Proceedings of the ASME 2024
Pressure Vessels & Piping Conference
PVP2024
July 29-August 2, 2024, Bellevue, Washington, USA

PVP2024-123471

EXPERIMENTAL VALIDATION OF THEORETICAL BURST STRENGTH SOLUTION FOR DEFECT-FREE THICK-WALLED PIPES

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ABSTRACT

The burst pressure of line pipes is an important strength property required in pipeline design and integrity management. Historically, the Barlow formula in conjunction with the ultimate tensile stress (UTS) of pipeline steels were utilized to estimate the burst strength of line pipes. However, the Barlow formula did not consider the plastic flow effect for ductile steels and is applicable only to thin-walled pipes. In 2006, the present author proposed a new multiaxial plastic yield theory and obtained a theoretical Zhu-Leis solution of burst strength for defect-free thin-walled pipes in term of UTS and strain hardening exponent n of pipeline steels. The Zhu-Leis solution has been validated by various burst test data for thin-walled pipelines for a wide range of steel grades from Grade B to X120. Recently, the present author extended the Zhu-Leis theory of plasticity to thick-walled pipes and obtained the Zhu-Leis solution of burst pressure for thick-walled pipes. The proposed burst pressure solution is applicable to both thin and thick-walled pipes.

To experimentally validate the proposed theoretical burst pressure solution, this paper obtains a set of burst test data for three thick-walled pipes in Grade B carbon steel with a nominal diameter of 2.375 inches and three nominal wall thicknesses, resulting in D/t = 15.4, 10.9, 6.9. Through comparisons, these burst data validate the theoretical burst pressure solution for thick-walled pipes. Moreover, two additional burst test datasets collected from literature for thin and thick-walled pipes further validate the proposed burst pressure solution for both thin and thick-walled pipes.

KEYWORDS: Burst strength, strength theory, pressure vessel, pipeline, Tresca, von Mises, Zhu-Leis criterion

1. INTRODUCTION

Pressure vessels and pipelines are the important national infrastructure widely utilized in storage or transportation of large volumes of fluids in the energy, petrochemical, and oil and gas industry. Facilities such as nuclear reactors, chemical reactors, plant piping, storage tanks, and transmission pipelines are dependent on these systems for safe and efficient operation. Thus, structural design, manufacture, construction, operation, and integrity management of pressure vessels and pipelines are essential to the national infrastructure.

To ensure the structural integrity of pressure vessels or pipelines, burst strength characterized by burst pressure is required in structural design and integrity management. In general, burst pressure was estimated using the classical strength theories, experimental tests, numerical simulations, empirical formulae, or industrial design codes. Christopher et al. [1] presented a good review on the burst pressure prediction models for thick-walled pressure vessels. Law and Bowie [2] discussed the burst pressure prediction for high-strength thin-walled line pipes. Zhu and Leis [3] evaluated a series of burst pressure prediction models for various pipeline steels in terms of strength theories and flow theories of plasticity. Zhu [4] performed a comparative study on the traditional strength criteria versus the modern plastic flow criteria used in the design and analysis of pressure vessels.

The literature reviews determined that most of the early models were developed from either a simple analysis or an empirical curve fit of experimental data for specific steels of interest, and thus no single model provided a robust, accurate prediction of burst pressure for all ductile steels. Some models focused on a conservative lower bound prediction, while other models reflected an upper bound prediction. Many early models were obtained using the hoop stress and one material strength property, such as yield stress (YS) or ultimate tensile stress (UTS). The effect of plastic flow or strain hardening was not considered. Steward and Klever [5] first showed a strong effect of plastic flow response on burst pressure test data of thin-walled pressure vessels for ductile steels within two bounds. The upper bound was defined by the von Mises solution of burst pressure [6, 7], while the lower bound was defined by the Tresca solution of burst pressure [5]. The averaged result of these two bound predictions provided a good fit to averaged test data.

This work was produced by Battelle Savannah River Alliance, LLC under Contract No. 89303321CEM000080 with the U.S. Department of Energy. Publisher acknowledges the U.S. Government license to provide public access under the DOE Public Access Plan (http://energy.gov/downloads/doe-public-access-plan).

To predict more accurate burst pressure, Zhu and Leis [8, 9] developed a new multiaxial plastic yield theory that was referred to as average shear stress yield criterion, or simply as the Zhu-Leis criterion in literature. For carbon steels, the Zhu-Leis flow solution of burst pressure agrees well with burst test data on average [3, 9]. Many other researchers [10-13] also validated the Zhu-Leis flow solution using different full-scale burst test data for a wide range of pipeline steels, including steel grades from Grade B to X120. All validations demonstrated that the Zhu-Leis solution is the best burst pressure prediction for thin-walled line pipes. However, this Zhu-Leis solution is not applicable to thick-walled pipes.

In the 1950s, Svensson [7] performed a detailed theoretical analysis for thick-walled cylinders based on the von Mises flow theory for determining the burst pressure of the thick-walled cylinders. However, this author failed to obtain an exact theoretical solution. Instead, an approximate closed-from burst pressure solution was provided. As such, a more accurate burst pressure solution for thick-walled pipes has been desired for more than one half of century in the pressure vessel industry. Recently, Zhu et al. [14-15] developed a modified strength theory and obtained a burst pressure solution for thick-walled pipes as a function of D_o/D_i, UTS and n. Moreover, Zhu [16-17] obtained an exact theoretical solution of burst pressure for thick-walled pipes using the flow theory of plasticity. Comparisons showed that this exact theoretical solution obtained in Ref [16-17] is identical to the burst pressure solution obtained in Ref [14-15] from the modified strength theory.

