

New Developments in the Mechanism for Core-Collapse Supernovae

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Abstract

Recent results indicate that the standard type-II supernova scenario in which the shock wave stagnates but is reenergized by neutrino heating fails to consistently produce supernova explosions having the required characteristics. We review the theory of convection and survey some recent calculations indicating the importance of convection operating on millisecond timescales in the protoneutron star. These calculations suggest that such convection is probably generic to the type-II scenario, that this produces a violent overturn of material below the stalled shock, and that this overturn could lead to significant alterations in the neutrino luminosity and energy. This provides a mechanism that could be effective in reenergizing the stalled shock and producing supernovae explosions having the quantitative characteristics demanded by observations. This mechanism implies, in turn, that the convection cannot be adequately described by the 1-dimensional hydrodynamics employed in most simulations. Thus, a full understanding of the supernova mechanism and the resulting heavy element production is likely to require 3-dimensional relativistic hydrodynamics and a comprehensive description of neutrino transport. The prospects for implementing such calculations using a new generation of massively parallel supercomputers and modern scalable algorithms are discussed.

1 Introduction

Considerable progress has been made over the past two decades in understanding the mechanisms responsible for type-II supernovae (see the review by Bethe [1] and the volume of Physics Reports edited by Brown [2]). This understanding was tested both qualitatively and quantitatively by the observation of Supernova 1987A in the nearby Magellanic Cloud.

massive star in which a degenerate iron core of approximately 1.4 solar masses collapses catastrophically on millisecond timescales. This gravitational collapse is reversed as the inner core exceeds nuclear densities and a pressure wave propagates outward, steepening into a shock wave as it passes into less dense material of the outer core. The most realistic simulations of this event indicate that for core masses of more than about 1.1 solar masses the shock stalls into an accretion shock several hundred kilometers from the center. For a decade there has been a growing consensus that the neutrinos, which are produced in prodigious quantities in the supernova and play a central role in the entire event, reenergize the shock, thereby allowing the explosion to proceed to a conclusion in accord with the observed properties of supernovae [3, 4]. In this lecture I wish to give a pedagogical review of the physics of supernova explosions, with particular emphasis on new developments associated with the role of convection and neutrinos in reenergizing the shock.

2 The Death of Massive Stars

Massive stars near the ends of their lives build up a layered structure of shells containing heavier and heavier elements with iron at the center. The iron core cannot produce energy by fusion, so it must be supported by electron degeneracy pressure. Electron degeneracy can support the iron core against gravitational collapse only if its mass remains below the Chandrasekhar limit, which depends on the electron fraction (the ratio of electrons to nucleons) but is approximately 1.4 solar masses for the typical case. When the iron core exceeds this critical mass, it begins to collapse because the electron degeneracy can no longer balance the gravitational forces.

At the point where the collapse begins, the iron core of a 25 solar mass star has a mass of about 1.4 solar masses and a diameter of several thousand km. The core density is about 6×10^9 g/cm³, the core temperature is approximately 6×10^9 K, and the entropy per baryon per Boltzmann constant is about 1, in dimensionless units. This entropy is remarkably low: the entropy of the original main-sequence star that produced this iron core is about 15–20 in these units. In ⁵⁶Fe, the 26 protons and 30 neutrons are highly ordered compared with 56 free nucleons in the original star, because they are constrained to move together as part of the iron nucleus. Thus, the core of the star becomes *more ordered* compared with the original star as the nuclear fuel is consumed. The entire universe becomes more disordered because the star radiates energy in the form of photons and neutrinos as it builds its ordered core.

3 Sequence of Events in Core Collapse

The instability of the iron core as the Chandrasekhar limit is exceeded triggers a catastrophic sequence of events that will occur in an elapsed time of less than a second.

