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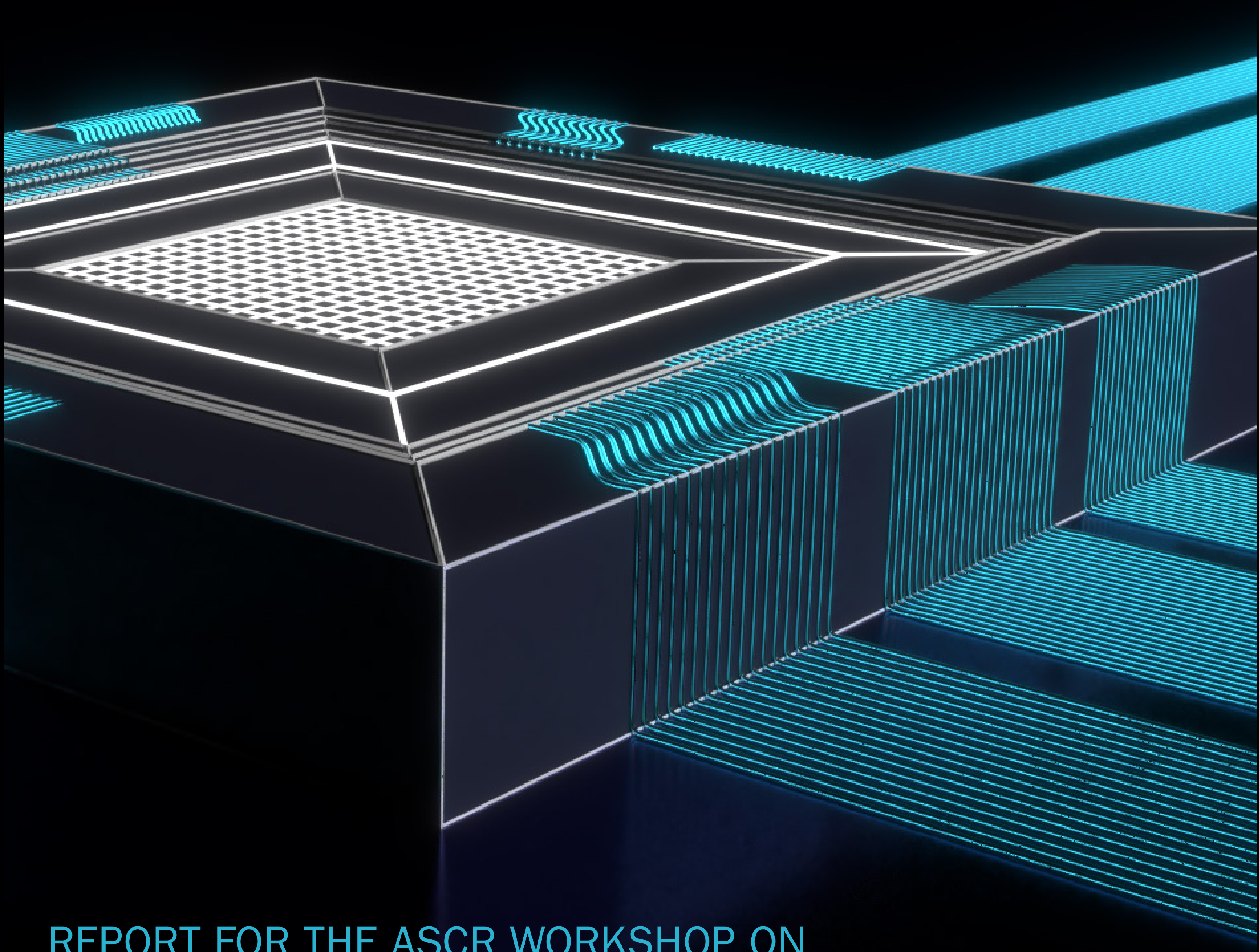

REPORT FOR THE ASCR WORKSHOP ON

Basic Research Needs in Quantum Computing and Networking

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PREPARED FOR THE U.S. DEPARTMENT OF ENERGY ADVANCED SCIENTIFIC COMPUTING RESEARCH

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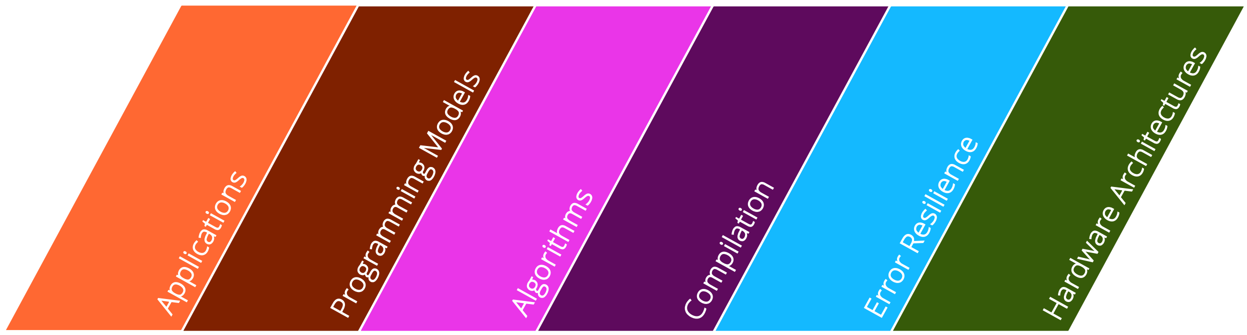

Figure 1: A stack depicting critical components of a quantum computing or networking system capable of end-to-end application impact.

1 Introduction and executive summary

Employing quantum mechanical resources in computing and networking opens the door to new computation and communication models and potential disruptive advantages over classical counterparts. However, quantifying and realizing such advantages face extensive scientific and engineering challenges. Investments by the Department of Energy (DOE) have driven progress toward addressing such challenges. Quantum algorithms have been recently developed, in some cases offering asymptotic exponential advantages in speed or accuracy, for fundamental scientific problems such as simulating physical systems, solving systems of linear equations, or solving differential equations. Empirical demonstrations on nascent quantum hardware suggest better performance than classical analogs on specialized computational tasks favorable to the quantum computing systems. However, demonstration of an end-to-end, substantial and rigorously quantifiable quantum performance advantage over classical analogs remains a grand challenge, especially for problems of practical value. The definition of requirements for quantum technologies to exhibit scalable, rigorous, and transformative performance advantages for practical applications also remains an outstanding open question, namely, what will be required to ultimately demonstrate practical quantum advantage?

In July 2023, DOE's Advanced Scientific Computing Research (ASCR) program in the Office of Science (SC) convened the Workshop on Basic Research Needs in Quantum Computing and Networking, where major opportunities and challenges were discussed and identified. Before the workshop, a pre-workshop report was prepared to seed potential research themes, and community input was solicited in the form of brief position papers. These formed the basis of panel and discussion sessions at the workshop. This report summarizes the findings of the workshop.

The workshop was framed in terms of six themes comprising layers of a broadly scoped end-to-end quantum computing and networking stack (Figure 1): applications, programming and computing models, algorithms, compilation, error resilience[1], and networking protocols and hardware architectures[2]. Bringing the potential of quantum computing and networking to bear on scientific applications will demand advancements in all layers of the stack, as well as crosscutting codesign and integration efforts across the stack.

1.1 Pre-workshop stack layer topics

The organizing committee of the workshop converged upon the following topics, in direct correspondence to Figure 1, to seed community input and workshop discussions.

[1]"Error resilience" here refers to techniques of error correction, error mitigation, error detection, or other techniques for reducing the physical errors in the system.

[2]Advancements in quantum computing architectures and hardware fall in the purview of the ASCR Quantum Computing Testbeds program, which was outside the scope of this workshop.

Stack Layer: Applications

Topics

- Fundamental mathematical kernels and standardized libraries
- New kinds of DOE science applications informed by quantum capabilities
- Assessment of realistic quantum advantages
- Tools for application performance modeling and estimation
- Application-inspired benchmarks and curated libraries of instances
- Applications of entanglement distribution networks

Stack Layer: Computing and programming models

Topics

- Design and analysis of quantum computing and programming models
- Models for hybrid quantum and classical computing
- Programming environments for expressing quantum algorithms
- Quantum network models and architectures
- Hybrid quantum and classical network design
- Models for distributed quantum computing

Stack Layer: Algorithms

Topics

- Quantum algorithms admitting theoretical or empirical evidence of advantage for fundamental domains, such as simulation, optimization, or machine learning
- Hybrid quantum and classical algorithms
- Quantum-inspired classical algorithms
- Classical algorithms and software systems to simulate quantum computers and networks, including tensor network and Monte Carlo simulations

Stack Layer: Compilation

Topics

- Expanding the scope, utility, efficiency, and robustness of software stacks for quantum computing
- Approaches, algorithms, and software systems for circuit compilation and qubit mapping, routing, parameter optimization, and scheduling

Stack Layer: Error Resilience

Topics

- Integration of error detection, prevention, protection, mitigation, and correction protocols across the quantum stack
- The role of noise characterization and noise modeling in targeted error resilience protocol design
- Assessing error propagation through the quantum hardware and software stacks
- Codesign of quantum algorithms and error resilience protocols

Stack Layer: Hardware architectures

Topics

- Codesign and integration across the stack
- Impact of application requirements across the stack
- Impact of noise, fidelity, and gate execution time on algorithms and applications

1.2 Priority research directions

Most of the presentations, discussions, and questions raised at the workshop were framed in terms of an implicit underlying grand challenge shared by the broader quantum information science community. As the workshop progressed, a clearer vision of this grand challenge emerged, as outlined in the following.

Grand Challenge: Demonstrate a rigorously quantifiable, end-to-end quantum advantage relative to state-of-the-art classical counterparts, particularly for problems with practical or scientific significance for which asymptotic exponential quantum advantages have been established.

Workshop participants discussed current obstacles and potential solutions related to the grand challenge from various perspectives, aligning with the workshop themes. In what follows, we summarize these discussions into five overarching and complementary priority research directions (PRDs). This report proposes these PRDs as guiding paths for a research program in quantum computing and networking.

Although the PRDs are based on our perspective of the computing and networking stack, the PRDs do *not* imply any direct correspondence with stack layers. Also, the numerical labels of the PRDs are meant

for reference only and are not intended to suggest any particular order of importance.

PRD 1. End-to-end software toolchains to program and control quantum systems and networks at scale

Driving questions How can we design expressive programming models and languages to attract broad user bases and facilitate quantum algorithm design and implementation? How can we incorporate these into end-to-end toolchains to produce resource-efficient quantum programs?

Quantum computing and networking systems continue to grow in scale and complexity and will place an increased burden on the software stack to program, control, and manage these systems effectively. Software toolchains that integrate programming models with hardware-level control systems will be needed to maximize the performance and fidelity of quantum systems and to facilitate codesign of hardware, control systems, and algorithms across different technology platforms. Integrating quantum networking systems with quantum computing systems will be critical in advancing the arrival of distributed quantum computing services. Achieving this goal necessitates a quantum networking stack that is compatible with the quantum computing software stack, and it is essential that the integrated system can be efficiently managed, controlled, and programmed.

PRD 2. Efficient algorithms delivering quantum advantages

Driving questions What classes of existing and understudied scientific applications admit substantial quantum advantages over conventional classical computing paradigms? How can we design novel algorithms and supporting mathematical models to realize such advantages? Are there any provable or empirical barriers to quantum advantages? What are the physical resource requirements of practical implementations of such algorithms, including numbers of physical qubits and quantum circuit depth?

Quantum computing is not expected to accelerate current computing tasks universally, so identifying problems with special structure amenable to quantum advantages is paramount. Taking a complementary perspective, broadening our understanding of foundational computational kernels admitting quantum advantages is equally important. While a variety of quantum advantages are currently known, they are subject to shortcomings such as a lack of known practical applications, near-term realization, rigorous provability of advantage, or efficient verifiability of advantage. In addition, quantum advantages have primarily focused on improving execution time. Advantages concerning other critical resources, such as quality/accuracy of solution, energy consumption[a], space/memory, or communication, are understudied, especially in the context of quantum networking.

[a]The energy consumption of quantum computing is an often-overlooked but critical parameter in developing sustainable computing ecosystems.

PRD 3. Benchmarking, verification, and simulation methods to assess quantum advantages

Driving questions How can we fairly assess quantum advantage relative to classical capabilities, especially as underlying technologies evolve and scale from the noisy intermediate-scale quantum (NISQ) to fault-tolerant paradigms? How can we measure progress of quantum systems toward demonstrating quantum advantage, across the computing and networking stacks? Which representative scientific use cases serve as insightful and scalable benchmarks for quantum computing and networking applications? How can we verify demonstrations of quantum advantage? How can we leverage numerical simulation of quantum systems to validate large-scale quantum applications?

Assessing progress toward quantum advantage is a challenging and multi-faceted endeavor. Empirical evidence of advantages is expected to continue to rely on large-scale classical simulations of quantum systems as quantum technologies mature. Forecasting scalable quantum advantage based on limited results obtained from relatively small near-term quantum systems and classical simulations is a considerable hurdle. On the one hand, while rigorous proofs of asymptotic quantum advantage are ultimately desirable and may be used to direct empirical studies, they often rely on abstract or specialized models of quantum computing or otherwise impose additional restrictions. On the other hand, the quantum advantage suggested by empirical assessments may not be sustainable as problems grow in scale or complexity or as better classical algorithms are developed. Bridging this gap between theory and practice is essential for establishing sound and practical quantum advantages. Rigorous, informative, and efficiently verifiable performance metrics at all levels of the quantum computing and networking stacks, from the application layer to the hardware itself, need to be defined and developed. Ideally, such metrics should be integrated across the stack so that progress toward quantum advantage can be quantified in a way that predicts practical performance.