In order to experimentally validate the proposed theoretical burst pressure solution for thick-walled pipes, this paper reports a set of burst pressure tests that were recently performed at the Savannah River National Laboratory (SRNL) for three thick-walled pipes in Grade B carbon steel with a nominal diameter of 2.375 inches and three nominal wall thicknesses, resulting in D/t = 15.4, 10.9, 6.9. The burst pressure test data were compared to the proposed theoretical solution of burst pressure and validated this theoretical solution for thick-walled pipes. In addition, two additional datasets of existing burst tests for thin and thick-walled pipes were collected from literature. These burst data further validated that the proposed burst pressure solution is more accurate for both thin and thick-walled pipes.

2. BURST PRESSURE PREDICTION MODELS FOR THIN AND THICK-WALLED PIPES

This section presents the representative burst pressure prediction models for both thin and thick-walled pipes or pressure vessels in the end-capped conditions.

2.1. Burst pressure models for thin-walled pipes

In the pipeline industry, most of line pipes are thin-walled cylindrical vessels with a large diameter to wall thickness ratio (i.e., D/t ≥ 20). The hoop stress of thin-walled pipes under internal pressure can be simply determined using the Barlow formula [15] that provides the basis for the regulation rules or design codes for pressure vessels and pipelines:

$$\sigma_h = \frac{PD}{2t} \tag{1}$$

where σ_h is the hoop stress of the pipe, *P* is the internal pressure, and *D* is the pipe diameter. In practice, outside diameter (OD), D_o, is frequently used as *D*. Sometimes the inside diameter (ID), D_i, or mean diameter (MD), $D_m = D_o - t$, is also used. The wall thickness is given as $t = (D_o - D_i)/2$. For pipeline steels, the Barlow formula determines the Barlow strength for thin-walled pipes in terms of the UTS:

$$P_b = \frac{2t}{D}\sigma_{uts} \tag{2}$$

where P_b is a burst pressure, and σ_{uts} is the UTS of pipe steel that is defined at its engineering value. In practice, OD is often used in Eq. (2) for a conservative estimation.

To improve the accuracy of the Barlow strength for thin-walled pipes, Zhu and Leis [8-9] considered the effect of the plastic flow response on burst pressure in terms of the flow theory of plasticity within the large deformation framework. In the thin-walled shell theory, stresses and strains are assumed constant through the wall thickness in a pipe, and thus the MD is more appropriate to use. For a power-law strain hardening steel, Zhu and Leis [8-9] obtained the following general flow solution of burst pressure for thin-walled pipes with regard to the Tresca, von Mises and Zhu-Leis yield criteria, respectively, as:

$$P_b = \left(\frac{C}{2}\right)^{n+1}\frac{4t}{D-t}\sigma_{uts} \tag{3}$$

where D is OD, D-t is MD, and *C* is a constant that depends on the yield criterion:

$$C = \begin{cases} 1, & \text{for Tresca yield criterion} \\ \frac{2}{\sqrt{3}}, & \text{for von Mises yield criterion} \\ \frac{1}{2}+\frac{1}{\sqrt{3}}, & \text{for Zhu-Leis yield criterion} \end{cases} \tag{4}$$

In which *n* is a strain hardening exponent of material that is measured from a simple tensile test or estimated from the YS and UTS [18]. For pipeline carbon steels, n typically ranges from 0.02 to 0.25. For a perfectly plastic material, n = 0. In this case, the burst pressure solutions in Eq. (3) reduce to those obtained using the traditional strength theories, i.e., the Tresca strength solution, von Mises strength solution, and Zhu-Leis strength solution, see Zhu [4] for details.

2.2. Burst pressure models for thick-walled Pipes

Historically, many valuable burst pressure tests were carried out for small diameter, thick-walled tubes (i.e., D/t < 20) in various structural steels by different investigators, such as Faupel [19, 20] in the 1950s. Based on burst test data, different simple and empirical burst pressure models were developed for thick-walled tubes with end-closed caps. Among them, the following three burst pressure models are often used for thick-walled pipes or pressure vessels in practice [17].

- Tresca strength solution:

$$P_b = \sigma_{uts} \ln\left(\frac{D_o}{D_i}\right) \tag{5}$$

- von Mises strength solution:

$$P_b = \frac{2}{\sqrt{3}} \sigma_{uts} \ln\left(\frac{D_o}{D_i}\right) \tag{6}$$

- Svensson approximate solution:

$$P_b = \left(\frac{0.25}{n+0.227}\right)\left(\frac{e}{n}\right)^n \sigma_{uts} \ln\left(\frac{D_o}{D_i}\right) \tag{7}$$

This approximate burst pressure solution was obtained by Svensson [7] using the von Mises flow theory of plasticity that considered the plastic flow response of steels.