1. When the mass of the iron core exceeds about 1.4 solar masses, it begins to collapse under the influence of gravity. This collapse is accelerated by two factors:
 - (a) As the core heats up, high-energy γ -rays are produced; these photodisintegrate some of the iron-peak nuclei into α -particles. The corresponding process is highly endothermic; for example, $^{56}\text{Fe} \rightarrow 13\alpha + 4n$ has a Q -value of -124.4 MeV. This decreases the kinetic energy of the electrons in the core, which lowers the pressure and hastens the collapse. This process is important for more massive stars.
 - (b) As the density and temperature increase in the core, the rate for the electron capture reaction $p^+ + e^- \rightarrow n + \nu$ is greatly enhanced. This reaction decreases the electron fraction Y_e of the core, and the neutrinos easily escape the core during the initial phases of the collapse because their mean free path is much larger than the initial radius of the core. These neutrinos carry energy with them, decreasing the core pressure and accelerating the collapse even further. This process is more important in less massive stars.
2. The accelerated core collapse proceeds on a timescale of milliseconds. The core separates into an *inner core* that collapses subsonically and homologously ($v \propto r$) and an *outer core* that collapses in near free-fall ($v \propto r^{-1/2}$), with a velocity exceeding the local velocity of sound in the medium.
3. This collapse is slow compared with the reaction rates and the core is approximately in equilibrium during all phases of the collapse. This implies that the *entropy is constant*, and the highly ordered iron core before collapse ($S \simeq 1$) remains ordered during the collapse.
4. As the collapse proceeds and the temperature and density rise, the neutrino mean free path becomes less than the radius of the core at a density of $10^{11-12} \text{ g/cm}^3$. The time for neutrinos to diffuse outward becomes longer than the characteristic time of the collapse and the neutrinos are effectively trapped in the collapsing core.
5. Because the collapse proceeds with low entropy, there is little excitation of the nuclei and the nucleons remain in the nuclei until nuclei begin to touch. Thus, the collapse cannot be stopped before the inner core reaches supernuclear densities. Somewhat

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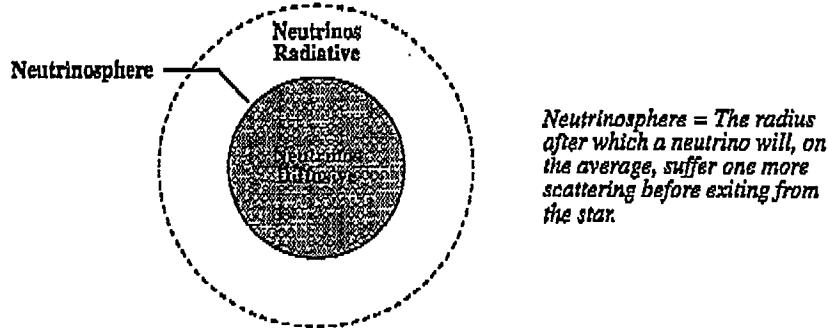


Figure 1: The neutrinosphere.

beyond nuclear density, the incompressible core of nearly degenerate nuclear matter rebounds violently and a pressure wave reflects from the center of the star and proceeds outward. This wave steepens into a shock wave as it moves through less and less dense material, with the shock wave forming near the boundary between the subsonic inner core and supersonic outer core. In the simplest picture this shock wave would eject the outer layers of the star, resulting in a supernova explosion. This is called the *prompt shock mechanism*.

6. Unfortunately, realistic calculations suggest that the prompt shock dissipates energy rapidly as it progresses through the outer core because of two unavoidable processes:
 - (a) The shock wave weakens as it dissociates Fe nuclei into nucleons in passing through the outer core.
 - (b) As the shock wave passes into less dense material, the mean free path for the trapped neutrinos increases until the neutrinos can once again be freely radiated from the core. The radius at which the neutrinos change from diffusive to radiative behavior is termed the *neutrinosphere* (see Fig. 1). When the shock wave penetrates the neutrinosphere, a burst of neutrinos is emitted from the core, carrying with it large amounts of energy and lowering the pressure behind the shock.

