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## Optimal Detection Of Short-Warning Near-Earth Object Threats

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### Abstract

Detection of near-Earth Objects (NEOs) has concentrated on long-warning threats. LPCs and smaller objects do not offer such warning. Their detection on final approach is a more demanding search problem. Improvements in ground- and space-based search sensors and strategies could provide adequate search capability.

### Introduction

Much of the work on the detection of NEOs has concentrated on long-warning threats from NEOs that are detected many decades before impact. Long period comets (LPCs), which apparently represent 20 to 40% of the threat, (Shoemaker 1995) and NEOs smaller than 1 km, which impact much more frequently than large NEOs, offer much less warning. They have to be detected on final approach, which is a much more demanding search problem. This note discusses improvements in ground- and space-based search sensors and strategies that could make adequate detection and warning possible. It discusses combinations that could provide adequate search capability and derives their search, detection, and completeness rates. It also discusses cost-optimized combinations appropriate for both long- and short-warning threats.

### Optimal ground-based detection

The preferred search strategies and technologies for the detection of NEOs has undergone a significant shift in the last few years—away from an earlier emphasis on the use of very large telescopes at maximum range toward the use of modest telescopes with very large charge coupled device (CCD) focal planes that cover the fresh sky completely each month. The reasons for this shift for large NEOs can be discussed simply; that also provides a useful background for discussion of the difficulties involved in detecting smaller objects. For low signal-to-noise ratio (SNR) detection, which necessitates long integration time  $t$ , the signal,  $S_{\text{req}}$ , required to achieve a given SNR is approximately

$$S_{\text{req}} = \sqrt{(\text{SNR}^2 B / Q t)}, \quad (1)$$

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where  $Q$  is the sensor quantum efficiency and  $SNR$  is the limiting ratio for detection, typically  $SNR \approx 4$ . The sensor's background is  $B \approx B'' A \theta^2$ , where  $A$  is its aperture and  $\theta$  its pixel diameter. The constant  $B'' \approx 0.25/\text{m}^2\text{-arcsec}^2$  for space-based systems;  $\approx 0.63/\text{m}^2\text{-arcsec}^2$  for ground-based systems. For ground-based sensors  $\theta$  is limited by atmospheric seeing to a few arcsec, so for ground sensors,  $B \propto A$ , and  $S_{req} \propto \sqrt{A/t}$ . Conversely, the signal received by a sensor of aperture  $A$  from an illuminated object of diameter  $D$  at range  $r$  is

$$S_{rec} = JAD^2/r^2(1+r)^2, \quad (2)$$

where  $J$  proportional to the solar illumination and  $r$  is in AU (Astronomical Unit  $\approx 150$  million km). Equating  $S_{req}$  to  $S_{rec}$  specifies the sensor parameters required for detection

$$\sqrt{(SNR^2 B'' \theta^2 / AQt)} = JD^2/r^2(1+r)^2, \quad (3)$$

so the area-exposure product scales as  $At \propto r^2(1+r)^2/D^2$ . Each exposure of duration  $t$  views a solid angle  $w$ ; thus, the sensor covers the sky at angular rate  $w/t$ . If it can detect objects out to range  $r$  in a volume of space that contains NEOs of density  $N \approx K/D^n$ , where  $K$  and  $n$  ( $\approx 2$  for  $0.1 < D < 2$  km) are constants it should NEOs at a rate

$$R \approx (w/t) \int dr r^2 N = (w/t) \int dr r^2 K/D^2 = (w/t) \sqrt{(QA t / SNR^2 B'' \theta^2)} \int dr r^2 KJ/r^2(1+r)^2$$

$$\approx (w/t) \sqrt{(QA t / SNR^2 B'' \theta^2)} KJ. \quad (4)$$

The rate  $R$  essentially scales as  $(w/t) \sqrt{(At)}$ , where the first factor is the rate of covering sky, and the second is the depth to which it is covered. For the sensor to cover the whole fresh sky each month requires a search rate  $W' \approx 100 \text{ deg}^2/\text{hr}$ . To achieve that with a sensor with a field of view (FOV)  $w$  requires an exposure time  $t \approx w/W'$ . Shorter exposure times would cover parts of the sky twice without producing additional discoveries. Longer exposures would leave parts of the sky uncovered. Thus,  $t \approx w/W'$  is the optimum time for maximizing the rate of detection. It is generally much shorter than the time for maximizing the range of detection, the usual metric.  $Q$ ,  $SNR$ ,  $B''$ , and  $\theta^2$  are relatively inflexible, so  $R$  scales as

$$R_{opt} \propto (w/t) \sqrt{(At)} \propto W' \sqrt{(Aw/W')} \propto \sqrt{(AwW')}. \quad (5)$$