Recently, Zhu et al. [15] proposed a modified strength theory and then obtained the following burst pressure solution for thick-walled pipes in terms of the Tresca, von Mises and Zhu-Leis yield criteria as:

$$P_b = 2\left(\frac{C}{2}\right)^{n+1} \sigma_{uts}\, ln\left(\frac{D_o}{D_i}\right) \tag{8}$$

where C is a yield criterion dependent constant, see definition in Eq. (3). This burst pressure solution is the same as the exact theoretical solution of burst pressure obtained by Zhu [16-17] for a strain hardening steel using the flow theory of plasticity.

2.3. Comparison of thin and thick-walled solutions

Comparing Eq. (8) and Eq. (3) shows that the material terms of the burst pressure solutions are the same for thin and thick-walled pipes, but their geometry terms are different. For thin-walled pipes, the geometry term is $2t/D_m$ ($2t/D_o$ or $2t/D_i$). For thick-walled pipes, the geometry term is $ln(D_o/D_i)$. Accordingly, the difference of the burst pressure solutions for thin and thick-walled pipes is determined by the geometry terms.

Figure 1 compares four geometry terms of $ln(D_o/D_i)$, $2t/D_m$, $2t/D_o$, and $2t/D_i$ for a wide range of the diameter ratios (D_o / D_i). In the pipeline industry, thin-walled pipes are defined as $D_o/t \geq 20$. At D/t = 20, the thin-walled geometry terms $2t/D_m$, $2t/D_o$, and $2t/D_i$ have an error of -0.1%, -5.1% and +5.5%, respectively compared to the thick-walled geometry term, i.e., the logarithmic function. Figure 1 shows that 2t / D_i overestimates the logarithmic function, 2t / D_o underestimates the logarithmic function, and 2t / D_m is close to the logarithmic function over the wide range of D_o / D_i ratios. At D/t = 7 for a thick-walled pipe, 2t / D_m has an error less than -1% compared to the logarithmic function. As a result, 2t / D_m is more accurate to use for thin-walled pipes. In the pipeline industry, the commonly used OD-based Barlow strength is conservative.

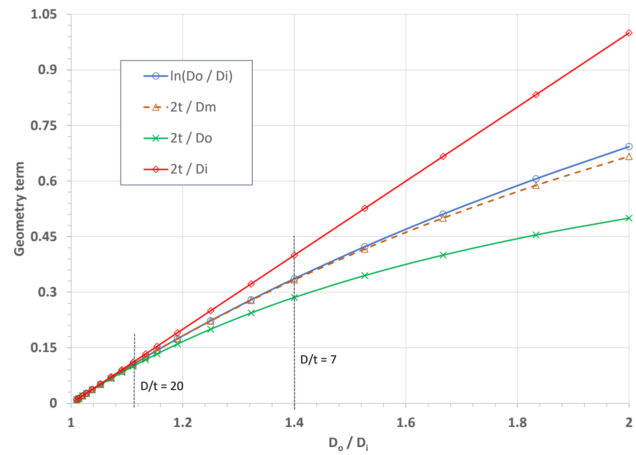

Figure 1. Comparison of four geometry terms of $ln(D_o/D_i)$, 2t / D_m, 2t / D_o, and 2t / D_i.

3. EXPERIMENTAL VALIDATION

3.1. Carbon steel pipes

In 2022, the U.S. Department of Energy (DOE) sponsored a R&D program at SRNL for developing an advanced plasticity theory and its experimental validation [21]. After the theoretical solutions of burst pressure were obtained for thick-walled pipes [14-17], a set of validation burst pressure tests were conducted.

Three 2-inch ASTM 106B (i.e., API 5L Grade B) black seamless carbon steel pipes were purchased from the Eastern Industrial Supplies, Inc in Augusta, Georgia. Each carbon steel pipe, as shown in Fig. 2, has a nominal diameter (OD) of 2.375 inches and a length of 21 feet. These steel pipes have three nominal wall thicknesses of 0.154, 0.218, and 0.344 inches that corresponds to SCH-40, SCH-80, and SCH-160 Grade B steels. This leads to three diameter-to-thickness ratios of D/t = 15.4, 10.9, and 6.9. Note that pipe schedule (SCH) is a standard measure of nominal wall thickness of a pipe. Tensile test and burst pipe specimens were cut and machined from these pipes.

Figure 2. API 5L Grade B black carbon steel pipe.

3.2. Tensile tests

Six tensile specimens were cut from the Grade B carbon steel pipes in the axial direction. Two specimens were machined for each schedule of the three pipes. Figure 3 shows these six tensile sheet specimens with full thicknesses of the steel pipes. All sheet specimen dimensions met the tensile test requirements described in ASTM E8 [22]. The strain gauge length is 2 inches (50.8 mm), and the specimen width in the strain gauge area ranges from 7.5 mm to 8.8 mm. With the guideline of ASTM E8, all six uniaxial tensile specimens were tested at room temperature in the quasi-static loading conditions.

Figure 3. Tensile test specimens.

Figure 4 shows the tensile specimen fixture, strain gage, and broken specimen in the tensile test.