Realistic calculations indicate that the shock wave stalls into an *accretion shock* (a standing shock wave at a constant radius) before it can exit the core, provided that the original iron core mass is larger than about 1.1 solar masses. In a typical calculation, the accretion shock forms at 200–300 km from the center of the star within about 10 ms of core bounce. Since SN1987A appears to have resulted from the collapse of a

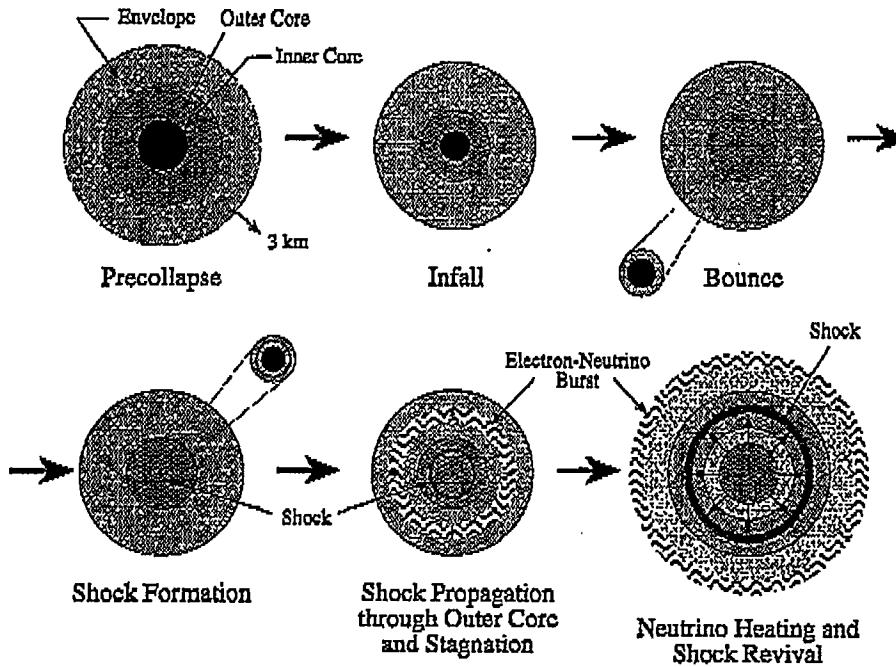


Figure 2: Illustration of the neutrino reheating mechanism for a supernova explosion (after Bruenn [5]). Figures are approximately to scale, except that the surface of the star would lie some 3 km from the center if represented on this scale.

core having 1.3–1.4 solar masses, serious doubt exists concerning the prompt shock mechanism as a generic explanation of type-II supernovae.

4 Neutrino Reheating

The idea that neutrinos might play a significant role in a supernova event is not new, but the failure of the prompt mechanism revived interest in such mechanisms [3, 4]. This evolved into what is generally termed the *delayed shock mechanism* or *neutrino reheating mechanism*, in which the stalled accretion shock is reenergized by neutrino heating of matter behind the shock. This raises the pressure sufficiently to impart an outward velocity to the shock on a timescale of approximately one second and complete the supernova explosion. The schematic mechanism for the supernova event thus becomes the two-stage process depicted in Fig. 2.

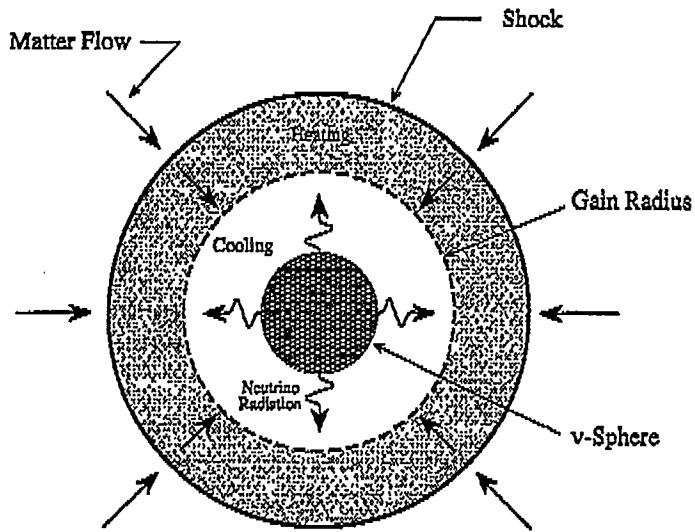


Figure 3: The supernova core during shock stagnation. The neutrinosphere and the gain radius are indicated.

