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Abstract

Sum-of-fractions is a method intended to make sure a subcritical margin for aqueous solutions and slurries of fissionable isotopes exists. The method indicates that a system is subcritical if the sum of the ratios of the mass of each isotope (in a mixture) to its individual minimum subcritical mass limit is less than or equal to one. Historically, the basis of the sum-of-fractions has been derived from allowances given in the American National Standards Institute (ANSI)/ American Nuclear Society (ANS)-8.15-1981. However, the allowance was removed in ANSI/ANS-8.15-2014 due to a lack of technical basis.

A methodology was developed to assess the validity of using the sum-of-fractions for water- or polyethylene-moderated systems for the following nuclides: ^{232}U , ^{233}U , ^{234}U , ^{235}U , ^{237}Np , ^{236}Pu , ^{238}Pu , ^{239}Pu , ^{240}Pu , ^{241}Pu , ^{242}Pu , ^{241}Am , $^{242\text{m}}\text{Am}$, ^{243}Am , ^{242}Cm , ^{243}Cm , ^{244}Cm , ^{245}Cm , ^{246}Cm , ^{247}Cm , ^{249}Cf , and ^{251}Cf . The methodology uses available benchmark data for mixtures of ^{233}U , ^{235}U , and ^{239}Pu to establish the calculational margin, and a mass limit reduction to establish the margin of subcriticality. Water- or polyethylene-moderated and -reflected mixtures containing the nuclides are evaluated with the code system, SCALE 6.2.4.

Including the calculational margin, subcritical mass limits for each nuclide were computed for optimally water- or polyethylene-moderated and fully reflected systems. These masses were used to create nuclide mixtures in which the sum of the mass to subcritical mass limit ratios is one. The various nuclide mixtures were modeled over a range of moderation and demonstrate the k_{eff} does not exceed the calculational margin. For additional assurance of subcriticality, a significant mass reduction is applied to each computed minimum critical mass of the nuclides without adequate benchmark data consistent with the method in ANSI/ANS-8.15-2014.

Summary

Sum-of-fractions is a method to make sure a subcritical margin for aqueous solutions and slurries of fissile and fissionable isotopes exists. The method suggests that, for mixtures containing one or more fissile or fissionable isotopes, if the sum of the ratios of the mass of each isotope in the mixture to its individual subcritical mass limit is less than or equal to one, the mixture is assumed to maintain a subcritical margin.

Little information on the safety and conservatisms of the limits produced using sum-of-fractions exists. This work provides a technical foundation for the use of sum-of-fractions for the following nuclides in optimally moderated and fully reflected systems of water and polyethylene: ^{232}U , ^{233}U , ^{234}U , ^{235}U , ^{237}Np , ^{236}Pu , ^{238}Pu , ^{239}Pu , ^{240}Pu , ^{241}Pu , ^{242}Pu , ^{241}Am , $^{242\text{m}}\text{Am}$, ^{243}Am , ^{242}Cm , ^{243}Cm , ^{244}Cm , ^{245}Cm , ^{246}Cm , ^{247}Cm , ^{249}Cf , and ^{251}Cf .

The process of developing the subcritical masses for use in sum-of-fractions involves first establishing a set of initial bias and uncertainty in the bias for the modeling method, in this case the code system, SCALE 6.2.4. From there, a set of minimum subcritical masses are computed for the actinides in optimally moderated and reflected systems. With the computed minimum subcritical masses, mixtures are established to evaluate if any would exceed critical, after considering the bias and uncertainty in bias. Once evaluated, a safety margin is applied through a mass reduction of the computed subcritical masses consistent with the method in ANSI/ANS-8.15-2014. The subcritical masses with the safety margin applied can then be used with the sum-of-fractions method to determine if an aqueous solutions and slurries of fissile and fissionable isotopes are subcritical.

Acknowledgments

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Acronyms and Abbreviations

ANS	American Nuclear Society
ANSI	American National Standards Institute
EALF	energy of average lethargy causing fission
NEA	Nuclear Energy Agency
ORNL	Oak Ridge National Laboratory
SDFs	sensitivity data files
SoF	sum-of-fractions
USL	upper subcritical limit
VALID	Verified, Archived Library of Inputs and Data

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1.0 Introduction

Over the years, methods for addressing multiple nuclides have been devised to minimize efforts in criticality safety analysis. Some of these methods stem from the concept detailed in the American National Standards Institute/American Nuclear Society (ANSI/ANS) standard 8.15-1981 (ANSI/ANS-8.15-1981) section 5.2 statement, "...the sum of the ratios of the mass of each fissile nuclide to its limit does not exceed unity...". In the context of the standard, the statement comes with caveats on the system composition and geometrical configuration; however, some of these methods that are derived extend the use of the sum of the ratios beyond what is permitted in the standard. Three methods of note that derive their basis from this statement are:

- Rule-of-fractions – Uses the ANS-8.15-1981 sum of ratios that applies to water-moderated and water-reflected systems.
- Fissile gram equivalent – A variant of the rule-of-fractions that is restricted to a specific moderated and reflected system.
- Sum-of-fractions (SoF) – A variant of the rule-of-fractions that can be extended to a variety of moderated and reflected systems.

SoF is a method to make sure that a subcritical margin exists for aqueous solutions and slurries of fissile and fissionable isotopes. SoF suggests that, for mixtures containing one or more fissile or fissionable isotopes, if the sum of the ratios of the mass of each isotope in the mixture to its individual subcritical mass limit is less than or equal to one, the mixture is assumed to maintain a subcritical margin. In equation form the method can be defined by:

$$\sum_i \frac{a_i}{A_i} \leq 1 \quad (1)$$

where a_i is the mass of isotope i present and A_i is the mass that corresponds to the minimum subcritical mass for isotope i .

The use of rule-of-fractions was permitted in ANSI/ANS-8.15-1981 for aqueous solutions and slurries reflected by water of unlimited thickness. In part, this was allowed as the mass limits in ANSI/ANS-8.15-1981 were felt to have significant margins that would compensate for any potential anomalous results¹.

However, due to limitations in determining if the limits produced using sum-of-fractions were safe/conservative, the allowance was removed in ANSI/ANS-8.15-2014. The current replacement is for each user to calculate their own subcritical limits for mixtures of materials, resulting in potentially repeated efforts among the criticality safety community. The work described in this analysis provides a technical basis and method for the criticality safety community and permits again the use of a commonly used method for generating limits for mixtures of nuclides. Bias, uncertainty in bias, and subcritical factors in line with ANSI/ANS-8.15-2014 Appendix C methodology for special actinides will be placed on the results to make sure that application of the SoF is subcritical.

¹ The rule-of-fractions assumes that using mixture of actinides based on the minimum subcritical mass will be less reactive than any minimum subcritical mass of actinide at the optimum concentration. There have been reports that sometimes the mix can be slightly more reactive.

The SoF extension is based on the idea that a minimum subcritical mass in a specific system can be calculated for a specific nuclide (i.e., ^{235}U) and the masses for the other nuclides can be included in the rule-of fractions approach if the minimum subcritical masses for all the nuclides are based on a system that is more reactive than the specific system evaluated. An example would be to calculate the minimum subcritical mass of ^{235}U for a water-moderated unreflected system, then using the minimum subcritical mass for water-moderated water-reflected systems for any of the other nuclides that might be mixed in the specific system being evaluated.

This work provides a technical foundation for the use of the SoF for the following nuclides in moderated and fully reflected systems: ^{232}U , ^{233}U , ^{234}U , ^{235}U , ^{237}Np , ^{236}Pu , ^{238}Pu , ^{239}Pu , ^{240}Pu , ^{241}Pu , ^{242}Pu , ^{241}Am , $^{242\text{m}}\text{Am}$, ^{243}Am , ^{242}Cm , ^{243}Cm , ^{244}Cm , ^{245}Cm , ^{246}Cm , ^{247}Cm , ^{249}Cf , and ^{251}Cf . Moderation and reflection are assessed for water and polyethylene.

2.0 Theory

The SoF method of generating a subcritical mass limit for mixtures applies a set of restrictions such that the mixture reactivity is less than the sum of the individual nuclide reactivities. The method uses the minimum subcritical mass limits for each nuclide, which occurs at an optimal moderator density. Mixtures of fissionable nuclides will have a different optimal moderator density than that of the individual nuclides and will result in the individual nuclides being over- or under-moderated in the system. As a result, the mixture should remain subcritical.

It is difficult to assess impacts to the absorption, scattering, and fission reaction rates of the mixtures, given the energy dependencies and potential shifts in the flux spectrum. As a result, additional limits are placed on the subcritical masses through establishing a validation methodology and conservative margins of subcriticality.

2.1 Codes and Tools

The tools used to establish the SoF method were developed using the SCALE code system. SCALE is an Oak Ridge National Laboratory- (ORNL-) managed code suite containing verified and validated tools for criticality safety, reactor physics, radiation shielding, radioactive source characterization, and sensitivity and uncertainty analysis. The work herein uses the criticality safety and sensitivity and uncertainty analysis tools from SCALE version 6.2.4 (Weiselquist et al. 2020) and 6.3.0 (Weiselquist and Lefebvre 2021). Excluding VADER, all modules are present and modeled in SCALE 6.2.4 using the codes' continuous energy cross-section library based on ENDFVII.1 and patch version 1.0.3 for the h-poly data for the polyethylene hydrogen thermal scattering. VADER uses SCALE version 6.3.0. Table 1 summarizes the modeling tools.

Table 1. Modeling tools.

Control Module	Description
CSAS6-KENO-VI	Performs k_{eff} calculations on a 3D system model.
VADER	A new module to SCALE 6.3 that allows the determination of bias and bias uncertainty for criticality safety computational methods.
TSUNAMI-1D and 3D	Provides sensitivity and uncertainty analysis of system responses using generalized perturbation theory either in 1D or 3D models.
TSUNAMI-IP	Utility module that uses sensitivity data generated by TSUNAMI-1D, -2D, or 3D, and cross section covariance data to generate relationships that can be used to determine similarities between two systems.

2.2 Methodology

The limitations in benchmark experiments prevent the direct determination of an upper subcritical limit (USL) for most actinides. To address this limitation, an approach was devised that used benchmarks where available to develop a subcritical margin in conjunction with biases on the actinide masses. The available benchmarks will provide evidence of the method's ability to handle mixtures of fissile isotopes via the calculational margin. The accuracies of the nuclides cross sections are then considered based on benchmark data availability.

First, a set of ^{233}U , ^{235}U , and ^{239}Pu mixtures were established to determine an appropriate bias and uncertainty in bias (This does not include the margin of subcriticality for code and data uncertainties nor the margin of subcriticality for validation applicability.) This is achieved by modeling mixes of the three isotopes near optimal moderation and full reflection. Sensitivity analyses are then performed on these mixes to provide the appropriate bias and bias uncertainty. Although the bias and uncertainty in bias originate from only a few isotopes, the intent of this approach is to demonstrate the code's ability to handle mixtures of fissile isotopes.

Secondly, the minimum subcritical mass for each actinide is computed for an optimally moderated and fully reflected system using SCALE 6.2.4. Calculations at various densities will be performed in the optimally moderated realm to determine the mass at the desired eigenvalue. Following the calculations, a polynomial fit will be used to determine the minimum subcritical mass. Benchmark data for the actinides of ANSI/ANS-8.15-2014 are limited or nonexistent, therefore the bias and bias uncertainty of the individual actinides cannot be evaluated. In lieu of this, the bias and bias uncertainty evaluated for the ^{233}U , ^{235}U , and ^{239}Pu mixture cases is applied when computing the minimum subcritical masses for the actinides. Given that these three isotopes are the most well-studied isotopes out of the actinides, it would be expected that at least the bias and bias uncertainty of ^{233}U , ^{235}U , and ^{239}Pu mixtures would need to be applied to the remaining actinides.

Thirdly, mixtures containing an actinide at 1/6 their evaluated minimum subcritical mass and 5/6 the minimum subcritical mass ^{233}U , ^{235}U , and ^{239}Pu are evaluated in the optimally moderated range. The resulting k_{eff} will be assessed to determine if it falls below critical including the bias and bias uncertainty of the ^{233}U , ^{235}U , and ^{239}Pu mixtures assessed. If it is above, the actinide minimum subcritical mass is reduced, and the case is recomputed. This step checks for and corrects for anomalous mixtures of actinides.

Finally, minimum subcritical masses verified in the previous step are taken and a safety margin is applied. This intends to consider the lack of experimental data as well as reservations associated with the uncertainties of the actinide cross sections. For actinides that do not have available experimental critical benchmarks, a mass penalty will be included on the minimum subcritical mass to ensure subcriticality; consistent with the approach used in Appendix C of ANSI/ANS 8.15-2014.

2.3 Applicability of the Sum-of-Fractions

Use of the SoF should be applied only when using subcritical masses that represent the minimum subcritical mass achievable by an individual fissionable isotope. The optimal densities of the nuclides can vary greatly, which can result in shifts where the optimal reactivity is for a mixture of various nuclides. Assessing mixture masses using critical masses that are not the minimum critical mass of the individual fissionable isotopes can therefore be nonconservative and the method described in this report should not be used for such applications.

The SoF approach is developed using homogenous mixtures of nuclides. Systems that contain heterogenous mixes may be more reactive. Additional evaluations would therefore be needed to determine the applicability of the SoF method for the heterogenous systems of interest.

2.4 Model Description

Models consist of a homogeneous spherical nuclide(s)/moderator mixture surrounded contiguously with a 12-inch-thick reflector. The moderator and reflector consist of either water or

polyethylene. Water theoretical density is set at 0.9982 g/cc. Polyethylene bulk densities vary based on fabrication techniques. Bulk densities of polyethylene range between 0.91 to 0.97 g/cc (Kupolati et al. 2017; Favaro et al. 2017; Suh and White 2007). The crystalline phase or particle densities of polyethylene can reach 1.00 g/cc (Suh and White 2007; Wada and Tsuge 1962; Muller et al. 2009). Therefore, a theoretical density is set at 1.00 g/cc for polyethylene. Theoretical densities of the nuclides were taken from Table 3 of ANSI/ANS-8.1-2014 and from the crystallographic densities in Table 1 of ANSI/ANS-8.15-2014 and are replicated in Table 2 below.

Table 2. Nuclide theoretical densities.

Nuclide	Density (g/cc)
U-233	18.650
U-235	18.810
Pu-239	19.820
U-232	18.681
U-234	18.842
Np-237	20.476
Pu-236	19.601
Pu-238	19.768
Pu-240	19.934
Pu-241	20.017
Pu-242	20.101
Am-241	13.660
Am-242m	13.717
Am-243	13.774
Cm-242	13.407
Cm-243	13.463
Cm-244	13.518
Cm-245	13.574
Cm-246	13.629
Cm-247	13.685
Cf-249	15.110
Cf-251	15.232

Density of the nuclide(s) and moderator mixtures were based on volumetric displacement of the nuclide(s) to the desired moderation for a given mass of nuclide(s). Calculation of the homogeneous mixture density $\rho_{solution}$ with a set of nuclides i of density, $\rho_{nuclide_i}$ is evaluated using the equation:

$$\rho_{solution} = \rho_{moderator} + \sum_i \rho_{nuclide_i}, \quad (2)$$

where the moderator density $\rho_{moderator}$, is evaluated using the equation:

$$\rho_{moderator} = \rho_{TD_{moderator}} \left(1 - \sum_i \frac{\rho_{nuclide_i}}{\rho_{TD_{nuclide_i}}} \right). \quad (3)$$

3.0 Validation

The validation efforts considered three different concentrations of mixtures containing ^{233}U , ^{235}U , and ^{239}Pu , each of which were examined to determine a subcritical limit based on critical experiment benchmark data. The results of this effort were then used to determine the area of applicability based on energy of average lethargy causing fission (EALF) and moderator/reflector type for generating subcritical mass limits.

3.1 Methodology

For validation of each of the concentration mixtures with water and polyethylene, the same general procedure was followed. First, model inputs generated by Pacific Northwest National Laboratory were used to calculate k_{eff} and sensitivity data files (SDFs) using the TSUNAMI-1D sequence in the SCALE 6.2.4 code system using the ENDF/B-VII.1 (Chadwick et al. 2011) 252-group library. Using the TSUNAMI-1D SDFs, the TSUNAMI-IP sequence was used along with the integral parameter c_k to assess the similarity and trends between an application system (a concentration mixture with water or polyethylene) and benchmark SDFs in the Verified, Archived Library of Inputs and Data (VALID) maintained by ORNL (Marshall and Reardon 2013) and the Nuclear Energy Agency (NEA) set of SDFs (NEA 2022). The SCALE 6.2.4 56-group covariance library based primarily on ENDF/B-VII.1 was used in all similarity assessments with no administrative margins applied.

Minimum values for c_k of 0.8 and 0.9 were used as thresholds to identify similar experiments to the application systems. Lists of similar experiments were then examined for statistical analysis of trends and normality with the VADER sequence in SCALE 6.3.0 as well as to generate a calculational margin for a specific concentration mixture. Statistical testing of the TSUNAMI-IP results guided the development of a specific calculational margin based on the description of the data. This process yielded calculational margins based on trending methods (i.e., USL-1), nontrending parametric methods, or nonparametric methods, as shown in Figure 1.

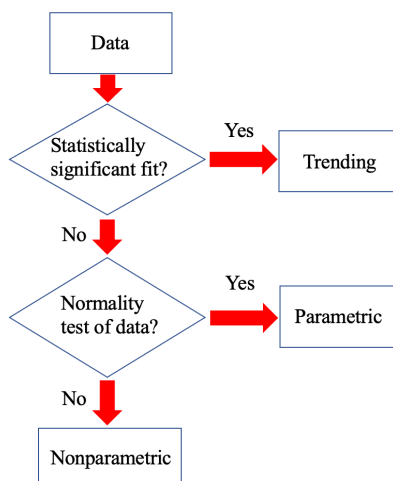


Figure 1. Calculational margin flow chart.

TSUNAMI-IP generates a list of benchmark experiment c_k values for an application system based on a specific threshold, for instance, 0.9. The generated list of c_k values is then examined statistically, using a t -test (Student 1908) to determine whether a significant trend with respect to c_k exists in the dataset. If the null hypothesis is rejected and a significant trend exists, then a trending method for generating the calculational margin is used. However, if the null hypothesis is accepted and no significant trend exists, then the dataset is assessed for normality using a χ^2 test (Pearson 1900). If the dataset is normally distributed, then a parametric method is used to generate the calculational margin; otherwise, a nonparametric method is used. The VADER sequence in SCALE 6.3.0 performs all statistical testing and allows for multiple calculational margins to be calculated based on the preferred validation method. Results from the validation are then used to generate specific nuclide mass limits.

