

# MOTION PLANNING FOR MOBILE MANIPULATORS USING THE FSP (FULL SPACE PARAMETERIZATION) APPROACH\*

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## ABSTRACT

The efficient utilization of the motion capabilities of mobile manipulators, i.e., manipulators mounted on mobile platforms, requires the resolution of the kinematically redundant system formed by the addition of the degrees of freedom (d.o.f.) of the platform to those of the manipulator. At the velocity level, the linearized Jacobian equation for such a redundant system represents an underspecified system of algebraic equations. In addition, constraints such as obstacle avoidance or joint limits may appear at any time during the trajectory of the system. A method, which we named the FSP (Full Space Parameterization), has recently been developed to resolve such underspecified systems with constraints that may vary in time and in number during a single trajectory. In this paper, we review the principles of the FSP and give analytical solutions for the constrained motion case, with a general optimization criterion for resolving the redundancy. We then focus on a solution to the problem introduced by the combined use of prismatic and revolute joints (a common occurrence in practical mobile manipulators) which makes the dimensions of the joint displacement vector components non-homogeneous. Successful applications to the motion planning of several large-payload mobile manipulators with up to 11 d.o.f. are discussed. Sample trajectories involving combined motions of the platform and manipulator under the time-varying occurrence of obstacle and joint limit constraints are presented to illustrate the use and efficiency of the FSP approach in complex motion planning problems.

## INTRODUCTION

For any robotic manipulator system, the forward kinematics are usually described by the equation

$$\bar{x} = F(\bar{q}) \quad (1)$$

where  $\bar{x}$  is the location of a point (generally the end-effector) of the manipulator in the world coordinate system,  $\bar{q}$  is the vector of joint angles measured in local coordinates, and  $F()$  is the transformation function. In general, desired motions are expressed as trajectories in end-effector space. For loop-rate control, these trajectories are broken up into finite steps of length  $\Delta\bar{x}$ . The relationship between end-effector steps  $\Delta\bar{x}$  and joint space steps  $\Delta\bar{q}$  is found by differentiating and linearizing Eq. (1):

$$\frac{\Delta \bar{x}}{\Delta t} = J \frac{\Delta \bar{q}}{\Delta t} \quad (2)$$

where  $J$  is the linearized system Jacobian over the current time step  $\Delta t$ . The equation with which we will be working is then

$$\Delta \bar{x} = J \Delta \bar{q} \quad (3)$$

In order to carry out trajectories, the robot must be given motions in terms of joint space variables. This task requires some type of inverse transformation to be made to convert from the known quantity  $\Delta \bar{x}$  to the desired quantity  $\Delta \bar{q}$ . When the dimension of  $\Delta \bar{q}$  (the number of joints in the system) is greater than that of  $\Delta \bar{x}$ , the system is kinematically redundant and Eq. (3) is underspecified. Several methods have been proposed for resolving underspecified systems of equations and [1] provides an excellent review of these methods for application to redundant manipulators. These methods, however, are quite varied and suffer from significant shortcomings (e.g., see discussions in [2] and [3]) when applied to real-time sensor-based systems. A novel approach, which we have named the Full Space Parameterization (FSP) method, has been recently developed [2], [3], [4], [5], [6] to remedy some of these shortcomings in cases where constraints and task criteria vary rapidly and unpredictably with time during a single trajectory.

## OVERVIEW OF FULL SPACE PARAMETERIZATION

The FSP method has been specifically designed to optimally solve the inverse kinematics problem for redundant systems in the presence of applied constraints and behavioral criterion that may vary at loop rate [2], [3], [4], [5], [6]. For a redundant system,  $J$  will have fewer rows ( $n$ ) than columns ( $m$ ), and the number of vectors  $\Delta \bar{q}$  which satisfy Eq. (3) will typically be infinite. This infinite set of solution vectors forms a subspace of the space spanned by  $m - n + 1$  linearly independent solution vectors  $\bar{g}_k$ , each of which satisfies the equation:

$$\Delta \bar{x} = J \bar{g}_k \quad (4)$$

The vectors  $\bar{g}_k$  can easily be found by inverting square submatrices  $J_k$  of the Jacobian  $J$  and inserting a 0 into the components corresponding to the columns of  $J$  that were removed to form  $J_k$ . The proof of existence and algorithms for the determination of the  $m - n + 1$  linearly independent solution vectors  $\bar{g}_k$  can be found in [2], [4], and [5].