PRD 4. Resilience through error detection, prevention, protection, mitigation, and correction

Driving questions How can we enhance the resilience of quantum systems to noise and errors to relieve scalability and quantum advantage bottlenecks? What kinds of quantum algorithm codesign techniques can aid in yielding resilient quantum systems?

Scientists and engineers in national laboratories, academia, and industry continue to improve quantum computing and networking hardware, yet, despite those steady advances, the hardware systems are expected to remain noisy and imperfect. In recent years, significant efforts characterizing errors and inserting error mitigation at various layers of the software stack have allowed the research community to address some of the noise and achieve reliable results in small-scale quantum experiments. To achieve reliable results with quantum systems at larger scale and complexity, more efficient and better methods characterizing, mitigating, preventing, or protecting against dynamical errors must be integrated into the critical layers of the software stack. Steps are needed toward fault tolerance, codesign, and early quantum error correction demonstrations that outperform the physical counterpart. Another approach that can be taken is identifying the error resistance mechanisms for quantum algorithms and codesigning new hardware-aware algorithms and hardware controls that lead to error resistance.

PRD 5. Hardware and protocols for next-generation quantum networks

Driving questions Can quantum repeater hardware be built to achieve entanglement distribution rates higher than those of "repeat-until-success" direct transmission experiments? What software and hardware, besides the repeaters, are needed to build scalable quantum networks? What applications and advantages will those networks enable? What kinds of distributed quantum computing models will result in novel quantum applications and advantages?

To date, non-error-corrected quantum memories and entanglement distribution between them have been demonstrated with multiple qubit technologies. Moving forward in enabling scalable entanglement distribution networks will require progress in multiple directions. Quantum memories must be augmented with error detection and correction functionality for designing and implementing fault-tolerant quantum repeaters. Photon sources, detectors, and time-tagging hardware will need to improve to increase the fidelity of entanglement-swapping operations. The quantum networking software stack must implement distributed error correction protocols and enable optimization across the stack. High-fidelity quantum information transduction methods and hardware need to be developed to enable the use of entanglement distribution networks in distributed quantum computing applications.

1.3 Summary and organization

With advancement toward quantum technology utility as an overarching goal, as well as a better understanding of the requirements for the utility both on the application and technology side, the DOE workshop, "Basic Research Needs in Quantum Computing and Networking" identified a grand challenge and five priority research directions (PRDs). Over the last decade, DOE investments in quantum computing software and hardware have laid the groundwork for the type of basic research that will underpin key advances in the five PRDs. Collaborative relationships among computer scientists, mathematicians, and physicists will be critical in making the advances across stack layers necessary to realize end-to-end demonstrations of disruptive quantum advantages for scientific applications.

The remainder of this report is organized into five sections corresponding to each of the five PRDs. Each section includes a brief introduction and discussions of driving questions, goals and desired outcomes, the current state of the art, and suggested research directions.

2 End-to-end software toolchains for quantum systems and networks

Quantum computing and networking systems continue to grow in scale and complexity and will place an increased burden on the software stack to effectively program, control, and manage these systems. Integrating quantum networking systems with quantum computing systems will be critical in advancing and delivering distributed quantum computing services (e.g., [1]). This will require compatible quantum networking and quantum computing software stacks so that the combined system can be efficiently managed, controlled, and programmed.

To bridge the complexity gap between quantum computing and networking systems, there is a pressing need for robust, scalable, and flexible quantum software frameworks. These frameworks must be sufficiently flexible to address the diversity of existing and future hardware architectures. Software toolchains will be needed that integrate programming models and compilers with hardware-level control systems to maximize the performance and fidelity of quantum systems, and to facilitate codesign of hardware, control systems, and algorithms across different technology platforms.

As quantum software and hardware increase in sophistication and complexity, there is a corresponding increase in the need for techniques and tools for designing the quantum stack. The evaluation of individual stack components and their ultimate integration can provide key insights into the propagation of errors [2–5], algorithm performance [6–13], benchmarking [14–16], and the estimation of classical and quantum resources [17–24]. When combined (e.g., [25–28]), this knowledge can facilitate the targeted honing of specific quantum technology stacks or enable cross-platform comparisons and the identification of broader principles for scalable quantum computing and networking architectures.

2.1 Driving questions

Realizing quantum computing will involve far more than just the writing of code, but the development of a broader ecosystem that includes programming, compilation, debugging, optimization, simulation, and deployment on quantum hardware. Such comprehensive toolchains will allow users to efficiently manage the limited but ever-changing (and, likely, distributed) quantum resources to minimize errors. While considerable progress has been made in the last 5-10 years, fundamental questions remain:

- How can we design expressive programming models and languages to attract broad user bases and facilitate quantum algorithm design and implementation?
- How can we incorporate these into end-to-end toolchains to produce resource-efficient quantum programs?

As end-to-end software toolchains are developed, how can we continue to build stronger collaborations among the quantum hardware developers, software developers, and end-users in academia, the national laboratory system, and industry to ensure that they are robust, user-friendly, and flexible for the fast-moving advances in quantum computing and networking technology and algorithms?

2.2 Goals and desired outcomes

To drive the development of an end-to-end software toolchain for quantum computing devices, networks, and complex integrated heterogeneous quantum systems, the participants at the workshop identified the following challenging tasks:

- Design programming models and languages that translate high-level scientific problems to the language of quantum computing and networking, while exploiting well-defined primitives at different levels of the software stack.
- Build scalable runtime systems and software toolchains that can be used to program, control, and compose/compile circuits for 10,000-1,000,000-qubit devices, and program, tune, and manage complex quantum networks, while transparently incorporating error correction.
- Develop software toolchains, protocols, and application programming interfaces (APIs) that can be used to program distributed, heterogeneous, quantum computing and networking devices.

The software toolchains capable of addressing these challenges will be able to take full advantage of quantum hardware technologies in the near future and long-term, while providing the scientific community with the critical tools to exploit these hardware technologies for quantum advantage and scientific discovery.

2.3 State of the art

The ability to express and efficiently execute complex quantum algorithms on quantum computing hardware is critical to enabling a broader adoption of quantum computing by the scientific computing community [29]. So far, programming quantum algorithms has been mostly limited to low-level gates and qubit resources or libraries specific to a single scientific domain, a specific programming model (such as digital/gate-based versus Hamiltonian, or analog versus photonic), and specific quantum hardware technology. There are analogs in classical computing where technologies based on central processing unit (CPU) or field-programmable gate array (FPGA) require distinctly different programming and compiling paradigms.

To achieve the efficient execution of algorithms, compilation tools tend to be closely tied with programming languages and hardware execution languages; therefore, the design of compilers capable of adapting to emerging trends in quantum programming should be an important consideration for software development [27, 30, 31]. Quantum compiling is a hybrid process, incorporating techniques from compilers for classical programming languages, transforming high-level language to assembly language, and hardware synthesis by hardware description language, where functions are automatically synthesized into customized hardware. This requires quantum programming methods and compiling techniques that are commensurate with today's sophisticated classical approaches. Compilers and their intermediate representations of hybrid programs are important for reasoning and optimizing execution in mixed machine models [32–35]. These include languages and compilers that extend conventional tools, including C/C++ and LLVM, to quantum settings [36–38], as well as executable languages that operate close to the hardware [39–43]. Domain-specific tools have also become essential for translating and compiling existing workflows to quantum technology targets [44–46].

Currently, the quantum software toolchains can support quantum hardware systems with less than 200 qubits. This scale enables researchers to explore complex quantum algorithms and offers a testing ground for quantum compilers and error correction techniques. Quantum error correction (essential for the practical use of quantum computers) is being developed concurrently with advances in control, compiler technologies, and numerical optimization methods for quantum systems. Current systems are closing in on demonstrating quantum advantage for specific problems, but are still some distance away from tackling large-scale practical computing tasks.

Critical research focused on developing software toolchains and runtime systems for the much larger, and potentially distributed, quantum computing and networking platforms of the future is limited. Looking to the future, research must focus on the important challenges outlined in Section 2.2 to unlock the full potential of quantum computing.

2.4 Research directions

Reflecting on the challenges outlined in Section 2.2, when viewed from the context of the current state of the art, considerable research will be required to achieve the desired outcome of an end-to-end software toolchain for quantum computing and networking for large, complex heterogeneous quantum devices. In this section, the connected challenges are translated into important research directions.

High-level programming models and languages. In recent years, a healthy diversity of quantum programming models has been developed, which has been both positive and negative in nature with respect to advancement. While it was negative in that it has led to fragmentation, it was also positive in that it has allowed for the exploration of ideas. Future programming models should include high-level, library-based approaches that expose application-level functionality. In classical computing, these would be comparable to Common Unified Data Access (CUDA) Math Libraries, or a vendor-optimized Basic Linear Algebra Subprograms (BLAS) Application Programming Interface (API) for quantum computing. The community should prioritize algorithmic primitives that serve as a basis for most (if not all) application codes (e.g., sampling, operator expectation values). There is a growing consensus that application- or domain-level data types will be necessary for engaging domain computational scientists. Hamiltonian-level language constructs

will be useful in moving from low-level to high-level programming of quantum circuits. Additionally, there is a need to integrate the emerging quantum programming models with existing application workflows. An important area is the specification and packaging of portable libraries. Modules such as the quantum Fourier transform (QFT) have completely different structures depending on their qubit usage. A single packaging of QFT that is able to generate programs across a configurable number of qubits is desirable. Furthermore, to avoid excessive compilation time, the library should provide for precompilation optimizations and hints. This will require scrutiny of how application requirements, including language and data types, impact the interfaces for quantum computational models. The community needs to start considering primitive types and APIs for distributed quantum computation, something akin to a quantum message-passing interface enabling cross-device entanglement. Finally, the community should start to explore runtime system software to include those systems that operate real-time, fault-tolerant and error-correcting architectures. These considerations are essential for scalable quantum computing applications but there is a significant gap in the analysis of these architectures for scientific applications. This work should include system-level programming models for managing the execution environments behind hybrid computation as well as the integration of these environments with conventional computational workflows. Related topics to consider are performant data transfer among CPUs, graphics processing units (GPUs) and quantum processing units (QPUs), dynamic control flow based on qubit measurement, and Pauli frame tracking, among many others. Furthermore, the portability and adoption of application programs across platforms should be considered essential for testing and evaluation. The creation of standardized quantum software interfaces and protocols will be vital in promoting interoperability between different quantum systems, paving the way for a cohesive and interconnected quantum computing ecosystem.

Software toolchains and runtime systems for large-scale quantum hardware. Compilation, also referred to as unitary synthesis or transpilation, of scientific programs is one of the critical components in the quantum software toolchain translating a quantum program into hardware executable operations for both digital, circuit-based, and analog quantum computers. Fundamentally, the optimization or minimization of the number of quantum operations is a Non-deterministic Polynomial (NP)-hard problem [47]. Novel methods for program partitioning, partial reasoning and distributed processing are required in all areas: optimization, mapping, resource allocation. These methods need to be complemented by rigorous mathematical foundations for reasoning about compositions of quantum programs. New mathematical or numerical methods and heuristics will need to be developed that will deliver (near-) optimal quantum operations. Classically, compilers rely on a significant amount of information about the problem. Similar information, such as knowledge about hardware constraints or requirements, is not yet fully exposed/developed for quantum compilation. Problem symmetries, constraints and sparsity have the potential to make compilation or synthesis for large numbers of qubits feasible. Different levels of abstraction will be needed to expose the problem structure to the optimization method. Repeating algorithmic patterns, such as a QFT, could be pre-compiled into BLAS-like libraries and inserted by the compiler when needed. New ways of reasoning about quantum operations need to be developed, especially when relating the impact of transformations to changes in fidelity/accuracy of the resulting output of quantum simulation. Approximate compilation methods should be considered, especially when quantum devices are noisy. Machine-learning approaches could be exploited to accelerate quantum compilation. Research is needed on new methods for code generation, optimization and resource estimation for logical qubits, which are likely to exist soon. To separate quantum error correction from the application defined by end-users of quantum computers, the community should explore runtime system software driving real-time error correcting architectures. New approaches are needed to perform resource management at compile-time and runtime.

Toward distributed quantum computing and networking. Distributed quantum computing will be a major driver in both quantum hardware, information science, and software in the next decade, as the scaling of computing devices is expected to encounter physical limits. While quantum networking is in its infancy [48], an early demonstration connecting two trapped ion quantum devices has shown the potential of distributed quantum computers [49]. Software toolchains, runtime systems and programming, tuning, and managing quantum networking systems, including a quantum internet stack, will need to be compatible with the quantum computing stack, to form the basis for distributed quantum computing services. Fundamental research will be needed on how to build a distributed shared qubit memory and how to distribute such resources, and how to distribute entanglement prior to computing. From a programming perspective, it will be important to define a quantum analog to the classical message passing interface (MPI) that enables

cross-device entanglement. Primitive types and functions and an associated API will need to be defined. As with distributed classical computer devices, the cost of entangling quantum information across devices will be non-negligible. It should be noted that even within a single quantum computing device, those costs cannot be ignored. Communication protocols will need to be developed that are resource-efficient, and can be readily integrated into quantum computing programming environments, compilers and runtime systems.

3 Quantum algorithms and advantages

While quantum computing offers a tantalizing potential for substantial speedups (exponential in some cases) over classical computing, a host of hurdles remain in meeting the grand challenge identified in Section 1.2. Exponential quantum advantages typically require problems with special structure [50, 51], and identifying such problems can be as challenging as crafting novel quantum algorithms for them. Even for well-understood problems, such as the solution of linear systems, idiosyncrasies in the way data needs to be input and output for quantum advantages can result in unfamiliar and understudied flavors of familiar problems. Quantum advantages may not be possible for all the current application drivers. Consequently, a better understanding of problems that are *not* currently being solved but are amenable to delivering quantum advantages will be necessary to open avenues to new kinds of solutions or applications.

