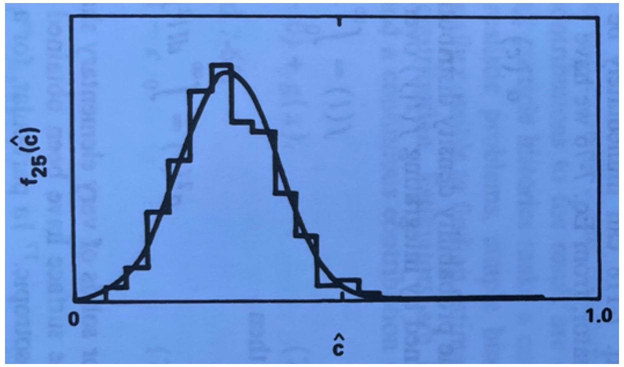




	Surface Tracker	Woodcock Tracker
Non-physical events:	• Boundary crossing	• Rejected pseudo-collisions
Tracking:	• More efficient	• Less efficient (usually)
Tallies:	• More efficient	• Less efficient (usually)

CHEETAH MC

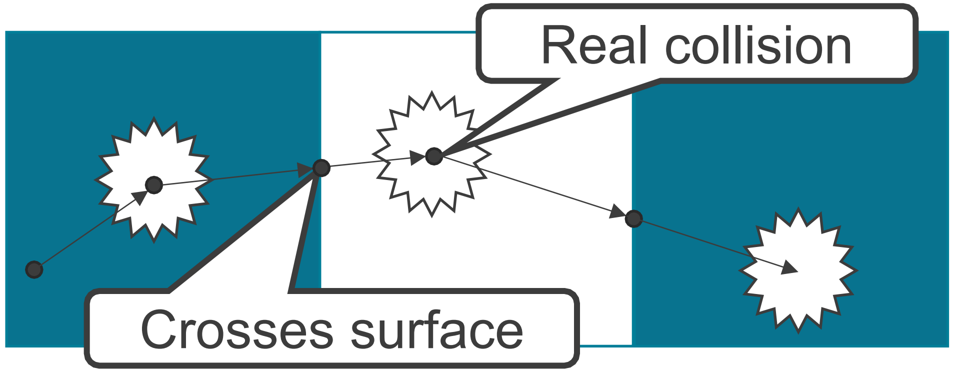




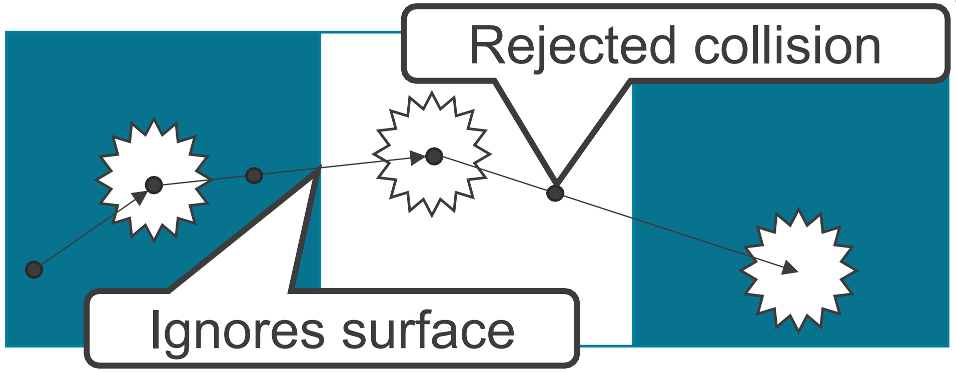
Lewis, E., and W. Miller Jr.
Computational Methods of Neutron Transport.
American Nuclear Society, Inc., 1993.

WHY HAVE A WOODCOCK-BASED METHOD?

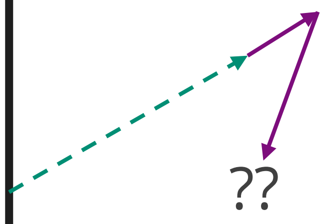
Surface Tracker



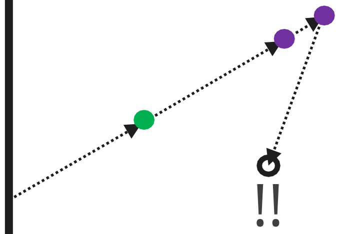
Woodcock Tracker



Memory of sampled features
+
Accurate conditional sampling
=
High accuracy
+
Beyond means

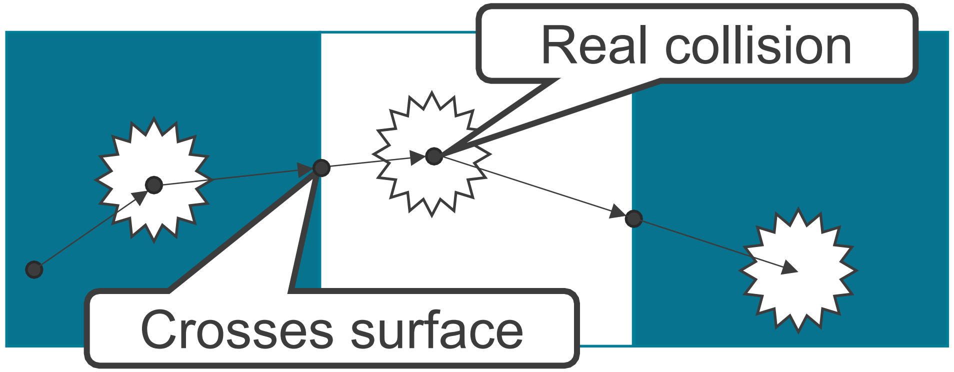
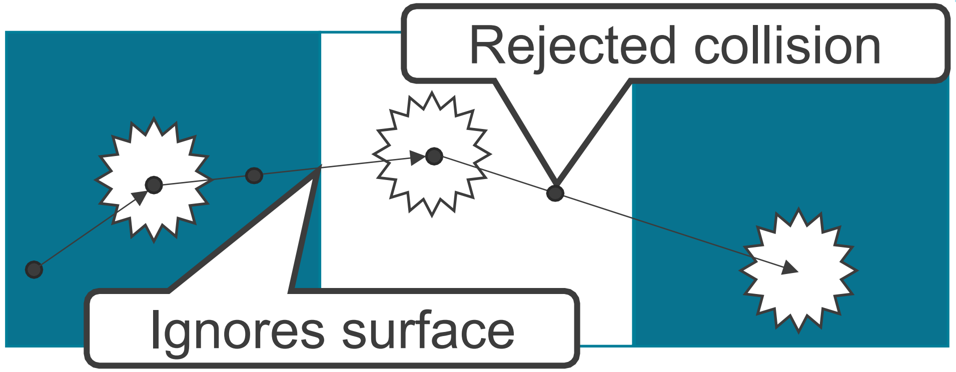


