

# Multi-Edge Graph Convolutional Networks for Power Systems

Abhijith Ravi, Linquan Bai

*University of North Carolina at Charlotte*

NC, USA

{aravi2,lbai4}@uncc.edu

Fei Ding

*National Renewable Energy Laboratory*

Golden, USA

fei.ding@nrel.gov

Hong Wang

*Oak Ridge National Laboratory*

TN, USA

wangh6@ornl.gov

**Abstract**—The exponential electrification of transportation has contributed to highly intermittent load variations in the distribution grid. This uncertainty has raised challenges for distribution system operation and control. Accurate nodal voltage estimation is highly essential for the safe and reliable operation of the grid. Graph convolutional networks have been used in machine-learning-based models for power grid applications like voltage estimation for their ability to capture the network topology of the grid. This paper presents a multi-edge graph convolutional layer that considers resistance and reactance as edge attributes. Then, this layer is used to create a multi-edge graph convolutional network-based surrogate model for estimating voltage in the distribution network with highly uncertain electric vehicle (EV) loads. Results indicate improved performance of the multi-edge graph convolutional network model when compared to a standard graph convolutional network model.

**Index Terms**—graph convolutional networks, electric vehicles, voltage prediction, smart grids, multi-edge graph convolutional networks

## I. INTRODUCTION

Electric vehicles (EVs) offer an interesting solution to reduce the dependency of the transportation sector on fossil fuels. However, the integration of EVs into existing power grids poses challenges such as increased electricity demand and voltage fluctuations. While some EV loads, like fleet depot charging stations, may be less uncertain, others associated with public charging infrastructure and highway fast charging stations may be highly uncertain. This uncertainty has raised concerns for short- and long-term load forecasting and voltage estimation in power distribution networks. Over time, the United States power distribution grids have evolved to integrate various smart grid technologies that leverage real-time data and advanced analytics to optimize grid performance and ensure reliable and efficient power delivery. Systems like the Advanced Distribution Management System (ADMS), Distributed Energy Resource Management Systems (DERMS), Fault Location Isolation and Service Restoration (FLISR), and Volt/VAR Optimization (VVO) have been crucial in managing the increasing complexity of the grid. These systems rely on accurate voltage estimation using conventional methods or data-based state estimation methods. All these systems have led to increased data collection from the grid. This data could be extremely valuable to train machine learning (ML) models.

### A. Literature Review

In recent years, power systems research has witnessed an interest in ML-based approaches for voltage estimation in power distribution networks. Traditional distribution system state estimation (DSSE) methods have been employed to estimate voltage using measurements collected by the sensors at different nodes in the power distribution network [1]. However, with the increase of highly uncertain loads like EVs, there is a need for adaptive voltage estimation techniques. The integration of ML-based techniques presents a promising solution to address this need [2]. Leveraging neural networks, can enhance voltage estimation accuracy by considering complex relationships between input variables and voltage levels in the active distribution network and accommodating reverse power flows due to distributed energy resources (DERs) [3]. Moreover, a continuously evolving grid with increasing smart devices requires real-time monitoring and control capabilities powered by ML-based voltage estimation methods [4]. Graph Neural Networks, such as Graph Convolutional Networks (GCN) can be employed in several applications in power systems such as fault scenario application, time-series prediction, power flow calculation, and data generation [5]. GCNs can capture the topology of the power distribution network, allowing for more accurate voltage estimation by considering spatial relationships between different buses and nodes.

### B. Motivation and Contributions

With increased EV adoption, it is essential to optimize the charging of EV loads for stable and reliable operation of the grid. With the development of ML technology, deep reinforcement learning algorithm has shown promise in the energy management strategy (EMS) of EVs [6]. Estimation of steady state parameters of the distribution network is a key step to learning in ML based-optimization algorithms [7]. Zhao et al. in [8] presents the use of GCN as a surrogate model for voltage estimation in a federated learning framework to implement optimal VVO control for DERs. GCNs when combined with federated architecture are perfect for the implementation of edge-intelligence-based distributed control systems that can continuously learn from real-time decision-making. Such an adaptive system could be highly efficient in dealing with EV load-induced uncertainty. However, conventional GCN fails to capture edge-related attributes in the power distribution