Table 3 presents all mixture combinations evaluated during the validation effort, ranging from pure fissile nuclide down to one-sixth volume fraction. Table 4 presents concentrations for each isotope in grams per cubic centimeter. Each mixture number in Table 3 was evaluated with each concentration in Table 4. For example, mixture 10 was evaluated with densities from concentrations 1, 2, and 3. This process yielded a total of 72 unique mixture–concentration combinations, which were separately evaluated for similarities with experimental benchmarks.

3.2 Sensitivity Coefficients

As noted previously, the TSUNAMI-1D sequence in SCALE 6.2.4 along with the ENDF/B-VII.1 252-group library was used to generate k_{eff} values and SDFs for all concentration–mixture combinations. These mixtures were moderated and reflected by either water or polyethylene in a simple spherical geometry, fitting the capability of the TSUNAMI-1D sequence. An additional limited study confirmed that the multigroup, deterministic methods used in XSDRN—the deterministic transport code used in TSUNAMI-1D—could generate accurate sensitivity data compared with those generated via Monte Carlo calculations with TSUNAMI-3D and the ENDF/B-VII.1 continuous-energy library. Direct perturbation calculations were also performed in both 1D and 3D to support the conclusion that both sequences generate accurate sensitivities. Table 5 presents results of mixture 15 (0.333 ^{233}U /0.333 ^{235}U /0.333 ^{239}Pu) for concentration 1 for 1D-3D sensitivity comparisons for water and polyethylene. In general, for both water and polyethylene systems at all three concentrations, the differences for all mixture–concentration combinations were less than 2% between 1D and 3D calculations, and most were less than 1%. However, for polyethylene mixtures with concentration 3, several mixtures had differences for H-poly greater than 5%.

Table 3. Volume fractions of mixtures used for validation.

Mixture	²³³ U	²³⁵ U	²³⁹ Pu
1	0	0.1667	0.8333
2	0	0	1
3	0	0.3333	0.6667
4	0	0.5	0.5
5	0	0.6667	0.3333
6	0	0.8333	0.1667
7	0	1	0
8	0.1667	0	0.8333
9	0.1667	0.1667	0.6667
10	0.1667	0.3333	0.5
11	0.1667	0.5	0.3333
12	0.1667	0.8333	0
13	0.3333	0	0.6667
14	0.3333	0.1667	0.5
15	0.3333	0.3333	0.3333
16	0.3333	0.6667	0
17	0.5	0	0.5
18	0.5	0.1667	0.3333
19	0.5	0.5	0
20	0.6667	0	0.3333
21	0.6667	0.3333	0
22	0.8333	0	0.1667
23	0.8333	0.1667	0
24	1	0	0

Table 4. Concentrations used for validation.

Concentration	Density (g/cm ³)		
	²³³ U	²³⁵ U	²³⁹ Pu
1	0.0800	0.0807	0.0851
2	0.0495	0.0499	0.0526
3	0.0306	0.0308	0.0323

Table 5. TSUNAMI-1D and -3D comparisons for energy-integrated total sensitivities for mixture 15, concentration 1.

Nuclide	TSUNAMI-1D	TSUNAMI-3D	Difference	Percent difference
Water	Sensitivity	Sensitivity	Δ	%
¹ H	0.44068	0.44028 $\pm$ 0.00156	0.00040	0.09
¹⁶ O	0.07303	0.07248 $\pm$ 0.00047	0.00056	0.76
²³³ U	0.05991	0.05985 $\pm$ 0.00016	0.00006	0.10
²³⁵ U	0.03492	0.03498 $\pm$ 0.00015	0.00006	0.18
²³⁹ Pu	0.04943	0.04904 $\pm$ 0.00021	0.00039	0.78
Polyethylene	Sensitivity	Sensitivity	Δ	%
C	0.08438	0.08299 $\pm$ 0.00053	0.00139	1.65
H-poly	0.40674	0.40011 $\pm$ 0.00168	0.00663	1.63
²³³ U	0.06502	0.06568 $\pm$ 0.00015	0.00066	1.02
²³⁵ U	0.04169	0.04212 $\pm$ 0.00016	0.00044	1.05
²³⁹ Pu	0.06369	0.06485 $\pm$ 0.00020	0.00117	1.83

3.3 Similarity Assessment

After all mixture-concentration combination SDFs were generated, the TSUNAMI-IP sequence was used to assess the similarity between the application models (water and polyethylene systems) and benchmark experiments available in VALID and in the International Criticality Safety Benchmark Evaluation Project (ICSBE) Handbook (NEA 2022) with the integral parameter c_k . Although some ²³³U-fueled benchmarks included in VALID have no associated SDFs, the SDFs have been previously generated and are available for use while being under review. The integral parameter c_k assesses the similarity between systems based on the shared cross section uncertainty and is represented as a correlation coefficient. For assessment and trending purposes, c_k threshold values of 0.8 and 0.9 were used to select benchmarks to be used for validation. The c_k values above 0.8 are thought to be marginally similar, whereas values above 0.9 are thought to have a high degree of similarity (Rearden et al. 2017).

3.4 Statistical Testing and Calculational Margin Generation

Once c_k lists were generated for all mixture-concentration combinations, the VADER sequence was used for statistical testing, including trending analysis on c_k and normality testing, and the generation of calculational margins needed for specific nuclide mass limits. This section describes the process for generating each specific calculational margin value.

First, a t -test assesses each c_k list for any significant trends in the data: the default α value of 0.05 represents a 95% confidence interval. If a specific c_k list has a statistically significant trend, then a calculational margin is generated using the trending method of USL-1, which is the confidence band with administrative margin method¹ (Dean and Tayloe 2001), using the default α value and no administrative margin for the VADER input.

If the initial t -test statistic is not significant, then the list is assessed for normality using a χ^2 test with the default parameter setting of an α value of 0.05. Lists that are normally distributed have calculational margins generated with the lower tolerance limit approach¹ (Dean and Tayloe 2001). If they are not normally distributed, then a nonparametric method as described elsewhere is used¹. Table 6 and Table 7 present results from water and polyethylene validation for both c_k thresholds of 0.8 and 0.9 using SDFs available from VALID. Similar trends were also noted in the NEA SDFs, but those results are excluded from this discussion for brevity.

As can be observed in both Table 6 and Table 7, several mixture values either have no similar corresponding experiments, or the c_k values are below the 0.8 or 0.9 cutoff for assessment. This trend is more noticeable in polyethylene systems for which VALID had only one mixture with any experiments above a c_k of 0.9. More experiments begin to appear as the c_k threshold decreases to 0.8. Although the NEA results were analogous to the VALID results for water-moderated and -reflected systems, several more applications had calculational margins generated for the polyethylene systems with the results found in Table 8. See Appendix A for a complete list of similarity assessments for all concentration-mixture combinations with water and polyethylene.

¹ J. B. Clarity. "Determination of Bias and Bias Uncertainty for Criticality Safety Computational Methods," In press

Table 6. Water validation for concentration 1 from VALID.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	Yes	Yes	0.9889	95	Yes	Yes	0.9891	86
2	Yes	Yes	0.9888	94	Yes	Yes	0.9893	86
3	Yes	Yes	0.9889	94	Yes	Yes	0.9877	85
4	Yes	Yes	0.9883	86	—	—	—	0
5	No	Yes*	0.9864 [†]	27	—	—	—	0
6	No	Yes	0.9869 [†]	69	No	Yes	0.9855 [†]	44
7	Yes	No	0.9901	83	No	Yes	0.9856 [†]	52
8	Yes	Yes	0.9880	90	Yes	Yes	0.9891	86
9	Yes	Yes	0.9888	94	Yes	Yes	0.9886	86
10	Yes	Yes	0.9873	87	No	Yes*	0.9837 [†]	28
11	—	—	—	0	—	—	—	0
12	No	Yes	0.9875 [†]	78	No	Yes	0.9855 [†]	48
13	Yes	Yes	0.9875	87	Yes	Yes	0.9878	85
14	Yes	Yes	0.9874	87	No	Yes*	0.9837 [†]	28
15	—	—	—	0	—	—	—	0
16	No	Yes	0.9859 [†]	59	No	Yes*	0.9864 [†]	20
17	Yes	Yes	0.9883	86	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	No	Yes	0.9863 [†]	59	—	—	—	0
21	Yes	No	0.9829	142	—	—	—	0
22	Yes	No	0.9820	170	Yes	No	0.9826	112
23	Yes	No	0.9818	174	Yes	No	0.9828	133
24	Yes	No	0.9815	175	Yes	No	0.9833	152

*Test for normality may be unreliable because of insufficient data; [†]parametric; [^]nonparametric; otherwise, uses USL-1.
CM = calculational margin

Table 7. Polyethylene validation for concentration 1 from VALID.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	Yes	Yes	0.9888	86	—	—	—	0
2	Yes	Yes	0.9888	86	—	—	—	0
3	—	—	—	0	—	—	—	0
4	—	—	—	0	—	—	—	0
5	—	—	—	0	—	—	—	0
6	No	Yes	0.9855 [†]	44	—	—	—	0
7	Yes	Yes	0.9882	48	—	—	—	0
8	Yes	Yes	0.9886	84	—	—	—	0
9	Yes	Yes*	0.9856	17	—	—	—	0
10	—	—	—	0	—	—	—	0
11	—	—	—	0	—	—	—	0
12	Yes	Yes	0.9877	46	—	—	—	0
13	—	—	—	0	—	—	—	0
14	—	—	—	0	—	—	—	0
15	—	—	—	0	—	—	—	0
16	—	—	—	0	—	—	—	0
17	—	—	—	0	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	—	—	—	0	—	—	—	0
21	N/A	Yes*	0.9209	3	—	—	—	0
22	Yes	Yes	0.9783	48	—	—	—	0
23	No	No	0.9555	140	—	—	—	1
24	Yes	No	0.9802	163	No	Yes*	0.9703 [†]	35

*Test for normality may be unreliable because of insufficient data; [†]parametric; ^nonparametric; otherwise, uses USL-1.
CM = calculational margin

3.5 Application to Minimum Actinide Mass Limits

Given the results from Table 6, Table 7, and Table 8, not all mixture combinations for both water and polyethylene systems had sufficient benchmarks to determine a bias and uncertainty in the bias. However, those mixtures with sufficient data available gave validity to determine calculational margins of 0.02 ($k_{eff}=0.98$) for water systems and 0.035 ($k_{eff}=0.965$) for polyethylene systems. The validation method described herein helps to demonstrate that mixtures of well-validated fissile nuclides can be subcritical provided additional adequate margins of subcriticality are included.

Table 8. Polyethylene validation for concentration 1 from NEA dataset.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	Yes	No	0.9862	432	Yes	No	0.9847	89
2	Yes	No	0.9855 [†]	453	No	No	0.9770 [^]	100
3	Yes	No	0.9797	116	Yes	No	0.9473	68
4	Yes	No	0.9855	75	—	—	—	0
5	—	—	—	0	—	—	—	0
6	No	Yes	0.9824 [†]	343	No	Yes*	0.9726 [†]	11
7	Yes	No	0.9851	600	No	Yes	0.9786 [†]	76
8	Yes	No	0.9861	424	Yes	No	0.9847	89
9	Yes	No	0.9835	229	Yes	No	0.9651	76
10	No	No	0.9771 [^]	102	—	—	—	0
11	—	—	—	0	—	—	—	0
12	Yes	No	0.9842	454	No	Yes*	0.9734 [†]	17
13	Yes	No	0.9797	114	Yes	No	0.9362	58
14	No	No	0.9771 [^]	102	—	—	—	0
15	—	—	—	0	—	—	—	0
16	Yes	Yes	0.9847	51	—	—	—	1
17	Yes	No	0.9855	77	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	—	—	—	0	—	—	—	0
21	No	Yes*	0.9694 [†]	9	—	—	—	0
22	Yes	Yes	0.9821	122	—	—	—	0
23	Yes	Yes	0.9816	182	No	Yes*	0.9675 [†]	8
24	Yes	Yes	0.9813	205	Yes	Yes	0.9762	68

*Test for normality may be unreliable because of insufficient data; [†]parametric; [^]nonparametric; otherwise, uses USL-1.

CM = calculational margin

3.6 Energy of Average Lethargy Causing Fission

From the validation cases, a validation applicability was determined for water and polyethylene systems based on a specific range of EALF values.

The EALFs ranged from 0.0349 to 0.0904 eV for the water moderated/reflected validation cases (Figure 2). For the polyethylene moderated/reflected cases, the EALF range was 0.0339 to 0.0819 eV (Figure 3). As shown in Figure 4, the total cross sections of the nuclides up to an energy of approximately 0.1 eV contain no resonances. Since the validation cases cover ranges prior to the resonance regions, the validation applicability is for energies up to 0.1 eV.

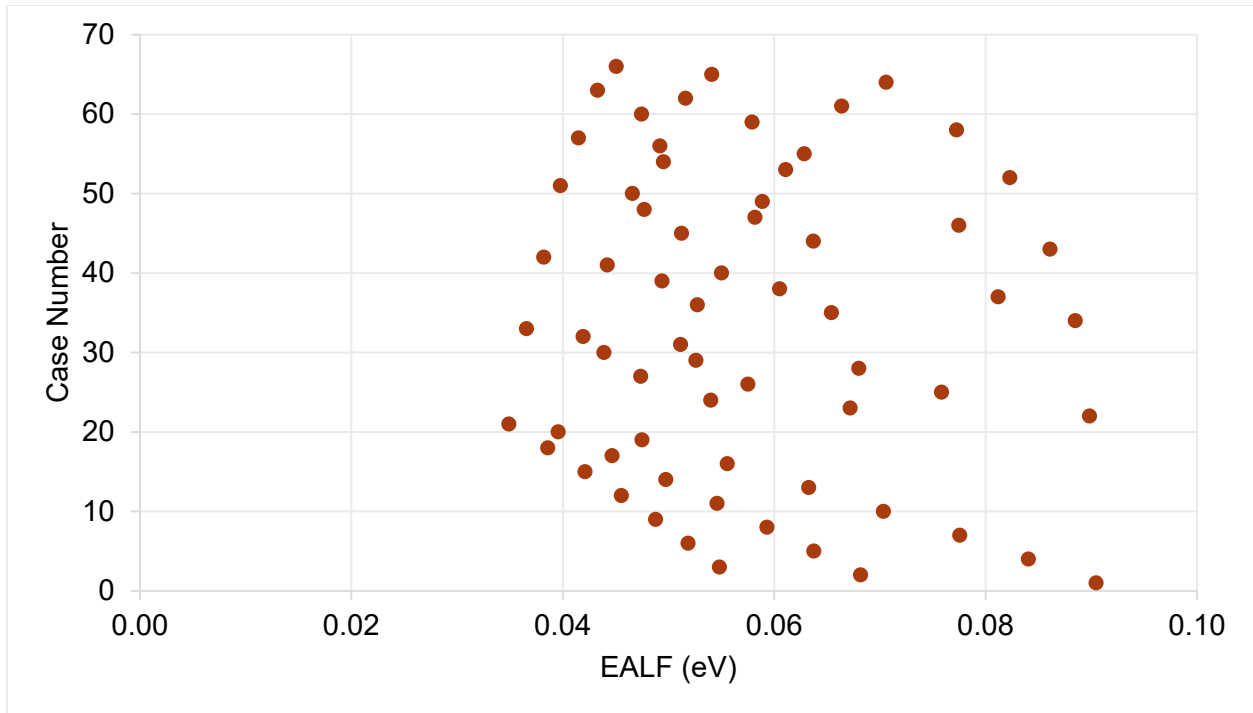


Figure 2. EALF values for water-moderated and -reflected validation cases.

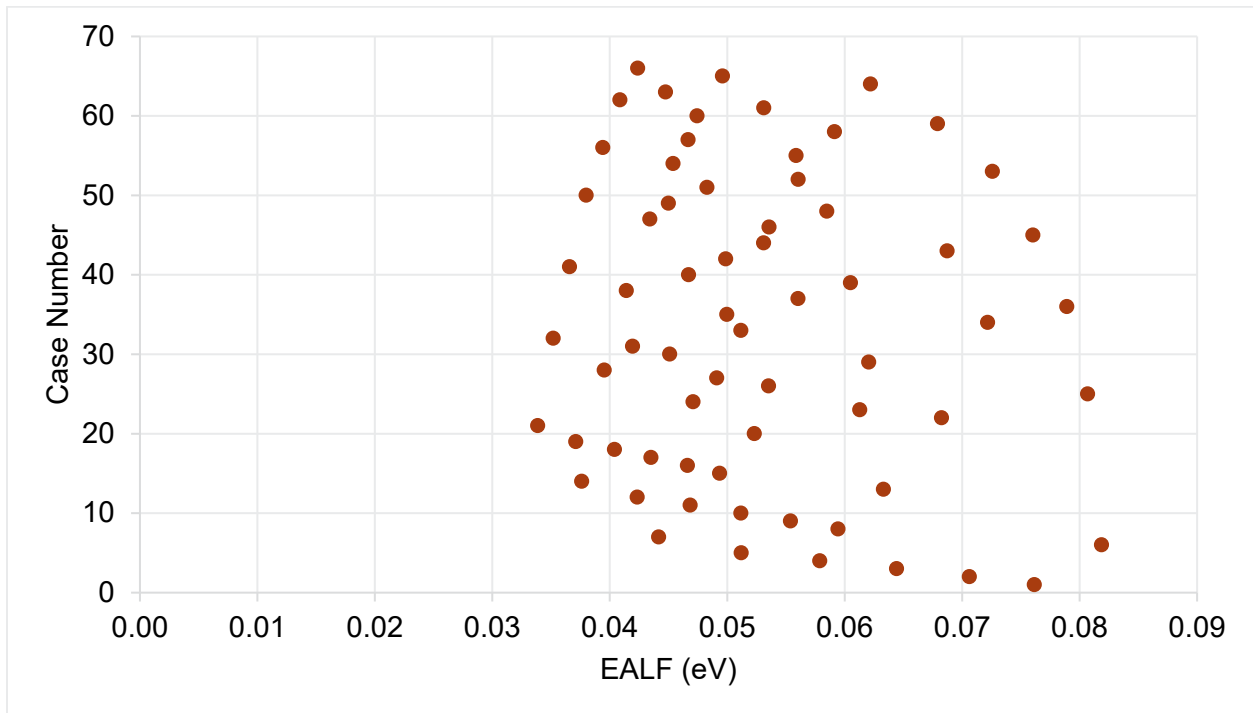


Figure 3. EALF values for polyethylene-moderated and -reflected validation cases.