The gold standard is the achievement of an exponential quantum advantage against the *best-possible* classical algorithm for a problem; however, it is extremely difficult to establish that an algorithm is the best possible. The *best-known* classical algorithm is a common substitute for the best-possible, as is the case for Shor's algorithm for factoring integers [52]. However, under this alternative, quantum advantages may vanish as and when improved classical algorithms are discovered. Continual research delineating the cutting edge of classical algorithms is therefore necessary to ensure soundness in claims of quantum advantages.

Although a variety of quantum algorithms and advantages are currently known [50, 51, 53], there are relatively few known exponential quantum advantages [50, 54–56], especially for fundamental problems and kernels to empower DOE science applications. There is a pressing need for quantum algorithms admitting theoretical or empirical evidence of advantage for fundamental domains such as simulation, optimization, and machine learning, which are likely to impact a broad spectrum of science applications.

3.1 Driving questions

The following are some of the questions that are expected to drive advancements in quantum algorithms and advantages.

- What classes of existing and understudied scientific applications admit substantial quantum advantages over conventional classical computing paradigms?

  Quantum computers are not expected to provide exponential speedups over conventional classical NP-hard problems, and they are unlikely to serve as drop-in replacements for classical counterparts in most existing applications. Even known quantum advantages against the best-known classical algorithms for scientific workhorse problems, such as optimization, solution of linear systems, solution of differential equations, and topological data analysis, impose strong requirements on how the input data is presented and how the output is accessed. This introduces new challenges in discovering and adapting applications to enjoy quantum advantages.

- How can we design novel algorithms and supporting mathematical models to realize such advantages?

  Many apparently disparate abstract computational models, such as quantum circuits or adiabatic quantum computing, are asymptotically equivalent up to polynomial factors. However, there may be practical and other advantages in selecting the right model for the job at hand. For example, some models may inspire algorithmic ideas that are obscure in others. Moreover, studying unconventional computational models has borne quantum advantages with respect to other critical resources beyond execution time, such as space/memory, quality/accuracy of solution, energy consumption, or communication.

- Are there any provable or empirical barriers for quantum advantages?

  Understanding the *limitations* as well as the potential power of quantum computing is essential for tempering unrealistic expectations and avoiding dead ends. While rigorously provable barriers are ideal, empirical evidence may be used as a guideline or to build intuition.

- What are the physical resource requirements of practical implementations of such algorithms, including numbers of physical qubits and quantum circuit depth?

Forecasting time frames for the realization of quantum advantages hinges on realistic assessments of resource requirements. The latter also may help predict and circumvent bottlenecks as well as facilitate end-to-end hardware, algorithm, and application codesign. For example, recent resource estimates suggest that polynomial quantum advantages may be undercut by error-correction overhead in the foreseeable future [57]; superpolynomial, or ideally exponential, quantum advantages are more likely to enable future practical impact [58, 59]. Future quantum computers delivering advantages may have little in common with those of today; hence, identifying critical but hardware-independent resources is crucial for progress.

3.2 Goals and desired outcomes

In order to make progress toward the grand challenge identified in Section 1.2, the participants at the workshop identified the following challenge areas:

- **Algorithms:** Design of novel quantum or quantum-inspired classical algorithms offering theoretical or empirical evidence of practical advantages
- **Models:** Development of quantum computing models enabling assessment of advantages for a range of computational resources (beyond execution time) and bridging gaps between empirical assessment and worst-case asymptotic analysis
- **Scientific applications:** Identification of new kinds of DOE science applications informed by a deeper understanding of sources of quantum advantages.

Such challenges will need to be addressed in some form in order to achieve eventual end-to-end quantum advantages for the DOE science mission. Consistency and clarity in understanding and explaining nuances of quantum advantages is also an essential ingredient. Examples of such nuances are highlighted below, along with a proposed classification of quantum advantages.

Asymptotic and empirical advantages. Asymptotic advantages speak to a world where both conventional and quantum computing are mature, highly optimized, and well understood. Asymptotic analysis focuses on relative scaling as instance size or complexity grows, rather than on wall-clock run time on specific instances. This makes for fairer comparisons between mature and emerging technologies. Asymptotic advantages are generally established with respect to the worst-case instance for a particular algorithm and may not reflect performance on real-world instances. Consequently, even when mature quantum computers offering exponential asymptotic quantum advantage are available, classical computers may solve some instances appreciably faster. Yet, as instances generally grow in size or complexity, a crossover favoring quantum computing would be expected for problems where asymptotic quantum advantages are known.

Empirical demonstrations of quantum advantage aim to measure practical advantages. Fairly establishing empirical advantages requires selecting appropriate problem instances, benchmarking metrics, hardware platforms, and software toolchains. Care must be taken to ensure that empirically assessed advantages do not engender misleading conclusions. For example, practical algorithmic performance can be highly sensitive to the types of instances selected. This poses a challenge for establishing a classical state of the art when comparing against a quantum algorithm, because no single classical algorithm is likely to be the best-known on all instances of interest for all performance metrics; no-free-lunch theorems [60] formalize these kinds of ideas. Currently, the biggest hurdle to empirical demonstrations of quantum advantage is the lack of scalable and error-resilient quantum computers. Classical simulation of quantum computing is also viable for assessing advantages; however, this is hampered by exponential resource requirements.

The grand challenge in Section 1.2 is formulated to take advantage of synergies between asymptotic and empirical advantages. Rigorously provable asymptotic advantages highlight features of quantum mechanics yielding computational gains. This may hint at ideas for practical quantum algorithms and types of problems amenable to practical quantum advantage. Empirical algorithmic assessments can help build intuition, expose counter-intuitive behavior, and suggest ideas for asymptotic advantages.

Maturity of quantum advantages. The three maturity levels for quantum advantage suggested below highlight essential features of quantum advantages as well as anticipated bottlenecks in achieving them. Goals are framed relative to the capabilities of conventional classical computing. While classical

capabilities are likely to continue maturing as quantum computing grows beyond infancy, it is generally believed that quantum computing will afford some kind of asymptotic exponential advantages against any computing paradigm employing only classical resources.

The maturity levels are presented in decreasing order of maturity (Level 3 representing the highest maturity level). Within each level the distinction between asymptotic and empirical goals is described. Together, the three levels may be viewed as the sequence of milestones that can be taken toward our grand challenge of Section 1.2.

Level 3: Practical quantum advantage

Asymptotic goal: Provable asymptotic exponential quantum advantage against the best-possible classical algorithm, for a problem with practical or scientific significance.

While unconditional proofs of advantage are ideal, they are also rare, and so proofs based on widely believed computational complexity conjectures are acceptable.

Empirical goal: Rigorously quantifiable and end-to-end quantum advantage against all best-known classical counterparts, for problem instances ranging in scale and with practical or scientific significance.

Ideally, solutions should be efficiently verifiable by a classical computer. An empirical practical quantum advantage for a problem should be sufficiently significant to suggest that stakeholders would opt to solve the problem using a quantum computer. "Rigorously quantifiable" refers to adherence to the best scientific practices in selecting performance metrics and executing comparative benchmarks. "End-to-end" entails measuring appropriate resources across all layers of a computing stack (Figure 1), such as I/O costs.

Level 2: Strong quantum advantage

Asymptotic goal: Provable asymptotic exponential quantum advantage against the best-possible classical algorithm, for a possibly contrived problem.

Empirical goal: Rigorously quantifiable quantum advantage against all best-known classical counterparts, for problem instances varying in scale.

Unlike a practical quantum advantage, a strong quantum advantage need not be "end-to-end" and may only focus on selected critical resources. Synthetic problem instances designed to expose quantum advantages may be employed instead of practical ones.

Level 1: Weak quantum advantage

Asymptotic goals: An asymptotic quantum advantage is achieved even if it fails to meet some of the conditions for a strong asymptotic quantum advantage.

Examples: Polynomial instead of exponential advantage, advantage against the best-known instead of the best-possible classical algorithm, advantage only on special cases of a problem, or uncommon assumptions on how input data is accessed.

Empirical goals: An empirical quantum advantage failing to meet some of the conditions for a strong empirical quantum advantage.

Examples: Advantage too modest to suggest practicality, advantage established against a limited set of classical algorithms, advantage established on one-off or unrelated instances that do not notably vary in scale, advantages hiding or ignoring critical resource costs, or advantages based on simulations with optimistically low estimates of errors.

3.3 State of the art

All currently known asymptotic or empirical demonstrations of quantum advantage with respect to run time are weak quantum advantages. Common weak asymptotic advantages are polynomial advantages, such as those based on Grover's algorithm [61], and exponential advantages, albeit under unrealistic black-box oracle input models, usually for contrived problems [50]. A variety of quantum advantages are weak because they are established with the unproven assumption that the best-possible classical algorithm requires exponential time. Examples include quantum advantages for factoring integers [52], simulating quantum systems [62], and topological data analysis [63]. These problems have been well-studied classically without producing polynomial-time classical algorithms, perhaps lending some credibility to strong exponential quantum advantages for them.

A related example is the solution of linear systems of equations, where a weak quantum advantage is known under a nonstandard input model that has not been widely studied classically [64]. This has the potential of becoming a practical quantum advantage if the best classical algorithm for it requires exponential time, and the nonstandard input model is determined to be of practical relevance. The consideration of resources beyond run time of an algorithm opens the door to new types of quantum advantages (see also Section 3.4). For example, memory is an especially critical resource for quantum computing as robust and scalable qubits are a scarce resource. A recent breakthrough in quantum streaming algorithms constitutes a strong and arguably practical quantum space advantage that requires exponentially less space that the best-possible classical counterpart [65].

Recent empirical attempts at demonstrating quantum advantage are encouraging and represent impressive feats of engineering, but they come with caveats [66–69]. The problems on which these are based are generally contrived instances of sampling problems with limited practical applications, where a quantum computer is expected to be able to efficiently sample from a distribution in a way that is not expected to be classically efficient. The output in such a setting consists of samples drawn from such a distribution, and it is challenging to verify that the samples indeed came from the claimed distribution. Indeed, it takes tremendous classical resources (asymptotically exponential time) to verify such a quantum advantage. This, along with current quantum computing scalability bottlenecks, limits the scalability of such attempts at achieving empirical quantum advantage. The latter have prompted improved classical algorithms for the particular sampling instances of interest, dramatically decreasing or eliminating quantum advantage gaps in some cases.

3.4 Research directions

Features of quantum mechanics that give rise to quantum advantages are not well understood, in part due to the lack of mature, fault-tolerant quantum computers to aid in empirically vetting different algorithmic ideas. Focusing on higher-level problems and kernels will enable quantum algorithm architects to impact a variety of potential applications. Discovering settings in which substantial quantum advantages are possible or provable remains a foremost goal, and this may benefit from considering resources beyond execution time, such as space or memory, quality or accuracy of the solution, the number of queries made to a data-access oracle, or the number of samples drawn from a distribution. Considerations regarding ways to improve energy efficiency of computations are also scarce.

Algorithms. Quantum advantages may take many forms; hybrid algorithms, through which quantum and classical computing may complement one another, are a viable strategy for leveraging near-term quantum computing. Variational algorithms [70] for quantum chemistry and discrete optimization applications have grown popular, although they are subject to limitations [70–72], and the advantages they may provide are unclear. Further research on the limitations of variational algorithms as well as broader NISQ algorithmic techniques is necessary to determine whether NISQ advantages are possible. Designing other kinds of hybrid quantum-classical computing schemes (not necessarily NISQ) remains a research challenge.

Quantum advantages cannot be divorced from classical algorithms. For sound quantum advantages, the classical state of the art must keep in pace with quantum algorithmic advances, especially for new types of quantum-inspired problems or quantum-inspired variants of existing problems. Quantum-inspired classical algorithms have been a serendipitous benefit of exploring quantum algorithms. A recent flurry of classical algorithms for machine learning and linear algebra has been fueled by "dequantizing" quantum algorithms [73, 74]. Dequantization is important, not only as a source of novel classical algorithms, but also for understanding the critical features of quantum advantages. However, care must be taken to ensure that dequantized algorithms employ realistic classical models and assumptions [75, 76]. New types of quantum-inspired classical algorithms and analyses are especially valuable, as are fair and rigorous models for "apples-to-apples" comparison of classical and quantum algorithms.

Empirical prototyping and evaluation of quantum algorithms and protocols are a challenge because emerging quantum computers are generally noisy and limited in size. Classical simulation of quantum computers and networks remains a viable approach, and classical algorithms and software systems to push the boundaries of simulation are critical. In addition to further advancing established techniques such as tensor network [11, 77, 78] and Monte Carlo [79–81] simulations, new ideas to accelerate simulation, perhaps for specialized settings where further assumptions are possible, warrant further exploration and development.

Models. The earliest and a consistent source of quantum advantages has been the black-box query model [50, 82], where the input data (exponentially large in the number of qubits in some settings) is only accessible via queries to individual elements. Algorithmic performance is measured solely in terms of the number of queries required, and exponential quantum advantages are possible because quantum algorithms are allowed to conduct queries in superposition. A recent notable example is an exponential quantum query advantage for simulating classical dynamics of coupled oscillators [83]. Black-box query models are generally unrealistic because input data must come from somewhere, and the source of the data imposes structure that is exploitable by algorithms. Exponential quantum query advantages for problems with scientific applications represents a reasonable step toward practical advantage. Models relaxing black-box assumptions and taking overall execution time into account will be important in bridging gaps to realistic application scenarios.