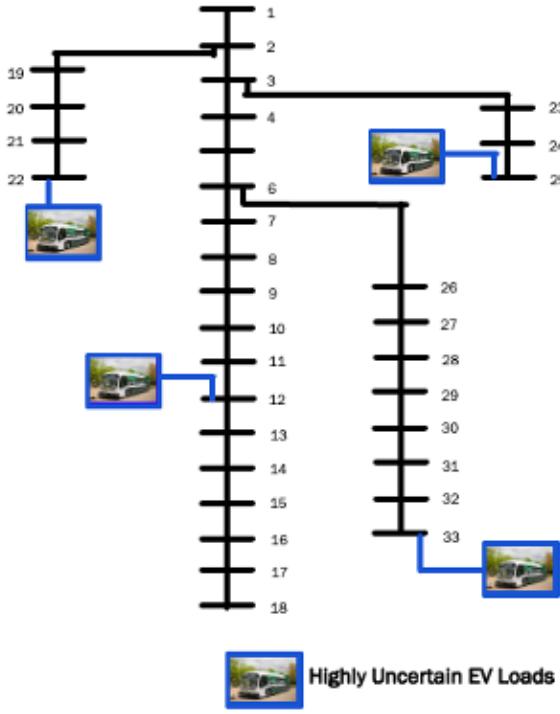


Fig. 1. Modified IEEE 33-node Distribution Network

network. Zhou et al. in [9] introduces the co-embedding of edges and nodes with deep graph convolutional neural networks to consider multiple edge attributes simultaneously. Such an approach, called Multi-Edge GCN (ME-GCN) can overcome the drawbacks of GCN. The contributions of this paper are listed below:

- The paper introduces a novel Multi-Edge GCN model customized for power systems. This model accepts the resistance and reactance of the distribution network as edge attributes.
- The proposed ME-GCN model is then compared with the traditional GCN model with a distribution network data with highly uncertain EV loads.

## II. PROBLEM STATEMENT

A distribution system operator (DSO) has to be prepared to deal with the uncertainty on the grid due to the integration of renewable energy sources and electric vehicles. EV charging loads can range from 3 kW - 4.5 MW in terms of power and 20 kWh - 1 MWh in terms of energy demand. Each EV load's demand curve is highly customer specific. Since most EV customers have an incentive in managing the charging to reduce the peak load drawn from the grid, EV charging data will be recorded by charging management systems. With increasing uncertainty, DSOs have to increase regulation and capacity reserves to ensure grid security and reliability for the customers. Moreover, with an ever-increasing number of EVs on the grid, it would be unwise to not use the available data to mitigate the impact of uncertainty. Therefore, it is

essential to explore and integrate advanced ML algorithms for power transmission and distribution systems. GCN-based surrogate models have been applied to estimate voltage in the distribution networks. With the ability to consider the topology of the distribution network during the model training, GCN has been explored for different applications on the power grid. This paper evaluates the performance of such a GCN model in a distribution network with highly erratic EV loads. Then, this model presents a novel ME-GCN layer with the ability to accept network parameters like resistance and reactance. Then, the introduced layer is utilized to build a surrogate model for the same application, for which the standard GCN-based surrogate model was used. The performances of both surrogate models are compared and analyzed.

## III. MODELING

### A. Data Generation Model for EVCS Load

To evaluate the performance of the GCN model used in this paper for voltage estimation in a distribution network, a high-uncertainty dataset was generated. For dataset generation, the EV charging station (EVCS) load was generated by several realistic assumptions. The rate of vehicle arrivals at the EVCS was modeled based on the time of day, leveraging a Poisson distribution to capture the stochastic nature of vehicle arrivals. While a higher arrival rate was selected during morning and evening commutes to account for peak hours, the arrival rates selected for off-peak and late-night hours were different. This model shows the variation of the rate of arrival of EVs to EVCS, reflecting real-world charging demand patterns [10]. The initial state of charge (SOC) for each vehicle was generated by using a normal distribution centered around 30% with a standard deviation of 10%. Furthermore, an inverted logistic function was used to model the dependency of the charging power of a battery on SOC [11]. The generated EVCS loads are shown in the Fig.2. Finally, these EV loads were assigned to the selected EVCS locations in the case.