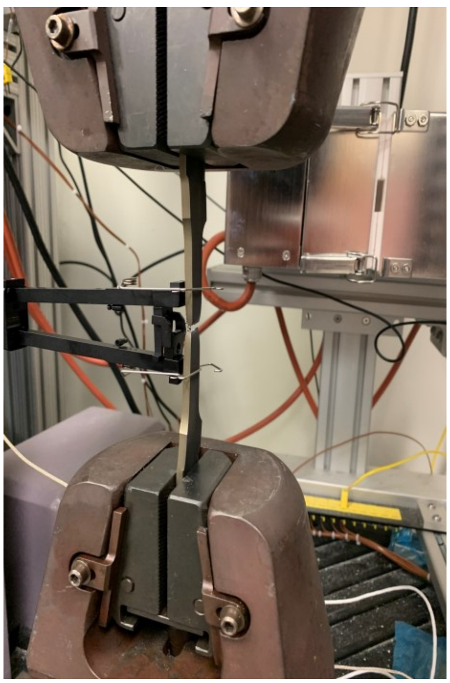

Figure 4. Tensile specimen fixture.

Figure 5 shows the representative engineering stress-strain curves measured for the three Grade B carbon steel pipes with wall thicknesses of 0.154, 0.218, and 0.344 inches.

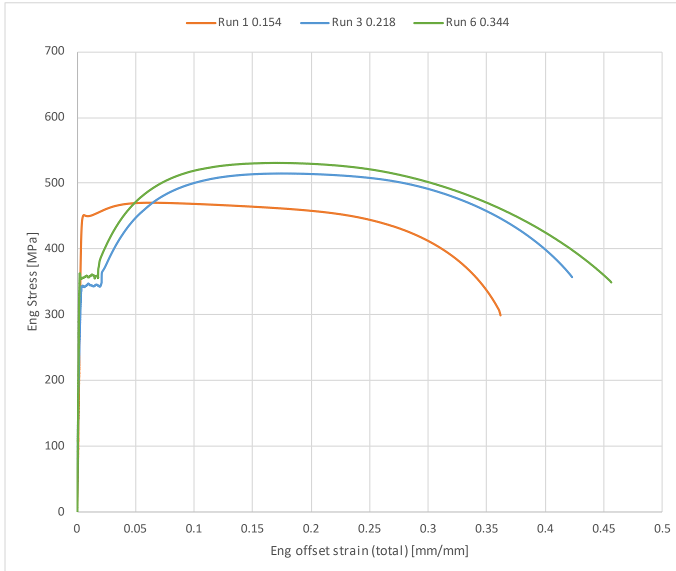

Figure 5. Engineering stress-strain curves of the three Grade B carbon steel pipes.

As shown in Fig. 5, the thin-walled (0.154 inch) steel pipe has significantly different stress-strain curve from those for the two thick-walled (0.218 and 0.344 inch) steel pipes. The two thick-walled steel pipes have the similar stress-strain curves. The reason for the difference is unknown. From Fig. 5, the yield stress defined at 0.5% total strain is determined as 450, 342, and 356 MPa (65.3, 49.6, and 51.6 ksi), respectively for SCH-40, SCH-80, and SCH-160 steel pipes. The UTS is determined as 470, 515, and 531 MPa (68.2, 74.7, and 77.0 ksi), respectively for the three steel pipes. The strain hardening exponent n is estimated as 0.062, 0.161, and 0.156, respectively for the three steel pipes. The elongation was measured as 37%, 43%, and 46%, respectively for the three steel pipes. These material properties are listed in Table 1.

Note that the material properties reported in Table 1 were measured in the axial direction of the steel pipes. The material properties can be different somehow in the circumferential direction of these steel pipes. Because of small diameters, standard tensile specimens are difficult to machine in the circumferential direction, and thus the mechanical properties in the circumferential direction were not measured.

Table 1. Mechanical properties of three carbon steels

SCH	YS (MPa)	YS (ksi)	UTS (MPa)	UTS (ksi)	n	Elon
Sch-40	450	65.3	470	68.2	0.062	37%
Sch-80	342	49.6	515	74.7	0.161	43%
Sch-160	356	51.6	531	77.0	0.156	46%

3.3. Pressure burst tests

A set of pressure burst tests were recently completed at SRNL to determine the pressure carry capacity of API 5L Grade B black seamless carbon steel pipes [23]. Six pipe specimens, two specimens for each pipe schedule, were instrumented and tested. Preparations of pipe specimens, instrumentation, test results and discussions were summarized in Report [23].

Figure 6 shows an example of pressure burst test setup for SCH-40 Pipe 1. The other burst test pipes have the similar experimental setup. All pipe specimens have a length of 10OD. Two 1-inch thick 3.5-inch circular plates were cut and welded to the ends of each pipe to ensure that burst failure will occur in the pipe body rather than the welded end caps. High pressure fittings were also fitted to the middle of each end plate.

Figure 6. Pressure burst test setup for SCH-40 Pipe 1.

The High Pressure Lab at SRNL adopted an "air-over-water" pressurization process for a pipe pressure burst test. Prior to pressurization, the pipe was filled with water. This was done by connecting one of the specimens' high-pressure fittings to a water line, and water fills until water was observed out of the second high pressure fitting. Since water is incompressible, it is easier to compress air over water to achieve high pressure. This pressurization system has the capacity to pressurize a pipe up to a high pressure of 30,000 psi.

Three pressure testing procedures were utilized throughout the pipe burst test task. The first testing procedure is a monotonic loading that increases pressure continuously to burst within three minutes. The first burst testing for SCH-40 Pipe 1 followed this monotonical loading procedure. Figure 7 shows the pressure-time records for the SCH-40 Pipe 1 test.

