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The Use of Fuzzy Mathematics in Subjective Uncertainty Analysis

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Abstract

Many analytical problems (for example, abnormal-environment safety analyses) depend on data that are partly or mostly subjective and may have dependence. Since fuzzy mathematics utilizes subjective operands, we have been investigating its applicability in safety assessment, particularly for portraying uncertainty and dependence in probabilistic safety assessments (PSAs). We have found that it is a very efficient approach, both in terms of methodology development time and program execution time. However, its most important attribute is that it processes subjective information subjectively, not as if it were based on measured data. It also provides for straightforward incorporation of dependence judgment, rather than requiring that all inputs be treated as if they were independent.

One of the most useful results of this work is that we have shown the potential for significant differences (especially in perceived margin relative to a decision threshold) between fuzzy mathematics analysis and conventional PSA analysis. This difference is due to subtle factors inherent in the choice of probability distributions for modeling uncertainty.

Since subjective uncertainty, stochastic variability, and dependence are all parts of most practical situations, a technique has been developed for combining the three effects. The methodology is based on hybrid numbers (ordered pairs of subjective variability fuzzy number functions and probability density or cumulative distribution functions) and on Frechet inequality Dependency Bounds Analysis. Some new results have also been obtained in the areas of efficient disjoint set representations and constrained (dependent input variations) uncertainty and variability analysis.

Introduction

There are many potential sources of quantitative variation in analyses. One form is classical variability, which is applicable to prescribed problems such as tossing dice and coins. Another form is subjective uncertainty, which means that available data are not definitive enough to completely describe the variations. The goal is to accommodate any potential mix of variability and uncertainty.

The challenge is enhanced by an accumulating collection of evidence that safety analyses, even those with a focus on extremes, do not reliably enough portray the chance of safety failure commensurate with the frequency of actual occurrences, most significantly when the inputs contain a degree of subjectivity or are dependent.

Subjective Uncertainty

Pure subjective uncertainty necessarily is based on opinion, and since the results of any analysis are only as good as the analysis inputs, the best possible inputs come from estimators who have the most applicable expertise. Fuzzy mathematics is appropriate for processing subjective information.

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The mathematical basis for combining fuzzy numbers is based on the Extension Principle of fuzzy logic. For addition and multiplication (basic to fault tree and event tree computations), this produces:

$$\mu_{A+B}(z) = \bigvee_{z=x+y} (\mu_A(x) \wedge \mu_B(y)) \tag{1}$$

$$\mu_{A\times B}(z) = \bigvee_{z=x\times y} (\mu_A(x) \wedge \mu_B(y)) \quad . \tag{2}$$

These are convolutions basically constructed like those used in probabilistic calculus.

Fuzzy mathematics gives a different perspective on extremes than conventional PSA, because it incorporates subjectivity differently and because the mathematical operations involving each abscissa value are not constrained probabilistically by the corresponding ordinate values. A somewhat focused view of safety analysis is that the important regime of interest in a spectrum of uncertainty is the extreme that indicates maximum potential loss of safety. This means that differences at the extremes should be carefully evaluated in safety assessment. There are many real-world examples of inputs to PSAs that are necessarily at least partly subjective. The concern is that modeling these based strictly on stochastic assumptions may be merely an assertion, rather than a reflection of reality.

Hybrid Numbers

Hybrid numbers allow addition and subtraction. The pairs can be added together by convolving the respective elements according to normal rules for fuzzy arithmetic and probability theory. For example, addition is defined as:

$$(f_1,p_1) + (f_2,p_2) = (f_1+f_2,p_1+p_2) \tag{3}$$

where f is a fuzzy number and p is a stochastic number, e.g., a pdf, and where the plus signs on the right side of the equation represent fuzzy max-min convolution and ordinary probabilistic sum-product convolution, respectively. This formulation of hybrid numbers does not directly allow multiplication or a full hybrid arithmetic (e.g., the product of a completely fuzzy number and a completely probabilistic number is undefined). Nevertheless, the construct is useful, when modified as follows.

The information available about an input is often partially based on limited data and partially based on subjective estimates. In order to combine subjective information with statistical information and to provide for smooth transitioning from subjective (fuzzy) characterization to stochastic characterization as information about inputs is gained, the fractional apportionment is made according to Equation 4:

$$h_i(x)= a_i \times p_i(x)+ (1 - a_i)\times f_i(x) \quad . \tag{4}$$

where) $p_i(x)$ is the probabilistic function (e.g., a probability density function), $f_i(x)$ is a fuzzy function (formally, a normal and convex fuzzy number), a_i is a "scale" factor (the relative fraction of statistical information available), $h_i(x)$ is a scaled hybrid number, and i is an index associated with the ith variable.

Since the operands of scaled hybrid numbers require that the probabilistic functions be combined, the fuzzy functions be combined, and the hybrid scale factors be combined, probabilistic calculus, fuzzy mathematics, and hybrid mathematics are included in the methodology. Dependence is accounted for through subjective measures of dependence to place values for each operation at a specified position in the interval bounded by values that would result from complete independence and complete dependence.

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