### B. Background: Fundamentals of GCNs

1) *Mathematical Representation of GCN Layer:* GCNs have become a notable deep learning approach in the field of graph-structured data. The GCN acquires a robust representation of the graph by exploiting the local neighborhood

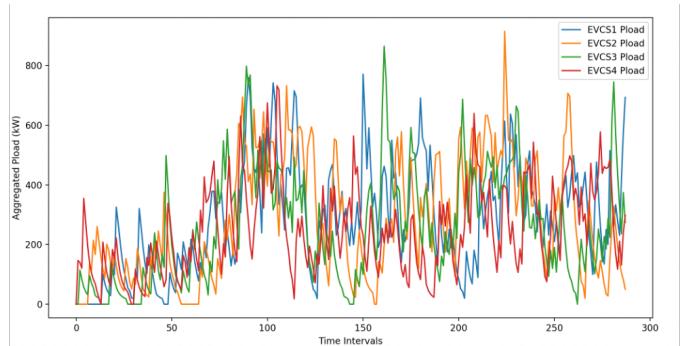


Fig. 2. Generated EVCS Load Profile

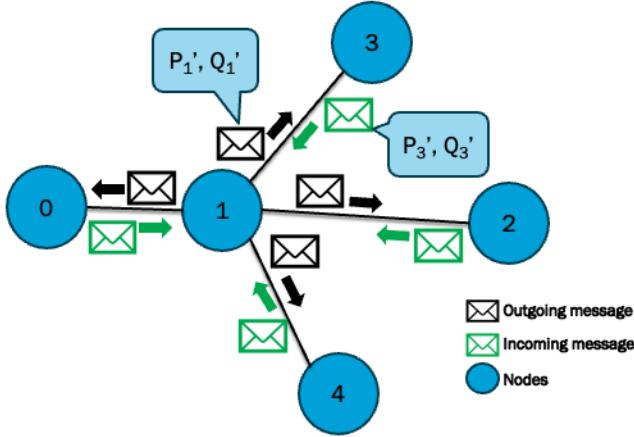


Fig. 3. Standard GCN Layer

information of nodes. The underlying concept of GCN is to combine neighboring nodes in order to capture both local and global graph structures [12]. Equation (1) presents the primary operation in a GCN layer.

$$H^{(l+1)} = \sigma(\tilde{D}^{-\frac{1}{2}} \tilde{A} \tilde{D}^{-\frac{1}{2}} H^{(l)} W^{(l)}) \quad (1)$$

where the matrix of node features at layer  $l$  is represented by  $H^{(l)}$ , the adjacency matrix with added self-loops is represented by  $\tilde{A}$ , the diagonal node degree matrix is represented by  $\tilde{D}$ , the weight matrix at layer  $l$  is represented by  $W^{(l)}$ , and  $\sigma$  is the activation function.

2) *Message Passing in GCN Layer*: The operation of a standard GCN layer for estimating voltage at a node utilizing both reactive and active power injection is shown in Fig. 3. From the perspective of node 1, the procedure commences by performing a linear transformation on the input features  $P_1$  and  $Q_1$ . In the ensuing stages, the transformed characteristics  $P'_1$  and  $Q'_1$  are disseminated to all adjacent nodes 0, 2, 3, and 4 by applying the adjacency matrix. In this phase, as shown in Figure. 3, Node 1 gathers transformed input features like  $P'_3$  and  $Q'_3$  from its neighboring nodes, thereby capturing the local graph topology. The node aggregates the data from all adjacent nodes. The data collected from the surrounding nodes, when paired with the modified data from node 1, is processed by a non-linear activation function, such as Rectified Linear Unit (ReLU), in order to capture intricate connections and patterns. By employing an additional transformation, the features that have been generated are utilized to make predictions for the voltage at node 1.