4.1 Reheating of Shocked Matter

In the post-bounce phase, the most important reactions leading to neutrino cooling are the capture reactions $e^- + p \rightarrow n + \bar{\nu}_e$ and $e^+ + n \rightarrow p + \bar{\nu}_e$, and the most important reactions leading to heating are the inverse reactions that absorb neutrinos. By these and other less important types of interactions the neutrinos that are produced in the core, and from the matter accreting on the core, can interact with the matter behind the shock wave. In discussing the details of that interaction, it is useful to introduce two characteristic radii. The first we have already met in conjunction with Fig. 1: The *neutrinosphere* defines a radius beyond which an average neutrino will suffer one more scattering before it leaves the star. Secondly, general considerations suggest that there is always a radius less than that of the shock, inside of which the net effect of the neutrino interactions is to cool the matter and outside of which the net effect of the neutrino interactions is to heat the matter. This break-even radius, beyond which the neutrino interactions can become effective in increasing the pressure behind the shock, is termed the *gain radius*. These radii are indicated schematically in Fig. 3.

4.2 Calculations with Neutrino Reheating

The result of a large number of calculations is that the neutrino reheating helps, and can often turn a failed explosion into a supernova. However, such calculations do not always succeed without artificial boosts of the neutrino luminosities. More importantly, even in the successfully reenergized shocks, the resulting supernova explosion often yields a factor of 3-10 less energy in the light and remnant kinetic energy than is observed. These persistent results suggest that there still are missing ingredients in the supernova mechanism that must be included to obtain a quantitative description. One suggestion is that convection in the region interior to the shock wave alters the neutrino emission in a non-negligible fashion. Let us now turn to a general discussion of the role that convection might play in supernova explosions.

5 Convection and Neutrino Reheating

We begin by examining the general phenomenon of convection, and by deriving some conditions under which we may expect a region of a fluid to become unstable against convective motion [6]. Let us first make a conceptual distinction between two categories of convection. The first may be termed *microconvection*, and applies when the convective blobs are small relative to the region that is unstable. The second may be termed *macroconvection*, and corresponds to convection in which the blobs are a substantial fraction of the size of the convective region. This distinction has an important practical implication. For microconvection, it is possible to introduce approximations (*mixing length theory*—see [1]) that allow one to retain spherical symmetry in the problem; thus, 1-dimensional hydrodynamics may suffice under such conditions. However, if macroconvection obtains, the spherical symmetry is strongly broken and one is forced to deal with multidimensional hydrodynamics.

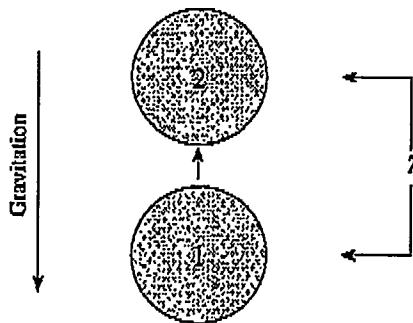
5.1 Conditions for Convective Instability

Consider the following thought experiment [6]: imagine a blob of matter in a fluid (in a gravitational field) which moves upward a distance λ from position 1 to position 2 because

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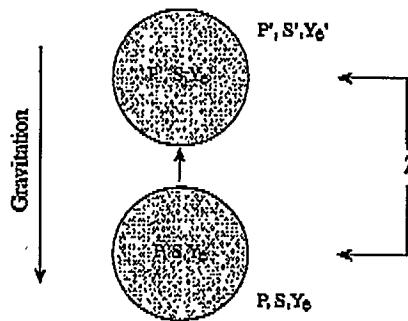
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of some infinitesimal stimulus.