Once the  $m - n + 1$  solution vectors  $\bar{g}_k$  have been found, any solution  $\Delta \bar{q}$  can be written [2] as:

$$\Delta \bar{q} = \sum_{i=1}^{m-n+1} t_i \bar{g}_i, \quad \sum_{i=1}^{m-n+1} t_i = 1 \quad , \quad (5)$$

where the parameters  $t_i$ ,  $i = 1, m - n + 1$ , can be found by minimizing the Lagrangian

$$L(t_i, \mu, v_j) = Q(t_i) + \mu \left( \sum_{i=1}^{m-n+1} t_i - 1 \right) + \sum_{j=1}^r v_j C^j(t_i) \quad (6)$$

in which  $Q$  is the optimization criterion to be satisfied by  $\Delta \bar{q}$  with a set of  $r$  constraints  $C^j$ . The optimality conditions to be solved for  $t_i$ ,  $i = 1, m - n + 1$ , are:

$$\frac{\partial L}{\partial t_i} = 0, i = [1, m-n+1]; \frac{\partial L}{\partial \mu} = 0; \frac{\partial L}{\partial v_j} = 0, j = [1, r]. \quad (7)$$

As an example, assume that the criterion  $Q$  can be expressed as

$$Q = \|\Delta \bar{Z}(\bar{q}, \Delta \bar{q}) - \bar{Z}r\|^2 \quad (8)$$

with

$$\Delta \bar{Z} = B(\bar{q}) \Delta \bar{q} \quad (9)$$

where  $B(\bar{q})$  is a matrix; and the constraints  $C^j$  can be written [3] as:

$$\bar{\beta}^{j^T} \bar{t} - 1 = 0; j = [1, r] \quad (10)$$

then the optimality conditions in Eq. (7) become:

$$\begin{cases} G\bar{t} + \bar{H} + \mu\bar{e} + \sum_{i=1}^r v_i \bar{\beta}^{i^T} \bar{t} = 0 \\ \bar{e}^T \bar{t} = 1 \\ \bar{\beta}^{j^T} \bar{t} = 1, j = [1, r] \end{cases} \quad (11)$$

with

$$\bar{H}, H_k = \Delta \bar{Z}^T B \bar{g}_k; k = [1, m-n+1] \quad (12)$$

$$G, G_{ij} = \bar{g}_i^T B^T B \bar{g}_j; i, j = [1, m-n+1] \quad (13)$$

$$\bar{e}, e_i = 1; i = [1, m-n+1] \quad (14)$$

Solving these equations gives (see [2] and [3])

$$\bar{v} = A^{-1}(\bar{d} - \bar{b}(1 + \bar{e}^T G^{-1} \bar{H})) \quad (15)$$

$$\mu = -(1 + \bar{v}^T \bar{b} + \bar{e}^T G^{-1} \bar{H})/a \quad (16)$$

$$\bar{t} = -G^{-1}(\mu \bar{e} + \mathcal{B} \bar{v} + \bar{H}) \quad (17)$$

for non-nullspace motion. For nullspace motion Eqs. (15) and (16) are replaced by

$$\bar{v} = A^{-1}(\bar{d} - \bar{b} \bar{e}^T G^{-1} \bar{H}) \quad (18)$$

$$\mu = -(\bar{e}^T G^{-1} \bar{H} + \bar{v}^T \bar{b})/a \quad (19)$$

where

$$a = \bar{e}^T G^{-1} \bar{e} \quad (20)$$

$$\bar{b}, b_i = \bar{e}^T G^{-1} \bar{\beta}^i = \bar{\beta}^{i^T} G^{-1} \bar{e}, i = [1, r] \quad (21)$$

$$\bar{d}, d_i = 1 + \bar{\beta}^{i^T} G^{-1} \bar{H}, i = [1, r] \quad (22)$$

$$A, A_{ij} = b_i b_j - a \bar{\beta}^{i^T} G^{-1} \bar{\beta}^j, i = [1, r], j = [1, r] \quad (23)$$

and  $\mathcal{B}$  is a matrix whose columns are  $\bar{\beta}^i$ .