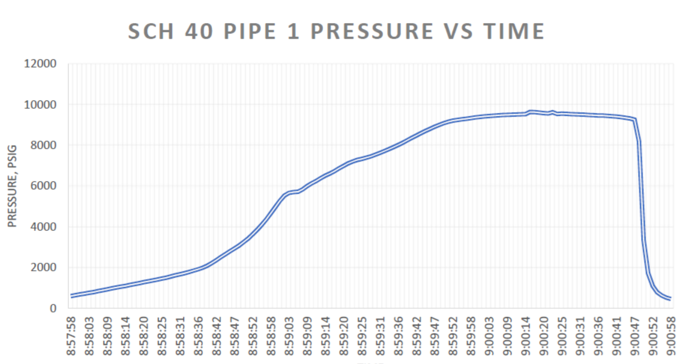

Figure 7. Pressure-time records for SCH-40 Pipe 1 test.

The second testing procedure is a multiple step loading approach: 1) increase pressure to 1/3 estimated burst pressure, 2) increase pressure by 2,000 psi every ½ hour and hold for equilibration in the elastic stage, 3) increase by 1,000 psi every ½ hour and hold in the plastic stage, and 4) test ends when pipe bursts. The second pressure burst testing for SCH-80 Pipe 2 followed this step loading procedure. The results showed that both the elastic and plastic pressure increments are too large. Thus, the second testing procedure was modified to the third one, where the elastic increase of pressure is reduced to 1,000 psi, the plastic increase of pressure is reduced to 500 psi, and the hold time is also reduced by monitoring an equilibrated strain value. Except for SCH-160 Pipe 2 (its burst test datum was not obtained because one high pressure fitting weld leaks), three remaining pipe burst tests (SCH-40 Pipe 2, SCH-80 Pipe 1, and SCH-160 Pipe 2) followed this third testing procedure. Figure 7 shows the pressure-time records for SCH-40 Pipe 2 test.

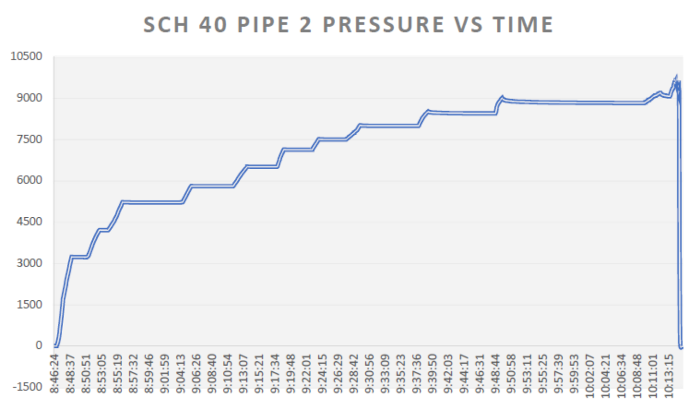

Figure 8. Pressure-time records for SCH-40 Pipe 2 test.

The failure appearance of five burst test pipes is given in Figures A1 to A5 in Appendix A, respectively for SCH-40 Pipe 1, SCH-40 Pipe 2, SCH-80 Pipe 1, SCH-80 Pipe 2, and SCH-160 Pipe 2. The test results are simply analyzed as follows:

- For SCH-40 Pipe 1, bursting occurred in the pipe body close the middle of the pipe, and burst pressure was achieved at 9,646.4 psig. Pipe failure location was ideal for this test.
- For SCH-40 Pipe 2, likewise, bursting occurred in the pipe body near the middle of the pipe, and burst pressure was measured as 9,634.94 psig. Pipe failure was ideal for this test.
- For SCH-80 Pipe 1, bursting occurred in the pipe body close to one end of the pipe, and burst pressure was measured as 14,369.81 psig. Pipe failure was ideal for this test.
- For SCH-80 Pipe 2, the test result was questionable. Pipe failure occurred in the seam weld. After the seam weld ruptured, the crack propagated into the end weld. The failure pressure was achieved at 14,110 psig. It may be less than that for the pipe body failure.

- For SCH-160 Pipe 1, failure around the high pressure fitting weld was present, and pressurization was not possible due to leaking. Thus, no burst data was present.
- For SCH-160 Pipe 2, failure occurred at the end cap weld. It demonstrated that the end cap welds used in this work may be inappropriate for thick-walled pipes, and stronger end cap welds are needed. The weld failure occurred at 24,886.65 psig.

With the measured material properties given in Table for the carbon steel pipes, the Zhu-Leis solution of burst pressure is predicted from Eq. (8) for each burst test pipe. Table 2 lists the measured and predicted burst pressures of all burst test pipes. The relative errors of the Zhu-Leis solution for thick-walled pipes compared to the measured burst data are also given in this table. The first three burst test pipes have body burst failure and the Zhu-Leis solution has a small error less than 3% of the measured burst data. The other two burst test pipes have seam weld or end cap weld failure, but the Zhu-Leis solution still has a relatively small error less than 5%.