How to sample new features based on previous samples?? (continuous)



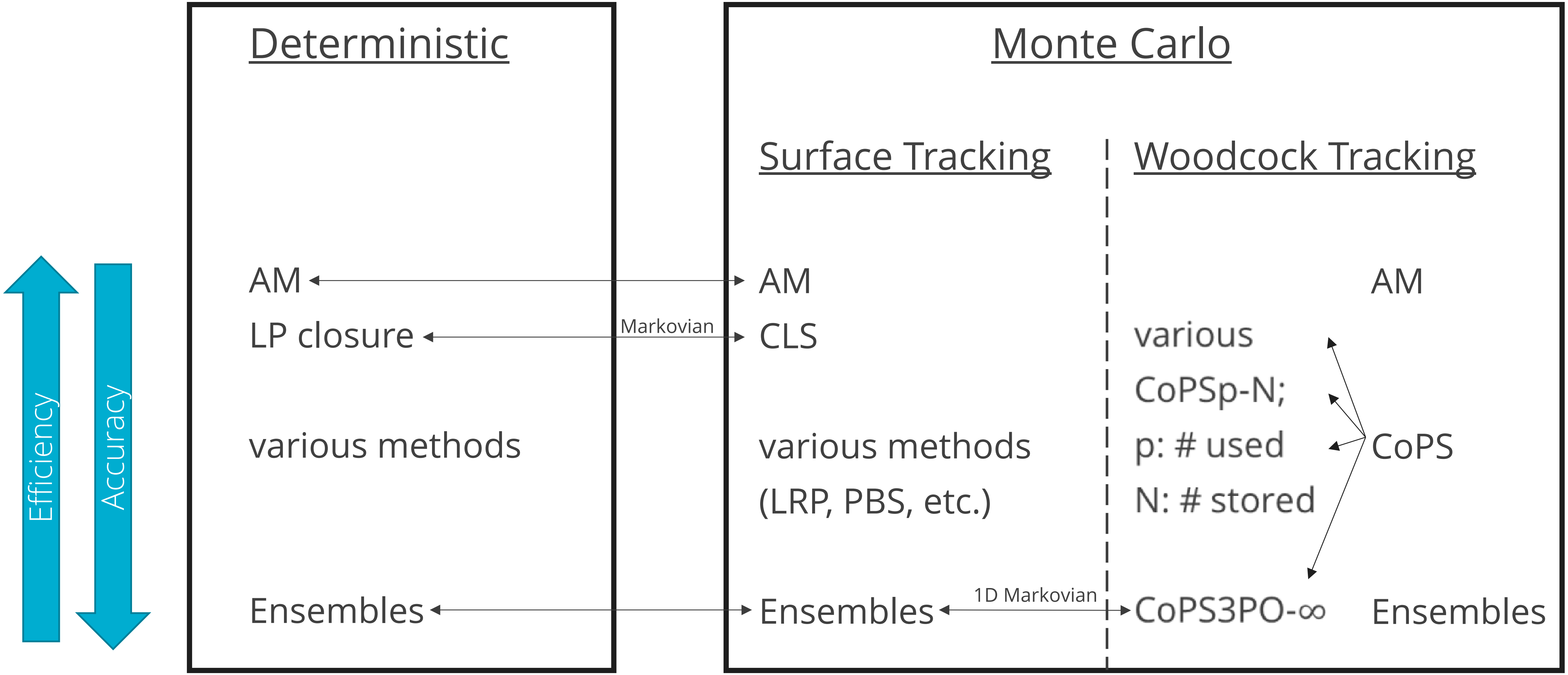
Can samples new features based on previous samples!! (discrete)

WHY HAVE A WOODCOCK-BASED METHOD?

	Surface Tracker	Woodcock Tracker
		
Non-physical events:	• Boundary crossing	• Rejected pseudo-collisions
Tracking:	• More efficient	• Less efficient (usually)
Tallies:	• More efficient	• Less efficient (usually)
SM Alg. Accuracy:	• Moderate accuracy	• High accuracy
SM Alg. Results:	• Mean only	• Mean, variance, PDFs, etc.
SM Alg. Flexibility:	• Some flexibility	• Adaptability and hierarchy

CHEETAH MC

LANDSCAPE OF SM TRANSPORT MODELS – W/ MF COPS



WHY DOES MULTI-FIDELITY COPS MATTER?

CoPS

Practical considerations:	Who good for?
• One computational framework	(developers)
• One conceptual framework	(developers & analysts)
• One user interface	(developers & analysts)
• Problem-specific accuracy/efficiency tradeoff	(analysts)

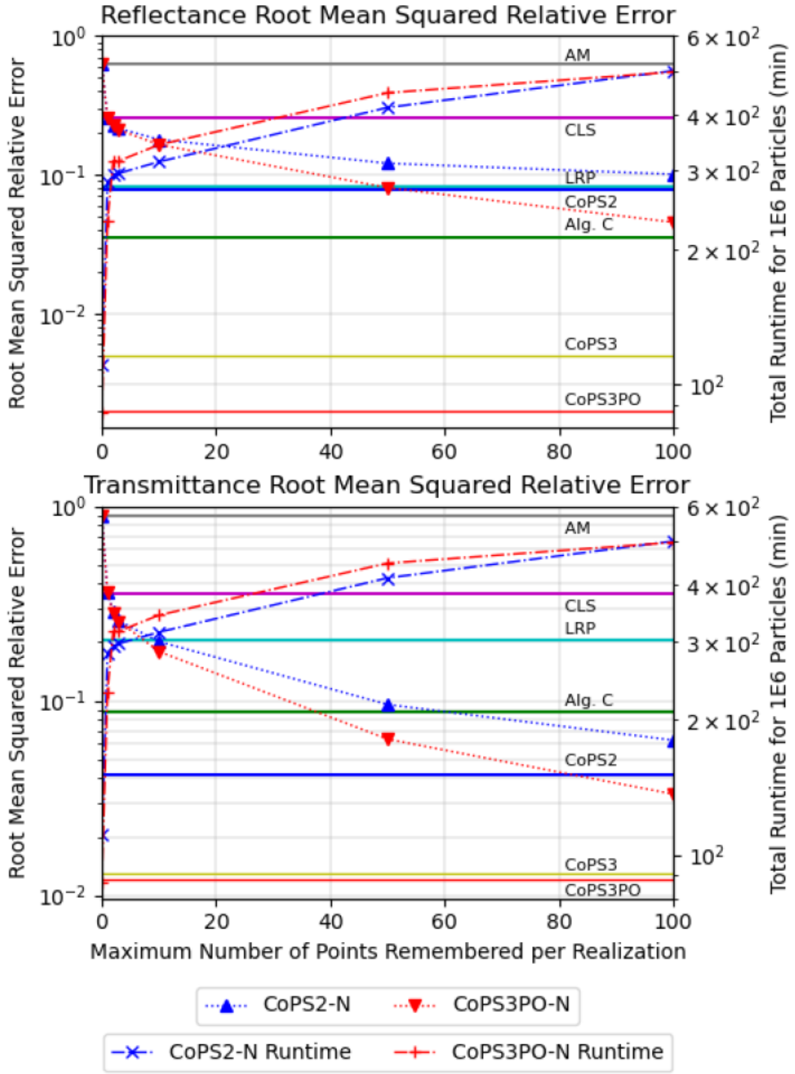
Formal multi-fidelity (MF) methods:

- Many cheap calculations + Few accurate calculations = Accurate answer efficiently
- Recent advances, e.g.,
  - B. Peherstorfer, et al. (2016) "Optimal model management for multifidelity Monte Carlo estimation." **38**(5): A3163-A3194.
  - G. Geraci, et al. (2023) "Multifidelity UQ methods for Monte Carlo radiation applications and stochastic media." *USNCCM17*

THE MULTI-FIDELITY BEHAVIOR OF COPS

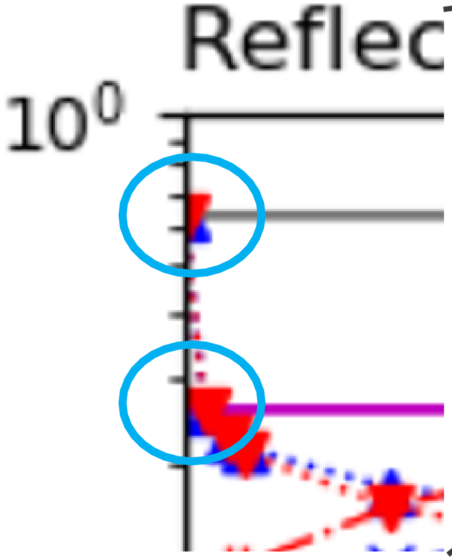
CoPS

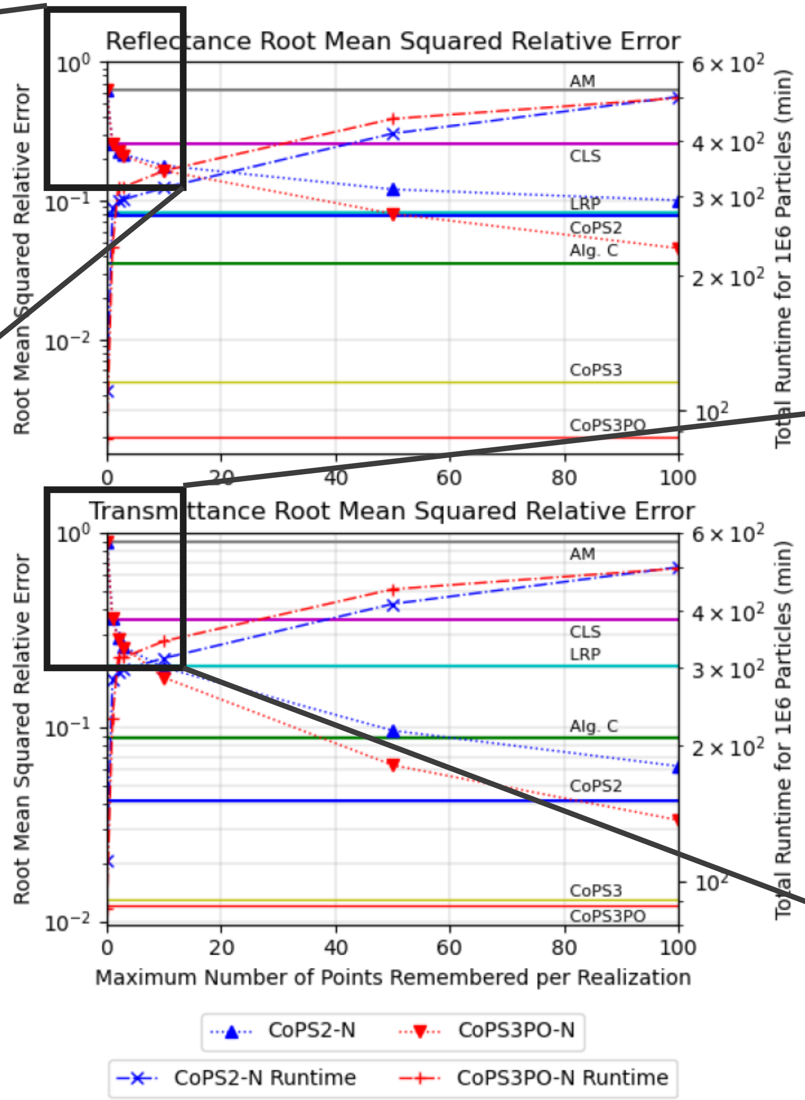
Multi-fidelity behavior seen in 2020

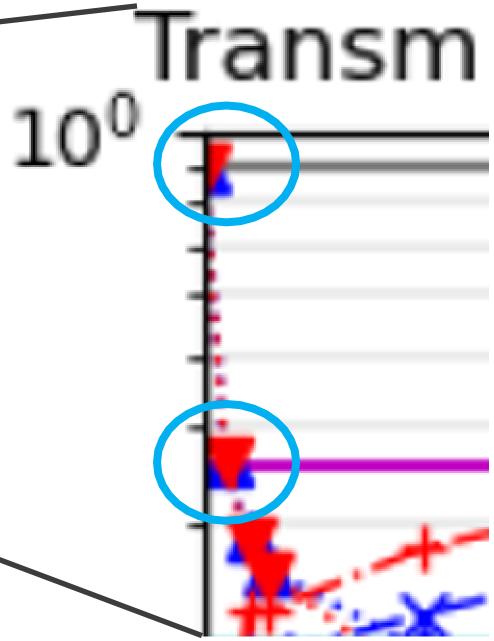


E.H. Vu, A.J. Olson, "Recent memory versions of Conditional Point Sampling for transport in 1D stochastic media" *Trans. ANS* (2020)

DOES COPS PROVIDE OPTIONS EQUIVALENT TO AM AND CLS?