If the blob of material at position 2 is less dense than the surrounding material, it will be driven upward by buoyancy forces, and the region is said to be convectively unstable. We may choose to impose particular physical conditions on how the blob of matter is moved, and these lead to three separate criteria for convective instability.

Schwarzschild Instability. Let us suppose that the blob moves adiabatically (constant entropy), but in pressure and composition equilibrium with the surrounding medium. Denoting the pressure, entropy, and electron fraction (composition) of the medium at position 1 by P , S , and Y_e , and at position 2 by the corresponding variables with a prime,



the condition for convective instability is

$$\rho(P', S', Y_e') - \rho(P', S, Y_e) \geq 0. \quad (1)$$

Expanding in a Taylor series,

$$\rho(P', S', Y_e') - \rho(P', S, Y_e) = \left. \frac{\partial \rho}{\partial S} \right|_{P, Y_e} \lambda \frac{dS}{dr}. \quad (2)$$

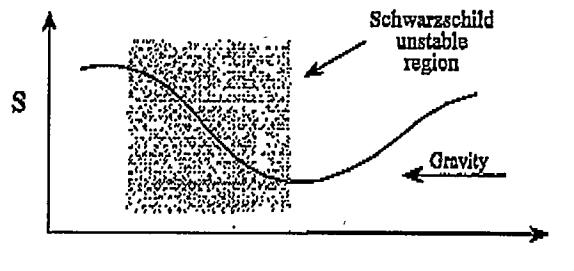
By introducing the specific heat at constant pressure C_p , we may exchange the entropy S for the temperature T as a variable and Eq. (1) becomes

$$\left. \frac{T}{C_p} \frac{\partial \rho}{\partial T} \right|_{P, Y_e} \lambda \frac{dS}{dr} \geq 0. \quad (3)$$

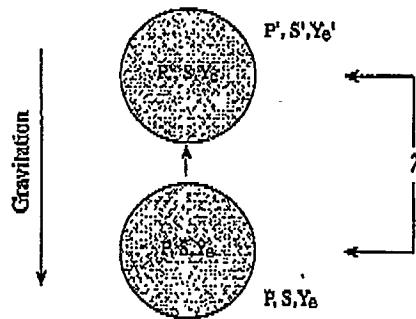
For normal equations of state the partial derivative is negative and Eq. (3) is equivalent to the *Schwarzschild condition* for convective instability,

$$\frac{dS}{dr} \leq 0 \quad \text{Schwarzschild condition.} \quad (4)$$

Thus, a region is unstable against Schwarzschild convection if there is a *negative entropy gradient*:



Ledoux Instability. Now suppose that the blob moves adiabatically with no composition change, but in pressure equilibrium.



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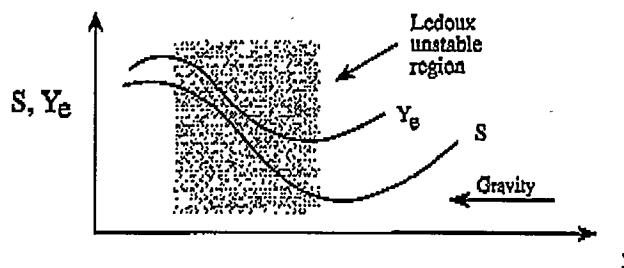
The condition for convective instability may now be expressed as

$$\frac{T}{C_p} \frac{\partial \rho}{\partial T} \Big|_{P, Y_e} \lambda \frac{dS}{dr} + \frac{\partial \rho}{\partial Y_e} \Big|_{P, S} \lambda \frac{dY_e}{dr} \geq 0 \quad (5)$$

where the first term is the same as for the Schwarzschild instability, and the second term arises because of the assumption that there is no composition change. For most cases of interest, both partial derivatives are negative and the Ledoux condition for instability is

