Optimizing the designs and operations of water networks: a decomposition approach

Yijiang Li [1], Santanu Dey [2], Nick Sahinidis [2], Naresh Susarla [3,4], Miguel Zamarripa-Perez [3,4], and Markus Drouven [3]

[1]Argonne National Laboratory [2]Georgia Institute of Technology [3]National Energy Technology Laboratory (NETL) [4]NETL site support contractors

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Water Network Design

What kind of water networks?

- Water can be produced from various sources with different characteristics
- Produced water co-produced together with oil and gas from reservoirs
  - Volume of water produced starts high and gradually decreases
  - Network is influenced by the elevations of the nodes
- The Produced Water Optimization Initiative (PARETO) [Drouven et al. 2022, `https://www.project-pareto.org`, and `https://github.com/project-pareto`]

PARETO
The Produced Water Optimization Initiative

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- Detailed review [Mala-Jetmarov et al. 2017, D'Ambrosio et al 2015]

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  - Linearization in a LP/NLP branch-and-bound; specialized leaf-node problems [Raghunathan 2013, Rajagopalan 2018]
  - Optimal placement of relief valves; linear relaxation and domain reduction [Pecci et al. 2019]

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  - Optimal placement of relief valves; linear relaxation and domain reduction [Pecci et al. 2019]
- Operation problem (on flowrates and pressures etc.)
  - Lagrangian decomposition to obtain smaller subproblems; simulation-based heuristic [Ghaddar et al. 2015]
  - Complex pump station modeling; linear relaxation in a LP/NLP branch-and-bound; primal heuristic [Bonvin et al. 2021]

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  - An arc is a pipe with multiple diameter choices $[n] := \{1, \ldots, n\}$
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  - Nodes include customers, reservoirs, and in-nodes
- Multiple time periods $\{1, \ldots, T\}$
- Flowrate $q_{a,t}$ for each arc $a \in \mathcal{A}$ in period t
- Pressure $p_{v,t}$ at each node $v \in \mathcal{V}$ in period t
  - Pressures at reservoirs are fixed to $p_{v,t}^{\text{src}}$

Modeling

Problem settings and Notations

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- Minimization of the total cost of selected diameter choices (a.k.a. the budget)

For a pipe $a \in \mathcal{A}$,

- Pressure drop for steady state flow due to friction via Hazen-Williams equation

$$\Delta_{p_t} \propto |q_{a,t}|^{0.852} q_{a,t}$$

- Elevation change induces pressure change, δ_a
- Binary variables for diameter selection
- Multiple copies of flowrate variables for each diameter choice
- Decomposition of flows into positive and negative flows, $q^+_{a,t}$ and $q^-_{a,t}$

- Variables are in blue while constants are in black

$$
\begin{aligned}
& 0 \leq q^{-}_{a,t,i},\ q^{+}_{a,t,i} \leq q^{\max}_{a,i} z_{a,i}, \quad a \in \mathcal{A}, i \in [n], t \in \{1, \ldots, T\} \\
& p_{v,t} - p_{w,t} = \sum_{i \in [n]} \alpha_{a,i} (q^{+}_{a,t,i})^{1.852} - \sum_{i \in [n]} \alpha_{a,i} (q^{-}_{a,t,i})^{1.852} + \delta_a, \\
& \quad a = (v, w) \in \mathcal{A}, t \in \{1, \ldots, T\} \\
& 0 \leq q^{+}_{a,t,i} \leq q^{\max}_{a,i} x^{\text{dir}}_{a,t}, \qquad a \in \mathcal{A}, i \in [n], t \in \{1, \ldots, T\} \\
& 0 \leq q^{-}_{a,t,i} \leq q^{\max}_{a,i} (1 - x^{\text{dir}}_{a,t}), \qquad a \in \mathcal{A}, i \in [n], t \in \{1, \ldots, T\} \\
& \sum_{i \in [n]} z_{a,i} = 1, \qquad a \in \mathcal{A} \\
& z_{a,i},\ x^{\text{dir}}_{a,t} \in \{0, 1\}, \qquad a \in \mathcal{A}, i \in [n], t \in \{1, \ldots, T\}.
\end{aligned}
$$