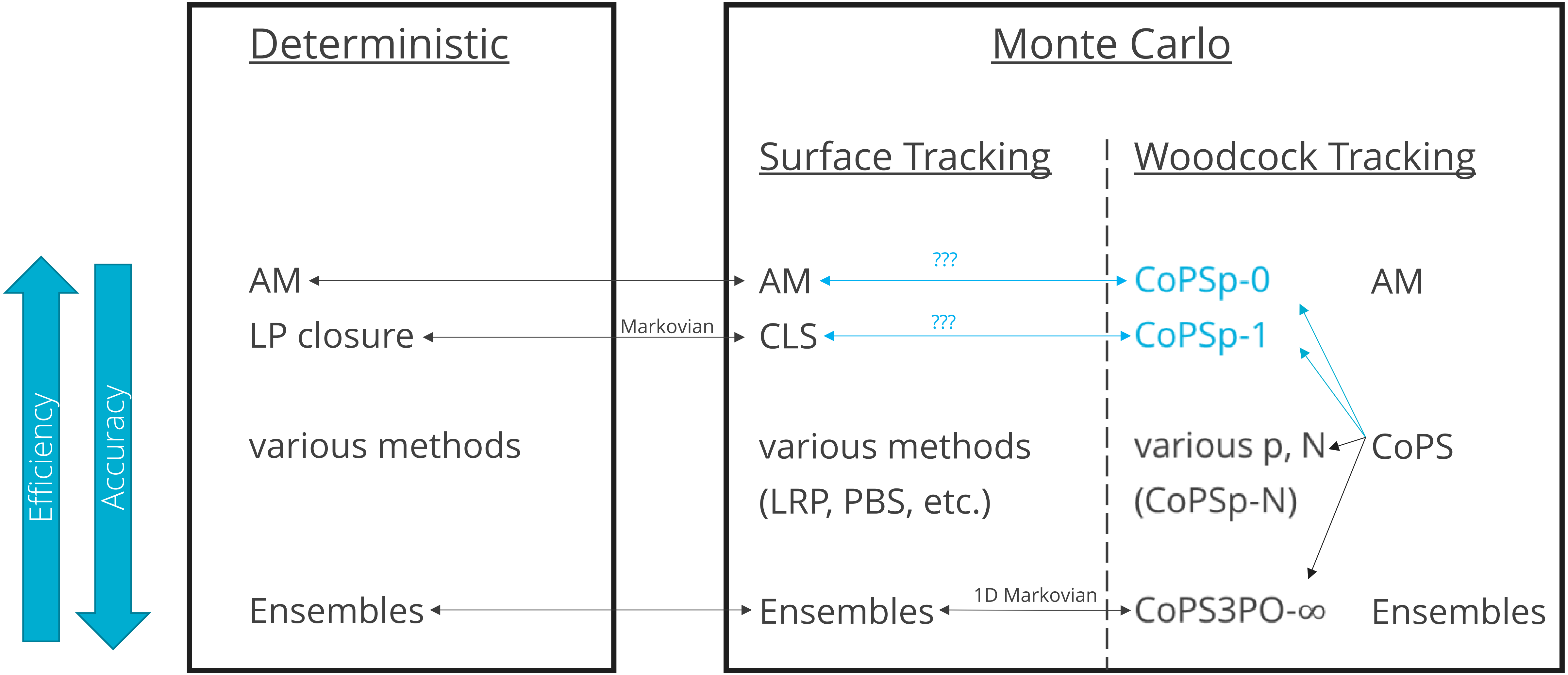


E.H. Vu, A.J. Olson, "Recent memory versions of Conditional Point Sampling for transport in 1D stochastic media" *Trans. ANS* (2020)

Multi-fidelity behavior seen in 2020

But do CoPSp-0 and CoPSp-1 match AM and CLS results?

LANDSCAPE OF SM TRANSPORT MODELS – W/ MF, MATCHED COPS?



WHAT ARE BE THE BENEFITS OF SHOWING EQUIVALENCY?

CoPS

Research benefits:

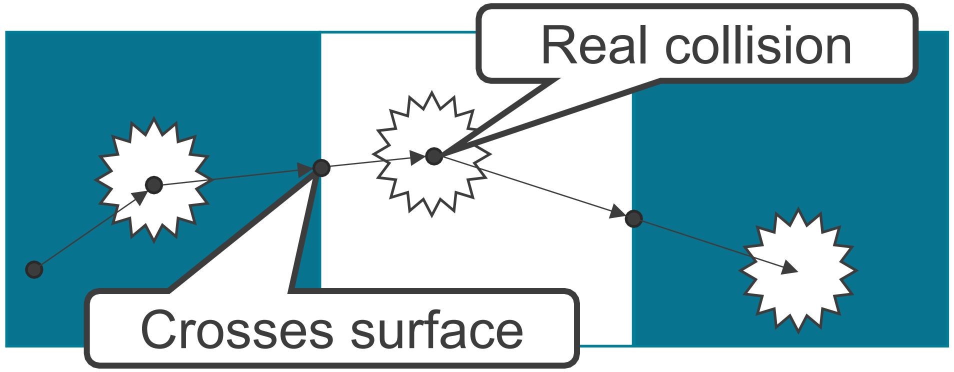
- Emulation: AM, CLS, or Ensemble methods emulate CoPS, and vice versa!
- Theory: New theoretical beachhead, e.g., point-based deterministic formulation?
- Accessibility: Mechanics *and results* analogous to accepted methods

Application benefits:

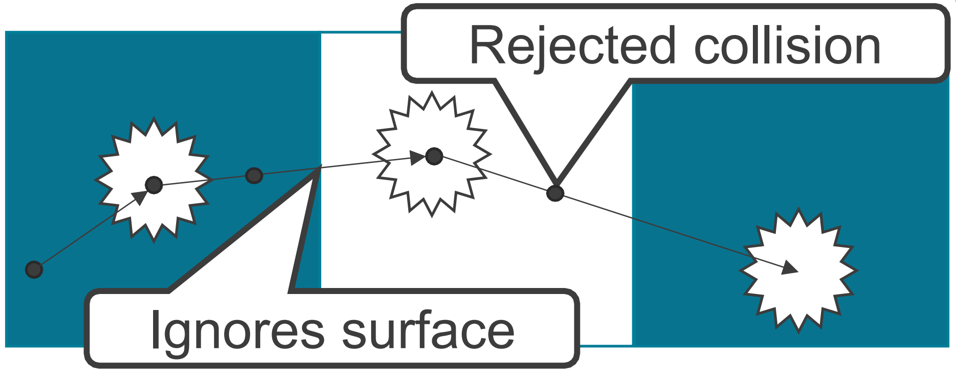
- Emulation: Know what accuracy other methods would provide
- Simplicity: One implementation provide variety of accuracy/efficiency fidelity options