Table 2. Measured and predicted burst pressures of all burst test pipes

Pipe #	Failure type	Measured burst P_b (psi)	Zhu-Leis bust P_b (psi)	Relative error (%)
SCH-40, Pipe 1	Body burst	9,646.4	9,816	1.76
SCH-40, Pipe 2	Body burst	9,634.95	9,816	1.88
SCH-80, Pipe 1	Body burst	14,369.81	14,774	2.81
SCH-80, Pipe 2	Seam failure	14,110.03	14,774	4.70
SCH-160, pipe 1	fitting leaks	N/A	25,763	N/A
SCH-160, Pipe 2	Weld failure	24,886.65	25,763	3.52

Figure 9 further compares all measured burst data with the burst pressure predictions from Eq. (8) for the Tresca, von Mises and Zhu-Leis yield criteria, respectively. As shown in this figure, the von Mises flow solution provides an upper bound prediction within +10% of the measured burst pressure, the Tresca flow solution provides a lower bound prediction within a -10% of the measured burst pressure, and the Zhu-Leis flow solution provides a more accurate prediction of burst pressure for all five burst test pipes compared to the measured burst pressure data on average.

In summary, the comparisons in Table 1 and Figure 9 demonstrated that the von Mises solution is an upper bound prediction, the Tresca solution is a lower bound prediction, and the Zhu-Leis solution of burst pressure is a more accurate prediction for thick-walled pipes.

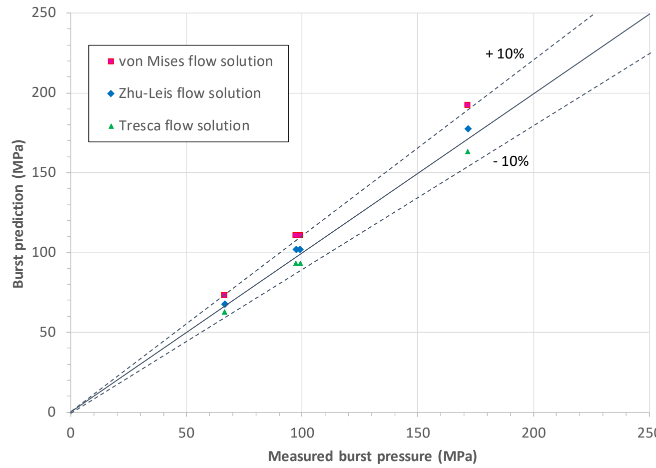

Figure 9. comparison of measured and predicted burst pressures.

4. ADDITIONAL EXPERIMENTAL VALIDATIONS

This section further evaluates four representatives of burst pressure models in Eqs. (5) to (8) using two additional datasets of burst pressure tests for thick-walled tubes or pipes. One dataset contains burst test data for small diameter, thick-walled tubes, and the other dataset contains burst test data for large diameter, thin and thick-walled cylinders.

4.1. Validation for thick-walled tubes

Faupel [19] conducted about one hundred burst pressure tests over seven years and published the burst test data for small diameter, thick-walled tubes in various metals, including plain carbon steels, stainless steels, low alloy steels, weld steels, and other metals. The pressure tubes were very thick, leading to a small D/t ratio in the range of $2.4 < D/t < 4.7$. From these burst tests, thirty burst test data for plain carbon steels designated as AISI 1025, AISI 1030, AISI 3130, AISI 3320, AISI 4130, AISI 4140, and AISI 4340 were selected and used in this work for evaluating the burst prediction models for thick-walled tubes. These materials include low, medium, and high carbon steels with the yield strength in the range of 244 to 1076 MPa and the tensile strength in the range of 459 to 1119 MPa. The strain hardening exponent n of the tube steels is less than 0.25. Note that the burst test data obtained by Faupel [19] were also reported by Christopher et al. [1] in their Table 5.

Figure 10 compares different burst pressure predictions with measured burst pressure data obtained by Faupel [19] for small diameter, thick-walled pressure tubes, where the y-axis denotes the burst pressure normalized by the Tresca strength solution, $P_b = \sigma_{uts} \ln\left(\frac{D_o}{D_i}\right)$, and the x-axis denotes the material strain hardening exponent n. The burst pressure predictions were obtained from the representative models for thick-walled pipes: the Tresca strength solution in Eq. (5), the von Mises strength solution in Eq. (6), the Svensson approximate solution in Eq. (7), and three proposed exact flow solutions in Eq. (8).

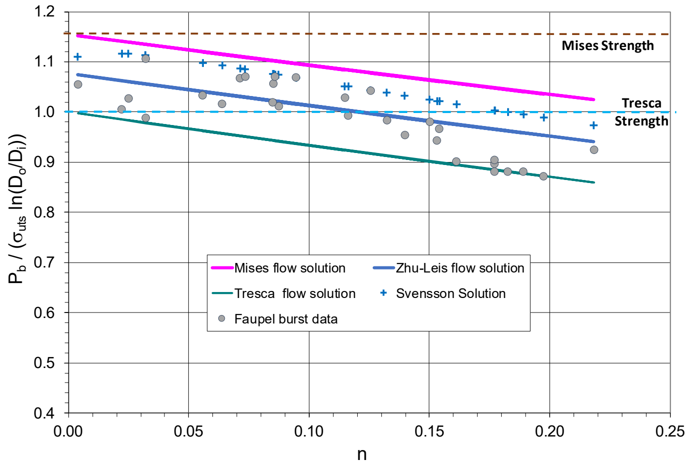

Figure 10. Comparison of different burst pressure predictions with burst test data for thick-walled tubes as a function of n.