- In addition to frictional pressure loss and elevation change:
  - A pump station induces pressure increase along a pipe
  - A relief valve induces pressure relief along a pipe
- Pump stations, if installed, have to satisfy additional operational constraints:
  - Minimum-up and minimum-down constraints
  - Operation cost upper limit

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$$p_{v,t} - p_{w,t} = \sum_{i \in [n]} \alpha_{a,i} (q^{+}_{a,t,i})^{1.852} - \sum_{i \in [n]} \alpha_{a,i} (q^{-}_{a,t,i})^{1.852} + \delta_a - \Delta_{I,a,t} + \Delta_{R,a,t},$$

$$a = (v, w) \in \mathcal{A}, t \in \{1, \ldots, T\}$$

$$0 \leq \Delta_{I,a,t} \leq \overline{\Delta}_{I,a} \xi_{a,t}, \quad a \in \mathcal{A}, t \in \{1, \ldots, T\}$$

$$0 \leq \Delta_{R,a,t}, \quad a \in \mathcal{A}, t \in \{1, \ldots, T\}$$

$$\xi_{a,t} \leq z_{I,a}, \quad a \in \mathcal{A}, t \in \{1, \ldots, T\}$$

$$\sum_{a \in \mathcal{A}} z_{I,a} \leq N$$

$$\sum_{t \in \{1, \ldots, T\}} \xi_{a,t} \leq M_a, \quad a \in \mathcal{A}$$

$$\xi_{a,t} - \xi_{a,t-1} \leq \xi_{a,\tau}, \quad a \in \mathcal{A}, t \in \{2, \ldots, T\}, \tau \in \{t+1, \ldots, \min\{t + \tau_o, T\}\}$$

$$\xi_{a,t-1} - \xi_{a,t} \leq 1 - \xi_{a,\tau}, \quad a \in \mathcal{A}, t \in \{2, \ldots, T\}, \tau \in \{t+1, \ldots, \min\{t + \tau_f, T\}\}$$

$$z_{I,a}, \xi_{a,t} \in \{0, 1\}, \quad a \in \mathcal{A}, t \in \{1, \ldots, T\}$$

Modeling

Challenges

1. Nonlinearity and nonconvexity in the pressure change constraints
   - There are solvers that can handle these constraints (e.g., BARON, SCIP)
2. Binary variables associated with the pipes and pump stations

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Solvers like BARON and SCIP can improve on the dual bounds, but can be slow in obtaining primal (feasible) solutions

Primal Solution
Two methods

To obtain primal solutions, we propose two methods,

- An iterative procedure involving a convex reformulation (based on CVXNLP)
- A time decomposition based method

Primal Solution

Background on CVXNLP

- Network analysis equations

$$p_v - p_w = \mathrm{sgn}(q_a)\phi(|q_a|),\ a \in \mathcal{A}$$

$$\sum_{a \in \mathrm{in}(v)} q_a - \sum_{a \in \mathrm{out}(v)} q_a = d_v,\ v \in \mathcal{V} \setminus V^{\mathrm{src}}$$

$$p_v = p_v^{\mathrm{src}},\ v \in V^{\mathrm{src}}$$

where $\mathrm{sgn}(\cdot)$ is the sign function

- CVXNLP [Cherry 1951]

$$\min \sum_{a \in \mathcal{A}} \Phi(q_a^+) + \Phi(q_a^-) - \sum_{v \in V^{\mathrm{src}}} \sum_{a \in \mathcal{A}} p_v^{\mathrm{src}}(q_a^+ - q_a^-)$$

$$\text{s.t.} \sum_{a \in \mathrm{in}(v)} (q_a^+ - q_a^-) - \sum_{a \in \mathrm{out}(v)} (q_a^+ - q_a^-)$$