The preceding discussion of asymptotic and empirical advantages suggests the possibility of investigating classical and quantum computing models together to bridge the gap between worst-case asymptotic analysis and empirical assessment on finite data sets. Such models speak directly to our grand challenge goal of empirically demonstrable practical advantages backed by rigorous analysis. Other types of semi-abstract models that incorporate realistic features of the quantum computing and networking stacks while supporting theoretical analysis need to be further explored. Recent work in this vein has considered models for NISQ computing [84]. As previously mentioned, models aware of computational resources likely to become bottlenecks, such as memory, data access, or energy consumption, will be critical in identifying new kinds of practical advantages and applications.

Aside from computing models, physically motivated scientific models will be a key ingredient for impact. An example is the quantum Heisenberg model, which is universal for quantum simulation [85] and has also

become a testbed for developing rigorous algorithms approximating ground states [86]. Quantum advantages for analog (and hybrid digital-analog) systems, open systems, and continuous variable systems are underdeveloped and may benefit from models focusing on key sources of potential advantage. Physically motivated models can be used across the quantum computing and networking stacks to inform architectures, compilers, error resilience schemes, algorithms, and applications.

Scientific applications. While the development of scientific applications around known quantum algorithms is important, the repertoire of quantum algorithmic techniques must be expanded to drive new and unforeseen applications. Even quantum advantages for familiar applications, such as solution of linear systems [64], may look utterly unfamiliar due to constraints on the way data must be accessed to yield a quantum advantage [76]. For example, a solution to a linear system is represented as a quantum state – prohibiting conventional random access. Finding applications for such unconventional versions of familiar problems, while retaining quantum advantages, is a major research challenge.

One path toward application-level impact is focusing on higher-level problem domains that are likely to benefit many kinds of scientific applications. Along with machine learning and simulation of physical systems, optimization is a prime example of such a domain. Quantum algorithms offering polynomial advantages over the best classical algorithms have recently been discovered for solving linear programs [87, 88] and more general convex optimization problems [89–91]. For optimization and related domains, the precision to which a solution is computed is important, and recent quantum algorithms for solving differential equations offer exponential improvements in precision [92]. Discrete optimization has enjoyed considerable interest from the quantum information science (QIS) community, due to the advent of quantum annealers and the Quantum Approximate Optimization Algorithm (QAOA) [93]. However, exponential quantum advantages for optimization remain largely elusive and should be more intensely studied. A recent body of work has unearthed new connections between finding approximate solutions of discrete optimization problems and approximating the ground states of physically relevant local Hamiltonians [94–97]. Deeper connections between optimization and quantum mechanics remain to be discovered and will mutually benefit many disciplines beyond QIS.

Realizing scientific impact from quantum computing will entail interdisciplinary efforts among quantum information scientists, computer scientists, applied mathematicians, and other domain scientists. For example, collaborative efforts will be required to produce scalable problem sets and meaningful benchmarking metrics to assess scientific application impact. Another essential element for enabling scientific impact is the dedicated access to physical or virtual quantum computing systems, such as through user programs, augmented by low-level understanding of hardware behaviors gleaned from testbeds.

To summarize, exponential quantum advantages are rare and beautiful beasts (fortunately they do not appear to be entirely mythical), and one must be creative, without imposing artificial biases, in searching for them.

4 Benchmarking, verification, and simulation methods

Evaluating the current performance of quantum computers and networks as well as forecasting their future performance goals are major concerns for tracking progress toward quantum advantage. However, assessing such progress is a challenging and multi-faceted endeavor. Measuring the performance of individual quantum applications is often coupled to evaluating the fidelity of the underlying technology, such as the QPU, the individual components, and the integrated logical system. Moreover, results obtained from relatively small, near-term quantum computers and networks offer limited insights into future performance expectations for larger and more complex systems. There is a pressing need to develop comprehensive approaches to performance analysis.

Additionally, the verification that quantum algorithms and protocols for these systems are correctly designed and the validation that applications accurately implement these designs are fundamental to providing assurance in performance measurements. On the one hand, while rigorous proofs of asymptotic quantum advantage are desirable and may be used to direct empirical studies, the former often rely on abstract or impractical models of quantum technology that impose unrealistic restrictions. Alternatively, a staple of performance assessment for classical technologies has been to compare empirical evidence against large-scale numerical simulations. However, the complexity of brute force approaches to simulating quantum (physical) systems stymies similar approaches to verifying design and validating behavior. Scalable approaches to the verification and validation of quantum systems are needed to track and forecast performance.

All of the above concerns for benchmarking performance are complicated by the presence of noise and errors during the operation. Noise and errors themselves affect the reliability of the quantum system (e.g., network or computer) and the reproducibility of observed results. This leads to an ill-characterized uncertainty in the system's performance. The addition of mitigation or correction of such errors before, during, and after execution introduces overheads that are substantial to performance assessment. Mixing logical and physical interactions between quantum and conventional technologies generates a hybrid model that impacts the methods for assessing performance. Ultimately, a complete calculus for assessing quantum advantage in hybrid systems is needed to match the broader evolution of (quantum) computer science and engineering.

4.1 Driving questions

The following are some of the questions that drive advancement in benchmarking, verification, and simulation methods.

- How can we fairly assess quantum advantage relative to classical capabilities, especially as underlying technologies evolve and scale from the NISQ to fault-tolerant paradigms?
- How can we measure progress of quantum systems toward demonstrating quantum advantage, across the computing and networking stacks?
- Which representative scientific use cases serve as insightful and scalable benchmarks for quantum computing and networking applications?
- How can we verify demonstrations of quantum advantage?
- How can we leverage numerical simulation of quantum systems to validate large-scale quantum applications?

Collectively, these questions drive a need to bridge the gap between the theory and the practice of evaluating applications of quantum computing and networking, and the ability to demonstrate those applications in a hardware system. It is essential to establish sound and practical contexts for assessing advantage that can be realized in near-term and future quantum systems.

4.2 Goals and desired outcomes

A leading outcome for this research will be a set of efficient, effective, and scalable methods to assess the performance and demonstration of quantum computing and networking applications. This will include methods to verify and validate the behavior of quantum computing and networking algorithms, protocols,

and applications. While near-term methods will address so-called "NISQ applications," intermediate-term methods must address the setting of fault-toleration operations. Additionally, the ideal long-term outcome is to measure the performance of applications implemented on quantum computing systems at increasing scale and complexity. This will address near-term methods for addressing individual levels of the hierarchy in physical and logical representations as well as the long-term goal to integrate these methods into a comprehensive analysis. Finally, near-term research outcomes should include the development of efficient and effective methods for numerical simulation of quantum computing systems, including noisy quantum circuits and fault-tolerant operation, that are widely accessible.

4.3 State of the art

Presently, benchmarking quantum computing systems has been dominated by varieties of small-sized NISQ applications that can be empirically demonstrated [14, 98–101]. This is due in part to the limits of available hardware systems but also the lack of benchmark methods that can be easily scaled across a range of problem sizes [15, 102, 103]. For present results, the reported metrics are often representative of the underlying physical hardware fidelity with some point cases connected to the demonstration of application-specific goals, such as chemical accuracy [98] or computational advantage [66]. Similarly, current methods for gathering and analyzing benchmark results are ad hoc due to the lack of a comprehensive context or framework by which results can be compared across different physical and logical representations [104]. Additionally, porting benchmarking methods across technologies and programming platforms represents a challenge to comparative analysis and tracking due in part to missing standards for software programming and hardware execution. There is also a lack of shared access to benchmark results that supports tracking state-of-the-art demonstrations.

Verification and validation of applications for the purpose of benchmarking is a nascent research topic [105–107]. Motivated in part by the recent surge in experimental demonstrations, the use of numerical simulation to validate results from known examples is a leading approach [108–110]. Alternatives based on formal verification are possible but do not readily extend to noisy or hybrid contexts.

State-of-the-art methods for numerical simulations of quantum computing, especially noisy quantum circuits, have only recently been developed in earnest with a few demonstrations on the most powerful (classical) computing systems [11, 111–113]. Method development has leveraged the unique representations afforded by tensor networks, randomized sampling, and algebraic formulations of quantum circuit execution.

4.4 Research directions

The leading priority research direction is to develop a holistic approach to assessing the performance of quantum computers and networks. This long-term activity will require formalizing the expectations for that moment in time when quantum applications indicate a cross-over point in performance relative to well-defined metrics for potential comparison against known conventional counterparts. Such methods for benchmarking performance must be tied rigorously to a framework by which current as well as future system behavior can be tracked and forecast. In the near term, these approaches may be focused on individual layers within the corresponding logical stack, but the long-term goal is to establish rigorous, informative, and efficiently verifiable performance metrics for all levels of the quantum computing and networking stacks. Ideally, these metrics will be integrated across the stack so that improvements can be quantified and predicted performance may be compared with observed behaviors.

The near-term development of individual benchmarking methodologies should extend to current quantum computers and networks and, in the intermediate term, to their future system. The latter should include advanced system architectures that integrate quantum and conventional processing into focused workflows. Composable approaches that address how individual devices and integrated components behave will provide insights into piece-wise performance. A near-term priority is establishing best practices for collecting and analyzing benchmark results that place bounds on uncertainty and reproducibility relative to observed performance. A long-term priority is to identify the requirements for component and system performance that ensure well-specified computational goals.

A near-term priority is to extend the scale over which simulation supports the verification of quantum algorithms and their validation in realistic quantum systems. This includes developing computational meth-

ods and heuristics that address the performance of specialized algorithmic structures as well as noisy and erroneous operations in quantum architectures. Concurrently, the development of numerical simulations will ideally validate benchmark outcomes as well-defined tests at larger problem sizes, even when exact simulations are infeasible. In the intermediate term, techniques are needed for validating system performance under the intended and realistic conditions at scale. Additionally, formal methods for validating quantum advantage beyond empirical assessments are needed to forecast performance and identify system requirements in the context of scientific computing. These methods are anticipated to require analysis of hybrid computational models that mix quantum and conventional logic.

5 Resilience through error detection, prevention, protection, mitigation, and correction

Achieving scalable and reliable quantum computation requires managing noise in quantum systems. Noise can originate from a variety of sources, such as fabrication imperfections, unwanted interactions between a system and its environment, and faulty gate operations. Leading to rapidly accumulating computational errors, noise can ultimately constrain large-scale quantum algorithms and realizations of quantum advantage. Thus, quantum processor resilience to noise, through a multitude of error detection, prevention, protection, mitigation, and correction protocols is essential for near- and long-term quantum computing. In the near term, these methods will enable the first milestones of practical quantum utility. In the long term, these methods will continually accelerate the ability of quantum systems to target the next frontier of quantum applications. Developing the most effective techniques for error resilience requires a thorough understanding of the requirements of target quantum applications and the limitations or characteristics of target quantum devices. Crucially, it demands a suite of scalable state-of-the-art, cross-stack strategies and technologies that can effectively bridge the gap between application requirements and device limitations.

5.1 Driving questions

The following are some of the questions that drive advancements dealing with error and noise.

- How can we enhance the resilience of quantum systems to noise and errors to relieve scalability and quantum advantage bottlenecks?

  Resilience is accomplished when errors are sufficiently addressed throughout the quantum stack such that the target performance accuracy of an application or set of applications is achieved. Resilience should bring about reliability and stability in the quantum device such that application performance remains consistent despite potential noise processes varying over time. Attaining this goal requires further progress within hardware and software layers to reduce the presence of noise and propagation of errors from the device to the application. Progress is required in the design of layer-specific and cross-stack error resilience protocols that optimally address noise in near-term devices while maintaining reasonably low overhead costs as devices scale.

- What kinds of quantum algorithm codesign techniques can aid in yielding resilient quantum systems?

  Traditionally, quantum algorithms are designed based solely on the application. They are not noise-aware, nor hardware-specific. In recent years, variational quantum algorithms [70] have emerged as a hardware-aware solution to near-term quantum computing, where algorithmic operations are chosen based on hardware limitations (e.g., sparse connectivity between qubits and high gate error rates). Although they are hardware-informed, they are not noise-informed in the sense that they are not designed to be inherently robust to dominant noise sources at the algorithmic layer. If one can identify such sources, there may be opportunities to redesign algorithms to be robust to noise. As a result, the algorithmic layer could alleviate demands typically placed on lower layers of the stack. In addition, algorithms and noise resilience protocols at the lower layers of the stack could be designed together to further optimize hardware performance.

  Furthermore, long-term quantum applications look to rely on fault-tolerant quantum computing by using quantum error-correcting (QEC) codes that are capable of arbitrarily suppressing error rates by increasing code distance. While there has been a field of study in improving the theoretical performance of these codes (e.g., thresholds and encoding rates), quantum computer systems that will be available in the near future will require error correction techniques to become concrete. For example, systems with thousands to tens of thousands of qubits can be designed and programmed in several ways architecturally. Beyond considering encodings of physical qubits into logical qubits via sophisticated, high distance codes, programmers will be interested in mixing low distance codes, error mitigation, and quantum control techniques, as well as new architectures to best use available quantum memory at the physical level. Implementing a quantum algorithm requires design at the logical level and, in the near term, will require informed decisions around factors such as the number of concatenations in

T-state factories, logical qubit connectivity and layout, as well as integration of classical processing performing low-latency error detection and short-distance quantum error correction.

5.2 Goals and desired outcomes

Error resilience protocols should reach a number of milestones on the path toward fault-tolerant quantum computation. Below, suggested milestones are categorized by hardware maturity. In particular, they span near-term devices, intermediate-scale devices that enable logical protection before achieving fault-tolerant thresholds, and future fault-tolerant systems.