### C. Proposed Method: Multi-Edge GCNs

In the proposed novel Multi-Edge GCN (ME-GCN) model, multiple edge attributes have been integrated into the layer. This allows the model to capture complex relationships between the training data and the edge attributes. The layer performs separate transformations for node and edge attributes, followed by a gating mechanism that combines these transformed features. The output is then passed through a non-

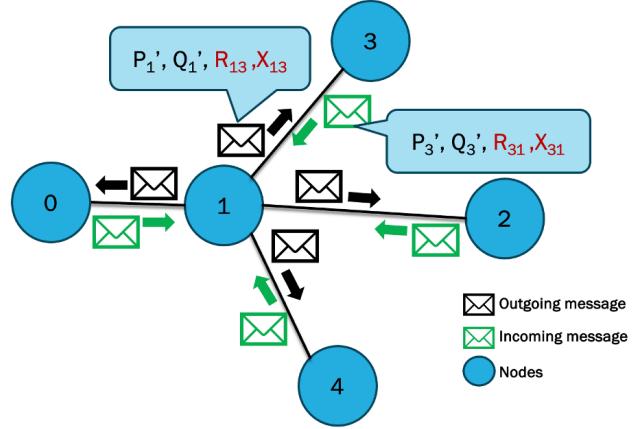


Fig. 4. Message Passing in the ME-GCN Layer

linear activation function like "Leaky RELU" to produce the final output.

1) *Mathematical Representation of ME-GCN*: The mathematical representation of the ME-GCN model is given (2).

$$H^{(l+1)} = \sigma(\text{CustomAggr}(\text{Concat}(H^{(l)}, E), W^{(l)})) \quad (2)$$

where  $H^{(l)}$  represents the matrix of node features at layer  $l$ ,  $E$  is the matrix of edge attributes associated with each edge in the graph,  $W^{(l)}$  is the weight matrix at layer  $l$ , which transforms the concatenated features, and  $\sigma$  is the activation function (e.g., LeakyReLU). CustomAggr denotes the custom aggregation function, typically a summation over the node's neighborhood after applying a transformation.  $\text{Concat}(H^{(l)}, E)$  signifies the concatenation of node features with corresponding edge attributes

2) *Message Passing in ME-GCN Layer*: The message passing of ME-GCN layer is shown in the Fig 4. When compared to a standard GCN layer, ME-GCN layers can accept more than one edge feature of the network. Resistance and reactance serve as edge attributes for the ME-GCN model presented in this paper. For a given node  $i$ , the ME-GCN layer starts with a linear transformation of the input features. Similar to a standard GCN layer, these transformed features are propagated to all adjacent nodes utilizing the adjacency matrix. However, the ME-GCN model incorporates edge attributes like resistance and reactance. For node 1, the edge attributes of all neighboring nodes are considered by the node.

The node  $i$  then concatenates these transformed features and edge attributes and disseminates them to the adjacent nodes based on the adjacency matrix. Similarly, node 1 would receive such features from all the neighboring nodes 0, 2, 3, and 4. Then, node 1 aggregates these node and edge attributes from neighboring nodes, which is then processed through a non-linear activation like *ReLU*. This step helps in capturing complex interactions and patterns in the data, considering both the node attributes and the characteristics of the connections between them. Finally, similar to a standard

GCN, an additional transformation is applied to these features to predict the nodal voltage.

3) *Model Data of ME-GCN*: The input features considered for the surrogate model for voltage estimation are nodal active power injection  $P$  and reactive power injection  $Q$ . The nature and dimension of the input feature matrix  $X$ , and adjacency matrix  $A$  are presented in [8]. The magnitudes of the resistance matrix and reactance matrix were extracted from the impedance matrix [13]. Within the model, each snapshot of power flow is represented as a graph with the topology  $A$  and with edge attributes  $R$  and  $X$ . Zero-mean normalization was done on input features and target values of voltage. The custom ME-GCN layer developed in this paper is presented as algorithm 1. The ME-GCN model used for the surrogate model is presented in algorithm 2. The hyper-parameters of the surrogate model is presented in the table. I. The training loop used for training both models is shown in algorithm 3.