For NEOs larger than a kilometer, it is possible to use telescopes a meter or so in diameter with CCD focal planes of a few million detectors to achieve ranges of a few AU and hence produce adequate detection rates. It is useful to examine the optimal sensors for this purpose. A sensor of f-number  $F$  ( $=$  focal length / primary diameter) with a CCD focal plane array containing  $N$  detectors of pitch  $d$  has a geometric FOV  $w \approx N(d/F)^2/A$ ; thus, its optimized search rate scales as

$$R_{opt} \propto \sqrt{(A[N(d/F)^2/A])} \propto (d/F) \sqrt{N}, \quad (6)$$

Since  $d$  decreases only with development and  $F$  cannot be decreased much below the current  $F \approx 2$  without degrading optical quality, increasing the number of detectors appears to be the only variable with significant potential for increasing detection rate.  $R_{opt}$  is independent of  $A$ , because for given  $N$ , the benefits in light gathering from increasing  $A$  is just offset by its reduction of FOV. Increasing  $A$  would just provide more flat space in the focal plane to put additional detectors—if needed. To the extent that space is available in the focal planes of existing telescopes, the most effective way to improve ground-based sensors is to improve their focal planes—not build bigger telescopes.

This result has an interesting corollary. Direct sensor costs scale as  $C \approx nN + aA$ , where  $n$  and  $a$  are constants. It is customary to maximize  $R$  while holding  $C$  fixed or to minimize  $C$  while holding  $R$  fixed. Either metric leads to the differentiation of  $C$  and  $R$  with respect to  $N$  and  $A$  to determine the optimum. Because in the limit of Eq. (6), performance depends only on  $N$ , there is no benefit from  $A$  to offset its cost, so optimal ground-based search systems would have large focal planes with apertures just large enough to hold them. This combination balances search rate and range to maximize detection rate, which is a different metric than earlier designs that had large telescopes, small focal planes, and long integration to maximize information from each exposure.

### Detection with reduced warning

Objects with reduced warning include LPCs, which pass infrequently, perhaps only once, and smaller objects that are less luminous. The smaller objects that hit more frequently yet with serious consequences illustrate the additional complications in a simple context. In contrast to NEOs, whose detection is expected many decades prior to impact, small objects would probably not be seen until their final approach. A one meter ground-based sensor that could detect a NEO at about an AU could detect a 100 m object at a few tenths of an AU. The objects approach at a relative velocity of roughly 6 AU per year, so detection at 0.1 AU would provide only a few weeks warning. Thus, small objects require almost constant monitoring of possible approaches. There,  $r \ll 1$ , so Eq. (2) reduces to  $S_{\text{rec}} \approx JAD^2/r^2$ , so that equating it to  $S_{\text{req}}$  produces the approximate scaling

$$1/\sqrt{(At)} \propto D^2/r^2, \quad (7)$$

from which the maximum range at which an object of diameter  $D$  can be detected scales as

$$r \propto D(At)^{1/4}. \quad (8)$$

If the sensor searches at rate  $W'$ , its detection rate is

$$R \approx W' \int dr' r'^2 N = W' \int_0^r dr' r'^2 K/D^2 = W' (K/D^2)(r^3/3) \approx W' KD(At)^{3/4}. \quad (9)$$

To search the whole sky requires  $t \approx w/W'$ , so that

$$R \approx KD(AwW')^{3/4/3} \quad (10)$$

If area on the focal plane is not a constraint,  $w \propto N/A$ , so that  $R \propto DN^{3/4}$ . The completeness,  $G$ , after search time  $T$  is the ratio of the number of objects discovered,  $RT$ , to the number of objects in the whole volume  $V$ ,  $NV$ , which is

$$G \propto TDN^{3/4} / (K/D^2) \propto TD^3N^{3/4}. \quad (11)$$

This indicates that a sensor that could achieve high completeness against 1 km objects would achieve only 0.1% completeness against 0.1 km objects. Conversely, to maintain a given completeness as  $D$  is decreased requires  $N$  to increase as  $N \propto 1/D^4$ . A factor of 10 decrease in  $D$  to 100 m would require  $N$  to increase by a factor of  $10^4$  from current levels of a few million.