$$= d_v,\ v \in \mathcal{V} \setminus V^{\mathrm{src}}$$

$$0 \leq q_a^-, q_a^+,\ a \in \mathcal{A}$$

where $\Phi(q) = \int_0^q \phi(q')dq'$

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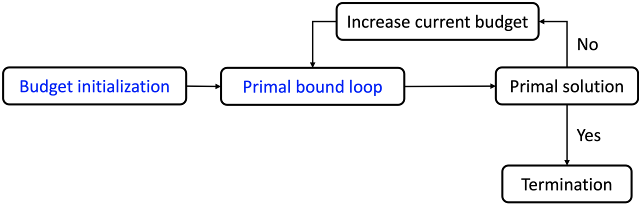
$$0 \le q_a^-, q_a^+,\ a \in \mathcal{A}$$

$$\text{where } \Phi(q) = \int_0^q \phi(q')dq'$$

Theorem. (c.f. [Raghunathan 2013]) If the pressure drop function $\phi(\cdot)$ is strictly monotonically increasing function of flowrate, q, with $\phi(0) = 0$, then there exists a solution (p, q) to the network analysis equations if and only if there exists a solution $(\hat{q}^+, \hat{q}^-, \hat{\lambda}, \hat{\mu}^+, \hat{\mu}^-)$ to (CVXNLP).

Primal Solution
CVXNLP based decomposition

- Two components:
  - Primal bound loop to validate if a given budget is feasible
  - Budget initialization to find a good initial budget



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- Master problem (P_m) is constructed based on CVXNLP that additionally includes
  - Binary variables for the diameter choices
  - A constraint on the cost of diameter selections
- Subproblem (P_s)
  - Diameter choices for pipes are fixed
  - A more computationally tractable feasibility MINLP

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- Outcomes for each iteration
  - Feasible: a primal (feasible) solution is obtained
  - Infeasible: add no-good cut and repeat

Primal Solution

Time decomposition

Motivations:

- Diameter choices that are optimal to a restricted problem for $\{1, \ldots, \tilde{T}\}$ where $\tilde{T} < T$ (or $\hat{T} \subset \{1, \ldots, T\}$) are likely feasible to the volume produced for the remaining periods
- A pump station is more likely needed with high flowrates

Primal Solution

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- $\tilde{T}$ and ε can be selected from a set $\mathcal{T}$ and a set $\mathcal{E}$ respectively

- All optimization problems are solved by SCIP
- CVXNLP based decomposition or time decomposition to obtain the first primal solution
- Provide the primal solution to SCIP as the initial point with 5 hours' time limit

Computational Experiments

Network

- An PARETO case study derived from a real-world network in the US
- Network characteristics

Type	Name	Count
Nodes	Sources	19
	Sinks	7
	In-nodes	29
Arcs	Pipes	58

- 4 discrete diameter choices for each pipe and 53 time periods
- Base demand scenario is scaled by stress factors from $\{0.1, 0.5, 1.0, 1.5, 2.0\}$ to obtain additional scenarios

Computational Experiments

Results

Stress	MINLP by SCIP			CVXNLP based decomposition			Time decomposition		
	Primal	Dual	Gap(%)	Primal	Dual	Gap (%)	Primal	Dual	Gap (%)
0.1	-	2443.39	-	2443.39	2443.39	0.00	2443.39	2443.39	0.00
0.5	-	2493.29	-	2721.64	2493.29	9.16	2645.06	2493.29	6.09
1.0	-	2599.72	-	4822.70	2583.69	86.66	2828.82	2582.85	9.52
1.5	-	2748.69	-	5288.98	2715.45	94.77	3054.41	2711.14	12.66
2.0	-	2857.61	-	-	-	-	3212.51	2829.65	13.53

- Time decomposition has the best performance ($5 \leq \tilde{T} \leq 8$ and $5\% \leq \varepsilon \leq 15\%$)
- CVXNLP based decomposition struggles as the stress factor increases
- No new primal solutions found by SCIP
- It can be difficult to solve even the restricted problem directly by SCIP; opportunity to combined the two methods

Summary

- Considered water network design and operation problems in a single problem
- Two methods to find primal (feasible) solutions
- Computational experiments on a PARETO case study

Thank you and any questions?

yijiang.li@anl.gov