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**Algorithm 1** Custom ME-GCN Convolutional Layer

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**Input:** in\_channels, out\_channels

**Output:** output

*Initialisation :*

1: Initialize linear transformation  $lin$  with input dimension  $in\_channels + 2$

*Propagation Process*

2: **for** each edge in edge\_index **do**

3:    concatenated  $\leftarrow \text{concat}(x_j, \text{edge\_weight})$

4:    output  $\leftarrow lin(concatenated)$

5: **end for**

6: **return** output

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**Algorithm 2** ME-GCN Model Architecture

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**Input:** data

**Output:** estimated voltage magnitude

*Initialisation :*

1: Extract  $x$ , edge\_index, edge\_weight from data

*Model Layers*

2:  $x \leftarrow \text{LeakyReLU}(\text{conv1}(x, \text{edge\_index}, \text{edge\_weight}))$

3:  $x \leftarrow \text{Dropout}(x)$

4:  $x \leftarrow \text{LeakyReLU}(\text{conv2}(x, \text{edge\_index}, \text{edge\_weight}))$

5:  $x \leftarrow \text{Dropout}(x)$

6:  $x \leftarrow \text{conv3}(x, \text{edge\_index}, \text{edge\_weight})$

7: **return**  $x.\text{squeeze}()$

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**Algorithm 3** Training Loop for ME-GCN Model

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**Input:** num\_epochs, patience, train\_loader, val\_loader

**Output:** best\_model

*Initialisation :*

1: Initialize best\_val\_loss and patience\_counter

*Training Process*

2: **for** epoch = 1 to num\_epochs **do**

3:    total\_loss  $\leftarrow 0$

4:    **for** each batch in train\_loader **do**

5:     Perform forward pass and loss computation

6:     Perform backpropagation and optimizer step

7:     total\_loss += current batch loss

8: **end for**

9: Compute average training loss

10: Evaluate on validation set

11: **if** validation loss  $\downarrow$  best\_val\_loss **then**

12:    Update best\_val\_loss and reset patience\_counter

13: **else**

14:    Increment patience\_counter

15:    **if** patience\_counter  $\geq$  patience **then**

16:     Trigger early stopping

17:     break

18:    **end if**

19: **end if**

20: **end for**

21: **return** best\_model

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#### IV. RESULTS AND ANALYSIS

The proposed local voltage estimation model for the ADN is tested using the modified IEEE 33-node distribution system in this section. The proposed method was implemented in Visual Studio Code using PyTorch. The numerical experiments were conducted on a computer with an Apple M2 Pro processor and 32 GB of RAM.

##### A. Case Description

A modified IEEE 33-node network shown in Figure 1 was employed to generate the data for training the model. For the generation of the dataset, the highly uncertain EV load was considered in the IEEE-33 network. The IEEE-33 node network was imported using the *matpower* module. EVCS load was generated based on the EV load data generation model presented in Section III-A. Then, the *pypower* module in Python was used to generate the power flow results of the model. The hyperparameters of the GCN and ME-GCN models are provided in Table I. The training data generated was for a 3-minute granularity for one year. The generated data was split into testing, training, and validation in the percentages of 70, 15, and 15, respectively. The training loss curve and validation loss curve of the ME-GCN model are shown in Fig. 5.

1) *Performance Evaluation Metrics*: Several performance metrics were employed to evaluate the accuracy and reliability of the models being compared. The Mean Absolute Error (MAE) reveals the average magnitude of errors in the prediction. Lower values of MAE indicate a closer alignment

TABLE I  
HYPERPARAMETERS FOR GCN AND ME-GCN

Hyperparameters	Data
Maximum Epoch	100
hidden dimension 1	128
hidden dimension 2	32
learning rate	0.01
batch size	1024
beta	0.01