Completeness could be increased by extending the duration of the search,  $T$ , but search times are already decades for kilometer NEOs, and as indicated above, smaller objects, which will be detected closer to impact, require shorter search times. If in addition to decreasing the object size by a factor of 10 the search duration was decreased from a decade to a month, the number of detectors would increase by a factor of  $N \propto 1/(TD^3)^{4/3} \propto 1/(0.01 \times 0.1^3)^{4/3} \approx 5 \times 10^6$ , which is clearly impractical.

If the sensor's flat field is already filled,  $w$  is fixed and  $N$  and  $A$  must increase together. Then Eq. (10) becomes  $R \propto DA^{3/4}$  and Eq. (11) becomes  $G \propto TD^3A^{3/4}$ , so that for a given completeness,  $A \propto N \propto 1/D^4$ . Since aperture and focal plane are comparably expensive, in the constrained case, the costs about twice those of the unconstrained case where only  $N$  increases.

It appears that appropriate technology is available for performing decadal searches for large NEOs. They are large enough to see at long distances, so an effective strategy is to wait and observe them as they pass by the Earth. For smaller objects, that does not appear to be the case. They are so dim that they would be seen only when they were very close to the Earth, which would make the duration of exhaustive searches many millennia. Thus, it would appear necessary to detect them on their final approach to Earth. But they are so dim that would be a stressing problem as well. The detection ranges and times would appear to be fractions of AU and periods of weeks. It is not clear what sorts of responses that degree of warning could support.

### Space-based detection

Sensors in space can relax the restrictions of ground-based sensors at some price in cost and complexity. The basis for this improved scaling is that in space it is possible to reduce the blur circle without reducing FOV, which is not the case for active optics sensors on the ground. On the ground, the sensor blur circle is proportional to  $(\lambda / r_0)^2$ , where  $\lambda$  is the wavelength of light and  $r_0$  is the coherence length of the atmosphere.  $r_0$  is typically 5 to 10 cm, so  $\lambda / r_0$  is typically a few arcsec, leading to  $\approx 10$ -20 sq. arcsec blur circles for the ground-based telescopes. When atmospheric distortion is removed, the blur circle can be reduced, which increases the detection rate. If the blur circle diameter is reduced from  $\lambda / r_0$ , to  $\lambda / D$ , where  $D = 1$  m, the increase in  $R$  is  $\approx \sqrt{100} \approx 10$  fold.

Operation in space restores the usefulness of large apertures, within limits. There is little benefit in making the blur circles smaller than the detectors. Thus, the benefits of space sensors are limited by the pitch of the detectors that can be produced in large arrays at any point in time. The present limit on volume arrays is about 5 micron, while the current detectors used are about 25 micron, so there is the potential for about a factor of 5 reduction in blur circle diameter, or 25 reduction in area, with space operation. Beyond that, improvements will be paced by technology.

For sensors in space, the pixel size  $\theta$  can be reduced to the diffraction limit,  $\lambda / \sqrt{A}$ , so  $B \approx B'' A \theta^2 \approx B'' A \lambda^2 / A$  is constant. Thus,  $S_{req} \propto 1/\sqrt{t}$ , so that the detection range for small objects scales as  $r \propto D \sqrt{A} t^{1/4}$ , for which the detection rate is

$$R \propto DA^{3/2}t^{3/4}. \quad (12)$$

For the optimal exposure time  $t = w/W'$ , the detection rate reduces to

$$R_{opt} \propto D(NA)^{3/4}. \quad (13)$$

This result differs in an essential way from the  $R \propto DN^{3/4}$  of ground-based sensors of Eq. (10). That result scales only on  $N$ ;  $A$  makes no contribution. Thus, it requires a large increase in  $N$  to offset a decrease in  $D$ . Equation (13) for space-based sensors scales equally strongly on  $N$  and  $A$ . Since they are equally effective in increasing  $R$  and about equally expensive, they would be increased proportionally to offset

increases in  $D$ . Thus, for space-based sensors,  $R_{\text{opt}} \propto DN^{3/2} \propto DA^{3/2}$ , and a 10-fold decrease in  $D$  would only require a  $10^{2/3} = 4.6$  increase in  $N$  and  $A$ .