Advantage before logical protection. In the near term, paths toward quantum advantage may be realized through the use of highly tuned, cross-stack error resilience protocols that utilize combinations of limited logical encoding with clever classical processing. They will rely on a set of complementary protocols that are chosen based on empirical performance, as evaluated on a set of benchmarks that are small-scale today, but are scalable and representative of real-world workloads. Key to the evaluation of error resiliency is evidence of improved system reliability, stability, and result consistency. Furthermore, comparisons should be made between unoptimized error resilience and cross-stack error resilience protocols, and against ideal noise-free results, where possible, to evaluate the true value of cross-stack optimization.

Faulty error-corrected quantum computation. In the intermediate term, design and evaluation of error resiliency should center on logical computation performed on 10s to 100s of logical qubits. Fully fault-tolerant, threshold-satisfying physical error rates are not expected in this period. However, the error rates should be low enough to demonstrate the advantage of logical computation over physical qubits under equivalent qubit resources. Error resiliency protocols should not solely focus on logical detection and correction, but leverage a wide range of physical-to-algorithmic protocols across the stack to improve performance. The ultimate goal would be to show evidence of quantum advantage prior to achieving fault-tolerant thresholds first on a specific application and then on a suite of algorithms. The latter would enable analyses of the relationships among applications, device noise characteristics, and error resilience capabilities.

Toward large-scale, fault-tolerant quantum computation. As initial stages of fault-tolerant error correction, evidence of logical qubits outperforming physical qubits through fault-tolerant quantum error correction should be demonstrated. Evaluation of fault-tolerant protocols should be compared against physical qubits utilizing optimized cross-stack error resilience protocols. Similarly, fault-tolerant protocols should be assessed with and without cross-stack optimization to identify benefits and potential points of failure in error resilience in the stack. The success of logical qubit error resilience should then be showcased through demonstrations of quantum advantage for a targeted application. This should be followed by a similar analysis on a suite of applications to evaluate the relationship between applications, device noise characterizations, and the capabilities of error correcting codes.

5.3 State of the art

Over the past two decades, a substantial number of protocols have been developed to address noise at various locations in the quantum stack. Broadly, protocols can be categorized as those that seek to protect, prevent, detect, correct, or mitigate errors. While many can be employed independently, their combined utility potentially offers enhanced performance as compared to their implementation in isolation. Theoretical studies have been complemented by experimental instantiations, particularly in recent years. The emergence of noisy quantum processors has enabled many demonstrations showcasing the potential utility of individual and combined approaches to improve quantum algorithmic performance.

Protection against errors is commonly captured by protocols designed to suppress or tailor noise. Noise suppression is typically afforded by quantum optimal control [114–116] and dynamical decoupling (DD) [117, 118]. Although these are not necessarily mutually exclusive, the former focuses on the design of quantum gate waveforms to suppress noise and implement high-fidelity logic operations, while the latter averages out noise via sequences of strong and fast gates. Both have a long-standing history with extensive experimental evidence highlighting their efficacy, most notably in currently available quantum devices. Suppression protocols, like DD, have shown utility at the physical [119–122], logical [123–125], and algorithmic [126–130] layers. At the algorithmic layer, pulse-based methods, such as Pauli twirling [131, 132] and its extensions [133, 134],

strive to tailor the noise, essentially transforming correlated errors into uncorrelated errors. While not requiring significant characterization of the noise like their suppression counterparts, such methods can incur high sampling overheads in order to ensure proper noise tailoring.

Error prevention can often be implemented as optimization passes in quantum compilers. These passes in today's devices involve strategies to identify the best mapping from the quantum circuit to the qubits, and identifying the best routing paths between qubits [135–138]. In contrast, future passes will focus on identifying optimal locations of logical qubits, placing magic state distillation factories, mapping logical qubits to physical qubits for the chosen QEC code, and more. All these optimizations suffer from considerable computational cost and will be unable to accurately scale to 1000s of qubits or will need to use approximating heuristics that significantly limit the capability of these techniques. Building more scalable but accurate strategies will be critical to consistent long-term benefits.

At the logical layer, quantum codes generally seek to detect and correct errors. This is accomplished by leveraging logical encodings of physical qubits in a variety of ways. Active QEC codes provide the ability to detect and correct specific error syndromes [139, 140]. Traditional codes are typically best suited for uncorrelated errors which have been shown to be at odds with realizations of quantum devices in recent years [141–151]. Nevertheless, a number of present-day proof-of-principle demonstrations have been realized despite the inability of current devices to satisfy error thresholds and qubit overhead costs required for fault tolerance [69, 123–125, 144, 152–157]. In contrast, passive quantum codes, like error-detecting codes [158, 159], decoherence-free subspaces [160–162], or minimal noise subsystems [163] seek to either detect or avoid errors via logical encoding. The latter relies on exploiting symmetries in the noise environment that can be difficult to achieve in practice and commonly require noise identification and/or engineering. While both detecting and avoiding codes have been realized experimentally, it is not known what role they will play in near-term and future systems.

Quantum error mitigation is an algorithmic approach that usually utilizes post-processing techniques on an ensemble of quantum circuits to reduce noise-biasing in expectation values. This process may include amplifying the noise or sampling the circuit ensemble at a specific noise strength in order to extract noiseless estimates of quantum observables [164]. Quantum error mitigation techniques like zero-noise extrapolation [165, 166], probability error cancellation [166, 167], and measurement error mitigation [168–172] have shown promising results on current hardware. Nevertheless, they suffer from significant sampling overhead that scales exponentially in the circuit size. Thus, while not incurring the qubit overhead associated with QEC codes, their scalability is currently a bottleneck.

5.4 Research directions

Scalable noise characterization and prediction. Knowledge of underlying noise processes paves the way for the development of targeted protocols for combating noise. Currently, noise characterization protocols, such as gate set tomography [173, 174], randomized benchmarking [175, 176], process tensor tomography [177], and quantum noise spectroscopy [178–182], are hindered by their scalability. There is a need to develop protocols that are resource-efficient and sufficiently informative to develop predictive error models at various layers of the stack. Such methods could leverage both quantum and classical resources (e.g., approximate Clifford simulation [183–186] or shadow tomography [187–189]) to reduce overhead costs. In addition to providing a utility to error-resilient protocol design, these techniques could elucidate important details about error propagation across the stack.

Hardware-aware noise resilience. Real hardware noise can be quite distinct from the typical Markovian assumption, possessing attributes of correlations in space [190–193] and time [194–198] and even non-stationarity [199, 200] and non-Gaussianity [201–203]. As a result, the underlying assumptions of various protocols for addressing noise are violated, thus limiting their efficacy. There is a need to envision protocols that are informed by hardware noise models and enable real-time updating as hardware characteristics change over time [204–210]. Realizing such schemes likely relies on a careful orchestration of characterization and protocol adaptation at key layers in the quantum stack.

Scalable compilers for near-term and future quantum devices. Optimally implementing tasks such as approximate unitary synthesis, pulse-level control, placement of logical qubits, selection of type, distance, concatenation of QEC codes, logical-to-physical qubit mapping, and selection of appropriate compiler passes is vital in improving the quality of quantum execution on imperfect devices. However,

designing these tasks optimally is challenging due to computational complexity and limited understanding of circuits and devices. Scalable compilers need to be designed which are a combination of rigorous formal methods, as well as heuristics and approximations. Building these compilers in a modular fashion is worthy of exploration – in such a scenario, the compiled output may be locally optimal but globally sub-optimal, but sub-optimality could be limited via good compiling strategies.

Application-aware noise resilience. Dominant noise sources may depend upon the application of interest. As such, it is less important to identify errors that are prominent at lower layers in the stack than those that are most detrimental to computational accuracy at the algorithmic and application layers. Assessing errors from a top-down perspective can potentially provide critical insight into the propagation of dominant noise sources and enable tailoring of quantum algorithms to specific noise sources. Furthermore, it could allow for improved error resilience at other layers of the stack or simultaneous design of algorithms and noise resilience protocols across the stack.

Error resilience across the stack. Instances of hybrid error protection schemes have been experimentally tested in recent years in the context of quantum error correction and quantum algorithms. Although promising results exist, there is little insight into why certain methods perform better than others. A comprehensive understanding of error resilience protocols that span the stack is required. This includes identifying intermediate metrics that are correlated with application performance. In addition, techniques are needed for synergistically optimizing application performance, logical and physical error resilience, and qubit mapping strategies while reducing overhead costs.

6 Hardware and protocols for next-generation quantum networks

Quantum networking is predominantly concerned with high-rate, high-fidelity entanglement generation, distribution, and storage in distributed environments. High-quality shared entanglement between distant users can be used for a plethora of impactful applications as discussed in the following. Current quantum networks have shown reasonable levels of quality entanglement distribution between several users separated by moderate distances. Moving forward toward more useful quantum networks enabling truly impactful applications requires ways to mitigate errors, including photon loss, which dominate at longer distances. To mitigate and correct these errors over arbitrary distances, quantum repeaters have been proposed using various architectures. All of the proposals for quantum repeaters are very demanding of the technology used and development is required to realize a functional quantum repeater.

In this section, we first discuss the driving questions for next-generation quantum network development, followed by the goals of quantum networking. The goals are organized in terms of the generations of quantum repeaters which are widely recognized. Here we follow a recent work [211], which employs the convention of classifying quantum repeaters in terms of how they handle errors, namely, loss and operational errors. Note that we consider these generations broadly to be inclusive of one-way and two-way all-optical quantum repeaters, which can also be classified into the generations organized by the way they correct errors. Furthermore, there is a discussion of the state of the art across quantum networking, which, in relation with the goals listed, motivates the research directions that conclude this section. The overall focus is weighted toward quantum repeaters because currently their lack of development is inhibiting further development and testing of additional, advanced aspects of quantum networking.

6.1 Driving questions

The following are some of the key questions that drive advancement in hardware and protocols for next-generation quantum networks.

- Can quantum repeater hardware be built to achieve entanglement distribution rates higher than those of repeat-until-success direct transmission experiments?

  The main challenge in communicating classical and quantum information is channel attenuation. In classical communications, the problem is solved by amplifying optical signals carrying information along the way. On the other hand, with quantum information, deterministic noiseless amplification of an unknown quantum state is fundamentally prohibited [212, 213]. Qubits cannot be physically transmitted over long distances without being hindered by the effects of signal loss (increasing exponentially with the distance) and other operational errors. To enable quantum networks that scale beyond tabletop demos, quantum repeater (QR) [211, 213–218] technologies are required. The major function of QRs is to attempt the correction of these loss and operational errors while preserving the unknown quantum state. Using QRs, the communication distance can be extended by dividing an end-to-end link into shorter intermediate segments connected by QRs to reduce the errors accumulated between intermediate nodes where they can be corrected. For this strategy to work, error rates in a channel with QR must be smaller than those in a direct transmission channel. Hence, the first step toward scalable quantum networks is to demonstrate QR technologies that enable better quantum communication performance than direct transmission losses.

- What software and hardware, besides repeaters, are needed to build scalable quantum networks?

  As the sizes and complexity of quantum networks increase, it is evident that quantum networks cannot be operated by manual control. The issues of quantum network abstraction, scaling of network architecture and protocols, and software automation are becoming increasingly important. In the open systems interconnection (OSI) reference model, the communications between nodes are split into different abstraction layers [219]. An analogous layered network stack should be constructed for quantum networks. The open issues are: i) what hardware/software functionalities are to be assigned to each layer, and ii) what interfaces between the layers should be defined in order to realize quantum communication and networking. In contrast to their classical counterparts, entanglement is the fundamental building block of quantum networks [220, 221]. Quantum-network-specific, entanglement-related operations include entanglement generation & distribution, entanglement routing, entanglement swapping,

and entanglement distillation. Therefore, determining how to build up a quantum network stack that can support various entanglement operations is critical. At the same time, quantum networks aim for high-fidelity quantum information transport. Therefore, it is necessary to incorporate quantum error correction into the quantum network protocol stack to achieve high-fidelity entangled links.

Quantum resources in quantum networks must be fully coherent and synchronized [222, 223]. Depending on the quantum protocols to be implemented and underlying quantum technologies, synchronization in time, frequency, phase, or their combinations are required. High-fidelity Bell state measurements, an essential function in quantum networks, require spectral, temporal, and polarization indistinguishability. Quantum network control tends to be decentralized and distributed as the size of the quantum network increases. Multiple synchronization references are likely to be deployed. Synchronization in such environments is challenging. While many synchronization technologies have already been developed and deployed in classical telecommunication networks, many of these still require significant improvements for quantum networks. One will need to build high-accuracy and high-precision quantum network control hardware, software, and protocols that can support various advanced synchronization mechanisms essential for the operation of quantum networks.

- What applications and advantages will those networks enable?

  At least three broad categories can be identified in a classification of the application space of quantum networks: quantum data transfer, distributed quantum computing, and distributed quantum sensing. To illustrate, envision a scenario where data stored in a quantum memory on a distant processor is needed at another facility for further processing or processing on a different computing scale. In this case, a quantum network (e.g., quantum-enabled Energy Sciences Network (ESnet)) could facilitate the necessary connection and data transfer. Establishing shared randomness for a variety of uses is another type of transfer protocol a quantum network could facilitate. High-fidelity entanglement distribution channels established between smaller quantum computers could help realize an advantage for distributed quantum computing if the resulting quantum volume is larger for the distributed quantum computer than any one of the individual processors. Similarly, entangled quantum sensors can collectively offer increased sensitivity because of their shared connections via a quantum network. Moreover, in a scenario involving federated quantum instruments, a system with edge quantum sensor(s) may benefit from, or need a connection to, a nearby quantum preprocessor with connections to a larger quantum processor for real-time data analysis facilitated by a quantum computer.

- What kinds of distributed quantum computing models will result in novel quantum applications and advantages?