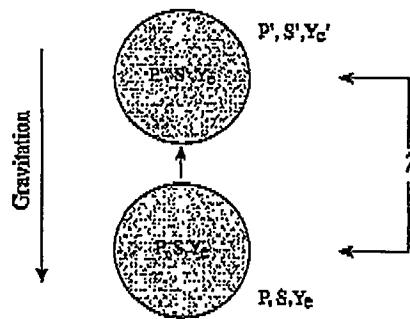
$$\frac{dS}{dr} + k \frac{dY_e}{dr} \leq 0 \quad \text{Ledoux condition} \quad (6)$$

where k is a positive constant. Thus, a region is unstable against Ledoux convection if both the entropy and the electron fraction have a negative gradient.



If the entropy gradient and electron fraction gradient have opposite signs, the stability of the region is dependent on the relative sizes of the two terms in Eq. (6). Thus, for example, a region could be Schwarzschild stable, but Ledoux unstable.

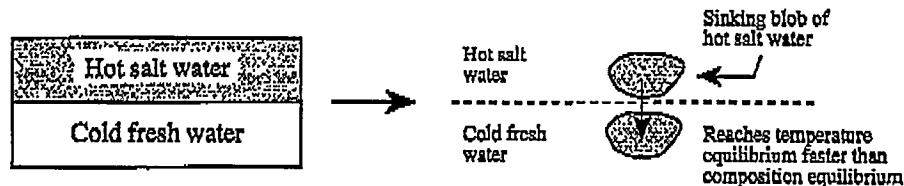
Salt-Finger Instability. Finally, let us consider a situation where the blob is in temperature and pressure equilibrium with the surrounding medium, but not in composition equilibrium:



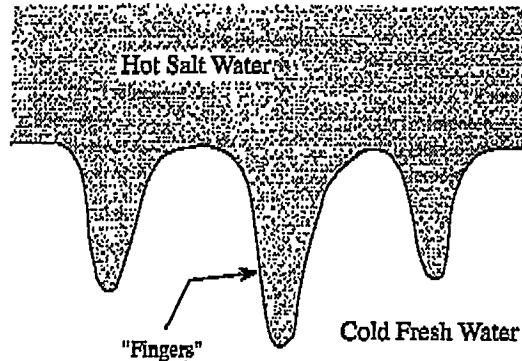
The condition for convective instability now takes the form,

$$\rho(P', T', Y_e) - \rho(P', T', Y_e) = \left. \frac{\partial \rho}{\partial Y_e} \right|_{P,T} \lambda \frac{dY_e}{dr} \geq 0 \quad (7)$$

We may imagine the following thought experiment in which such an instability could occur. Consider a layer of hot salt water that lies over a layer of cold fresh water, and imagine a blob of the hot salt water that begins to sink into the underlying cold fresh water:



This blob of sinking material will come into heat equilibrium with its surroundings faster than it will come into composition equilibrium, because the transfer of heat by means of molecular collisions is faster than the motion of the Na and Cl ions that causes the composition to equilibrate. Thus, such a blob may be in approximate temperature equilibrium but remain out of composition equilibrium. The heat diffusion will cool the blob of salt water, and since salt water is more dense than fresh water at the same temperature, the blob continues to sink in the surrounding fresh water. As this motion continues, the medium develops "fingers" of salt water reaching down into the fresh water,



This is a prototype of a convective instability that is commonly termed a *salt-finger instability*.

The analog of the salt-finger instability may occur in a supernova. Consider a collapsed supernova progenitor shortly after core bounce; calculations suggest that near shock