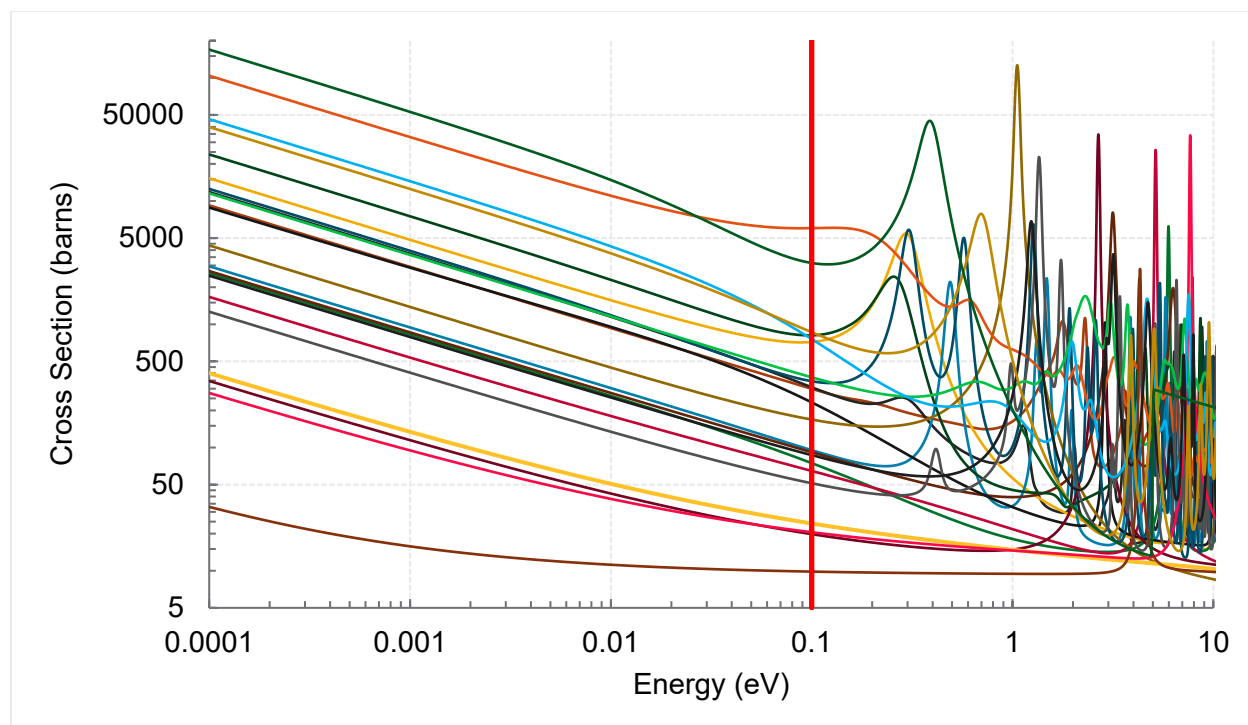


Figure 4. Nuclides total cross sections. Red vertical line indicates upper EALF range for validation applicability.

It is important to note that the cross section data for some of the nuclides present in ANSI/ANS 8.15 are limited at higher energies, resulting in large uncertainties. The number of cross section data points below 0.1 eV, however, are consistent among all the ANSI/ANS-8.1 and -8.15 nuclides. As an example, take the nuclides ^{235}U and ^{251}Cf . The SCALE ENDF/B-VII.1 ^{235}U total cross section library contains over 110,000 data points with approximately 200 data points up to 0.1 eV. The ENDF/B-VII.1 ^{251}Cf total cross section library contains approximately 1,000 data points with approximately 200 data points up to 0.1 eV. The visualization of the differences can be seen in Figure 5, where only one resonance is visible for ^{251}Cf . Figure 6 provides the number of cross section data points below 0.1 eV in relation to the cumulative number of data points over the entire energy spectrum for all nuclides.

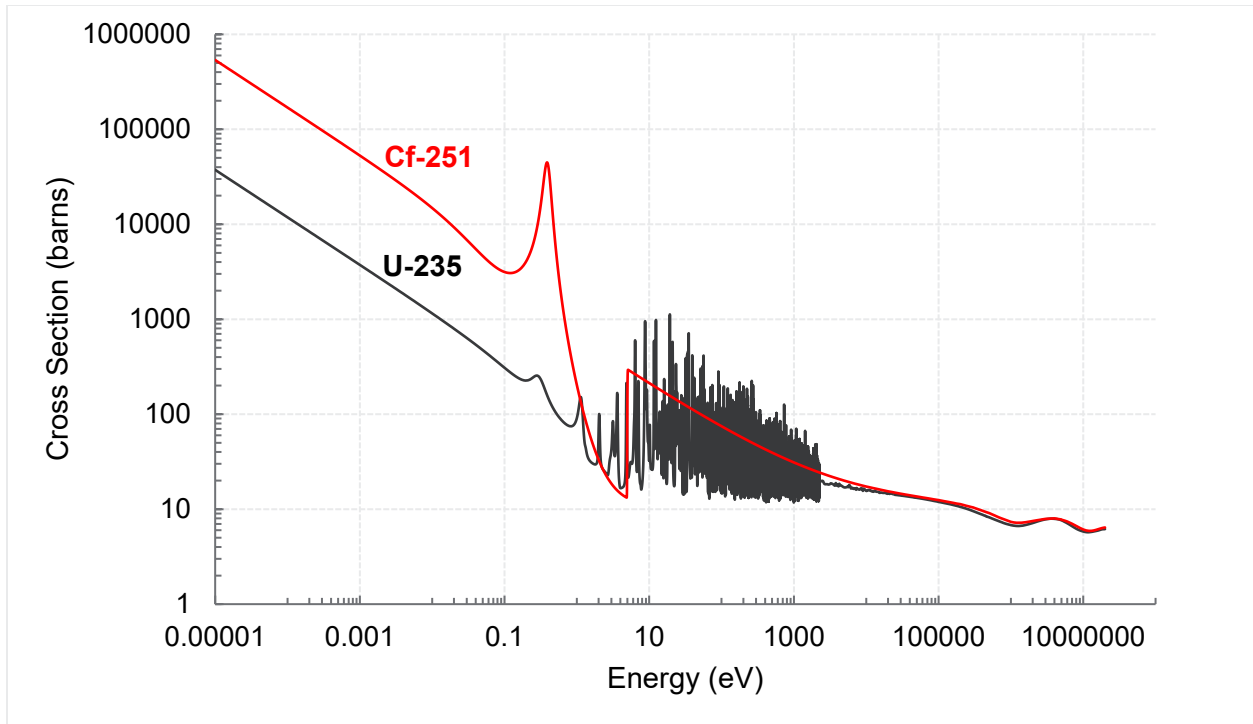


Figure 5. ^{235}U and ^{251}Cf total cross sections.

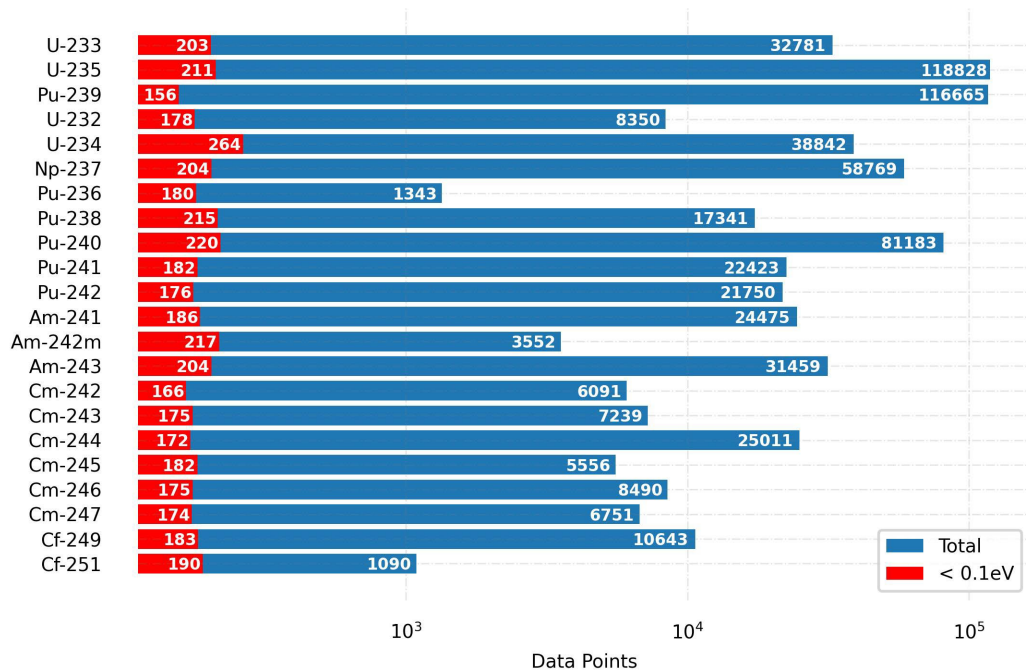


Figure 6. Number of data points for total cross sections of nuclides at 293 Kelvin in ENDF/B-VII.1.

The consistency in data below 0.1 eV (in conjunction with low uncertainties) provides some degree of confidence in results with systems containing EALFs falling below 0.1 eV, however two concerns remain. The first is few to no benchmarks for these nuclides demonstrate the accuracy of the cross sections. The second being that the minimum critical masses in ANSI/ANS-8.15 occur in moderated or unmoderated systems. As a result, some of the computed minimum critical masses will be in systems above an EALF of 0.1 eV and fall outside of the validation applicability. While this does not affect the reliability of the results computed in the validation applicability, the computed minimum critical masses falling in unmoderated systems may be dependent on cross-sectional data with large uncertainties. To combat these two concerns, a mass penalty will be applied to the computed minimum subcritical masses of the nuclides.

4.0 Minimum Subcritical Masses

Minimum subcritical masses were computed using 10,000 neutrons per generation for 200 active generations and 50 skipped generations. Masses were computed for an eigenvalue of 0.9775 ± 0.0025 for water-moderated and -reflected systems and 0.9625 ± 0.0025 for polyethylene-moderated and -reflected systems. Minimum subcritical masses for fissile isotopes were estimated based on a polynomial fit to data over the data range of densities. These masses are used to generate minimum subcritical mass mixtures computed and those estimated with the sum of fractions. Values presented are rounded down to the nearest 1000 grams for isotopes with subcritical masses that exceed 10,000 grams, 100 grams for isotopes with subcritical masses that exceed 1000 grams, and 50 grams for isotopes subcritical masses that exceed 100 grams. The isotopes with subcritical masses below 100 grams are rounded down to the nearest gram. The resulting minimum subcritical masses are presented in Table 9.

Excluding ^{232}U , all fissile nuclides minimum critical masses were in moderated systems and the fissible¹ nuclide minimum critical masses were in unmoderated systems. Although ^{232}U is a fissile nuclide, the minimum critical mass was in the unmoderated region.

Table 9. Subcritical masses for water- and polyethylene-moderated and -reflected systems.

Nuclide	Water Mass (grams)	Polyethylene Mass (grams)
U-233	500	250
U-235	700	400
Pu-239	450	250
U-232	3300	2900
U-234	100000	90000
Np-237	47000	44000
Pu-236	1000	600
Pu-238	6300	5800
Pu-240	31000	29000
Pu-241	250	100
Pu-242	59000	55000
Am-241	56000	52000
Am-242m	21	11
Am-243	120000	108000
Cm-242	9800	9000
Cm-243	200	100
Cm-244	20000	19000
Cm-245	58	33
Cm-246	65000	60000
Cm-247	1100	650
Cf-249	58	33
Cf-251	27	15

¹ Fissible refers to a nuclide that cannot support a slow-neutron chain reaction but is only capable of a fast neutron chain reaction, provided that the effective fast-neutron production cross section exceeds the effective fast neutron removal cross section (Pruvost et al. 2004).

5.0 Mixture Evaluation

Calculations were performed where the fissionable mass of the system was set such that the sum of fractions equates to one. Three sets of two or more fissionable isotopes were assessed.

5.1.1 Two Nuclides

The first set consisted of mixtures containing 5/6 of the minimum subcritical mass of either ^{233}U , ^{235}U , or ^{239}Pu and 1/6 the minimum subcritical mass of one of the nuclides from ANSI/ANS-8.15. Results as indicated in Table 10 and Table 11 show that the peak $k_{\text{eff}} + 2\sigma$ does not exceed the calculational margins for the water- or polyethylene-moderated and -reflected systems.

Table 10. Peak $k_{\text{eff}} + 2\sigma$ for water-moderated and -reflected systems.

Nuclides	Peak $k_{\text{eff}} + 2\sigma$		
	U-233	U-235	Pu-239
U-232	0.94460	0.95325	0.95936
U-234	0.69193	0.68034	0.68686
Np-237	0.68804	0.67331	0.68838
Pu-236	0.97117	0.97020	0.97235
Pu-238	0.66346	0.61320	0.66999
Pu-240	0.68379	0.65930	0.67622
Pu-241	0.97272	0.97275	0.97387
Pu-242	0.75434	0.73505	0.74328
Am-241	0.66221	0.65124	0.66054
Am-242m	0.96940	0.96659	0.96620
Am-243	0.68449	0.68054	0.68890
Cm-242	0.86693	0.88840	0.90021
Cm-243	0.97070	0.96670	0.96969
Cm-244	0.78007	0.81321	0.83216
Cm-245	0.96671	0.96500	0.96487
Cm-246	0.86514	0.88055	0.88915
Cm-247	0.97597	0.97401	0.97720
Cf-249	0.96783	0.96444	0.96513
Cf-251	0.97240	0.96958	0.97093

Table 11. Peak $k_{\text{eff}}+2\sigma$ for polyethylene-moderated and -reflected systems.

Nuclides	Peak $k_{\text{eff}}+2\sigma$		
	U-233	U-235	Pu-239
U-232	0.91788	0.95464	0.95785
U-234	0.65213	0.65001	0.65339
Np-237	0.65988	0.65371	0.66329
Pu-236	0.93390	0.96216	0.96080
Pu-238	0.63505	0.60340	0.63979
Pu-240	0.65067	0.63757	0.64615
Pu-241	0.92016	0.95183	0.95114
Pu-242	0.71013	0.70779	0.71319
Am-241	0.64177	0.63613	0.64149
Am-242m	0.92844	0.95487	0.95351
Am-243	0.65719	0.65839	0.66441
Cm-242	0.79658	0.85772	0.87052
Cm-243	0.92501	0.95282	0.95283
Cm-244	0.71151	0.75483	0.77501
Cm-245	0.92613	0.95339	0.95102
Cm-246	0.80447	0.85556	0.86265
Cm-247	0.93719	0.96479	0.96445
Cf-249	0.92713	0.95415	0.95145
Cf-251	0.93252	0.96118	0.95740

Three examples of the reactivity curves for water-moderated and -reflected systems as a function of density are shown in Figure 7 through Figure 9. For systems containing two fissile materials such as ^{239}Pu and ^{241}Pu , the minimum critical mass is in the moderated regime with a reactivity close to the calculational margin (Figure 7). However, for cases in which the critical mass occurs at moderations that vary greatly, such as for ^{235}U and ^{237}Np , the maximum reactivity shifts towards the unmoderated region (Figure 8).

One unique mixture is a fissile-only mixture containing ^{232}U . Although the minimum critical mass of ^{232}U is highest when unmoderated, it has a local minimum critical mass at the moderated range that is relatively close to the minimum critical mass. This has the potential to result in a mixture containing ^{232}U to have a minimum critical mass in a moderated region as opposed to the other nuclides where minimum critical mass is in the unmoderated region. An example of the reactivity curve for a water-moderated and -reflected system containing ^{233}U and ^{232}U is provided in Figure 9.

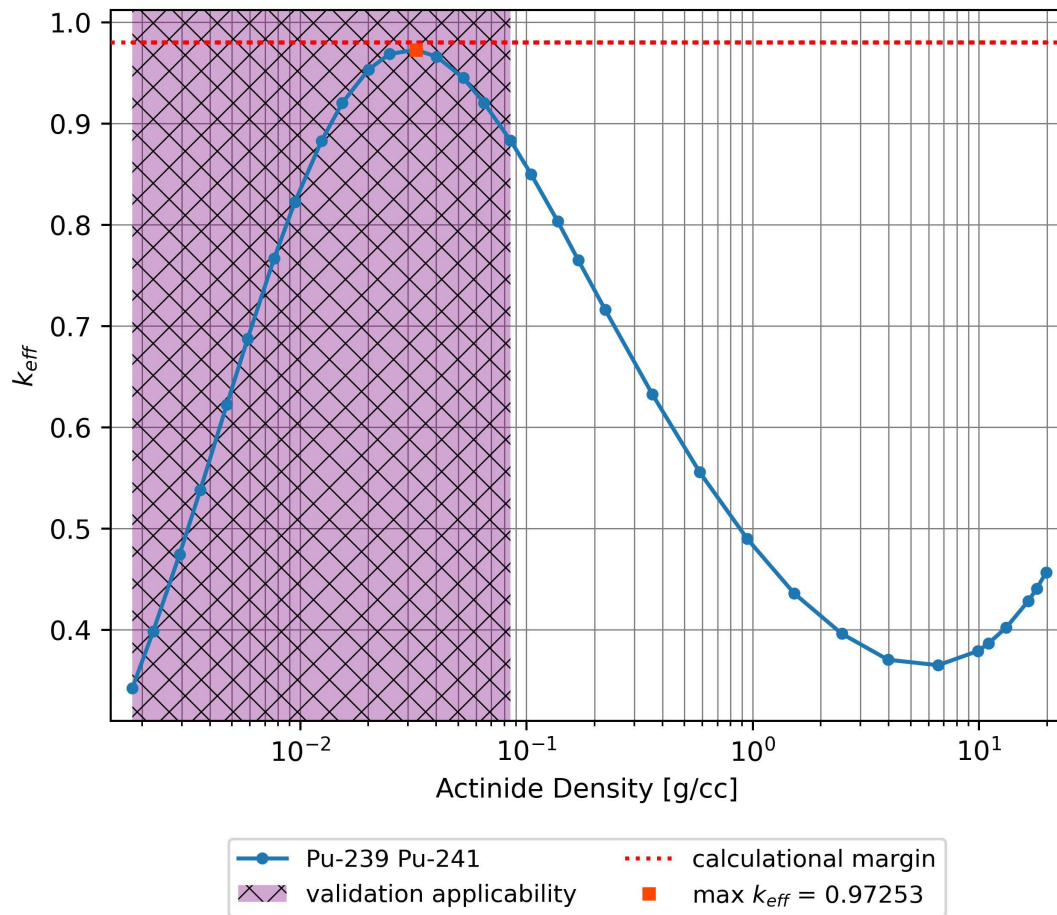


Figure 7. Reactivity curve of ^{239}Pu and ^{241}Pu mixture.