The approach for calculating the coefficient vectors  $\bar{\beta}^i$  expressing the constraints has been described in detail in [3]. In particular, applications to redundant manipulators for the cases of joint limit and obstacle avoidance, and bounded joint accelerations were presented in [3] and [6], respectively. Figures 1 and 2 show two of the experimental testbeds on which implementations and tests of the FSP approach have been performed, initially using only the manipulator, then considering the complete mobile platform and manipulator system as discussed below. Figure 1 shows the ATLAS vehicle which includes a planar, 5 d.o.f. manipulator, mounted on an all-steerable wheeled platform. The two degrees of redundancy (d.o.r.) provide the system with added dexterity in the handling of various palletized cargo. Figure 2 shows the Next Generation Munition Handler (see paper in this conference) which utilizes advanced robotic technologies, including a fully omnidirectional platform (see [7]) and a 8 d.o.f. manipulator system to allow rapid aircraft reload turnaround. Including their platform, these systems involve 8 and 11 d.o.f., respectively. For a variety of tasks, some of these d.o.f. (i.e., joints) can be enabled or disabled, leading to systems with varying configurations and joint space dimensions (from 3 to 8 and 11, respectively). Also note that both systems include at least one prismatic joint.



Figure 1. The ATLAS mobile manipulator for palletized cargo handling.

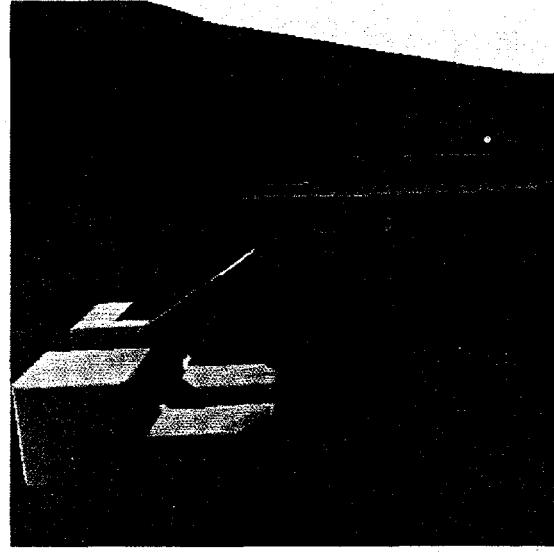


Figure 2. The U.S. Air Force Next Generation Munition Handler for rapid aircraft turnaround.

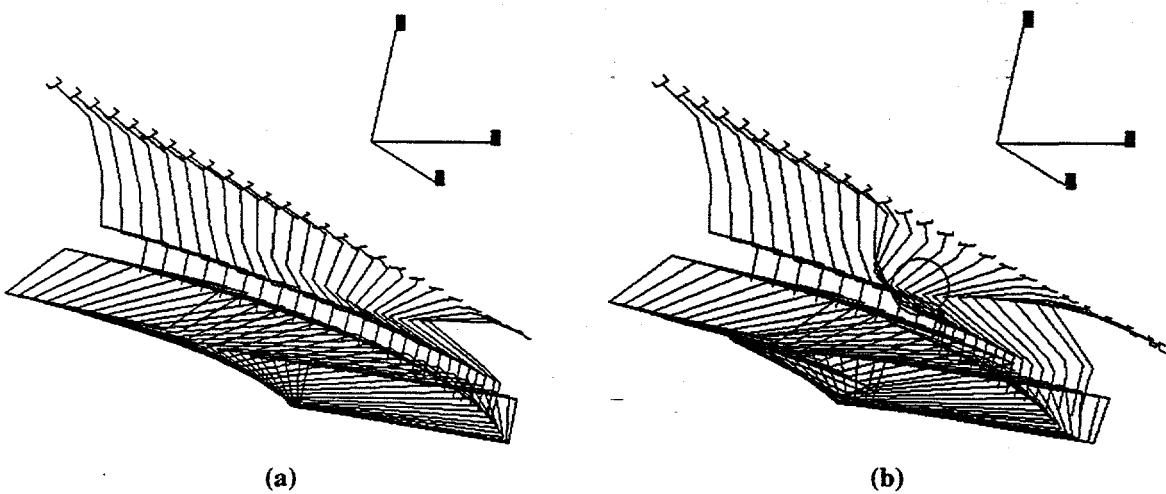
## APPLICATION TO MOBILE MANIPULATOR

For application to redundancy resolution for mobile manipulators under time-varying constraints, task requirements, and active configurations, unique advantages of the FSP method can be utilized. First, the FSP code [4], [5] has been developed to allow treatment of any dimension of the joint space or of the (Cartesian) control space; i.e., the Jacobian matrix can be an  $n \times m$  matrix with any value for  $n$  (typically 3 or 6 for robot control applications) and  $m$  (the number of joints), and these values can change at every loop rate if necessary. Second, the FSP allows implementation of the most common constraints encountered in robotic motion planning (e.g., joint limits and obstacle avoidance as described in [3], non-holonomic constraints as described in [8]). The number and expression of these constraints (e.g., see Eq. (10) for one of the most common

forms [3]) can vary at loop rate, i.e., can be based on sensor information in dynamic or *a priori* unknown environments. These aspects of the FSP have been treated in companion papers [2], [3], [4], [5], [8] and are only recalled here for completeness.