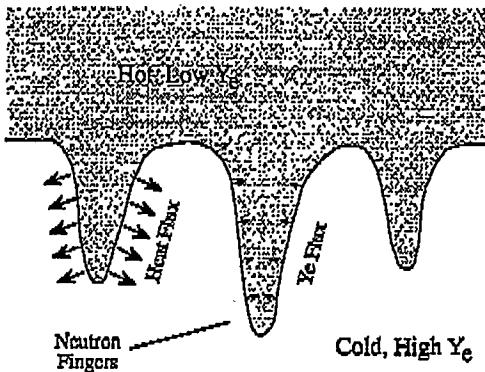
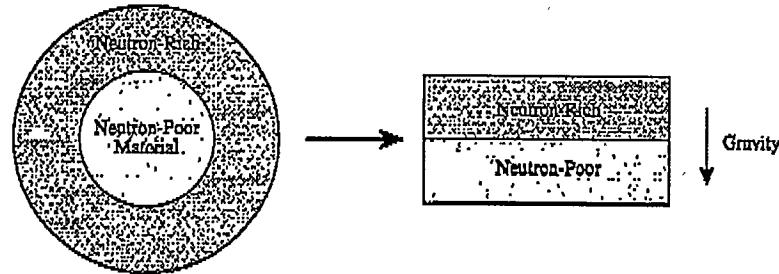


Figure 4: Neutron-finger convective instability.

stagnation a neutron-rich region surrounds a central region that is less neutron rich:



By analogy with the salt-finger example just discussed, a blob of sinking neutron-rich material will be brought quickly to thermal equilibrium by the bath of neutrinos and antineutrinos of all flavors. However, the approach to composition equilibrium will be slower because it depends on the the difference in the numbers of neutrinos and antineutrinos, $N_\nu - N_{\bar{\nu}}$. Furthermore, a sinking neutron-rich blob of matter (in temperature equilibrium but not composition equilibrium with the surroundings) is *more dense* than the surrounding neutron-poor matter. This situation is analogous to the instability just discussed for salt fingers, with heat diffusing rapidly and electron fraction playing the role of the (slowly diffusing) salt concentration. Thus, We may expect an instability toward the formation of *neutron fingers*, as illustrated in Fig. 4.

5.2 Convectively Unstable Regions in Supernovae

Armed with our pedagogical results from the preceding section for predicting which regions may be convectively unstable, let us now examine the lepton fraction and entropy gradients

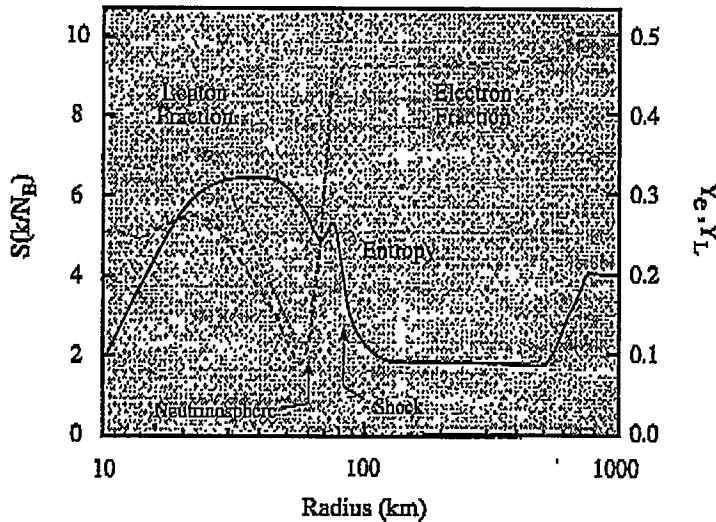


Figure 5: Composition fractions and entropy following the bounce in a typical supernova calculation. The progenitor had a mass of $15 M_{\odot}$, and the calculation is described in Ref. [5].

produced during the period of shock stagnation in typical supernova calculations. In Fig. 5 we show results due to Bruenn for a $15 M_{\odot}$ star at 6 ms after the bounce. There are regions lying inside the shock front where either the entropy or the lepton fraction, or both, exhibit strong negative gradients and thus may be convectively unstable. Such features are commonly found in supernova simulations.