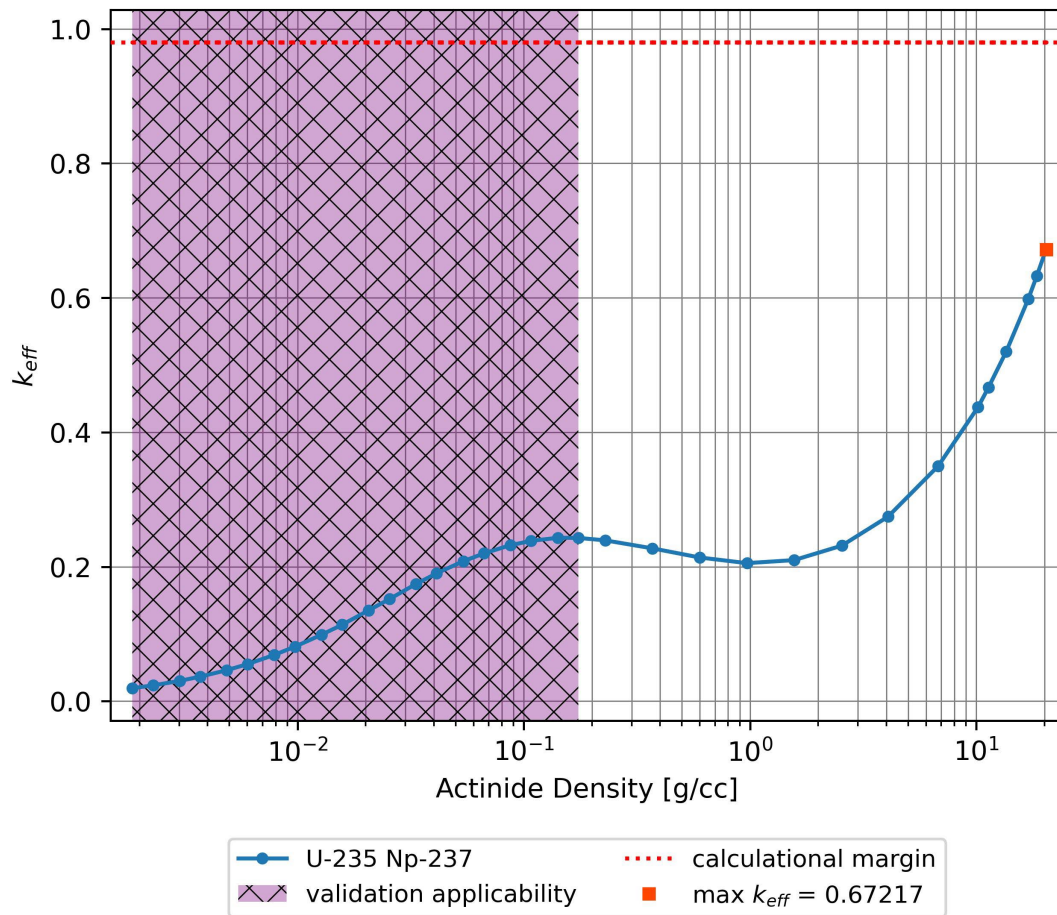


Figure 8. Reactivity curve of ^{235}U and ^{237}Np mixture.

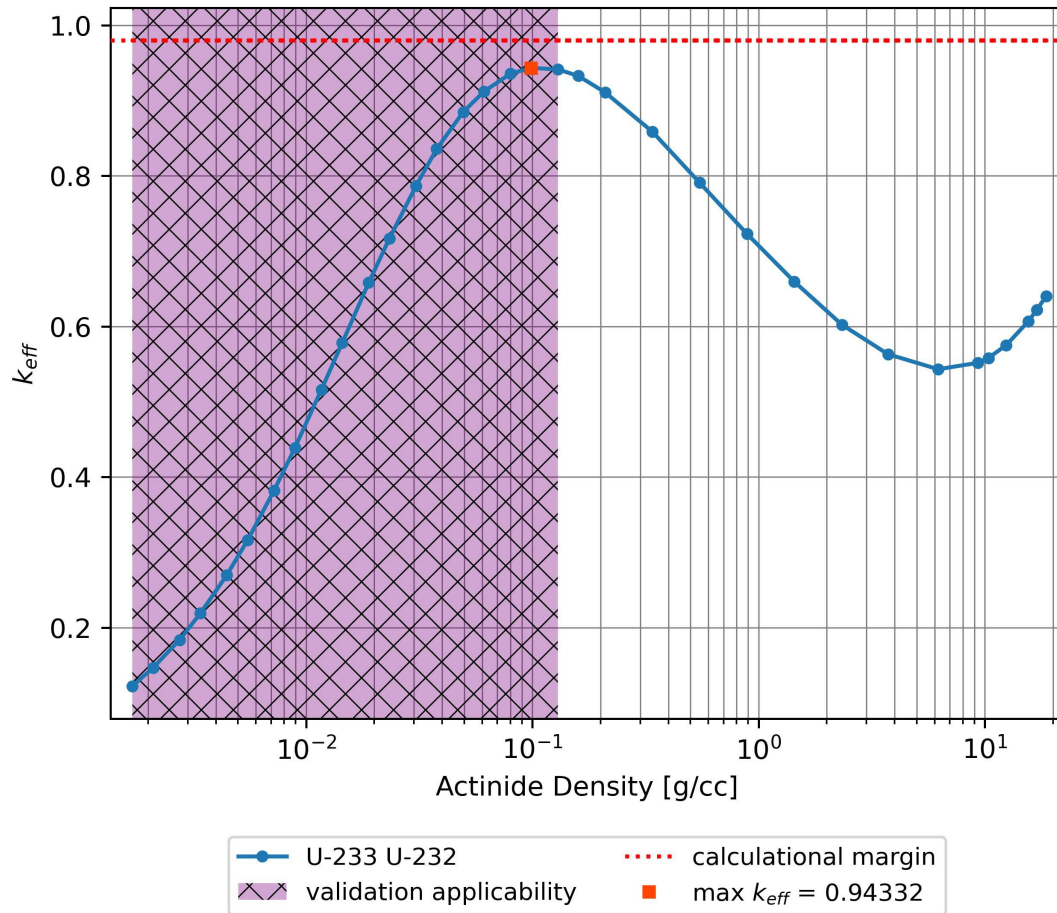


Figure 9. Reactivity curve of ^{233}U and ^{232}U mixture.

5.1.2 Mixture Evaluation – All Nuclides

The second set consisted of 100 random mixtures of all fissionable nuclides in which the SoF mass equates to 1 (see Appendix B for mass fractions of nuclides). The random set of mixes were modeled in both water- and polyethylene-moderated and -reflected systems. The reactivity curves as a function of actinide density are shown in Figure 10 and Figure 11. The peak reactivity of the mixtures occurs in the unmoderated region indicating that the fissionable nuclides drive the reactivity of the systems due to their large subcritical masses in relation to the fissile masses. No mixtures were found to be above the calculational margin.

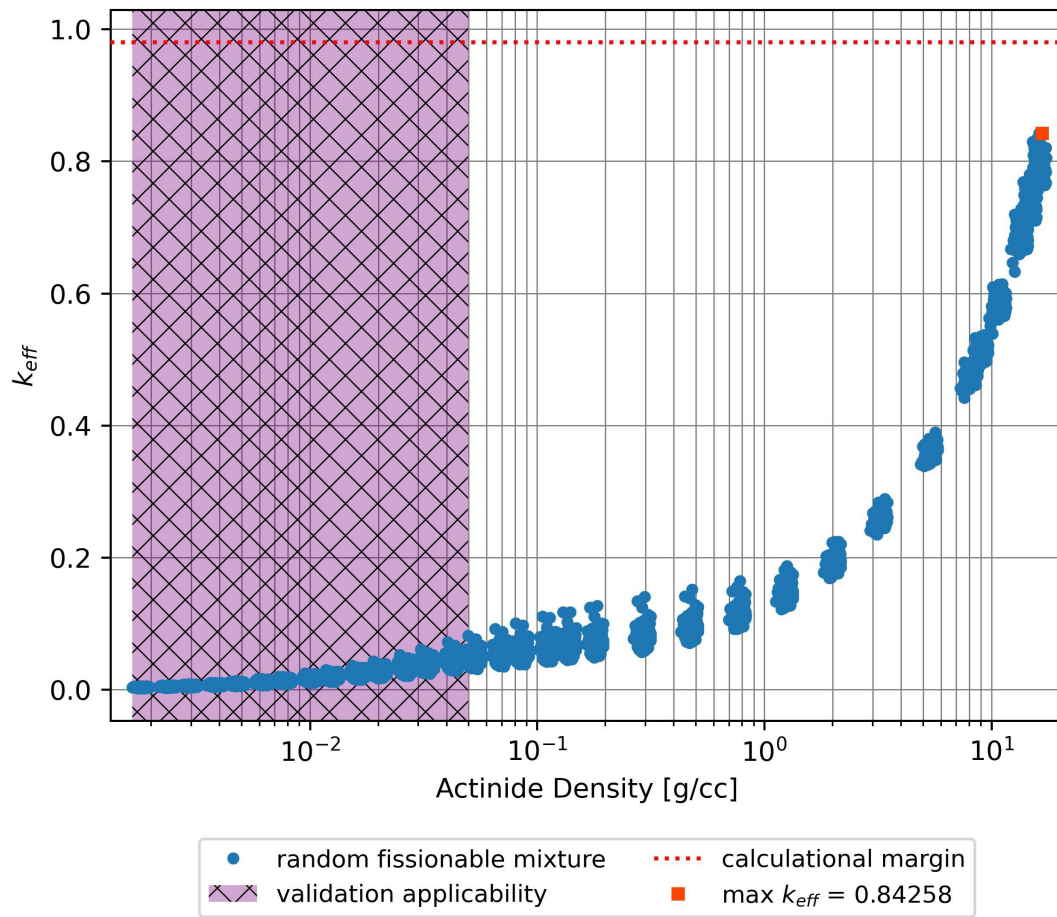


Figure 10. Reactivity curve for water-moderated and -reflected fissionable mixtures.

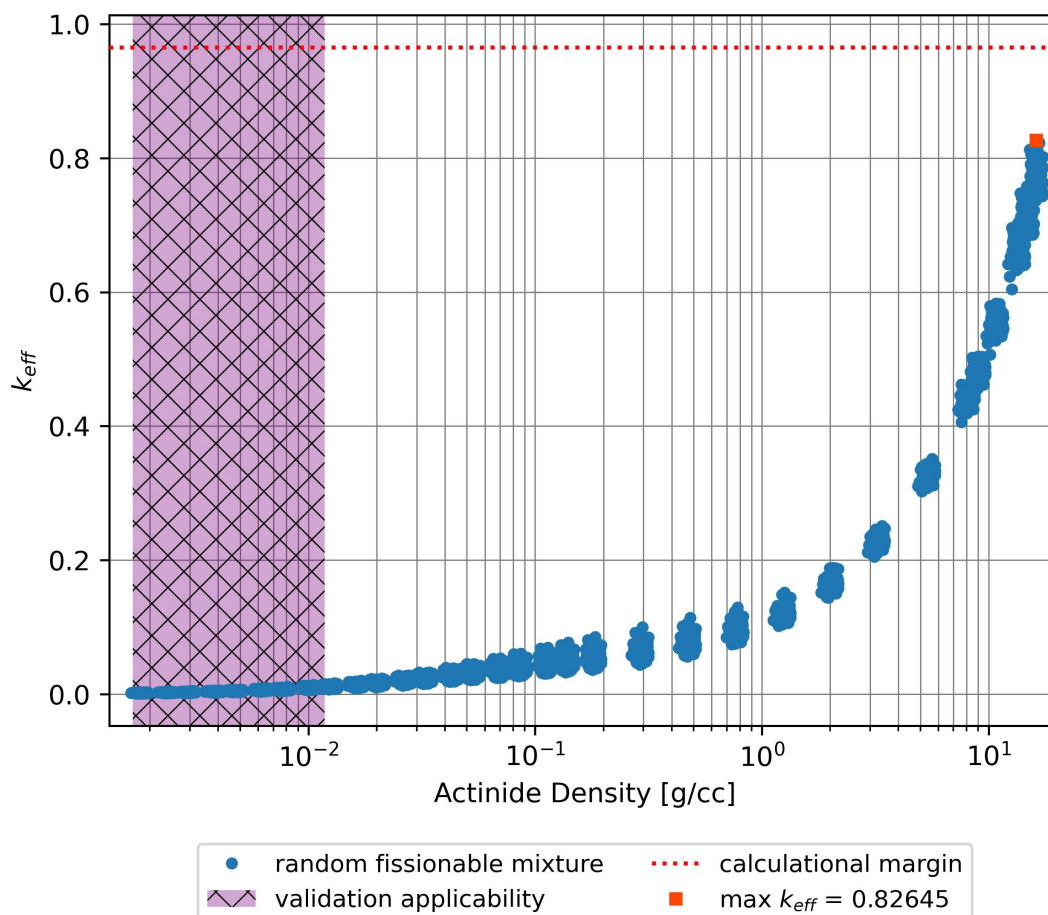


Figure 11. Reactivity curve for polyethylene-moderated and -reflected fissionable mixtures.

5.1.3 Mixture Evaluation – Fissile Nuclides

The third set consisted of 100 random mixes of fissile nuclides in which the SoF mass equates to 1 (see Appendix B for mass fractions of nuclides). The random set of mixes were modeled in both water- and polyethylene-moderated and -reflected systems. The reactivity curves as a function of actinide density are shown in Figure 12 and Figure 13. The reactivity of the mixtures has a peak reactivity in the moderated region. Variation in the mixture composition can result in reactivity changes of 30%, depending on the moderation present. As the moderation approaches optimal for the mixtures, the variation in the reactivities falls to around 10%. Regardless of the variations in the reactivity, no mixtures were found to be above the calculational margin.

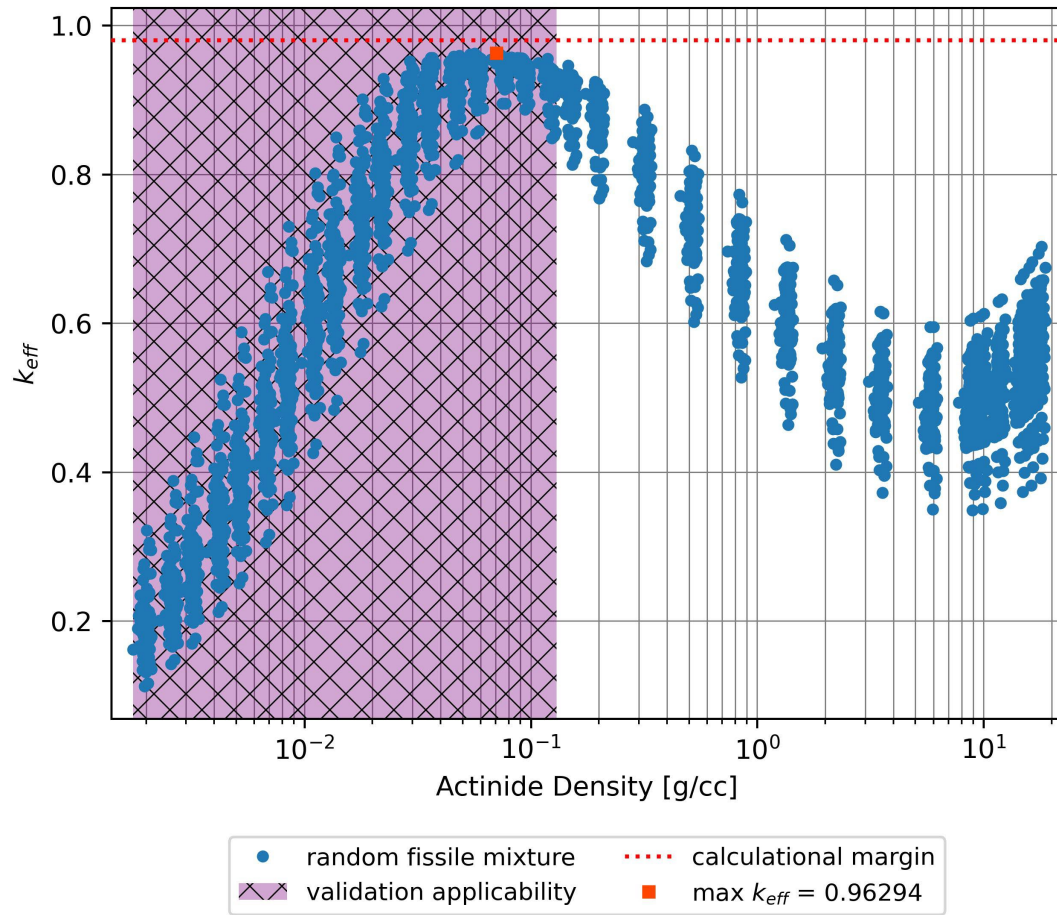


Figure 12. Reactivity curve for water-moderated and -reflected fissile mixtures.