For space-based sensors, completeness is  $G \propto D^3(NA)^{3/4}$ , from which  $NA \propto 1/D^4$  for fixed  $G$ . Thus,  $N \propto A \propto 1/D^2$ , so a 100-fold increase in  $N$  and  $A$  would compensate for a 10-fold decrease in  $D$ . Overall, space-basing would reduce the detector count and aperture for space basing about a factor of 100 from that for ground basing to more practical levels.

The estimates above have concentrated on small objects that are detected at short range, but the analysis can readily be extended to LPCs, which are brighter and hence can be detected at greater range—indeed, they must be since proportionally greater range is required to deflect them (Canavan 1994). The stronger range dependence of the received signal complicates the analysis somewhat, but the main element of the analysis is the fact that the ability to use smaller blur circles—or even diffraction limited optics—again produces the favorable scaling noted above for smaller objects. Thus, the general benefit of space basing is the ability to take advantage of both detector count and aperture area in increasing sensitivity, search, and detection rate.

### Combinations

Ground- and space-based systems could perform decadal search for large NEOs and prompt search for small and large objects that give less warning. Ground-based sensors are relatively cheap, and can achieve the search rates required for the detection of most large NEOs in a few decades. However, their detection rates are proportional to object size, so the completeness of their searches decreases with the cube of diameter. Since their detection rate scales on detector count, it would take impractically large focal planes to search for objects much smaller than a kilometer. However, as detector technology improves, it should be possible to extend search down to diameters varying inversely with  $N^{1/4}$ .

The detection rate for space-based sensors scales on the product of detectors and aperture, so the optimized rate scales as  $R_{\text{opt}} \propto DN^{3/2}$ . Thus, for fixed completeness, the object diameter that can be searched scales as  $D \propto 1/\sqrt{N}$ . Since this is a much stronger scaling than the  $1/N^{1/4}$  of ground-based sensors, at some diameter space-based sensors should become the preferred solution. Where that tradeoff occurs for small objects is determined by a set of performance and cost parameters that are defined but not estimated above. Of those, the most sensitive are the relative costs of ground and space basing. They are poorly known and somewhat speculative, so the detailed estimate of the crossover is not attempted here.

### Summary

Much of the work on detection has concentrated on long-warning threats from NEOs. LPCs and smaller objects represent a significant fraction of the threat and present additional challenges in short warning and dim targets. Their detection on final approach is a more demanding search problem, which improvements in ground- and space-based search sensors and strategies could address. Appropriate technology is available for performing decadal searches for large NEOs. They are large enough to see at long distances, so it is an effective strategy to wait and observe them as they pass by the Earth. For smaller objects, that is not the case. They would be seen only when they were very close to the Earth, which would make searches millennial. Thus,

it is necessary to detect them on their final approach . But they are so dim that is also a stressing problem. The detection ranges and times appear to be fractions of AU and periods of weeks, so it is not clear what responses such warning could support.

Ground-based sensors are dominated by atmospheric seeing, which reduces the effects of aperture and produces optimal search rates sensitive only to detector count. Since search completeness depends on the product of  $D^3$  and  $N^{3/4}$ , a large increase in  $N$  is required to search for smaller objects, and the sensors required for rapid search appear impractical. Space-based sensors relax these restrictions at a price in cost and complexity. They do so by avoiding the limits of atmospheric turbulence. Their search rates depend on the product  $NA$ , so that increasing  $N$  and  $A$  in proportion produces a quadratic increase in  $R$ . That advantage also applies to completeness, for which a 100-fold increase in  $N$  and  $A$  could decrease the minimum  $D$  detected 10-fold. Essentially the same advantages apply to the scaling of space-based sensors for LPCs.

Cost-optimized combinations of sensors appropriate for long- and short-warning threats can be derived directly from the physical arguments above. Combinations of ground- and space-based systems could perform the combination of decadal search for large NEOs and prompt search for small and large objects with less warning. Because of their relative inexpensiveness, ground-based sensors are suited for long-duration searches for large objects, and because of their high sensitivity, space-based sensors are best suited for rapid searches for dim objects that give little warning and require rapid response.

## References

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