  As current quantum computing systems are smaller in scale while trying to be scaled up, networking smaller quantum processors together to synthesize a larger interconnected processor may help quantum computers scale faster than focusing on scaling the individual processors. Moreover, for tasks similar to homomorphic classical computing (e.g., blind quantum computing), these computing models are naturally distributed and provide a model of distributed quantum computing for those applications. There are likely to be different computing models optimized for intra-processor communication as mentioned previously, where there may be multiple tiles or cores within a quantum processor and a scenario where the physical separation among multiple quantum computers is so large that quantum transduction to the quantum networking signals and/or quantum repeaters is needed in the link between the quantum computers. Intra-processor communication is likely to benefit from higher qubit connectivity and rates, with quantum links of high quality, whereas connections involving long-distance quantum networking will likely exhibit reduced channel capacity and qubit connectivity, as well as longer delays between successive rounds of connection establishment. Optimizing computational performance in these two scenarios is likely to result in substantial differences; however, each preceding model (among others) has the potential to enable different novel quantum applications and advantages.

6.2 Goals and desired outcomes

In the following, organized by the generations of quantum repeaters, are summaries of generational capabilities, enabled applications, and near-term development needs. Three generations (1G, 2G, and 3G) are

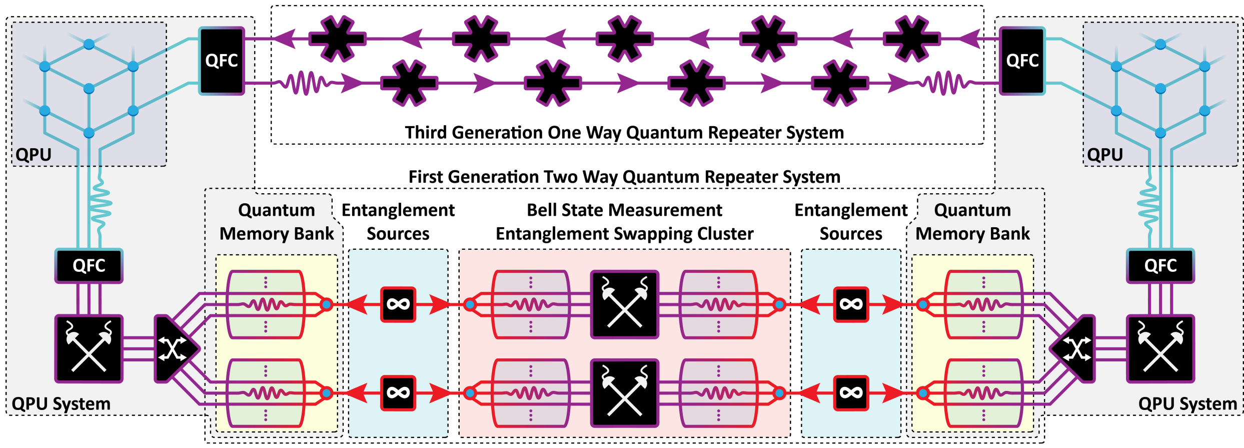

Figure 2: A conceptual example of a distributed quantum network illustrating quantum devices and components necessary to establish communication between quantum processing systems (QPUs) using entanglement distribution via quantum repeaters (QR). The key components permitting the interconnection between QPUs are quantum transduction and frequency conversion units (QFC) bridging the energy differences between the different qubit systems. (Bottom) Two-way QR system composed of high-repetition entanglement sources (ES), quantum memory banks (QMBs) and entanglement swapping clusters (ESCs), allowing entanglement distribution with high repetition rates. (Top) One-way QR system composed of error corrected quantum-gate-type light-matter interfaces.

envisioned, organized by their methods for correction of loss and operational errors [211, 213–218].

1G and 2G rely on heralded entanglement generation via entanglement swapping which requires two-way classical communications between separate nodes, resulting in reduced performance. 1G repeaters also rely on heralded entanglement purification (also requiring additional two-way classical communications), whereas 2G repeaters begin to employ quantum error correction, which does not need two-way classical communications, resulting in higher performance. 3G uses quantum error correction on a transmitted resource state (e.g., cluster state [224] or Gottesman, Kitaev, Preskill (GKP) state [225]) and does not need any two-way classical communications, thereby delivering the highest performance while also requiring the lowest loss and error from its constituent parts [211]. All-photonic approaches [226–229] do not require the quantum memories needed in 1G and 2G repeaters; nevertheless, various proposals have similar structures to 1G, 2G, or 3G depending on their methods for correcting errors and, for broader coverage, are included within the respective generations.

Generation 1: Two-way heralded entanglement generation and purification

Hardware Capabilities: Multiplexed high-efficiency long-coherence-time quantum memory, memory-compatible high-rate high-efficiency photon sources, efficient entanglement-purification hardware, sufficient-bandwidth low-latency classical signaling, high-efficiency quantum transduction, high-rate and high-fidelity light-matter interfaces, high-efficiency quantum detectors, high-fidelity long-coherence-time qubit technologies, advanced quantum frequency conversion and quantum transduction technologies, etc.

Applications: Low-rate quantum data or randomness transfer protocols, low-performance distributed entangled quantum sensing, limited-connectivity distributed quantum computing.

Near-term development: Improved quantum memory coupling to fiber-optic cable, near-deterministic memory-compatible entangled photon sources and single-photon sources, demonstration of efficient strong coupling and/or quantum non-demolition operation between atomic system and photonic mode for entanglement purification, demonstration of efficient entanglement purification, improvements in efficiency, noise-reduction of quantum transduction/conversion, large scale network live low-latency coordination and synchronization.

Generation 2: Two-way heralded entanglement generation and quantum error correction

Hardware Capabilities: Multiplexed high-efficiency long-coherence-time quantum memory, memory-compatible high-rate high-efficiency photon sources, efficient quantum error correction hardware, sufficient-bandwidth low-latency classical signaling, high-efficiency quantum transduction, high-rate and high-fidelity light-matter interfaces, high-efficiency quantum detectors, high-fidelity long-coherence-time qubit technologies, advanced quantum frequency conversion and quantum transduction technologies, etc.

Applications: Medium-rate quantum data or randomness transfer protocols, medium-performance distributed entangled quantum sensing, distributed quantum computing.

Near-term development: Generations 1 developments (except purification), demonstrated quantum error correction in quantum processors with network-compatible quantum memory or measurement-based quantum error correction for all-optical implementations.

Generation 3: One-way entangled resource states preserved by quantum error correction

Hardware Capabilities: High-rate, high-efficiency, high-quality resource-state sources, efficient quantum error correction hardware, high-efficiency high-quality quantum transduction, efficient variable-delay quantum buffers, etc.

Applications: Quantum data or randomness transfer protocols, distributed quantum sensing, distributed quantum computing.

Near-term development: Demonstrations of optical resource-state generation and demonstrated quantum error correction compatible with a proposed resource state.

6.3 State of the art

Today, quantum networks are in their infancy. Like the Internet, quantum networks are expected to undergo different stages of research and development before they reach a level of production maturity. Several fundamental quantum technologies are required to build practical quantum repeaters, which are essential to scaling testbed quantum communication systems into distributed complex networks at large distances. In general, these fundamental quantum technologies can be put into several broad categories: (1) sources and measurement/detection of optical quantum states; (2) quantum memories and buffers; (3) light-matter interactions and interfaces; (4) purification and error correction; (5) component integration and demonstration of quantum repeater node; (6) quantum resource routing, switching, transduction/conversion, and allocation; (7) quantum network stack and architecture; (8) protocols and applications.

Quantum light sources. Sources of quantum light can broadly be broken down into *probabilistic* and *near-deterministic* sources, corresponding to probability $P < 0.5$ and $P \gg 0.5$, respectively. Probabilistic sources are most often based on spontaneous parametric down-conversion (SPDC) but also based on spontaneous four-wave mixing and other processes [230]. Since the advent of SPDC-based entangled photon sources nearly 30 years ago [231], they have been the dominant source of optical entanglement due to their accessibility (relatively simple optics and no cryogenics) and their high versatility leading to a plethora of demonstrations using a wide variety of spectral and temporal combinations [232, 233]. For the common discrete-variable encoding, the main deficiency besides being probabilistic is that they can spontaneously emit multiple pairs of photons, which can result in reduced state quality [233–235]. On the other hand, these multiple pairs are exactly what are needed to generate squeezed states; thus, for continuous-variable encodings, SPDC is a high-fidelity method for generating Gaussian entanglement from squeezed vacuum states produced by SPDC [236]; namely, it is a *deterministic* source of Gaussian entanglement [237]. For discrete-variable encodings, researchers are designing near-deterministic sources of single photons or entangled photons, such as those based on quantum dots [238] or atomic-based (often atomic-memory-based) entangled photon sources [239–241]. These sources are much better at producing single photons (or single photon pairs) but usually have lower collection efficiency compared to SPDC, although collection efficiency in the near-deterministic region has been recently shown [238, 242]. In addition, certain atomic systems coupled to a cavity can also be used to directly generate matter-photon entanglement and store the matter qubit until the photon is used for entanglement swapping [243].

With all these sources, there are often trade-offs made between state fidelity, collection efficiency, generation rate, external resources (e.g., cryogenics), and other criteria such that there has not yet been an ideal quantum photon source developed that (near-)deterministically generates single or entangled photons on demand with high fidelity and high collection probability into fiber. For quantum repeaters based on matter-based memories, multiplexing many of these ideal sources together will be crucial for generating internode entanglement at high rates in conjunction with long-storage quantum memories [244, 245]. Moreover, certain all-optical quantum repeaters need many of these types of ideal sources to generate entangled cluster

states [213]. On the other hand, other all-optical quantum repeaters call for (GKP) states [225, 228], which have not been generated optically yet, but numerous proposals (e.g., [246–250]) for their generation often include squeezed light as a precursor.

Quantum-grade detectors. As for detectors, single-photon detectors (normally used with discrete-variable encodings), primarily those based on superconducting nanowire single-photon detectors, have been demonstrated and commercialized to detect single-photons at various wavelengths with high efficiency ($>$ 90%), high count rates ($>$10 Mcps), and low false-count rates ($<$10 cps) in a fiber-coupled package; however, this approach needs cooling to $\approx$ 1K temperatures [251]. On the other hand, homodyne detection (normally used with continuous-variable encodings), has been demonstrated with high quantum efficiency ($>$ 99%) photodiodes for visible and IR wavelengths (including telecommunication band) and low electronics noise compared to shot noise (< -15 dB) [252], and compatible with both the frequency- and time-domains [253]. Notably, the mixing of the signal and local oscillator, especially with a narrowband local oscillator, enables the filtering of significant amounts of nearby noise (e.g., noise from coexisting fiber communication signals), whereas, without external filters, single-photon detectors do not have such an ability [254].

Quantum memories and buffers. Quantum memories or buffers can be broadly categorized by whether they store the state in some way (e.g., absorb a photon to change a stationary atomic state) or only delay the state temporarily using *a priori* semi-fixed delay (e.g., fiber delay or echo phenomena), respectively. Matter-based quantum memories are common using a variety of platforms: warm atomic vapor, cooled trapped atoms, defect centers, and rare-earth-ion-doped crystals, among others. Each has its strengths which have been compared in other analyses [213, 255, 256]. Moreover, quantum buffers are often based on fiber or free-space delay [257, 258] and in those implementations have large delay-bandwidth products but are limited by propagation loss and are best for short delays ($< 1\mu$s), whereas echo-based systems [259] provide a programmable delay using an atomic system. Ideally, quantum networking, and quantum repeaters more specifically, need a quantum memory with capabilities of this sort or better: heralded storage, on-demand re-emission, high ($>$90%) collection efficiency into the fiber, storage times $\gtrapprox$ 1 ms, emission times $\lessapprox$ 100 ns, compatible with spectral and/or temporal multiplexing, and compatible with continuous variable (CV) and/or discrete variable (DV) encodings. Certain quantum repeaters, especially the 3rd-generation one-way versions or all-optical implementations, will benefit from low-loss reconfigurable quantum buffers as well.

Light-matter interactions and interfaces. Photons are excellent carriers of quantum information because they do not readily decohere during transmission, owing to their extremely low interaction with their surrounding environment. However this presents a challenge for interacting multiple photons, as for example, in entanglement generation, resource-state generation, or error correction [260] or for constructing efficient light-matter interfaces between photons and matter qubits. Memory-based quantum repeaters depend on interactions among entangled photons via entanglement swapping based on a Bell-state measurement (BSM) [261]. Using simple linear optics, this BSM is limited to 50% efficiency [262], although it can asymptotically approach unity with significant additional complexity [263] or can be accomplished completely within non-linear atomic systems [264]. Moreover, to herald the memory storage and entanglement generation, quantum non-demolition measurement would be enabling [265, 266]. Again, using linear optics alone, this is limited to a low probability of success [267]. Similarly, proposals for all-optical repeaters leverage detection-induced non-linearity to generate the resource states needed. This non-linearity has been demonstrated to probabilistically generate small cluster states [268, 269] or non-Gaussian states (precursor to GKP states), such as cat states [270–272]. Linear optics is not sufficient to deterministically interact photons [224, 260], and single-photon non-linear optics has a low success probability [273]. Nevertheless, systems with a strong coupling between an atomic system (e.g., trapped atom or quantum dot) and a cavity mode, have been demonstrated to generate deterministic interactions between atoms and photons [239, 243, 245, 274], which can also be used for creating interactions between successive photons [275].