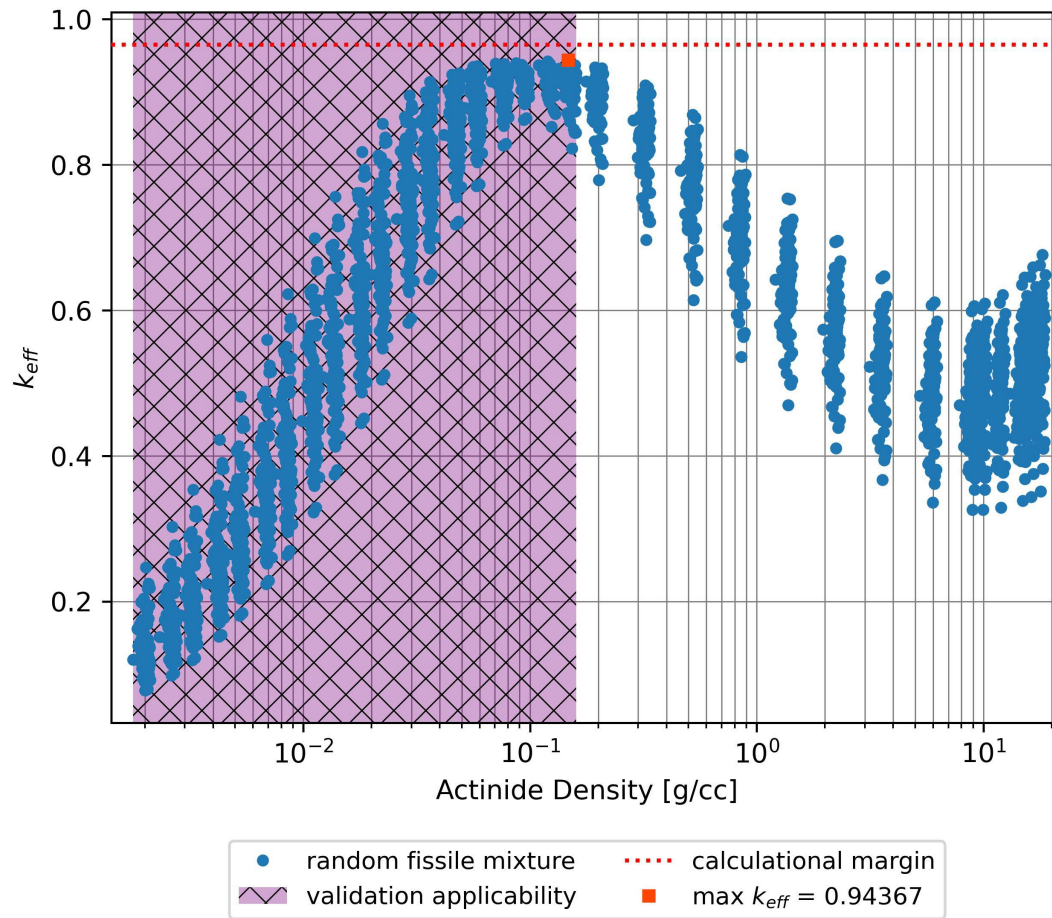


Figure 13. Reactivity curve for polyethylene-moderated and -reflected fissile mixtures.

6.0 Margin of Subcriticality

To address the lack of benchmarks and uncertainties in cross sections for the ANSI/ANS-8.15 nuclides, a mass penalty is applied to the computed subcritical masses via a subcritical factor consistent with Appendix C of ANSI/ANS-8.15-2014. This subcritical factor is extended to polyethylene-moderated and -reflected systems. The subcritical masses computed are then compared to the subcritical masses in ANSI/ANS-8.15-2014 and the lower minimum critical mass for each given actinide is selected as the subcritical mass for use with SoF. Table 12 and Table 13 provide the resulting masses.

Table 12. Subcritical masses for sum-of-fractions in water-moderated and -reflected systems.

Nuclide	Computed mass (g)	Factors	Computed w/ factors (g)	ANSI/ANS 8.15 (g)	SoF mass (g)
U-233	500	1	500	--	500
U-235	700	1	700	--	700
Pu-239	450	1	450	--	450
U-232	3300	0.5	1650	1000	1000
U-234	100000	0.5	50000	59000	50000
Np-237	47000	0.7	32900	35000	32900
Pu-236	1000	0.5	500	600	500
Pu-238	6300	0.7	4410	5100	4410
Pu-240	31000	0.7	21700	20000	20000
Pu-241	250	0.7	175	185	175
Pu-242	59000	0.7	41300	55000	41300
Am-241	56000	0.5	28000	24000	24000
Am-242m	21	0.5	10.5	11	10.5
Am-243	120000	0.5	60000	65000	60000
Cm-242	9800	0.5	4900	6000	4900
Cm-243	200	0.5	100	90	90
Cm-244	20000	0.5	10000	11000	10000
Cm-245	58	0.5	29	23	23
Cm-246	65000	0.5	32500	16000	16000
Cm-247	1100	0.5	550	500	500
Cf-249	58	0.5	29	10	10
Cf-251	27	0.5	13.5	5	5
ANSI/ANS = American National Standards Institute/American Nuclear Society; SoF = sum-of-fractions					

Table 13. Subcritical masses for sum-of-fractions in polyethylene-moderated and -reflected systems.

Nuclide	Computed mass (g)	Factors	Computed w/ factors (g)	ANSI/ANS 8.15 (g)	SoF mass (g)
U-233	250	1	250	--	250
U-235	400	1	400	--	400
Pu-239	250	1	250	--	250
U-232	2900	0.5	1450	1000	1000
U-234	90000	0.5	45000	59000	45000
Np-237	44000	0.7	30800	35000	30800
Pu-236	600	0.5	300	600	300
Pu-238	5800	0.7	4060	5100	4060
Pu-240	29000	0.7	20300	20000	20000
Pu-241	100	0.7	70	185	70
Pu-242	55000	0.7	38500	55000	38500
Am-241	52000	0.5	26000	24000	24000
Am-242m	11	0.5	5.5	11	5.5
Am-243	108000	0.5	54000	65000	54000
Cm-242	9000	0.5	4500	6000	4500
Cm-243	100	0.5	50	90	50
Cm-244	19000	0.5	9500	11000	9500
Cm-245	33	0.5	16.5	23	16.5
Cm-246	60000	0.5	30000	16000	16000
Cm-247	650	0.5	325	500	325
Cf-249	33	0.5	16.5	10	10
Cf-251	15	0.5	7.5	5	5
ANSI/ANS = American National Standards Institute/American Nuclear Society; SoF = sum-of-fractions					

7.0 Application – Californium Production

The SoF method is applied to a nuclide mixture like irradiated targets used for californium production (Table 14). The weight fraction of each isotope in the mixture is used in conjunction with the SoF subcritical masses to determine the mixture subcritical mass of 768 grams. Use of ANSI/ANS-8.15-2014 would restrict the subcritical mass of the mixture to the most limiting nuclide subcritical mass, which is the ^{251}Cf mass limit of 5 grams.

Table 14. Example of subcritical masses determined by the SoF method for californium production.

Nuclide	Weight percent	Nuclide masses (g) at SoF = 1
Pu-238	0.060	0.461
Pu-239	0.036	0.276
Pu-240	33.581	257.902
Pu-241	0.121	0.929
Pu-242	5.023	38.577
Am-241	0.591	4.539
Am-243	3.829	29.407
Cm-244	10.914	83.820
Cm-245	2.563	19.684
Cm-246	40.955	314.534
Cm-247	2.261	17.364
Cf-249	0.054	0.415
Cf-251	0.012	0.092
SoF = sum-of-fractions		

To demonstrate the 768-gram limits applicability, the mixture mass reactivity is evaluated for various moderations at a mass of 1977 grams. The mass represents the computed subcritical masses, which do not include the subcritical factors. As can be seen in Figure 14, the peak reactivity is 0.47716 ($k_{\text{eff}} + 2\sigma = 0.47818$) in the unmoderated region of the reactivity curve. Assessing the mixture with fissile nuclides removed results in an increase in reactivity in the moderated region and results in a peak k_{eff} of 0.94968 ($k_{\text{eff}} + 2\sigma = 0.95140$) as seen in Figure 15. For both methods of assessment, the peak k_{eff} falls below the calculational margin of 0.980.

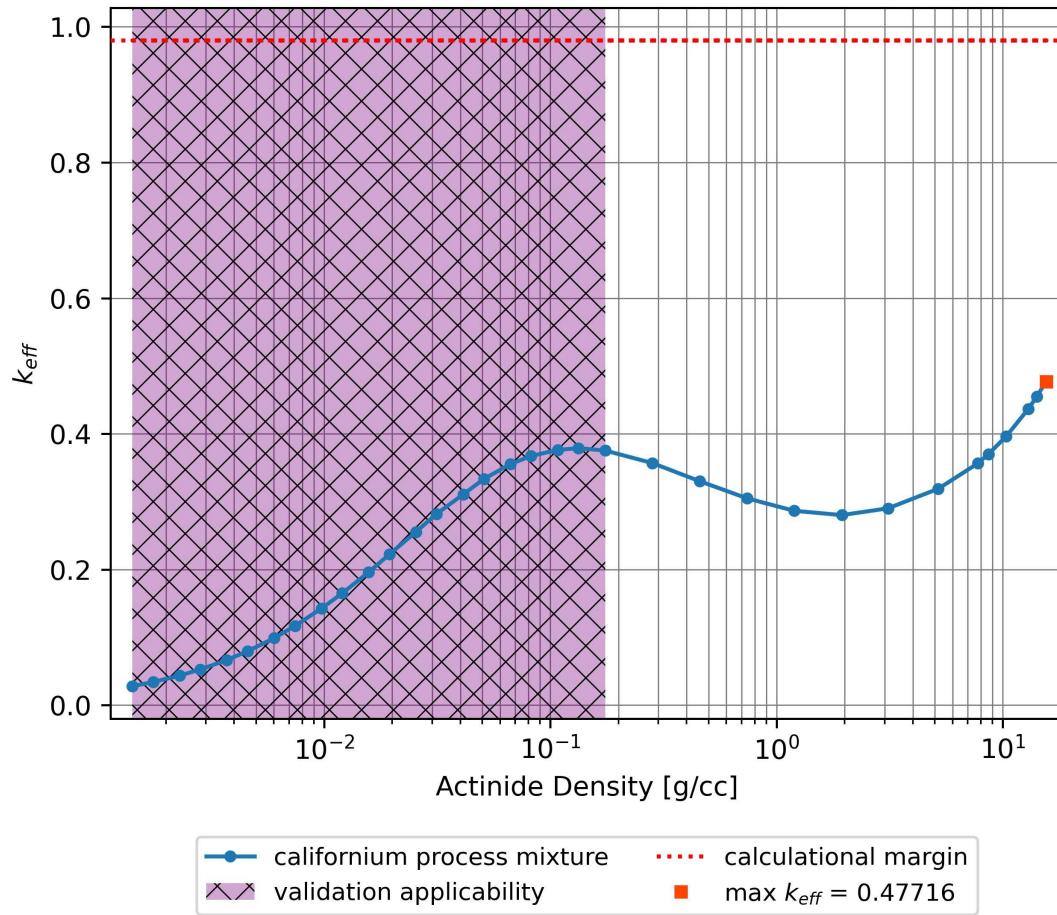


Figure 14. Reactivity of californium process mixture at SoF of one for various moderations; fissile and fissionable nuclides.

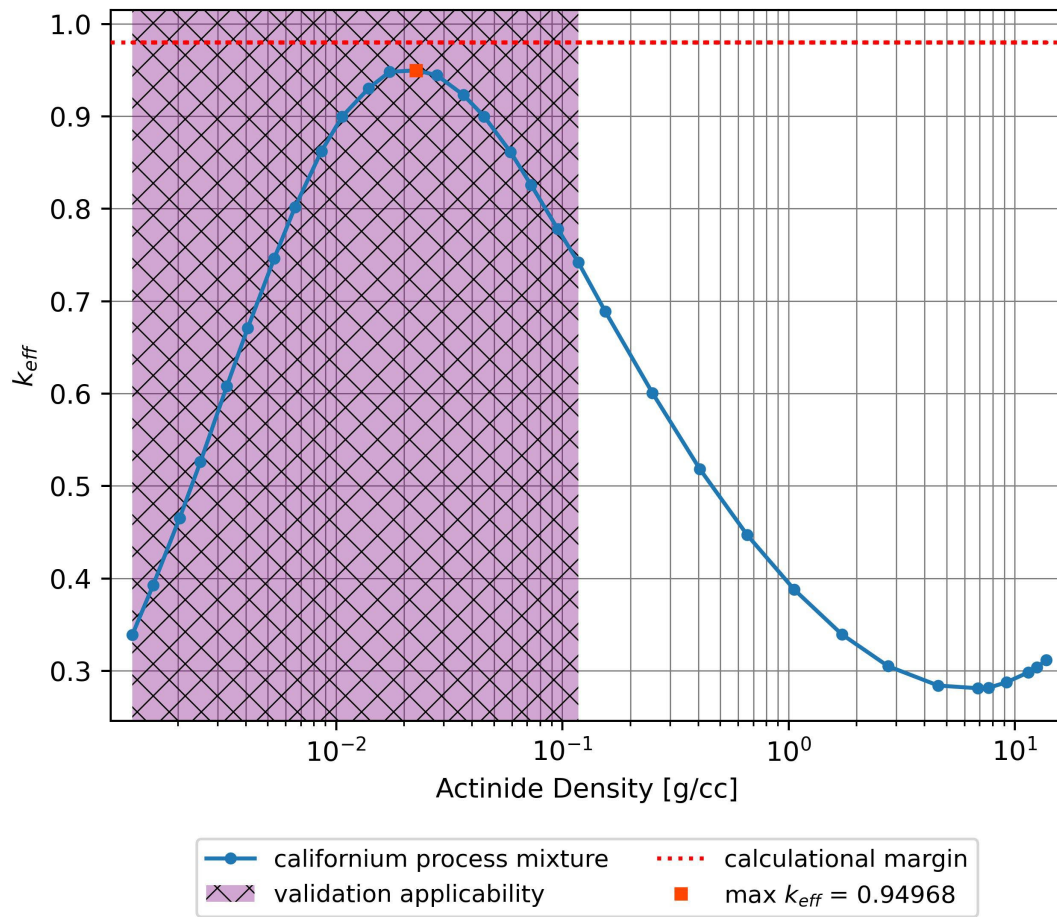


Figure 15. Reactivity of californium process mixture at SoF of one for various moderations; fissile-only nuclides.

8.0 Conclusion

A sum-of-fractions method was developed in this work for water- and polyethylene-moderated and -reflected systems containing a mixture of nuclides in ANSI/ANS-8.15-2014. The method addressed uncertainties with the models as well as cross section libraries. This is accomplished through a calculational margin derived through sensitivity and uncertainty analyses using SCALE/TSUNAMI and a margin of safety using subcritical factors that are consistent with Appendix C of ANSI/ANS-8.15-2014. Selection of the final subcritical masses for the nuclides was based on the minimum critical mass between the evaluated values and the ANSI/ANS-8.15-2014 values for water-moderated and -reflected systems.

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Appendix A – Similarity Assessments

This appendix provides a complete list of similarity assessments for all concentration-mixture combinations with water and polyethylene.

Table A.1. Water validation for concentration 1 from VALID.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	Yes	Yes	0.9889	95	Yes	Yes	0.9891	86
2	Yes	Yes	0.9888	94	Yes	Yes	0.9893	86
3	Yes	Yes	0.9889	94	Yes	Yes	0.9877	85
4	Yes	Yes	0.9883	86	—	—	—	0
5	No	Yes*	0.9864 [†]	27	—	—	—	0
6	No	Yes	0.9869 [†]	69	No	Yes	0.9855 [†]	44
7	Yes	No	0.9901	83	No	Yes	0.9856 [†]	52
8	Yes	Yes	0.9880	90	Yes	Yes	0.9891	86
9	Yes	Yes	0.9888	94	Yes	Yes	0.9886	86
10	Yes	Yes	0.9873	87	No	Yes*	0.9837 [†]	28
11	—	—	—	0	—	—	—	0
12	No	Yes	0.9875 [†]	78	No	Yes	0.9855 [†]	48
13	Yes	Yes	0.9875	87	Yes	Yes	0.9878	85
14	Yes	Yes	0.9874	87	No	Yes*	0.9837 [†]	28
15	—	—	—	0	—	—	—	0
16	No	Yes	0.9859 [†]	59	No	Yes*	0.9864 [†]	20
17	Yes	Yes	0.9883	86	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	No	Yes	0.9863 [†]	59	—	—	—	0
21	Yes	No	0.9829	142	—	—	—	0
22	Yes	No	0.9820	170	Yes	No	0.9826	112
23	Yes	No	0.9818	174	Yes	No	0.9828	133
24	Yes	No	0.9815	175	Yes	No	0.9833	152

*Test for normality may be unreliable because of insufficient data; [†]parametric; [^]nonparametric; otherwise, uses USL-1. CM = calculational margin

Table A.2. Water validation for concentration 2 from VALID.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	Yes	Yes	0.9930	103	Yes	Yes	0.9888	86
2	Yes	Yes	0.9931	103	Yes	Yes	0.9890	86
3	Yes	Yes	0.9929	102	No	Yes	0.9872 [†]	85
4	No	Yes	0.9917 [†]	89	—	—	—	0
5	No	Yes*	0.9909 [†]	15	—	—	—	0
6	Yes	Yes	0.9921	77	No	Yes	0.9858 [†]	47
7	No	No	0.9915 [^]	97	No	Yes	0.9861 [†]	58
8	Yes	Yes	0.9931	103	Yes	Yes	0.9888	86
9	Yes	Yes	0.9930	102	No	Yes	0.9872 [†]	86
10	Yes	Yes	0.9927	96	Yes	Yes	0.9410	55
11	—	—	—	0	—	—	—	0
12	Yes	Yes	0.9947	82	No	Yes	0.9859 [†]	55
13	Yes	Yes	0.9926	93	No	Yes	0.9872 [†]	85
14	Yes	Yes	0.9923	90	Yes	Yes	0.9609	67
15	—	—	—	0	—	—	—	0
16	No	Yes	0.9920 [†]	70	No	Yes	0.9864 [†]	15
17	No	Yes	0.9916 [†]	86	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	No	Yes*	0.9875 [†]	15	—	—	—	0
21	Yes	Yes	0.9874	121	—	—	—	0
22	Yes	Yes	0.9863	164	No	Yes	0.9836 [†]	90
23	Yes	No	0.9866	171	No	No	0.9328 [†]	107
24	Yes	No	0.9863	174	Yes	No	0.9831	135

*Test for normality may be unreliable because of insufficient data; [†]parametric; [^]nonparametric; otherwise, uses USL-1. CM = calculational margin