A particular aspect that derives from the formulation of Eqs. (8) and (9) is the capability to handle mixtures of revolute and prismatic joints in the system. This is particularly important since, as mentioned above, most practical mobile manipulators typically include prismatic joints in their "boom" or arm, but also because the platform motion can be seen as analogous to a combination of prismatic motions (with, of course, non-holonomic constraints between them, as appropriate [8]). The problem in Eq. (8) comes from the different dimensions (e.g., meters vs. radians) of the components of  $\Delta\bar{q}$  for prismatic and revolute joints, making the *optimization* of the norm highly dependent on the choice of *relative* units in joint space. The solution involves using Eq. (9) to make the dimensions of the norm components uniform, or essentially dimensionless. The matrix  $B$  can therefore be expressed as  $B = B_c B_d$ , where  $B_c$  relates to the particular task criterion considered (e.g., if  $B$  is the identity matrix and the system only includes revolute joints, Eq. (8) with  $\bar{Z}_r = 0$  provides the least norm of the joint displacements for comparison with the pseudo-inverse, as described in [2]) and  $B_d$  is a diagonal matrix used to "uniformize" the dimensions of the norm components. This essentially corresponds to a *relative* weighing of the joint motions, and it is important to note that this weighing is not arbitrary but actually expresses a desired *relative behavior* between the prismatic and revolute joints. In the system shown in Figure 2, for example, the components of the diagonal matrix  $B_d$  were selected so that displacement of the prismatic joint (joint 3) over its entire range (.6 m) was "equivalent" to a motion of joint 1 over its entire range (1.52 rd). Thus, using units of meters and radians in the system,  $B_{11} \times 1.52 = B_{33} \times .6$ , or  $B_{33} = 2.53B_{11}$ . Other schemes can, of course, be implemented and *changed at loop rate* (e.g., see [9]), as desired or required by the time-varying task requirements.

The various aspects of the FSP described above were implemented and tested on several systems consisting mainly of manipulator arms (e.g., see [2], [3], [9]). Several implementations and tests were also performed on mobile manipulators and Figure 3 shows examples of trajectories that were created using FSP for the HERMIES-III mobile manipulator. The HERMIES-III system [10] is composed of a three d.o.f. (in Cartesian Space) omnidirectional platform and a seven d.o.f. manipulator. The trajectories were created by specifying the start and finish location of the end-effector. Orientation control was not utilized to create these examples, i.e., only 3-D end-effector position is controlled, leading to a  $3 \times 10$  Jacobian, or 7 degrees of redundancy. Both trajectories were specified



**Figure 3.** Examples of the FSP-produced motions for a mobile manipulator (a) with joint limits, (b) with joint limits and obstacle avoidance.

to use the least norm as the optimization criteria. As can be seen in Figure 3(a), the motions of both the platform and manipulator are simultaneous and smooth. The perspective from which the image was captured causes the dimensions of the platform to appear distorted as it rotates approximately 130 degrees in a clockwise direction from start to finish. In the second trajectory, a spherical obstacle was placed directly in the path that had been followed by the manipulator in the first example. As the manipulator moves to a location that would cause it to impact the object, the platform backs up slightly, and the links of the manipulator near the obstacle move into positions that prevent impact. The distortion effect due to the viewing perspective of the platform is present in the second image as it was in the first. Even with the sudden addition of an obstacle (and corresponding constraint) to the environment through which the robot moves, the motions of both the platform and the manipulator are smooth and simultaneous, and the end-effector reaches the desired position.

## CONCLUSION

An approach to the motion planning of highly kinematically redundant mobile manipulators under time-varying constraints and task criterion has been presented. This approach is based on the use of the FSP method to resolve the constrained, underspecified, system of velocity equations. Emphasis in this paper has been placed on the treatment of particular features specific to practical mobile manipulators, in particular the handling of mixtures of prismatic and revolute joints. Sample trajectories for one of our mobile manipulator testbeds have also been presented to illustrate the use of the FSP approach in cases with time-varying obstacle and joint limit constraints.

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