Purification and error correction. Entanglement purification is necessary for 1G quantum repeaters (as mentioned previously in Sec. 6.2) and is possible to realize in theory but has been practically very hard to demonstrate. After several protocols were theorized, the first demonstration using SPDC was achieved [276]. Although there have been numerous proposals for purification protocols [277], there have not been many follow-up demonstrations to date. A notable recent one [278] still uses SPDC but demonstrates nested purification, with very modest improvements. Notably these are achieved using probabilistic methods and are not leveraging non-linear interactions or strong light-matter interactions for deterministic operations

because those too are relatively immature as described above. Demonstrations of quantum error correction are likewise few due to their performance demands on the host system. There have been several demonstrations of quantum error correction, for example, using superconducting circuits [154, 279], ions [157, 280], or spin qubits [281]. Notably, there have not been demonstrations of quantum error correction hosted in the leading quantum memory platforms which demonstrate quantum error correction and quantum memory capabilities in the same platform, needed especially for 2G quantum repeaters. As for quantum error correction on photonic resource states for 3G one-way quantum repeaters (e.g., cluster states or GKP states), there have been several low-performance error-correction demonstrations using cluster states [269, 282] but relatively little for optical GKP states because optical GKP states have not been demonstrated yet. The closest experiment to demonstrate optical GKP qubits was recently done with the interference of two non-squeezed kitten states [283]. However, error correction on GKP states in superconducting circuits has been recently shown to exceed the break-even point [154].

Integration into repeater node. Furthermore, these technologies discussed so far need to be integrated together into functional network nodes to enable quantum repeater functionality that surpasses the rate-loss limit [284]. To date, there have been several demonstrations of repeater subsystems, although of limited functionality, between only a few nodes [285–288]. A full demonstration is yet to be seen of any of the quantum repeater protocols. For a functional quantum repeater, all the components such as the sources, light-matter interactions, and memories/buffers approaching the idealized versions described previously will be needed. They need to be compatible with one another and in sufficient quantity to orchestrate entanglement generation and purification/error correction between distant nodes connected by quantum repeater nodes of a sufficiently high quality and rate to surpass the rate-loss limit.

Quantum routing and switching. As for quantum information routing and switching, there has been limited work in these areas, except for some development of low-loss, all-optical switches [289, 290]. Routing, as it relates to network topology, has begun to be addressed in several demonstrations and proposals [291–294]. This area is less mature because it is a functionality needed for more complex quantum networks than currently demonstrated.

Quantum transduction/conversion. Quantum transduction and state conversion are other important abilities when needing to connect heterogeneous systems that are expected in a larger-scale future quantum network. Specifically, most leading quantum computing platforms are based on systems with microwave or visible wavelength transitions. In this sense, the needed conversion translates to a frequency/energy-scale conversion because quantum networks have operated primarily in the telecommunications bands due to low fiber-based propagation loss. Demonstrations include microwave-optical conversion [295] and quantum frequency conversion between various relatively nearby wavelengths [288, 296, 297]. These demonstrations are limited by noise injected by the conversion process, inefficiency of the conversion process, and/or low transmission through the system; in a notable exception [296] (and similar more recent work [298]), the conversion performs very well but the conversion direction is away from the telecommunication band instead of toward the telecommunication band. An ideal quantum transducer would match the wavelength and bandwidth requirements of both sides with unit efficiency and add no noise to the process.

Architecture and protocol design and testing. Ideally, research on quantum network architecture and protocol stacks should be carried out in real network environments. However, building quantum network testbeds is expensive and time-consuming. To date, only a few quantum network testbeds have been, or are being, built around the world. These testbeds are typically not open to external researchers. In particular, the work on these testbeds is focused on the physical and link layers of quantum networks, layers which are currently realizable to a certain degree. On the other hand, quantum network modeling, simulation, and mathematical analysis offer a powerful and cost-effective alternative for studying and researching quantum network architecture and protocol stacks without requiring physical networks while relying on some reasonable assumptions about future network operations. A few quantum network modeling and simulation tools, such as QuISP [299], NetSquid [300], SeQueNCe [301], and QuNetSim [302] have been developed. Researchers have used these tools for numerical simulations to study quantum network architectures and protocols. These studies include quantum network design [303], protocols [304, 305], routing [306, 307], capacity [308], and benchmarking [309].

Protocols and applications. Multi-partite entanglement distribution is a foundational protocol for quantum networking. This has been widely implemented to date using entangled photon pairs with discrete

variable encoding over various distances using various degrees of freedom to host the entangled qubit(s). Without quantum repeaters, the longest distances to date have been achieved using satellite-based links [310]. On the other hand, a 50-km deployed fiber-based entanglement distribution has recently been shown coexisting with classical communications separated by about 200 nm [311]. CV entanglement, i.e., two-mode squeezing, has been confined to laboratory experiments until recently when it was distributed on a 1-km deployed fiber coexisting with classical communications separated by about 1 nm [312]. Leveraging entanglement distribution, other protocols and applications can be categorized into: (1) Characterization; (2) Physical tests, e.g., of non-locality; (3) Quantum cryptography; (4) Entanglement and state transfer.

Protocol and application categories. To characterize quantum state and channels in quantum networking, quantum state tomography [313–320] and quantum process tomography [321–324] have been implemented using a plethora of optimization techniques to reconstruct physical states given noisy data. Entanglement distribution has also enabled numerous tests of fundamental physics surrounding local realism and non-locality. Notably, several loop-hole-free Bell tests were measured [325–327], while others have probed the limits of quantum theory with high-quality entanglement distribution [328]. Still others have formulated and demonstrated higher dimensional (beyond qubits, including hyperentanglement) Bell tests [329–331] and quantum steering [331–333]. Quantum cryptography is more commonly implemented using attenuated lasers but entanglement distribution can also be leveraged for numerous interesting quantum cryptographic protocols, including quantum key distribution from satellites [334], using high-dimensional entanglement [335], or using hyperentanglement [336]. Also quantum digital signatures [337, 338] and quantum secret sharing [339, 340], among others [341], have been implemented using entanglement distribution. Finally, state-of-the-art demonstrations of entanglement- and state-transfer protocols include quantum teleportation [342, 343], entanglement swapping [286, 344, 345], remote state preparation [346], superdense teleportation [347], among others [348, 349].

6.4 Research directions

Quantum repeater generation overview. Although quantum repeaters were first theorized over 25 years ago, a comparison of Section 6.2 with Section 6.3 reveals that more research and development is needed in realizing functional and practical quantum repeaters and networks that use them. While the quantum repeater generations do reflect advancements in underlying technological capabilities, they do not imply that they will be implemented in a progressive way. Each generation of quantum repeaters in fact may be best suited for a specific type of underlying quantum technology, for a particular scale of quantum network, and for a specific regime of operational parameters such as local gate speed and gate fidelity [211]. As such there are important research directions which can be presently investigated relating to all three generations of quantum repeaters to determine technological viability and further technological development in the process. Moreover, not all near-term efforts should focus solely on 1G repeaters. For example, 1G (and 2G) repeaters need high-efficiency quantum memory, with non-deterministic (deterministic) gates, compatible with an efficient heralded entanglement generation method [350]. 1G repeaters also need demonstrations of efficient implementations of heralded entanglement purification. 2G and 3G repeaters need demonstrations of efficient quantum error correction methods. 3G quantum repeaters need deterministic sources of the required resource states created with high fidelity.

Quantum memory. Depending on the underlying quantum technologies, certain types of 1G and 2G repeaters will require high-performance quantum memories due to the delays for the two-way classical communication protocol steps and the BSM temporal indistinguishability requirement. As remarked earlier, these memories will need performance of this degree or better: heralded storage, on-demand emission, high (>90%) collection efficiency into fiber, storage times $\gtrapprox$ 1 ms, emission times $\lessapprox$ 100 ns, compatible with spectral and/or temporal multiplexing, and compatible with CV and/or DV encodings. Probably one of the largest technological gaps for quantum memories is their collection efficiency into fiber, especially considering that most current memory technologies emit at visible wavelengths thereby needing quantum frequency conversion to be compatible with the telecommunications infrastructure.

Quantum interfaces. Other types of 1G and 2G repeaters such as those based on trapped-ion or Nitrogen Vacancy (NV)-center technologies will not require additional quantum memories because matter qubits in such systems can function not only as quantum computing qubits and light-matter entanglement sources, but also as quantum memory. Each type of quantum repeaters offers its own unique advantage. A

future quantum network will likely consist of different types of quantum repeaters, interfaced with optical photons. Optical connection of different quantum repeaters over long distance is challenging due to the incompatibility of certain matter-qubit wavelengths with low-loss spectral window of fiber-optic and free-space channels as well as the mismatch between the wavelength and bandwidth of different QR technologies. While the traditional tools of quantum photonics offer efficient optical wavelength-conversion techniques over a wide spectral range for alleviating the wavelength mismatch and lossy transmission, there is a vast temporal wavepacket-length mismatch between the commonly studied qubit platforms. Therefore, it is necessary to find solutions to match the length of single-photon wavepackets via coherent conversion processes which optimize the interference visibility in heterogeneous quantum networks.

Deterministic operation. Across all generations of quantum repeaters, whether for state generation, purification, heralding, or error correction, there is a need for strong light-light and light-matter interactions (light-matter interactions are often proposed to mediate strong light-light interaction) that can enable deterministic multi-qubit operations (including purification and error correction) and deterministic state generation (including matter-photon entanglement [245], cluster [351], and GKP states [247, 352]). To realize the interactions needed for quantum repeaters, more development is needed to increase the light-matter coupling strengths and overall system efficiency. In particular, the strong confinement of integrated photonic cavities may help finally enable a scalable method of strong light-matter [353] and light-light [354, 355] interactions. Nonetheless, even probabilistic versions of the capabilities discussed above (purification, error correction, and resource-state generation) offer value for quantum repeaters and protocol refinement. However, limited demonstrations of these have been carried out, although there are many theoretical proposals [224, 248, 250, 277]. Leveraging recent technological advancements, new demonstrations of these capabilities advancing the state-of-the-art would be very valuable to further advance the development of quantum repeaters. Achieving deterministic interactions can then open a wide door to many novel capabilities sought after for quantum repeaters, including efficient state preparation, quantum non-demolition measurement, two-photon interactions, purification, error correction and more. Demonstrations of deterministic interactions should be then applied to demonstrate one (or more) of these novel capabilities needed for quantum repeaters.

Supporting classical hardware. Besides quantum repeater technology, there are supporting needs in systems development. It is often assumed in quantum repeater protocols that there are very low-latency, sufficient-bandwidth classical communication channels available to support the repeater protocol. In reality, the Internet is slower than light's propagation times through fiber and has latency on the scale of milliseconds to seconds even for relatively short links, which is significantly longer than numerous quantum-memory decoherence times. Development and demonstration of low-latency classical control plane communications would be beneficial for facilitating quantum repeater protocols. Moreover, depending on the protocol, some type of synchronization, be it phase and/or clock synchronization, is required for nearly all quantum networking protocols. Demonstrations achieved to date [356, 357] could be improved by lowering the resource overhead needed to achieve the synchronization (either in the form of extra optical fibers or as extra equipment) and the synchronization errors will need to be further reduced for higher repetition rate demonstrations and/or demonstrations over longer delay time scales/distances. To avoid duplication of control plane development that is specific to a particular implementation, there is value in the development of hardware-agnostic control and coordination that leverage specialized quantum drivers, with a standardized interface for compatibility. This could enable the development of a control system that could be shared across the community and increase the portability of equipment to different networks.

Quantum transduction/conversion. To enable applications and connections of the quantum repeater network to other heterogeneous platforms, transduction and/or frequency conversion will be necessary. Within quantum networks, quantum frequency conversion may be needed to bridge quantum memory transition wavelengths and telecommunications band wavelengths. Besides that, applications of quantum networks will require connections to quantum computers of various platforms most of which do not readily emit telecommunication-wavelength photons. In addition, quantum sensors connected to the network are not likely to directly emit into the telecommunications band. Thus, transduction between different qubit technologies and quantum frequency conversion between different wavelengths will be required to make the connections. More work is needed to take us from the current inefficient and noisy transducers to the ideal transducer that matches the wavelength and bandwidth requirements of both sides with unit efficiency and adds no noise in the process. In the short term, focus can be placed on improving the efficiency of quantum

frequency conversion and reducing the noise of quantum transduction, which efforts are critical to those respective thrusts of development. They will likely be mutually beneficial due to similarities between certain implementations of quantum transduction and quantum frequency conversion. Moreover, quantum repeater technology, including frequency conversion and transduction, are usually bulk systems, which, for scalability, will need to move toward more integrated systems likely involving photonic integrated circuits. Research is needed to further develop the components and capabilities of these integrated platforms and the efficiency of their interfaces to fiber [358, 359].

Quantum networking applications. As for the application of distributed quantum computing, research is needed to investigate the optimal distributed quantum computing models for different types of interconnections (e.g., ranging from local inter-processor connectivity to distant connections through quantum repeaters). Moreover, analysis should investigate the thresholds for quality and type of connections where there is a benefit from distributed computation compared to local computation on smaller processors, and investigate benchmarking adaptions for distributed quantum computing. There should be an investigation into a model, of the federated-instruments kind, for quantum processors and quantum sensors connected by a quantum network, to ascertain if a special distributed quantum computing model is necessary and what types of sensing modalities require or benefit from that model.