APPROACH TO SHOWING EQUIVALENCY

Surface Tracker



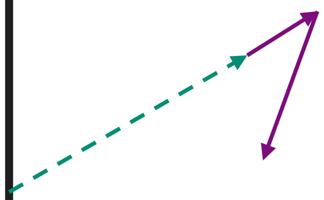
Woodcock Tracker



Non-physical events:

- Boundary crossing
- Rejected pseudo-collisions

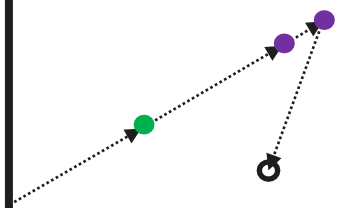
AM/CLS



Interface crossing:

Simulated, but not physical

CoPS



Rejected pseudo-collision:

Simulated, but not physical

Equivalent if:

Distribution of physical events the same

$$f_i(x)dx = p_i(x)\Sigma_{t,i}\exp\left[-\int_{x'}^{x} dx''\Sigma_t(x'')\right]dx$$

APPROACH TO SHOWING EQUIVALENCY – AM VS. COPSP-0

CoPS

AM

- Stream based on $\langle\Sigma_t\rangle = \sum p_i \Sigma_{t,i}$
- Sample collision material based on $\frac{p_i \Sigma_{t,i}}{\langle\Sigma_t\rangle}$

CoPSp-0

- Stream based on $\Sigma_t^* = \max(\Sigma_{t,i})$
- Sample local material based on p_i
- Accept collision in material i based on $\frac{\Sigma_{t,i}}{\Sigma_t^*}$

CoPSp-0: a way to perform AM with Woodcock tracking

$$f_i(x)dx = p_i \Sigma_{t,i} \exp\left[-\langle\Sigma_t\rangle(x - x')\right] dx$$

$$f(x)dx = \langle\Sigma_t\rangle \exp\left[-\langle\Sigma_t\rangle(x - x')\right] dx$$

APPROACH TO SHOWING EQUIVALENCY – CLS VS. COPSP-1

CoPS

CLS

- Incorporate stochastic mixing via d_i
  - Sampled distance to material interface
- Markov property:
  - Only current material affects d_i

CoPSp-0

- Incorporate stochastic mixing via $f(k, r)$
  - Sampled number of material interfaces k on r
- Markov property:
  - Only current material affects $f(k, r)$

CLS and CoPSp-1: two ways to account for memoryless interface crossing

$$f_i(x)dx = p_i(x - x')\Sigma_{t,i} \exp\left[-\int_{x'}^{x} dx'' \sum_{i=0}^{N_{mat}-1} \left(p_i(x'' - x')\Sigma_{t,i}\right) \right] dx$$

AM VS. COPSP-0 COLLISION DISTRIBUTIONS

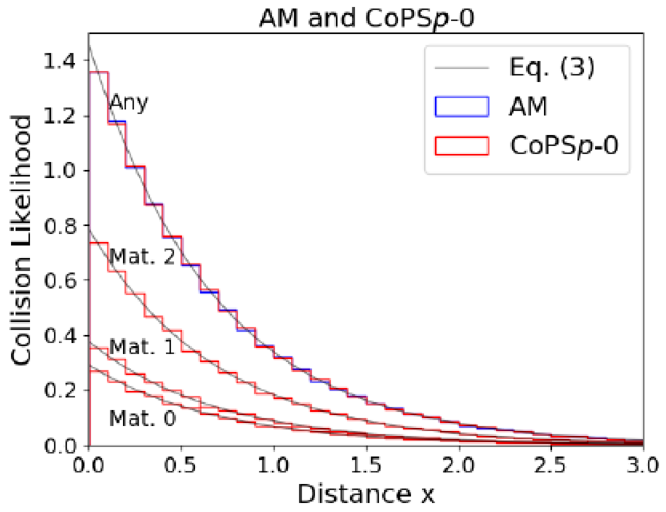
Problem Descriptions:

- Problem 1:
  - $\lambda_0, \lambda_1, \lambda_2 = 1.0, 1.45, 1.9 \;\; (p_0, p_1, p_2 = 9.7\%, 37.8\%, 52.5\%)$
  - $\Sigma_0, \Sigma_1, \Sigma_2 = 3.0, 1.0 \;, 1.5$
- Problem 2:
  - $\lambda_0, \lambda_1, \lambda_2 = 1.0, 1.45, 1.9$
  - $\Sigma_0, \Sigma_1, \Sigma_2 = 1.0, 1.0 \;, 1.0$

Numerical Experiment:

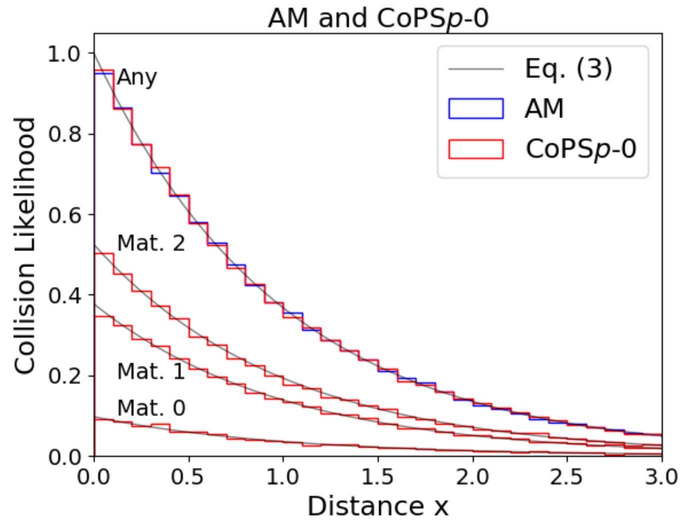
1. Source particle in purely scattering material
2. Simulate until second physically real collision
3. Tally distances to collisions and material types
4. Process tallies into collision distributions