Table A.3. Water validation for concentration 3 from VALID.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	Yes	Yes	0.9933	131	No	Yes	0.9871 [†]	86
2	Yes	Yes	0.9936	129	No	Yes	0.9871 [†]	86
3	Yes	Yes	0.9933	129	No	Yes	0.9873 [†]	82
4	No	Yes	0.9920 [†]	95	—	—	—	0
5			--	0	—	—	—	0
6	Yes	Yes	0.9949	80	No	Yes*	0.9874 [†]	27
7	Yes	No	0.9940	96	No	Yes	0.9866 [†]	65
8	Yes	Yes	0.9935	129	No	Yes	0.9871 [†]	86
9	Yes	Yes	0.9934	128	No	Yes	0.9873 [†]	83
10	Yes	Yes	0.9932	127	Yes	Yes	0.9344	47
11			--	0	—	—	—	0
12	Yes	No	0.9949	86	Yes	Yes	0.9847	61
13	No	Yes	0.9920 [†]	102	No	Yes	0.9872 [†]	82
14	No	Yes	0.9918 [†]	99	Yes	Yes	0.9490	54
15	No	Yes*	0.9917 [†]	12	—	—	—	0
16	No	Yes	0.9923 [†]	75	**	Yes*	-0.1159	3
17	No	Yes	0.9919 [†]	88	—	—	—	0
18			--	0	—	—	—	0
19			--	0	—	—	—	0
20			--	0	—	—	—	0
21	No	Yes	0.9872 [†]	60	—	—	—	0
22	Yes	No	0.9871	132	No	Yes*	0.9817 [†]	32
23	Yes	No	0.9866	145	No	Yes	0.9824 [†]	50
24	Yes	No	0.9869	166	No	No	0.9750 [^]	100

*Test for normality may be unreliable because of insufficient data; [†]parametric; [^]nonparametric; otherwise, uses USL-1. CM = calculational margin

Table A.4. Polyethylene validation for concentration 1 from VALID.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	Yes	Yes	0.9888	86	—	—	—	0
2	Yes	Yes	0.9888	86	—	—	—	0
3	—	—	—	0	—	—	—	0
4	—	—	—	0	—	—	—	0
5	—	—	—	0	—	—	—	0
6	No	Yes	0.9855 [†]	44	—	—	—	0
7	Yes	Yes	0.9882	48	—	—	—	0
8	Yes	Yes	0.9886	84	—	—	—	0
9	Yes	Yes*	0.9856	17	—	—	—	0
10	—	—	—	0	—	—	—	0
11	—	—	—	0	—	—	—	0
12	Yes	Yes	0.9877	46	—	—	—	0
13	—	—	—	0	—	—	—	0
14	—	—	—	0	—	—	—	0
15	—	—	—	0	—	—	—	0
16	—	—	—	0	—	—	—	0
17	—	—	—	0	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	—	—	—	0	—	—	—	0
21	N/A	Yes*	0.9209	3	—	—	—	0
22	Yes	Yes	0.9783	48	—	—	—	0
23	No	No	0.9555	140	—	—	—	1
24	Yes	No	0.9802	163	No	Yes*	0.9703 [†]	35

*Test for normality may be unreliable because of insufficient data; [†]parametric; ^nonparametric; otherwise, uses USL-1. CM = calculational margin

Table A.5. Polyethylene validation for concentration 2 from VALID.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	No	Yes	0.9917 [†]	82	—	—	—	0
2	No	Yes	0.9915 [†]	86	—	—	—	0
3	—	—	—	0	—	—	—	0
4	—	—	—	0	—	—	—	0
5	—	—	—	0	—	—	—	0
6	—	—	—	0	—	—	—	0
7	Yes	Yes	0.9938	48	—	—	—	0
8	Yes	Yes	0.9926	82	—	—	—	0
9	—	—	—	0	—	—	—	0
10	—	—	—	0	—	—	—	0
11	—	—	—	0	—	—	—	0
12	No	Yes	0.9921 [†]	44	—	—	—	0
13	—	—	—	0	—	—	—	0
14	—	—	—	0	—	—	—	0
15	—	—	—	0	—	—	—	0
16	—	—	—	0	—	—	—	0
17	—	—	—	0	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	—	—	—	0	—	—	—	0
21	—	—	—	0	—	—	—	0
22	No	Yes*	0.9797 [†]	34	—	—	—	0
23	Yes	Yes	0.9830	42	—	—	—	0
24	Yes	No	0.9846	137	No	No*	0.9794 [^]	8

*Test for normality may be unreliable because of insufficient data; [†]parametric; [^]nonparametric; otherwise, uses USL-1. CM = calculational margin

Table A.6. Polyethylene validation for concentration 3 from VALID

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	No	Yes	0.9915 [†]	62	—	—	—	0
2	No	Yes	0.9916 [†]	80	—	—	—	0
3	—	—	—	0	—	—	—	0
4	—	—	—	0	—	—	—	0
5	—	—	—	0	—	—	—	0
6	—	—	—	0	—	—	—	0
7	—	—	—	1	—	—	—	0
8	No	Yes	0.9915 [†]	62	—	—	—	0
9	—	—	—	0	—	—	—	0
10	—	—	—	0	—	—	—	0
11	—	—	—	0	—	—	—	0
12	—	—	—	0	—	—	—	0
13	—	—	—	0	—	—	—	0
14	—	—	—	0	—	—	—	0
15	—	—	—	0	—	—	—	0
16	—	—	—	0	—	—	—	0
17	—	—	—	0	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	—	—	—	0	—	—	—	0
21	—	—	—	0	—	—	—	0
22	—	—	—	0	—	—	—	0
23	—	—	—	0	—	—	—	0
24	No	Yes *	0.9815 [†]	32	—	—	—	0

*Test for normality may be unreliable because of insufficient data; [†]parametric; ^nonparametric; otherwise, uses USL-1. CM = calculational margin

Table A.7. Water validation for concentration 1 from NEA.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	Yes	No	0.9882	453	Yes	No	0.9900	335
2	Yes	No	0.9889	452	Yes	No	0.9900	338
3	Yes	No	0.9912	475	Yes	No	0.9890	290
4	Yes	No	0.9908	411	—	—	—	0
5	No	No	0.9394 [^]	148	—	—	—	0
6	No	No	0.9353 [^]	876	No	No	0.9758 [^]	378
7	No	No	0.9361 [^]	989	No	No	0.9758 [^]	535
8	Yes	No	0.9898	511	Yes	No	0.9899	331
9	Yes	No	0.9912	456	Yes	No	0.9896	315
10	Yes	No	0.9897	345	Yes	Yes *	0.7835	28
11	—	—	—	0	—	—	—	0
12	No	No	0.9357 [^]	946	No	No	0.9758 [^]	459
13	Yes	No	0.9910	441	Yes	No	0.9888	277
14	Yes	No	0.9912	438	Yes	Yes*	0.7907	28
15	—	—	—	0	—	—	—	0
16	No	No	0.9333 [^]	711	No	No	0.9789 [^]	111
17	Yes	No	0.9910	383	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	Yes	No*	0.9918	33	—	—	—	0
21	Yes	Yes	0.9850	184	—	—	—	0
22	Yes	Yes	0.9847	211	Yes	Yes	0.9823	146
23	Yes	Yes	0.9848	216	Yes	Yes	0.9817	175
24	Yes	Yes	0.9845	217	Yes	Yes	0.9825	194

*Test for normality may be unreliable because of insufficient data; †parametric; ^nonparametric; otherwise, uses USL-1. CM = calculational margin

Table A.8. Water validation for concentration 2 from NEA.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	Yes	No	0.9874	605	Yes	No	0.9899	344
2	Yes	No	0.9870	608	Yes	No	0.9900	345
3	Yes	No	0.9921	531	No	No	0.9813 [^]	321
4	Yes	No	0.9914	402	—	—	—	0
5	No	No	0.9387 [^]	123	—	—	—	0
6	No	No	0.9353 [^]	915	No	No	0.9758 [^]	362
7	No	No	0.9365 [^]	1051	No	No	0.9758 [^]	582
8	Yes	No	0.9880	567	Yes	No	0.9899	344
9	Yes	No	0.9892	532	Yes	No	0.9897	338
10	Yes	No	0.9918	497	No	No*	0.9937 [^]	47
11	—	—	—	0	—	—	—	0
12	No	No	0.9637 [^]	989	No	No	0.9758 [^]	513
13	Yes	No	0.9917	463	No	No	0.9813 [^]	319
14	Yes	No	0.9919	442	No	Yes*	0.9859 [†]	54
15	—	—	—	0	—	—	—	0
16	Yes	No	0.9878	787	No	No	0.9899 [^]	109
17	Yes	No	0.9919	411	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	No	Yes*	0.9422 [†]	8	—	—	—	0
21	Yes	Yes	0.9865	165	—	—	—	0
22	Yes	Yes	0.9857	207	No	Yes	0.9817 [†]	81
23	Yes	Yes	0.9861	213	Yes	Yes	0.9832	153
24	Yes	Yes	0.9860	216	Yes	Yes	0.9825	177

*Test for normality may be unreliable because of insufficient data; [†]parametric; [^]nonparametric; otherwise, uses USL-1. CM = calculational margin

Table A.9. Water validation for concentration 3 from NEA.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	No	No	0.9387 [^]	625	No	No	0.9813 [^]	343
2	No	No	0.9383 [^]	636	No	No	0.9813 [^]	345
3	Yes	No	0.9921	559	Yes	Yes	0.9890	219
4	Yes	No	0.9903	360	—	—	—	0
5	Yes	No*	0.9874	51	—	—	—	0
6	Yes	No	0.9883	912	Yes	No	0.9877	235
7	Yes	No	0.9884	1070	Yes	No	0.9859	590
8	No	No	0.9389 [^]	611	No	No	0.9813 [^]	343
9	Yes	No	0.9923	560	No	No	0.9813 [^]	313
10	Yes	No	0.9914	527	Yes	Yes	0.9140	41
11	—	—	—	0	—	—	—	0
12	Yes	No	0.9885	1020	Yes	No	0.9874	409
13	Yes	No	0.9923	529	Yes	Yes	0.9891	227
14	Yes	No	0.9917	492	Yes	Yes	0.9586	56
15	No	Yes*	0.9435 [†]	11	—	—	—	0
16	Yes	No	0.9883	734	No	No*	0.9902 [^]	50
17	Yes	No	0.9923	357	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	—	—	—	0	—	—	—	0
21	No	Yes	0.9370 [†]	54	—	—	—	0
22	Yes	Yes	0.9866	174	No	Yes*	0.9788 [†]	25
23	Yes	Yes	0.9863	186	No	Yes	0.9819 [†]	50
24	Yes	Yes	0.9867	208	No	Yes	0.9830 [†]	118

*Test for normality may be unreliable because of insufficient data; [†]parametric; [^]nonparametric; otherwise, uses USL-1. CM = calculational margin

Table A.10. Polyethylene validation for concentration 1 from NEA.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	Yes	No	0.9862	432	Yes	No*	0.9847	89
2	Yes	No	0.9855 [†]	453	No	No	0.9770 [^]	100
3	Yes	No	0.9797	116	Yes	No*	0.9473	68
4	Yes	No	0.9855	75	—	—	—	0
5	—	—	—	0	—	—	—	0
6	No	Yes	0.9824 [†]	343	No	Yes*	0.9726 [†]	11
7	Yes	No	0.9851	600	No	Yes	0.9786 [†]	76
8	Yes	No	0.9861	424	Yes	No*	0.9847	89
9	Yes	No	0.9835	229	Yes	No*	0.9651	76
10	No	No	0.9771 [^]	102	—	—	—	0
11	—	—	—	0	—	—	—	0
12	Yes	No	0.9842	454	No	Yes*	0.9734 [†]	17
13	Yes	No	0.9797	114	Yes	No*	0.9362	58
14	No	No	0.9771 [^]	102	—	—	—	0
15	—	—	—	0	—	—	—	0
16	Yes	Yes	0.9847	51	—	—	—	0
17	Yes	No	0.9855	77	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	—	—	—	0	—	—	—	0
21	No	Yes*	0.9694 [†]	9	—	—	—	0
22	Yes	Yes	0.9821	122	—	—	—	0
23	Yes	Yes	0.9816	182	No	Yes*	0.9675 [†]	8
24	Yes	Yes	0.9813	205	Yes	Yes	0.9762	68

*Test for normality may be unreliable because of insufficient data; [†]parametric; [^]nonparametric; otherwise, uses USL-1. CM = calculational margin

Table A.11. Polyethylene validation for concentration 2 from NEA.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	Yes	No	0.9887	458	No	No*	0.9729 [^]	81
2	Yes	No	0.9888	510	No	No*	0.9729 [^]	83
3	Yes	No	0.9820	114	No	No*	0.9797 [^]	45
4	No	No*	0.9832 [^]	68	—	—	—	0
5	—	—	—	0	—	—	—	0
6	Yes	Yes	0.9866	111	No	Yes*	0.9719 [†]	12
7	Yes	No	0.9866	634	No	Yes*	0.9760 [†]	19
8	Yes	No	0.9887	446	No	No*	0.9729 [^]	81
9	Yes	No	0.9836	140	No	No*	0.9784 [^]	68
10	Yes	No	0.9874	90	—	—	—	0
11	—	—	—	0	—	—	—	0
12	Yes	Yes	0.9864	465	No	Yes*	0.9747 [†]	15
13	Yes	No	0.9821	114	No	No*	0.9799 [^]	43
14	No	No	0.9813 [^]	93	—	—	—	0
15	—	—	—	0	—	—	—	0
16	No	Yes*	0.9816 [†]	20	No	Yes*	0.9521 [†]	3
17	No	No*	0.9832 [^]	68	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	—	—	—	0	—	—	—	0
21	—	—	—	0	—	—	—	0
22	Yes	Yes	0.9858	55	—	—	—	0
23	Yes	Yes	0.9868	107	—	—	—	0
24	Yes	Yes	0.9857	180	No	Yes*	0.9225 [†]	15

*Test for normality may be unreliable because of insufficient data; †parametric; ^nonparametric; otherwise, uses USL-1. CM = calculational margin

Table A.12. Polyethylene validation for concentration 3 from NEA.

Mixture	$c_k > 0.8$				$c_k > 0.9$			
	Trend	Normal	CM	Cases	Trend	Normal	CM	Cases
1	Yes	No	0.9865	268	No	No*	0.9784 [^]	68
2	Yes	No	0.9883	413	No	No*	0.9783 [^]	76
3	Yes	No	0.9823	102	No	Yes*	0.9806 [†]	7
4	Yes	No*	0.9886	40	—	—	—	0
5	—	—	—	0	—	—	—	0
6	Yes	Yes*	0.9911	16	No	Yes*	0.9640 [†]	4
7	Yes	No	0.9873	179	No	No*	0.9808 [^]	14
8	Yes	No	0.9865	263	No	No*	0.9784 [^]	68
9	Yes	No	0.9819	113	Yes	Yes*	0.9820	29
10	Yes	No*	0.9892	73	—	—	—	0
11	—	—	—	0	—	—	—	0
12	Yes	Yes*	0.9790	20	No	Yes*	0.9760 [†]	9
13	Yes	No	0.9823	105	No	Yes*	0.9806 [†]	7
14	Yes	No*	0.9888	74	—	—	—	0
15	—	—	—	0	—	—	—	0
16	Yes	Yes*	0.9817	11	—	—	—	1
17	Yes	No *	0.9882	49	—	—	—	0
18	—	—	—	0	—	—	—	0
19	—	—	—	0	—	—	—	0
20	—	—	—	0	—	—	—	0
21	—	—	—	0	—	—	—	0
22	—	—	—	1	—	—	—	0
23	No	Yes*	0.9031 [†]	5	—	—	—	0
24	Yes	Yes	0.9860	53	—	—	—	0

*Test for normality may be unreliable because of insufficient data; †parametric; ^nonparametric; otherwise, uses USL-1. CM = calculational margin

Appendix B – Random Nuclides

The mass fractions of the nuclides used for the 100 random mixtures calculations in which the SoF mass equates to one for fissile and fissionable nuclides are provided in Table B.1. and Table B.2., respectively.

Table B.1. Random fissile nuclides mass fractions.