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A Workshop agenda

Tuesday, July 11, 2023	
Time	**Event**
8:45 - 9:00 am	Welcome and Opening Remarks
9:00 - 10:00 am	Keynote "Physics-Aware Full-Stack Software Optimizations" by Fred Chong (University of Chicago, USA)
10:00 - 10:40 am	Panel Discussion Topic: Quantum computing models, programming, and software stack. Panelists: Robin Blume-Kohout, Costin Iancu, Alex McCaskey, Xiaodi Wu. Moderator: Bert de Jong
10:40 - 11:00 am	Break
11:00 - 12:15 pm	**Breakout Sessions**
	Session A: Novel quantum computing models
	Session B: Programming models and environments
	Session C: Compilation approaches, algorithms, and software tools
	Session D: Benchmarking and verification methodologies
	Session E: Impact and mitigation of errors across the quantum software stack
12:15 - 1:30 pm	Lunch
1:30 - 1:55 pm	Contributed Talk "Challenges and Opportunities in Quantum Networking for Future Distributed Quantum Processing: New Perspective by Quantum Wrapper Networking" Speaker: S. J. Ben Yoo (UC Davis)
2:00 - 3:00 pm	Keynote Speaker: Andrew Childs (University of Maryland, College Park, USA)
3:00 - 3:20 pm	Break
3:20 - 4:00 pm	Panel Discussion Topic: Quantum algorithms and applications Panelists: Andrew Childs (UMD), Antonio Mezzacapo (IBM), Will Zeng (Unitary Fund) Moderator: Ojas Parekh
4:00 - 5:00 pm	**Breakout Sessions**
	Session A: Quantum algorithms and advantages
	Session B: Hybrid quantum-classical algorithms and quantum-inspired classical algorithm
	Session C: Classical and hybrid simulation of quantum computing
	Session D: Scientific applications informed by quantum capabilities
	Session E: Application benchmarks and performance modeling and estimation
5:00 - 5:30 pm	Organizing Committee Meeting

Wednesday, July 12, 2023	
Time	**Event**
8:00 - 9:00 am	Registration and Breakfast
9:00 - 10:00 am	Keynote "Towards near-term quantum networks" Speaker: David Elkouss (Okinawa Institute of Science and Technology, Japan)
10:00 - 10:40 am	Panel Discussion Topic: Quantum network design, algorithms, and applications Panelists: Paul Kwiat (UIUC), David Levonian (AWS), Nicholas Peters (ORNL) Moderator: Pavel Lougovski
10:40 - 11:00 am	Break
11:00 - 12:15 pm	**Breakout Sessions**
	Session A Topic: Quantum network models and architectures
	Session B Topic: Applications of entanglement distribution and quantum networks
	Session C Topic: Enabling technologies for quantum networking
	Session D Topic: Distributed quantum computing
12:15 - 1:30 pm	Lunch
1:30 - 1:55 pm	Contributed Talk "Quantum Co-design and Integration for Science Applications" Speaker: Sue Mniszewski (LANL)
1:55 - 2:20 pm	Contributed Talk "Advanced noise handling capabilities: from error mitigation to error correction" Speaker: Sarah Sheldon (IBM)
2:20 - 2:40 pm	Break
2:40 - 4:00 pm	All-hands Discussion: Viewpoints from academia, industry, and the labs Moderator: Travis Humble (ORNL)
4:00 - 4:30 pm	Organizing Committee Meeting

Thursday, July 13, 2023	
Time	**Event**
8:00 - 9:00 am	Registration and Breakfast
9:00 - 10:00 am	**Breakout Sessions**
	Session A: Topic: Codesign benefits, challenges, and opportunities
	Session B: Topic: Hardware-informed resource estimation for quantum algorithms
	Session C: Topic: Impact of noise on algorithms and applications
10:00 - 11:00 am	All-hands Closing Discussion: Research priorities in quantum computing and networking
11:00 - 11:30 am	Break
11:30 - 1:00 pm	Organizing Committee Meeting: Working lunch for report planning

B Workshop participants

Name	Institution
Yuri Alexeev	Argonne National Laboratory
Tushar Athawale	Oak Ridge National Laboratory
Andrew Baczewski	Sandia National Laboratories
Kristin Beck	Lawrence Livermore National Laboratory
Ryan Bennink	Oak Ridge National Laboratory
Robin Blume-Kohout	Sandia National Laboratories
Michael Brodsky	DEVCOM U.S. Army Research Laboratory
Joseph Broz	IBM
Spencer Bryngelson	Georgia Institute of Technology
Mark Byrd	Southern Illinois University
Daan Camps	Lawrence Berkeley National Laboratory
Gabriella Carini	Brookhaven National Laboratory
Matthew Cha	General Atomics
Joseph Chapman	Oak Ridge National Laboratory
Lali Chatterjee	U.S. Department of Energy
Andrew Childs	University of Maryland
Joaquin Chung Miranda	Argonne National Laboratory
Daniel Claudino	Oak Ridge National Laboratory
Connor Clayton	University of Maryland
La Vida Cooper	DEVCOM U.S. Army Research Laboratory
Antonio Corcoles	IBM
Claire Cramer	U.S. Department of Energy
Tanner Crowder	White House Office of Science and Technology Policy
Wibe de Jong	Lawrence Berkeley National Laboratory
Andrea Delgado	Oak Ridge National Laboratory
Yufei Ding	University of California, Santa Barbara
Shengwang Du	University of Texas at Dallas
Yuhua Duan	National Energy Technology Laboratory
Eugene Dumitrescu	Oak Ridge National Laboratory
Avik Dutt	University of Maryland
Stephan Eidenbenz	Los Alamos National Laboratory
Dirk Englund	Massachusetts Institute of Technology
Eden Figueroa	Stony Brook University
Hal Finkel	U.S. Department of Energy
Marco Fornari	U.S. Department of Energy
James Freericks	Georgetown University
April Gillens	U.S. Department of Energy
Pranav Gokhale	Infleqtion
Alexey Gorshkov	National Institute of Standards and Technology/University of Maryland
Peter Graf	National Renewable Energy Laboratory
Stefanie Guenther	Lawrence Livermore National Laboratory
Salman Habib	Argonne National Laboratory
Tobias Hagge	Pacific Northwest National Laboratory
Kathleen Hamilton	Oak Ridge National Laboratory
Akihiro Hayashi	Georgia Institute of Technology
Aaron Holder	U.S. Department of Energy
Daniel Huang	San Francisco State University
Travis Humble	Oak Ridge National Laboratory
Costin Iancu	Lawrence Berkeley National Laboratory
Joshua Izaac	Xanadu

Name	Institution
Dieter Jaksch	University of Hamburg
David Jennings	PsiQuantum
Shantenu Jha	Brookhaven National Laboratory
Weiwen Jiang	George Mason University
Nicholas Johnson	Riverlane
Kyungseon Joo	University of Connecticut
Pejman Jouzdani	General Atomics
Dimitrios Katramatos	Brookhaven National Laboratory
Rajkumar Kettimuthu	Argonne National Laboratory
Stefan Krastanov	University of Massachusetts Amherst
Resham Kulkarni	U.S. Department of Energy
Prem Kumar	Northwestern University
Paul Kwiat	University of Illinois at Urbana Champaign
Ying-Cheng Lai	Arizona State University
Jeffrey Larson	Argonne National Laboratory
Tyler LeBlond	Oak Ridge National Laboratory
Margaret Lentz	U.S. Department of Energy
David Levonian	Amazon
Ang Li	Pacific Northwest National Laboratory
Benjamin Lienhard	Princeton University
Meifeng Lin	Brookhaven National Laboratory
Ji Liu	Argonne National Laboratory
Phillip Lotshaw	Oak Ridge National Laboratory
Pavel Lougovski	Amazon Web Services
Michael Martin	Los Alamos National Laboratory
Alexander McCaskey	NVIDIA Corporation
Kristina Meier	Los Alamos National Laboratory
Claudia Mewes	U.S. Department of Energy
Antonio Mezzacapo	IBM
Susan Mniszewski	Los Alamos National Laboratory
Johannes Muelmenstaedt	Pacific Northwest National Laboratory
Harsha Nagarajan	Los Alamos National Laboratory
Ivan Novikau	Princeton Plasma Physics Laboratory
Kasra Nowrouzi	Lawrence Berkeley National Laboratory
Santiago Núñez-Corrales	University of Illinois, Urbana-Champaign
mehmet berkay on	University of California, Davis
Ojas Parekh	Sandia National Laboratories
Abid Patwa	U.S. Department of Energy
Kalyan Perumalla	U.S. Department of Energy
Nicholas Peters	Oak Ridge National Laboratory
Timothy Proctor	Sandia National Laboratories
Chen Qian	University of California, Santa Cruz
Chunming Qiao	University at Buffalo - SUNY
Gregory Quiroz	Johns Hopkins University
David Rabson	U.S. Department of Energy
Steve Reinhardt	Quantum Machines
Samah Saeed	City College of New York, City University of New York
Erhan Saglamyurek	Lawrence Berkeley National Laboratory
Paul Sammak	U.S. Department of Energy
Mohan Sarovar	Sandia National Laboratories
Ruslan Shaydulin	JPMorgan Chase & Co
Sarah Sheldon	IBM
Andrew Sornborger	Los Alamos National Laboratory

Name	Institution
William Spotz	U.S. Department of Energy
Omer Subasi	Pacific Northwest National Laboratory
Ceren Susut	U.S. Department of Energy
Jennifer Tamsiran-Zurakowski	Oak Ridge Institute for Science and Education
Deneise Terry	Oak Ridge Institute for Science and Education
Himanshu Thapliyal	University of Tennessee, Knoxville
Devesh Tiwari	Northwestern University
Venkata Ramana Raju Valivarthi	Caltech
Roel Van Beeumen	Lawrence Berkeley National Laboratory
Marc Vuffray	Los Alamos National Laboratory
Yan Wang	Oak Ridge National Laboratory
Fei Wang	George Mason University
Bruce Warford	Oak Ridge Institute for Science and Education
Tom Wong	U.S. Department of Energy
Clement Wong	Science Applications International Corporation
Wenji Wu	Lawrence Berkeley National Laboratory
Xiaodi Wu	University of Maryland, College Park
Yufeng Xin	University of North Carolina at Chapel Hill
Bin Yan	Los Alamos National Laboratory
Kübra Yeter-Aydeniz	The MITRE Corporation
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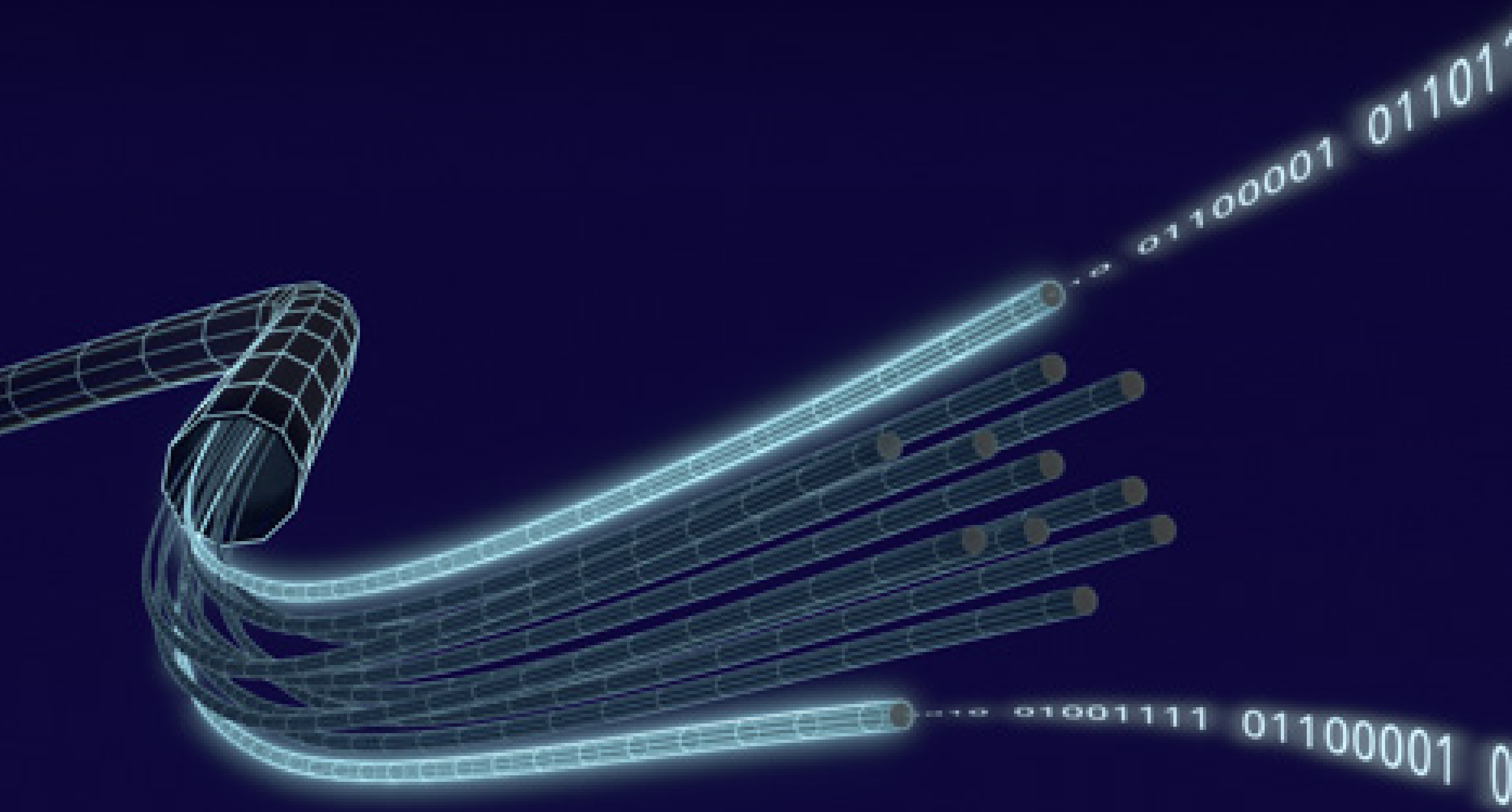
Meng Yue	Brookhaven National Laboratory
William Zeng	Unitary Fund
Liang Zhang	Lawrence Berkeley National Laboratory
Junyao Zhang	Duke University



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