Problem 1

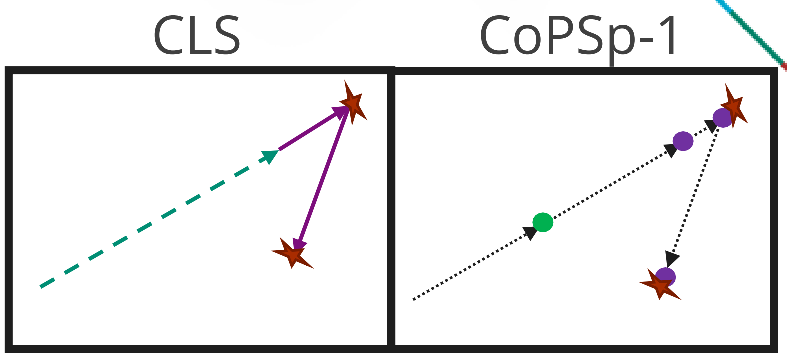


(a) AM and CoPSp-0: Collision following instantiation or collision

Problem 2



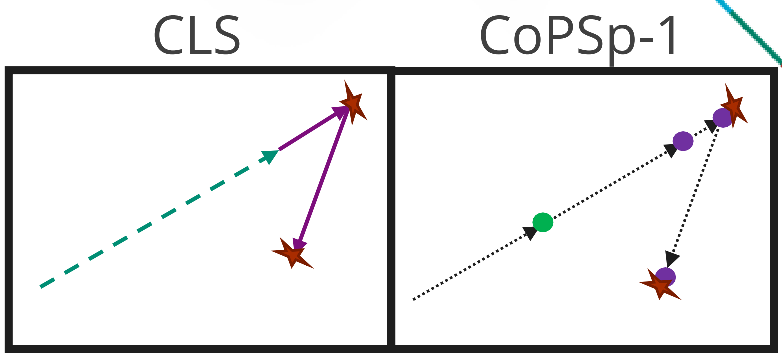
CLS VS. COPSP-1 COLLISION DISTRIBUTIONS



First collision

Second collision

CLS VS. COPSP-1 COLLISION DISTRIBUTIONS

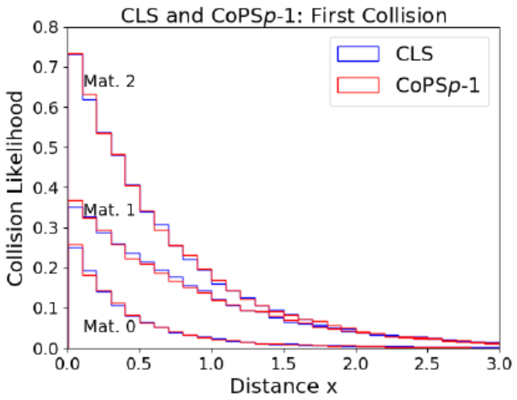


CoPS

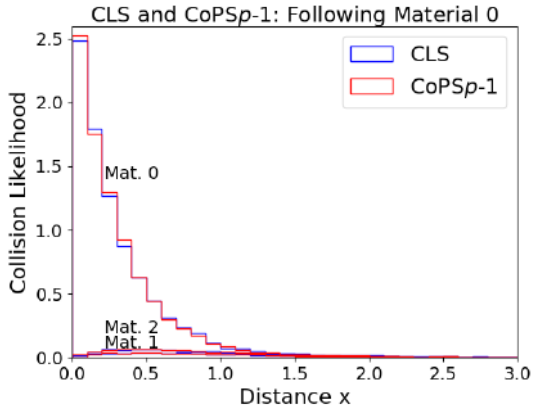
First collision

Second collision

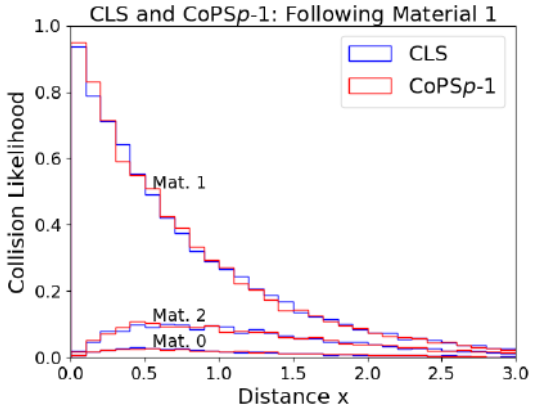
Problem 1



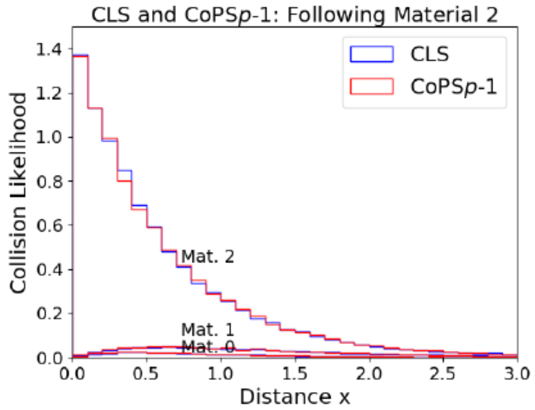
(b) CLS and CoPSp-1: First collision after instantiation



(c) CLS and CoPSp-1: Previous collision in Material 0

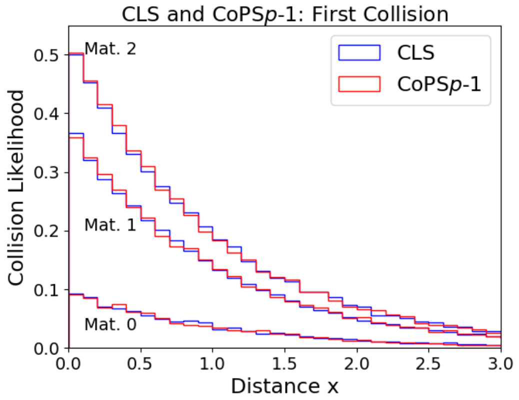


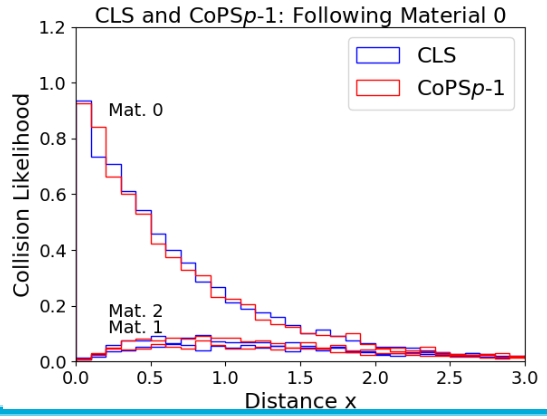
(d) CLS and CoPSp-1: Previous collision in Material 1