U ²³²	U ²³³	U ²³⁵	Pu ²³⁶	Pu ²³⁹	Pu ²⁴¹	Am ^{242m}	Cm ²⁴³	Cm ²⁴⁵	Cm ²⁴⁷	Cf ²⁴⁹	Cf ²⁵¹
0.04132	0.07316	0.21054	0.01137	0.04098	0.09331	0.04115	0.01110	0.09710	0.17272	0.09697	0.11030
0.01962	0.09548	0.09330	0.15747	0.05630	0.14218	0.03083	0.12079	0.15548	0.07726	0.03725	0.01404
0.00145	0.17198	0.12522	0.06543	0.06097	0.01271	0.01124	0.16484	0.08942	0.13103	0.05574	0.10998
0.02105	0.15894	0.01578	0.05090	0.11254	0.11024	0.03552	0.00869	0.10218	0.10263	0.14133	0.14020
0.02624	0.08789	0.01332	0.04424	0.03547	0.00465	0.02500	0.17722	0.07440	0.20417	0.21901	0.08840
0.10476	0.08111	0.05855	0.08775	0.04493	0.00895	0.01366	0.02168	0.09510	0.17832	0.16645	0.13875
0.13227	0.09296	0.09896	0.11009	0.09007	0.09779	0.01010	0.01935	0.10337	0.10874	0.07390	0.06240
0.12685	0.15330	0.13630	0.02653	0.13485	0.00086	0.02243	0.13688	0.01535	0.07512	0.05143	0.12010
0.07401	0.02481	0.09513	0.13672	0.00862	0.16025	0.02560	0.04179	0.10013	0.10130	0.14509	0.08656
0.10639	0.08786	0.10932	0.05409	0.04161	0.09363	0.07745	0.12819	0.06989	0.12867	0.00869	0.09420
0.01183	0.04411	0.17955	0.00542	0.06456	0.14281	0.16728	0.16074	0.00822	0.04774	0.07142	0.09631
0.02262	0.06396	0.06457	0.06069	0.07824	0.07751	0.19512	0.08787	0.09197	0.00154	0.09791	0.15800
0.03405	0.03128	0.04104	0.06792	0.17147	0.10733	0.04552	0.00529	0.09945	0.16692	0.07336	0.15636
0.09334	0.01113	0.15625	0.15747	0.04020	0.04815	0.11122	0.09316	0.06249	0.07018	0.05857	0.09782
0.02660	0.10318	0.05996	0.09120	0.08713	0.13315	0.12588	0.10289	0.02828	0.11613	0.07209	0.05350
0.00939	0.13603	0.10308	0.09466	0.04607	0.08753	0.07882	0.03201	0.11467	0.08733	0.07919	0.13123
0.15820	0.16202	0.10177	0.03750	0.01555	0.07583	0.02005	0.02779	0.14964	0.12043	0.12032	0.01090
0.05545	0.12911	0.11648	0.10076	0.13501	0.09179	0.08766	0.05410	0.05908	0.13845	0.01167	0.02042
0.05563	0.12059	0.11307	0.06441	0.01577	0.03996	0.10277	0.12882	0.09131	0.09041	0.13088	0.04637
0.06821	0.10218	0.00404	0.12203	0.06967	0.12394	0.09019	0.07180	0.13900	0.00846	0.11250	0.08798
0.05588	0.05648	0.04565	0.15777	0.13642	0.07265	0.02932	0.09925	0.12905	0.14956	0.04916	0.01881
0.03441	0.03724	0.17094	0.02320	0.07757	0.13588	0.02190	0.08882	0.09866	0.17900	0.01431	0.11808
0.02582	0.10437	0.11936	0.01056	0.09958	0.04986	0.06519	0.12695	0.13927	0.10523	0.14373	0.01009
0.08339	0.11588	0.02229	0.06158	0.10253	0.10350	0.11848	0.10038	0.07181	0.01176	0.11779	0.09060
0.01665	0.12265	0.00766	0.14381	0.14059	0.11633	0.15871	0.09451	0.00059	0.13071	0.06120	0.00660
0.09537	0.00867	0.17632	0.12008	0.09228	0.16952	0.08502	0.11324	0.02608	0.06885	0.02948	0.01509
0.13217	0.05092	0.09850	0.07824	0.05969	0.09619	0.13131	0.12555	0.02445	0.07953	0.11491	0.00853
0.04926	0.15390	0.05101	0.04338	0.08388	0.08981	0.09849	0.11042	0.06700	0.10591	0.09854	0.04840
0.06977	0.08262	0.12670	0.01221	0.08005	0.07078	0.12582	0.10500	0.06780	0.11532	0.08400	0.05992
0.07681	0.06332	0.12578	0.05650	0.13629	0.04812	0.06862	0.10240	0.04623	0.13175	0.09022	0.05396
0.11816	0.08482	0.07633	0.08697	0.14973	0.08454	0.05914	0.02658	0.13307	0.11586	0.06133	0.00347
0.04338	0.10212	0.11356	0.19924	0.10672	0.01646	0.01757	0.07640	0.02889	0.18851	0.09919	0.00797
0.03839	0.00492	0.13673	0.02810	0.02032	0.01134	0.10706	0.15602	0.15379	0.12821	0.13931	0.07582
0.02824	0.10126	0.07272	0.00749	0.07995	0.01605	0.12615	0.14765	0.13066	0.02251	0.16459	0.10274
0.09923	0.19404	0.00554	0.00031	0.19668	0.00551	0.02389	0.09208	0.03731	0.05702	0.09305	0.19534
0.07039	0.07477	0.11955	0.09296	0.12014	0.10232	0.05510	0.09951	0.01045	0.11063	0.10235	0.04183
0.08801	0.02191	0.09626	0.09856	0.11462	0.13581	0.05618	0.13147	0.01461	0.03762	0.09523	0.10973
0.10138	0.14045	0.06862	0.12956	0.01788	0.05175	0.13466	0.09522	0.08242	0.06853	0.06580	0.04374
0.04185	0.05642	0.03863	0.10917	0.14043	0.13950	0.15790	0.05427	0.00947	0.15373	0.07161	0.02702
0.19957	0.01162	0.05187	0.13597	0.05624	0.03569	0.05267	0.21586	0.07962	0.06246	0.04942	0.04903
0.08122	0.16810	0.12434	0.08719	0.14481	0.04326	0.12696	0.03768	0.06000	0.06799	0.04584	0.01260
0.08965	0.03597	0.13290	0.06601	0.11376	0.06478	0.00942	0.12129	0.13383	0.12162	0.02075	0.09004
0.06008	0.09476	0.02799	0.05092	0.13819	0.07372	0.10475	0.16762	0.10819	0.04255	0.03682	0.09441
0.01154	0.10134	0.10471	0.10379	0.08436	0.13801	0.13910	0.01243	0.05388	0.13179	0.07702	0.04203
0.08677	0.11542	0.05513	0.01317	0.11021	0.04784	0.13448	0.15710	0.12077	0.00691	0.08281	0.06938
0.17362	0.09738	0.13507	0.17686	0.07244	0.02238	0.01632	0.06455	0.02701	0.00579	0.15926	0.04933
0.05787	0.12018	0.09995	0.08738	0.06654	0.14533	0.08705	0.11721	0.00087	0.09533	0.01083	0.11144
0.04568	0.08289	0.09696	0.14525	0.13536	0.06452	0.05524	0.04136	0.02207	0.12273	0.13321	0.05473
0.07089	0.08167	0.02898	0.08924	0.09158	0.07749	0.08681	0.09121	0.11429	0.12047	0.10622	0.04116
0.02210	0.14353	0.01766	0.02381	0.09254	0.11788	0.14645	0.01331	0.07211	0.07028	0.11942	0.16091
0.05924	0.10985	0.12300	0.00093	0.14672	0.06160	0.13491	0.06189	0.01783	0.08029	0.09517	0.10857
0.02537	0.17502	0.07574	0.13689	0.06934	0.11499	0.04192	0.03468	0.10357	0.08378	0.11157	0.02713
0.05546	0.14945	0.01634	0.04619	0.07253	0.01544	0.04136	0.15727	0.05612	0.16546	0.03031	0.19406
0.04165	0.19154	0.04196	0.02445	0.06061	0.02312	0.17792	0.13930	0.07619	0.00220	0.05180	0.16926
0.01822	0.12675	0.12789	0.08377	0.08253	0.04461	0.14621	0.15237	0.00157	0.09420	0.07791	0.04397
0.07313	0.11154	0.14566	0.00640	0.01965	0.08213	0.16385	0.06259	0.02698	0.08790	0.09661	0.12355
0.00646	0.11985	0.10931	0.04134	0.15498	0.04454	0.15988	0.01110	0.14554	0.09115	0.09236	0.02349
0.01941	0.12200	0.08115	0.14596	0.14659	0.07034	0.02270	0.12080	0.03455	0.11670	0.07909	0.04072
0.03761	0.15011	0.02446	0.12425	0.12168	0.07402	0.02412	0.05516	0.12592	0.08688	0.13340	0.04238
0.11163	0.05487	0.12957	0.05855	0.07943	0.11962	0.08347	0.06436	0.12779	0.02556	0.10810	0.03705
0.04637	0.16131	0.12774	0.13061	0.01493	0.08222	0.02842	0.07746	0.09817	0.05768	0.11362	0.06148

Table B.1 contd.

²³² U	²³³ U	²³⁵ U	²³⁶ Pu	²³⁹ Pu	²⁴¹ Pu	^{242m} Am	²⁴³ Cm	²⁴⁵ Cm	²⁴⁷ Cm	²⁴⁹ Cf	²⁵¹ Cf
0.06551	0.10607	0.00576	0.11887	0.08757	0.13332	0.02309	0.13637	0.07178	0.10830	0.11379	0.02958
0.11448	0.15574	0.16339	0.00900	0.00786	0.15439	0.04995	0.00598	0.11239	0.06956	0.10650	0.05074
0.16929	0.11192	0.00319	0.10748	0.05467	0.02499	0.01225	0.14657	0.11313	0.14776	0.09499	0.01377
0.08856	0.14285	0.12342	0.03090	0.13679	0.01825	0.05691	0.03794	0.10066	0.09307	0.09104	0.07961
0.03492	0.14801	0.15041	0.06455	0.10075	0.08882	0.12029	0.05542	0.04642	0.03616	0.15304	0.00122
0.06739	0.14464	0.12281	0.13125	0.02844	0.06032	0.08931	0.00424	0.07764	0.01364	0.13168	0.12864
0.02109	0.05668	0.08820	0.06871	0.15282	0.14401	0.14980	0.05875	0.15588	0.00435	0.05421	0.04549
0.03474	0.12683	0.13503	0.00430	0.10655	0.10364	0.04020	0.10532	0.09951	0.10345	0.01533	0.12510
0.15125	0.03587	0.15964	0.02619	0.00284	0.11534	0.08649	0.04004	0.13649	0.15617	0.05233	0.03734
0.15866	0.07351	0.13614	0.00798	0.11662	0.14902	0.00544	0.02557	0.05825	0.00003	0.10994	0.15884
0.06539	0.02674	0.11108	0.13972	0.00498	0.07526	0.12796	0.12184	0.02337	0.13906	0.05236	0.11225
0.14197	0.10453	0.06814	0.00026	0.01121	0.07909	0.12253	0.12199	0.06142	0.08889	0.10161	0.09836
0.06060	0.09716	0.03383	0.11043	0.12678	0.02695	0.00792	0.04431	0.17140	0.04471	0.16960	0.10630
0.08869	0.15378	0.08884	0.15326	0.14695	0.06703	0.01336	0.08818	0.10882	0.02942	0.02185	0.03982
0.07174	0.14311	0.10627	0.08270	0.02528	0.13450	0.05602	0.07502	0.07540	0.06938	0.04269	0.11788
0.07035	0.10915	0.07844	0.08516	0.08730	0.12976	0.11872	0.09366	0.00817	0.05513	0.13253	0.03163
0.01784	0.00450	0.03683	0.16866	0.11210	0.13759	0.12648	0.04822	0.15496	0.01629	0.02686	0.14966
0.00490	0.01051	0.12182	0.15356	0.15980	0.07018	0.04422	0.07457	0.13382	0.05751	0.04313	0.12597
0.12207	0.04769	0.02168	0.08163	0.19138	0.06042	0.10637	0.01191	0.04601	0.06994	0.12347	0.11741
0.05376	0.06617	0.08229	0.09981	0.08118	0.07340	0.09352	0.07975	0.10188	0.09832	0.08462	0.08528
0.02592	0.04101	0.01248	0.05434	0.06644	0.12114	0.13948	0.07444	0.12162	0.09827	0.11053	0.13433
0.11759	0.05167	0.08221	0.04613	0.06090	0.07201	0.13408	0.12445	0.09594	0.06224	0.10512	0.04766
0.04326	0.14197	0.01120	0.04354	0.01236	0.13330	0.09791	0.12280	0.11605	0.05167	0.07701	0.14893
0.07015	0.07898	0.09808	0.11445	0.10602	0.11016	0.04181	0.07345	0.02535	0.06034	0.11573	0.10549
0.12574	0.08640	0.09907	0.11367	0.06735	0.07681	0.14091	0.07876	0.01562	0.02895	0.09620	0.07053
0.17025	0.00782	0.23513	0.11598	0.16510	0.00997	0.03163	0.02034	0.13796	0.05833	0.03625	0.01123
0.12681	0.01882	0.18802	0.01492	0.14772	0.04967	0.03516	0.08213	0.09057	0.06639	0.09769	0.08209
0.05145	0.07398	0.15276	0.09498	0.07801	0.10831	0.00271	0.15399	0.00762	0.09100	0.03550	0.14969
0.07864	0.04834	0.17009	0.03059	0.14337	0.02585	0.06974	0.01439	0.17037	0.03109	0.11814	0.09938
0.14018	0.04883	0.10444	0.11407	0.14016	0.07245	0.09534	0.00889	0.10422	0.11054	0.04635	0.01454
0.09290	0.08190	0.03764	0.06941	0.06066	0.07305	0.11494	0.06482	0.12254	0.11871	0.06301	0.10042
0.11480	0.11421	0.09210	0.08317	0.12308	0.03720	0.07519	0.09808	0.02106	0.12019	0.08364	0.03730
0.13167	0.05703	0.11996	0.12939	0.04087	0.12315	0.07526	0.05463	0.12432	0.05580	0.01405	0.07388
0.02237	0.15696	0.15990	0.01030	0.01731	0.15363	0.12291	0.09668	0.06085	0.00658	0.01551	0.17700
0.10709	0.05442	0.13070	0.03577	0.04857	0.11965	0.05831	0.09663	0.01053	0.12408	0.09006	0.12419
0.08315	0.10525	0.10052	0.11904	0.00082	0.08858	0.08912	0.12344	0.09723	0.04332	0.12914	0.02038
0.11843	0.02033	0.00890	0.16378	0.06595	0.19555	0.09283	0.09056	0.01642	0.10211	0.12083	0.00431
0.02547	0.13784	0.08817	0.12721	0.13577	0.04791	0.13079	0.13469	0.03116	0.01868	0.01808	0.10424
0.13293	0.00479	0.05562	0.01039	0.13961	0.11402	0.11233	0.07358	0.11661	0.07544	0.13917	0.02551

Table B.2. Random fissionable nuclides mass fractions.

U ²³²	U ²³³	U ²³⁴	U ²³⁵	Np ²³⁷	Pu ²³⁶	Pu ²³⁸	Pu ²³⁹	Pu ²⁴⁰	Pu ²⁴¹	Pu ²⁴²	Am ²⁴¹	Am ^{242m}	Am ²⁴³	Cm ²⁴²	Cm ²⁴³	Cm ²⁴⁴	Cm ²⁴⁵	Cm ²⁴⁶	Cm ²⁴⁷	Cf ²⁴⁹	Cf ²⁵¹
0.05794	0.05070	0.07938	0.05237	0.05588	0.03344	0.05068	0.02821	0.02535	0.03203	0.06705	0.01588	0.00887	0.06605	0.00663	0.07762	0.05526	0.00148	0.07170	0.03500	0.05347	0.07502
0.06385	0.06292	0.08772	0.02033	0.00905	0.02750	0.02797	0.08089	0.05273	0.04033	0.05272	0.06491	0.04506	0.04384	0.03457	0.00907	0.02997	0.05735	0.04156	0.09212	0.05380	0.00176
0.00458	0.08413	0.04292	0.05469	0.06488	0.05994	0.08836	0.07441	0.06325	0.03829	0.04201	0.01365	0.07575	0.04036	0.00155	0.01358	0.04419	0.00894	0.01652	0.01509	0.06340	0.08950
0.06152	0.00116	0.06994	0.03980	0.03488	0.02041	0.04106	0.06397	0.05805	0.01802	0.07126	0.04654	0.00743	0.07192	0.04564	0.04518	0.06543	0.06193	0.06968	0.05323	0.01075	0.04219
0.04700	0.00613	0.03781	0.03317	0.02150	0.06955	0.05471	0.03714	0.04034	0.05560	0.05634	0.06630	0.06323	0.03955	0.05336	0.06316	0.06444	0.04730	0.05062	0.06672	0.00784	0.01818
0.04596	0.00534	0.05064	0.02897	0.07211	0.07802	0.01849	0.08026	0.06848	0.00542	0.07038	0.00901	0.05719	0.05221	0.06336	0.07021	0.04051	0.06685	0.00142	0.05274	0.05181	0.01064
0.09065	0.06386	0.00628	0.01219	0.09325	0.01292	0.01377	0.06846	0.05927	0.01530	0.04287	0.08953	0.04107	0.08452	0.08105	0.09734	0.01755	0.01295	0.02360	0.02111	0.04225	0.01021
0.00503	0.03293	0.06031	0.01625	0.07169	0.02192	0.02491	0.07230	0.02301	0.04808	0.00622	0.05103	0.06676	0.07077	0.04746	0.07560	0.02498	0.08438	0.04797	0.08414	0.01895	0.04532
0.01651	0.05333	0.04811	0.03245	0.03421	0.01149	0.01264	0.06092	0.00311	0.06870	0.04599	0.09334	0.06324	0.01704	0.07603	0.04447	0.06269	0.03270	0.06738	0.07013	0.04163	0.04389
0.07616	0.05495	0.05834	0.00217	0.03282	0.00329	0.10789	0.04718	0.06829	0.08195	0.00510	0.02813	0.01116	0.01696	0.04300	0.02145	0.04781	0.00391	0.01313	0.10507	0.10465	0.06662
0.00717	0.05156	0.03191	0.04548	0.04584	0.05394	0.07415	0.06592	0.05066	0.00501	0.01615	0.06688	0.02459	0.01734	0.05479	0.00449	0.07539	0.03084	0.08201	0.07225	0.05800	0.06562
0.06771	0.05281	0.01802	0.03196	0.01365	0.04299	0.04208	0.06337	0.09055	0.02151	0.04717	0.09433	0.01386	0.03626	0.07668	0.02222	0.04898	0.05452	0.09640	0.01684	0.01984	0.02824
0.08353	0.04528	0.02324	0.02812	0.08419	0.03667	0.07025	0.01961	0.05174	0.06752	0.03627	0.05957	0.07772	0.00626	0.06982	0.02912	0.05271	0.02380	0.01279	0.01442	0.06753	0.03983
0.01022	0.05867	0.03092	0.00739	0.09222	0.01785	0.07419	0.01541	0.07451	0.05601	0.01250	0.08900	0.00841	0.04860	0.00068	0.12222	0.04370	0.12147	0.02754	0.04213	0.02952	0.01684
0.02859	0.00214	0.02131	0.08623	0.06860	0.03814	0.02273	0.03536	0.01018	0.06283	0.06265	0.00543	0.00118	0.01116	0.07271	0.06892	0.06734	0.06171	0.07249	0.06151	0.06060	0.07819
0.02235	0.04001	0.00670	0.01400	0.02575	0.05512	0.05985	0.01355	0.06645	0.07612	0.00777	0.04066	0.04722	0.08440	0.00367	0.04598	0.02843	0.09365	0.05356	0.00253	0.04167	0.09054
0.06529	0.04858	0.00973	0.05838	0.01961	0.08283	0.03380	0.07221	0.01885	0.03522	0.04600	0.08322	0.03808	0.02789	0.06897	0.06441	0.01519	0.01518	0.08366	0.06037	0.03092	0.02162
0.00894	0.07040	0.06669	0.02782	0.02594	0.04598	0.01233	0.00625	0.03227	0.04132	0.06970	0.04268	0.03196	0.08665	0.03177	0.02340	0.05348	0.09437	0.04390	0.08438	0.06558	0.03418
0.03832	0.01930	0.05889	0.03040	0.02675	0.07788	0.01764	0.02875	0.03244	0.06091	0.00425	0.07245	0.07741	0.02898	0.01603	0.07579	0.09372	0.01386	0.06133	0.04306	0.07100	0.05082
0.05546	0.01087	0.09051	0.02193	0.07475	0.05603	0.00630	0.02176	0.05378	0.02865	0.07926	0.07668	0.08053	0.01441	0.05944	0.00853	0.01853	0.01809	0.08349	0.06158	0.06519	0.01423
0.07816	0.06159	0.01274	0.08675	0.07717	0.03647	0.06535	0.05699	0.05863	0.04323	0.00108	0.00872	0.02067	0.06554	0.04491	0.01717	0.03831	0.04502	0.04744	0.01313	0.03328	0.08766
0.02727	0.01064	0.05935	0.08305	0.04610	0.06010	0.07477	0.02593	0.01679	0.08300	0.01473	0.07819	0.05421	0.03800	0.08970	0.04955	0.06007	0.07275	0.01453	0.01732	0.01938	0.00460
0.05534	0.01107	0.07370	0.07335	0.07682	0.00525	0.08410	0.03389	0.06230	0.01089	0.03285	0.02291	0.06674	0.04635	0.02163	0.00547	0.07911	0.06938	0.07104	0.03536	0.02691	0.03557
0.05986	0.01620	0.04707	0.04747	0.00843	0.06890	0.03904	0.07269	0.02847	0.05664	0.06131	0.00593	0.01731	0.04032	0.07019	0.06531	0.07305	0.05111	0.06116	0.05641	0.00399	0.04914
0.05218	0.04239	0.03856	0.02391	0.01297	0.02854	0.04142	0.05858	0.08705	0.07116	0.03762	0.05256	0.04114	0.03333	0.05560	0.03535	0.01369	0.07114	0.05721	0.08038	0.04408	0.02115
0.03552	0.05313	0.00535	0.07008	0.07744	0.04150	0.05736	0.04996	0.07900	0.07929	0.03926	0.07080	0.08413	0.02914	0.04303	0.03603	0.00648	0.00872	0.03697	0.05015	0.01242	0.03426
0.07468	0.01027	0.02162	0.08705	0.01374	0.01738	0.08599	0.00171	0.06309	0.05587	0.06862	0.06787	0.07129	0.07097	0.02013	0.06153	0.04459	0.01939	0.05978	0.03059	0.01639	0.03745
0.02187	0.01817	0.04176	0.02742	0.06552	0.02325	0.01069	0.02512	0.02714	0.02767	0.02060	0.04145	0.05620	0.09352	0.02403	0.11438	0.03622	0.00526	0.04425	0.09134	0.07720	0.01695
0.08327	0.04223	0.02843	0.02797	0.08112	0.02203	0.08055	0.05186	0.04951	0.01607	0.07014	0.04650	0.06374	0.05325	0.06921	0.01055	0.00943	0.00921	0.01023	0.04682	0.04662	0.08124
0.03230	0.07684	0.03877	0.08652	0.08506	0.00263	0.06468	0.06995	0.03845	0.07679	0.08279	0.02446	0.06277	0.04469	0.00754	0.01691	0.03776	0.02099	0.03858	0.02010	0.02529	0.04614
0.05370	0.00657	0.05102	0.02756	0.05365	0.07989	0.04013	0.04958	0.01263	0.02941	0.06376	0.05272	0.07581	0.01114	0.06522	0.01518	0.02339	0.08224	0.06614	0.06050	0.04412	0.03565
0.01384	0.01647	0.08875	0.02942	0.08833	0.02439	0.03395	0.07979	0.02634	0.07745	0.03888	0.00098	0.05352	0.05510	0.03291	0.09062	0.02540	0.03239	0.00942	0.09093	0.07753	0.01360
0.00265	0.05646	0.07088	0.00731	0.00583	0.04910	0.00798	0.04356	0.08043	0.06618	0.02282	0.04415	0.07829	0.05027	0.00258	0.07284	0.06277	0.05955	0.07301	0.08052	0.03931	0.02350
0.08301	0.05819	0.08530	0.04859	0.06581	0.07927	0.02196	0.03895	0.06326	0.01741	0.02356	0.00132	0.01022	0.06177	0.05475	0.05309	0.08211	0.05640	0.00782	0.00413	0.04492	0.03815
0.02047	0.02413	0.04098	0.10213	0.02779	0.05129	0.07105	0.00995	0.05656	0.00438	0.04126	0.11238	0.06179	0.02167	0.04707	0.00530	0.02904	0.00404	0.10618	0.11189	0.02702	0.02362
0.00275	0.02162	0.07073	0.01389	0.06706	0.06383	0.04391	0.05193	0.01548	0.01286	0.02572	0.05091	0.04846	0.01351	0.07777	0.07659	0.06457	0.04932	0.07555	0.05145	0.07117	0.03092
0.00877	0.07150	0.00394	0.05339	0.06369	0.02355	0.01802	0.09189	0.02099	0.08688	0.01699	0.00270	0.06907	0.06258	0.04384	0.09093	0.01445	0.02151	0.05716	0.05296	0.08375	0.04142
0.03591	0.07124	0.05050	0.03721	0.00784	0.02759	0.05583	0.06213	0.07962	0.03211	0.06289	0.07431	0.05978	0.07343	0.03928	0.05958	0.01924	0.02089	0.03972	0.03927	0.02780	0.02382
0.07301	0.02286	0.03783	0.06887	0.00087	0.05369	0.00723	0.01557	0.05906	0.04631	0.06710	0.04020	0.07313	0.03774	0.05414	0.04749	0.02773	0.03774	0.07290	0.01827	0.06697	0.07127
0.02013	0.05232	0.05292	0.04926	0.06933	0.04753	0.07324	0.07218	0.04709	0.04856	0.02051	0.03740	0.05282	0.03257	0.03315	0.01521	0.00782	0.02580	0.06710	0.02686	0.07344	0.07476
0.01797	0.02921	0.05478	0.07176	0.08770	0.09870	0.01358	0.01049	0.04290	0.07226	0.00934	0.03842	0.05304	0.03310	0.07379	0.03143	0.04269	0.07489	0.04913	0.00441	0.06367	0.02673
0.06034	0.04015	0.11060	0.04123	0.02113	0.03211	0.06076	0.07594	0.01985	0.02225	0.10022	0.06469	0.00868	0.00572	0.03366	0.02310	0.02228	0.05649	0.02926	0.01888	0.04083	0.11184
0.01243	0.01329	0.07719	0.06039	0.05877	0.01358	0.04813	0.03851	0.06272	0.05470	0.03726	0.02393	0.04827	0.08777	0.06875	0.05881	0.07579	0.02489	0.04481	0.01801	0.00752	0.04650
0.01456	0.05434	0.07502	0.05380	0.07173	0.01712	0.03986	0.05086	0.05369	0.06064	0.07364	0.07623	0.06483	0.03148	0.04410	0.00113	0.03318	0.06259	0.00677	0.05509	0.02269	0.03664
0.05616	0.05155	0.00501	0.08130	0.05888	0.05383	0.07040	0.01706	0.04597	0.03												