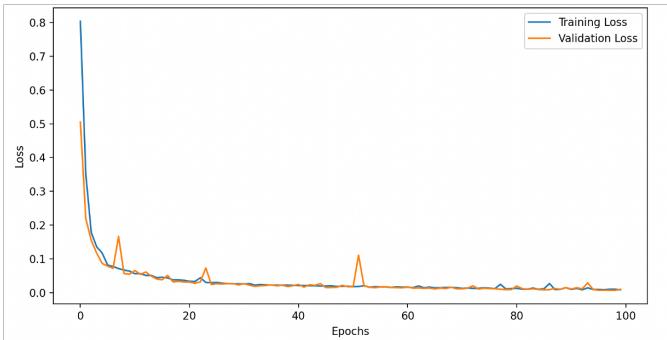


Fig. 5. Training and Validation Loss Curve for ME-GCN Model

with the observed data. While Mean Squared Error (MSE) highlights large errors by squaring deviations, Root Mean Squared Error (RMSE) shows errors in the magnitude of the target variable, presenting the average extent of errors. The coefficient of determination, or R-squared metric, quantifies the percentage of output variance explained by the model's input variables. A higher value of explained variance denotes a model's enhanced explanatory power. Explained variance measures the proportion of the dataset's total variance that is captured by the model. The Mean Absolute Percentage Error (MAPE) is useful for assessing the relative accuracy of the model using a percentage-based value of error magnitude. These metrics together provide a complete evaluation of the model. The value of training loss and validation loss is decreasing as the number of epochs increases. There are a few spikes in the validation loss at epochs 7, 23, 51, 72, 80, and 93. However, since the training loss and the validation loss are reducing until the final epoch, the model is learning as the number of epochs increases.

#### B. Comparison of Performance metrics of GCN and ME-GCN Models

MAE, MSE, RMSE, R-squared, explained variance, and MAPE were used to evaluate and compare the performance of GCN and ME-GCN models. Table II shows the values of the performance indicators of both models. The ME-GCN model achieves significantly lower MAE, MSE, and RMSE values. This proves that the ME-GCN model predicts values that are much closer to the actual values. Moreover, the ME-GCN model exhibits higher R-squared and explained variance values, indicating its ability to capture the variance in the data. Collectively, these metrics indicate the enhanced ability of the ME-GCN model to capture the dataset's underlying structure.

Figure 6 shows the scatter plot of predicted values to actual values from the GCN model. The plot for the GCN model shows that the points are around the 45-degree line from the origin, which shows that it does a good job at predicting the voltage values. However, there are some points that are scattered away from the diagonal line. On the other hand, the scatter plot of actual and predicted values for the ME-GCN model shown in Fig. 7 indicates a better distribution of the data points. The variance of the predicted value from the actual

TABLE II  
COMPARISON OF PERFORMANCE METRICS

Performance Metrics	ME-GCN	GCN
MAE	1.90e-04	3.60e-04
MSE	7.99e-08	2.74e-07
RMSE	2.82e-04	5.23e-04
R-squared	0.9934	0.9775
Explained Variance	0.9934	0.9776
MAPE	0.0200	0.0362

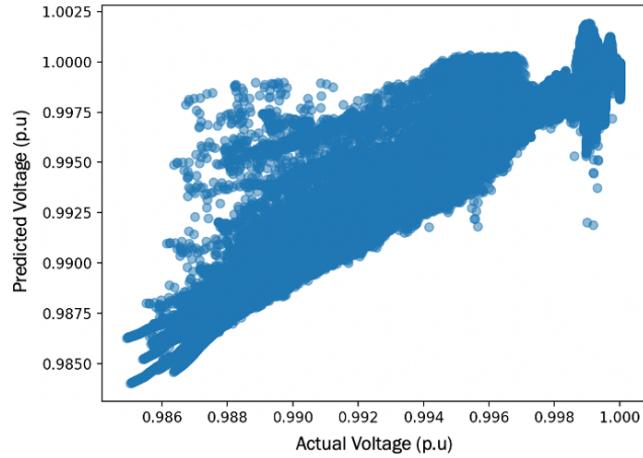


Fig. 6. Actual vs Predicted Voltage for GCN Model

value is highly reduced for the ME-GCN model. Similarly, the error distribution of the predicted voltage values from the GCN model is presented in Fig. 8. The error ranges from negative 0.0025 to positive 0.0025 p.u voltage. Whereas the range of the ME-GCN model's error distribution illustrated in Fig. 9 is in the range of negative 0.001 to positive 0.001 p.u. voltage.