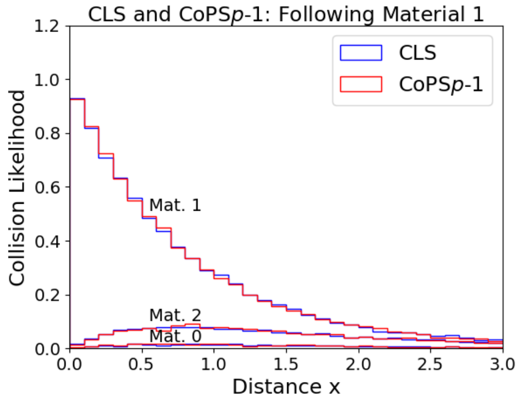


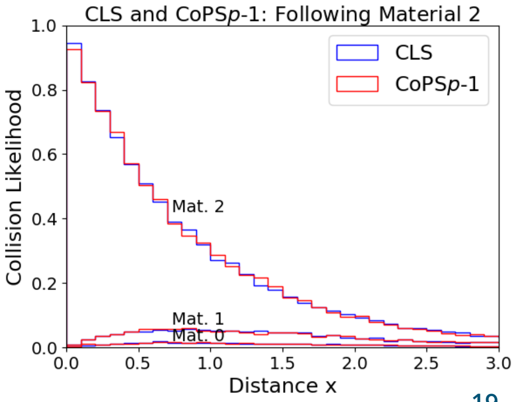
(e) CLS and CoPSp-1: Previous collision in Material 2