Table B.2. contd.

U ²³²	U ²³³	U ²³⁴	U ²³⁵	Np ²³⁷	Pu ²³⁶	Pu ²³⁸	Pu ²³⁹	Pu ²⁴⁰	Pu ²⁴¹	Pu ²⁴²	Am ²⁴¹	Am ^{242m}	Am ²⁴³	Cm ²⁴²	Cm ²⁴³	Cm ²⁴⁴	Cm ²⁴⁵	Cm ²⁴⁶	Cm ²⁴⁷	Cf ²⁴⁹	Cf ²⁵¹
0.02557	0.02370	0.08070	0.01857	0.00040	0.08242	0.01470	0.03580	0.02714	0.07672	0.07844	0.06236	0.02389	0.07707	0.07037	0.07527	0.04350	0.04182	0.02112	0.02252	0.05019	0.04775
0.01559	0.00874	0.07858	0.06803	0.07240	0.07064	0.02348	0.06140	0.02440	0.00550	0.06493	0.05905	0.08193	0.05727	0.02316	0.00277	0.04591	0.02634	0.07129	0.08001	0.03911	0.01948
0.03706	0.01651	0.06275	0.07075	0.08165	0.03595	0.03243	0.01859	0.05970	0.07055	0.07693	0.07443	0.06496	0.04804	0.01791	0.02007	0.07343	0.00997	0.04461	0.01667	0.02315	0.04388
0.03803	0.01170	0.06695	0.06432	0.00533	0.04506	0.00306	0.04616	0.09620	0.04009	0.04505	0.07355	0.01352	0.08172	0.03061	0.09598	0.08110	0.02693	0.03140	0.03080	0.04503	0.02742
0.04961	0.00324	0.02897	0.05864	0.06056	0.05376	0.02424	0.06026	0.08667	0.07541	0.07076	0.00591	0.06925	0.09640	0.00784	0.00521	0.04907	0.04102	0.03900	0.00442	0.01145	0.09829
0.06084	0.09312	0.09376	0.04724	0.06144	0.03050	0.01818	0.01894	0.00582	0.10072	0.08676	0.06348	0.00003	0.01232	0.00078	0.03993	0.00782	0.02766	0.02644	0.08630	0.04138	0.07654
0.00851	0.01827	0.09622	0.03358	0.07328	0.02699	0.09213	0.03279	0.06787	0.05324	0.04010	0.00477	0.09198	0.03105	0.08249	0.08101	0.07381	0.02918	0.01449	0.02696	0.00477	0.01650
0.05576	0.01088	0.06418	0.07488	0.04099	0.06240	0.01926	0.01163	0.08323	0.08779	0.06618	0.03188	0.04985	0.07321	0.05863	0.04559	0.07632	0.01817	0.01751	0.04511	0.00587	0.00068
0.04422	0.02689	0.03273	0.01572	0.07545	0.05013	0.03897	0.01777	0.06292	0.04758	0.02478	0.03994	0.07796	0.07880	0.05907	0.00321	0.05939	0.07230	0.02578	0.06539	0.05897	0.02204
0.00300	0.02445	0.08249	0.06214	0.04143	0.07942	0.07939	0.02622	0.02824	0.07083	0.05962	0.01613	0.01765	0.00414	0.01659	0.02070	0.05574	0.07626	0.05504	0.06752	0.08105	0.03195
0.07497	0.01032	0.05014	0.05929	0.07211	0.06327	0.03590	0.06267	0.08112	0.07167	0.01914	0.07609	0.05501	0.01913	0.04155	0.03340	0.01826	0.05373	0.00200	0.01070	0.01808	0.07145
0.07454	0.01369	0.04316	0.06499	0.06312	0.00497	0.04933	0.03341	0.08747	0.03641	0.06777	0.03028	0.00823	0.04359	0.07239	0.00954	0.03606	0.01583	0.05406	0.06660	0.07143	0.05313
0.06009	0.05616	0.02111	0.04638	0.08296	0.00242	0.08185	0.07920	0.00169	0.07926	0.03538	0.01607	0.01456	0.08186	0.03770	0.03763	0.04211	0.05663	0.05844	0.05964	0.01926	0.02960
0.07933	0.01724	0.07107	0.03134	0.03757	0.07823	0.06698	0.03531	0.00358	0.01130	0.05590	0.03656	0.02861	0.06012	0.08182	0.01100	0.06152	0.04577	0.05478	0.02302	0.06502	0.04394
0.02466	0.02674	0.06476	0.04855	0.02859	0.06141	0.02227	0.05934	0.03336	0.00052	0.09851	0.00210	0.06456	0.03975	0.03304	0.03173	0.04851	0.01675	0.08517	0.10036	0.05648	0.05283
0.01576	0.04108	0.02864	0.03166	0.01302	0.04072	0.04935	0.05969	0.06635	0.08598	0.06767	0.01755	0.07949	0.02489	0.06116	0.01614	0.03377	0.06817	0.05208	0.05395	0.02778	0.06508
0.05151	0.07984	0.02386	0.03584	0.01883	0.04216	0.00014	0.06530	0.04357	0.03311	0.00757	0.05515	0.04709	0.08823	0.04460	0.02554	0.02161	0.06465	0.07964	0.03006	0.06642	0.07529
0.04337	0.01469	0.05930	0.05501	0.06304	0.03305	0.03387	0.02115	0.04817	0.06264	0.00016	0.01893	0.07148	0.01822	0.06545	0.06643	0.03017	0.03103	0.07940	0.05424	0.06174	0.06845
0.01582	0.06646	0.06037	0.05881	0.02568	0.07730	0.08399	0.00199	0.06196	0.02089	0.05686	0.05750	0.03209	0.07372	0.00924	0.03343	0.05294	0.02951	0.05903	0.06347	0.00338	0.05557
0.04256	0.05516	0.07631	0.03597	0.03331	0.07221	0.08346	0.05796	0.02709	0.08297	0.04941	0.02650	0.04092	0.00471	0.03975	0.03558	0.02887	0.06550	0.04064	0.00582	0.03465	0.09166
0.02051	0.07835	0.07533	0.04349	0.04710	0.05633	0.03029	0.04371	0.05840	0.00150	0.00798	0.02785	0.03250	0.06763	0.03072	0.05270	0.05904	0.02907	0.08904	0.03331	0.02716	0.08800
0.00703	0.00024	0.03387	0.06961	0.04291	0.07625	0.04251	0.06946	0.02978	0.08670	0.03358	0.05784	0.01461	0.04335	0.07921	0.08059	0.00588	0.06431	0.02041	0.07762	0.05010	0.01415
0.08895	0.06072	0.00519	0.07021	0.02631	0.03022	0.06338	0.03653	0.03392	0.00194	0.01755	0.03012	0.05523	0.07876	0.04360	0.07340	0.00750	0.05145	0.04183	0.02656	0.07329	0.08334
0.03735	0.02699	0.05398	0.00719	0.02804	0.06601	0.08039	0.07863	0.04453	0.08403	0.00407	0.06588	0.08227	0.02210	0.01190	0.03139	0.05225	0.08641	0.03671	0.04613	0.01644	0.03732
0.06173	0.04755	0.07018	0.05282	0.05906	0.05852	0.00975	0.05540	0.05437	0.01791	0.00864	0.05513	0.03213	0.03211	0.05193	0.06157	0.01604	0.04072	0.07101	0.03652	0.05550	0.05140
0.00467	0.05036	0.07222	0.03491	0.01756	0.08114	0.05692	0.07413	0.01251	0.04334	0.03565	0.06588	0.00113	0.04963	0.04982	0.02378	0.04563	0.07462	0.07863	0.04765	0.07047	0.00935
0.01252	0.04501	0.02996	0.08469	0.00276	0.03575	0.08131	0.05470	0.03196	0.07123	0.07837	0.05986	0.01736	0.00155	0.03533	0.07858	0.04256	0.03155	0.07084	0.07828	0.00113	0.05468
0.08686	0.03642	0.03938	0.07510	0.02124	0.04191	0.05404	0.00873	0.08316	0.02851	0.08942	0.01466	0.02023	0.02635	0.07694	0.00948	0.03615	0.07337	0.03843	0.06384	0.03896	0.03680
0.02138	0.06428	0.06884	0.00009	0.00897	0.07571	0.04197	0.08533	0.00392	0.05029	0.06584	0.03894	0.04869	0.08433	0.00184	0.00340	0.03712	0.01801	0.07343	0.04647	0.07929	0.08186
0.02383	0.07362	0.01751	0.02432	0.08195	0.04299	0.03929	0.08413	0.05408	0.00437	0.02024	0.01565	0.05088	0.02816	0.02705	0.03359	0.01000	0.05059	0.07548	0.07072	0.09556	0.07601
0.01394	0.04879	0.01957	0.00695	0.03929	0.03621	0.05961	0.08173	0.05234	0.06201	0.00463	0.02724	0.07787	0.02854	0.05494	0.03983	0.08376	0.06394	0.06431	0.05186	0.07110	0.01153
0.02325	0.07752	0.11213	0.00289	0.05525	0.10130	0.10686	0.05607	0.01268	0.02081	0.02189	0.03215	0.01461	0.03975	0.06843	0.00343	0.09732	0.01024	0.00713	0.00558	0.07524	0.05547
0.05276	0.08253	0.00486	0.06950	0.04654	0.00307	0.01288	0.08074	0.06510	0.03691	0.07600	0.06487	0.03297	0.06858	0.00861	0.02447	0.03855	0.01772	0.07965	0.08435	0.00694	0.04239
0.00265	0.04980	0.00654	0.01285	0.07992	0.03175	0.08465	0.07406	0.06277	0.03614	0.00311	0.01375	0.00576	0.08072	0.06245	0.06968	0.05342	0.01332	0.05392	0.07536	0.09111	0.03627
0.03776	0.01305	0.01299	0.06114	0.00996	0.03720	0.02771	0.08570	0.05868	0.03711	0.04874	0.05107	0.04776	0.07511	0.00440	0.08465	0.09971	0.03724	0.02944	0.03200	0.09987	0.00870
0.00386	0.00940	0.08799	0.07060	0.01048	0.00675	0.05056	0.08364	0.07310	0.01564	0.00339	0.07805	0.00103	0.06084	0.01032	0.04576	0.05515	0.08015	0.08685	0.05005	0.06141	0.05497
0.00259	0.01597	0.03742	0.00301	0.03149	0.04565	0.00652	0.07356	0.02983	0.05919	0.06845	0.04047	0.02601	0.05946	0.10310	0.00817	0.04495	0.06810	0.06916	0.09826	0.08136	0.02728
0.02527	0.08226	0.07800	0.02827	0.04839	0.04413	0.03485	0.01302	0.05221	0.03411	0.01241	0.01139	0.04046	0.07277	0.03317	0.08404	0.00137	0.08431	0.04369	0.06263	0.05286	0.06038
0.04714	0.00860	0.06045	0.01831	0.07558	0.07875	0.00842	0.04305	0.04148	0.01906	0.02413	0.02720	0.00658	0.02217	0.04930	0.05177	0.10398	0.04305	0.04142	0.04548	0.10180	0.08228
0.02973	0.01748	0.05477	0.08696	0.04276	0.08325	0.06344	0.00632	0.05540	0.02281	0.05706	0.00640	0.04621	0.01959	0.08526	0.07546	0.03556	0.03630	0.05815	0.03855	0.07381	0.00473
0.02425	0.01162	0.03039	0.07207	0.07237	0.04987	0.06025	0.09068	0.02219	0.06179	0.04229	0.00772	0.01402	0.07315	0.10357	0.03399	0.01637	0.02855	0.08614	0.07898	0.00819	0.01156
0.06389	0.08414	0.03500	0.00984	0.04271	0.05588	0.05563	0.02883	0.09023	0.05442	0.04640	0.02714	0.09036	0.02125	0.04528	0.08213	0.01902	0.04088	0.01507	0.03007	0.00656	0.05525
0.07356	0.04151	0.09098	0.05689	0.00793	0.04420	0.04624	0.02634	0.05390	0.02919	0.05667	0.01156	0.05911	0.00268	0.00490	0.06163	0.02857	0.06878	0.08861	0.06173	0.00034	0.08468
0.03031	0.06588	0.02111	0.03518	0.00955	0.05846	0.10701	0.07466	0.10306	0.00889	0.00012	0.02177	0.09099	0.09272	0.00360	0.04120	0.05505	0.01827	0.02909	0.02345	0.06423	0.04541
0.00243	0.06852	0.02434	0.08586	0.07622	0.05778	0.01435	0.07909	0.04151	0.04358												

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