Our arguments to this point only identify regions that are favorable for convective motion. Whether such regions develop convection, the timescale for that convection, and the quantitative implications for supernova explosions may only be settled by detailed calculations (see the next section). Nevertheless, we may conjecture that substantial convection inside the stalled shock could have significant influence on the possibility of neutrinos reenergizing the shock, and on the quantitative characteristics of a reenergized shock. In particular, we note that to boost the stalled shock it is necessary for the neutrinos to interact with matter outside the gain radius and inside the shock front. Convective motion inside the shock front could, by overturning hot and cooler matter, cause more neutrino production. The convection could also move neutrino-producing matter beyond the neutrinosphere, so that the neutrinos that are produced would have a better chance to propagate into the region closely behind the shock where deposition of energy would have the most favorable influence in increasing the pressure and reenergizing the shock. This would provide a possible method to produce a supernova explosion with the required $\simeq 10^{51}$ ergs of energy, thereby

solving the "supernova problem".

5.3 Convection in Multidimensional Hydrodynamics

In Fig. 6 we illustrate a recent calculation that has used 2-dimensional hydrodynamics to test the preceding conjectures about convective instabilities and the role that they might have in reenergizing the stalled shock of a type-II supernova [7]. Proceeding from the upper right, each quadrant represents lepton fraction distributions in one quadrant of the protoneutron star at successive 5 ms intervals after shock stagnation. One sees evidence for rapid and large-scale convection on this timescale. This convection quickly engulfs much of the region below the shock, including the neutrinosphere. Although such calculations are not yet realistic enough to provide conclusive evidence, they suggest that strong convection on millisecond timescales should play a generic role in the stalled supernova. Although the full implications for the neutrinos are not addressed by these calculations, approximate neutrino transport coupled to multidimensional hydrodynamics in at least some more recent calculations do find enhanced neutrino reheating in the presence of macroscopic convection.

These results suggest the necessity of employing multidimensional hydrodynamics with sophisticated neutrino transport for any realistic simulation of a type-II supernova. Such calculations are orders of magnitude more difficult than traditional supernova calculations. Fortunately, recent advances in massively parallel computation promise that orders of magnitude increases in computational power may be available for scalable algorithms implemented on such computers. We are presently investigating the potential of a new 3-dimensional Eulerian hydrodynamics code written for parallel systems to solve this problem.

6 Summary

In conclusion, type-II supernovae obviously occur, and almost certainly correspond to a stellar core collapse in some form. However, there is an increasing feeling that neither the prompt shock, nor simple neutrino reheating with 1-dimensional hydrodynamics, can account for the observational characteristics of type-II supernovae. Model calculations suggest that macroscopic convection operating on millisecond timescales may be a generic feature behind the stalled shock, and this could alter neutrino production in a way to produce a satisfactory delayed explosion. These developments suggest that an adequate model for a supernova will require 3-dimensional Eulerian hydrodynamics with a comprehensive treatment of neutrino transport. There is hope that such calculations can be implemented on a new generation of massively parallel supercomputers, where orders of magnitude

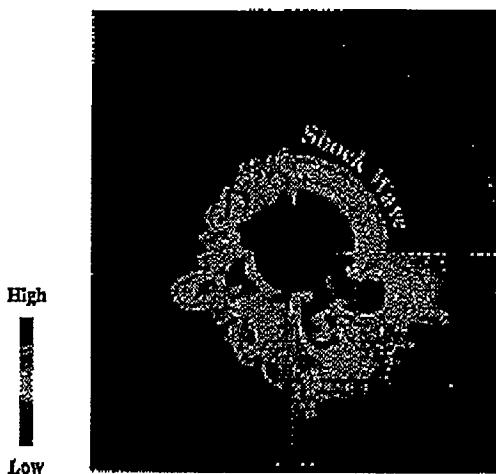


Figure 6: Lepton fraction distribution in protoneutron star at 5, 10, 15, and 20 ms after shock stagnation. Distributions at the four times are shown in counter-clockwise order beginning from the upper right quadrant. The position of the shock front at about 100 km is noted. The region inside of 50 km was replaced by a boundary condition. Calculations are from Burrows and Fryxell [7].

enhancement in computational power may be realized for sufficiently scalable algorithms. We are presently developing such algorithms for the supernova problem.

7 Acknowledgments

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