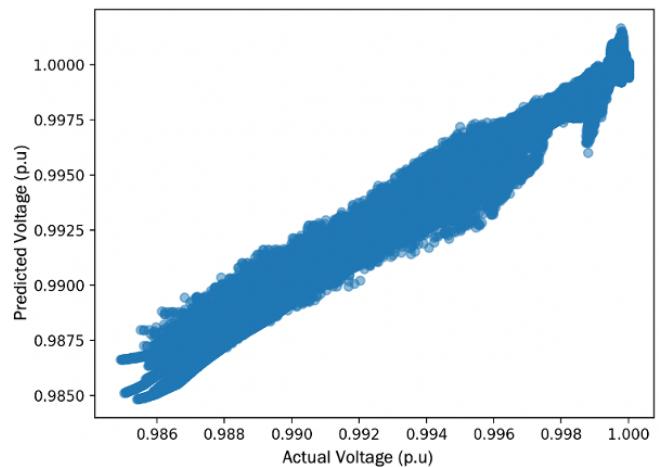


Fig. 7. Actual vs Predicted Voltage for ME-GCN Model

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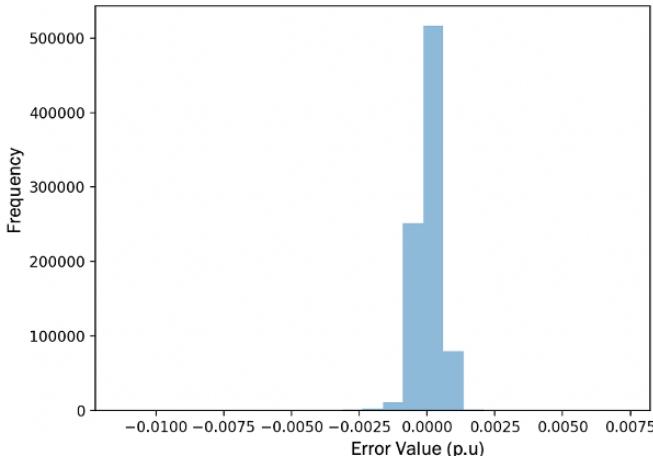


Fig. 8. Error Distribution for GCN Model

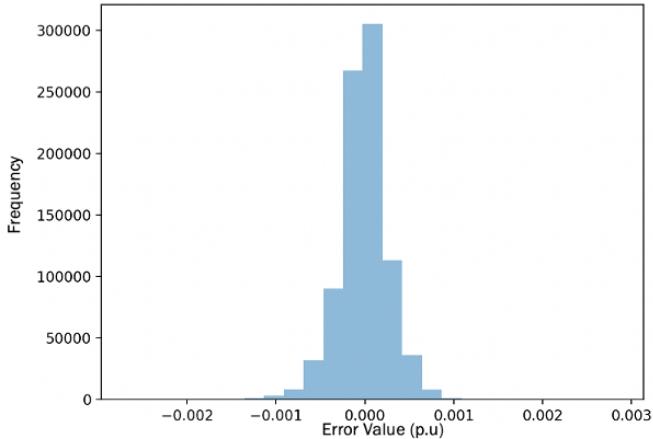


Fig. 9. Error Distribution for ME-GCN Model

## V. CONCLUSION

An ever-evolving power grid with rapid digitization of the grid has created challenges and opportunities to the grid operation. The advancement in machine-learning-based techniques can help capture the complex relationships between known and unknown parameters of the grid. In this paper, an ME-GCN layer customized for the power grid is presented. The layer is then used to build an ME-GCN-based surrogate model for voltage estimation for a distribution network for highly uncertain EV loads. Moreover, a comparative study demonstrates higher accuracy of the ME-GCN model compared to a standard GCN-based surrogate model under the system with high EV charging loads using the same dataset. [8] have demonstrated the ability of GCN to leverage the grid's topology for accurate voltage estimation. Multi-Edge GCN further enhances the capability of GCN by integrating edge attributes. The proposed ME-GCN model is an effective neural network model to represent the static power grid operation in ML-based surrogate models designed for grid applications.