Problem 2

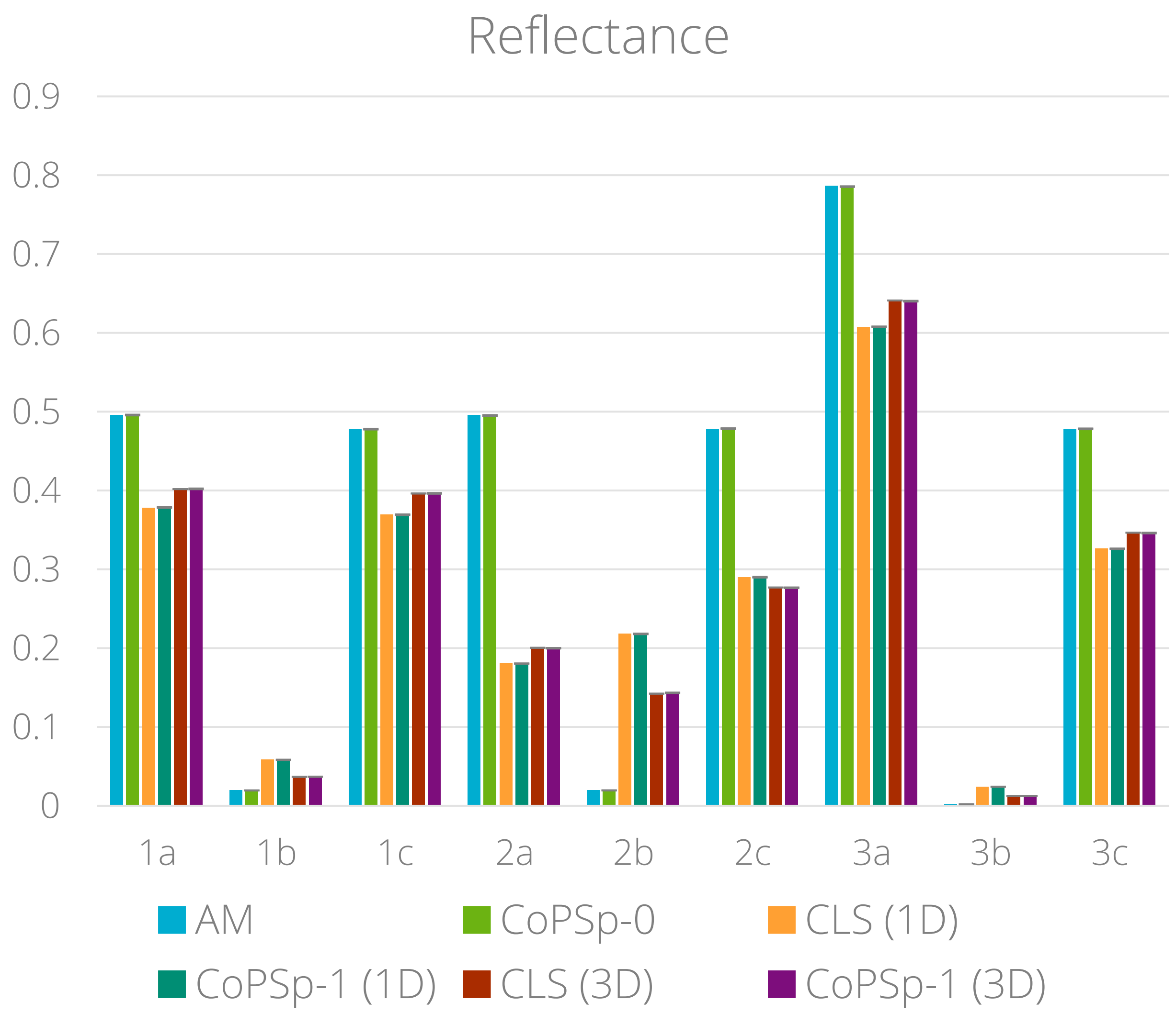


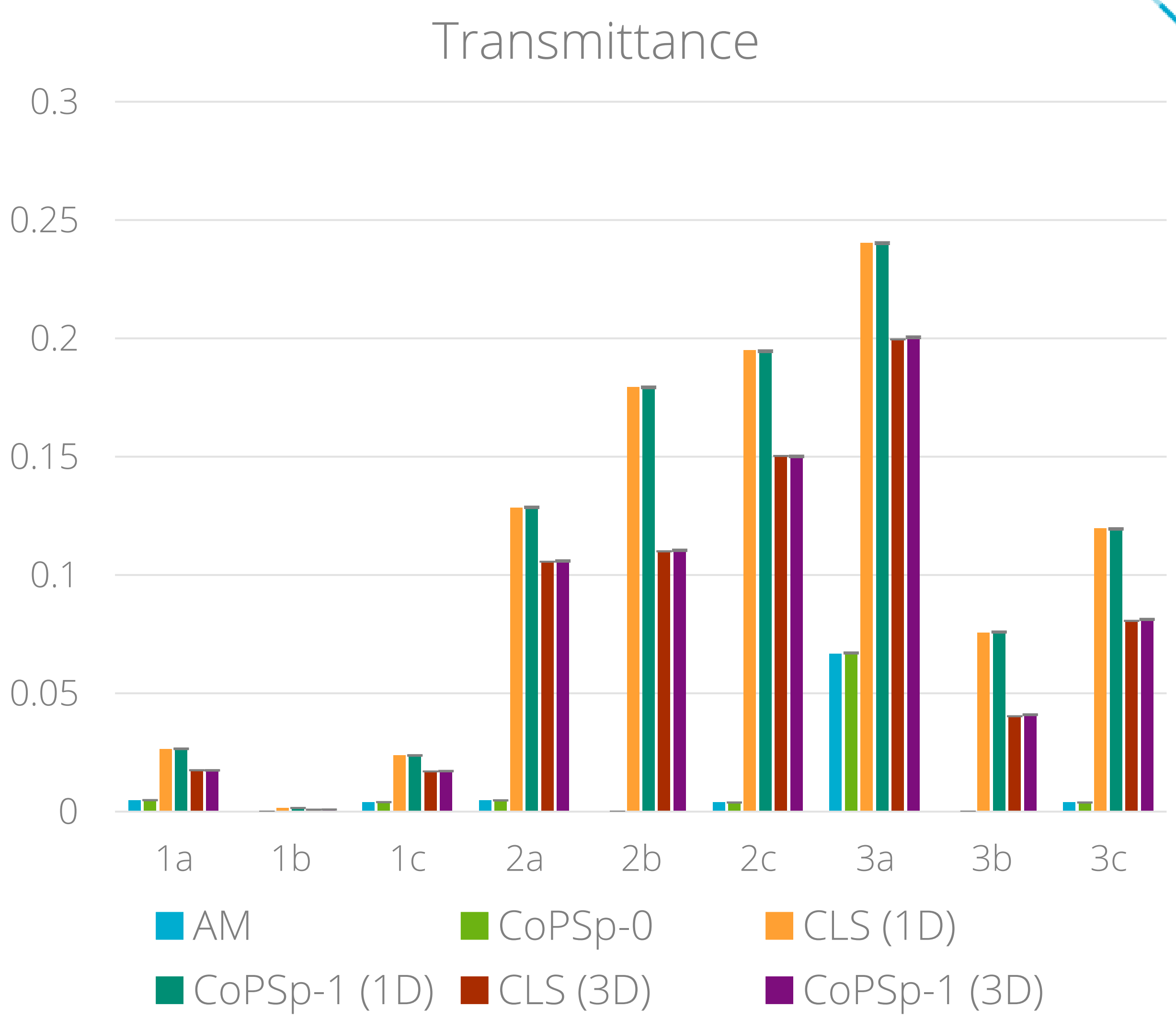






FULL-ALGORITHM – BINARY MIXTURES



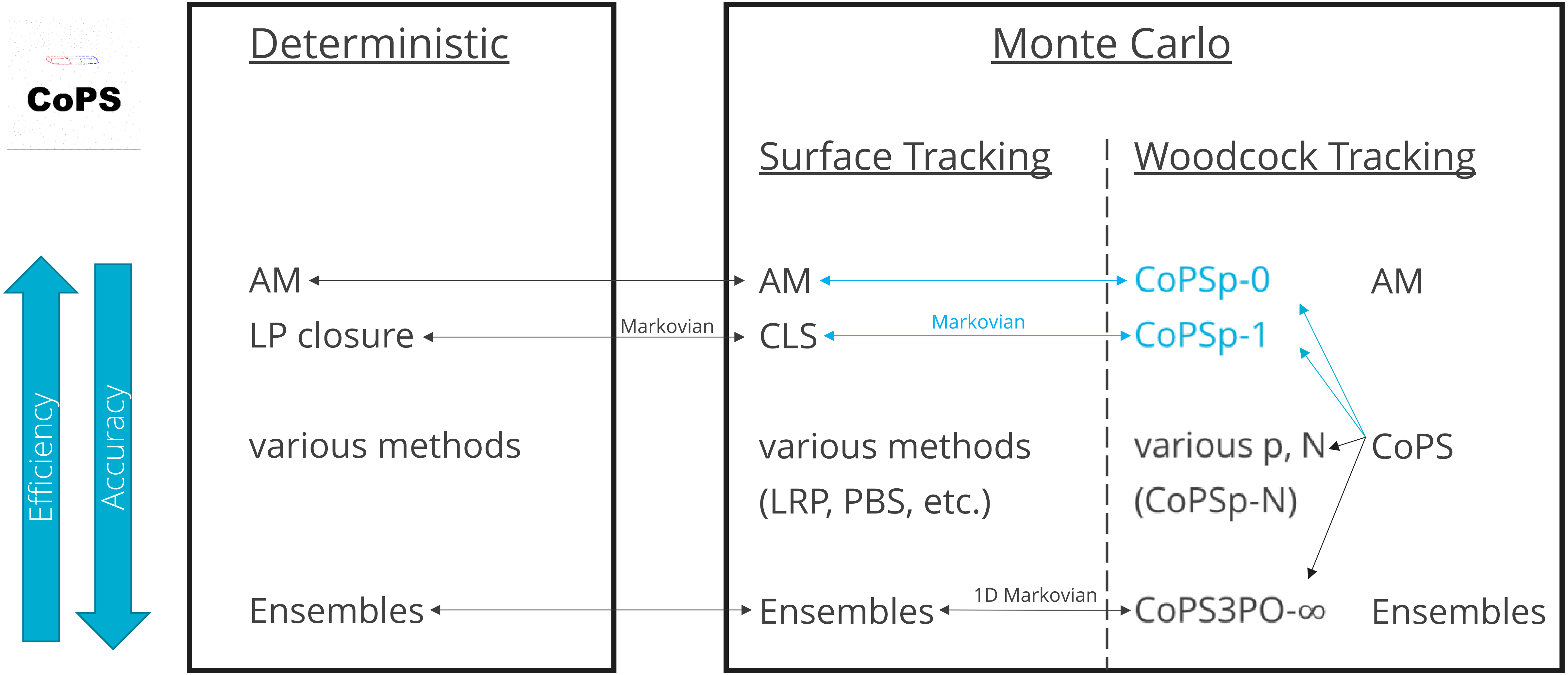


FULL-ALGORITHM – QUATERNARY MIXTURES

CoPS

LANDSCAPE OF SM TRANSPORT MODELS – W/ MF, MATCHED COPS

CoPS



CONCLUSIONS AND FUTURE WORK

CoPS

Conclusions:

- CoPSp-0 produces AM quantities
- CoPSp-1 produces CLS quantities (in 1D, Markovian media)
- Equivalencies enable emulation, accessibility, simplicity, and a new theoretical beachhead

Future work:

- Mathematical proof of equivalency
- Runtime comparisons
- Examination of LRP-like version of CoPS