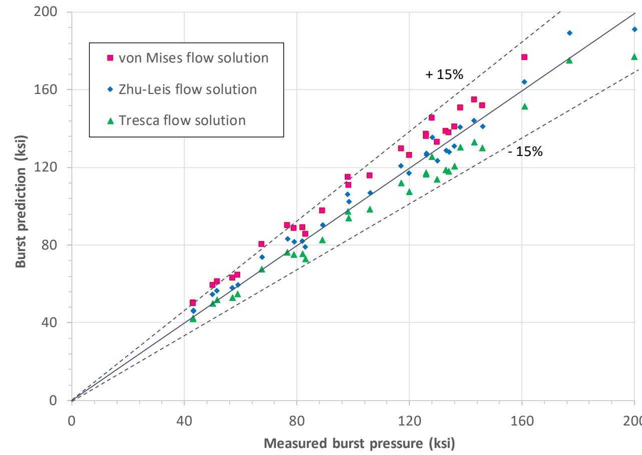

Figure 11. Comparison of the proposed burst pressure predictions with measured burst test data [19].

From Fig. 10, the following observations are determined:

1) The von Mises strength solution is independent of n and provides an absolute upper bound prediction of the burst data.
2) The Tresca strength solution is independent of n, but gives an adequate burst pressure prediction, particularly for n < 0.12.
3) The von Mises flow solution is an upper bound prediction of the burst data for all n values.
4) The Tresca flow solution is a lower bound prediction of the burst data for all n values.
5) The approximate Svensson model predicts outcomes that are smaller than and close to the Mises flow solutions for all n values. However, this Svensson model still significantly overestimates most of the burst data because it is an approximate form of the von Mises flow solution.
6) The Zhu-Leis flow solution is an intermediate prediction and agrees well with most burst data for all n values. Thus, this flow solution is the best prediction of burst data on average.

On the other hand, from the design point of view, a good prediction model should match most of burst test data. In engineering, a good model is usually defined as a goodness of fit data within an error of ± 5%. From Fig. 11, the von Mises flow solution overpredicts the burst pressure data and fits only about 10% of the burst data. The Tresca flow solution underpredicts the burst pressure data and fits about 30% of the burst data. The Zhu-Leis flow solution provides an intermediate prediction and fits about 60% of the burst data. As a result, the Zhu-Leis flow solution is the best prediction model for use as a design curve.

From the above-noted model evaluations, it is concluded that for thick-walled pipes, 1) the von Mises strength solution absolutely overpredicts the burst data, 2) the Tresca strength solution overall determines an adequate prediction of all burst data, 3) the Mises flow solution (or Svensson approximate solution) is an upper bound prediction, 4) the Tresca flow solution is a lower bound prediction, and 5) the Zhu-Leis flow solution is an intermediate prediction that serves a more accurate estimate of the burst data on average.

4.2. Validation for thin and thick-walled cylinders

Zimmermann et al. [19] collected 37 burst test data for large diameter, thin and thick-walled pressure vessels that were obtained by the European Pipeline Research Group (EPRG) for evaluating a set of burst prediction models. The vessel materials are either pipeline steels or structural carbon steels, and the diameter to thickness ratio varies in the range of $10 < D/t < 80$. The vessel steels have yield strengths in the range of 264 to 807 MPa and tensile strengths in the range of 392 to 869 MPa. The strain hardening exponents n of the vessel steels are all less than 0.20.

Figure 10 compares the burst pressure predictions with measured burst pressure data that were given by Zimmermann et al. [10] for large diameter, thin and thick-walled pressure cylinders, where the y-axis represents burst pressure normalized by the Tresca strength solution, $P_b = \sigma_{uts} \ln\left(\frac{D_o}{D_i}\right)$, for thick-walled cylinders, and the x-axis denotes the strain hardening exponent n. The burst pressure predictions were obtained from four representative models for thick-walled pipes, that is: the Tresca strength solution in Eq. (5), the von Mises strength solution in Eq. (6), the Svensson approximate solution in Eq. (7) and the proposed exact flow solutions in Eq. (8) that include the von Mises flow solution, the Tresca flow solution and the Zhu-Leis flow solution.

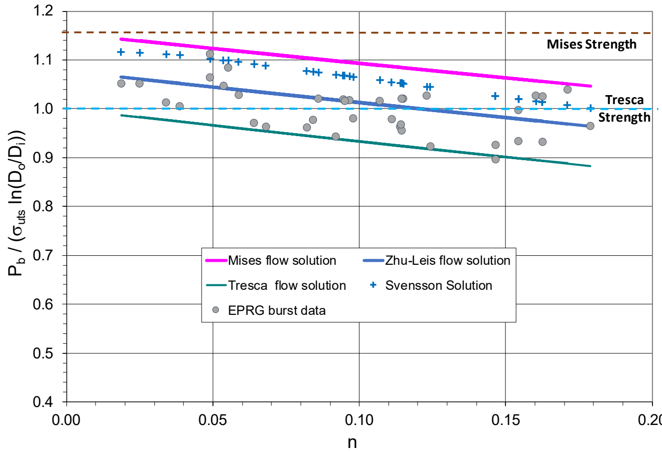

Figure 12. Comparison of burst pressure predictions with burst test data for thin and thick-walled cylinders as a function of n.

Comparison of Fig. 12 with Fig. 10 shows all model predictions in these two figures are similar, and thus all observations and conclusions determined in Fig. 10 are the same as those determined in Fig. 12 for the EPRG burst data.

Figure 13 shows a direct comparison of burst pressure predictions with measured burst data for thin and thick-walled cylinders, where the prediction models include the von Mises, Tresca and Zhu-Leis flow solutions. Again, it is observed that 1) the von Mises flow solution overestimates the burst data; 2) the Tresca flow solution underestimates the burst data; 3) the Zhu-Leis flow solution provides an intermediate prediction that match best with the burst data on average.

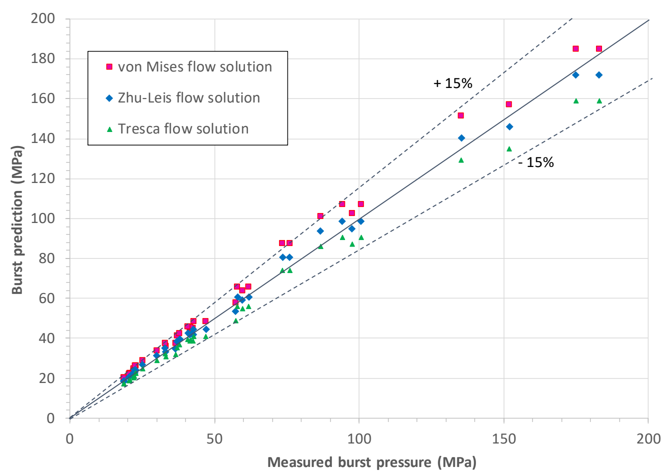

Figure 13. Comparison of the proposed burst pressure predictions with measured burst test data [10].

In summary, all experimental evaluations validated the Zhu-Leis flow solution for thick-walled pipes to more accurately predict burst pressure for both thin and thick-walled cylinders. In contrast, the Mises and Tresca flow solutions are an upper and a lower bound prediction, respectively for these cylinders.

5. CONCLUSIONS

This paper briefly discussed four representatives of burst pressure prediction models for thick-walled pipes, including the Tresca strength solution, von Mises strength solution, Svensson approximate solution, and the newly proposed exact theoretical solutions for thick-walled pipes in terms of the Tresca, von Mises and Zhu-Leis yield criteria. To validate the proposed Zhu-Leis flow solution for thick-walled pipes, a set of pressure burst tests were recently conducted for API 5L Grade B carbon steel pipes with a nominal diameter of 2.375 inches and three nominal wall thicknesses of 0.154, 0.218, and 0.344 inches. Comparisons of burst pressure predictions with burst pressure data measured in this work and obtained from literature all validated the proposed Zhu-Leis flow solution for thin and thick-walled pipes. The following results and conclusions are obtained:

(1). Three exact flow solutions of burst pressure in terms of the Tresca, von Mises and Zhu-Leis yield criteria are functions of UTS, n and $\ln(D_o/D_i)$ for thick-walled pipes. Among them, the Tresca solution provides a lower bound prediction of burst pressure, von Mise solution provide an upper bound prediction, and the Zhu-Leis solution provide an intermediate prediction.

(2). For three Grade B carbon steel pipes, the material mechanical properties were obtained from the uniaxial tensile tests. The experimental stress-strain curves are similar to each other for two thicker pipes (SCH-80 and SCH-160), but they are different from that for the thinner one (SCH-40). The reason is unknown for the difference of the same purchased pipes.

(3). The "air over water" pressurization process was successfully unitized in all pressure burst tests at SRNL. Both monotonical loading and step loading approaches were applied to the pipe pressure burst tests. The slow step loading approach seemed more reasonable for a pressure burst test.

(4). The outside circular welds used to join end caps for pipe pressure test specimens works fine for thinner walls of the pipes (SCH-40 and SCH-80) but are inappropriate for thick walls (SCH-160). In the case for thick walls, circular welds failed first.

(5). The burst pressure test data measured at SRNL and obtained from literature all validated that the exact Zhu-Leis solution is the best prediction of burst data for thin and thick-walled pipes, while the von Mises solution is an upper bound prediction, and the Tresca solution is a lower bond prediction for both thin and thick-walled pipes.

ACKNOWLEDGEMENTS

The pressure burst tests were conducted by Mrs. Will Baggett, Michael Restivo, and Greg Sides at the Savannah River National Laboratory (SRNL). The tensile tests were conducted by Dr. Tim Krentz at SRNL. Dr. Bruce Wiersma supported steel pipe purchase. Mr. Martin Johnson led specimen machining and test pipe caps welding.

This work was sponsored by U.S. DOE Laboratory Directed Research and Development (LDRD) program within SRNL. This document was prepared in conjunction with work accomplished under Contract No. 89303321CEM000080 with the U.S. Department of Energy (DOE) Office of Environmental Management (EM).

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APPENDIX A. FAILURE APPEARANCE OF PRESSURE BURST TEST PIPES

This Appendix reports the failure appearance of pressure burst test pipes after bursting.

Figure A1. Schedule 40 Pipe 1 after bursting

Figure A2. Schedule 40 Pipe 2 after bursting

Figure A3. Schedule 80 Pipe 1 after bursting

Figure A4. Schedule 80 Pipe 2 after bursting

Figure A5. Schedule 160 Pipe 2after bursting